Computer Aided Design of Handwriting Trajectories with the Kinematic Theory of Rapid Human Movements

Daniel BERIO,[a] Frederic FOL LEYMARIE[a] and Réjean PLAMONDON[b]

[a] *Goldsmiths, University of London, United Kingdom*

[b] *École Polytechnique de Montréal, Canada.*

Abstract. We present the Sigma Lognormal model as a potential tool for curve generation in computer graphics related applications. We discuss its extension and parameterisations for the interactive definition of handwriting, drawing and calligraphic trajectories. This results in an efficient trajectory synthesis method, that has a user interface similar to the ones commonly used with Bézier curves or splines, but with the added benefit of capturing the kinematics of human drawing or writing movements. Such kinematics produced by the model can then be exploited to generate realistic stroke animations or to facilitate expressive rendering methods.

1. Introduction and background

Years of graphonomics research have produced numerous results that have contributed to our understanding of the neural, cognititive and bio-mechanical aspects of human handwriting and drawing. In parallel, the computer graphics (CG) field, especially within the sub-field of non-photorealistic rendering and animation (NPAR), has produced a wealth of algorithms for the rendition of images that resemble diverse artistic styles and techniques. Noticeably, these methods seldom take the *motor act* of writing/drawing into account (e.g., cf. Kyprianidis et al. 2013), and often rely on methods such as poly-lines or polynomial based curve generation methods including the classic Bézier curves and related splines.

In this paper, we propose a *movement centric* approach to curve generation, specifically aimed at CG applications that require the simulation of traces such as the ones seen in drawing, painting, writing and calligraphy. Here a curve is defined by the dynamics underlying its production rather than by solely an explicit definition of its geometry. We argue that this makes it easier to capture the subtle observable qualities and variations that are typical of hand drawn traces. With an appropriate parameterisation such a method can be used in similitude to established popular spline-based methods, with the additional advantage of capturing both the geometry and dynamics of a human made trace with a single integrated representation.

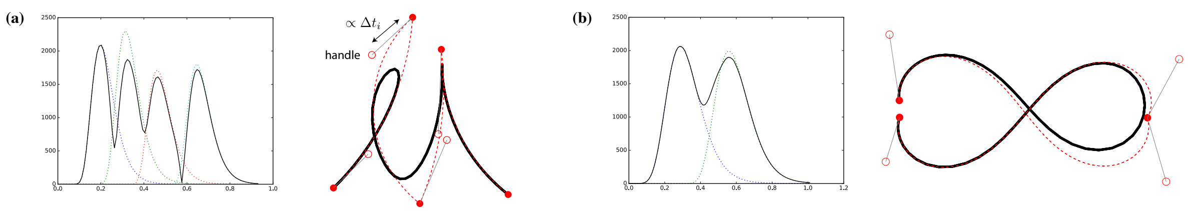

Fig. 1. Interactive editing of Sigma-Lognormal trajectories with the corresponding speed profiles (to the left) using (a) circular arc strokes and (b) Euler spiral strokes. The handles determine the tangent angles defining the shape of each stroke, and the length of the handle is proportional to the stroke time overlap (Δt_i).

For the task at hand, we rely on the *Kinematic Theory of Rapid Human Movements* (Plamondon, 1995), a family of models of reaching and handwriting motions, in which a movement is described as the result of the parallel and hierarchical interaction of a large number of coupled neuromuscular components. Plamondon et al. (2003) have shown that the impulse response of such a system to a centrally generated command is given by a *lognormal* (eq. 2.), which accurately describes the variably asymmetric bell shape that commonly characterises human movements (Plamondon et al., 1993; Rohrer, Hogan, 2003). Arbitrarily complex movement trajectories are generated with the space-time superimposition of a discrete number of lognormal movement primitives or *strokes*, where each such stroke is aimed at an imaginary location referred to as *virtual target*. The sequence of virtual targets describes a high level *action plan* for the trajectory, which in an interactive setting can be manipulated in a manner similar to the "control polygon" typically used in Computer Aided Design (CAD) applications (Fig. 1).

In the following sections we describe three variations of the Sigma Lognormal ($\Sigma\Lambda$) model (Plamondon et al., 2014), one of the Kinematic Theory models that is particularly aimed at describing human handwriting motions. First we describe the $\Sigma\Lambda$ model in its original formulation (Section 2.). Then we describe two extensions: the Weighted $\Sigma\Lambda$ (Section 3.) and the Spiral $\Sigma\Lambda$ (Section 4.) models, which are both particularly aimed at interactive CG and CAD applications. Finally we discuss use cases of the models for the interactive generation of trajectories (Section 5.). Towards this application, we demonstrate a parametrisation that explicitly defines the asymmetry and duration of each lognormal stroke profile.

2. The Sigma Lognormal ($\Sigma\Lambda$) Model

On the basis of the Kinematic Theory, the Sigma Lognormal ($\Sigma\Lambda$) model (Plamondon et al., 2014) describes complex handwriting trajectories via the vectorial superimposition of N time shifted stroke primitives. The pentip velocity is calculated with:

$$\boldsymbol{v}(t) = \sum_{i=1}^{N} \boldsymbol{D}(t)_i \Lambda(t)_i, \tag{1}$$

where $\boldsymbol{D}(t)_i$ describes the direction and amplitude of each stroke (towards a virtual target) and

$$\Lambda_i(t) = -\frac{1}{\sigma_i\sqrt{2\pi}(t-t_{0i})}\exp\left(\frac{(ln(t-t_{0i})-\mu_i)^2}{2\sigma_i{}^2}\right), \tag{2}$$

describes the lognormal impulse response of each stroke to a centrally generated command occurring at time t_{0i}. The parameters μ_i and σ_i respectively describe the stroke delay and response time in a logarithmic time scale, and determine the shape and asymmetry of the lognormal. With the assumption that handwriting movements are made with rotations of the elbow or wrist, the curvilinear evolution of a stroke can initially be described by a circular arc. The angular evolution of a stroke is described by using the time integral of eq. 2.:

$$\phi_i(t) = \theta_{si} + (\theta_{ei} - \theta_{si})\int_0^t \Lambda_i(u)du = \theta_{si} + \frac{(\theta_{ei}-\theta_{si})}{2}\left[1+\text{erf}\left(\frac{\log(t-t_{0i})-\mu_i}{\sigma_i\sqrt{2}}\right)\right], \tag{3}$$

which can be computed efficiently using the error function (erf) and where θ_{si} and θ_{ei} describe the starting and ending angle of the circular arc.

3. The Weighted Sigma Lognormal ($\omega\Sigma\Lambda$) Model

While the traditional $\Sigma\Lambda$ formulation describes the velocity of a handwriting trajectory (eq. 1) that is integrated to generate the corresponding trace, here we describe each stroke through a *weight* that is computed similarly to eq. 3 with:

$$w(t) = \int_0^t \Lambda_i(u)du = \frac{1}{2}\left[1+\text{erf}\left(\frac{\log(t-t_{0i})-\mu_i}{\sigma_i\sqrt{2}}\right)\right] \quad \in [0,1]. \tag{4}$$

This formulation avoids the need for numerical integration in the computer implementation and consequently provides a performance advantage, since the error function (erf) is a special function, and an optimised implementation is readily available in most modern programming languages. Furthermore it becomes possible to sample a trajectory at a given time step without the necessity to integrate from a given initial condition.

The planar evolution of a trajectory with m strokes can be described with an initial position $\boldsymbol{p}_0$ and a sequence of virtual targets $(\boldsymbol{p}_1, \dots, \boldsymbol{p}_m)$. Each stroke evolves between a pair of consecutive virtual targets $\boldsymbol{p}_{i-1}, \boldsymbol{p}_i$ and is parametrised with an amplitude D_i and a direction θ_i given by the norm and orientation of the vector $\boldsymbol{p}_i - \boldsymbol{p}_{i-1}$. The curvilinear evolution of a stroke is then described with a circular arc shape with a central angle δ_i. Each stroke is described by a *weighted displacement* at time t computed with:

$$\boldsymbol{d}_i(t) = \begin{cases} D_i \begin{bmatrix} (\cos(\theta_{0i} - \delta_i w_i(t)) - \cos(\theta_{0i}))\,(2\sin(\delta_i/2))^{-1} \\ (\sin(\theta_{0i} - \delta_i w_i(t)) - \sin(\theta_{0i}))\,(2\sin(\delta_i/2))^{-1} \end{bmatrix} & \text{if } |\delta_i| > \epsilon, \\ D_i \begin{bmatrix} \cos(\theta_i)\, w_i(t) \\ \sin(\theta_i)\, w_i(t) \end{bmatrix} & \text{otherwise} \end{cases} \tag{5}$$

where ϵ is a small value (1×10^{-10}) used to avoid divisions by zero and numerical precision issues when the arc internal angle δ_i is close to zero, and where

$$\theta_{0i} = \theta_i + \frac{\pi + \delta_i}{2} \tag{6}$$

is the initial angle on the circle used to compute the circular arc for each stroke. The position along the trajectory is then computed using a vectorial sum of each stroke with:

$$\boldsymbol{p}(t) = \boldsymbol{p}_0 + \sum_{i=1}^{m} \boldsymbol{d}_i(t). \tag{7}$$

4. The Weighted Euler Spiral Sigma Lognormal ($\omega\mathscr{E}\Sigma\Lambda$) Model

In an alternative formulation of the weighted $\Sigma\Lambda$ model specifically aimed at interactive curve editing, we define strokes using an Euler spiral primitive. Euler spirals (aka Cornu spirals, or clothoids or spiros) are such that their curvature varies linearly with arc length, also permitting the description of variably curved segments as well as inflections. At the expense of adding a supplementary parameter per stroke to the model, the use of Euler spirals reduces the number of virtual targets needed to define a trajectory — such as when defining a doubly looping eight ("8") figure (Berio, Fol Leymarie, 2015) — and provides an additional level of editing flexibility. The resulting method can also be used to define trajectories that are identical to the standard $\Sigma\Lambda$ model, since in the limit an Euler spiral segment converges to a circular arc (Walton, Meek, 2008).

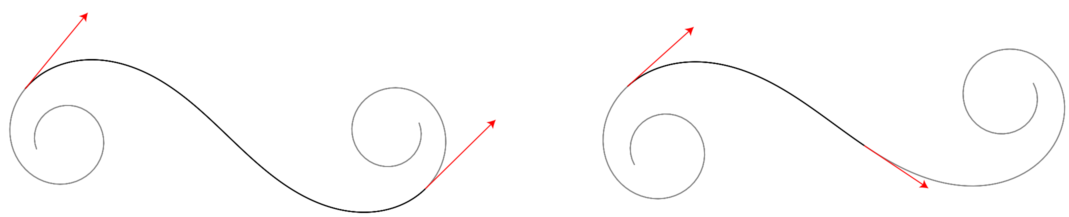

Fig. 2. Examples of Euler spiral stroke primitives using different Hermite constraints (in red).

To describe a spiral we use the following parametrisation of the cosine ($C(s)$) and sine ($S(s)$) Fresnel integrals (Levien, 2009):

$$C(s) = \int_0^s \cos\left(\pi u^2/2\right) du \quad \text{and} \quad S(s) = \int_0^s \sin\left(\pi u^2/2\right) du\ , \tag{8}$$

which we compute using a fast numerical approximation described by Heald (1985). We define the curvilinear evolution of each stroke with *two* parameters s_{0i} and s_{1i}, which parameterise the arc-length along the spiral. While the spiral parameters s_{0i} and s_{1i} are not intuitive to specify, these can be uniquely computed given the orientation of two tangents (Hermite constraints) with respect to the chord of the spiral (Fig. 2). A number of methods exist for this task (Kimia et al., 2003; Walton, Meek, 2008; Levien, 2009; Bertolazzi, Frego, 2015); for our study we choose the method defined by Levien (2009), which has proven experimentally to be fast and robust. We then define the arc length evolution of the spiral for each stroke with:

$$\phi_{si}(t) = s_{0i} + (s_{1i} - s_{0i}) w_i(t) \tag{9}$$

and the integrated displacement of each stroke with:

$$\boldsymbol{d}_i(t) = D_i l_i^{-1} \begin{bmatrix} (C(\phi_{si}(t)) - C(s_{0i}))\cos(\theta_i - \theta_{ci}) - S(h_i\phi_{si}(t)) - S(h_i s_{0i}))\cos(\theta_i - \theta_{ci}) \\ (C(\phi_{si}(t)) - C(s_{0i}))\sin(\theta_i - \theta_{ci}) + S(h_i\phi_{si}(t)) - S(h_i s_{0i}))\cos(\theta_i - \theta_{ci}) \end{bmatrix}, \tag{10}$$

where

$$l_i = \sqrt{(C(s_{1i}) - C(s_{0i}))^2 + (S(s_{1i}) - S(s_{0i}))^2} \quad \text{and} \quad \theta_{ci} = \tan^1\left(\frac{S(s_{1i}) - S(s_{0i})}{C(s_{1i}) - C(s_{0i})}\right) \tag{11}$$

are used to respectively correct for the chord length and orientation of the spiral in its canonical form, and where

$$h_i = \operatorname{sgn}\left(\frac{\pi s_{1i}^3}{2|s_{1i}|} - \frac{\pi s_{0i}^3}{2|s_{0i}|}\right) \tag{12}$$

takes care of flipping the spiral along the horizontal axis depending on its curvature and thus allows to capture different combinations of user defined Hermite constraints.

5. User Interaction and Rendering

The $\omega\Sigma\Lambda$ and $\omega\mathscr{E}\Sigma\Lambda$ models provide flexible trajectory generation tools that are particularly well suited for interactive (point and click) editing procedures. For example, the user can easily edit the spatial evolution of the trajectory by dragging virtual targets, and modify the stroke shapes and timing properties by manipulating handles placed in correspondence with the virtual targets. The resulting editing procedure is similar to the more traditional CAD methods based on Bézier curves or splines, but allows to edit both the geometry as well as the physiologically plausible kinematics of the trajectory. This provides an additional layer of information that can be exploited to facilitate expressive rendering methods that take the pen dynamics into account (Fig. 3.(b)). Furthermore, it becomes trivial to generate realistic stroke animations, or to use the resulting trajectories as smooth motion paths for virtual character animations, or even to drive the motions of a robotic drawing system (Berio et al., 2016).

An additional advantage with respect to traditional curve generation methods is that the parameters of such models correspond with physically and biologically interpretable aspects of the trajectory planning and formation processes (Plamondon et al., 2014). As a result, slight random perturbations to the virtual target positions and the remaining stroke parameters result in variations of the trajectory (Fig. 3.(a)) that are alike the ones that can be observed in instances of a human drawing or writing (Diaz et al., 2016). This allows a user to define a whole *family of trajectories*, rather than just a single path, this in one editing session.

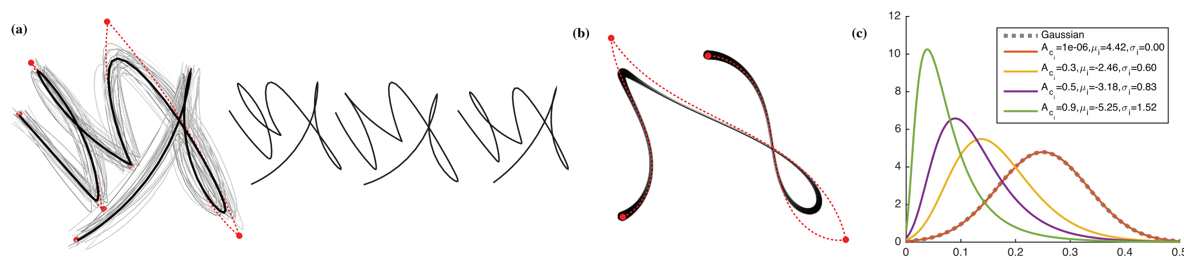

Fig. 3. **(a)** Variations of a trajectory using $\omega\Sigma\Lambda$ parameter perturbations. **(b)** Rendering brush effects using the $\omega\mathscr{E}\Sigma\Lambda$ model. **(c)** Lognormals with different skewedness generated by varying the shape parameter A_{c_i}; note that with $A_{c_i} \approx 0$ the lognormal closely approximates a (symmetric) Gaussian kernel.

Intermediate parametrisation. In order to facilitate the precise specification of timing and profile shape of each stroke, we rely upon an intermediate parametrisation that takes advantage of a few known properties of the lognormal (Djioua, Plamondon, 2008). We define each stroke with: (i) a stroke duration T_i, (ii) a relative time offset Δt_i with respect to the previous stroke time occurrence and duration, and (iii) a shape parameter $A_{c_i} \in (0, 1)$, which defines the skewedness of the lognormal (Plamondon et al., 2003). The corresponding $\Sigma\Lambda$ parameters $\{t_{0i}, \mu_i, \sigma_i\}$ can be then computed with:

$$\sigma_i = \sqrt{-\log(1 - A_{c_i})}, \qquad \mu_i = 3\sigma_i - \log\left(\frac{-1 + e^{6\sigma_i}}{T_i}\right) \tag{13}$$

$$t_{0i} = t_{1i} - e^{\mu - 3\sigma} \qquad t_{1i} = t_{1(i-1)} + d_{i-1}\Delta t_i \qquad t_{1(0)} = 0, \tag{14}$$

where t_{1i} is the onset time of the lognormal stroke profile. As A_{c_i} approaches 0, the shape of the lognormal converges to a Gaussian (Fig. 3.(c)), with mean $t_{0i} + e^{\mu_i - \sigma_i^2}$ (the mode of the corresponding lognormal) and standard deviation $\frac{T_i}{6}$.

6. Conclusion

We presented a novel way to adapt the $\Sigma\Lambda$ model as a powerful tool for the interactive generation of handwriting, drawing or calligraphic trajectories. We presented some possible applications and summarised the advantages of our proposed implementation, including (i) the removal of the need of performing explicit numerical integration and (ii) the use of Euler spirals to capture and edit stroke primitives with inflections.

Our longer term goal is to explore the hypothesis that viewing a static image of a hand-made trace involves the mental recovery of its underlying movement (Pignocchi, 2010; Freedberg, Gallese, 2007; Longcamp et al., 2003) and that such recovery influences its aesthetic appreciation (Leder et al., 2012). As a result, we also hypothesise that a precise enough simulation of movement may trigger similar responses in an observer of artificially rendered traces.

Acknowledgements

This work has been partly supported by UK's EPSRC Centre for Doctoral Training in Intelligent Games and Game Intelligence (IGGI; grant EP/L015846/1).

References

Berio D., Calinon S., Fol Leymarie F. Learning dynamic graffiti strokes with a compliant robot // Proc. IEEE/RSJ Intl Conf. on Intelligent Robots and Systems (IROS). Daejeon, Korea, October 2016. 3981–6. https://doi.org/10.1109/IROS.2016.7759586.

Berio D., Fol Leymarie F. Computational Models for the Analysis and Synthesis of Graffiti Tag Strokes // Proceedings of Computational Aesthetics (CAe). Istanbul, Turkey: Eurographics, June 2015. 35–47. http://dx.doi.org/10.2312/exp.20151177.

Bertolazzi E., Frego M. G^1 fitting with clothoids // Mathematical Methods in the Applied Sciences. March 2015. 38, 5. 881–897. http://dx.doi.org/10.1002/mma.3114.

Diaz M., Fischer A., Ferrer M. A., Plamondon R. Dynamic Signature Verification System Based on One Real Signature // IEEE Transactions on Cybernetics. December 2016. early access. 12 pages. https://doi.org/10.1109/TCYB.2016.2630419.

Djioua M., Plamondon R. A new methodology to improve myoelectric signal processing using handwriting // Proceedings of the International Conference on Frontiers in Handwriting Recognition. Montreal, Canada, 2008. 112–117.

Freedberg D., Gallese V. Motion, emotion and empathy in esthetic experience // Trends in cognitive sciences. May 2007. 11, 5. 197–203. https://doi.org/10.1016/j.tics.2007.02.003.

Heald M. A. Rational approximations for the Fresnel integrals // Mathematics of Computation. 1985. 44, 170. 459–461. https://doi.org/10.1090/S0025-5718-1985-0777277-6.

Kimia B., Frankel I., Popescu A. Euler spiral for shape completion // International Journal of Computer Vision. August 2003. 54, 1. 159–182. https://doi.org/10.1023/A:1023713602895.

Kyprianidis J.E., Collomosse J., Wang T., Isenberg T. State of the “Art”: A Taxonomy of Artistic Stylization Techniques for Images and Video // IEEE Transactions on Visualization and Computer Graphics. 2013. 19, 5. 866–885. https://doi.org/10.1109/TVCG.2012.160.

Leder H., Bär S., Topolinski S. Covert painting simulations influence aesthetic appreciation of artworks // Psychological Science. 2012. 23, 12. 1479–1481. http://dx.doi.org/10.1177

Levien R.L. From Spiral to Spline: Optimal Techniques in Interactive Curve Design. December 2009. 191 pages. PhD thesis, EECS Department, University of California, Berkeley.

Longcamp M., Anton J.L., Roth M., Velay J.L. Visual Presentation of Single Letters Activates a Premotor area Involved in Writing // NeuroImage. August 2003. 19, 4. 1492–1500. https://doi.org/10.1016/S1053-8119(03)00088-0.

Pignocchi A. How the Intentions of the Draftsman Shape Perception of a Drawing // Consciousness and Cognition. 2010. 19, 4. 887–898. https://doi.org/10.1016/j.concog.2010.04.009.

Plamondon R. A Kinematic Theory of Rapid Human Movements. Part I. Movement Representation and Generation // Biological Cybernetics. March 1995. 72, 4. 295–307. http://dx.doi.org/10.1007/BF00202785.

Plamondon R., Alimi A.M., Yergeau P., Leclerc F. Modelling velocity profiles of rapid movements: A comparative study // Biological Cybernetics. June 1993. 69, 2. 119–128. http://dx.doi.org/10.1007/BF00226195.

Plamondon R., Feng C., Woch A. A kinematic theory of rapid human movement. Part IV: A formal mathematical proof and new insights // Biological Cybernetics. August 2003. 89, 2. 126–138. http://dx.doi.org/10.1007/s00422-003-0407-9.

Plamondon R., O'reilly C., Galbally J., Almaksour A., Anquetil A. Recent developments in the study of rapid human movements with the kinematic theory: Applications to handwriting and signature synthesis // Pattern Recognition Letters. January 2014. 35, 1. 225–235. https://doi.org/10.1016/j.patrec.2012.06.004.

Rohrer B., Hogan N. Avoiding spurious submovement decompositions: A globally optimal algorithm // Biological Cybernetics. September 2003. 89, 3. 190–199. http://dx.doi.org/10.1007/s00422-003-0428-.

Walton D.J., Meek D.S. An Improved Euler Spiral Algorithm for Shape Completion // Proceedings of the Canadian Conference on Computer and Robot Vision (CRV). Windsor, Canada, May 2008. 237–244. https://doi.org/10.1109/CRV.2008.11.