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Working Paper N. 01/2016

DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze, Italia
www.disei.unifi.it

Network Calibration and Metamodeling of a Financial Accelerator Agent Based Model

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January 8, 2016

Abstract

We allow firms and banks to entertain multiple credit connections in a financially constrained production framework, resorting to a random network model whose parameters are calibrated with real data. The calibration is successful since the network model is able to reproduce the degree and strength (debt and loan) distributions of the Japanese credit market. We run simulations over the parameter space using an efficient design, and compare a number of alternative statistical metamodels in order to select the best specification for the relationship between the parameters and a set of endogenous variables of the model. We show that the metamodeling approach can be usefully extended to economic models in order to bridge the gap between micro and macro variables through a rigorous statistical analysis of ABMs, without imposing unrealistic restrictions on the micro model such as the representative agent hypothesis.

1 Introduction

The relationship between ABM and empirics is a widely discussed topic among scholars in the field. On the one hand, ABMs provide a more faithful representation of economic reality, introducing more realistic behavioral assumptions with respect to mainstream models. Thus, they should potentially provide a better agreement with empirical data. Indeed numerous contributions have underlined the success of ABM in replicating "stylized facts" thanks to the introduction of agent heterogeneity, bounded rationality and learning, decentralized out-of-equilibrium interactions[1]. On the other hand, there is still little consensus in the field on how to evaluate the agreement between models and facts. Some ABM scholars support the view that a systematic calibration / validation with real data, not to speak of forecasting exercises, are neither possible nor desirable (Valente, 2005). Instead, they advocate for a loose comparison with historical data and other descriptive evidence. They underline that social phenomena are inherently complex and not stationary, involving too many dimensions. Thus any quantitative exercise of the sort mentioned above is bound to fail when applied to models for which the ergodic hypothesis does not hold.

Most researchers do not share this view, underlining in particular that empirics imposes a much needed discipline on model building, and that ABMs should accept the challenge of a stringent comparison with empirical evidence (Fagiolo *et al.*, 2007). Nonetheless, existing works follow diverse approaches, of which it is even difficult to work out a shared characterization. For instance, alternative taxonomies have been proposed which are at first sight not consistent with each other (Fagiolo *et al.*, 2007; Brenner and Werker, 2007). On the other hand, a growing number of contributions tackle the issue of econometric estimation of agent-based models, although these exercises are exclusively confined to relatively simple models of financial markets Alfarano *et al.* (2005); Manzan and Westerhoff (2007).

Before we proceed further, we believe it is important to underline the specificity of ABM within the larger field of simulated models. In particular, we would like to stress that ABMs cannot be likened to simulated statistical models. Indeed the mathematical structure of the latter is completely specified by the modeler, e.g. by choosing regressors, functional shapes, lags and by making assumptions on the structure of errors. What is left to estimate from data is the "strength" of the assumed relationships between variables. The ABM modeler instead does not have an equivalent control over all the properties emerging from her model. Not coincidentally, unexpected simulation results are a common experience for researchers in this field. Once

[1]For a recent review see Chen *et al.* (2012)

statistical models are estimated, they can be used to provide computationally inexpensive predictions over endogenous variables conditioned to variations of the exogenous variables. Instead, ABM modelers are always bound to replicate computationally expensive simulations runs over each point of the model parameter space to provide such predictions. This necessity follows from the fact that the functional form of the relationship between endogenous and exogenous variables in ABMs is unknown.

To make things a bit more complicated, there is also some terminological uncertainty at stake. While econometricians usually write about model "estimation" or "evaluation", ABM scholars usually opt for the terms "calibration" and "validation", which are employed both individually and jointly, sometimes as synonyms. Then we feel it is necessary to clarify the meaning we attach to these terms for the purpose of this paper. By "model calibration" we mean the attainment of the maximum alignment of a set of model outputs (i.e. endogenous variables) with empirical evidence, specifically under the form of numerical data ("calibration dataset"). From the definition it's clear that calibrating a model involves some kind of optimization. Thus "model calibration" is a synonym of "model estimation" as intended by the econometricians. Instead, we distinguish "model calibration" as just defined from "parameter calibration", which refers to the imposition of a restriction over the domain of model parameters, which is derived from a statistical estimation or from prior assumptions which must be specified *before* model calibration (Brenner and Werker, 2007)[2]. Finally, by "model validation" we mean the comparison of the conditional expectations of a set of model outputs, computed from the calibrated/estimated model, with data other than those used for calibration ("validation dataset").

All these procedures should be regarded as stages of a more general procedure of model selection, by which we try to detect the best model among a set of alternatives according to some criterion. In most works, models are selected at the calibration stage by means of various goodness-of-fit measures and/or information criteria. Some authors have underlined instead that model selection should be performed on the validation dataset in order to avoid overfitting (Hassan *et al.*, 2013). In the latter case, forecasting exercises are performed on a third set of data. Apart from this differences, we believe that a comparative approach is of fundamental importance for

[2]It is implicit in the definition that unobserved parameters cannot undergo a calibration procedure of this sort.

ABMs since simulations by themselves can only prove existence assertions, namely that some model behavior follows from model assumptions at a given set of parameter values. In order to strengthen our confidence in these assumptions, we would need to compare for instance their forecasting power with other, incompatible, assumptions, e.g. those underlying DSGE models. Although our purpose in this paper is more modest as detailed below, we believe that, generally speaking, this is the path to follow.

The calibration procedure of ABMs must face multiple hurdles. In a classical statistical framework, the calibration problem can be formulated as follows

$$\theta^* = \underset{\theta \in \Theta}{\arg\min}\ F(x, y(\theta)) \tag{1}$$

where F is a criterion function, x is a vector of statistics computed from real data, and $y(\theta)$ is a vector of the same statistics produced from a model characterized by the parameter vector θ taking values in the domain Θ[3]. If x and y result from the estimation of the same "auxiliary" statistical model over real and simulated data, this approach is usually termed "indirect inference" (Gouriéroux C. and Monfort, 1996). A frequent choice for F, in case of over-identified models, is a quadratic loss function, while y typically includes some aggregate or distributional statistics, e.g. regarding the size distribution of agents (see e. g. Bianchi *et al.* (2007)).

It generally happens with ABMs that y is a possibly non-linear function of θ whose likelihood function is unknown. Thus we can employ neither ML estimators nor standard approximations of the likelihood function, like the Laplace approximation, which are generally employed for both classical and Bayesian inference. Moreover, since the model is simulated, y is itself a random variable, and standard numerical optimization algorithms, such as gradient methods or the Nelder-Mead simplex algorithm, are bound to fail even if we take averages over a large number of simulations runs in order to obtain an estimate of $\mathbb{E}[y|\theta]$. One option is to employ adapted optimization algorithms, such as the one proposed by Gilli and Winker (2003), which may work on these estimates. A shortcoming of this option is that we cannot avoid extensive Montecarlo replications. To overcome this problem, we may use instead a deterministic approximation of the model, as in Recchioni *et*

[3]In a Bayesian framework, instead, calibration is attained by updating the probability distribution of parameters in accordance with data. We are not going to investigate the Bayesan approach in this paper.

al. (2015), if it is available. In both cases, numerical algorithms provide local solutions, therefore we cannot exclude that the model is unidentifiable.

In order to address the identification issue, we should explore systematically the parameter space of ABMs before we attempt any calibration / validation exercise. Normally, this is an impossible task because of the curse of dimensionality. In order to overcome this problem, we propose to estimate the influence of θ over y by means of a *metamodel*, i.e. a statistical auxiliary model of the following form:

$$y(\theta) = \mu(\theta) + u \tag{2}$$

where $\mu(\theta)$ is a deterministic, possibly non linear, term, and u is a second-order stationary, zero mean, potentially heteroskedastic, random term with given covariance matrix (Salle and Yildizoglu, 2014). The metamodel is estimated from a sample of points in the parameter space, which still represents a computationally costly exercise for ABMs that can be made more efficient by an appropriate choice of evaluation points, e.g. with latin hypercube designs or other parsimonious sampling designs (see below). Furthermore, the parameter space may be eventually restricted through the calibration of at least some of them, as explained above, following the suggestion of Brenner and Werker (2007).

We remark that the result obtained in this way is analogous, for a simulated model, to the reduced form of an analytically solvable model. We can employ this reduced form, if its forecasting power with respect to the original ABM is good, for a variety of purposes, like sensitivity analysis (Campolongo *et al.*, 2000), model calibration and validation.

Given the general framework just outlined, in this paper we focus only on a subset of the numerous problems mentioned above. By taking as workhorse the ABM described in sec. 2, we provide an example of parameter calibration aimed at matching some fundamental properties of real credit networks with those of networks simulated in the model (sec. 3). After having specified a suitable sampling design for the remaining parameters of the model, we turn to simulations and discuss shortly some basic results of the ABM (sec. 4). Then we compare a number of alternative statistical models which could serve as "reduced form" of the ABM and, after having selected our model of choice by means of cross validation, we analyze the role of each parameter through regressions, and employ the same model for sensitivity analysis, with the specific purpose of assessing its consistency with the ABM (sec. 5). Sec.

6 provides some conclusions along with considerations regarding the long standing issue of aggregation.

2 Model Description

The most faithful microscopic description should depict the economic system as a network of networks, each corresponding to a market, which are populated by large numbers of heterogeneous, interacting agents evolving over time. In each of these networks, agents entertain multiple connections, which are endogenously adjusted according to their goals and behavioral procedures. The latter are not necessarily optimal, and outcomes are affected by a variety of endogenous and exogenous sources of uncertainty. In order to get a bit closer to this general picture, we extend the "Network-based financial accelerator" model of Riccetti *et al.* (2013) allowing firms to have multiple credit suppliers. Firms' financial structure adjusts towards a time dependent, endogenous, leverage target, assuming that firms follow a sort of Dynamic Trade-off Theory.

Firms produce their output with a linear production function where labor is the only input. They set their production plans at the maximum level allowed by their target leverage $\lambda_i = \frac{D_i}{E_i}$. We can use the balance sheet constraint $(1+\lambda_i)E_i = W_i = wN_i$ and the production function $Y_i = \alpha\, N_i$ to obtain[4]

$$\hat{Y}_i = \frac{1+\lambda_i}{c}\, E_i \tag{3}$$

where $c = w\,\alpha^{-1}$ is the unit labor cost. The effective production of final firm i is $Y_i = \min(\hat{Y}_i, \check{Y}_i)$, where $Y_i = \check{Y}_i$ in case of credit rationing, i.e. $\check{Y}_i$ represents the maximal production level of i which can be financed by the credit sector. Agents are matched on the credit markets by means of a random network model which can be calibrated with real data as shown in sec. 3. The extension to multiple connections, indeed, allows to calibrate our model with respect to fundamental network observables like degrees, i.e. the number of first neighbors. The actual leverage of firms in each simulation period is endogenously determined as the outcome of calibrated interactions on the credit market. In particular, following empirical data (see Sec. 3),

[4] Time indices are omitted whenever they are not strictly necessary.

we assume that firms' debt is proportional to their equity, $D_f \propto E_f$. The interest rate charged from banks to firm f is set in the following manner:

$$r_f = r_{cb}\,(1 + \delta\,\lambda_f) \tag{4}$$

where r_{cb} is the benchmark policy rate and $\delta \geqslant 0$ is a parameter which reflects the sensitivity of lenders to borrowers creditworthiness. Firms are subject to a random price shock defined with respect to labor unit cost :

$$p_f = c\,(1 + \epsilon_f) \tag{5}$$

where $\epsilon_f \sim N(\mu, \sigma)$. In general, we may view the distribution of price shocks as reflecting demand conditions, within a framework of price adjustment to market imbalances. Thus, a higher μ and a lower σ stand, ceteris paribus, for a stronger final demand. Firms' equity is updated according to profits, assuming that no dividends are distributed:

$$E_f^{t+1} = E_f^t + \pi_f \tag{6}$$

The profits of firms π_f are given by the following equation

$$\Pi_f = \left[p_f - \left(c + r_f \frac{D_f}{Y_f}\right)\right]\,Y_f \tag{7}$$

where $D_i = \sum_b L_{fb}$ is the total debt of firm f and L_{fb} is the amount of loan extended from bank b to firm f. Substituting Y_f, p_f and r_f respectively with eqs. (3),(5) and (4), after some simplifications we obtain

$$\pi_f = \frac{\Pi_f}{E_f + D_f} = \epsilon_f - r_{cb}\,\frac{D_f}{E_f}\,\frac{(E_f + \delta\,D_f)}{(E_f +\,D_f)} = \epsilon_f - r_{cb}\,\frac{1 + \delta\lambda_f}{\lambda_f^{-1} + 1} \tag{8}$$

From this expression we see that the profit rate of firms is independent of c. Instead it depends positively on the parameters of the price shock μ, σ, negatively on the parameters of the interest rates r_{cb}, δ and on λ_f, which is an endogenous variable since D_f depends on the net worth of f and on the net worth of banks. Thus all the relevant economic variables for firms (expected profits, loss risk, bankruptcy risk) depend on a firm specific, time varying endogenous component. For instance, bankruptcy risk may be written as follows

$$P\left(\Pi_f + E_f \leqslant 0\right) = P\left(\pi_f + \frac{1}{1+\lambda_f} \leqslant 0\right) = \Phi\left(r_{cb}\,\frac{1+\delta\lambda_f}{\lambda_f^{-1}+1} + \frac{1}{1+\lambda_f}\right) \tag{9}$$

where Φ is the normal cdf. The profits of bank are computed as follows

$$\Pi_b = \sum_f H(E_f + \Pi_f)\, r_f\, L_{fb} - \sum_f \left[1 - H(E_f + \Pi_f)\right]\, L_{fb} \tag{10}$$

where $H(E_f + \Pi_f) = 1$ if $E_f + \Pi_f > 0$ and $H(E_f + \Pi_f) = 0$ otherwise. We also assume that the supply of credit is proportional to banks' equity $S_b \propto E_b$ and that banks don't distribute dividends:

$$E_b^{t+1} = E_b^t + \pi_b \tag{11}$$

With the changes above the model becomes more parsimonious in terms of parameters with respect to the original formulation, thereby making it easier to sample the parameter space as required by our metamodeling exercise. On the other hand, our setting is indeed simplistic from an economic point view, and it's easy to argue that a more realistic model is likely to be required to obtain convincing results from a model calibration exercise which is, at any rate, beyond the scope of this paper.

3 Credit Market Interaction

We opt for a representation of market interactions by means of a random network model. We follow this path for two reasons. In the first place, it is less expensive in terms of computational time. Since we allow multiple credit connections, standard AB simulations should go through all potential links, i.e. cycle over $n \times m$ steps, where n and m are the number of firms and banks respectively. This is a slow, inefficient, solution for the most widely used languages, like Matlab, R or Python. Using a random network model we perform, instead, the following two operations: firstly we compute the parameters of a set of $n \times m$ probability distributions; secondly, we draw $n \times m$ random variables from these distributions[5].

[5]Both operations can be accomplished quickly thanks to vectorized, high performance, libraries like LAPACK or ARPACK.

In the second place, a random network model provides a general framework for describing economic interactions, which can be easily calibrated with economic data. In particular, an important set of network measures is related to connectivity, since most economic and social networks are found to be sparse. A credit network involving n firms and m banks connected by l links is said to be sparse when $l \ll n \times m$, otherwise the network is said to be dense. The former case is typical of real networks, e.g. the Japanese credit market studied in Bargigli and Gallegati (2011), whose most recent data are employed in this paper, had $l = 21,811$ connections over a maximum of $n \times m = 2,674 \times 182 = 486,668$ in 2005.

If a network is sparse, its topological properties[6] become non trivial. For instance, the size of the neighborhood of an agent, i.e. her *degree*, becomes very important. Agents with a high number of neighbors, called *hubs*, are typically conducive of large systemic effects, in particular they can potentially trigger bankruptcy avalanches, if affected by external shocks, through balance-sheet effects on many other agents (Shin, 2008).

Indeed, real credit markets display a high fraction of such agents, since their degree distributions are typically right-skewed. We wish to replicate this property of real credit markets in our model. A well known solution for this task in network theory is to build a statistical ensemble of random networks for which the average degree of each node is equal to the degree of the same node in the real network (Park and Newman, 2004). Random networks drawn from this ensemble trivially replicate the degree distribution of the original network. Here we follow a different route, because we wish to connect the degree distribution with the economic variables of the model. In particular, since we have assumed that credit demand and supply are determined as multiples of the net worth of firms and banks respectively (sec. 2), we wish to make their respective degrees dependent just on these variables (see sec. 3.1).

Once obtained a model for the activation of the link between firm i and

[6]By topological property we mean any observable which is defined on a binary network or on the binary representation of a weighted network. A network G is specified by the couple (V, E), where V is the vertex or node set, typically mapped onto a subset of $\mathbb{N}$, while E is the edge or link set, with $E \subset V \times V$ with elements $e_{ij} = (i, j)$. If the elements of E map onto $\{0, 1\}$ we say that the network is *binary*. If they map onto a subset D of $\mathbb{R}$ or $\mathbb{N}$ the network is *weighted*, and w_{ij} is the weight associated with (i, j). The binary representation of a weighted network is defined as $H(w_{ij})$ for each link (i, j), where H is the Heaviside function.

bank j, conditioned on their respective net worth, we need also a model to determine the size of loans L_{ij}, conditioned on the same variables, with the ambition to replicate the debt and loan size distributions of some real market. In order to control for topological properties and assign loan amounts at the same time, we need to follow an approach in two stages, i.e. we constrain L_{ij} to be positive only whenever a link between i and j has been activated (see sec. 3.2). In fact, random weighted network models that replicate only the strength distribution of some real network are not bound to follow any topological property, including the degree distribution. Recent papers (Mastromatteo *et al.*, 2012; Musmeci *et al.*, 2013) have highlighted that random networks in these ensemble are, with high probability, dense.

To sum up, the purpose of the estimations presented in the following sections is to calibrate our simulated credit market with real markets taking as reference a set of properties of choice (degree, debt distributions). For this exercise we employ the dataset for the banks-firms lending-borrowing links from 1980 to 2012 in Japan, maintained by the Econophysics Group at the University of Kyoto. The dataset includes balance sheet data on commercial banks and other credit institutions, as well as on listed companies[7].

3.1 Links

We suppose that connections between banks and firms on the credit market are binary random variables dependent on respective equity:

$$a_{fb} \sim F(E_f, E_b) \tag{12}$$

From network theory Park and Newman (2004) we know that the maximum entropy distribution of the value of the link between nodes i, j, in a statistical ensemble of binary networks $\mathcal{G}$, is associated with an expectation of the following form:

$$\mathbb{E}\left[a_{ij}\right] = \frac{1}{1 + \exp(-\epsilon_{ij})} \tag{13}$$

where ϵ_{ij} embodies a set of constraints imposed on $\mathcal{G}$. Then it is natural to explore the relationship (12) by means of a logistic regression model. In practice, we make the following specification in (13):

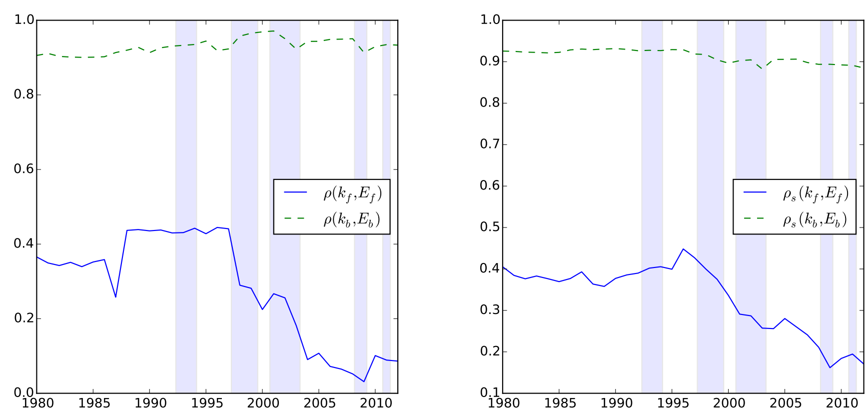
[7] For more details see `http://www.econophysics.jp/foc_kyoto/index.php?FOC%20Kyoto`.

$$\epsilon_{fb} = \alpha + \beta \log E_f + \gamma \log E_b \tag{14}$$

where f, b stand for firms and banks indices respectively. From the Japanese dataset we see that the two regressors behave differently over time. Fig. 1 highlights that there is a strong and stable linear relationship between the degree of banks k_b and their equity E_b. The relationship between companies' equity E_f and degree k_f is significant but weaker, less stable. This suggests that our specification does not include some relevant variables on the firms' side, like sector classification, which are not available in the Japanese dataset. Since we expect the logistic model to be misspecified, at least on the firms' side, we try to improve its fitness by means of random effects.

Figure 1: Correlations between degree and equity, Japanese dataset (Shaded areas are recessions)

(a) Pearson ρ (b) Spearman ρ



It might be worth to discuss the relationship between the topological properties of networks and the maximum likelihood (ML) logistic estimator. The number of links in the observed network is given by $l = \sum_{fb} a_{fb}$, while the model expectation is $\mathbb{E}[l|\theta^*] = \sum_{fb} \mathbb{E}[a_{fb}|\theta^*]$, where $\theta^* = (\alpha^*, \beta^*, \gamma^*)$ is estimated from data and $\mathbb{E}[a_{fb}|.]$ is specified by (13) and (14). The FOC for the ML problem are

$$\frac{\partial \ln \mathcal{L}}{\partial \theta} = \sum_{fb} \left(a_{fb} - \mathbb{E}[a_{fb}|\theta, x_{fb}] \right) x_{fb} = 0 \tag{15}$$

where $x_{fb} = (1, D_f, E_b)$ and $\theta = (\alpha, \beta, \gamma)$. This implies

$$L = \sum_{fb} a_{fb} = \sum_{fb} \mathbb{E}\left[a_{fb}|\theta, x_{fb}\right] = \mathbb{E}\left[L|\theta, x\right] \tag{16}$$

Thus we see that the ML estimator automatically recovers the connectivity of the original network. The same is not true for the two (firms,banks) observed degree distributions. In fact from the FOC we obtain

$$\sum_{f} E_f k_f = \sum_{f} E_f \mathbb{E}\left[k_f|\theta, x_f\right] \tag{17}$$

$$\sum_{b} E_b k_b = \sum_{b} E_b \mathbb{E}\left[k_b|\theta, x_b\right] \tag{18}$$

while the purpose of our calibration is to minimize the $n+m$ expressions

$$\|k_f - \mathbb{E}\left[k_f|\theta^*\right]\| \qquad f = 1, \dots, n \tag{19}$$

$$\|k_b - \mathbb{E}\left[k_b|\theta^*\right]\| \qquad b = 1, \dots, m \tag{20}$$

It's clear that the conditions (19)-(20) are more restrictive than (17)-(18). Thus the success of our calibration exercise depends on the effectiveness of the specifications that we adopt.

We estimate three models using the most recent data available in the dataset (2011): model 1 is given by eqs. (13) - (14); model 2 adds firm-specific random effects; model 3 includes both firm and bank-specific random effects. We have opted for random effects instead of fixed effects because the conditional log-likelihood estimation method of Chamberlain (1980) for logistic models with fixed effects does not provide the coefficients of the latter, which are needed for simulations (see sec. 4). From Tab. 1 we see that the coefficients of the three models are always significant and their magnitude is similar. From the goodness-of-fit measures in the table we see that the introduction of random effects improves the estimation.

Fig. 2 compares the distribution of degrees in actual data with the one obtained from a sample of 1,000 random networks simulated with calibrated parameters. We see that model 1 provides a poor fit for the degree distribution of firms, while model 3 provides the best fit to the degree distribution of banks. In order to test for conditions (19)-(20), we perform the two-sample KS test comparing the actual and simulated degree distributions. From Tab. 2 we see that the null hypothesis of equal distributions cannot be rejected for models 2 and 3, and that the latter provides the best approximation for both distributions. For this reason we select this model for our simulations.

Table 1: Logistic estimation on the Japanese credit market data (2011), ***($p < 0.01$)

	Model 1	Model 2	Model 3
(Intercept)	-3.95951***	-4.20761***	-4.35155***
$\log E_f$	1.53145***	1.62358***	1.60026***
$\log E_b$	0.19733***	0.17885***	0.18615***
Firms RE	No	Yes	Yes
Banks RE	No	No	Yes
Null Dev.	75,004		
Resid. Dev.	53,576	51,686	49,750
AIC	53,582	51,694	49,760
Pseudo R^2	0.286	0.312	0.337

Table 2: Kolmogorov-Smirnov 2-sample test for real and simulated degree distributions

		Model 1	Model 2	Model 3
k_f	KS stat.	0.111	0.021	0.016
	p-value	0.000	0.460	0.804
k_b	KS stat.	0.081	0.067	0.035
	p-value	0.410	0.643	0.998

3.2 Loans

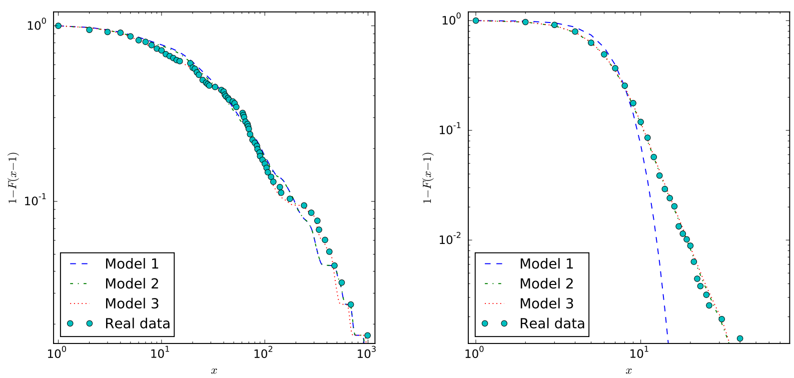
We want to employ the same regressors in order to explain the value of loans *conditioned* to the existence of a link between a firm f and a bank b:

$$(L_{fb}|a_{fb} = 1) \sim F(E_f, E_b) \tag{21}$$

From Fig. 3 we see that E_f and E_b are correlated in the Japanese dataset respectively with corporate bank debt D and the loan assets of banks S, i.e.

Figure 2: Simulated *vs.* real degree distribution (artificial network sample size $R = 1,000$)

(a) k_b, 2011 (b) k_f, 2011



the sum of loans extended to firms. In particular, the correlation between E_b and S is very high and stable over time.

We estimate the following model firstly with OLS, all log-variables are rescaled:

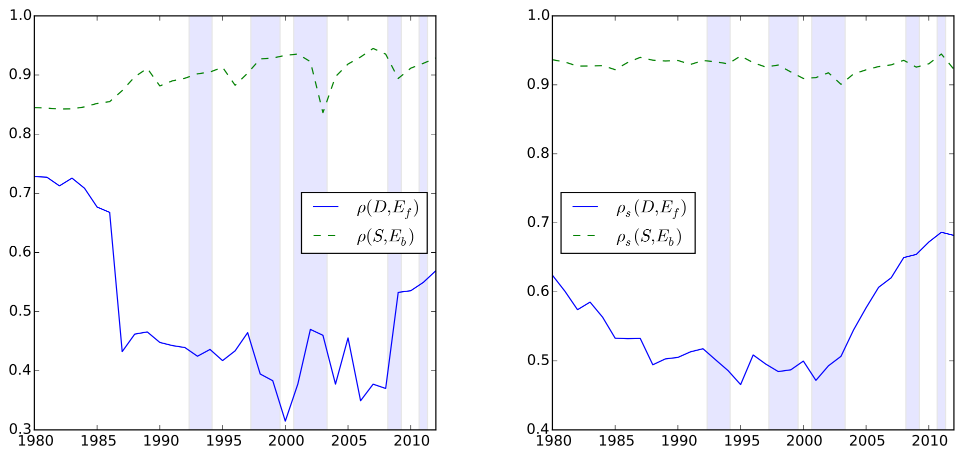
$$\log(L_{fb}|a_{fb} = 1) = \alpha + \beta \log E_f + \gamma \log E_b + u \tag{22}$$

With standard tests we detect both firm and bank specific effects in the data. In order to take these into account, we estimate distinct models with clustered errors. Finally, we add an interaction term between regressors. From Tab. 3 we see that the coefficients are always significant with expected sign. We also detect a significant influence of the interaction term. The inclusion of random effects improves the fitness of the estimation, as shown by the decrease of the AIC measure, and by the increase of the conditional R^2 proposed by Nakagawa and Schielzeth (2013), which is equal to the proportion of variance explained by both the fixed and random factors, while the marginal R^2 accounts for the variance explained by fixed factors alone.

In Fig. 4 we compare real data with simulated values obtained from models by randomly drawing from the residuals. We see that all models provide at first sight a good approximation to the distributions of D and S across firms and banks respectively. From Tab. 4 we see that the hypothesis

Figure 3: Correlations between firms' debt and equity, Japanese dataset (Shaded areas are recessions)

(a) Pearson ρ (b) Spearman ρ



of equal distribution cannot be rejected for models 5 and 6, with the latter providing a significantly better agreement with data for S. In the overall, we opt for this model in our AB simulations.

4 Simulation

The initial conditions of the model are given by the equity of firms and banks. For our simulations we employ as initial conditions the observed values of both variables taken from the Japanese dataset in 2011, which includes $n = 1572$ firms and $m = 117$ banks[8]. During simulations, the equity of agents evolves endogenously according to eqs. (6)-(11). When a firm / bank goes bankrupt, it is re-initialized with the median equity of survived firms / banks. The parameters needed for credit market interactions are fixed at the values estimated respectively from model 3 of Sec. 3.1 and model 6 of Sec. 3.2. In the simulations we draw samples from the distribution of the

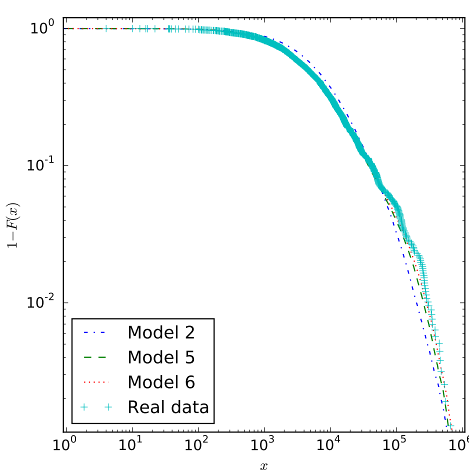
[8]For a detailed explanation of the dataset see `http://www.econophysics.jp/foc_kyoto/index.php?FOC%20Kyoto`

Table 3: Loan estimation (2011), ***($p < 0.01$)

	M1	M2	M3	M4	M5	M6
(Int.)	0.000	-0.000	0.010	0.009	0.010	0.000
$\log E_f$	0.564***	0.571***	0.587***	0.560***	0.591***	0.580***
$\log E_b$	0.234***	0.236***	0.225***	0.241***	0.227***	0.228***
$\log E_f \times \log E_b$		0.126***	-	-	-	0.095***
Firms RE	no	no	yes	no	yes	yes
Banks RE	no	no	no	yes	yes	yes
AIC	22,562	22,306	19,844	22,416	19,664	19,485
marg. R^2	0.374	0.391	0.381	0.371	0.383	0.384
cond. R^2	-	-	0.655	0.391	0.670	0.667

Figure 4: Loan estimation: models *versus* real data, 2011 sample size $R = 1,000$)

(a) D



(b) S

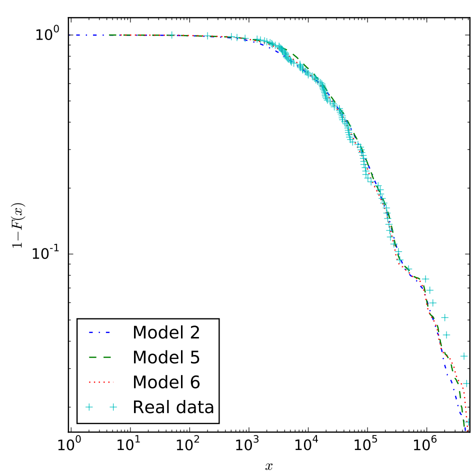


Table 4: Kolmogorov-Smirnov 2-sample test for real and simulated distributions

		Model 2	Model 5	Model 6
D	KS stat.	0.094	0.017	0.019
	p-value	0.000	0.714	0.560
S	KS stat.	0.060	0.084	0.042
	p-value	0.784	0.367	0.981

estimated random effects from these models in order to take into account of their influence over market interaction.

The range of the remaining parameters is presented in Tab. 5. In general, the dynamics of the model is determined by the losses of firms and the resulting bankruptcies. For each firm the probability of bankruptcy depends on the one hand on an endogenous threshold, which is independent of c but dependent on r_{cb} and δ, and on the other hand on the parameters of the distribution of price shocks (see eq. 9), i.e. the probability increases if the probability mass that falls below the endogenous threshold increases. So if, for example, we decrease μ, the distribution of price shocks moves leftwards and bankruptcies increase *ceteris paribus*. Otherwise, if we increase σ, the probability mass in the tails increases and the effect is the same. However, the interpretation is different, since in the former case the increase in bankruptcies is associated with a decrease in the expected profit at constant uncertainty, which we may call a "first order" effect of price shocks, in the latter case instead uncertainty increases with expected profit unchanged, which we may call a "second order" effect of price shocks. Mixing the two effects would make the results more difficult to interpret. Thus, for sake of simplicity, we confine ourselves to first order effects of price shocks by varying μ while keeping σ constant, and leave for the future the analysis of second order effects.

The sampling scheme we adopt for the subspace of varying parameters (r_{cb}, δ, μ) is the one suggested by Cioppa and Lucas (2007) and employed by Salle and Yildizoglu (2014). Random sampling from an uniform distribution, which is a common choice in Montecarlo exercises, is inefficient because it generates a high number of redundant sampling points (points very close to each other), while leaving some parts of the parameter space unexplored. A common alternative is importance sampling, which however requires prior information. A proper "design of experiment" (DOE) delivers instead a parsimonious sample which is nevertheless representative of the parameter space. In particular, representative samples are said to be "space filling", since they cover as uniformly as possible the domain of variation.

Table 5: Parameter space explored in the simulations

r_{cb}	$[0.001, 0.05]$
δ	$[0.1, 1]$
μ	$[0.001, 0.1]$
σ	0.001
c	0.8

The scheme of Cioppa and Lucas (2007) is based on Nearly Orthogonal Latin Hypercubes (NOLH). In the context of sampling theory, a square grid representing the location of sample points for a couple of parameters is a Latin square if there is only one sample point in each row and each column. A Latin hypercube is the generalization of this concept to an arbitrary number of dimensions, whereby each sample point is the only one in each axis-aligned hyperplane containing it. This property ensures that sample points are non collapsing, i.e. that the 1-dimensional projections of sample points along each axis are space filling. In fact, with this scheme, the sampled values of each parameter appear once and only once. Basic Latin Hypercube schemes may display correlations between the columns of the $k \times n$ design matrix X, where k is the number of parameters and n is the sample size for each parameter, especially when $k \lessapprox n$. Instead, an orthogonal design is convenient because it gives uncorrelated estimates of the coefficients in linear regression models and improves the performance of statistical estimation in general. In practice, in orthogonal sampling, the sample space is divided into equally probable subspaces. All sample points in the orthogonal LH scheme are then chosen simultaneously making sure that the total ensemble of sample points is a Latin Hypercube and that each subspace is sampled with the same density. The NOLH scheme of Cioppa and Lucas (2007), in particular, improves the space filling properties of the resulting sample when $k \lessapprox n$ at the cost of introducing a small maximal correlation of 0.03 between the columns of X. Furthermore, no assumptions regarding the homoskedasticy of errors or the shape of the response surface (like linearity) are required to obtain this scheme.

For each of the 17 points of the NOLH scheme reproduced in Appendix, we replicate 10 simulations over $T = 500$ periods, after an initial run of 200 periods. For each simulation run, we compute the average and standard deviation of the time log-differences of the following aggregate variables: production (`y`), firms' equity (`feq`), banks' equity (`beq`), firms' debt (`dbt`), firms' leverage (`lev`). Furthermore, we compute the average of firms' and banks' bankruptcies by simulation step (`fbkr` and `bbkr` respectively).

A summary of the results is presented in Fig. 5 through correlations between model outputs obtained at different points of the parameter sample. We see that `avg_y` is highly correlated both with `avg_feq` and `avg_dbt`, which is a consequence of eq. (3). This equation is also at work in the high correlation of the `std` variables. The common factor in both cases is given by price shocks. Growth (`avg_y`) is also significantly correlated with leverage (`avg_lev`), which means that aggregate leverage is pro-cyclical in the model, at least within the subspace Ω of Tab. 5. On their part, the growth of debt and leverage are highly correlated with the growth of bank equity (`avg_beq`), which suggests, given the credit market mechanism we have assumed, that debt growth is mostly driven by supply, while the linkage with credit demand is weaker as shown by the fact that leverage is negatively correlated with firms' equity. Growth and bankruptcies are negatively correlated, as we might expect, since higher growth is associated with a stronger equity base for firms. Banks become instead more risky when they lend more and are more profitable (indeed `bbkr` is positively correlated with `avg_dbt`, `avg_lev` and `avg_beq`), as well as with higher volatility in general. We also find a significant clue of contagion effects in the positive correlation of `bbkr` and `fbkr`.

We underline that the negative correlation between `feq` and `lev` is a consequence of the network calibration of the model. Indeed

$$L_{fb} = (L_{fb}|a_{fb} = 1) \times a_{fb} = A \times E_f^{\beta} \times E_b^{\gamma} \times a_{fb} \tag{23}$$

where $A = \exp(\alpha + u)$. By taking expectations we obtain

$$\mathbb{E}[L_{fb}] = A' \times E_f^{\beta} \times E_b^{\gamma} \times \frac{1}{1 + \exp(-\epsilon_{fb})} \tag{24}$$

where ϵ_{fb} is specified as in eq. (14) and $A' = \exp(\alpha)\mathbb{E}[\exp(u)]$. We see that the r.h.s. of eq. (24) is an increasing function of E_f and E_b, since the denominator is decreasing in both variables. Thus $\mathbb{E}[D_f]$ is of course also an increasing function of net worths. Regarding leverage we may proceed as follows

$$\mathbb{E}[\lambda_f] = \frac{\mathbb{E}[D_f]}{E_f} = A' \times E_f^{\beta-1} \times \sum_b \frac{E_b^\gamma}{1+\exp(-\epsilon_{fb})} \leqslant A' \times E_f^{\beta-1} \times \left(\sum_b E_b^\gamma \right) \quad (25)$$

We see that λ_f is decreasing in E_f whenever $\beta < 1$, which is indeed the case in our estimations (Tab. 3), while it is always increasing in banks' equity.

5 Metamodels

We compare a number of metamodels. On the one hand we estimate various Kriging spatial models, which are generalized regression models, potentially allowing for heteroskedastic and correlated errors, whose general form is given by eq. (2) above. This approach is widely used for ABM metamodeling in various fields (see e.g. Salle and Yildizoglu (2014), Dancik *et al.* (2010) and references therein). Using generalized regression is convenient since some of the parameters of our model are related to random distributions (μ, σ, which are related to price shocks), which naturally affect the variability of model output. The Kriging approach (Roustant *et al.*, 2012) resorts to feasible generalized least squares by assuming a stationary correlation kernel $K(h) = K(\theta_i - \theta_j)$, where θ_i, θ_j are points in the parameter space Θ. $K(h)$ takes the following general form:

$$K(h) = \prod_{j=1}^{d} g(h_j, \lambda_j) \quad (26)$$

where d is the dimension of Θ, and $\lambda = (\lambda_1, \ldots, \lambda_d)$ is a vector of parameters to be determined. In particular, we employ for g the specifications of Tab. 6.

Since we work with noisy, potentially heteroskedastic observations, in our estimation the covariance matrix is determined as follows:

$$C = \sigma^2 R + \mathrm{diag}(\tau) \quad (27)$$

Table 6: Covariance kernels (Roustant *et al.*, 2012)

`K1`	Matérn ($\nu = 5/2$)	$g(h) = \left(1 + \frac{\sqrt{5}\lvert h\rvert}{\lambda} + \frac{5h^2}{3\lambda^2}\right) \exp\left(-\frac{\sqrt{5}\lvert h\rvert}{\lambda}\right)$
`K2`	Matérn ($\nu = 3/2$)	$g(h) = \left(1 + \frac{\sqrt{3}\lvert h\rvert}{\lambda}\right) \exp\left(-\frac{\sqrt{3}\lvert h\rvert}{\lambda}\right)$
`K3`	Gaussian	$g(h) = \exp\left(-\frac{h^2}{2\lambda^2}\right)$
`K4`	Power-Exponential	$g(h) = \exp\left(-\left(\frac{\lvert h\rvert}{\lambda}\right)^t\right)$
`K5`	Exponential	$g(h) = \exp\left(-\frac{\lvert h\rvert}{\lambda}\right)$

where R is the correlation matrix with elements $R_{ij} = K(\theta_i - \theta_j)$ and $\tau = (\tau_1^2, \ldots, \tau_n^2)$ is the vector containing the observed variance of model output at fixed points of the parameter space, where n is the size of the NOLH design. The ML estimation is performed on the "concentrated" multivariate Gaussian log-likelihood, obtained by substituting the vector of coefficients β with its generalized least square estimator. The "concentrated" log-likelihood is a function of σ and λ, which are the optimization variables of the estimation. The solution is obtained numerically through the quasi-Newton algorithm provided by the DiceKriging `R` (2015) package (Roustant *et al.*, 2012).

In our exercise we compare the fitness of the same metamodel estimated with different Kriging kernels and with OLS. The metamodel reads as follows:

$$y = \beta x + u(\theta) \tag{28}$$

$$x = (1, r_{cb}, \delta, \mu, r_{cb}^2, \delta^2, \mu^2, r_{cb} \times \delta, r_{cb} \times \mu, \delta \times \mu) \tag{29}$$

$$y = (\texttt{avg\_y}, \texttt{std\_y}, \texttt{fbkr}) \tag{30}$$

$$\theta = (r_{cb}, \delta, \mu) \tag{31}$$

where β is the matrix of coefficients which can be estimated equation by equation with the methods mentioned above, and the dependency of the error term from θ, due to heteroskedasticity and cross correlations, is made explicit. The higher order terms in the model are introduced because we expect the ABM to display non linear behavior. In particular, we wish to capture the combined effect of parameters regulating credit cost (r_{cb}, δ) and price shocks or final demand conditions as explained in Sec. 2 (μ). The components of y

are chosen among model outputs that are not highly correlated (see previous section), in order to obtain independent equations.

In a first estimation exercise, we compute a simple OLS regression with the specification (28)-(29) for each model output in y. Since the hypothesis of constant variance is rejected by the Breusch-Pagan test, we introduce robust weights computed from the interquartile distance of model outputs at fixed values of the parameters, and observe an improved fitness of the estimation. Then we opt for weighted regression in the comparison with Kriging models.

The fitness of alternative estimations is computed by means of k-fold cross validation, i.e. the models are used to predict the response variables on k random sections of the experiment design after being estimated on the rest of it. In particular, we set $k = 5$. Fitness is compared through RMSE, MAE and Q^2, which is a R^2 statistics computed out of sample (thus it can take negative values). The values of Tab. 7 are means over 100 replications of the procedure. We see that the weighted OLS regression always performs better than the alternative Kriging models. It is remarkable that the fitness of all models is very low when the dependent variable is `avg_y`. We regard this result as a consequence of the complex behavior of ABMs (see Sec. 6).

After having estimated the full model (28)-(29) over the entire simulation sample, we find high variance inflation factors (`vifs`) for all regressors, which are not eliminated after centering data with respect to the mean. High `vifs` are expected since we employ higher order and interaction terms, and we wish to obtain reliable regression coefficients for a consistent economic interpretation of results. Then we select, for each model output, distinct submodels which guarantee a higher fitness in terms of R^2 coupled with low vifs.

Table 8 summarizes the WLS estimation of these models. Since the hypothesis of constant variance is still rejected for `avg_y`, we employ heteroskedastic consistent standard errors for inference purposes in this case. It is noteworthy that we find no significant effect of μ over growth. This can be explained as follows. A higher μ implies a higher rate of accumulation for firms but also for banks. If accumulation is unbalanced in favor of the latter, credit supply expands too much, pushing up debt and leverage, see eq. (25). But a higher leverage makes credit more costly, thereby making firms more likely to make losses, see eq. (7). If many firms go bust, a large number of banks (if not all) are likely to go bust as well. In this case, overall bank equity is reduced drastically, cutting credit supply and finally hampering growth itself. This is confirmed by the volatility enhancing effect of μ

Table 7: Cross $k-$validation of Metamodels, $k = 5$

(a) `avg_y`

	`WLM`	`K1`	`K2`	`K3`	`K4`	`K5`
Q^2	-0.066	-15.064	-15.782	-15.677	-13.995	-17.772
	(0.002)	(0.535)	(0.597)	(0.671)	(0.507)	(0.889)
RMSE	0.004	0.009	0.009	0.009	0.009	0.010
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
MAE	0.003	0.007	0.007	0.007	0.007	0.007
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

(b) `std_y`

	`WLM`	`K1`	`K2`	`K3`	`K4`	`K5`
Q^2	0.954	0.598	0.638	0.546	0.598	0.609
	(0.000)	(0.014)	(0.008)	(0.032)	(0.010)	(0.014)
RMSE	0.196	0.556	0.538	0.562	0.565	0.548
	(0.000)	(0.007)	(0.005)	(0.011)	(0.006)	(0.007)
MAE	0.123	0.403	0.390	0.402	0.412	0.394
	(0.000)	(0.004)	(0.004)	(0.006)	(0.004)	(0.005)

(c) `fbkr`

	`WLM`	`K1`	`K2`	`K3`	`K4`	`K5`
Q^2	0.765	0.244	-0.178	0.118	0.245	0.235
	(0.001)	(0.029)	(0.160)	(0.057)	(0.039)	(0.044)
RMSE	8.930	15.249	16.978	15.841	15.029	15.058
	(0.011)	(0.280)	(0.675)	(0.428)	(0.327)	(0.343)
MAE	5.155	11.571	12.292	11.852	11.330	11.427
	(0.008)	(0.169)	(0.358)	(0.272)	(0.206)	(0.205)

`WLM` = weighted OLS estimation; `K1` = Kriging est., Matern(5/2); `K2` = Kriging est., Matern(3/2); `K3` = Kriging est., Gaussian; `K4` = Kriging est., power-exponential; `K5` = Kriging est., exponential. Standard errors in parentheses.

over growth. A typical evolution of the ABM for high μ is reproduced in Fig. 6. On the other hand, from the same figure we see that r_{cb} and δ have a stabilizing effect since if rates are high and μ is low, firms' equity decreases, keeping bank accumulation at bay and thus preventing the explosive behavior of leverage. The signs of the interaction terms for `fbkr` are consistent with this framework, since the effect of higher r_{cb}, δ combined with the explosive behavior of leverage triggered by a higher μ, makes bankruptcy more likely. Finally, the low fitness for `avg_y` is also explained since, if widespread bankruptcies occur when growth-related parameters are at first sight more favorable, equity and thus production is determined by the ad hoc replacement mechanism explained in Sec. 2, which works independently from the parameters of the model.

Table 8: WLS estimation of metamodels

	Dependent variable:		
	`avg_y`	`std_y`	`fbkr`
r_{cb}	-0.037^{**}	-21.113^{***}	21.453^{***}
	(0.015)	(1.169)	(1.350)
δ	-0.002^{***}	-0.893^{***}	-0.057
	(0.0005)	(0.069)	(0.081)
μ	0.002	22.395^{***}	11.473^{***}
	(0.005)	(0.549)	(0.722)
r_{cb}^2		-17.715^{***}	
		(4.564)	
δ^2		61.025	
		(57.495)	
μ^2		-24.138^{***}	
		(1.372)	
$r_{cb} \times \delta$			-122.680
			(82.289)
$r_{cb} \times \mu$			2.695^{***}
			(0.401)
$\delta \times \mu$			172.055^{***}
			(28.298)
Obs.	170	170	170
R^2	0.125	0.963	0.909
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Since some of the effects captured by the metamodel appear counterintuitive, we would like to verify in a systematic way that the metamodels reproduce adequately the behavior of the underlying ABM. We accomplish this task by means of sensitivity analysis.

Campolongo *et al.* (2000) define sensitivity analysis (SA) as the study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input. With this respect, SA techniques should satisfy the two main requirements of being global and model free. By global, one means that SA must take into consideration the entire joint distribution of parameters. Global methods are opposed to local methods, which take into consideration the variation of one parameter at a time, e.g. by computing marginal effects of each parameter. By model independent, one means that no assumptions on the model functional relationship with its inputs, such as linearity, is required. Campolongo *et al.* (2000) propose a global approach based on the decomposition of variance:

$$\begin{aligned}
V(y) &= \sum_{i}^{k} V_i + \sum_{i<j} V_{ij} + \sum_{i<j<m} V_{ijm} + \cdots + V_{12\ldots k} \\
V_i &= \mathbb{V}\left[\mathbb{E}\left(y|\theta_i = x_i\right)\right] \\
V_{ij} &= \mathbb{V}\left[\mathbb{E}\left(y|\theta_i = x_i, \theta_j = x_j\right)\right] - \mathbb{V}\left[\mathbb{E}\left(y|\theta_i = x\right)\right] - \mathbb{V}\left[\mathbb{E}\left(y|\theta_j = x\right)\right] \\
&\ldots
\end{aligned}$$

We see that V_i represents the variance of the main effect of parameter i, while all the other terms are related to interactions effects. From this general formula we can obtain the contribution of interaction effects S_{Ii} involving the parameter θ_i in $y = f(\theta)$, with f square integrable, as

$$\begin{aligned}
S_{Ii} &= S_{Ti} - S_i \\
S_i &= \frac{\mathbb{V}_{\theta_i}\left[\mathbb{E}_{\theta_{-i}}(y|\theta_i)\right]}{V} \\
S_{Ti} &= \frac{\mathbb{E}_{x_{-i}}\left[\mathbb{V}_{x_i}(y|x_{-i})\right]}{V} = 1 - \frac{\mathbb{V}_{\theta_{-i}}\left[\mathbb{E}_{x_i}(y|x_{-i})\right]}{V}
\end{aligned}$$

The multidimensional integral of the last line can be evaluated numerically using the extended FAST method described in Campolongo *et al.* (2000). This method requires a specific sampling scheme, involving unfortunately a much larger number of model runs than the design of sec. 4. Thus we are more interested in a SA method which may work with a given sampling scheme, like the one proposed by Plischke *et al.* (2015), at the cost of losing some precision of estimates. They introduce a global, moment independent, uncertainty indicator whose definition, to the contrary of the previous example, is well posed also in the presence of correlations among the parameters, since their distributions are not required to be independent. This indicator is always between 0 and 1, it equals 0 if the output is not dependent upon a parameter, it is readily defined for parameter groups and it equals unity if the group of all inputs is considered. Its definition is as follows:

$$d(\theta_i) = \frac{1}{2}\,\mathbb{E}_{\theta_i}\left[s(\theta_i = x)\right] \tag{32}$$

$$s(\theta_i = x) = \int \left| f\left[y(\theta)\right] - f\left[y(\theta)\right|\theta_i = x\right]\right| p(\theta) d\theta \tag{33}$$

where f is the probability density of y. Eq. (33) provides the variational distance between the unconditional density of y and its conditional density once θ_i is fixed. $d(\theta_i)$ can be viewed as the reduction of uncertainty regarding the model output which is consequent to knowing the value of θ_i. Plischke *et al.* (2015) provide an estimator for this quantity, which works with any given sample. Thanks to this property, we can compare the response of the metamodels with the one of the original ABM. Thus we see from Fig. 7 that the full metamodel of eq. (29) is able to mimic the ABM's behavior for `std_y` and `fbkr`, while we observe significant difference in the two responses in the case of `avg_y`, as we might expect from the low fitness of this metamodel. Moreover, we see that there is some loss of accuracy, at least for `fbkr`, if we use the metamodel of Tab. 8, which comes along with the benefit of more reliable estimates for the regression coefficients.

6 Conclusions

In this paper we have extended the ABM of Riccetti *et al.* (2013) by allowing firms and banks to entertain multiple connections in a stylized credit market model. In practice, we resort to a random network model whose parameters are calibrated with real data. The calibration is successful in the sense that the network model is able to reproduce the degree and strength (debt and loan) distributions of the Japanese credit market. At the same time, we reduce the number of parameters of the ABM, in order to make it easier to explore globally the parameter space, which is sampled with the efficient design proposed by Cioppa and Lucas (2007). Subsequently we compare a number of alternative statistical models in order to select the best specification for the relationship between the parameters and a set of endogenous variables of the model. These statistical metamodels are estimated on AB simulations, and we observe a high fitness for two endogenous variables out of three, both using in-sample criteria, like R^2 and sensitivity analysis, and out of sample criteria like cross validation.

The selected metamodels, one for each endogenous variable under consideration, allow to identify the effect of each parameter, taking into account non linearities, by looking at the significance and sign of the corresponding regression coefficients. While some results are in line with expectations, others appear at first sight counterintuitive. In particular, the average growth of production is not increasing in μ, to the contrary of what would suggest eq. (8), and we also observe a low fitness of the metamodel for this variable. This results follow from the fact that the growth of the two sectors (firms and banks) is not necessarily balanced in the model: if the net worth of banks grows faster than the net worth of firms, credit supply expands too much, pushing up leverage and making defaults more likely, thereby hampering growth. If defaults are numerous, net worth and production are determined mostly by the replacement mechanism described in Sec. 4 and not by the value of the parameters, a circumstance which explains the low fitness of the metamodel in this case.

From this discussion we see that metamodeling can be a source of theoretical discipline for ABMs. In particular, our analysis shows that some mechanism to balance the accumulation process of firms and banks is required to obtain more convincing results at the aggregate level. Following the general approach of agent based modeling, any solution should avoid theoretical shortcuts, like assuming balanced growth *ex ante*, which are typical of neoclassical models. For instance, we could introduce simple dividend policies in both sectors or, at least, in one in of them, to control for equity growth.

Some final considerations are in order at this point. Aggregation is a long standing issue in economic theory. In the case of mainstream macro models, the gap between micro and macro is bridged by imposing ex ante strong theoretical restrictions (such as doing away with agent heterogeneity) which allow to derive a log-linear mathematical state-space representation of the macro observables directly from the micro model. ABMs instead generally lack analytical solutions, and most ABM modelers are not favorable to select their assumptions on the basis of analytical tractability. Moreover, as soon as heterogeneity is allowed for, the relationship between micro and macro variables becomes more complex to handle, as underlined in many contributions (e.g. Kirman (1992), Stocker (1993) and Gallegati *et al.* (2006)).

Our results show that we can bridge micro and macro through a rigorous statistical analysis of ABMs. In agent based simulations, aggregate statistics are directly collected from microsimulations. With this bottom-up approach we are extempted from the need to impose restrictions on the micro model (Stocker, 1993) or to take approximations (Gallegati *et al.*, 2006). On the other hand, we see that a positive result is not guaranteed, since some aggregate variables (like growth in our model) might elude our efforts to encapsulate them in a mathematical relationship with the parameters of the model. This limitation could be possibly overcome employing as metamodel a state space formulation instead of a reduced form formulation like the one employed in this paper. To begin with, we could make growth at t depend on the number of defaults in the previous period, since the latter affect aggregate net worth of firms and banks which on their turn affect production. Otherwise, we may turn to indirect inference, and seek a parametrization for which the ABM is able to reproduce a set of key statistical properties of real aggregate time series, e.g. by fitting some densely parametrized, unrestricted model like a VAR model both on real and simulated data (Winker *et al.*, 2007). As explained in the introduction, we leave these issues for future research.

Acknowledgments: we thank all the participants to the Department seminar of DISEI held on November 17 for their useful comments. All the usual disclaimers apply.

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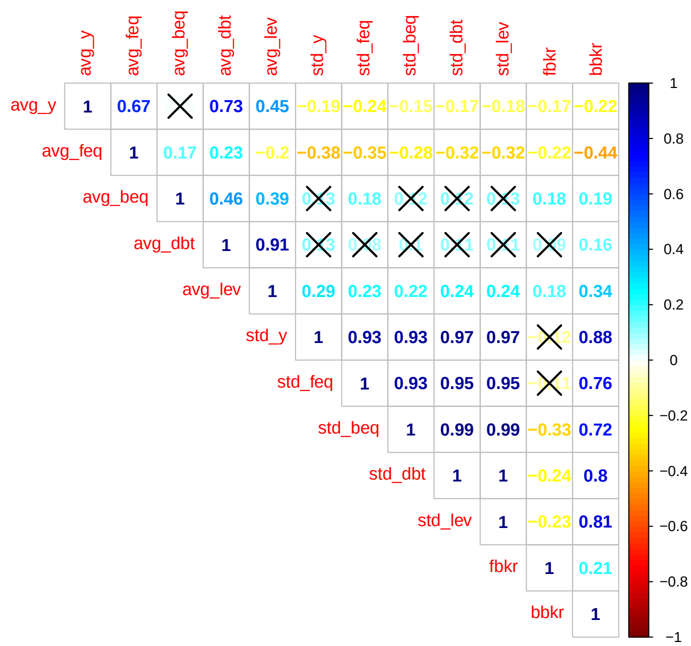
A Appendix: DOE

Table A.1: DOE

r_{cb}	δ	μ
0.019	0.381	0.026
0.029	0.156	0.001
0.013	0.213	0.063
0.050	0.269	0.057
0.007	0.775	0.032
0.041	1.000	0.007
0.016	0.663	0.088
0.047	0.606	0.081
0.026	0.550	0.050
0.032	0.719	0.075
0.022	0.944	0.100
0.038	0.888	0.038
0.001	0.831	0.044
0.044	0.325	0.069
0.010	0.100	0.094
0.035	0.438	0.013
0.004	0.494	0.020

Figure 5: Correlation plot of model outputs (non crossed values 5% significant)



`y` = production; `feq` = firms' equity; `beq` = banks' equity; `dbt` = firms' debt; `fbkr` = firms' bankruptcies; `bbkr` = banks' bankruptcies, `lev` = firms' leverage; `avg` = (time) average; `std` = (time) standard deviation. `avg_y`, `avg_feq`, `avg_beq`, `avg_dbt`,`lev` are the time average of log-differences. `std_y`, `std_feq`, `std_beq`, `std_dbt` are the standard deviation of log-differences. `fbkr` and `bbkr` are computed as the average number of failures per period.

Figure 6: Simulation runs

(a) $r_{cb} = 0.01$, $\delta = 0.1$, $\mu = 0.094$ (b) $r_{cb} = 0.041$, $\delta = 1$, $\mu = 0.007$

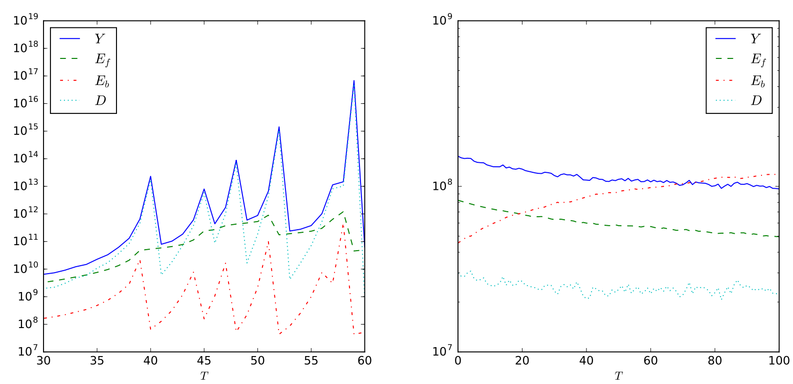
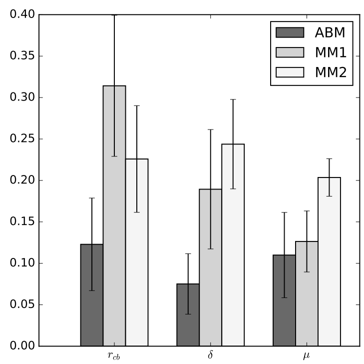
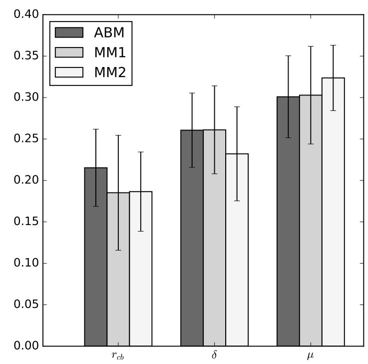


Figure 7: Density based sensitivity analysis

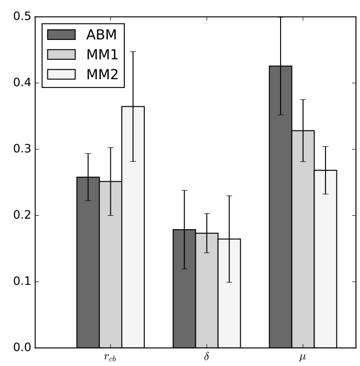
(a) `avg_y`



(b) `std_y`



(c) `fbkr`



ABM = agent based model; MM1 = full metamodel of eq. (29); MM2 = metamodels of Tab. 8