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# Age-based preferences in paired kidney exchange ☆

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A B S T R A C T

We consider a Paired Kidney Exchange (PKE) model in which patients' preferences are restricted so that patients prefer kidneys from compatible younger donors to kidneys from older ones. We propose a family of rules, sequential priority rules, that only allow for pairwise exchanges and satisfy individual rationality, efficiency, strategy-proofness, and non-bossiness. These rules allocate kidneys according to a priority algorithm that gives priority to patients with younger donors and assign kidneys from younger donors first. We extend the analysis to rules that allow multiple ways exchanges and to the case of patients who have more than one potential donor.



## 1. Introduction

Paired Kidney Exchange (PKE) programs seek to overcome any incompatibility (of blood or tissue types)[1] of living donor–patient pairs by arranging swaps of donors among several pairs (Delmonico, 2004; Delmonico et al., 2004; Roth et al., 2004). PKE programs work as clearing houses that periodically search for mutually compatible exchanges of donors in a pool of donor–patient pairs. In order to find such mutually compatible exchanges, PKE programs need to elicit relevant information from patients (and their doctors) and to overcome feasibility constraints that are absent in standard problems of allocation of indivisible goods. Specifically, PKE programs involve the cooperation and the coordination of several transplantation units at different medical centers. Thus, the complexity of the logistics makes exchanges involving too

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[1] A patient and a donor are incompatible if the patient's body will immediately reject the donor's kidney after the graft, and thus the transplantation is deemed not viable.

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many donor–patient pairs unfeasible.[2] For this reason, real-life PKE programs have generally focused on maximizing the number of simultaneous compatible organ exchanges between two donor–patient pairs, although swaps involving more than two pairs are also carried out. To deal with situations where a donor–patient pair may be necessary in more than one compatible exchange of donors, real life PKE programs usually give priority to particular patients in much the same way as it happens in the allocation of kidneys obtained from cadaveric donors.[3]

Living donor kidney transplantation yields excellent results in terms of life expectancy of the graft compared to kidney transplantation from cadaveric organs (Delmonico, 2004; Gjertson and Cecka, 2000). This fact explains the prevalent approach in PKE programs, which assumes that patients only care about receiving a compatible kidney. Recent medical research, however, supports the idea that different compatible kidneys can have substantially different outcomes. The age and the health status of the donor, in fact, have a major impact on the expected survival of the graft (Su et al., 2004; Gentry et al., 2007; Gjertson, 2004; Øien et al., 2007).[4] This observation has important implications for modeling PKE. First, the heterogeneity of transplantation outcomes may affect participants' incentives. Secondly, it provides a justification for the participation in PKE programs of patients with a compatible willing donor. These pairs may have incentives to participate in PKE programs since the patient could obtain a kidney that results in higher life-expectancy than that of her donor's kidney. The participation of compatible pairs can dramatically increase the chances of finding compatible swaps for incompatible pairs, and boost the transplantation rate (Roth et al., 2004, 2005b, 2006; Gentry et al., 2007).[5]

We model PKE clearing houses as rules that assign kidneys to patients through kidney exchanges taking into account patients' preferences over available kidneys. Preferences of patients depend on the donors' characteristics that determine compatibility as well as life expectancy after transplantation. The pool of available kidneys can be partitioned into groups of kidneys of similar quality corresponding to the age of the donors. Patients are interested in receiving a compatible kidney, but they prefer a kidney from a younger donor. This observation suggests the analysis of a restricted domain of preferences: the *age-based preference domain*. In this domain, we study rules that satisfy individual rationality,[6] efficiency restricted by the logistic constraints, and strategy-proofness.[7] Individual rationality is an essential property of a rule for the obvious reason that no patient can be forced to perform a transplant. In our setting, it also aims to guarantee to compatible pairs that they can never regret for enrolling in a PKE program. Strategy-proofness is compelling in our setting, because it implies that patients of compatible pairs truthfully reveal which is the minimum gain in term of higher life-expectancy that persuades them to accept a paired kidney exchange.

We first focus on rules that only allow for pairwise exchanges among two donor–patient pairs.[8] Rules that satisfy the above properties and the auxiliary property of non-bossiness select assignments that maximize the number of pairwise exchanges among pairs with the youngest donors and, sequentially among pairs in different age groups according to an ordering based on donors' age.[9] We call such rules sequential maximizing rules. It results illustrative to describe the intuition behind sequential maximizing rules for the simplest case in which donors are partitioned in two age groups, young and mature donors. A sequential maximizing rule identifies a set of patients who maximizes the number of compatible pairwise exchanges between pairs with a young donor. Having fixed the exchanges performed by these pairs, it proceeds by identifying a set of patients who maximizes the number of compatible pairwise exchanges between unmatched patients with a young donor and patients with a mature donor. Finally, keeping fixed all swaps identified in the previous stages, it identifies a set of patients who maximize the number of compatible pairwise exchanges between unmatched patients with a mature donor.

In addition to the previous necessary condition, we also provide an algorithm that defines a family of rules, sequential priority rules, that satisfy our axioms. Patients are prioritized according to their donor age, and among patients with donors



[2] There is growing interest in the creation of non-simultaneous, extended non-directed altruistic anonymous donor chains that help to avoid such limitations. A donor chain starts with an anonymous altruistic donor willing to donate to anyone needing a kidney transplant without having a related direct recipient (Rees et al., 2009; Ünver, 2010; Ausubel and Morrill, 2014). In some cases, the incompatible donor of the recipient starts a chain as a new anonymous altruistic donor. Based on Organ Procurement and Transplantation Network data as of January 23, 2017; in 2016, 612 living donation kidney transplants have been obtained through standard paired donation and 217 by anonymous donors (https://optn.transplant.hrsa.gov/data/view-data-reports/national-data/ and https://optn.transplant.hrsa.gov/resources/ethics/living-non-directed-organ-donation/).

[3] This is the case of the New England PKE program (Roth et al., 2005a, 2005b). Similar protocols are adopted by other centralized PKE programs implemented in countries like Korea (Park et al., 2004), the Netherlands (Keizer et al., 2005), the United Kingdom (NHS Blood and Transplant, 2016), Spain (Organización Nacional de Transplantes, 2015), and the United States with its UNOS National Pilot Program for Kidney Paired Donation (Ashlagi and Roth, 2012, 2014).

[4] For instance, Øien et al. (2007) confirm that the donor's age and health status have a crucial role in the case of living donations. A donor over 65 years old is associated with a higher risk of graft loss at every time point after transplantation. There is more controversy in the medical literature regarding the effects of other characteristics such as the similarity of tissue types between patients and donors (Delmonico, 2004; Gjertson and Cecka, 2000; Opelz, 1997).

[5] In the most recent updates of the Spanish PKE program (Organización Nacional de Transplantes, 2015), the problem of how to favor compatible pairs' participation is explicitly acknowledged.

[6] A rule satisfies individual rationality if patients never prefer the initial assignment where no kidney swap is performed to the outcome prescribed by the rule.

[7] A rule satisfies strategy-proofness if patients never have incentives to misrepresent their preferences.

[8] While PKE programs perform kidney exchanges involving more than two donor–patient pairs, pairwise exchanges are prevalent. We discuss the possibility of multiple ways exchanges in Section 5.2.

[9] A rule satisfies non-bossiness if, whenever a change in a patient's preferences does not modify her assignment, that is, the kidney she receives, it does not modify any other patient's assignment.

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in the same age-group, using additional criteria (like time in waiting list). In the case of two age groups, at the first stage, patients select, following the priority ordering, their favorite assignments in the set of individually rational assignments that only include pairwise exchanges in which at least one donor is young. At the end of the first stage, some patients are matched with a donor and some patients remain unmatched. At the second stage, the unmatched patients (sequentially) select their favorite assignments in the set of individually rational assignments that include the exchanges prescribed at the first stage. This logic may be immediately extended to any number of age groups. Sequential priority rules combine the aim of maximizing the number of compatible exchanges with that of assigning priorities to patients based on their medical characteristics that are common in PKE protocols used in practice. The additional constraint imposed by strategy-proofness in our domain is that the main criterion to prioritize the patients is the age of their donor, which reflects the value for the other patients of their endowment. Moreover, those rules maintain their properties when multiple ways exchanges are admitted, but with the restriction that multiple ways exchanges should only involve donors in the same age group.

Our positive results rely on the analysis of the existence of strict core assignments in the non-monetary exchange problem defined by PKE.[10] Under strict preferences and no feasibility constraints, the strict core is single-valued, it can be attained through the celebrated Top Trade Cycling algorithm (Shapley and Scarf, 1974), and the rule that selects the unique core assignment satisfies strategy-proofness and many other desirable properties (Roth, 1982; Bird, 1984; Miyagawa, 2002; Ma, 1994; Sönmez, 1999). This is not the case however, when indifferences are allowed and/or there are feasibility constraints (Ehlers, 2002; Alcalde-Unzu and Molis, 2011; Jaramillo and Manjunath, 2012). Under our domain restriction, we show that there are pairwise exchange rules that satisfy those desirable properties, and such rules are naturally related to sequential priority rules.

This paper contributes to the kidney exchange literature initiated by Roth et al. (2004). In recent years, PKE has received a considerable interest from both a theoretical and a practical design point of view. Most works have considered the framework that incorporates specific features consistent with the medical approach to PKE in New England (Roth et al., 2005a, 2005b, 2007; Hatfield, 2005; Ünver, 2010; Yılmaz, 2011, 2014). That approach assumes that only incompatible pairs participate in PKE, and that patients are indifferent between two compatible kidneys. In this paper, we introduce a restricted domain of preferences, which allows for strict preferences over compatible kidneys and provides incentives for compatible pairs to enroll on PKE programs, but inevitably introduces additional structure for satisfactory rules. Finally, we refer to few recent papers very related to ours. Sönmez and Ünver (2014) analyze the structure of Pareto efficient matchings when compatible pairs are admitted into PKE in the New England PKE framework. In that paper, the sole motivation for the participation of compatible pairs is altruism. Sönmez and Ünver (2015) propose to provide incentives to a compatible pair to enroll in PKE programs, by giving priority to the patient of this pair on the deceased-donor queue in case her transplant fails. Nicolò and Rodríguez-Álvarez (2012) consider a model where patients' preferences are unrestricted, but the only private information is the minimal quality of the kidney that each patient requires to undergo transplantation. In that framework, under any arbitrary restriction on the number of pairs involved in exchanges, no rule satisfies individual rationality, efficiency, and strategy-proofness.[11]

Before proceeding with the formal analysis, we briefly outline the contents of the remainder of this paper. In Section 2, we present basic notation and definitions. In Section 3, we introduce the concept of age-based preferences. In Section 4, we state our results for pairwise exchanges. In Section 5, we discuss computability issues, the extension to multiple ways exchanges, and the possibility that patients may have more than one potential donor. We provide all the proofs in Appendix A.

## 2. Basic notation

Consider a finite society consisting of a set $N = \{1, \ldots, n\}$ of patients ($n \geq 3$) who need a kidney for transplantation. Each patient has a potential donor, and $\Omega = \{\omega_1, \ldots, \omega_n\}$ denotes the set of kidneys available for transplantation. For each patient $i$, $\omega_i$ refers to the kidney of patient $i$'s donor. We assume that all available kidneys are obtained from living donors and each patient has only one potential donor.[12]

Each patient $i$ is equipped with a complete, reflexive, and transitive preference relation $\succsim_i$ on $\Omega$. We denote by $\succ_i$ the associated strict preference relation and by $\sim_i$ the associated indifference relation. Let $\mathcal{P}$ denote the set of all preferences. We call $\succsim = (\succsim_i)_{i \in N} \in \mathcal{P}^N$ a preference profile. For each patient $i$ and each $\succsim \in \mathcal{P}^N$, $\succsim_{-i} \in \mathcal{P}^{n-1}$ denotes the restriction of the profile $\succsim$ for all the patients excluding $i$. We assume that patients' preferences are further restricted, so that for each patient $i$ her preferences belong to a subset $\mathcal{D}_i \subset \mathcal{P}$. We denote by $\mathcal{D} \equiv \times_{i \in N} \mathcal{D}_i \subseteq \mathcal{P}^N$ a domain of preference profiles over kidneys.

An ***assignment*** $a$ is a bijection from kidneys to patients. For each patient $i$ and each assignment $a$, $a_i$ denotes the kidney assigned to $i$ according to $a$. Whenever $a_i = \omega_i$, we consider that either patient $i$ continues in dialysis or – if she is compatible with her donor – she receives her donor's kidney. Let $\mathcal{A}$ be the set of all assignments.

[10] A kidney assignment is in the strict core if no group of patients can (weakly) benefit by swapping donors among themselves.

[11] Zenios (2002) also considers PKE where patients care about the quality of the outcome in a dynamic setting but there is no information to be elicited from the patients. The focus is on the optimal assignment of donor–patient pairs to direct exchange programs or indirect exchange programs, where patients may swap their incompatible donor to gain priority on the waiting list.

[12] We dispense with the later assumption in Section 5.3.



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In an assignment, kidneys are assigned to patients by forming cycles of patient–donor pairs. In each cycle, every patient receives a kidney from the donor of some patient, and her donor's kidney is transplanted to another patient. In this paper, we firstly focus on the most binding feasibility constraints, and consider pairwise assignments such that only exchanges between two donor–patient pairs are admitted.[13] That is, an assignment $a \in \mathcal{A}$ is a pairwise assignment if for each $i, j \in N$, $a_i = \omega_j$ implies $a_j = \omega_i$. Let $\mathcal{A}_2$ be the set of all pairwise assignments. In general, for each $k \in \mathbb{N}$, $k \leq n$, we say that the assignment $a$ is ***k-feasible*** if it is formed by cycles involving at most $k$ donor–patient pairs, and $\mathcal{A}_k$ denotes the set of all $k$-feasible assignments.

Given a preference domain $\mathcal{D}$, a rule is a mapping $\varphi : \mathcal{D} \to \mathcal{A}$, and we denote by $r_\varphi$, the range of $\varphi$, that is

$$r_\varphi = \left\{ a \in \mathcal{A} \mid \text{ for some } \succsim \in \mathcal{D},\ a = \varphi(\succsim) \right\}.$$

A rule $\varphi$ is a pairwise exchange rule if $\varphi : \mathcal{D} \to \mathcal{A}_2$ and $r_\varphi = \mathcal{A}_2$.

We are interested in rules that satisfy the properties of *individual rationality*, *efficiency*, and *strategy-proofness*.

***Individual rationality.*** For each $i \in N$ and each $\succsim \in \mathcal{D}$, $\varphi_i(\succsim) \succsim_i \omega_i$.

***Efficiency.*** For each $\succsim \in \mathcal{D}$, there is no $a \in r_\varphi$ such that for each $i \in N$ $a_i \succsim_i \varphi_i(\succsim)$ and for some $j \in N$, $a_j \succ_j \varphi_j(\succsim)$.

***Strategy-proofness.*** For each $i \in N$, each $\succsim \in \mathcal{D}$, and each $\succsim'_i \in \mathcal{D}_i$, $\varphi_i(\succsim) \succsim_i \varphi_i(\succsim'_i, \succsim_{-i})$.

In addition to these basic properties, we consider one additional desirable property.

***Non-bossiness.*** For each $i \in N$, each $\succsim \in \mathcal{D}$, and each $\succsim'_i \in \mathcal{D}_i$, $\varphi_i(\succsim) = \varphi_i(\succsim'_i, \succsim_{-i})$ implies $\varphi(\succsim) = \varphi(\succsim'_i, \succsim_{-i})$.

*Non-Bossiness* is a standard technical property in environments that admit indifferences. Since monetary transactions related to organ donation are almost universally banned, *non-bossiness* has also a reasonable normative justification in PKE problems. A rule that violates *non-bossiness* may give rise to illegal bribes among donor–patient pairs. If a patient $i$ changes her preference report and affects the outcome of patient $j$, then $i$ may have incentives to accept a monetary compensation from $j$ in order to reverse her report.

## 3. Age-based preferences

In this section we present a new domain restriction that is directly inspired by the specifics of PKE. We start by introducing some useful notation.

For each patient $i$ and each preference $\succsim_i \in \mathcal{P}$, let $D(\succsim_i) \equiv \{\omega \in \Omega \setminus \{\omega_i\} \mid \omega \succ_i \omega_i\}$ define the ***set of desirable kidneys for patient*** $i$. Let $ND(\succsim_i) \equiv \{\omega \in \Omega \setminus \{\omega_i\} \mid \omega_i \succsim_i \omega\}$ define the ***set of undesirable kidneys for patient*** $i$.

The set $D(\succsim_i)$ contains all the kidneys which lead to an improvement with respect to $i$'s outside option $\omega_i$: that is, either staying in dialysis or receiving her donor's kidney. Conversely, $ND(\succsim_i)$ contains all incompatible kidneys that lead to $i$'s rejection of the graft as well as those kidneys that may lead to a poor transplantation outcome, and hence such that patient $i$ prefers to keep $\omega_i$ rather than receiving one of these organs.

The viability (non-rejection) of transplantation depends on the compatibility of tissue and blood types that are idiosyncratic to each patient and donor. Other factors crucially affect the expected quality of a transplant. In case of living donation, it turns out that donor's age and health status are the most important characteristics in determining the probability of long term graft survival. Both characteristics are quite closely correlated and directly observable by transplant coordinators, and they affect all the patients in the same way. The age of the donors does not determine the compatibility between a kidney and a patient. However, when a patient compares two desirable kidneys, she prefers the kidney from the youngest donor. If the donors of both kidneys are (approximately) of the same age, then she is indifferent between them. We therefore assume that there is a characteristic, namely the age of the donor, according to which available kidneys can be partitioned into groups of kidneys of the same quality. The following notation formalizes this idea.

An ***age structure*** is a partition $\Pi = \{\Pi(1), \ldots, \Pi(m)\}$ of $\Omega$. The age structure partitions the set of available organs according to the age of the donors. Each subset contains donors who approximately have the same age. Thus, we call each element of the age structure an ***age group***.

Let $\Pi = \{\Pi(1), \ldots, \Pi(m)\}$ be an age structure. For each patient $i \in N$, the preference relation $\succsim_i \in \mathcal{P}$ is a $\Pi$***-age-based preference*** if for each $\omega, \omega' \in D(\succsim_i)$ and for each $\bar{\omega} \in ND(\succsim_i)$:

(i) $\omega \in \Pi(j)$ and $\omega' \in \Pi(k)$ and $j < k$ imply $\omega \succ_i \omega'$,
(ii) $\omega, \omega' \in \Pi(j)$ implies $\omega \sim_i \omega'$, and
(iii) $\omega_i \succ_i \bar{\omega}$.

[13] We leave the discussion of general feasibility constraints to Section 5.2.



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Let $\mathcal{D}_i^{\Pi}$ denote the set of all $\Pi$-age-based preferences for patient $i$ and let $\mathcal{D}^{\Pi} \equiv \times_{i \in N} \mathcal{D}_i^{\Pi}$ denote the domain of $\Pi$-age-based preferences.

Without any loss of generality and to simplify notation, we henceforth assume that for each age structure $\Pi$, for each $i, j \in N$, if $i < j$, $\omega_i \in \Pi(l)$, and $\omega_j \in \Pi(l')$, then $l \leq l'$.

We denote with $\Pi^*$ the ***strict age structure*** such that for each $l \leq n$, $\Pi^*(l) = \{\omega_l\}$. According to $\Pi^*$, kidneys are strictly ordered according to the natural ordering that corresponds to donors' ages. Hence, according to preferences in the strict domain $\mathcal{D}^{\Pi^*}$, patients are never indifferent between two desirable kidneys. Alternatively, let $\bar{\Pi}$ denote the coarsest partition such that $\bar{\Pi} = \{\bar{\Pi}(1)\}$ with $\bar{\Pi}(1) = \Omega$. The domain $\mathcal{D}^{\bar{\Pi}}$ corresponds to the domain of dichotomous preferences introduced by Roth et al. (2005a, 2005b). According to preferences in the dichotomous domain $\mathcal{D}^{\bar{\Pi}}$, patients are always indifferent between two desirable kidneys.

## 4. Pairwise exchange rules for age-based preferences

In this section we restrict the attention to rules that only allow pairwise exchanges. It is worth noticing that both rules that play a central role in the literature of allocation of indivisible objects, Top Trade Cycle (TTC) rule and serial dictatorship, are not satisfactory in the PKE environment. On the one hand, the rule defined by the Top Trade Cycle algorithm (Shapley and Scarf, 1974) usually proposes assignments involving large exchange cycles that are not feasible. On the other hand, a serial dictatorship (Hylland and Zeckhauser, 1979; Svensson, 1999; Pápai, 2001), in which patients sequentially choose their preferred alternative from the set of feasible assignments, cannot secure the selection of mutually desirable exchanges, violating *individual rationality*.

We start the formal analysis introducing a necessary condition for pairwise exchange rules that satisfy *individual rationality*, *efficiency*, *strategy-proofness*, and *non-bossiness*. When pairs of patients have mutually compatible donors, then one of them has to receive an organ at least as good as the one she could receive performing this pairwise exchange.

**Lemma 1.** *For each age structure $\Pi$, if a pairwise exchange rule $\varphi : \mathcal{D}^{\Pi} \to \mathcal{A}_2$ satisfies* individual rationality, efficiency, strategy-proofness, *and* non-bossiness, *then for each $i, j \in N$ and $\succsim \in \mathcal{D}^{\Pi}$, $\omega_i \in D(\succsim_j)$ and $\omega_j \in D(\succsim_i)$ imply either $\varphi_i(\succsim) \succsim_i \omega_j$ or $\varphi_j(\succsim) \succsim_j \omega_i$ (or both).*

An immediate consequence of Lemma 1 is that our properties preclude the maximization of the number of patients that receive a desirable kidney for age structures with at least two age groups. This is in sharp contrast with the results in the dichotomous domain framework presented by Roth et al. (2005a). Every pairwise exchange rule defined in the domain $\mathcal{D}^{\bar{\Pi}}$ that satisfies *individual rationality* and *efficiency* maximizes the number of transplants. When patients may have strict preferences over compatible kidneys, efficiency does not require to maximize the number of transplants and the additional properties make impossible to get such result. We illustrate this fact in the following example.

**Example 1.** *Let $N = \{1, 2, 3, 4\}$, $\Pi(1) = \{\omega_1, \omega_2\}$, and $\Pi(2) = \{\omega_3, \omega_4\}$. Consider the preference profile $\succsim \in \mathcal{D}^{\Pi}$ such that $D(\succsim_1) = \{\omega_2, \omega_3, \omega_4\}$, $D(\succsim_2) = \{\omega_1, \omega_3, \omega_4\}$, $D(\succsim_3) = \{\omega_1\}$, and $D(\succsim_4) = \{\omega_2\}$. Any pairwise exchange rule that maximizes the number of transplants selects the following assignment $a = (\omega_3, \omega_4, \omega_1, \omega_2)$, according to which four transplants are performed. However, by Lemma 1, every pairwise exchange rule $\varphi : \mathcal{D}^{\Pi} \to \mathcal{A}_2$ that satisfies* individual rationality, efficiency, strategy-proofness, *and* non-bossiness *select the assignment $\varphi(\succsim) = (\omega_2, \omega_1, \omega_3, \omega_4)$.*

Example 1 provides an intuitive proof of Lemma 1. Suppose by contradiction that $\varphi(\succsim) = (\omega_3, \omega_4, \omega_1, \omega_2)$; patients 1 and 2 could report preferences $\succsim_1'$ and $\succsim_2'$ such that $D(\succsim_1') = \{\omega_2\}$, and $D(\succsim_2') = \{\omega_1\}$. *Individual rationality* and *efficiency* imply that at profile $(\succsim_1', \succsim_2', \succsim_3 \succsim_4)$ patients 1 and 2 exchange their donors. Hence, $\varphi_1(\succsim_1', \succsim_2', \succsim_3 \succsim_4) = \omega_2$ and $\varphi_2(\succsim_1', \succsim_2', \succsim_3 \succsim_4) = \omega_1$. Since $\omega_2$ is the preferred organ for patient 1 according to her preference $\succsim_1$, and $\omega_1$ is the preferred organ for patient 2 according to her preferences $\succsim_2$, an immediate application of *strategy-proofness* and *non-bossiness* (changing sequentially preferences of patients 1 and 2 from $\succsim'$ to $\succsim$) leads to conclude that $\varphi_1(\succsim) = \omega_2$ and $\varphi_2(\succsim) = \omega_1$.

Lemma 1 and Example 1 suggest that, under a binary age structure $\Pi = \{\Pi(1), \Pi(2)\}$, any exchange rule satisfying the above properties has first to maximize the number of exchanges among pairs of patients with donors in $\Pi(1)$, then the number of exchanges among remaining pairs of patients with a donor in $\Pi(1)$ and a donor in $\Pi(2)$ and finally among remaining patients with donors in $\Pi(2)$. We define a class of pairwise exchange rules that extend this intuition to all age groups. For that purpose it is useful to have in mind the construction of an assignment as a multi-stage process involving exchanges among pairs of patients with donors belonging to different age groups.

For each $a \in \mathcal{A}$, each $t, t' \in \mathbb{N}$ with $t \leq t' \leq m$, let

$$M_{t,t'}(a) \equiv \left\{ i \in N \,\middle|\, a_i \neq \omega_i \;\&\; \begin{array}{c} \text{either } \omega_i \in \Pi(t) \text{ and } a_i \in \Pi(t') \\ \text{or } \omega_i \in \Pi(t') \text{ and } a_i \in \Pi(t) \end{array} \right\}.$$

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That is, $M_{t,t'}(a)$ contains patients with a donor in $\Pi(t)$ who receive a kidney in $\Pi(t')$ and patients with a donor in $\Pi(t')$ who receive a kidney in $\Pi(t)$.

For an arbitrary pairwise assignment $a \in \mathcal{A}_2$, we can interpret the pairwise assignment $a$ as the result of a sequential procedure that starts by matching the patients in $M_{1,1}(a)$. Then, the procedure proceeds by matching patients in $M_{1,2}(a)$, and sequentially with the remaining age groups till $M_{1,m}(a)$. The process continues with exchanges among patients in $M_{2,2}(a)$, and so on.

Next, at each stage $t, t'$, of the process, we can define the group of patients who have received an organ in previous stages, $P_{t,t'}(a)$. For each $a \in \mathcal{A}_2$, let $P_{1,1}(a) \equiv \varnothing$ and iteratively for each $t, t' \in N$ with $\{t, t'\} \neq \{1, 1\}$, $t \leq t' \leq m$:

$$P_{t,t'}(a) \equiv \begin{cases} P_{t,t'-1}(a) \cup M_{t,t'-1}(a) & \text{if } t < t', \\ P_{t-1,m}(a) \cup M_{t-1,m}(a) & \text{if } t = t'. \end{cases}$$

Finally, for each $a \in \mathcal{A}$ and each $t, t' \in \mathbb{N}$ with $t \leq t' \leq m$, let $\mathcal{R}_{1,1}(a) = \mathcal{A}$, and for each $t, t' \in N$ with $\{t, t'\} \neq \{1, 1\}$,

$$\mathcal{R}_{t,t'}(a) = \left\{a' \in \mathcal{A} \mid \text{ for each } i \in P_{t,t'}(a),\ a'_i = a_i\right\}.$$

The set $\mathcal{R}_{t,t'}(a)$ contains all the assignments in which every patient in $P_{t,t'}(a)$ receives the same kidney that she receives in assignment $a$.

For each $\succsim \in \mathcal{P}^n$, let $\mathcal{I}(\succsim) \equiv \{a \in \mathcal{A}_2 \mid \text{for each } i \in N\ a_i \succsim_i \omega_i\}$ denote the ***set of individually rational pairwise assignments***.

A pairwise exchange rule $\varphi : \mathcal{D}^{\Pi} \to \mathcal{A}_2$ is a ***sequentially maximizing rule*** if for each $\succsim \in \mathcal{D}^{\Pi}$,

i) $\varphi(\succsim) \in \mathcal{I}(\succsim)$, and
ii) for each $t, t' \in \mathbb{N}$ with $t \leq t' \leq m$, for each $a \in \mathcal{R}_{t,t'}(\varphi(\succsim)) \cap \mathcal{I}(\succsim)$,

$$\#M_{t,t'}(\varphi(\succsim)) \geq \#M_{t,t'}(a).$$

Sequentially maximizing rules propose assignments that maximize the number of exchanges within each age group and among different age groups following the ordering induced by the age structure. Sequentially maximizing rules follow the logic previously conjectured for a binary age structure. Thus, $\Pi(1)$ is the set of young donors, and $\Pi(2)$ is the set of mature donors. In this case, a sequential maximizing rule maximizes among the assignments in $\mathcal{I}(\succsim)$ the number of transplants involving only donor–patient pairs with a young donor. Then, given the exchanges between pairs of patients with a young donor, it maximizes the number of exchanges between patients who have a young donor and those with a mature donor. Finally, given the exchanges arranged in the previous stages, the rule maximizes the number of exchanges involving donor–patient pairs with mature donors. For more general age structures, a sequential maximizing rule proceeds in the same fashion, maximizing the number of pairwise exchanges among patients with the youngest donors, then among remaining patients with the youngest donors and patients with donors of elder age groups, following the ordering of the age groups. Then, it applies the same logic maximizing pairwise exchanges among pairs of patients with donors in the second age group, and so on.

**Theorem 1.** *If a pairwise exchange rule $\varphi : D^{\Pi} \to \mathcal{A}_2$ satisfies* individual rationality, efficiency, strategy-proofness, *and* non-bossiness, *then $\varphi$ is a sequential maximizing rule.*

In order to provide a tidier description of the rules belonging to this class we start by analyzing the domains generated by the two extreme age structures, the dichotomous domain $\mathcal{D}^{\bar{\Pi}}$ and the strict domain $\mathcal{D}^{\Pi^*}$ induced by the strict age structure.

In the dichotomous domain $\mathcal{D}^{\bar{\Pi}}$, the literature on PKE has focused on priority mechanisms that resemble the protocols commonly used to allocate cadaveric organs. Priority rules (Roth et al., 2005a, 2005b) proceed as serial dictatorship rules in which patients sequentially select the assignments in which they receive their preferred kidneys among all individually rational assignments.[14] To preserve *efficiency*, whenever a patient is indifferent among several available kidneys, priority rules break ties taking into account the preferences of the remaining patients in the sequence.[15] After having defined priority rules, we show that in the domain $\mathcal{D}^{\Pi^*}$, pairwise exchange rules satisfy *individual rationality*, *efficiency*, and *strategy-proofness* if and only if they are priority rules with the priorities of each patient is determined by the age of her potential donor. However, we show that priority rules fail to satisfy *strategy-proofness* in age-based domains that admit strict and weak preferences over desirable kidneys.

A ***priority ordering of the patients*** $\sigma : N \to N$ is a permutation such that the $k$-th patient in the permutation is the patient with the $k$-th priority. Let $\sigma^*$ denote the ***natural priority ordering*** such that for each $i \in N$, $\sigma^*(i) = i$. For each age

[14] Bogomolnaia and Moulin (2002) consider a problem of allocation of objects under common preferences similar to age-based preferences. In that paper, a random version of serial dictatorship is dubbed a random priority mechanism.

[15] It is immediate to check that serial dictatorship rules that select individually rational assignments using arbitrary (or random) tie-breaking procedures do not satisfy *efficiency*.

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structure $\Pi$ and each priority ordering $\sigma$, we say that $\sigma$ ***respects*** $\Pi$ if for every $i, j \in N$, $\omega_i \in \Pi(l)$, $\omega_j \in \Pi(l')$, and $l < l'$ imply $\sigma(i) < \sigma(j)$.

***Priority algorithm.*** Fix a priority ordering of the patients $\sigma$, a preference profile $\succsim \in \mathcal{P}^N$, and a set of assignments $A \subset \mathcal{A}$:

- Let $\mathcal{M}_0^\sigma(\succsim, A) = A$.
- For each $t \leq n$, let $\mathcal{M}_t^\sigma(\succsim, A) \subseteq \mathcal{M}_{t-1}^\sigma(\succsim, A)$ be such that:

$$\mathcal{M}_t^\sigma(\succsim, A) = \left\{a \in \mathcal{M}_{t-1}^\sigma(\succsim, A) \mid \text{for no } b \in \mathcal{M}_{t-1}^\sigma(\succsim, A),\ b_{\sigma^{-1}(t)} \succ_{\sigma^{-1}(t)} a_{\sigma^{-1}(t)}\right\}.$$

The set $\mathcal{M}_1^\sigma(\succsim, A)$ selects among all assignments in $A$ those who are weakly preferred by the agent who has the highest priority according to the priority ordering $\sigma$; the selection proceeds according to priority $\sigma$ and at any step $t$, patient $\sigma^{-1}(t)$ selects her preferred assignments among those that have survived in the previous steps. The set $\mathcal{M}_n^\sigma(\succsim, A)$ is well defined, non-empty, and essentially single-valued.[16] A pairwise exchange rule $\varphi : \mathcal{D}^\Pi \to \mathcal{A}_2$ is an ***age-based priority rule*** if there is a priority ordering $\sigma$ that respects $\Pi$ such that for each $\succsim \in \mathcal{D}^\Pi$, $\varphi(\succsim) \in \mathcal{M}_n^\sigma(\succsim, \mathcal{I}(\succsim))$.

The following result is implied by a result in Sönmez (1999).

**Theorem 2.** *A pairwise exchange rule $\varphi : \mathcal{D}^{\Pi^*} \to \mathcal{A}_2$ satisfies* individual rationality, efficiency, *and* strategy-proofness *if and only if it is the age-based priority rule for the natural priority ordering $\sigma^*$.*

For general assignments problems with non-empty strict core, Sönmez (1999) shows that there are rules that satisfy *individual rationality*, *efficiency*, and *strategy-proofness* only if the strict core is essentially unique. Moreover, such a pairwise exchange rule always selects assignments in the core. For our analysis under the strict age structure $\Pi^*$, the strict core coincides with the assignment selected by the age-based priority rule.

Could the previous result be generalized to age structures $\Pi$ in which some age groups may contain more than one donor? The following example provides an immediate negative answer. Age-based priority rules may violate *strategy-proofness* when patients have age-based preferences in circumstances with a minimal conflict of interest among the preferences of the patients.

**Example 2.** *Let $\Pi(1) = \{\omega_1\}$, $\Pi(2) = \{\omega_2\}$, $\Pi(3) = \{\omega_3, \omega_4\}$ and $\succsim \in D^\Pi$ be such that $D(\succsim_1) = \{\omega_3, \omega_4\}$, $D(\succsim_2) = \{\omega_3\}$, $D(\succsim_3) = \{\omega_1, \omega_2\}$, and $D(\succsim_4) = \{\omega_1\}$. Consider the natural priority ordering $\sigma^*$. Let $\varphi^*$ be the age-based priority rule defined by the natural priority ordering $\sigma^*$. Notice that $\varphi^*(\succsim) = (\omega_4, \omega_3, \omega_2, \omega_1)$. Let $\succsim' \in \mathcal{D}^\Pi$ be such that $\succsim'_{-3} = \succsim_{-3}$ and $D(\succsim'_3) = \{\omega_1\}$. Then $\varphi^*(\succsim'_3, \succsim_{-3}) = (\omega_3, \omega_2, \omega_1, \omega_4)$. Since $\varphi_3^*(\succsim'_3, \succsim_{-3}) \succ_3 \varphi_3^*(\succsim)$, $\varphi^*$ violates* strategy-proofness.

In the above example whether patient 1 exchanges her donors with patient 3 or with patient 4, it depends on patient 2's preferences. However, while patient 1 is indifferent between $\omega_3$ and $\omega_4$, patient 3 strictly prefers $\omega_1$ to $\omega_2$ and therefore she misreports her preferences to be matched with her favorite donor. *Strategy-proofness* requires that which pairwise exchange patient 1 is involved to, either with patient 2 or with patient 4, should only depend on the preferences of patients 1, 3, and 4, and not on the preferences of patient 2, who is not directly involved in these exchanges.

We present now an alternative definition of sequential priorities that elaborates this intuition and is able to generate pairwise exchange rules that satisfy *strategy-proofness*.

***Sequential priority algorithm.*** Fix a priority ordering $\sigma$ over patients and a preference profile $\succsim \in \mathcal{P}^N$.

- Let $\mathcal{F}_0^\sigma(\succsim) = \mathcal{I}(\succsim)$.
- For each $1 \leq t \leq m$ let

$$\begin{aligned}
\mathcal{S}_t^\sigma(\succsim) &\equiv \left\{a' \in \mathcal{I}(\succsim) \,\middle|\, \begin{array}{l} \exists a \in \mathcal{F}_{t-1}^\sigma(\succsim), \forall i \in P_{t,m}(a) \cup M_{t,m}(a), a'_i = a_i, \\ \forall j \notin P_{t,m}(a) \cup M_{t,m}(a), a'_j = \omega_j \end{array}\right\}, \\
\mathcal{F}_t^\sigma(\succsim) &\equiv \{a \in \mathcal{I}(\succsim) \mid \exists a' \in \mathcal{M}_n^\sigma(\succsim, \mathcal{S}_t^\sigma(\succsim)), \forall i \in P_{t,m}(a) \cup M_{t,m}(a), a_i = a'_i\}.
\end{aligned}$$

Note that for each $a, a' \in \mathcal{M}_n^\sigma(\succsim, \mathcal{S}_t^\sigma(\succsim))$ for each $1 \leq t \leq t' \leq m$, $M_{t,t'}(a) = M_{t,t'}(a')$, and for each $i$ with $\omega_i \in \Pi(1)$ and $i \notin \cup_{t=1}^m M_{1,t}(a)$, $a_i = a'_i = \omega_i$.

We say that the pairwise exchange rule $\psi^\sigma : \mathcal{D}^\Pi \to \mathcal{A}_2$ is a ***sequential priority rule*** if for each $\succsim \in \mathcal{D}^\Pi$, $\psi^\sigma(\succsim) \in \mathcal{F}_m^\sigma(\succsim)$ and $\sigma$ respects $\Pi$.

The first step of a sequential priority rules work as follows. The set $\mathcal{S}_1^\sigma(\succsim)$ contains all individually rational assignments in which patients with donors in $\Pi(1)$ exchange their donors among themselves and sequentially with patients with elder

[16] A set is essentially single-valued either if it is single-valued or if it contains more than one element and all the patients are indifferent between any two elements in the set. That is, for each patient $i$, each $\succsim \in \mathcal{P}^N$, and each $a, a' \in \mathcal{M}_n^\sigma(\succsim, A)$, $a_i \sim_i a'_i$.

donors, and all patients who are not involved in such exchanges keep their donor. The set $\mathcal{M}_n^\sigma(\succsim, \mathcal{S}_1^\sigma(\succsim))$ selects a subset of assignments in $\mathcal{S}_1^\sigma(\succsim)$ following the priority ordering $\sigma$. Therefore, it identifies a (unique) list of pairwise exchanges among patients with donors in $\Pi(1)$ and all the other patients. The set $\mathcal{F}_1^\sigma(\succsim)$ contains all individually rational assignments in which these exchanges are performed. The algorithm proceeds looking for further exchanges among patients with donors in $\Pi(2)$ and other patients with elder donors who were not involved in the exchanges identified in the previous step. The main difference between sequential priority rules and age-based priority rules is that, while both use the logic of serial dictatorships starting from the set of individually rational assignments, sequential priority rules fix the exchanges involving the donors in $\Pi(1)$ before (sequentially) looking for exchanges among patients in the remaining age groups. In this way *strategy-proofness* is preserved. It is also not hard to prove that sequential priority rules satisfy the other desirable properties we have imposed.

**Theorem 3.** *For each age structure $\Pi$ and each priority ordering $\sigma$ that respects $\Pi$, the sequential priority rule $\psi^\sigma : \mathcal{D}^\Pi \to \mathcal{A}_2$ satisfies* individual rationality, efficiency, strategy-proofness, *and* non-bossiness.

It is worth to note that for arbitrary age structures, a sequential priority rule does not necessarily maximize the number of transplants at every preference profile. Actually, the number of transplants carried out by a sequential priority rule depends on the specific priority ordering of the patients, as shown in the example below.

**Example 3.** *Let $\Pi = \{\Pi(1), \Pi(2)\}$, such that $\Pi(1) = \{1\}$, $\Pi(2) = \{2, 3, 4\}$. Let the priority orderings $\sigma, \sigma'$ be such that $\sigma = \sigma^*$, and $\sigma'(i) = \sigma(i)$ for each $i \notin \{2, 3\}$, $\sigma'(2) = 3$, $\sigma'(3) = 2$. Consider the associated sequential priority rules $\psi^\sigma$ and $\psi^{\sigma'}$. Let $\succsim \in \mathcal{D}^\Pi$ be such that $D(\succsim_1) = \{\omega_2, \omega_3\}$, $D(\succsim_2) = \{\omega_1\}$, $D(\succsim_3) = \{\omega_1, \omega_4\}$, and $D(\succsim_4) = \{\omega_3\}$. Notice that $\psi^\sigma(\succsim) = (\omega_2, \omega_1, \omega_4, \omega_3)$ while $\psi^{\sigma'}(\succsim) = (\omega_3, \omega_2, \omega_1, \omega_4)$, then at profile $\succsim$ more transplants are performed under $\psi^\sigma$ than under $\psi^{\sigma'}$.*

Before concluding this section, two remarks are in order. Theorems 1 and 3 do not provide a complete characterization of the family of exchange rules that satisfy our axioms. It is easy to construct other rules that satisfy those axioms simply by applying different priority orderings of the patients at each stage of the sequential priority algorithm. Finally, we want to highlight that in the dichotomous domain $\mathcal{D}^{\bar{\Pi}}$ and in the strict age-based domain $\mathcal{D}^{\Pi^*}$, age-based priority rules and sequential priority rules select the same assignments.

## 5. Discussion and extensions

In this section, we discuss some relevant issues regarding the computability of the proposed algorithms and some possible extensions.

### 5.1. Computability of sequential priority rules outcomes

A crucial issue for real life applications of PKE programs is the computational feasibility of finding an optimal solution. Finding an assignment that maximizes the number of mutually compatible exchanges when they are restricted to be pairwise is equivalent to find a maximal weight assignment in a bipartite graph. It is a well established fact that solving such problem is computationally feasible in polynomial time (Edmonds, 1965; Gabow, 1990). However, sequential priority rules do not necessarily maximize the total number of exchanges even when the age structure is binary, as we have seen in Examples 1 and 3.[17] Moreover, while the sequential priority algorithm employed by sequential priority rules is computationally simple in the domain without indifferences induced by the strict age structure, it is hardly computable when age groups contain many elements and therefore patients are indifferent among several compatible pairs of kidneys. Nevertheless, for each sequential priority rule we are able to provide an alternative algorithm that at each preference profile selects the same assignment, and such that its solution is computationally feasible in polynomial time.

***Sequential maximizing with fixed priorities algorithm.*** Fix a priority ordering $\sigma$ and a preference profile $\succsim \in \mathcal{P}^N$,

- Let $\mathcal{F}_{0,m}^\sigma(\succsim) = \mathcal{I}(\succsim)$.
- For each $t, t'$, $1 \le t \le t' \le m$ let:

$$\mathcal{S}_{t,t'}^\sigma(\succsim) \equiv \left\{ a' \in \mathcal{I}(\succsim) \;\middle|\; \begin{array}{l} \exists a \in \mathcal{F}_{\tau,\tau'}^\sigma(\succsim), \forall i \in P_{t,t'}(a) \cup M_{t,t'}(a), a'_i = a_i \\ \forall j \notin P_{t,t'}(a) \cup M_{t,t'}(a), a'_j = \omega_j, \end{array} \right\},$$

where $\{\tau, \tau'\} = \{t-1, m\}$ if $t = t'$, and $\{\tau, \tau'\} = \{t, t'-1\}$ otherwise; and

[17] This fact is true only for the dichotomous domain age structure $\bar{\Pi}$.

$$\mathcal{B}^{\sigma}_{t,t'}(\succsim) \equiv \{a \in \mathcal{S}^{\sigma}_{t,t'}(\succsim) \mid \nexists a' \in \mathcal{S}^{\sigma}_{t,t'}(\succsim), \#M_{t,t'}(a') > \#M_{t,t'}(a)\},$$
$$\mathcal{F}^{\sigma}_{t,t'}(\succsim) \equiv \{a \in \mathcal{I}(\succsim) \mid \exists a' \in \mathcal{M}^{\sigma}_{n}(\succsim, \mathcal{B}^{\sigma}_{t,t'}(\succsim)), \forall i \in P_{t,t'}(a) \cup M_{t,t'}(a), a_i = a'_i\}.$$

The sequential maximizing with fixed priorities algorithm follows a logic similar to the rational behind the sequential priority algorithm. The set $\mathcal{S}^{\sigma}_{1,1}(\succsim)$ contains all individually rational assignments in which patients with donors in $\Pi(1)$ exchange their donors among themselves, and all patients who are not involved in such exchanges keep their donors. From the set $\mathcal{S}^{\sigma}_{1,1}(\succsim)$, the algorithm picks the assignments that maximize the number of exchanges, $\mathcal{B}^{\sigma}_{1,1}(\succsim)$, and breaks ties in that set of assignments using the priority algorithm $\mathcal{M}^{\sigma}_{n}(\succsim, \mathcal{B}^{\sigma}_{t,t'}(\succsim))$. Since all assignments in $\mathcal{M}^{\sigma}_{n}(\succsim, \mathcal{B}^{\sigma}_{t,t'}(\succsim))$ are essentially equivalent, we can interpret that the assignments in $\mathcal{M}^{\sigma}_{n}(\succsim, \mathcal{B}^{\sigma}_{t,t'}(\succsim))$ as a list of exchanges involving only patients with donor in $\Pi(1)$. The set $\mathcal{F}^{\sigma}_{1,1}(\succsim)$ contains all individually rational assignments in which those exchanges are performed. The algorithm proceeds in the following step selecting assignments with further exchanges among patients with donor in $\Pi(1)$ and patients with donor in $\Pi(2)$ who were not involved in the exchanges identified in the previous step, and so on.

In the following lemma we show that when restricted to consider only pairwise exchanges the sequential priority algorithm and the sequential maximizing algorithm with fixed priorities yield the same assignment.

**Lemma 2.** *For each age structure $\Pi$, each priority ordering $\sigma$ that respects $\Pi$, and each $\succsim \in \mathcal{D}^{\Pi}$, $\psi^{\sigma}(\succsim) \in \mathcal{F}^{\sigma}_{m,m}(\succsim)$.*

By Lemma 2 we can interpret the assignment selected by the age-based sequential priority rule $\psi^{\sigma}$ as the outcome of a sequential maximizing process. Theorem 2 and Corollary 4 in Okumura (2014) show that finding the maximal number of possible exchanges when ties are broken according to a priority ordering can be solved in polynomial time by using the efficient algorithm provided by Gabow (1990). Since age-based sequential priority rules repeat the process $\frac{m(m+1)}{2}$ times, we have the following result.

**Corollary 1.** *Let $\Pi = \{\Pi(1), \dots, \Pi(m)\}$ and let $\psi^{\sigma}$ be sequential priority rule. Let $g = \max_{l,l'} \frac{\#\Pi(l) \times (\#\Pi(l')+1)}{2} < \frac{n(n+1)}{2}$ and $\mu = \max_l \#\Pi(l) < n$. For each $\succsim \in \mathcal{D}^{\Pi}$, $\psi^{\sigma}(\succsim)$ can be correctly determined on $O\left(\frac{m(m+1)}{2}\left[\mu g + \mu^2 \log \mu\right]\right)$ time.*

### *5.2. Beyond pairwise exchanges*

In this paper we have presented a domain restriction under which we can construct exchange rules that satisfy *individual rationality*, *efficiency*, and *strategy-proofness*, when chains of crossed donations are restricted to pairwise exchanges. Feasibility restrictions are an inherent feature of PKE programs. Although pairwise exchanges are prevalent in PKE programs, larger exchanges are also usual. In this section, we justify our focus on pairwise exchanges and discuss whether and to which extent such feasibility restrictions can be relaxed.

Roth et al. (2005a) and Hatfield (2005) have shown that, in the dichotomous age-based preference domain, a plethora of rules satisfy our requirements for any arbitrary feasibility constraint. On the contrary, in the age-based domain when the length of exchange cycles forming an assignment are restricted, those possibility results do not longer hold. The arguments in the proof of Theorem 1 in Nicolò and Rodríguez-Álvarez (2012) immediately apply to age-based domains. Thus, whenever there are constraints on the size of feasible cycles and exchanges involving at least three donor–patient pairs are admitted, those properties are not generally compatible in age-based domains.[18]

**Theorem 4.** *Let $\Pi = \{\Pi(1), \dots, \Pi(m)\}$ be an age structure such that either $m \geq 3$ or there are at least two donors in both $\Pi(1)$ and $\Pi(2)$. For each $k \in \mathbb{N}$ with $2 < k < n$, there is no exchange rule $\varphi : \mathcal{D}^{\Pi} \to \mathcal{A}_k$ with $r_{\varphi} = \mathcal{A}_k$ that satisfies* individual rationality, efficiency, strategy-proofness.

To obtain the previous impossibility result it is not necessary to employ the full strength of *efficiency*. In fact, it suffices to admit some multiple ways exchanges involving donor–patient pairs belonging to different age groups to obtain the negative result. However, the arguments in the proof do not use multiple ways exchanges involving patients with donors in the same age group. Hence, it becomes natural to check whether the impossibility result can be overcome if only multiple way exchanges among patients with donors in the same age group are allowed.

We define the set of ***age-based multiple ways assignments*** $\tilde{\mathcal{A}} \subset \mathcal{A}$ as the set of assignments that does not involve multiple ways exchanges among donor–patient pairs in different age groups. That is, an assignment $a \in \tilde{\mathcal{A}}$ if and only if for each $i, j \in N$, $a_i = \omega_j$ implies that either $a_j = \omega_i$ or there is $l \leq m$ such that $\{\omega_i, \omega_j\} \subset \Pi(l)$. A ***multiple ways exchange rule*** $\tilde{\varphi}$ is a rule $\tilde{\varphi} : \mathcal{D}^{\Pi} \to \tilde{\mathcal{A}}$ such that only selects age-based multiple ways assignments and $r_{\tilde{\varphi}} = \tilde{\mathcal{A}}$. For each $\succsim \in \mathcal{P}^N$, let $\tilde{\mathcal{I}}(\succsim) \equiv \{a \in \mathcal{A}_2 \mid \text{for each } i \in N \; a_i \succsim_i \omega_i\}$ denote the *set of individually rational age-based multiple ways assignments*.

[18] With similar arguments to those in Nicolò and Rodríguez-Álvarez (2013), we can reinforce the result replacing *strategy-proofness* with the weaker notion of Ordinal Bayesian Incentive Compatibility.

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*Individual rationality*, *efficiency*, *strategy-proofness* and *non-bossiness* impose conditions on multiple ways exchange that are similar to those that define sequential maximizing rules in the pairwise exchange scenario, but admit some additional flexibility. The structure of such rules requires a sequential process involving exchanges among donor–patient pairs in different age groups.[19] At each stage of the sequential process, the set of patients who receive a kidney is maximal with respect to inclusion. Thus, it is not necessary that the number of patients with a donor in $\Pi(1)$ who receive a kidney in $\Pi(1)$ is maximized, but for the stages where exchanges between different age groups are restricted to be pairwise exchanges the number of exchanges is maximized.

To illustrate the additional flexibility that allows the possibility of age-based multiple ways exchanges, we can immediately define the exchange rules defined with the Sequential Priority Algorithm and the Sequential Maximizing Algorithm with Fixed Priorities when the initial set is $\tilde{\mathcal{I}}(\succsim)$. It turns out that the algorithms define different exchange rules. For each structure $\Pi$, we say a rule $\tilde{\varphi}:\mathcal{D}^{\Pi}\to\tilde{\mathcal{A}}$ is a ***multiple ways sequential priority rule*** if there is a priority ordering $\sigma$ that respects $\Pi$ such that for each $\succsim\in\mathcal{D}$, $\tilde{\varphi}(\succsim)\in\mathcal{F}^{\sigma}_{m}(\succsim)$ when $\mathcal{F}^{\sigma}_{0}(\succsim)=\tilde{\mathcal{I}}(\succsim)$. We denote by $\tilde{\psi}^{\sigma}$, the ***multiple ways sequential priority rule with priority ordering*** $\sigma$. Alternatively, a rule $\tilde{\varphi}:\mathcal{D}^{\Pi}\to\tilde{\mathcal{A}}$ is a ***multiple ways sequential maximizing rule with fixed priority*** $\sigma$ if there is a priority ordering $\sigma$ that respects $\Pi$ such that for each $\succsim\in\mathcal{D}$, $\tilde{\varphi}(\succsim)\in\mathcal{F}^{\sigma}_{m,m}(\succsim)$ when $\mathcal{F}^{\sigma}_{0,m}(\succsim)=\tilde{\mathcal{I}}(\succsim)$.

Multiple ways sequential priority rules do not necessary maximize the number of compatible exchanges among patients with donors in $\Pi(1)$ if there are at least 4 donor–patient pairs in the same age group. Moreover, since the arguments in the proof of Theorem 3 do not depend on the fact that only pairwise exchanges are admitted, it follows immediately that every multiple ways sequential priority rule satisfies *individual rationality*, *efficiency*, *strategy-proofness*, and *non-bossiness*. Similar arguments apply to multiple ways sequential maximizing with fixed priorities rules. Hence, two different generalizations of the priority rules proposed by Roth et al. (2005a) would work in the multiple ways scenario under age-based preferences.

**Theorem 5.** *Let $\sigma$ be a priority ordering $\sigma$ that respects $\Pi$. Let $\tilde{\psi}^{\sigma}$ and $\tilde{\varphi}$ be respectively the* multiple ways sequential priority rule *and the* multiple ways sequential maximizing rule with fixed priority *$\sigma$, then*

(i) *$\tilde{\psi}^{\sigma}$ and $\tilde{\varphi}$ satisfy* individual rationality, efficiency, strategy-proofness, *and* non-bossiness,
(ii) *if there is $l\le m$ such that $\#\Pi(l)\ge 4$, then there are $\succsim\in\mathcal{D}^{\Pi}$ such that $\tilde{\varphi}(\succsim)\neq\tilde{\psi}^{\sigma}(\succsim)$.*

Theorem 5 shows that, in the age-based domain, to maximize of the number of transplants and to prioritize specific patients are feasible objectives if multiple ways exchanges are restricted to involve donors with kidneys in the same age group, but no longer equivalent.

On practical grounds, we should mention that the additional flexibility that we have just described for multiple ways exchanges may be difficult to implement. The problem of finding assignments that maximize the number of exchanges is not solvable in polynomial time, but there exist algorithms that find an exact solution to the problem (Abraham et al., 2007). With the arguments in the previous section, we can obtain the assignments proposed by multiple ways sequential maximizing rules with fixed priorities. Unfortunately, exact algorithms for multiple ways sequential priority rules are not available.

### 5.3. Multiple donors

Patients in the waiting list often find more than one potential donor. The fact that a patient may have many potential donors can greatly increase the chances to find mutually compatible pairs. Hence, it could seem apparent that patients have incentives to present all their potential donors. The analysis of the general scenario with multiple donors is not however, an immediate extension of the single donor case we have dealt in previous sections. In fact, there is no one-to-one relation between donors and patients, and patients may have potential donors with kidneys in different age groups. Therefore, it should be checked that patients cannot improve by withdrawing some potential donors from the pool of available donors. We next address these issues.

Since patients may present multiple donors, we need to slightly change notation and definition of an assignment and a pairwise assignment. Let $N=\{i,\dots,n\}$ and $\Omega=\{\omega,\omega',\dots\}$ be a finite set of available kidneys from living donors, $n\#\Omega=q$. For each patient $i$ let $\Omega_i$ denote the set of kidneys from patient $i$'s donors, with $\bigcup_{i\in N}\Omega_i=\Omega$. We assume that for each patient $j\neq i$, $\Omega_i\cap\Omega_j=\varnothing$ and for each $i\in N$, $\#\Omega_i\ge 1$. We denote by $(\Omega_i)_{i\in N}$ the partition of $\Omega$ induced by the donor–patient relation and we call it ***donor structure***. Given an age structure $\Pi=\{\Pi(1),\dots,\Pi(m)\}$, let $\Omega^{l}_{i}=\Omega_i\cap\Pi(l)$ be the set of patient $i$'s donors who belong to the age group $\Pi(l)$.

An assignment $a$ with multiple donors is a $n$-vector of kidneys $(a_1,\dots,a_n)$ such that if $a_i\in\Omega_j$, then for each $k\in N\setminus\{i\}$, $a_k\notin\Omega_j$. Therefore, for each patient $i\in N$ at most one of her potential donors donates her organ. A pairwise assignment with multiple donors is an assignment such that if $a_i\in\Omega_j$, then $a_j\in\Omega_i$. We denote by $\mathcal{A}^{M}$ the set

[19] The additional details and the exact statement of the necessary conditions are available from the authors under request.

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of assignments with multiple donors, and by $\mathcal{A}_2^M$ the set of pairwise assignments with multiple donors. A ***multiple donors pairwise exchange rule*** is a mapping $\Phi : \mathcal{D}^\Pi \to \mathcal{A}_2^M$. Throughout this section, we assume that for each $i \in N$ whenever $\Phi_i(\succsim) \in \Omega_i$, then there is no $\omega' \in \Omega_i$ such that $\omega' \succ_i \Phi_i(\succsim)$. For each patient $i \in N$, and each preference $\succsim_i \in \mathcal{D}_i^\Pi$, the set of desirable kidneys for patient $i$ is now defined as $D(\succsim_i) = \{\omega \in \Omega \mid \text{for each}\, \omega' \in \Omega_i, \omega \succ_i \omega'\}$. Finally, let $\mathcal{I}^M(\succsim) =\equiv \left\{a \in \mathcal{A}_2^M \mid \text{for each } i \in N, \text{ for each } \omega \in \Omega_i;\ a_i \succsim_i \omega \text{ for each } \omega \in \Omega_i\right\}$ denote the set of *individually rational assignments with multiple donors*.

The axioms we presented in Section 2 admit a straightforward extension to the case with multiple donors. In order to analyze patients' incentives to withdraw some potential donors, we introduce a new axiom for multiple donors pairwise exchange rules.

**Donor monotonicity.** For each $i \in N$, each $\omega \in \Omega_i$, and each $\succsim, \succsim' \in \mathcal{D}^\Pi$ such that $\succsim_i = \succsim'_i$, for each $j$ with $\omega \notin D(\succsim_j)$, $\succsim_j = \succsim'_j$, and for each $j'$ such that $\omega \in D(\succsim'_j)$, $D(\succsim'_{j'}) = D(\succsim_{j'}) \setminus \{\omega\}$, $\Phi_i(\succsim) \succsim_i \Phi_i(\succsim')$.

*Donor monotonicity* incorporates the idea that when a donor becomes useless because no patient considers her kidney desirable, then the patient initially attached to her cannot receive a better kidney. Note that for PKE programs in which the selected assignment uniquely depends on patients' preferences over individually rational assignments, and it is independent of the pool of donors, *donor monotonicity* implies that a patient cannot improve by withdrawing a donor from her initial pool of available donors.[20]

Since the multiple donors framework is a generalization of the single donor case, we could expect that our axioms would have similar implications. There is however, a crucial issue that is not present in the single donor scenario. The logic behind Lemma 1 still applies if patients may have multiple donors. Hence, two patients with mutually compatible donors should not receive less preferred kidneys than those they could exchange. In the single donor scenario, this observation implies that in order to satisfy our axioms, kidneys in the youngest age groups are offered first. In the multiple donors scenario this fact may be problematic since a patient may prefer the outcome she obtains when the exchange involves one of her elder donors to the kidney she receives in exchanges with one of her youngest donors. It turns out that in rather general situations our axioms are incompatible.

**Theorem 6.** *There are donor structures $(\Omega)_{i \in N}$ and age structures $\Pi = \{\Pi(1), \ldots, \Pi(m)\}$ with $m \geq 3$ such that no pairwise exchange rule with multiple donors satisfies* individual rationality, efficiency, *and* strategy-proofness.

We have to point out that there are two relevant situations where the negative result of the previous theorem does not apply. An immediate possibility arises when for each patient all her potential donors belong to the same age group. In that case, sequential priority rules can be immediately extended, since all patients consider all (compatible) donors of each patient as equally desirable. Alternatively, the negative result requires the existence of at least three age groups in the age structure, which opens the possibility for age structures consisting of only two age groups. It turns out that in the simple and practically relevant case with binary age structures, sequential priority rules can be extended to the multiple donors framework and still satisfy our axioms. The extension is not trivial and requires additional notation and definitions, since we have to take into account the possibility of patients with several donors in different age groups. Let $\pi : \Omega \to \mathbb{N}$ denote a priority ordering over the set of donors. Given an age structure $\Pi$ and a priority ordering over donors $\pi$, we say that $\pi$ respects $\Pi$ if for every pair $\omega, \omega' \in \Omega$ such that $\omega \in \Pi(l)$, $\omega' \in \Pi(l')$ with $l < l'$, $\pi_\omega < \pi_{\omega'}$.

Next, we need provide a generalized version of the priority algorithm.

***Generalized priority algorithm with multiple donors.*** Fix a priority ordering $\pi$ over $\Omega$, a preference profile $\succsim \in \mathcal{P}^N$, and a subset $A \subseteq \mathcal{A}^M$. Let $\rho : \mathbb{N} \to N$ be the mapping such that for each $k$, $\rho(k) = i$ if and only if $\pi^{-1}(k) \in \Omega_i$.

- Let $\mathcal{GM}_0^\sigma(\succsim, A) = A$.
- For each $1 \leq k \leq q$, let $i = \rho(k)$ and

$$\begin{aligned}
\mathcal{E}_k^\pi(\succsim, A) &\equiv \left\{a \in \mathcal{GM}_{k-1}^\pi(\succsim, A) \,\middle|\, \begin{array}{ll} \text{either} & a_i \in \Omega_i, \\ \text{or} & \text{for some } j \neq i, a_j = \pi^{-1}(t) \end{array}\right\}, \\
\mathcal{GM}_k^\pi(\succsim, A) &\equiv \left\{a \in \mathcal{GM}_{k-1}^\pi(\succsim, A) \mid \forall b \in \mathcal{E}_k^\pi(\succsim, A), a_i \succsim_i b_i\right\}.
\end{aligned}$$

We can interpret the generalized priority algorithm as a sequential veto process. Kidneys are offered sequentially according to a priority ordering $\pi$. At each stage $k$, kidney $\pi(k)$ is offered, and patient $\rho(k)$ can veto all the assignments that she can improve upon with an exchange involving $\pi(k)$.

Since all patients have a finite number of potential donors, for every $\succsim \in \mathcal{P}$ and $A \in \mathcal{A}^M$, $\mathcal{GM}_q^\pi(\succsim, A)$ is essentially single-valued. With the generalized version of the priority algorithm at hand, we can define the general version of the sequential priority algorithm with multiple donors when there are only two age groups.

[20] Roth et al. (2005a) analyze rules that only depend on the set of individually rational assignments and consider that a rule satisfies *donor monotonicity* if patients do not improve by misrepresenting her preferences over available kidneys or by concealing some potential donors.

***Sequential priority algorithm with multiple donors.*** Fix a priority ordering $\pi$ over $\Omega$ that respects $\Pi$ and a preference profile $\succsim \in \mathcal{P}^N$:

- Let $\mathcal{GF}_0^{\sigma}(\succsim) = \mathcal{I}^M(\succsim)$,
- For each $t = 1, 2$, let

$$\begin{aligned} \mathcal{GS}_t^{\pi}(\succsim) &\equiv \left\{ a' \in \mathcal{I}^*(\succsim) \,\middle|\, \begin{array}{l} \exists a \in \mathcal{GF}_{t-1}^{\pi}(\succsim), \forall i \in P_{t,2} \cup M_{t,2}(a), a'_i = a_i, \\ \forall j \notin P_{t,2} \cup M_{t,2}(a), a'_j \in \omega_j \end{array} \right\}, \\ \mathcal{GF}_t^{\pi}(\succsim) &\equiv \left\{ a \in \mathcal{I}^*(\succsim) \,\middle|\, \begin{array}{l} \exists a' \in \mathcal{GM}_q^{\pi}(\succsim, \mathcal{GS}_t^{\pi}(\succsim)), \\ \forall i \in P_{t,2} \cup M_{t,2}(a), a_i = a'_i \end{array} \right\}. \end{aligned}$$

Note that for each $a, a' \in \mathcal{GM}_q^{\pi}(\succsim, \mathcal{GS}_1^{\pi}(\succsim))$ for each $t \leq 2$, $M_{1,t}(a) = M_{1,t}(a')$.

We say that $\Psi^{\pi} : \mathcal{D}^{\Pi} \to \mathcal{A}_2^M$ is a ***multiple donors sequential priority rule*** if for each $\succsim \in \mathcal{D}^{\Pi}$, $\Psi^{\pi}(\succsim) \in \mathcal{GF}_m^{\pi}(\succsim)$ and $\pi$ respects $\Pi$.

It is not difficult to prove that multiple donors sequential priority rules satisfy *individual rationality*, *efficiency*, *strategy-proofness*, and *non-bossiness*. The logic behind the negative result in Theorem 6 does not apply to age structures with only two age groups. With only two age groups, if a patient receives a kidney in the first stage in return for one of her young donors, she cannot get a more desirable kidney in the second stage in exchange for one of her mature donors. In fact, it is also immediate to prove that multiple donors sequential priority rules also satisfy *donor monotonicity*.

**Theorem 7.** *Let $\Pi = \{\Pi(1), \Pi(2)\}$. A multiple donors sequential priority rule $\Psi^{\pi}$ satisfies* individual rationality, efficiency, strategy-proofness, non-bossiness, *and* donor monotonicity.

As a consequence of Theorem 7, multiple donors sequential priority rules provide incentives for the patients to reveal the right information about their compatibility and their full set of available donors.

## 6. Final remarks

To conclude, we have analyzed a framework that incorporates a novel feature of PKE, the existence of an observable characteristic, specifically the age of the donors, that affects the expected living standards of the recipients and consequently patients' preferences over compatible organs. Our model provides new insights for the design of PKE programs. The main justification for our framework is the design of PKE protocols that encourage the participation of compatible donor–patient pairs. In the standard model, the motivation of compatible donor–patient pairs to participate in PKE programs is entirely altruistic (Roth et al., 2005b; Sönmez and Ünver, 2014). In our model, compatible donor–patient pairs with relatively old donors may be willing to enroll PKE programs that adopt sequential priority rules because their patients can receive organs with longer expected graft survival. The relevant case we have in mind is a compatible pair formed, for instance, by an A blood-type patient and a mature O blood-type donor. This pair could be willing to enroll in a PKE program if the patient could be matched with an incompatible pair formed by a O blood-type patient and a young A blood-type donor.[21]

Recent studies (Gentry et al., 2007; Sönmez and Ünver, 2014) suggest that the participation of compatible pairs may represent the most important factor in expanding the number of kidney paired exchanges. Using simulated data, Gentry et al. (2007) prove that there may be large benefits for both incompatible pairs and compatible pairs if compatible pairs are willing to participate in PKE programs. The participation of compatible pairs could dramatically reduce blood group imbalances in the pool of compatible pairs. As a result, the match rate for incompatible pairs would double from 28.2 to 64.5 percent for a single-center program and from 37.4 to 75.4 percent for a national program.

## Appendix A. Proofs

**Proof of Lemma 1.** The proof is directly implied by *individual rationality* and *efficiency* for the age structure $\bar{\Pi}$. Hence, we consider an arbitrary age structure $\Pi = \{\Pi(1), \ldots, \Pi(m)\}$ with $m \geq 2$. Assume, to the contrary, that there are $i, j \in N$ and $\succsim \in \mathcal{D}^{\Pi}$ such that $\omega_i \in D(\succsim_j)$ and $\omega_j \in D(\succsim_i)$ but $\omega_i \succ_j \varphi_j(\succsim)$ and $\omega_j \succ_i \varphi_i(\succsim)$. Let $\succsim^1 \in \mathcal{D}^{\Pi}$ be such that $D(\succsim_i^1) = \{\omega_j, \varphi_i(\succsim)\}$ and $\succsim_{-j}^1 = \succsim_{-j}$. By *individual rationality* and *strategy-proofness*, $\varphi_i(\succsim^1) = \varphi_i(\succsim)$. By *non-bossiness*, $\varphi(\succsim^1) = \varphi(\succsim)$. Let $\succsim^2 \in \mathcal{D}^{\Pi}$ be such that $D(\succsim_j^2) = \{\omega_i\}$ and for each $h \neq j$ $\succsim_h^2 = \succsim_h^1$. By *individual rationality* and *strategy-proofness*, $\varphi_j(\succsim^2) = \omega_j$. Let $\succsim^3 \in \mathcal{D}^{\Pi}$ be such that $D(\succsim_i^3) = \{\omega_j\}$ and $\succsim_{-i}^3 = \succsim_{-i}^2$. By *individual rationality* and *strategy-proofness*, $\varphi_i(\succsim^3) = \omega_i$ and by *individual rationality* also $\varphi_j(\succsim^3) = \omega_j$. Let $b \in \mathcal{A}_2$ be such that $b_i = \omega_j$, $b_j = \omega_i$, and for each $h \notin \{i, j\}$, $b_h = \varphi_h(\succsim^3)$. Note that $b_i \succ_i^3 \varphi_i(\succsim^3)$, $b_j \succ_j^3 \varphi_j(\succsim^3)$, and for each $h \notin \{i, j\}$, $b_h \sim_h^3 \varphi_h(\succsim^3)$, which contradicts *efficiency*. $\square$

[21] Most of O blood-type donors can directly donate to their intended recipients. Hence, O blood-type patients in the incompatible donor–patient pool must rely on a scant number of O blood-type donors and can rarely find a match.

**Proof of Theorem 1.** The result immediately follows from *individual rationality* and *efficiency* for the age structure $\bar{\Pi}$ (Roth et al., 2005a). Thus, we consider an arbitrary age structure $\Pi = \{\Pi(1), \ldots, \Pi(m)\}$ with $m \geq 2$. Let the pairwise exchange rule $\varphi$ that satisfies *individual rationality*, *efficiency*, *strategy-proofness*, and *non-bossiness*. By *individual rationality*, for each $\succsim \in \mathcal{D}^{\Pi}$, $\varphi(\succsim) \in \mathcal{I}(\succsim)$. We prove (ii) of the definition of sequential maximizing rules by a series of steps.

*Step 1: $t = 1$ and $t' = 1$.* Assume, to the contrary, that there are $\succsim \in \mathcal{D}^{\Pi}$ and $a \in \mathcal{I}(\succsim)$ such that $\#M_{1,1}(a) > \#M_{1,1}(\varphi(\succsim))$. For each $a' \in \mathcal{A}_2$, $P_{1,1}(a') = \varnothing$. There exist a set $T \subset N$ and $h, h' \in N \setminus T$ such that:

(i) For each $i \in T \cup \{h, h'\}$, $\omega_i \in \Pi(1)$, $\varphi_h(\succsim) \notin \Pi(1)$, $\varphi_{h'}(\succsim) \notin \Pi(1)$.
(ii) For each $i \in T \cup \{h, h'\}$, there exists $i' \in \big(T \cup \{h, h'\}\big) \setminus \{i\}$ such that $a_i = \omega_{i'}$.
(iii) For each $j \in T$, there exists $j' \in T \setminus \{j\}$ such that $\varphi_j(\succsim) = \omega_{j'}$.

There are two cases.

**Case i).** $T = \varnothing$. Clearly, $a_h = \omega_{h'}$ and $a_{h'} = \omega_h$. Since $a \in \mathcal{I}(\succsim)$, then $\omega_h \in D(\succsim_{h'})$ and $\omega_{h'} \in D(\succsim_h)$. Because $\varphi_h(\succsim) \notin \Pi(1)$ and $\varphi_{h'}(\succsim) \notin \Pi(1)$, this contradicts Lemma 1.

**Case ii).** $T \neq \varnothing$. Let $\succsim' \in \mathcal{D}^{\Pi}$ be such that for each $i \in T \cup \{h, h'\}$, $\succsim'_i = \succsim_i$, and for each $j \notin T \cup \{h, h'\}$, $D(\succsim'_j) = \{\varphi_j(\succsim)\}$. Let $j \notin T \cup \{h, h'\}$. By *individual rationality*, $\varphi_j(\succsim'_j, \succsim_{-j}) \in \{\omega_j, \varphi_j(\succsim)\}$. By *strategy-proofness*, $\varphi_j(\succsim'_j, \succsim_{-j}) \succsim'_j \varphi_i(\succsim)$. Then, $\varphi_j(\succsim'_j, \succsim_{-j}) = \varphi_j(\succsim)$ and by *non-bossiness*, $\varphi(\succsim'_j, \succsim_{-j}) = \varphi(\succsim)$. Repeating the same argument switching the preference of each patient one at a time, we obtain $\varphi(\succsim') = \varphi(\succsim)$. Let $\succsim'' \in \mathcal{D}^{\Pi}$ be such that for each $j \notin \{h, h'\}$, $\succsim''_j = \succsim_j$, $D(\succsim''_h) = \{a_h\}$, and $D(\succsim''_{h'}) = \{a_{h'}\}$. By *individual rationality*, $\varphi_h(\succsim''_h, \succsim'_{-h}) \in \{\omega_h, a_h\}$. By *strategy-proofness*, $\varphi_h(\succsim') \succsim'_h \varphi_h(\succsim''_h, \succsim'_{-h})$. Hence, $\varphi_h(\succsim''_h, \succsim'_{-h}) = \omega_h$. By *individual rationality*, for each $j \notin T \cup \{h, h'\}$, $\varphi_j(\succsim''_h, \succsim'_{-h}) \in \{\omega_j, \varphi_j(\succsim)\}$. By *efficiency*, for each $j \notin T \cup \{h, h'\}$ such that $\varphi_j(\succsim) \neq \omega_h$, we have $\varphi_j(\succsim''_h, \succsim'_{-h}) = \varphi_j(\succsim)$. For each $j' \in T \cup \{h'\}$, either there is $i' \in T \cup \{h'\}$ such that $\varphi_{j'}(\succsim''_h \succsim'_{-h}) = \omega_{i'}$, or $\varphi_{j'}(\succsim''_h, \succsim'_{-h}) = \omega_{j'}$. Repeating the argument with patient $h'$, we obtain that $\varphi_{h'}(\succsim'') = \omega_{h'}$, for each $j \notin T \cup \{h, h'\}$ such that $\varphi_j(\succsim) \notin \{\omega_h, \omega_{h'}\}$, $\varphi_j(\succsim'') = \varphi_j(\succsim)$, and for each $j' \in T \cup \{h\}$, either there is $i' \in T \cup \{h\}$ such that $\varphi_{j'}(\succsim'') = \omega_{i'}$, or $\varphi_{j'}(\succsim'') = \omega_{j'}$. Moreover, since $\varphi(\succsim'') \in \mathcal{A}_2$ and $\varphi_{h'}(\succsim'') = \omega_{h'}$, there is $h'' \in T \cup \{h'\}$ such that $\varphi_{h''}(\succsim'') = \omega_{h''}$. Let $b \in \mathcal{A}_2$ be such that for each $i \in T \cup \{h, h'\}$, $b_i = a_i$ and for each $j \notin T \cup \{h, h'\}$, $b_j = \varphi_j(\succsim'')$. Note that for each $i \in T \cup \{h, h'\}$, $b_i \in \Pi(1)$. Then, for each $i' \in N \setminus \{h, h''\}$, $b_{i'} \succsim'_{i'} \varphi_{i'}(\succsim'')$, and $b_h \succ''_h \varphi_h(\succsim'')$, which contradicts *efficiency*.

*Step 2: $t = 1$, $t' = 2$.* Assume, to the contrary, that there are $\succsim \in \mathcal{D}^{\Pi}$ and $a \in \mathcal{I}(\succsim)$ such that for each $i \in P_{1,2}\big(\varphi(\succsim)\big)$, $\varphi_i(\succsim) = a_i$ and $\#M_{1,2}(a) > \#M_{1,2}\big(\varphi(\succsim)\big)$. Then, there are a set $T \subset N \setminus P_{1,2}(\varphi(\succsim))$ and a pair of patients $h, h' \in N \setminus (T \cup P_{1,2}(\varphi(\succsim))$ such that:

(i) For each $i \in T$, $\omega_i \in (\Pi(1) \cup \Pi(2))$, $\omega_h \in \Pi(1)$, $\omega_{h'} \in \Pi(2)$.
(ii) For each $i \in T$ with $\omega_i \in \Pi(1)$ there exists $i' \in T$ with $\omega_{i'} \in \Pi(2)$ such that $\varphi_i(\succsim) = \omega_{i'}$.
(iii) For each $j \in T \cup \{h, h'\}$ with $\omega_j \in \Pi(1)$, there is $j' \in T \cup \{h, h'\}$ with $\omega_{h'} \in \Pi(2)$ such that $a_j = \omega_{j'}$.

Notice that $h, h' \notin P_{1,2}(\varphi(\succsim))$. There are two cases.

**Case i).** $T = \varnothing$, with the arguments in the proof for $t = 1$ and $t' = 1$, we obtain a contradiction with Lemma 1.

**Case ii).** $T \neq \varnothing$. Let $\succsim' \in \mathcal{D}^{\Pi}$ be such that for each $i \in T \cup \{h, h'\}$, $\succsim'_i = \succsim_i$ and for each $j \notin T \cup \{h, h'\}$, $D(\succsim_j) = \{\varphi_j(\succsim)\}$. By *individual rationality*, $\varphi_j(\succsim'_j, \succsim_{-j}) \in \{\omega_j, \varphi_j(\succsim)\}$. By *strategy-proofness*, $\varphi_j(\succsim'_j, \succsim_{-j}) \succsim'_j \varphi_i(\succsim)$. Then, $\varphi_j(\succsim'_j, \succsim_{-j}) = \varphi_j(\succsim)$ and by *non-bossiness*, $\varphi(\succsim'_j, \succsim_{-j}) = \varphi(\succsim)$. Repeating the same argument exchanging the preference of each patient one at a time, we obtain $\varphi(\succsim') = \varphi(\succsim)$. Let $\succsim''_{h'} \in \mathcal{D}^{\Pi}_{h'}$ be such that $D(\succsim''_{h'}) = \{a_{h'}\}$. By *individual rationality*, $\varphi_{h'}(\succsim''_{h'}, \succsim'_{-h'}) \in \{\omega_{h'}, a_{h'}\}$. By *strategy-proofness*, $\varphi_{h'}(\succsim') \succsim'_{h'} \varphi_{h'}(\succsim''_{h'}, \succsim'_{-h'})$. Therefore, $\varphi_{h'}(\succsim''_{h'}, \succsim'_{-h'}) = \omega_{h'}$. By *individual rationality*, for each $j \notin T \cup \{h, h'\}$, $\varphi_j(\succsim''_h, \succsim'_{-h}) \in \{\omega_j, \varphi_j(\succsim)\}$. By *efficiency*, for each $j \notin T \cup \{h, h'\}$, $\varphi_j(\succsim''_{h'}, \succsim'_{-h'}) = \varphi_j(\succsim)$. Notice that for each $j' \in T \cup \{h\}$, either there is $i' \in T \cup \{h\}$ such that $\varphi_{j'}(\succsim''_{h'} \succsim'_{-h'}) = \omega_{i'}$, or $\varphi_{j'}(\succsim''_{h'}, \succsim'_{-h'}) = \omega_{j'}$. Since $\varphi(\succsim''_{h'}, \succsim'_{-h'}) \in \mathcal{A}_2$ and $\varphi_{h'}(\succsim''_{h'}, \succsim'_{-h'}) = \omega_{h'}$, there is $h'' \in T \cup \{h\}$ such that $\varphi_{h''}(\succsim''_h, \succsim'_{-h}) = \omega_{h''}$. Since for each $a \in \mathcal{I}(\succsim)$, $\#M_{1,1}(\varphi(\succsim)) \geq \#M_{1,1}(a)$, we have that for each $j' \in T \cup \{h\}$ with $\omega_{j'} \in \Pi(1)$, $\varphi_{j'}(\succsim''_{h'}, \succsim'_{-h'}) \in \Pi(2) \cup \{\omega_{j'}\}$. Let $b \in \mathcal{A}_2$ be such that for each $i \in T \cup \{h, h'\}$, $b_i = a_i$ and for each $j \notin T \cup \{h, h'\}$, $b_j = \varphi_j(\succsim''_{h'}, \succsim'_{-h'}) = \varphi_j(\succsim)$. Note that for each $i \notin T \cup \{h, h'\}$ with $\omega_i \in \Pi(1)$, $b_i \in \Pi(2)$ and for each $i' \in T \cup \{h, h'\}$ with $\omega_{i'} \in \Pi(2)$, $b_{i'} \in \Pi(1)$. Then, for each $j \in N \setminus \{h', h''\}$, $b_j \succsim'_j \varphi_j(\succsim''_{h'}, \succsim'_{-h'})$, $b_{h'} \succ''_{h'} \varphi_{h'}(\succsim''_{h'}, \succsim'_{-h'})$, and $b_{h''} \succ''_{h''} \varphi_{h'}(\succsim''_{h'}, \succsim'_{-h'})$, which contradicts *efficiency*.



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In order to conclude the proof, we can apply iteratively the arguments in Step 1, to prove the result for $t = 1$ and $t' = 3, \ldots, m$. Then, given that the result is true for $t = 1$ and $t' = m$, the arguments in Step 2 directly apply to prove the result for $t = 2$ and $t' = 2$, and we can proceed iteratively with all the remaining steps till we reach $t = m$ and $t' = m$. □

**Proof of Theorem 2.** Let $\varphi^*$ denote the *age-based priority rule*. By construction, $\varphi^*$ satisfies *individual rationality* and *efficiency*. Consequently, we only show that $\varphi^*$ satisfies *strategy-proofness*. Let $i \in N$, and $\succsim, \succsim' \in \mathcal{D}^{\Pi^*}$ be such that $\succsim'_{-i} = \succsim_{-i}$. Assume first that, $\varphi^*_i(\succsim) = \varphi^*_i(\succsim')$, then $\varphi^*_i(\succsim) \succsim_i \varphi^*_i(\succsim')$. Assume now that $\varphi^*_i(\succsim) \neq \varphi^*_i(\succsim')$. Let $j, j' \in N$ be such that $\varphi^*_i(\succsim) = \omega_j$ and $\varphi^*_i(\succsim') = \omega_{j'}$. There are two cases:

**Case i).** $i = j$. $\varphi^*_i(\succsim) = \omega_i$ implies that for each $a \in \mathcal{M}^{\sigma^*}_{i-1}(\succsim, \mathcal{I}(\succsim))$, $a_i = \omega_i$. Since only patient $i$ has changed her preferences, $\varphi^*_i(\succsim') \in \mathcal{M}^{\sigma^*}_{i-1}((\succsim'), \mathcal{I}(\succsim'))$, and $\varphi^*_i(\succsim') = \omega_{j'}$, imply $\omega_{j'} \in D(\succsim'_i) \setminus D(\succsim_i)$. Hence, $\varphi^*_i(\succsim) \succsim_i \varphi^*_i(\succsim')$.

**Case ii).** $i \neq j$. Since $\varphi^*(\succsim) \in \mathcal{I}(\succsim)$, $\omega_j \in D(\succsim_i)$ and therefore $\omega_j \succ_i \omega_i$. Since $\varphi^*_i(\succsim) \neq \varphi^*_i(\succsim')$, then either (by the arguments in the previous case) $j' < j$ with $\omega_{j'} \in D(\succsim'_i) \setminus D(\succsim_i)$, or $j < j'$, or $i = j'$. In either case $\varphi^*_i(\succsim) \succsim_i \varphi^*_i(\succsim')$.

Since both cases are exhaustive, $\varphi^*$ satisfies *strategy-proofness*.

Let $\varphi$ be a rule that satisfies *individual rationality*, *efficiency*, and *strategy-proofness* in $\mathcal{D}^{\Pi^*}$. Note that for each $i \in N$ and $\succsim_i \in \mathcal{D}^{\Pi^*}$, by the definition of $\Pi^*$-age-based preferences, for each $a \in \mathcal{A}_2$, $a_i \sim_i \omega_i$ if and only if $a_i = \omega_i$. Moreover, for each $a \in \mathcal{A}_2$ such that $a_i \succsim_i \omega_i$, there is $\succsim'_i \in \mathcal{D}^{\Pi^*}$ such that $D(\succsim'_i) = \{b_i \in D(\succsim_i) \mid b_i \succsim_i a_i\}$. Hence, the domain $\mathcal{D}^{\Pi^*}$ satisfies Assumptions A and B on the domain of preferences proposed by Sönmez (1999). By Theorem 1 in Sönmez (1999), if there is a rule $\varphi$ that satisfies *individual rationality*, *efficiency*, and *strategy-proofness* in $\mathcal{D}^{\Pi^*}$, then the strict core is essentially unique and $\varphi$ always selects assignments in the strict core. By the arguments in the previous paragraph, $\varphi^*$ satisfies *individual rationality*, *efficiency*, and *strategy-proofness* in $\mathcal{D}^{\Pi^*}$. Thus, $\varphi = \varphi^*$. □

**Proof of Theorem 3.** By construction, $\psi^\sigma$ satisfies *individual rationality*. Next we check *efficiency*. Let $\succsim \in \mathcal{D}^{\Pi}$ and $a = \psi^\sigma(\succsim)$. Note first that $a \in \mathcal{I}(\succsim)$. Let $i \in N$ be such that $a_i \in \Pi(1)$. If $a_i \neq \omega_i$, then for each $t = 1, \ldots, m$ and each $a' \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}\left(\succsim, \mathcal{S}^{\sigma}_t(\succsim)\right)$, $a'_i = \omega_i$. Hence, for each $b \in \mathcal{A}_2 \cap \mathcal{I}(\succsim)$, such that $b_i \succ_i a_i$, then there is some $j$ with $\sigma(j) < \sigma(i)$, $a_j \succ_j b_j$. We can replicate the arguments for the remaining age groups. Let $i \in \cup_{t'=t}^{m} M_{t,t'}(a) \cup \{j \in N \mid \omega_j \in \Pi(t), a_j = \omega_j\}$. For each $b \in \mathcal{I}(\succsim)$ such that for some $l < t$, $b_i \in \Pi(l)$ there is $j'$ with $\sigma(j') < \sigma(j)$ with $a_{j'} \succ_{j'} b_{j'}$. If $a_i \neq \omega_i$ and $a_i \in \Pi(t)$, then there is no $b \in \mathcal{A} \setminus \cup_{l=1}^{t-1}\Pi(l)$ such that $b_i \succ_i a_i$. If $a_i = \omega_i$, for each $t = 1, \ldots, m$ and each $a' \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}\left(\succsim, \mathcal{S}^{\sigma}_t(\succsim)\right)$, $a'_i = \omega_i$. Hence, for each $b \in \mathcal{A}_2 \cap \mathcal{I}(\succsim)$, such that $b_i \succ_i a_i$, there is some $j$ with $\sigma(j) < \sigma(i)$, $a_j \succ_j b_j$. Therefore, there is no $b \in \mathcal{A}_2$ such that for each $i \in N$ $b_i \succsim_i a_i$ and for some $j$ $b_j \succ_j a_j$, and $\psi^\sigma$ satisfies *efficiency*.

In order to check *strategy-proofness*, let $i \in N$, and $\succsim, \succsim' \in \mathcal{D}^{\Pi}$ be such that $\succsim'_{-i} = \succsim_{-i}$. Assume first that, $\psi^\sigma_i(\succsim) = \psi^\sigma_i(\succsim')$, then $\psi^\sigma_i(\succsim) \succsim_i \psi^\sigma_i(\succsim')$. Assume now that $\psi^\sigma_i(\succsim) \neq \psi^\sigma_i(\succsim')$. Let $j, j' \in N$ be such that $\psi^\sigma_i(\succsim) = \omega_j$ and $\psi^\sigma_i(\succsim') = \omega_{j'}$. There are three cases:

**Case i).** $i = j$ and $i \neq j'$. $\psi^\sigma(\succsim) = \omega_i$ implies that for each $t \leq m$ there is no $a \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim, \mathcal{S}^{\sigma}_t(\succsim))$ with $a_i \neq \omega_i$. That is, at every stage $t$ of the sequential process, and each $\omega \in D(\succsim_i) \cap \Pi(t)$, $\omega$ is assigned to another patient $j$ with higher priority ($\sigma(j) < \sigma(i)$). Let $t \leq m$ be the smallest integer such that there is $a \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_t(\succsim'))$ with $a_i \neq \omega_i$. Notice that $\psi^\sigma(\succsim') \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_t(\succsim'))$ and $\psi^\sigma_i(\succsim') \neq \omega_i$. Since only patient $i$ has changed her preferences, for each $a \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_t(\succsim')) \setminus \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim, \mathcal{S}^{\sigma}_t(\succsim))$, $a_i \in D(\succsim'_i) \setminus D(\succsim_i)$, and $\psi^\sigma_i(\succsim) \succsim_i \psi^\sigma_i(\succsim')$.

**Case ii).** $i \neq j$ and $i = j'$. Since $\psi^\sigma_i(\succsim) = \omega_j \neq \omega_i = \psi^*_i(\succsim')$, $\omega_j \in D(\succsim_i)$, and therefore $\psi^\sigma_i(\succsim) \succsim_i \psi^\sigma_i(\succsim')$.

**Case iii).** $i \neq j$ and $i \neq j'$. Let $\omega_j \in \Pi(l), \omega_{j'} \in \Pi(l')$. If $l \leq l'$, since $\psi^\sigma(\succsim) \in \mathcal{I}(\succsim)$, $\psi^\sigma_i(\succsim) \succsim_i \psi^\sigma_i(\succsim')$. If $l' < l$, we consider two parallel cases. Assume first that $\omega_i \in \Pi(l'')$ with $l' < l''$. Then, $\psi^\sigma(\succsim') \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_{l'}(\succsim')) \setminus \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim, \mathcal{S}^{\sigma}_{l'}(\succsim))$. By the same reasoning of the previous cases, since only patient $i$ has changed her preferences, we have that for each $a \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_{l'}(\succsim')) \setminus \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim, \mathcal{S}^{\sigma}_{l'}(\succsim))$ with $a_i \neq \omega_i$, $a_i \in D(\succsim'_i) \setminus D(\succsim_i)$, and $\psi^\sigma_i(\succsim) \succsim_i \psi^\sigma_i(\succsim')$. Finally, assume that $\omega_i \in \Pi(l'')$ with $l'' \leq l'$. The fact that $\psi^\sigma(\succsim') \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_{l''}(\succsim')) \setminus \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim, \mathcal{S}^{\sigma}_{l''}(\succsim))$ and the already familiar reasoning imply that $\omega_{j'} \notin D(\succsim_i)$, and $\psi^\sigma_i(\succsim) \succsim_i \psi^\sigma_i(\succsim')$.

Since the three cases are exhaustive, $\psi^\sigma$ satisfies *strategy-proofness*.

Finally we prove that $\psi^\sigma$ satisfies *non-bossiness*. Let $i \in N$, and $\succsim, \succsim' \in \mathcal{D}^{\Pi}$ such that $\succsim_{-i} = \succsim'_{-i}$ and such that $\psi^\sigma_i(\succsim) = \psi^\sigma_i(\succsim')$. Let $t^*$ be the smallest number $t$ such that for some $a \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim, \mathcal{S}^{\sigma}_t(\succsim))$, $a_i \neq \omega_i$. Since $\psi^\sigma_i(\succsim) = \psi^\sigma_i(\succsim')$, $t^*$ is the smallest number $t'$ such that for some $a' \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_{t'}(\succsim'))$, $a'_i \neq \omega_i$. Since only patient $i$ has changed her preferences, then for each patient $j$ who receives a kidney at any stage $t < t^*$ or at stage $t^*$ and $\sigma(j) < \sigma(i)$, $\psi^\sigma_j(\succsim) = \psi^\sigma_j(\succsim')$. Thus, $\psi^\sigma(\succsim) \in \mathcal{M}^{\sigma}_{\sigma^{-1}(i)-1}(\succsim', \mathcal{S}^{\sigma}_{t^*}(\succsim'))$, and since only patient $i$ has changed her preferences, $\psi^\sigma(\succsim) = \psi^\sigma(\succsim')$. □



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**Proof of Lemma 2.** Notice that $\psi^\sigma$ satisfies *individual rationality*, *efficiency*, *strategy-proofness*, and *non-bossiness*, then $\psi^\sigma$ is a sequential maximizing rule. Hence, for each $\succsim \in \mathcal{D}^\Pi$, for each $a \in \mathcal{F}^\sigma_{m,m}(\succsim)$, $\# M_{1,1}(\psi^\sigma(\succsim)) = \# M_{1,1}(a)$. Let $a' \in \mathcal{B}^\sigma_{1,1}(\succsim)$ such that for each $i \in M_{1,1}(\psi^\sigma(\succsim))$, $a'_i \neq \omega_i$, for each $j \notin M_{1,1}(\psi^\sigma(\succsim))$, $a'_j = \omega_j$. Let $i$ be such that $\sigma(i) = 1$. Assume first that $i \in M_{1,1}(\psi^\sigma(\succsim))$. Since $a' \in \mathcal{B}^\sigma_{1,1}(\succsim)$, $a' \in \mathcal{M}^\sigma_0(\succsim, \mathcal{B}^\sigma_{1,1}(\succsim))$ and $a' \in \mathcal{M}^\sigma_1(\succsim, \mathcal{B}^\sigma_{1,1}(\succsim))$. Thus, for each $a \in \mathcal{F}^\sigma_{m,m}(\succsim)$, $i \in M_{1,1}(a)$. Assume, now that $i \notin M_{1,1}(\psi^\sigma(\succsim))$, then for each $a \in \mathcal{M}^\sigma_0(\succsim, \mathcal{B}^\sigma_{1,1}(\succsim))$, either $a_i = \omega_i$ or $a_i \notin \Pi(1)$. Hence for each $a' \in \mathcal{M}^\sigma_0(\succsim, \mathcal{B}^\sigma_{1,1}(\succsim))$, $a_i = \omega_i$. Thus, $\mathcal{M}^\sigma_0(\succsim, \mathcal{B}^\sigma_{1,1}(\succsim)) = \mathcal{M}^\sigma_1(\succsim, \mathcal{B}^\sigma_{1,1}(\succsim))$, and for each $a \in \mathcal{F}^\sigma_{m,m}(\succsim)$, $i \notin M_{1,1}(a)$. We can replicate the argument for the remaining patients with a donor in $M_{1,1}(\psi^\sigma(\succsim))$ and for subsequent $t, t'$ to obtain the result. □

**Proof of Theorem 4.** It follows directly from the proof of Theorem 1 in Nicolò and Rodríguez-Álvarez (2012). The arguments in the proof of that theorem use age-based preferences for an arbitrary age structure $\Pi$ that either contains three age groups, or only two age groups with at least two kidneys in each age group. □

**Proof of Theorem 5.** Since the arguments in the proof of Theorem 3 do not use the assumption that assignments are pairwise, they directly apply to prove that $\tilde{\psi}^\sigma$ satisfies *individual rationality*, *efficiency*, *strategy-proofness*, and *non-bossiness*. Actually, we can use slightly modified arguments to prove that $\tilde{\varphi}$ also satisfies the axioms. For completeness, we include the proof for *strategy-proofness*.

Consider an arbitrary partition $\Pi$. Given that for each $t, t'$ with $1 \leq t \leq t' \leq m$ and $\succsim \in \mathcal{D}^\Pi$, $\mathcal{B}^\sigma_{t,t'}(\succsim) \subseteq \mathcal{M}^\sigma_{t,t'}(\succsim)$, the proof follows the same arguments that those of Theorem 3. Let $i \in N$, and $\succsim, \succsim' \in \mathcal{D}^{\Pi^*}$ be such that $\succsim'_{-i} = \succsim_{-i}$. Let $\omega_i \in \Pi(t_i)$. Assume first that, $\tilde{\varphi}_i(\succsim) = \tilde{\varphi}_i(\succsim')$, then $\tilde{\varphi}_i(\succsim) \succsim_i \tilde{\varphi}_i(\succsim')$. Assume now that $\tilde{\varphi}_i(\succsim) \neq \tilde{\varphi}_i(\succsim')$. Let $j, j' \in N$ be such that $\tilde{\varphi}_i(\succsim) = \omega_j$ and $\tilde{\varphi}_i(\succsim') = \omega_{j'}$. There are three cases:

**Case i).** $i = j$ and $i \neq j'$; $\tilde{\varphi}(\succsim) = \omega_i$ implies that for each $t$ with $t \leq t_i \leq m$ there is no $a \in \mathcal{M}^\sigma_{i-1}(\succsim, \mathcal{B}^\sigma_{t,t_i}(\succsim))$ with $a_i \neq \omega_i$, and for each $t'$ with $t_i \leq t' \leq m$, there is no $a' \in \mathcal{M}^\sigma_{i-1}(\succsim, \mathcal{B}^\sigma_{t_i,t'}(\succsim))$ with $a'_i \neq \omega_i$. Assume that $\tilde{\varphi}(\succsim') \in \Pi(t)$ with $t \leq t_i$. Let $t \leq t_i \leq m$ be the smallest integer such that there is $a \in \mathcal{M}^\sigma_{i-1}(\succsim', \mathcal{B}^\sigma_{t,t_i}(\succsim'))$ with $a_i \neq \omega_i$. Notice then that $\tilde{\varphi}(\succsim') \in \mathcal{M}^\sigma_{i-1}(\succsim', \mathcal{S}^\sigma_{t,t_i}(\succsim'))$ and $\tilde{\varphi}(\succsim') \neq \omega_i$. Since only patient $i$ has changed her preferences, and $\mathcal{B}^\sigma_{t,t_i}(\succsim) \subseteq \mathcal{S}^\sigma_{t,t_i}(\succsim)$, then for each $a \in \mathcal{M}^\sigma_{i-1}(\succsim', \mathcal{B}^\sigma_{t,t_i}(\succsim')) \setminus \mathcal{M}^\sigma_{i-1}(\succsim, \mathcal{B}^\sigma_{t,t_i}(\succsim))$, $a_i \in D(\succsim'_i) \setminus D(\succsim_i)$, and $\tilde{\varphi}_i(\succsim) \succsim_i \tilde{\varphi}_i(\succsim')$. The same arguments apply for the case $\tilde{\varphi}(\succsim') \in \Pi(t')$ with $t' > t_i$.

**Case ii).** $i \neq j$ and $i = j'$. Since $\tilde{\varphi}_i(\succsim) = \omega_j \neq \omega_i = \tilde{\varphi}_i(\succsim')$, $\omega_j \in D(\succsim_i)$, and therefore $\tilde{\varphi}_i(\succsim) \succsim_i \tilde{\varphi}_i(\succsim')$.

**Case iii).** $i \neq j$ and $i \neq j'$. Let $\omega_j \in \Pi(l), \omega_{j'} \in \Pi(l')$. If $l \leq l'$, since $\tilde{\varphi}(\succsim) \in \tilde{\mathcal{I}}(\succsim)$, $\tilde{\varphi}_i(\succsim) \succsim_i \tilde{\varphi}_i(\succsim')$. Assume finally, $l' < l$. We have to consider two cases. Assume first that $l' < t_i$. Then, $\tilde{\varphi}(\succsim') \in \mathcal{M}^\sigma_{i-1}(\succsim', \mathcal{B}^\sigma_{l',t_i}(\succsim')) \setminus \mathcal{M}^\sigma_{i-1}(\succsim, \mathcal{B}^\sigma_{l',t_i}(\succsim))$. By the same reasoning of the initial case, since only patient $i$ has changed her preferences, for each $a \in \mathcal{M}^\sigma_{i-1}(\succsim', \mathcal{B}^\sigma_{l',t_i}(\succsim')) \setminus \mathcal{M}^\sigma_{i-1}(\succsim, \mathcal{B}^\sigma_{l',t_i}(\succsim))$ with $a_i \neq \omega_i$, $a_i \in D(\succsim'_i) \setminus D(\succsim_i)$, and $\tilde{\varphi}_i(\succsim) \succsim_i \tilde{\varphi}_i(\succsim')$. Finally, if $t_i \leq l'$, since and $\tilde{\varphi}(\succsim') \in \mathcal{M}^\sigma_{i-1}(\succsim', \mathcal{B}^\sigma_{t_i,l'}(\succsim')) \setminus \mathcal{M}^\sigma_{i-1}(\succsim, \mathcal{B}^\sigma_{t_i,l'}(\succsim))$, the already familiar reasoning implies that $\omega_{j'} \notin D(\succsim_i)$, and $\tilde{\varphi}_i(\succsim) \succsim_i \tilde{\varphi}_i(\succsim')$.

Finally we prove that for every priority ordering that respects $\Pi$, the multiple ways age-based sequential priority rule and the multiple ways age-based sequential maximizing rule with fixed priorities do not coincide at some preference profile if there is an age group with at least four elements. Without loss of generality, consider $\Pi$ such that $\{\omega_1, \omega_2, \omega_3, \omega_4\} \subseteq \Pi(1)$ and the natural priority ordering $\sigma^*$. Let $\succsim \in \mathcal{D}^\Pi$ be such that $D(\succsim_1) = \{\omega_2\}$, $D(\succsim_2) = \{\omega_1, \omega_4\}$, $D(\succsim_3) = \{\omega_2\}$, and $D(\succsim_4) = \{\omega_3\}$. Note that $\psi^{\sigma^*}(\succsim) = (\omega_2, \omega_1, \omega_3, \omega_4, \ldots)$ and $\tilde{\varphi}(\succsim) = (\omega_1, \omega_4, \omega_2, \omega_3, \ldots)$. □

**Proof of Theorem 6.** Let $N = \{1, 2, 3, 4\}$, $\Pi$ and $(\Omega_i)_{i \in N}$ be such that:

| | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ | $\Omega_4$ |
|---|---|---|---|---|
| $\Pi(1)$ | $\omega_a$ | | | $\omega_g$ |
| $\Pi(2)$ | | $\omega_c$ | $\omega_e$ | |
| $\Pi(3)$ | $\omega_b$ | $\omega_d$ | $\omega_f$ | $\omega_h$ |

Let $\succsim \in \mathcal{D}^\Pi$ be such that:

$$D(\succsim_1) = \{\omega_d, \omega_e\},$$
$$D(\succsim_2) = \{\omega_a, \omega_h\},$$
$$D(\succsim_3) = \{\omega_b, \omega_g\},$$
$$D(\succsim_4) = \{\omega_c, \omega_f\}.$$

Note that the assignments $\alpha = (\omega_d, \omega_a, \omega_g, \omega_f)$ and $\alpha' = (\omega_e, \omega_h, \omega_b, \omega_c)$ are such that $\alpha'_1 \succ_1 \alpha_1$, $\alpha_2 \succ_2 \alpha'_2$, $\alpha_3 \succ_3 \alpha'_3$, and $\alpha'_4 \succ_4 \alpha_4$. Hence, both $\alpha$ and $\alpha'$ are not weakly blocked and belong to the strict core of the associated pairwise assignment game. By Theorem 1 in Sönmez (1999), since we have two assignments in the strict core, there is no $\Phi : \mathcal{D}^\Pi \to \mathcal{A}^M_2$ that satisfies *individual rationality*, *strategy-proofness*, and *efficiency*. □

**Proof of Theorem 7.** The arguments in the proof of Theorem 3 directly apply to the multiple donors framework to prove that $\Psi^\pi$ satisfies *individual rationality*, *efficiency*, and *non-bossiness*. So we focus on *strategy-proofness* and *donor monotonicity*.

We first check *strategy-proofness*. Let $i \in N$, and $\succsim, \succsim' \in \mathcal{D}^{\Pi^*}$ such that $\succsim'_{-i} = \succsim_{-i}$. Clearly, if $\Psi_i^\pi(\succsim) \in \Pi(1)$, since $\Psi^\pi(\succsim) \in \mathcal{I}^*(\succsim)$, then $\Psi_i^\pi(\succsim) \succsim_i \Psi_i^\pi(\succsim')$. Assume that $\Psi_i^\pi(\succsim) \in \Omega_i$, then for each $k$ such that $\rho(k) = i$, for each $t = 1, 2$, and each $a \in \mathcal{GM}_{k-1}^\pi\left(\succsim, \mathcal{GS}_t^\pi(\succsim)\right)$, $a_i \in \Omega_i$. If $\Psi_i^\pi(\succsim') \notin \Omega_i$, since only $i$ has changed her preferences, replicating the arguments in the proof of Theorem 1, $\Psi_i^\pi(\succsim') \in D(\succsim'_i) \setminus D(\succsim_i)$, and $\Psi_i^\pi(\succsim) \succsim_i \Psi_i^\pi(\succsim')$. Finally, assume that $\Psi_i^\pi(\succsim) \in \Pi(2)$, the previous argument implies that if there is $k$ with $\rho(k) = i$ and $a \in \mathcal{GM}_{k-1}^\pi(\succsim', \mathcal{GS}_1(\succsim'))$ with $a_i \notin \Omega_i$, then $a_i \in D(\succsim'_i) \setminus D(\succsim_i)$. Hence, either $\Psi_i^\pi(\succsim') \in \Omega_i$, $\Psi_i^\pi(\succsim') \in \Pi(2)$, or $\Psi_i^\pi(\succsim') \in \Pi(1) \cap \left(D(\succsim'_i) \setminus D(\succsim_i)\right)$. In either case, $\Psi_i^\pi(\succsim) \succsim_i \Psi_i^\pi(\succsim')$, which suffices to prove *strategy-proofness*.

We conclude by checking *donor monotonicity*. Let $i, j \in N$, $\omega \in \Omega_i$, and $\succsim, \succsim' \in \mathcal{D}^{\Pi^*}$ be such that $\succsim'_{-j} = \succsim_{-j}$, and $\omega \in D(\succsim_j)$, $D(\succsim'_j) = D(\succsim_j) \setminus \{\omega\}$. Let $k = \pi(\omega)$. Assume first that $\Psi_i^\pi(\succsim) \in \Pi(1) \setminus \Omega_i$, then $\Psi_i^\pi(\succsim) \succsim_i \Psi^\pi(\succsim')$. Assume next that $\Psi_i^\pi(\succsim) \in \Omega_i$. Then, for each $k'$ such that $\rho(k') = i$, for each $t = 1, 2$, and each $a \in \mathcal{GM}_{k'-1}^\pi(\succsim, \mathcal{GS}_t^\pi(\succsim))$, $a_i \in \Omega_i$ and $\mathcal{GM}_{k'-1}^\pi(\succsim, \mathcal{GS}_t^\pi(\succsim)) = \mathcal{GM}_{k'}^\pi(\succsim, \mathcal{GS}_t^\pi(\succsim))$. Since only patient $j$ has changed her preferences, and $D(\succsim'_j) = D(\succsim_j) \setminus \{\omega\}$, for each $k'$ such that $\rho(k') = i$, for each $t = 1, 2$, and each $a' \in \mathcal{GM}_{k'-1}^\pi(\succsim', \mathcal{GS}_t^\pi(\succsim'))$, $a'_i \in \Omega_i$ and $\mathcal{GM}_{k'-1}^\pi(\succsim', \mathcal{GS}_t^\pi(\succsim')) = \mathcal{GM}_{k'}^\pi(\succsim', \mathcal{GS}_t^\pi(\succsim'))$. Finally, assume that $\Psi_i^\pi(\succsim) \in \Pi(2)$, then for each $k'$ such that $\rho(k') = i$, and each $a \in \mathcal{GM}_{k'-1}^\pi(\succsim, \mathcal{GS}_1^\pi(\succsim))$, $a_i \in \Omega_i$. With an already familiar argument, for each $a' \in \mathcal{GM}_{k'-1}^\pi(\succsim, \mathcal{GS}_1^\pi(\succsim'))$, $a'_i \in \Omega_i$, and $\Psi_i^\pi(\succsim') \notin \Pi(1) \setminus \Omega_i$. Hence, $\Psi_i^\pi(\succsim) \succsim_i \Psi^\pi(\succsim')$. The argument can be iterated as many times as necessary to conclude that $\Psi^\pi$ satisfies *donor monotonicity*. □

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## Highlights

- We model Paired Kidney Exchange (PKE) under logistic feasibility constraints.
- Patients prefer kidneys from compatible younger donors to kidneys from older ones.
- Sequential priority rules satisfy individual rationality, (constrained) efficiency, strategy-proofness, and non-bossiness.
- Sequential priority rules allocate kidneys according to a priority algorithm that gives priority to patients with younger donors.



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