X-ray Tomographic System Behavior Prediction Based on a Mathematical Model

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Abstract. There appear certain challenges in defining the dependence of the X-ray radiation intensity change in passing through the material (as fixed by the detector) conditioned by various parameters of an X-ray optical system while designing new modifications of X-ray tomographs. At present, this problem is experimentally solved by selection of voltage corresponding parameter values on an X-ray tube with thickness and type of the studied material considered. To reduce the design time and complexity, a mathematical model of parameter behavior is required to characterize the X-ray optical system in the major working range of values. The present paper investigates the X-ray optical system behavior using methods of mathematical statistics. A regression model has been obtained which matches the change of the X-ray intensity value to the intensity in the X-ray tube. The research has defined the further study direction of X -ray optical system parameters.

1. Introduction

X-ray tomographs are regarded the most widespread systems for industrial non-destructive testing (NDT). X-ray tomographs are classified into several types according to the studied value sizes, their technical characteristics, the field of application, etc [1]. Currently, microtomographs are considered the most popular direction of NDT system development [2]. These systems allow investigating internal structure of objects with micron sizes and are characterized by voltage on the X-ray tube in the range from 10 to 160 [kV] and a small size of the focal spot (from 1 to 10 microns). As betatrons belong to another X-ray system class due to an alternative implementation principle and their technical characteristics, they are not considered in this work.

An X-ray optical system for NDT consists of the following four elements: an X-ray tube, a detector, a worktable positioning system and the test material [3]. The X-ray tube and detectors (Hammatsu, Japan) produced in Tomsk have been used in microtomographs. These elements fully comply with modern advanced requirements in the field of energy saving, functionality, measurement accuracy and an X-ray beam [3]. These tomographs compete with the tomographs produced by such leading companies as Matrix, SkyScan, and Feinfocus.

The X-ray tomograph system design as a fundamental stage of the process involves basic technical system parameters calculations according to the specification, fundamentally new technical solution development as well as the system layout.

As X-ray system development and design is of experimental nature, the equipment and system parameters are selected according to the theoretical formulas of the X-ray intensity change in passing

through material and intuitive component selection conforming to these parameters. These calculations are considered labor-consuming and mostly characterize a situational (static) condition of an X-ray optical system within specifically chosen factor values and do not allow complete system behavior (change of output parameter) estimating at factor value changing [4].

2. Problem statement
To avoid such complexities, a new method of determining mathematical relationships has been required to enable the process simulation and system behavior analysis, to reduce time costs and to improve the developed system quality and accuracy [5]. To achieve this goal, the tools and features of a complete factorial experiment have been applied. A complete factorial experiment allows to vary all factors simultaneously and to obtain a regression model of the studied system [1]. In addition, it enables the search of optimum conditions of functioning of the system based on the adequate models.

3. Research questions
The following tasks have been set and solved to achieve the abovementioned goals:
• to define the X-ray optical system layout;
• to determine the system optimal values;
• to conduct a complete factorial experiment 2^3;
• to conduct a regression analysis;
• to obtain a mathematical relationship for predicting the process of X-ray intensity changing for the basic operation modes and studied materials;
• to evaluate the obtained results and their applicability in practice.

This value of response is considered significant as it characterizes the quality of shadow images obtaining and 3D model building, uniformity and quality of the X-ray tube emitting beam, the material parameters and its uniformity, etc.

The experiment has covered three basic factors of the process:
1. the voltage at the X-ray tube;
2. the thickness of the illuminated material;
3. the type of material.

The basic factor levels have been formed; they characterize the main modes of the device operation. The variation intervals have been formed on the basis of the real range of factor acceptable values. The tables of the parameter actual values and their intervals have been filled in according to the experiment plan X.

Table 1 demonstrates the two materials (titanium and iron) selected for the experiment.

Table 1. Factor levels and variation intervals.

Factor levels	Designation	U, [kV] $\tilde{x}_1$	Thickness, [mm] $\tilde{x}_2$	Type of material $\tilde{x}_3$
Lower	-1	100	1	iron
Upper	+1	140	2	titanium

Iron is one of the most frequent materials applied in various industrial detail production and parts in mechanical engineering, space engineering as well as in the radio-electronic equipment. Titan has a broad application in the aircraft equipment, rocket production, instrument making, in chemical industry and shipbuilding [6].

4. Research methods
The work aims are to improve the process of new X-ray optical system design; to reduce time and labor input due to forecasting of X-ray optical system behavior based on a mathematical model.

The applied cover methods and tools for X-ray system design, theoretical calculation methods of X-ray radiation intensity changes, the two-level, complete factorial experiment for three factors, namely the 2^3 design, dispersive and regression analyses are used as the research methods.

First, the X-ray microtomograph was plugged into the electrical network. After 2-3 minute waring up, the check run of X-ray radiation pulse without material was produced.

The value of the X-ray intensity was measured. The experiment could be further conducted if the value observed was comparable to the device technical documentation or the X-ray intensity deviation cause had been eliminated. With the system working properly, the material translucence started. First, the material was placed on the device worktable and the lid was closed for safety reasons. Then, the voltage value was set on the X-ray tube and after the signal of material transmission the result presented on the display device was recorded. The studied material, its size and the X-ray tube voltage value were to be chosen according the experiment plan.

The experiment aimed to study the process of changing X-ray intensity obtained in the X-ray microtomograph in passing through the material (iron and titanium) of two sizes 1 and 2 mm respectively. Factor alternating was carried out for obtaining a set of parameter combination values listed in the rows of the plan table. The experience was repeated 3 times at each point of the experiment plan. Table 2 shows the test results (value of X-ray intensity).

Table 2. Experimental results.

No Exp.	Factors in the natural scale			Factors in the dimensionless coordinate system				Output parameter		
	Z_1	Z_2	Z_3	X_0	X_1	X_2	X_3	Y_1	Y_2	Y_3
	[kV]	[cm]						[keV]	[keV]	[keV]
1	100	0.1	1	+1	-1	-1	-1	30.4	30	29.6
2	140	0.1	1	+1	+1	-1	-1	65.6	66.4	63.8
3	100	0.2	1	+1	-1	+1	-1	14	13	13.4
4	140	0.2	1	+1	+1	+1	-1	46.31	46.2	45.7
5	100	0.1	2	+1	-1	-1	+1	49.6	48.2	48.4
6	140	0.1	2	+1	+1	-1	+1	79.8	80.6	80.7
7	100	0.2	2	+1	-1	+1	+1	37	37.1	36.8
8	140	0.2	2	+1	+1	+1	+1	69.8	70	68.6

5. Findings

Multiple measurements under identical conditions were carried out to reduce experimental errors. Dispersion homogeneity testing was performed by using the Cochran's criterion.

As a result, the regression equations were received:

- a linear equation, which considered only the coefficient factors, $Y=48.8+16.5 x_1 - 7.3 x_2+10.1 x_3$;

- a nonlinear one, which regarded the coefficients of individual factors and their interaction with each other, $Y=48.8+16.5 x -7.3 x_2+10.1 x_3 - 0.47 x_1 x_3 +1.65 x_2 x_3+0.458 x_1 x_2 x_3$.

The equations presented the research results: the larger the absolute value of the regression coefficient b_i, the greater its influence on the optimization criterion in a specified range factor values.

The model adequacy was then checked by the Fisher criterion which led to the conclusion that the two obtained equations were not adequate. The model poorly described the experimental data in the specified range factor values. If a linear model was inadequate, then that could not be approximated by a plane surface response. Some other factors affecting the model have not been considered in the experiment described. There is a suggestion that the materials selected for the experiment possess different physical properties. The parameter value dispersion characterizing the linear attenuation coefficient of X-rays in the material is equal to 0.6 cm^{-1} for iron up to 1.9 cm^{-1} for titanium respectively when the energy is about 0.7 MeV. Furthermore, titanium has a more complicated three-dimensional internal structure and an X-ray attenuation dependence which differs from that of iron.

The decisions generally taken to build up an adequate model might be the following: to change intervals of varying factors, to shift the focus of the experiment plan, to extend the experimental design.

Among these, the most common method is to adjust the range of values of varying factors. This method is based on carrying out a new series of experiments [7]. However, this solution does not seem to be appropriate in this situation, because the intervals of variation have been defined on the experiment planning stage and include important materials in modern industry and meet the basic operating conditions of an X-ray system. Changing this interval may result in changing the range of values, which is of the greatest interest from the point of view of practical applications, including the construction of a model to predict the results. Altering the range of values of varying factors might turn possible, if materials with similar physical properties are selected [8].

The study continued as univariate regression analysis. Then, the impact of the effect of an X-ray tube voltage value on the value of the output intensity after passing through the material under investigation is considered.

An additional experiment was conducted under the following conditions. The values of thickness and type of the material were fixed, as well as the of X-ray tube power. The selected material was iron, as one of the most frequently encountered in practice. The investigated iron plate had a thickness of 2 mm. the Power of the X-ray tube was kept the same, 0.7 [MeV].

The following variables were identified for analysis:

1. The output power intensity after passing through the material under investigation, [keV] – Y;

2. The voltage on the X-ray tube, [kV] – X.

Regression analysis is usually used in order to determine the correlation ratio between indicators of the voltage and the output intensity of the evaluation factors that have the greatest impact on output intensity. It also used to establish the degree of the output intensity on the stress influences and to select the type and form of unknown causal relationships, as well as to define the calculated values of the output intensity (regression function). Regression analysis allows predicting the value of the output intensity for better construction of a 3D shadow image when working with the instrument. The most important stage of building a regression model (regression equation) is the selection of the mathematical function form that most accurately defines the existing relations between the analyzed features [9–11]. In this case, intensity has been assigned as the depended attribute of the magnitude, and stress as the independent attribute respectively.

The raw data analysis allows suggesting that the pair linear correlation equation is represented by the function:

$$y = bx_0 + bx_1.$$

Figure 1 shows a dispersion diagram. The correlation has a positive trend.

Performing the corresponding calculations in Excel the following results have been obtained. The sums of squares calculated in Excel stand for $SSR = 3890.653$; $SSE = 107.728$; $SST = 3998.3811$. R-squared (coefficient of determination) is a utility assessment of the regression equation and estimates the proportion of the Y variation of the variable, which is explained by the independent variable in the regression model X. This value is equal to 0.973. It describes that 97.3% of the variation is due to the intensity of the output voltage value and only 2.7% by other factors. Large values tending to 1, indicate that X can be used to explain the behavior of Y.

The standard error is equal to 3.6696. This shows that 95% of the values may fall within ±3.6696 from the regression line.

As a result, a linear regression equation has been obtained: $Y = -55.9916 + 0.6867 \cdot x$. This equation can be applied to the selected range of operating systems values. The positive slope of the linear regression indicates that an increase in voltage in one unit leads to the rise value of the output intensity at 0.6867. For example, for a voltage of 120 [kV] the output intensity value is equal to 26.41 [keV].
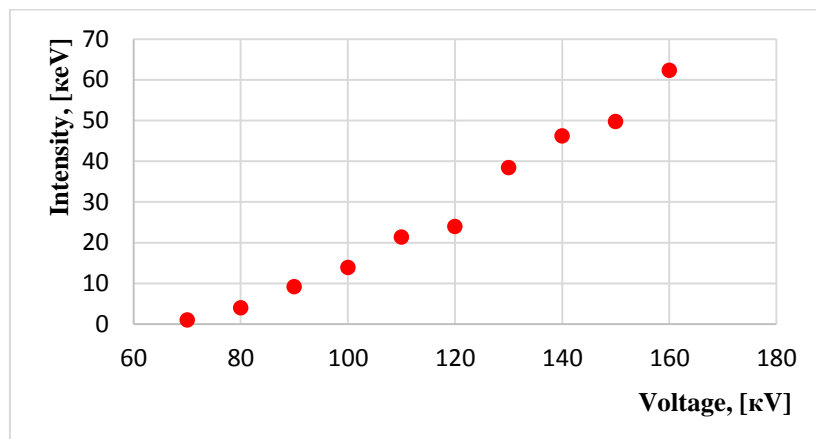
Figure 1. Dispersion diagram.

6. Conclusion

The complete factorial experiment has proved that the resulting linear model is inadequate. This means that the model poorly describes the experimental data in the specified range factor. To use these equations in order to predict the intensity value changes as it passes through the material under investigation is unfeasible. The solution to the problem is to continue the research on the model of a two-factor experiment, excluding the factor of the material. Thus, the principle factors are considered the X-ray tube voltage and the thickness of the material as the. X-ray optical system developers might get interested in the experiment results.

Due to the obtained result there is a need to continue research in order to study this process and provide useful information for the development and design of new versions of X-ray scanners. For this purpose some additional experiments have been conducted. Univariate regression analysis has been carried out where the response of the system as a complete factorial experiment has been the intensity value after passing through the thickness of the material, and the an X-ray tube voltage has been taken as a factor. All the remaining parameters of the system are a group of «experimental conditions».

Due to the univariate regression analysis, the form of the linear regression equation as been determined, it has been proved that this model could be applied in practice, and there is a correlation between variables X and Y. The practical application of the linear regression equation facilitates predicting the value of the output intensity fast and easily according to the selected voltage on the X-ray tube for this range of values. The system can be described most accurately and completely within the prescribed framework with the dependence derived from practical data. This fact confirms the mathematical relationship from the available theoretical calculation tools. This model has a practical significance, since it allows obtaining better shadow images to construct a 3D image, and facilitates the work of engineers and constructs to develop new versions of X-ray systems.

References

[1] Landisa E N *and* Keane D T 2010 *Materials Characterization* **61(12)** 1305–1316 DOI: 10.1016/j.matchar.2010.09.012

[2] Dunsmuir J *et al* 2006 *Powder Diffraction* **21(2)** 125–131 DOI: 10.1154/1.2204956

[3] Syryamkin V I *et al* 2011 *Russian Journal of Nondestructive Testing* **47(11)** 754–764 DOI: 10.1134/S1061830911110076

[4] Marusina M *and* Kaznacheeva A 2006 *Modern Tomography Types* (St. Petersburg: SPbGU ITMO) (in Russian)

[5] Natterrer F 2001 *The Mathematics of Computerized Tomography* DOI: 10.1137/1.9780898719284

[6] Plotnikova I *et al* 2016 IOP Conf. Series: Materials Science and Engineering 132 012023 DOI: 10.1088/1757-899X/132/1/012023

[7] Adler Ju *et al* 1976 *Planning of Experiment by Search of Optimum Conditions* (Moscow: Nauka) (in Russian)

[8]   Box G *and* Wilson K 1951 *J. Roy. Statist. Soc. Ser. B Metho*. **13** 1–45
[9]   Masood   I   *et   al*   2014   *Applied   Mechanics   and   Materials*   **465-466**   1149–1154   DOI:
        10.4028/www.scientific.net/AMM.465-466.1119
[10]   Rafidah   A   *et   al*   2014   *Applied   Mechanics   and   Materials*   **660**   275–279   DOI:
        10.4028/www.scientific.net/AMM.660.275
[11]   Redko A *et al* 2015 *EpSBS* **VII** 319–324 DOI: 10.15405/epsbs.2016.02.24