# Optimal design of queueing systems for using communication channels with multiple access

**D.M. Sonkin[1], A.A. Nazarov[2], S.A. Khrul[3] and A.V. Kudryashova[1]**

[1] National Research Tomsk Polytechnic University, 30, Lenina Ave., Tomsk, 634050, Russia
[2] National Research Tomsk State University, 36, Lenin Ave., Tomsk, 634050, Russia
[3] Ltd. "INCOM", 14a, Rosa Luxemburg Str., Tomsk, 634009, Russia

E-mail: sonkind@tpu.ru

**Abstract.** This article considers the mathematical model of queueing systems for the communication network with multiple access, which is used in the automated group of moving objects control systems. The dynamic communication model in the form of a single-line queuing model with the batch input claims was researched. The method of asymptotic analysis of the communication channel under a heavy load conditions was applied. As a result, the basic probabilistic characteristics of the system were obtained, including: the probability distribution of the waiting time values of the virtual claims, the average length of the queue claims at random time. It has been discovered that the characteristics of the communications models allow performing parametric optimization of communication networks, establishing the most appropriate values of the network parameters.

## 1. Introduction

Nowadays, there is a tendency towards the use of distributed robotics. However, the communication channels for each element of such a system are not always dedicated to performing such functions. It can be illustrated by the example of developed communication along a finite set of alternative communication channels with a group of AUV (automated underwater vehicle) while making research at a great depth and under the conditions of the bottom rugged relief. For such systems, it is required to use ad hoc queueing networks enabling to maintain concurrent control of a number of missions by various machineries grouping for providing communication between a control center (CC) and autonomous underwater vehicles. Currently, the expanding traffic volume is being defined by the demand of simultaneous multiple access of various groups of control objects, such as AUV, an unmanned aerial vehicle (UAV), control centers on convoy vessels, research vessels, sensor values and other information sources of CC [1-3]. The major channels of communication with the groups of control objects are as follows:

- hydroacoustic channels (communication speed is up to 2 Kbit/sec.);
- radioelectronic channels, communicating mainly within the ultra-short band (communication speed is up to 9.6 Kbit/sec.).

## 2. Characteristics of mobile objects of ad hoc communication networks

A distinguishing characteristic of exchanging data with AUV being under water is the use of nothing but a hydroacoustic channel strongly limited in data transmission speed and, consequently, in the possibility of information exchange.

A hydroacoustic system consists of a transmitter, a telecommunication line and a receiver. A signal sender influences the transmitter while signaling a message. It is significant that the reception integrity in every real communication channel is affected by noise, which energetic level can sometimes exceed wanted signals. Such excess is especially typical of hydroacoustic communication. Actually, the signal which gets to the receiver is not clear, wanted and transmitted, because of its superposition to the sea noises, which have random allocation of phases and amplitudes. The function of a receiving unit can be accomplished by the antenna of a hydroacoustic station, the receptors of any animal, a hydrophone of a measuring unit, etc. A receiving unit typically operates in a passive mode, perceiving only the signals which enter the receptors from the environment. It is a standard operation mode of the communication system connecting ships, a sound locating station, the station of biological control over the situation in fishing area, etc. However, one more operation mode called an active one is possible. In this case, an energy source controlled by a receiving unit (simultaneously, a receiver and a transmitter, either directional or non-directional, get involved into the system operation) is used for receiving information. The transmitter receives only that part of the energy which was reflected by external environment, more specifically, those objects in water which were covered by a soundwave from the transmitter and which reflected some part of this wave energy [4-6].

It is important to emphasize that receiving information from the apparatus should in some cases be done on a real time basis. In addition, it is necessary to protect the transmitting channel and data from compromising in the optimum way, which would meet safety and productivity requirements.

Thus, the problem of effective communication under the conditions of resource limitation (channel traffic, a large amount of information, noise) is rather relevant.

A large amount of work is devoted to the development of organization of the structure of interaction (communications network models) between AUV and the basic control center through the available communication channels [1–13].

In particular, a widely used topology for such communication networks is a star topology (Figure 1), which host node fulfills the function of controlling the groups and is, in this respect, a network shareable resource. A shareable resource can be as well represented by a coastal AUV group monitoring center, Mission control center (MCC), any other control center which interacts with vehicles (underwater, ground-base or, off-the-ground ones) [4-6].

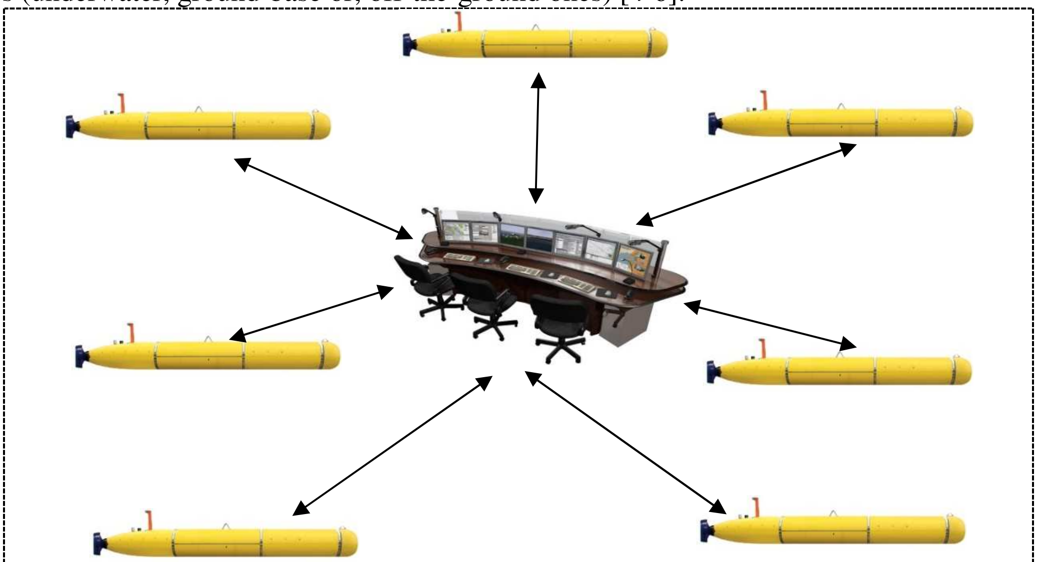

**Figure 1.** The star network topology.

In the process of performing tasks (measurement, monitoring, shooting), the AUVs, included in a group, transmit the accumulated data to the CC. The transmission is carried out through the multiple access protocol. Such protocols can be represented by widely used simulation models of the Retrial Queueing System random-access protocols [7-10], and for the cyclic protocols, these are the polling systems [11, 12]. Let us define the mathematic multiple access network model as a single-line Queueing System (QS) with the grouped incoming request flow where the incoming applications group corresponds to the messages from the AUV group (Figure 2).

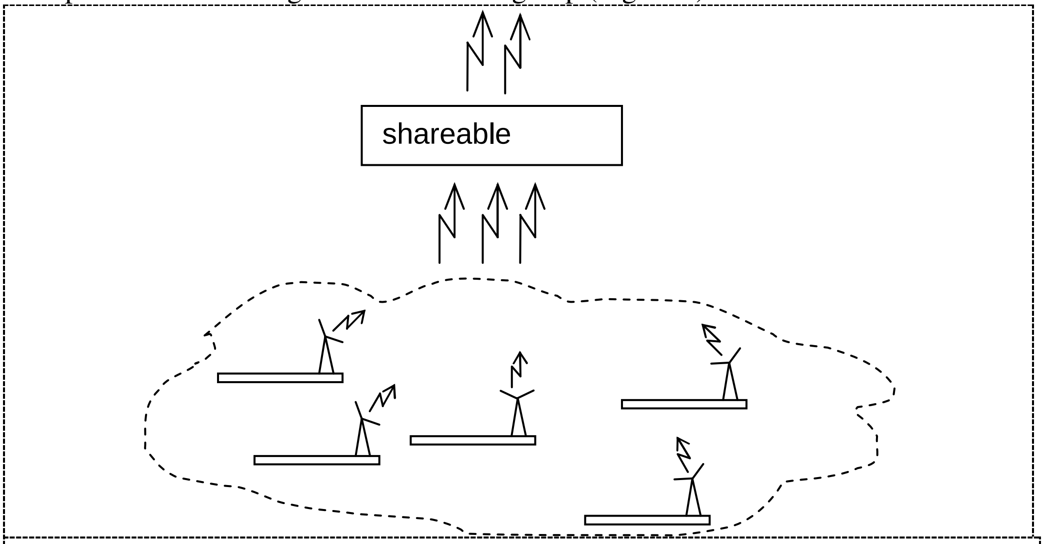


**Figure 2.** The telecommunication network with the shareable resource.

This paper studies the cyclic system, in which the cycle is represented by the sum of intervals of access to the shareable resource of each network user. The characteristics of such intervals are random time, uncorrelatedness between each other, independence from arrival intensity and the presence of the incoming requests flow.

The objective is to define the characteristics of requests timeout in such a queuing system, i.e. the message delivery time in the telecommunication network of cyclic multiple access.

The task is fulfilled by a classical approach “systems with server vacation” [13]. The stated algorithm of allocating the access intervals independent from the incoming flow and time spent on attending the incoming requests solves the key polling problem, which is as follows: the strategies considered in [11] give the stochastically dependent timeouts of the separate moments. For the stated strategy these intervals are independent. Consequently, a multivariable distribution of the probabilities of clients’ waiting time is factored, and the method “systems with vacations”, applied in the work, solves the task in hand successfully.

## 3. The mathematical model of the communication network

Let us consider the procedure of incoming and processing messages in greater detail. The process is represented by a single-line queueing system (Figure 3), from which the input enters N Poisson arrival of requests with intensity values λn for the n flow [14]. If the block and the unit device are free at the moment of the requests inflow, requests processing starts. If the server is engaged, the request gets into a queue (storage). According to Basharin-Kendall's notation, the queueing system under consideration corresponds to type $\boldsymbol{M^x|G|1|}\infty$.

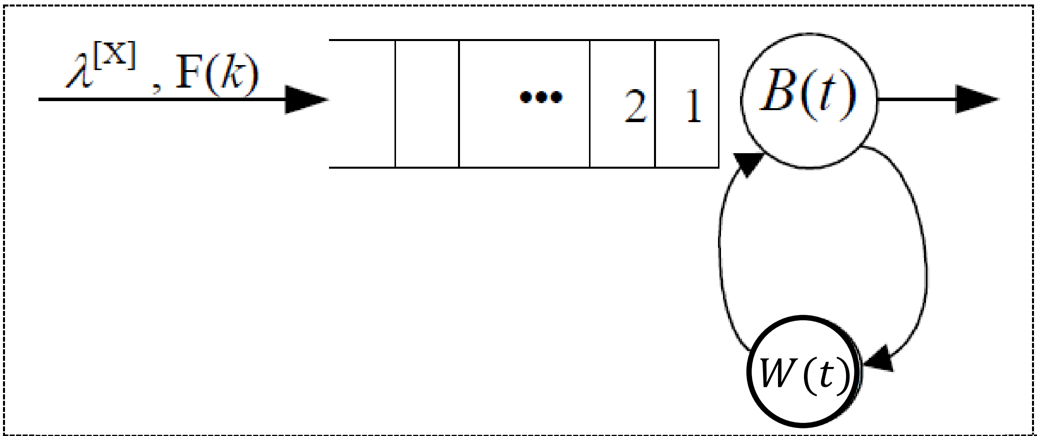


**Figure 3.** The model of the requests inflow to the system.

Let us consider the $n$ customer flow, whose requests are served on the server during its access interval. Let us name the aggregate duration of all other intervals as the time of server vacation or time of service waiting $W_n(t)$, during which the requests of the current flow are not served, but are in the storage waiting for the initiation of their access intervals.

Owing to the suggested structure of the service cycle with the intervals of the request access to the server, components $W_n(t)$ of $N$-dimensional random probability are

$W(t) = \{ W_1(t), W_2(t), \ldots, W_N(t)\}$,

which are independent from the incoming flows and processing intervals and are stochastically independent as well. Consequently, to determine their multivariable probabilities distribution, it is sufficient to determine marginal distribution of probabilities' values of single component $W_n(t)$.

The intervals of $n$-flow requests servicing are random, independent and are determined by distribution function $B_n(x)$.

Let us determine the distribution function of the interval length of its requests access to the server as $T_1(x)$ and the distribution function of the server vacation time as $T_2(x)$.

The group length of the requests being in the storage at moment $t$ is a random variable with distribution function $F(k)$, where $k$ is a nonnegative integral parameter. Let us define the existing possibilities as $f\ (k)$.

To study waiting time $W(t)$ in the "system with server vacation", let us find the characteristic function and the value $V(t)$ probabilities distribution. Here, $V(t)$ is the value of work on servicing all requests being in the system at moment $t$. Besides, the work has not been finished by moment $t$. Further, it will be shown how to find the probabilities distribution of virtual waiting time $W(t)$, being aware of the probabilities distribution of value $V(t)$, which is the value of unfinished work.

## 4. Researching the waiting time in a cyclic system

Let us mark the server state as k(t): 1 – the server is in the mode of input flow requests availability, 2 – the server is in the mode of vacation.

z(t) is the remaining time of the server being in a corresponding mode.

To calculate V(t), let us view three dimensional process {V(t), k(t), z(t)} and derive the system of Kolmogorov differential equations for the stationary distribution of probabilities Pk(v, z) = P{ V(t) < v, k(t) = k, z(t) < z}.

$$\begin{cases} \dfrac{\partial P_1(v,z)}{\partial v} + \dfrac{\partial P_1(v,z)}{\partial z} - \dfrac{\partial P_1(v,0)}{\partial z} - \lambda P_1(v,z) + \lambda \int\limits_0^v B(v-x)dP_1(x,z) + \dfrac{\partial P_2(v,0)}{\partial z} T_1(z) = 0, \\ \dfrac{\partial P_2(v,z)}{\partial z} - \dfrac{\partial P_1(v,0)}{\partial z} - \lambda P_2(v,z) + \lambda \int\limits_0^v B(v-x)dP_2(x,z) + \dfrac{\partial P_1(v,0)}{\partial z} T_2(z) = 0. \end{cases} \quad (1)$$

Here, the following notations are used: $\left.\frac{\partial P_k(v,z)}{\partial z}\right|_{z=0} = \frac{\partial P_k(v,0)}{\partial z}$.

Let us introduce the finite characteristic functions: $H_k(u,z) = \int_0^{\infty} e^{juv} dP_k(v,z)$,

for which Kolmogorov's equations system (1) will be presented as follows:

$$\begin{cases} \frac{\partial H_1(u,z)}{\partial z} - \frac{\partial H_1(u,0)}{\partial z} - [\lambda(1-\beta(u)) + ju]H_1(u,z) + juP_1(0,z) + \frac{\partial H_2(u,0)}{\partial z} T_1(z) = 0, \\ \frac{\partial H_2(u,z)}{\partial z} - \frac{\partial H_2(u,0)}{\partial z} - \lambda(1-\beta(u))H_2(u,z) + \frac{\partial H_1(u,0)}{\partial z} T_2(z) = 0. \end{cases} \quad (2)$$

System of equations (2) will be solved using the method of asymptotic analysis [15-19] under the conditions of the queueing system heavy loading [20].

Let us set the value of transmission capacity by S and the small positive indicator (which is considered as ε→0 in theoretical research) by ε. Let us make the following substitutions in the system of equations (2):

Let us set the value of the transmission capacity of the system "server with vacation" by S,

λ = (1 – ε)S, u = εw, Hk(u, z) = Fk(w, z, ε), P1(0, z) = επ1(z, ε) (3)

Then system (2) will be rewritten as follows:

$$\begin{cases} \frac{\partial F_1(w,z,\varepsilon)}{\partial z} - \frac{\partial F_1(w,0,\varepsilon)}{\partial z} - \\ -((1-\varepsilon)S(1-\beta(\varepsilon w)) + j\varepsilon w)F_1(w,z,\varepsilon) + j\varepsilon^2 w\pi_1(z,\varepsilon) + \frac{\partial F_2(w,0,\varepsilon)}{\partial z} T_1(z) = 0, \\ \frac{\partial F_2(w,z,\varepsilon)}{\partial z} - \frac{\partial F_2(w,0,\varepsilon)}{\partial z} - (1-\varepsilon)S(1-\beta(\varepsilon w))F_2(w,z,\varepsilon) + \frac{\partial F_1(w,0,\varepsilon)}{\partial z} T_2(z) = 0. \end{cases} \quad (4)$$

Let us set the first and second moments of random variables, determined by distribution functions B(x), T1(x), T2(x) for servicing time b and b2, and for the length of availability intervals T2 and T2(2).

According to theorem 1 [20], as ε→0, limit value Fk(w) = Fk(w, ∞) of solution Fk(w, z, ε) of equations system (4) is represented as Fk(w,z) = Rk(z)F(w), where

$$R_1 = Sb,\ R_2 = 1 - Sb,\ S = \frac{R_1}{b} = \frac{T_1}{(T_1 + T_2)b}, \quad (5)$$

$$\begin{cases} R_1(z) = \int_0^z \left(R_1'(0) - R_2'(0)T_1(x)\right)dx, \\ R_2(z) = \int_0^z \left(R_2'(0) - R_1'(0)T_2(x)\right)dx. \end{cases} \quad (6)$$

At that, $\{R_1(z), R_2(z)\}$ is the two-dimension distribution of the state of the server and the value of the time remaining for the server to be in it.

$$\text{If} \int_0^{\infty} (1 - T_k(x))dx = T_k \quad (7)$$

is an average time of having a particular state, then $R_1(\infty) + R_2(\infty) = 1$, however, on the other side,

$$R_1(\infty) + R_2(\infty) = R'(0)\int_0^{\infty}(1 - T_1(x))dx + \int_0^{\infty}(1 - T_2(x))dx = R'(0)(T_1 + T_2) \cdot \quad (8)$$

Then, the probability value of the server being at the first mode will be as follows: $R_1(\infty) = Sb$.

Let us set $R_1 = R_1(\infty)$, $R_2 = R_2(\infty)$, then, the probability distribution of the server state is given by (5): $R_1 = Sb$, $R_2 = 1 - Sb$.

Let us further study virtual waiting time W(t), by considering its conventional characteristic function.

Let us assign y(t) to the time period which is elapsed by moment t characterized by the server, being in mode k, where k = 1, if the server is in the mode of input flow requests availability and k = 2, if the server is vacant. Then,

$$P\{y(t) < y + \Delta t / y(t) \geq y\} = \frac{T_k(y + \Delta t) - T_k(y)}{1 - T_k(y)} = \frac{T_k'(y)}{1 - T_k(y)} \Delta t + o(\Delta t). \tag{9}$$

Let us set

$$\mu_k(y) = \frac{T_k'(y)}{1 - T_k(y)}, \tag{10}$$

i.e. the following equation is fulfilled as $P\{y(t) < y + \Delta t / y(t) \geq y\} = \mu_k(y)\Delta t + o(\Delta t)$.

Let us consider the conventional characteristic function of process W(t):

$$M\left\{e^{juW(t)} \middle| k(t) = k, V(t) = v, y(t)\right\} = G_k(u, v, y, t). \tag{11}$$

For this functional, let us form the backward Kolmogorov differential equations system:

$$\begin{cases} \dfrac{\partial G_1(u,v,y)}{\partial v} = \dfrac{\partial G_1(u,v,y)}{\partial y} + (ju - \mu_1(y))G_1(u,v,y) + \mu_1(y)G_2(u,v,0), \\ 0 = \dfrac{\partial G_2(u,v,y)}{\partial y} + (ju - \mu_2(y))G_2(u,v,y) + \mu_2(y)G_1(u,v,0). \end{cases} \tag{12}$$

Likewise (3), let us make substitutions in system (12)

$$u = \varepsilon w,\ \varepsilon v = x,\ G_k(u, v, y) = F_k(w, x, y, \varepsilon), \tag{13}$$

The substitutions correspond to the marginal condition of heavy loading, where ε = 1 – λ/S, then for function Fk(w, x, y, ε), the following system is deduced:

$$\begin{cases} \dfrac{\partial F_1(w,x,y,\varepsilon)}{\partial v} = \dfrac{\partial F_1(w,x,y,\varepsilon)}{\partial y} + (j\varepsilon w - \mu_1(y))F_1(w,x,y,\varepsilon) + \mu_1(y)F_2(w,x,0,\varepsilon), \\ 0 = \dfrac{\partial F_2(w,x,y,\varepsilon)}{\partial y} + (j\varepsilon w - \mu_2(y))F_2(w,x,y,\varepsilon) + \mu_2(y)F_1(w,x,0,\varepsilon). \end{cases} \tag{14}$$

Here, μk(y) is determined by equation (11).

Calculation of Fk(w, x, y, ε) amounts to solving the equation under the marginal conditions when ε→0 of value Fk(w, x, y) of equations system (14) is determined by equation [20]:

$$F_1(w,x,y) = F_2(w,x,y) = F(w,x) = \exp\left\{ jw \frac{(T_1 + T_2)}{T_1} x \right\}. \tag{15}$$

In other words, (15) is the characteristics function of determinated value $\frac{(T_1 + T_2)}{T_1} x$, where x, due to the substitution (13), is limitary when ε→0 of the value of random variable εV(t). And, consequently, under the conditions of heavy loading, virtual waiting time W(t) is $\frac{(T_1 + T_2)}{T_1}$ times larger than value V(t) of the incomplete procedure on servicing all requests being in a cyclic system at timepoint *t*. Thus, in virtue of equation (8), the stated assertion is obvious.

## 5. Results

A mathematical model of such network is considered to be a communication dynamical model by means of a single-line queueing network with the group requests input. Application of the system of the server with vacation and asymptotic analysis under the conditions of heavy loading has led to finding the system's main probability characteristics.

Taking into account the structure of the cycle of public resource allocation, i.e. the access intervals of the requests belonging to different flows proceeding to the server, it has been justifies that multivariable distribution is factored and each multiplicand is determined by the exponential distribution described above.

## 6. Conclusion

This paper describes the research of the mathematical model of the cyclic communication network with multiple access. The dynamic communication model in the form of a single-line queuing model with the batch input claims was investigated. The method of asymptotic analysis of the communication channel under a heavy load conditions was applied.

It is shown that distribution of the considered model is exponential with parameters defined by (15). The authors has found that distribution allows performing parametric optimization of communication networks, establishing the most appropriate values of the network parameters.

The revealed characteristics of the communications models allow one to perform parametric optimization of communication networks, establishing the most appropriate values of the network parameters.

## References

[1] Sonkin M and Yampolsky V 2008 Generalized properties of special communication and monitoring systems for remote and mobile objects *Bulletin of the TPU* **312(2)** 154-56
[2] Sonkin M, Pogrebnoy V and Pogrebnoy A 2008 Optimizing the use of communication resources in a surface weather observation network *Bulletin of the TPU* **313(5)** 46-50
[3] Sonkin M and Shamin A 2008 Optimization of operation of multi-channel distributed information and telecommunication systems for remote objects *Bulletin of the TPU* **313(5)** 51-54
[4] Saaty T 1971 Elements of queuing theory and its applications (Moscow, Soviet radio press)
[5] Bronstein O 1976 Spiritual IM priority service models in information and computing systems (Moscow, Nauka press)
[6] Leibowitz M 1961 An Approximate Method for Treating a Class of Multiqueue Problems *IBM J. Research Dev* **5(3)** 204-9
[7] Sonkin M and Yampolsky V 2011 The navigation of mobile objects monitoring system, mobile units and control centers *Problems of Informatics* **2(10)** 4-10
[8] Miroshnikov A, Sergeichik S and Kharlamov S 2005 Integrated system documented and data communication *Thematic collection of Communication and automation MVD Russia*
[9] Sonkin M, Pecherskaya E and Novikov S 2012 Technical solutions for the creation of public warning systems on the basis of information and telecommunication systems P-166 ITK OS *COO "International STS instrumentation and metrology"* **12**
[10] Artalejo J 1999 Accessible bibliography on retrial queues *Mathematical and Computer Modelling* **30(3-4)** 1-6
[11] Artalejo J 1999 A classified bibliography of research on retrial queues *Progress in 1990-1999 Top 7* 187-211
[12] Artalejo J 1999 Retrial Queuing Systems *Mathematical and Computer Modelling* **30(3-4)** 100-228
[13] Artalejo J 2006 Algorithmic Methods in Retrial Queues *Annals of Operations Research* **141** 10-301
[14] Vishnevskiy V and Semenova O 2007 Polling systems: theory and applications in broadband wireless networks (Moscow, Technosphere press) 312
[15] Vishnevsky V ans Semenova O 2006 Mathematical methods of research polling systems *Automation and Remote Control* **2** 3-56

[16] Nazarov A and Terpugov A 2004 Queueing theory: a tutorial (Tomsk, NTL press) 228
[17] Gnedenko B and Kovalenko I 2007 Introduction to queuing theory ed D (Moscow, Publishing House LKI) 400
[18] Borovkov A 1980 Asymptotic methods in queuing theory (Moscow, Nauka press) 381
[19] Anisimov V1984 Asymptotic methods of analysis of stochastic systems (Tbilisi, Mitsniereba Press) 178
[20] Moses E and Nazarov A 2013 Investigation RQ-system MMPP|GI|1 method of asymptotic analysis in a high load *Vestnik TSU. Management, Computer Science and Informatics* **4(25)** 84-94