

An inventory model with random demand

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Abstract. The article describes a three-product inventory model with random demand at equal frequencies of delivery. A feature of this model is that the additional purchase of resources required is carried out within the scope of their deficit. This fact allows reducing their storage costs. A simulation based on the data on arrival of raw and materials at an enterprise in Kazakhstan has been prepared. The proposed model is shown to enable savings up to 40.8% of working capital.

1. Introduction

Any company is interested in minimizing inventory storage costs. A number of different stochastic inventory models have been created that allow optimizing enterprises in one way or another. A summary of the inventory theory is presented in the monograph by D Buchan and E Koenigsberg [1].

In [2], the authors conventionally divide the inventory model to the single-stage and multi-stage models. The single-stage models are used mostly in commercial enterprises which functioning is seasonal in nature, and therefore one product purchase is made only once from a single supplier with certain delivery conditions, whereas the other purchase is carried out from another specific provider. The multi-stage models are used in exactly opposite situations when every stage is characterized by the purchase of each product under different conditions of buying. Therefore, multi-stage models are more relevant in the current market context, since one and the same product is offered by a variety of suppliers. However, the model proposed in [2] is limited by the fact that the product can not be ordered from several suppliers at the same time at the same stage. It is also worth noting that to improve the rapidness of decision-making, a multiple launch of the effective solution search procedure is required.

O A Kosorukov and O A Sviridov in their works [3,4] propose a stochastic model which is based on the principle of balancing costs at enterprises of different types. It also takes into account the uncertainty regarding not only the demand, but also the time of delivery. The publication [4] aims at identifying time points of order delivery, which will support the stocks at optimal levels and reduce storage costs and losses resulting from the deficit of goods. This model takes into account the uncertainty of demand, simultaneously considering a case when the order is delivered right on time without delay.



In [5], the authors use a database on the history of sales of several thousand types of products, many of which refer to the group of panic buying goods. To forecast the time series values, the paper proposes the adaptive method.

A mathematical model of inventory management at random seasonal demand is described in [6,7].

A special type of the inventory model based on an assumption of getting additional revenue by optimizing the assortment set of products is formulated in [8]. Here, the demand and inventory stocks are also random variables. It should also be noted that in the case of unsteady demand, a division of the forecast period of time into several intervals is made, in which the stationary condition is observed.

In [9,10], the authors describe the problem of managing multiproduct stocks in a possible nonstationary demand, while its statistical characteristics are unknown. At the same time, there is information about an observation of changes at the level of demand for goods. The author specifically proposes to emphasize trends of changes in demand and then classify goods according to the ABC-method. The author uses the expert-statistical approach (the analog method) for prediction. In the end, the deterministic multiproduct inventory management problem.

Paper [11] is interesting for its decision diagram making procedure. This diagram is constructed for the random delay time and random demand in the (s, S) -strategy. The essence of the strategy lies in the fact that the value of stock y after satisfying the requirements each time is compared with level s and, on condition of $s > y$, an order in the amount of $(S-y)$ is made. Thus, the task of determining the optimum levels of s, S for random delay and random demand is set. Solving to the problem is based on the adaptive method, and optimum levels s, S are determined by the condition of the minimum in the mean-square deviation of the current costs from the required optimum. Also, such a method is used in [12,13].

Another optimization criterion is suggested by the authors of [14]. The company's average profit from sales of purchased goods in response to changes in demand for goods is used as the optimization criterion here. In this way, to solve the problem of a multiproduct order, it is necessary to maximize the average net income. This criterion differs from the conventional ones in that it is not required to minimize losses, but to maximize the average profit taking into account the costs for the enterprise to create the stock.

In solving the inventory management problem, there may be a number of limitations that should be considered. The authors of [15] approach this problem with regard to the restrictions on the supply and demand, storage capacity, etc. One of limitations in solving the inventory problem could be the amount of capital [1]. This limitation means that the cost of all the resources may not exceed sum Y_m . For this reason, let us introduce normalization factor K , which is equal to the ratio of the maximum total value of resources, to the sum of their maximum values. Also, the inventory problem with a limited amount of capital and a study of normalization factor K is viewed in other research works [16,17].

The author of [18] describes the results of developing theoretical and practical methods for solving the inventory problem. He focuses on the fact that there is no universal method for solving this problem in terms of demand uncertainty, and argues that the inventory management problem in terms unsteadiness remains relevant because without additional experimental studies it can not be argued that the relatively simple models and algorithms are of a sufficient quality.

In our paper, we propose one more variant of the model with random demand, which feature is that the missing resources are replenished to the amount of the deficit in a specific resource. This model will allow significant savings for stockpiling.

2. The Model

A company buys three types of resources. The amount of the first resource is q_1 in physical indicators, the cost per resource unit is d_1 of currency units; the amounts of the second resource and price are q_2 and d_2 , respectively; the value and the price of the third resource are q_3 and d_3 , respectively. Let us assume periods of use for each resource (cycle) type to be the same and equal to $T_1 = T_2 = T_3 = T$. The

amount of funds for the purchase of resources is limited to the value of $Y_m \leq d_1q_1 + d_2q_2 + d_3q_3$. For definiteness, let us assume that Resource 1 is completely purchased at the beginning of a period, Resource 2 – partially to extent k_2q_2 , where $k_2 \leq 1$ is the share of the second resource, and Resource 3 is bought partially to extent k_3q_3 where $k_3 \leq 1$ is the share of the third resource. Then

$$d_1q_1 + k_2d_2q_2 + k_3d_3q_3 = Y_m. \quad (1)$$

In contrast to [19], we will assume that the expenditure of resources means random processes subjected to stochastic equations

$$\begin{aligned} dq_1 &= \mu_1 dt + \sigma_1 dw_1, \\ dq_2 &= \mu_2 dt + \sigma_2 dw_2, \\ dq_3 &= \mu_3 dt + \sigma_3 dw_3. \end{aligned} \quad (2)$$

Here, μ_1, μ_2, μ_3 are the average values per time unit of random variables x_1, x_2, x_3 , so the average expenditure rates are $\mu_1 = \frac{q_1}{T}, \mu_2 = \frac{q_2}{T}, \mu_3 = \frac{q_3}{T}$; whereas $\sigma_1^2, \sigma_2^2, \sigma_3^2$ are the dispersions per time unit for random variables x_1, x_2, x_3 , respectively (the dimension is q^2/T); and dw_1, dw_2, dw_3 are standard Wiener processes.

Solutions of equations (2), provided that the resources are expended simultaneously in the same proportion, but at random, are [20]:

$$\begin{aligned} x_1(t) &= \mu_1 t + \sigma_1 \sqrt{t} \cdot \varepsilon, \\ x_2(t) &= \mu_2 t + \sigma_2 \sqrt{t} \cdot \varepsilon, \\ x_3(t) &= \mu_3 t + \sigma_3 \sqrt{t} \cdot \varepsilon, \end{aligned} \quad (3)$$

where ε is the standard normal random variable with mean $M(\varepsilon)=0$ and second-initial moment $M(\varepsilon^2)=1$.

With $t_0 = 0$, we will obtain the solution: $x_1(t_0) = 0, x_2(t_0) = 0, x_3(t_0) = 0$.

We will find the t_2 and t_3 time points and replenishment of the second resource to the value of $q_2(1 - k_2)$ and the third resource to the value of $q_3(1 - k_3)$ from conditions (4):

$$\begin{aligned} d_1x_1(t_2) + d_2x_2(t_2) + d_3x_3(t_2) &= a_2(1 - k_2) + a_3(1 - k_3); \\ d_1x_1(t_3) + d_2x_2(t_3) + d_3x_3(t_3) &= a_3(1 - k_3), \end{aligned} \quad (4)$$

where k_2 is the share of the second resource at the initial time point; k_3 is the share of the third resource at the initial time point; $a_2(1 - k_2)$ is the deficit of the second resource in value terms; and $a_3(1 - k_3)$ is the deficit of the third resource in value terms.

As a result, we will obtain the following equations to determine coefficients k_2, k_3 and moments for additional purchase of the second and third resources:

$$\begin{aligned} bt_2 + c\varepsilon\sqrt{t_2} &= a_2(1 - k_2) + a_3(1 - k_3), \\ k_2a_2 &= b_2t_2 + c_2\varepsilon\sqrt{t_2}, \\ bt_3 + c\varepsilon\sqrt{t_3} &= a_3(1 - k_3), \\ k_3a_3 &= b_3t_3 + c_3\varepsilon\sqrt{t_3}. \end{aligned} \quad (5)$$

Here $a_1 = d_1q_1, a_2 = d_2q_2, a_3 = d_3q_3, a = a_1 + a_2 + a_3; b_1 = d_1\mu_1, b_2 = d_2\mu_2, b_3 = d_3\mu_3, b = b_1 + b_2 + b_3; c_1 = d_1\sigma_1, c_2 = d_2\sigma_2, c_3 = d_3\sigma_3, c = c_1 + c_2 + c_3$.

Values k_2, k_3, t_2, t_3 found from the solution of system (5) are random. We are interested in the statistical characteristics of these values. Let us put the expressions for the first two moments of values t_2, t_3 :

$$M(t_2) = \frac{a_2 + a_3(1 - k_3)}{b + b_2} + \frac{(c + c_2)^2}{2(b + b_2)^2}, \quad (6)$$

$$M(t_3) = \frac{a_3}{b + b_3} + \frac{(c + c_3)^2}{2(b + b_3)^2}, \quad (7)$$

$$\sqrt{D(t_2)} = \sqrt{\frac{(c + c_2)^2}{4(b + b_2)^2} (5(c + c_2)^2 + (a_2 + a_3(1 - k_3))(b + b_2))}, \quad (8)$$

$$\sqrt{D(t_3)} = \sqrt{\frac{(c + c_3)^2}{4(b + b_3)^4} (5(c + c_3)^2 + 4(b + b_3)a_3)}, \quad (9)$$

and the mean value and standard deviation of the time for resource replenishment (cycle termination time) are as follows:

$$M(t_c) = \frac{Y_\Sigma}{b} + \frac{c^2}{2b^2}, \quad \sqrt{D(t_c)} = \frac{c}{2b^2} \sqrt{5c^2 + 4bY_\Sigma}. \quad (10)$$

The average value of normalization factor $K_m = Y_m / Y_\Sigma$ is as follows:

$$M(K_m) = \frac{M(Y_m)}{Y_\Sigma} = \frac{M(Y_m)}{a}. \quad (11)$$

3. Numerical simulation results based on business data

The data on June, 2014, on the arrival of raw and materials at the enterprise LLP "JV HS-Plast (SP VG-Plast, Kazakhstan, <http://www.gammagroup.kz/64/>)" have been used as a model of initial data. The company's products include PVC window boards, PVC wall panels, etc.

Table 1 provides data on stocks of raw and materials on June, 2014. All the funds are in Kazakh Tenge (KZT).

Table 1. Data on the arrival of raw and materials

Source, number	Material	Measurement unit	Amount	Price, KZT
1	Additives	kg	17900	11827160
2	Microsuspension	kg	58000	10234204
3	Hydrophobic chalk	kg	201915	7505878

Let us calculate the working capital spent for the case when three kinds of resources are purchased (see Table 1). We assume that at the cycle beginning, the first resource is purchased in full, whereas the second and third resources in part. In addition, we consider $a_1 > a_2 > a_3$ (see the last two columns of Table 1). Let us presume all resources to be consumed simultaneously in the same proportions, but randomly.

Table 2. Working capital to stockpile

The real expired funds, KZT	The minimally required funds, KZT	Benefit, %
29567242	17517594	40.8

Table 2 shows the real working capital for creating stocks and minimally required funds. It also shows the profit in % to create the stock of working capital.

Thus, according to the model, Resource 1 should be purchased in its entirety at the beginning of the period, and the second and third resources should be purchased in volumes $k_2=0.408$ and $k_3=0.202$, respectively. Then the time point for the additional purchase of Resource 2, on condition of the working capital deficit at the beginning of the cycle, is $t_2=12$ days and the time point for the additional purchase of Resource 3 is $t_3=6$ days.

4. Conclusion

The proposed model provides that resources are replenished to the amount of the deficit of the given resource. In addition, the demand for products is random, and the supply frequency of all resources is the same. In the simulation with real data, it was revealed that the benefit when using the constructed model is 40.8% when three types of resources are purchased.

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References

- [1] Bukan D and Kenigsberg E 1967 *Scientific inventory management* (Moscow: Nauka)
- [2] Kuznetsov D N and Tolstykh S S 2007 *Upravlenie obshchestvennymi i ekonomicheskimi sistemami* **1** 1-6 (in Russian)
- [3] Sviridova O A 2011 *Logistics* **4** 28-30 (in Russian)
- [4] Kosorukov O A and Sviridova O A 2012 *Logistics* **6** 12-13 (in Russian)
- [5] Belyakov A G, Lapin A V and Mandel' A S 2005 *Problemy upravleniya* **6** 40-45 (in Russian)
- [6] Kozak Yu A and Kolchin R V 2004 *Works of Odessa Polyt. Univ.* **1(21)** 133-136 (in Russian)
- [7] Dombrovskiy V V and Chausova E V 2000 *Tomsk State Univ. J.* **271** 141-146 (in Russian)
- [8] Petrenko S V and Karlova M Yu 2010 *Science Prospects* **5(07)** 120-124 (in Russian)
- [9] Mandel' A S 2011 *Problemy upravleniya* **6** 47-51 (in Russian)
- [10] Mandel' A S 2012 *Problemy upravleniya* **1** 42-6 (in Russian)
- [11] Antipenko V S and Kats G B 1974 *Avtomat. i Telemekh.* **7** 178-182 (in Russian)
- [12] Chen X and Simchi-Levi D 2004 *Math. of Oper. Research* **29** 3 698-723
[dx.doi.org/10.1287/moor.1040.0093](https://doi.org/10.1287/moor.1040.0093)
- [13] Arreola-Risa A and DeCroix G A 1998 *Naval Research Log.* **45** 7 687-703 DOI: 10.1002/(SICI)1520-6750(199810)45:7<687::AID-NAV3>3.0.CO;2-7
- [14] Seraya O V, Klimenko T A and Samorodov V B 2009 *Vestnik Khar'kovskogo natsional'nogo avtomobil'no-dorozhnogo universiteta* **45** 31-4 (in Russian)
- [15] Bochkarev A A and Bochkarev P A 2014 *Logistika i upravlenie tsepyami postavok* **1** 37-42 (in Russian)
- [16] Kulakova Yu N 2012 *Logistika i upravlenie tsepyami postavok* **3** 76-83 (in Russian)
- [17] Kulakov A B and Kulakova Yu N 2013 *Economic Analysis: Theory and Practice* **29(332)** 58-62 (in Russian)
- [18] Dzenzelyuk N S 2011 *Vestnik Yuzhno-Ural'skogo gosudarstvennogo universiteta Seriya: Ekonomika i menedzhment* **21(238)** 27-31 (in Russian)
- [19] Mitsel A A and Alimkhanova D A 2015 *Economic Analysis: Theory and Practice* **40(439)** 55-66 (in Russian)
- [20] Shiryaev A N 1999 *Essentials of Stochastic Finance: Facts, Models, Theory* vol 1 (Singapore: World Scientific Publishing Company)