The model and the planning method of volume and variety assessment of innovative products in an industrial enterprise

V G Anisimov[1], **E G Anisimov**[2], **T N Saurenko**[3] **and M A Sonkin**[4]

[1] Peter the Great St. Petersburg Polytechnic University, 29, Polytechnic St., St. Petersburg, 195251, Russia
[2] Russian customs academy, 4, Komsomolsky Ave., Lyubertsy, 140009, Russia
[3] Peoples' Friendship University of Russia, 6, Miklukho-Maklaya St., Moscow, 117198, Russia
[4] Tomsk Polytechnic University, 30, Lenina Ave., Tomsk, 634050, Russia

E-mail: sonkin@tpu.ru

Abstract. In the long term, the innovative development strategy efficiency is considered as the most crucial condition for assurance of economic system competitiveness in market conditions. It determines the problem relevance of such justification strategies with regard to specific systems features and conditions of their operation. The problem solution for industrial enterprises can be based on mathematical models of supporting the decision-making on the elements of the innovative manufacturing program. An optimization model and the planning method of innovative products volume and variety are suggested. The feature of the suggested model lies in the nonlinear nature of the objective function. It allows taking into consideration the law of diminishing marginal utility. The suggested method of optimization takes into account the system features and enables the effective implementation of manufacturing capabilities in modern conditions of production organization and sales in terms of market saturation.

1. Introduction

Regardless of particular enterprises characteristics, the establishment of the appropriate volume and the variety of innovative products is the most important task in product strategies and manufacturing programs justification. The effective method of solving this problem with regard to high volumes and large variety is mathematical modeling [1 - 3]. Various types of linear programming models or discrete programming models with linear objective function are used mostly in the innovative management system of supporting the decision-making [4-6]. Such models allow describing the process of product development, manufacturing and consuming in the absence of market saturation [7]. At the same time, enterprise management needs models, which will allow taking into consideration the fact that the market is getting saturated, while increasing the volumes as a result of the law of diminishing marginal utility, and the linear nature of the utility function is disrupted [6,8 - 10]. In this case, the use of traditional models for supporting the decision-making on innovative product strategy formation and manufacturing program can lead to some significant errors [6, 7, 11 - 14] and, finally, does not allow effective implementation of enterprise manufacturing capabilities [15 - 17]. Model formation, which considers conditions listed above, presents the main purpose of this article.

2. Model description

The problem of the appropriate innovative products range and volumes establishment can be formalized as following.

Let us determine set $J = \{1,2, \ldots j, \ldots, N\}$ as types of needs (demand), set $I = \{1,2, \ldots i, \ldots, M\}$ as product items (the variety of products), which are able to meet the whole set of needs. It is necessary to choose from set I of acceptable product types such subset $I^* \subset I$ (the optimum variety of items), which provides the maximum needs satisfaction with regard to limited costs for development, manufacturing and sales.

It determines:

c_i^0 – fixed costs of development and manufacturing for the i-th type of product (initial costs);

$\alpha_{i,j}$ – the quantity of the i-th type of product, which is necessary for meeting the needs of one unit j –th type of product;

c_i – the cost of production and sales of one i-th type product;

b_j – the demand of the j-th type of product;

M_0 – the number of items types that can be included in the optimal row of I^*;

R_i – the maximum possible quantity of the i–th type product according to the manufacturing plan.

The relationship between elements of sets I and J is described as matrix applications $\|\beta_{i,j}\|$, in which $\beta_{i,j} = 1$, if the product of the i-th type can ensure the j-th type of needs, and $\beta_{i,j} = 0$ otherwise. Regarding the listed definitions, the costs associated with meeting the demand of the j-th type of needs and the i-th type of product are determined by ratio

$$c_{i,j} = \begin{cases} \alpha_{i,j} c_i b_j, & if\ \beta_{i,j} = 1 \\ \infty, & if\ \beta_{i,j} = 0 \end{cases}, \tag{1}$$

Control variables are determined in the form:

$$y_i = \begin{cases} 1, & \text{if the } i - \text{th type of product is included in the range, so that } \sum_{j=1}^{N} \alpha_{i,j} \geq 1 \\ & 0, \text{ otherwise} \end{cases}, \tag{2}$$

$$x_{i,j} = \begin{cases} 1, & \text{if the i} - \text{th type of product meets the j} - \text{th type of need, so that } \alpha_{i,j} \geq 1 \\ & 0, \text{ otherwise} \end{cases}, \tag{3}$$

As to the determined notation, the problem of manufactured products volumes and variety optimization can be formalized as follows.

To determine plan

$$a^* = \|a_{i,j}\|, a_{i,j} = \alpha_{i,j}^* x_{i,j},\ i = 1,2, \ldots M, j = 1,2 \ldots N, \tag{4}$$

Item production from variety I and its distribution to meet the needs are defined by set J, for which

$$W(a^*) = \sum_{j=1}^{N} W(a_j^*) \geq (1 - \mu) \cdot \max_{\alpha_j} \sum_{j=1}^{N} W_j(a_j), \tag{5}$$

With restrictions:

$$0 \leq \mu \leq 1, \tag{6}$$

$$\alpha_{i,j} = 0,1,2 \ldots, \tag{7}$$

$$\sum_{i=1}^{M} \alpha_{i,j} x_{i,j} \geq 1,\ j = 1,2, \ldots N \tag{8}$$

$$x_{i,j} \leq y_i,\ i = 1,2, \ldots M, j = 1,2, \ldots N \tag{9}$$

$$\sum_{i=1}^{M} y_i \leq M_0,\ M_0 \leq M, \tag{10}$$

$$C = \left(\sum_{i=1}^{M} c_i^0 y_i + \sum_{i=1}^{M} \sum_{j=1}^{N} c_{i,j} x_{i,j}\right) \leq C^{max}, \tag{11}$$

$$\sum_{j=1}^{N} a_{i,j} \leq R_i,\ i = 1,2, \ldots I, \tag{12}$$

where:

$a_j,\ j = 1,2, \ldots N$ is the j-th column of matrix (4);

$W(a)$ – a positive non-decreasing concave function, reflecting the full effect of the products used in accordance to the plan (4);

$W(a_j)$ – a positive non-decreasing concave function, reflecting the effect of the α_j products used to meet the j-th needs;

μ - allowed relative deviation of function $W(a^*)$ from its maximum possible value;

C – the cost of included in the variety products development, production and sales;

C^{max}- the maximum allowable value of included in the variety products development, production and sales.

Condition (5) formalizes the goal of the commercial strategy and the production program.

Condition (6) shows the requirement for the accuracy of optimization problem's solution.

Constraint (7) takes into account the indivisibility of goods;

Constraint (8) means that all needs of the J set must be met.

Constraint (9) means that meeting a certain type of need, just those included in the product range products, can be assigned.

Constraint (10) does not allow including more than M_0 products in optimal range I^*.

Constraint (11) imposes a limit on the cost of the product development, production and sales.

Constraint (12) takes into account the limited production possibilities and allows including only the number of i-th type of products which can be produced in the plan from set I of needs defined by set J.

Ratios (1) - (12) formalize the analyzed problem as a nonlinear discrete optimization model of innovative products manufacturing volumes and range regarding the conditions of deterministic demand. This model belongs to the class of polynomial hard (NP - hard) problems [2, 6, 7, 18]. Effective optimization techniques for such models can be created only by taking into account the features of each of them [6]. In particular, no current effective method of optimization for model (1) – (12) determined the need of its creation in the framework of this article.

3. Method description

To solve problem (1) - (12), we propose a method that implements the procedure of directed search. The basis of the iteration algorithm is a schematic diagram of the branch and border method [4-7, 10, 19]. Building the tree of possible options is carried out in accordance with the dichotomous scheme. Each branch of the tree options is set $S = \{\delta_{i,j}\}, i = 1,2, \dots M, j = 1,2, \dots N$ of such variables $\delta_{i,j} = \{0, 1\}$ that the elements of the corresponding S–th fragment $a^s = \|a^s_{i,j}\|$ branch of plan (4) satisfy the condition:

$$a^s_{i,j} = \sum_{\delta_{i,j} \in S} \delta_{i,j},\ i = 1,2, \dots M, j = 1,2, \dots N, \tag{13}$$

In terms of the product range formation problem, the variables have the following meanings:

$$x_{i,j} = \begin{cases} 1, & \text{if the i} - \text{th type of product is distinguished at a specific process stage} \\ & \text{for meeting the j} - \text{th type of need} \\ 0, & \text{otherwise} \end{cases}$$

It determines:

G_S - the set of variables $\delta_{i,j}, i = 1,2, \dots M, j = 1,2, \dots N$, which can be included in the S-th branch of the tree options without constraint violations (8), (9), (10), (11), (12);

$W(a^s)$ - the implementation of the effect of the relevant S-th branch of the α^s fragment to meet product needs;

$\Delta W(\alpha^s + \delta_{i,j})$ - increment function $W(\alpha^s)$ for inclusion of variable $\delta_{i,j} = 1$ in the S-th branch of the tree variants;

$Q(G_S)$ - the estimate of the increment upper bound effect by inclusion variables from set G_S in the plan;

$P_s(\delta_{i,j} = 0)$ - the estimate of the upper bound decision for the S-th branch solution regarding the earlier variable inclusion of $\delta_{i,j} = 0$;

$P_s(\delta_{i,j} = 1)$ the estimate of the upper bound decision for the S-th branch solution regarding the earlier variable inclusion of $\delta_{i,j} = 1$.

According to the notations for estimating the upper bounds of the solutions regarding the inclusion of $\delta_{i,j} = 0$, $\delta_{i,j} = 1$ variables in the S-th branch, the following ratios are used

$$P_s(\delta_{i,j} = 0) = W(a^s) + Q(G_S^1)\,, \tag{14}$$

$$P_s\big(\delta_{i,j} = 1\big) = W(a^s) + \Delta W\big(a^s + \delta_{i,j}\big) + Q(G_S^1)\,, \tag{15}$$

where G_s^1 is the set of variables $\delta_{i,j}, i = 1,2, \dots M,\ j = 1,2, \dots N$ which, after setting selected $\delta_{i,j}$ variable value $\delta_{i,j} = 0$ or $\delta_{i,j} = 1$ in the next step of branching, can be added to this branch without violating restrictions (8), (9), (10), (11), (12). It is obtained by removing the selected variable $\delta_{i,j}$ from G_s;

$Q(G_s^1)$ - the estimate of the upper bound of the increment effect by inclusion of variables from set G_s^1in the plan.

Value $Q(G_s^1)$ can be represented in the form of

$$Q(G_s^1) = \sum_{i=1}^{M} Q_i(G_s^1)\,, \tag{16}$$

where $Q_i(G_s^1)$ - the estimate of the upper bound of the increment effect by the inclusion in the plan of all i-th type variables from set G_s^1.

Calculation $Q_i(G_s^1)$, $i = 1,2, \dots M$, is performed using the following algorithm:

1. Put $a^\alpha = a^s$.
2. Put $Q_i(G_s^1) = 0$.
3. Verify the fulfillment of conditions (9), (10), (11), (12)

If, at least, one is not met, go to step 9. Otherwise, go to the next one.

4. Build matrix $W^a = \left\|\Delta W(a^\alpha + \delta_{i,j}\right\|, \delta_{i,j} \in G_S^1$.
5. Determine maximum element $\Delta W_i^* = \max_{\delta_{i,j}}\left\{\Delta W(a^\alpha + \delta_{i,j})\right\}, j = 1,2, \dots N$ and corresponding index j^*.
6. Put $Q_i(G_S^1) = Q_i(G_S^1) + \Delta W_i^*$
7. Put $a^\alpha = a^\alpha + \delta_{i,j^*}$, where $\delta_{i,j^*} = 1$.
8. Go to step 3.
9. The end.

This algorithm allows determining the upper estimate of the increment effect for the i-th type of product ($i = 1,2, \dots M$), which design and application does not violate constraints (9), (10), (11), (12) at each step of branching.

Obviously, $W_j\big(a_j\big)$, $j = 1,2, \dots N$ is a positive non-decreasing concave function; consequently, $W(a^s) = \sum_{j=1}^{N} W_j(a_j^s)$ is also a positive non-decreasing concave function, and their increments $\Delta W(a^s + \delta_{i,j})$ are non-increasing. So we have ratios

$$\forall\big(G_S^*: G_S^* \subseteq G_S^1\big) \to Q_i(G_S^1) \geq Q_i(G_S^*), \tag{17}$$

$$Q_i(G_S^1) \leq R_i^s \max_{\delta_{i,j} \in G_S^1}\left\{\Delta W(a^s + \delta_{i,j)}\right\}, \tag{18}$$

From (16), (17) with (15) it follows that

$$\forall\big(G_S^*: G_S^* \subseteq G_S^1\big) \to W(G_S^1) \geq W(G_S^*), \tag{19}$$

$$Q(G_S^1) \leq \sum_{i=1}^{I} R_i^S \max_{\delta_{i,j} \in G_S^1}\left\{\Delta W(a^s + \delta_{i,j)}\right\}, \tag{20}$$

where $R_i^S = R_i - \sum_{j-1}^{N} a_{i,j}^s,\ i = 1,2, \dots M$.

From (19) it directly follows that amount $Q(G_s^1)$ determined by formula (16) based on the procedures, characterizes the upper estimate of possible increment function $W(a^s)$ on set G_S^1, and relations (14), (15) - upper boundary solutions for the respective sequels S-th tree branch of options.

An important element of the branch-and-bound algorithm, significantly affecting the average convergence of the optimization problem solution is the method of selecting the next variable $\delta_{i,j} \in G_s^1$ for inclusion in the S-th branch of the tree options at each step of branching. In the proposed method let us select matrix

$$\Delta W^S = \left\|\Delta W_{i,j}^S\right\|, \ i = 1,2, \dots M, j = 1,2, \dots N, \tag{21}$$

the elements of which are determined by formula

$$\Delta W_{i,j}^S = \begin{cases} \Delta W(a^s + \delta_{i,j)}, & if \ \delta_{i,j} \in G_S^1 \\ 0, & if \ \delta_{i,j} \notin G_S^1, \end{cases} \tag{22}$$

The rule of row selection is the following: for each row of matrix (21), the magnitude is determined by

$$\left.\begin{array}{r} \max \quad \Delta W_{i,j^*}^S = \max_j\{\Delta W_{i,j}^S\} \\ d_i = \min_r\{\max \quad \Delta W_{i,j^*}^S - \Delta W_{i,r}^S\} \end{array}\right\}, i = 1,2, \dots M, r = 1,2, \dots N, r \neq j^* \tag{23}$$

maximum element max $\Delta W_{i,j^*}^S$ and difference d_i between it and the nearest largest element is determined.

Similarly, for each column of matrix (21), the magnitude is determined by

$$\left.\begin{array}{r} \max \Delta W_{i^*,j}^S = \max\limits_i\{\Delta W_{i,j}^S\} \\ b_j = \min\limits_r\left\{\max \Delta W_{i^*,j}^S - \Delta W_{r,j}^S\right\} \end{array}\right\}, r = 1,2, \dots M, r = 1,2, \dots N, r \neq i^*$$

Then let us choose indices i, j for which $d_i = b_j = c$, where

$$c = \max\left\{\max_i[d_i], \max_j[d_j]\right\}, i = 1,2 \dots M, j = 1,2, \dots N$$

Among the maximum elements allocated in the rows (columns) we choose variable $\delta_{i,j}$, which is the largest in magnitude and includes the step of branching in the S-th branch of the tree options.

Each s branch of the tree ends, if $G_S^1 = \emptyset$ (i.e., received a valid decision) or if

$$P_S(\cdot) \leq W_0(1 - \mu), \tag{24}$$

where W_0 is the objective function value for the best previously obtained feasible solution (record).

The procedure of finding a solution ends if condition (24) is fulfilled for all the remaining branches. Last record W_0 is the desired optimal value of the objective function, and matrix (4), the elements of which are defined by formula (14) for branches S_0 of the tree, corresponding to record W_0, is the optimal plan for the innovative products production.

In practice, the tree traversal options method should be organized in accordance with the rule, "go to the right". The rule means that when building each branch S of set G_S^1, many possible variables are selected $\delta_{i,j} = 1$. When the branch ends, it returns to last $\{\delta_{i,j}\} \subset S$ in the list variable $\delta_{i,j} = 1$, value $\delta_{i,j} = 0$ is assigned and the expansion of the new branch includes variables $\delta_{i,j} = 1$ from set G_S^1. With the tree traversal options method of establishing the optimal solution (condition (24) for all remaining branches) corresponds to the second return branching in the root node.

The rule enables the analysis of all possible plan options and eliminates the reviewing. For its implementation, it is sufficient to store only the current plan fragment, the smallest of the previously obtained objective function values and corresponding to this value of the tree options branch in computer memory.

The tree traversal options method in combination with the method of selecting variables at each step of the branching algorithm is an approximate solution of problem (1) - (12), allowing obtaining a first valid solution in a finite number of steps. Final matrix (5) reflects the set of products included in optimal assortment I^*:

$$\forall(i: a_{i,j} \geq 1) \rightarrow (y_i = 1, i \in I^*), \tag{25}$$

And the optimal plan for their application to meet the needs is determined by set J.

4. Conclusion

A model and planning method for innovative products volume and variety at an enterprise is suggested in this article. The nonlinear nature of the subject function, compared with popular models, allows taking into consideration the law of diminishing marginal utility. The suggested method of optimization takes into account the system features and allows obtaining accurate solutions. This creates preconditions for improvement of the quality of innovative development strategy decisions.

The considered model and the optimization problem solution method of innovative products volumes and variety eliminate a significant disadvantage of the popular models, which consists in the assumption of target function linearity. This assumption does not optimize the volume and the range of products in the typical modern organization when the market is saturated.

5. Acknowledgments

The proposed model is relatively easily integrated in a specific system to support decision-making [6, 7, 13, 15, 20, 21], as key performance indices and limitations of model (1) – (12) are quite general, which allows creating the basis for a wide range of specific techniques for rationalizing strategy elements for innovative development at industrial enterprises to ensure their competitiveness in modern market conditions.

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