

DYNAMICAL EFFECTS IN INVARIANT  
COORDINATES FOR  $dp$  BREAKUP\*I. SKWIRA-CHALOT<sup>a</sup>, I. CIEPAŁ<sup>b</sup>, ST. KISTRYN<sup>c</sup>  
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Regular studies of few-nucleon systems reveal various dynamical components, such as three-nucleon force, Coulomb force and relativistic effects, which play an important role in correct description of nuclear interaction. A large set of existing experimental data for  ${}^1\text{H}(d, pp)n$  reaction allows for systematic investigations of these dynamical effects, which vary with energy and appear with different strength in certain observables and phase space regions. In order to perform systematic comparisons with precise theoretical calculations, the experimental data are transformed to the variables based on the Lorentz invariants.

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## 1. Introduction

One of the simplest processes for testing the dynamics of three nucleons is deuteron breakup in collision with a proton. The rich kinematics of such process makes it selective with respect to the used model of interaction. At low- and medium-energy range, the general properties of few-nucleon systems are successfully described by realistic nucleon–nucleon ( $NN$ ) potentials, coupled-channel (CC) calculations with realistic potential including the excitation of a single nucleon to a  $\Delta$  isobar [1] or Chiral Perturbation Theory (ChPT) [2]. With increasing energy, the dynamical effects of few-nucleons, like three-nucleon force (3NF) [3, 4] and the relativistic component [5], start

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playing an important role and must be included in theoretical calculations. In the case of  $dp$  breakup reaction, the Coulomb force has also a crucial influence on the cross section, and also should be supplemented in theory. Over the last few years, a big effort for including all dynamical ingredients in theoretical calculations has been made. Currently calculations combining the 3NF and Coulomb effects are available [6, 7] as well as relativistic calculations including 3NF [8, 9]. High precision experimental data for the breakup process and precise calculations give an opportunity to study these very subtle dynamical effects.

The kinematics of  $dp$  reaction can be described in many different ways, *e.g.* using information about the energies and emission angles of registered nucleons or with the Jacobi momenta, which is a very practical method for description of reactions with few nucleons in the final state [10]. In order to regularly study a large set of experimental data collected in a wide range of energies, the description of the breakup kinematics, based on Lorentz-invariants, has been proposed [11]. In this paper, the experiment–theory comparison of 3NF and Coulomb force effects is done for cross section presented as a function of invariant coordinates.

## 2. Invariant coordinates

The Mandelstam variables for two-body reaction have been rewritten in a proper way for a breakup reaction  $p + d \rightarrow p^{(1)} + p^{(2)} + n$ . All three particles exist in the entrance channel, because a deuteron is treated as a proton–neutron pair flying together. In the exit channel, there are two identical protons, and one cannot say which of them,  $p^{(1)}$  or  $p^{(2)}$ , was free (or bound) before the interaction. Based on 4-momentum of proton  $p_p$  and deuteron  $p_d$  in the entrance channel and 4-momentum of two protons  $p_{p^{(1)}}$ ,  $p_{p^{(2)}}$  and neutron  $p_n$  in the exit channel, the four invariant coordinates have been instituted:

- the kinetic energy of relative motion of two nucleons (proton–proton ( $E_{\text{rel}}^{pp}$ ) and proton–neutron ( $E_{\text{rel}}^{pn}$ )) in the final state:

$$E_{\text{rel}}^{pp} = \sqrt{(p_{p^{(1)}} + p_{p^{(2)}})^2} - 2m_p, \quad E_{\text{rel}}^{pn} = \sqrt{(p_{p^{(1)}} + p_n)^2} - m_p - m_n,$$

- the energy transfer from a bound neutron or free proton in the entrance channel to a neutron ( $E_{\text{tr}}^n$ ) or one of two protons ( $E_{\text{tr}}^p$ ), respectively, in the exit channel:

$$E_{\text{tr}}^n = \frac{-(p_d/2 - p_n)^2}{2m_n}, \quad E_{\text{tr}}^p = \frac{-(p_p - p_{p^{(2)}})^2}{2m_p}.$$

### 3. Results

The breakup of a deuteron in collision with a proton at medium-energy range has been carried out at KVI in Groningen, using SALAD detector [11–13], which covered a wide range of phase space. The experimental cross sections of about 90 kinematical configurations compared with theoretical calculations show the significant role of the 3NF [14, 15] and the Coulomb force [15] in correct description of the differential cross section for the breakup reaction at 130 MeV. In this paper, the data are revisited to study the 3NF and Coulomb effects in terms of invariant coordinates.

Figure 1 presents the comparison of the net effects of 3NF predicted in theory (full circles and squares) and observed in the breakup experiment (open circles and squares) at 130 MeV. The  $y$ -axis represents the ratio  $\frac{\sigma_i - \sigma_j}{\sigma_j}$ , where  $\sigma_i$  denotes the theoretical (with the Tucson Melbourne (TM99) 3NF included or with the Urbana IX (UIX) 3NF and the Coulomb force included) or experimental differential cross section,  $\sigma_j$  indicates the theoretical calculations for pure CD-Bonn  $NN$  potential or Argonne V18 potential supplemented by the Coulomb force. By comparing the theoretical calculations with (AV18+UIX+C, full squares) and without (CDB + TM99, full circles) Coulomb force included, one can conclude that the net effects of the 3NF do not depend significantly on the chosen model of three-nucleon force and

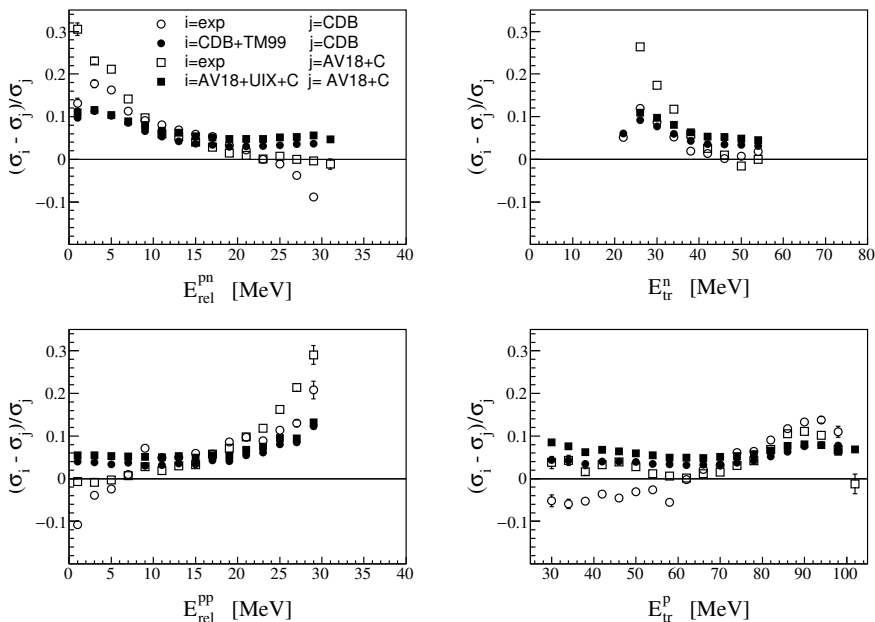


Fig. 1. Net effects of 3NF in the differential cross section of the  $dp$  breakup at 130 MeV, presented as a function of four invariants.

are practically the same for  $p$ - $d$  and  $n$ - $d$  systems. Comparing the open and full circles one can see that generally, experimental data reveal effects beyond the pure CD-Bonn  $NN$  potential, which are consistent with predicted influence of the 3NF. The disagreement between the data and theory for  $E_{\text{rel}}^{pp} < 10$  MeV and  $E_{\text{tr}}^p < 60$  MeV is due to the missing Coulomb force in the calculations. These differences decrease for the computations supplemented by the Coulomb force (open and full squares). Focusing on these results, one can see that, for example, for  $E_{\text{tr}}^n \approx 30$  MeV, the 3NF effect observed in the experiment (open squares) is almost twice larger than the predicted one (full squares) and for  $E_{\text{tr}}^n \approx 48$  MeV, this effect disappears in data while the theory predicts influence of three-nucleon force at the level of 5% (level of normalization accuracy). This observation shows that the changes of a magnitude of the 3NF with  $E_{\text{tr}}^n$  are more rapid in the experimental data than in the theoretical calculations. (A similar behaviour is observed for  $E_{\text{rel}}^{pn}$ .) This suggests, in turn, that either the models of three-nucleon force need improvement or the relativistic effects should be included in the calculations to obtain correct description of the data.

Figure 2 presents the distribution of the experimental (full circles) and theoretical (open circles and squares) cross sections entangled with acceptance of  $dp$  breakup experiment at 130 MeV, which were obtained for four invariant coordinates. (The experimental uncertainties are too small to be

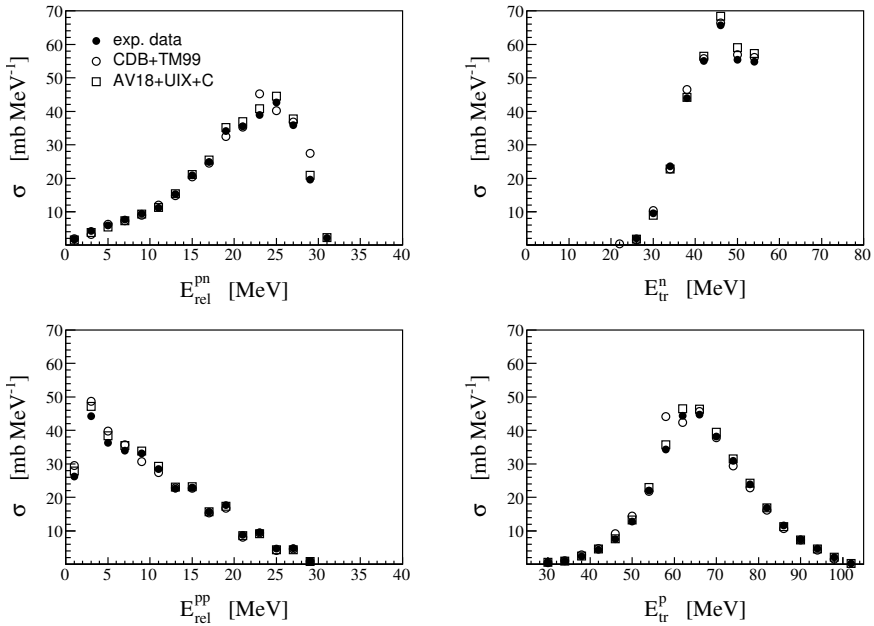


Fig. 2. Cross section of the  $dp$  breakup at 130 MeV, presented as a function of four invariants.

visible.) One can see that the strongest influence of the Coulomb interaction is observed for the largest values of  $E_{\text{rel}}^{pn}$  and the smallest  $E_{\text{rel}}^{pp}$ . Moreover, for an almost whole range of  $E_{\text{tr}}^p$ , the theoretical calculations including 3NF and the Coulomb force provides the best description of the experimental data. The remaining small discrepancies can be ascribed to systematic uncertainties of the experimental data or to contribution of the relativistic effects.

In order to study the electromagnetic interaction, a very forward part of available phase space has been analysed. Figure 3 presents the magnitude of the Coulomb effect observed in the experiment (open squares) and predicted in theory (full squares). The  $y$ -axis shows the ratio  $\frac{\sigma_i - \sigma_{\text{AV18+UIX}}}{\sigma_{\text{AV18+UIX}}}$ , where  $\sigma_i$  denotes the theoretical (with Coulomb force included) or experimental differential cross section. The theoretical calculations are based on the Argonne V18 potential supplemented by Urbana IX 3NF with ( $\sigma_i$ ) and without ( $\sigma_{\text{AV18+UIX}}$ ) Coulomb force included. It is well-seen that shapes of distributions for the experiment and theory are similar, what means that the Coulomb force effect is very well-reconstructed. A slight shift of these distributions relative to each other may arise either from a systematic uncertainty of normalization or deficiency of description of the  $n$ - $d$  systems by the theoretical calculations based on AV18+UIX potential.

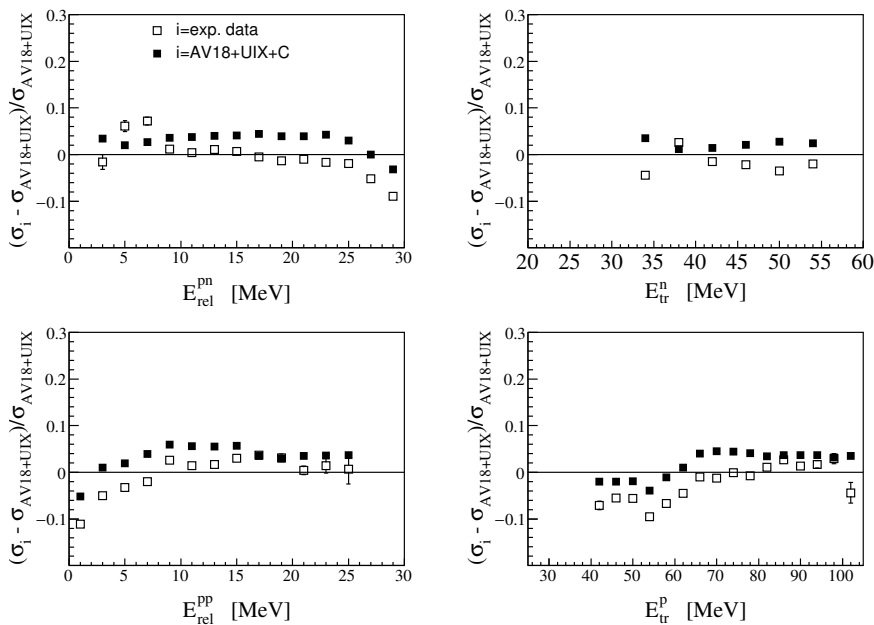


Fig. 3. Net effects of the Coulomb force in the differential cross section of the  $dp$  breakup at 130 MeV, presented as a function of four invariants.

For better localization of Coulomb force effects, two-dimensional spectra have been constructed. Figure 4 presents the net effects in function of  $E_{tr}^n$  and  $E_{rel}^{pp}$  coordinates. Shapes of these spectra correspond to the selected phase space of the  $dp$  breakup experiment at 130 MeV, while their colours code a magnitude of the effect. These results are consistent with the previous analysis of the Coulomb effect based on data originating from a dedicated  $dp$  breakup experiment with deuteron beam of energy of 130 MeV, which has been done at FZ-Juelich, using Germanium Wall (GeWall) setup (for more information see [11, 16, 17, 19]).

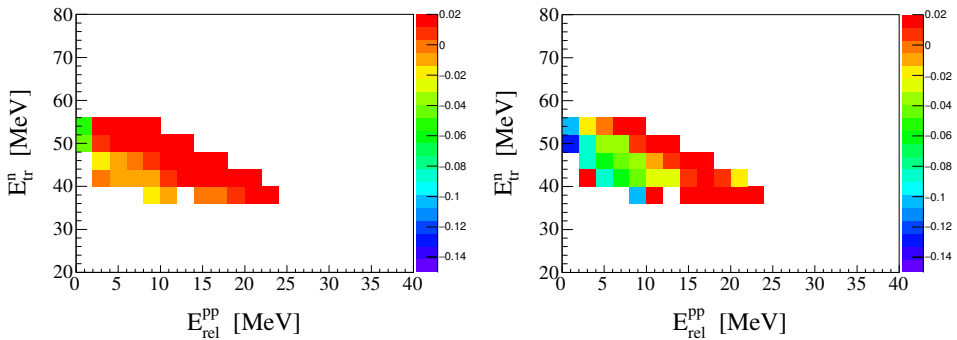


Fig. 4. Net effects of the Coulomb force in the differential cross section of the  $dp$  breakup at 130 MeV, presented as a function of two out of four invariants. Left panel: Difference of theoretical predictions (by Deltuva [7]) obtained for Argonne V18 potential combined with UIX 3NF with and without the Coulomb force, relatively normalized to AV18+UIX calculations. Right panel: Difference between experimental data and calculations with AV18+UIX alone, normalized in the same way.

#### 4. Outlook

Systematic studies of the breakup reaction in a wide range of the phase space are very important for understanding the interaction between nucleons in few-nucleon systems. In order to verify and expand the existing theoretical approaches, a large and exact experimental database is needed. The variety of dynamical effects present in the cross section of the breakup reaction requires comparing the data with calculations including all of these ingredients, either separately or, if only possible, combined to get a complete picture of the process.

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