On cosmological constants from α'-corrections

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ABSTRACT: We examine to what extent perturbative α'-corrections can generate a small cosmological constant in warped string compactifications. Focusing on the heterotic string at lowest order in the string loop expansion, we show that, for a maximally symmetric spacetime, the α'-corrected 4D scalar potential has no effect on the cosmological constant. The only relevant terms are instead higher order products of 4D Riemann tensors, which, however, are found to vanish in the usual perturbative regime of the α'-expansion. The heterotic string therefore only allows for 4D Minkowski vacua to all orders in α', unless one also introduces string loop and/or nonperturbative corrections or allows for curvatures or field strengths that are large in string units. In particular, we find that perturbative α'-effects cannot induce weakly curved AdS$_4$ solutions.

Contents

1 Introduction

At sufficiently low energies and for small string coupling, perturbative string theory is well approximated by an effective two-derivative supergravity Lagrangian supplemented by small corrections coming from a double expansion in the slope parameter α' and the string coupling g_s. The terms of the α'-expansion are higher derivative corrections to the supergravity action that account for the extended nature of the strings. They are negligible if the curvature of the background manifold and derivatives of the fields are small in units of α'. The terms coming from the g_s-expansion are loop corrections due to nontrivial topologies of the string world sheet, which are negligible in the semi-classical regime when the string coupling is small.

From a phenomenological point of view, such sub-leading corrections can have important consequences, as they may allow for solutions with properties that are forbidden at the two-derivative supergravity level. A well-known example in type IIB string theory are the AdS$_4$ solutions at large internal volume [1], where α'-corrections [2] break the no-scale structure of the leading order Minkowski solutions found in [3] and contribute to a nonzero cosmological constant. In this example, however, the α'-corrections alone are not sufficient, and also non-perturbative quantum corrections from localized sources are needed in order to generate the AdS vacuum.

For the heterotic string, an analogous scenario was investigated in [4], where the authors found that an interplay of the lowest order α'-correction [5] and non-perturbative

effects could give rise to a similar large volume AdS vacuum in 4D, while the classical two-derivative supergravity action only admits Minkowski ground states.

In view of these constructions, one might wonder whether there could also be situations where the perturbative α'-corrections alone already suffice to generate a small non-vanishing cosmological constant in a controlled compactification scheme. This question should be easiest to study for the heterotic string, where D-branes and orientifold planes are absent, and the leading α'-corrections are completely known and already appear at order $\mathcal{O}(\alpha')$. Looking at the heterotic effective action at string tree-level, however, one might quickly conclude that α'-corrections alone can never suffice to generate vacua other than Minkowski space. The apparent reason is that, in the absence of string loop or non-perturbative corrections, all terms in the heterotic effective action come from world sheets with spherical topology so that the action scales uniformly with the dilaton ϕ:

$$S = \int \mathrm{d}^{10}x\sqrt{-g}\,\mathrm{e}^{-2\phi}\{\ldots\} \tag{1.1}$$

(cf. (2.2)). As a consequence, the four-dimensional effective scalar potential likewise scales uniformly with the dilaton zero mode, and one would expect the 4D dilaton equation to be solved either if the potential vanishes on the solution or if there is a runaway to a free vacuum. It therefore seems obvious that heterotic string theory at string tree level can only lead to Minkowski solutions, and that a non-vanishing cosmological constant also requires string loop or non-perturbative quantum corrections. A related argument was employed by Dine and Seiberg in [6, 7] to suggest that realistic string vacua might be strongly coupled.[1]

It is the purpose of this paper to re-address this question and in particular the seemingly trivial counter-argument against non-Minkowski vacua sketched in the previous paragraph. The reason is that the higher curvature terms among the α'-corrections (e.g. the $\alpha'\mathrm{tr}|R^+|^2$-terms in the heterotic string) also lead to contributions to the four-dimensional Einstein equation and the equations of motion for the moduli that involve higher powers of external Riemann tensors and hence can *not* be interpreted as a part of the effective scalar potential. It is therefore a priori not clear whether the scaling argument sketched above is still valid or whether nontrivial effects might emerge from such higher order terms.

That these effects exist follows from explicitly known AdS$_4$-compactifications of the heterotic string when the effective action is truncated after the lowest order α'-corrections (see e.g. [17, 18]). In these solutions, the 4D cosmological constant turns out large, $\Lambda \sim \frac{1}{\alpha'}$, so that the effects of even higher α'-corrections are difficult to estimate offhand and would require more explicit calculations [17].

In an interesting recent paper [19], on the other hand, it was investigated whether the α'-corrections of the heterotic string could also give rise to a *small* cosmological constant $\Lambda \sim \alpha'C$, where C is a 6D integral over fields such as the dilaton or the warp factor with

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[1]A priori, all this also applies to the oriented closed string sector of the type II theories, but the inclusion of orientifold planes and D-branes leads to terms with a different dilaton scaling already at string tree-level. It is these different scalings that allow, e.g., for the classical AdS vacua of [8] and that have been exploited in attempts to construct "classical" de Sitter vacua in type II supergravity with (smeared) orientifold planes (see e.g. [9–11] for early discussions) that could evade the "no-go" theorems discussed in [12–16].

four internal derivatives. Intriguingly, the authors found that de Sitter vacua of this type are excluded, but raised the possibility of warped AdS_4 compactifications as an $\mathcal{O}(\alpha')$-effect. Proving this requires only some of the 10D field equations, and it was left as an open problem to check whether really all field equations could be satisfied at the considered order in the α'-expansion.

In this paper we investigate to what extent the usual scaling analysis of the 4D effective potential is invalidated by higher curvature terms in the α'-expansion and check whether this expansion can yield perturbatively small cosmological constants of order $\mathcal{O}(\alpha')$ or higher. The main result of our analysis is that this is in general not possible at string tree-level. This follows from the four-dimensional Einstein equation and the dilaton equation, which can be combined to yield a constraint of the form

$$\Lambda = \sum_{m,n} c_{mn} \alpha'^m \Lambda^n, \qquad m, n > 0, \tag{1.2}$$

where c_{mn} are numerical coefficients containing integrals over internal fields and their derivatives. Assuming a perturbative α'-expansion for Λ, one then obtains $\Lambda = 0$ as the only solution to all orders in α', as we will explain in more detail below.

This paper is organized as follows. In section 2.1 we establish our notation and detail a simple argument (cf. [20]) showing that heterotic supergravity with the first order α'-corrections does not yield solutions with a nonzero cosmological constant to that order. In appendix B we investigate the proposed warped AdS solutions of [19] at order $\mathcal{O}(\alpha')$ more directly, showing explicitly that the given $\mathcal{O}(\alpha')$-expression for the cosmological constant is really of higher order and thus could compete with neglected terms in the action. In section 2.2 we then show how the argument of section 2.1 can be extended to all orders in the α'-expansion and that it is completely independent of the details of the α'-corrected 4D scalar potential. We conclude with section 3, where we discuss several ways to circumvent this "no-go theorem", its relation to the Dine-Seiberg problem, and possible effects of violations of the effective potential description. Appendix A collects some useful identities for the Riemann tensor, and appendix C contains the ten-dimensional version of the argument of section 2.2.

2 A "no-go theorem"

In this section, we discuss a simple argument showing that tree-level heterotic string theory with its first order α'-corrections does not have 4D de Sitter or anti-de Sitter vacua with a perturbatively small cosmological constant at this order [20]. We then show that the argument can in fact be extended to all orders in the α'-expansion. Our assumptions throughout the paper are as follows:

- We consider compactifications to four dimensions that respect maximal four-dimensional spacetime symmetry, i.e.:

    - The 10D metric is a warped product of a maximally symmetric 4D spacetime (parameterized by coordinates x^μ; $\mu, \nu, \ldots = 0, \ldots, 3$) and a 6D compact mani-

fold (parameterized by y^m; $m, n, \ldots = 4, \ldots, 9$),

$$\mathrm{d}s^2 = \mathrm{e}^{2A}\mathrm{d}s_4^2 + \mathrm{d}s_6^2, \tag{2.1}$$

where e^{2A} depends on the 6D coordinates only, and $\mathrm{d}s_4^2$ describes an unwarped 4D Minkowski, de Sitter or anti-de Sitter spacetime.

- All 4D parts of tensor and spinor fields vanish (up to gauge choices) except for combinations that can be built from the 4D (unwarped) metric, its Riemann tensor or its volume form. This means, in particular, that there are no spacetime-filling fluxes[2] and that all 4D covariant derivatives of all tensor fields, including the dilaton and the Riemann tensor, can be set to zero on the solution.[3] Furthermore, the Lorentz-Chern-Simons 3-form does not contribute to the equations of motion in maximally symmetric backgrounds [21].

- String-loop and/or non-perturbative corrections to the action are disregarded.

- α' is a meaningful expansion parameter in the sense that all field variations are small over a string length and the α'-corrections can be organized in a perturbative expansion about the zero-slope limit.[4]

2.1 Heterotic supergravity with leading α'-corrections

In string frame, the heterotic supergravity action with leading α'-corrections reads (for simplicity we set the 10D gravitational coupling $\kappa^2 = \frac{1}{2}$)

$$S = \int \mathrm{d}^{10}x\sqrt{-g}\,\mathrm{e}^{-2\phi}\left\{R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\left[\mathrm{tr}|F|^2 - \mathrm{tr}|R^+|^2\right] + \mathcal{O}(\alpha'^2)\right\} \tag{2.2}$$

with $|H|^2 = \frac{1}{6}H_{MNL}H^{MNL}$, $\mathrm{tr}|F|^2 = \frac{1}{2}\mathrm{tr}F_{MN}F^{MN}$ and $\mathrm{tr}|R^+|^2 = \frac{1}{2}R^+_{MNPQ}R^{+MNPQ}$. Here, ϕ denotes the dilaton, F is the Yang-Mills field strength, and $R^+_{MNP}{}^Q$ is the Riemann tensor constructed from the torsionful connection $\Gamma^{+M}_{NL} = \Gamma^M_{NL} - \frac{1}{2}H_{NL}{}^M$. H is the α'-corrected 3-form field strength of the NS 2-form potential B,

$$H = \mathrm{d}B + \frac{\alpha'}{4}\left(\omega_{3\mathrm{L}} - \omega_{3\mathrm{Y}}\right), \tag{2.3}$$

where $\omega_{3\mathrm{L}}$ and $\omega_{3\mathrm{Y}}$ denote the Chern-Simons 3-forms formed from the spin connection and the Yang-Mills gauge field, respectively.

For our argument, it is sufficient to look at the field equations of the dilaton and the external metric. This can be done either by using a 4D effective action approach or by

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[2]We express everything in terms of the Yang-Mills field strength F and the NS 3-form H, which have a too small rank to be spacetime-filling in 4D. The Hodge duals of purely 6D fluxes of these fields would of course generically have spacetime-filling components, but they do not appear explicitly in our formalism.

[3]Note that for maximally symmetric spaces, the Riemann tensor becomes an algebraic combination of metric tensors, and therefore its covariant derivative vanishes.

[4]The α'-expansion differs from the derivative expansion in that some terms appear at higher orders than suggested by the number of their derivatives. An example is the term $\mathrm{tr}|F|^2$ which, although a two derivative term, appears at $\mathcal{O}(\alpha')$. It should be noted though that our analysis does not depend on which of the two expansion schemes is used.

working directly with the 10D field equations. We describe the 4D effective action approach here and sketch the analogous 10D argument in appendix C.

For the 4D argument, we can restrict our attention to the zero mode, τ, of the dilaton, which we define by separating off the higher Kaluza-Klein modes,

$$\mathrm{e}^{-\phi} = \tau \mathrm{e}^{-\phi_{\mathrm{KK}}}. \tag{2.4}$$

Here ϕ_{KK} denotes the sum of all remaining KK-modes, which we integrate out by simply setting them equal to their on-shell values. It does not matter for our argument whether τ or one of the KK modes has the lowest mass (or whether they even combine with other degrees of freedom in the low energy EFT as suggested in [22]) as can be seen directly from the equivalent ten-dimensional analysis in appendix C.

On-shell, all fields in 4D must be covariantly constant by maximal symmetry, so we can henceforth ignore any x^μ-dependence of τ and only need to keep track of τ itself in the action, but not of its derivatives.

The only other field whose dynamics we need to consider is the external metric $g_{\mu\nu}$. Switching to four-dimensional Einstein frame, we define a new 4D metric $\tilde{g}_{\mu\nu}$ by

$$\tilde{g}_{\mu\nu} \equiv \mathcal{V}\tau^2 \mathrm{e}^{-2A} g_{\mu\nu}. \tag{2.5}$$

Here

$$\mathcal{V} \equiv \int \mathrm{d}^6 y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}+2A}, \tag{2.6}$$

which can again be treated as constant in 4D by maximal symmetry.

Performing this rescaling, we then obtain an effective 4D action for $\tilde{g}_{\mu\nu}$ and τ of the form

$$S = \int \mathrm{d}^4 x \sqrt{-\tilde{g}_4} \left\{ \tilde{R}_4 - V + W \right\}, \tag{2.7}$$

where we have split the action into the Einstein-Hilbert term and two extra contributions. V contains all terms that are constructed from fields without external indices, whereas W contains all terms that include fields with 4D spacetime indices. In the absence of W, V is just the usual effective potential.

Using (2.2), these two terms are given by

$$V = - \int \mathrm{d}^6 y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}+4A} \frac{1}{\tau^2 \mathcal{V}^2}$$
$$\times \left[R_6 - 20(\partial A)^2 - 8\nabla^2 A + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\left(\mathrm{tr}|F|^2 - |R_6^+|^2\right) \right] + \mathcal{O}(\alpha'^2) \tag{2.8}$$

and

$$W = \int \mathrm{d}^6 y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}} \left[\frac{\alpha'\tau^2}{4}|\tilde{R}_{\mu\nu\lambda}{}^\rho|^2 - \frac{\alpha'}{2\mathcal{V}}\mathrm{e}^{2A}\tilde{R}_4(\partial A)^2 \right] + \mathcal{O}(\alpha'^2), \tag{2.9}$$

where we have evaluated the curvature terms R and $\mathrm{tr}|R^+|^2$ for the tilded metric (2.5) and expressed them in terms of $\tilde{R}_4$ and $|\tilde{R}_{\mu\nu\lambda}{}^\rho|^2 = \frac{1}{2}\tilde{R}_{\mu\nu\lambda\rho}\tilde{R}^{\mu\nu\lambda\rho}$ as well as a term $|R_6^+|^2$

containing various internal fields. Further details and the definition of $|R_6^+|^2$ can be found in appendix A.

Using the scaling $V \sim \tau^{-2}$, one finds the 4D dilaton equation,

$$2V + \frac{\alpha'\tau^2}{2}|\tilde{R}_{\mu\nu\lambda}{}^\rho|^2 \int \mathrm{d}^6y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}} = 0, \tag{2.10}$$

and the trace of the four-dimensional Einstein equation,

$$\tilde{R}_4 - 2V - \frac{\alpha'}{2\mathcal{V}}\tilde{R}_4 \int \mathrm{d}^6y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}+2A}\, (\partial A)^2 = 0, \tag{2.11}$$

where we have neglected the variation with respect to the connection as it would give rise to covariant derivatives upon partial integration, which vanish due to maximal symmetry. Combining the two equations such that V cancels out and substituting $\tilde{R}_{\mu\nu\lambda\rho} = \frac{2}{3}\Lambda \tilde{g}_{\lambda[\mu}\tilde{g}_{\nu]\rho}$ then yields an equation of the form

$$\Lambda = \alpha' \left(c_{11}\Lambda + c_{12}\Lambda^2\right) + \mathcal{O}(\alpha'^2), \tag{2.12}$$

where c_{11} and c_{12} are given by

$$c_{11} = \frac{1}{2\mathcal{V}} \int \mathrm{d}^6y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}+2A}\, (\partial A)^2, \qquad c_{12} = -\frac{\tau^2}{3} \int \mathrm{d}^6y \sqrt{g_6}\, \mathrm{e}^{-2\phi_{\mathrm{KK}}}. \tag{2.13}$$

Given our assumption that we are in the regime of validity of the perturbative α'-expansion, (2.12) must be solved order by order with an ansatz of the form

$$\Lambda = \Lambda_0 + \alpha'\Lambda_1 + \mathcal{O}(\alpha'^2) \tag{2.14}$$

for the cosmological constant, where Λ_0 denotes the solution of the leading order supergravity equations without α'-corrections, $\alpha'\Lambda_1$ is a correction due to next-to-leading order terms in the α'-expansion, and so on. It is straightforward to see that plugging this ansatz into (2.12) yields

$$\Lambda = \mathcal{O}(\alpha'^2) \tag{2.15}$$

as the only solution. Thus, perturbative heterotic string theory does not yield solutions with a nonzero cosmological constant up to corrections of order $\mathcal{O}(\alpha'^2)$.

Let us now compare this to the result of [19], where it was suggested that warped AdS vacua might be allowed in heterotic string theory as an $\mathcal{O}(\alpha')$-effect, i.e.,

$$\Lambda = -\alpha'C + \mathcal{O}(\alpha'^2), \tag{2.16}$$

where C is a non-negative constant which is built from a sum of squares of internal fields integrated over the internal manifold. At first sight, this seems to contradict the above argument that solutions with nonzero cosmological constant are not allowed at order $\mathcal{O}(\alpha'^1)$. However, one can show directly by means of the supergravity equations of motion that the terms contained in C actually vanish at the order considered here such that $C = \mathcal{O}(\alpha')$. The right hand sides of (2.15) and (2.16) are therefore equal up to corrections of order $\mathcal{O}(\alpha'^2)$. For convenience, we give the details in appendix B.

2.2 General argument

Let us now generalize the above argument to the heterotic string with α'-corrections of arbitrarily high order. The effective action for the massless fields then reads

$$S = \int \mathrm{d}^{10}x\sqrt{-g}\,\mathrm{e}^{-2\phi}\left\{R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 + \alpha'\text{-corrections}\right\}, \qquad (2.17)$$

where all terms scale identically with respect to the dilaton if we neglect string loop or non-perturbative corrections as initially stated.

Rescaling the metric as in (2.5), we obtain the action in four-dimensional Einstein frame

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}_4}\left\{\tilde{R}_4 - V + W\right\}. \qquad (2.18)$$

As in the previous section, we have split the action into an Einstein-Hilbert term $\tilde{R}_4$, a term V containing all terms that are constructed from fields without external spacetime indices, and a term W containing everything else.

In the absence of string loop or non-perturbative corrections, all terms in V scale again as $V \sim \tau^{-2}$ such that the dilaton equation yields

$$2V + \tau\partial_\tau W = 0. \qquad (2.19)$$

Taking the trace of the four-dimensional Einstein equation, we furthermore find

$$\tilde{R}_4 - 2V - W' = 0, \qquad W' \equiv \frac{\tilde{g}^{\mu\nu}}{\sqrt{-\tilde{g}_4}}\frac{\delta}{\delta\tilde{g}^{\mu\nu}}\left(\int \mathrm{d}^4x\sqrt{-\tilde{g}_4}\,W\right), \qquad (2.20)$$

where, as indicated, W' denotes all terms that are due to the variation of W with respect to the external metric.

Combining the two equations (2.19) and (2.20), we then find

$$\tilde{R}_4 = -\tau\partial_\tau W + W'. \qquad (2.21)$$

Although an explicit expression for the right hand side of this equation is only known for the first few orders in the α'-expansion, the general structure is rather simple: it is a sum of positive powers of the cosmological constant with coefficients built from integrals over internal fields and their derivatives.

To see this, recall that our assumption of maximal 4D spacetime symmetry implies that only the metric, the epsilon tensor and the Riemann tensor are nontrivial, all with vanishing covariant derivative. Considering first the metric variations of W that come from variations of connections (either within covariant derivatives or curvature tensors or Lorentz-Chern-Simons forms), one sees that these variations do not contribute to the right hand side of (2.21), as they would lead to terms with a total 4D covariant derivative, which vanish by assumption. The only contributions to W' are therefore from variations of metric tensors that appear algebraically in W or in the metric determinant. As there are no nontrivial contractions of just the epsilon tensor and/or the metric, all these terms must

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contain at least one Riemann tensor.[5] Similar remarks also apply to the dilaton variation of W, so that the right hand side of (2.21) is a sum of terms that each involves at least one Riemann tensor. Because of $\tilde{R}_{\mu\nu\lambda\rho} = \frac{2}{3}\Lambda\tilde{g}_{\lambda[\mu}\tilde{g}_{\nu]\rho}$, these then translate into positive powers of the cosmological constant, as claimed.

Since at leading order the supergravity action does not contain any terms that depend on the Riemann tensor except for the Einstein-Hilbert term, the terms in W and W' are of order $\mathcal{O}(\alpha')$ or higher. We can therefore schematically rewrite (2.21) as

$$\Lambda = \sum_{m,n} c_{mn}\alpha'^m\Lambda^n, \qquad m, n > 0, \tag{2.22}$$

with some numerical coefficients c_{mn} that in general contain integrals over contractions of warp factor terms, internal field strengths and curvatures, and so on.

Assuming again the validity of a perturbative α'-expansion, we need to solve (2.22) order by order with an ansatz of the form

$$\Lambda = \Lambda_0 + \alpha'\Lambda_1 + \alpha'^2\Lambda_2 + \ldots \tag{2.23}$$

as in section 2.1. This yields

$$\Lambda = 0 \tag{2.24}$$

as the only solution to all orders in the perturbative α'-expansion.[6] Hence, heterotic string theory yields Minkowski spacetime as the only maximally symmetric solution to all orders in the perturbative α'-expansion, unless one introduces loop and/or non-perturbative corrections. In particular, we don't find α'-generated AdS_4 vacua with perturbatively small curvatures to be possible.

3 Discussion

Let us now discuss several implications of our findings. In particular, we will discuss possibilities to evade our above no-go argument, its relation to the Dine-Seiberg problem and the violation of the effective potential description due to higher order corrections to the supergravity action.

3.1 Evading the no-go theorem

In section 2.2, we have shown that heterotic string compactifications at string tree-level yield 4D Minkowski spacetime as the only maximally symmetric solution to all orders in a perturbative α'-expansion, unless one violates one of our initial assumptions. Let us now discuss these possible violations and how they evade our argument.

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[5]Note that there is no constant term in W: a constant has no external spacetime indices and hence would be part of V, which however cancels out in (2.21).

[6]We might also try to solve (2.22) without expanding Λ as in (2.23). Assuming that $\Lambda \neq 0$, we can then divide by Λ to get $1 \leq \sum |c_{mn}\alpha'^m\Lambda^{n-1}|$. But this is again a contradiction to the assumption made in the beginning of section 2.

Loop and non-perturbative corrections/extended sources. An obvious possibility to circumvent the argument of section 2.2 is the inclusion of terms that scale differently with respect to the dilaton than the tree-level terms considered here. Natural candidates are string loop or non-perturbative corrections e.g. from gaugino condensation [23, 24]. With such terms turned on, the dilaton and Einstein equations read

$$-\tau\partial_\tau V + \tau\partial_\tau W = 0, \qquad \tilde{R}_4 - 2V - W' = 0 \qquad (3.1)$$

and can in general not be combined such that V cancels out. The right hand side of (2.22) may then contain terms which are independent of Λ, making solutions other than $\Lambda = 0$ possible. It would be interesting to see whether including the first loop correction at order $\mathcal{O}(\alpha'^3 g_s)$ could allow for purely perturbative solutions with a non-zero cosmological constant for the heterotic string.

A different dilaton scaling may also be introduced if one includes extended sources such as the various types of D-branes and orientifold planes in type II string theory. Being an open string tree-level action, the DBI action scales only with $\mathrm{e}^{-\phi}$ and so would in general also invalidate our argument. In fact, in type II string theory, a large number of compactification scenarios with a nonzero cosmological constant have been proposed using D-branes and orientifolds as well as non-perturbative quantum corrections starting with [25]. Heterotic string theory, on the other hand, is much more limited in this respect, as it does not contain D-branes and O-planes but would require dealing with less common extended objects.

Spacetime-filling fluxes. Since spacetime-filling fluxes are in general not forbidden by maximal symmetry, they can be used to invalidate our argument around (2.21), where we explained that all terms in W are contractions of Riemann tensors and must therefore contain factors of the cosmological constant. In heterotic string theory, there are no spacetime-filling fluxes if spacetime is assumed to be four-dimensional. Compactifying to three dimensions, however, allows for solutions with a nonzero cosmological constant, if spacetime components of H are turned on (see e.g. [26]). In type II string theory, spacetime-filling RR-fluxes may also be present in compactifications to four dimensions and may lead to solutions with a nonzero cosmological constant.

Large higher derivative terms. Another way to circumvent our no-go theorem is to leave the perturbative regime of the α'-expansion and consider solutions for which higher derivative terms are not small in units of α'. A truncation of the action at a finite order is then in general not guaranteed to be a good approximation to the full theory, because higher order terms are not automatically suppressed.[7] This problem does of course not apply when supergravity is studied in its own right instead of being considered the low energy effective field theory of string theory. In any case, allowing curvature and derivatives of the fields to be large in units of α', it is indeed possible to construct solutions with a nonzero cosmological constant that is large in units of α'. A good example are the heterotic AdS

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[7]This does of course not rule out that the truncated action could still capture the essential features of a solution or that the higher order terms happen to be small or even vanish in certain cases.

compactifications studied in [17], which are solutions to the heterotic supergravity action with linear α'-corrections that feature a curvature of order $\mathcal{O}(\frac{1}{\alpha'})$. By construction, our argument does not make statements in this regime.

Breaking maximal symmetry. Requiring spacetime to be maximally symmetric implies a very limited field content such that, in the absence of spacetime-filling fluxes, all terms showing up on the right hand side of (2.21) contain contractions of spacetime components of the Riemann tensor. All of these terms can then be rewritten as a power of Λ times some numerical factor, regardless of how the Riemann tensors are contracted. As explained earlier, this property ensures that the higher derivative curvature terms on the right hand side of (2.21) are much smaller than the Ricci scalar on the left hand side, leading to constraint (2.22) and the conclusion that only Minkowski solutions are possible.

For spacetimes without maximal symmetry, however, this need not be the case. The presence of various (spacetime) tensor fields then leads to new terms in (2.21) which can be of the same order as the 4D Ricci scalar and thus generate a nonzero cosmological constant. Furthermore, it is not guaranteed anymore that higher derivative curvature terms in (2.21) are negligible, since whether they are much smaller than the 4D Ricci scalar or can compete with it depends on how they are contracted. This is due to the well-known fact that for general spaces the magnitude of individual components of the Riemann tensor and the Ricci scalar need not be the same, so that the Riemann tensor can have large components even when the Ricci scalar is very small. In heterotic string theory, the Ricci scalar may then compete, for example, with the $\alpha'|\tilde{R}_{\mu\nu\lambda}{}^{\rho}|^2$ term and thus become nonzero.

This is also the reason why it is not possible to extend our analysis to make a statement about the curvature of the internal space. An exception are compactifications on maximally symmetric spaces such as the six-sphere, which can be ruled out using an argument along the lines of section 2.2, unless there exist six-form fluxes filling internal space. Since this only concerns a very restricted class of compactifications, our discussion does unfortunately not add much to the discussion of [27], where it is suggested that higher derivative corrections (or strong warping, see also [28]) could in principle support an everywhere negative internal Ricci scalar, which is difficult to realize otherwise.

3.2 The Dine-Seiberg problem

In [6, 7], Dine and Seiberg used the dilaton behavior of the effective 4D scalar potential in the weak coupling limit to argue that, unless the effective potential is identically zero, there must in general either be a runaway to the free vacuum or a minimum at strong coupling. Using an analogous scaling analysis for the universal volume modulus, one may argue for similar difficulties regarding compactifications at large volume (cf. e.g. [29] for a recent discussion). Progress in moduli stabilization techniques have since then led to many interesting scenarios where an interplay of various scalar potential contributions suggest the existence of weakly coupled minima at controllably large volumes. Still many of the difficulties and complexities one encounters in these attempts can be traced back to the issues pointed out in [6, 7].

The argument given in the present paper, although somewhat similar in its consequences, differs from the argument of [6, 7] in several ways. First of all we do not really use or discuss moduli stabilization. Nor do we trace the dependence of the scalar potential on the volume modulus. In fact, the detailed form of the scalar potential and its moduli dependence play no role for our argument (except that we exploit the overall dilaton scaling to eliminate the scalar potential completely from the equation of interest (eq. (2.21))). Instead, the only terms that matter for our argument are higher order products of 4D Riemann tensors, which did not play a role for the arguments in [6, 7].

Moreover, it could have been the case that terms that appear to be of lower order in the α'-expansion compete with terms that are explicitly of higher order in α' without that the perturbative α'-expansion breaks down. An example for this are the $|H|^2$ and $|F|^2$ terms appearing in the heterotic supergravity action or gradient terms of the warp factor or the dilaton. As reviewed in appendix B, they are forced to be zero by the leading order equations of motion, if our initially stated assumptions hold. Including α'-corrections to the action, however, the equations of motion are modified such that the above terms can become nonzero and thus compete with higher order terms in the α'-expansion. This could have postponed the emergence of a nontrivial cosmological constant to a higher order than suggested by [19]. Our argument from section 2.2, however, shows that this can not happen at any order in α', regardless of the scalar potential.

3.3 Violation of effective scalar potential description

The effective scalar potential description is a standard tool in effective field theory which is widely used in the moduli stabilization literature. For solutions yielding a maximally symmetric spacetime, the effective potential is usually expected to fulfill two assumptions:

- The equations of motion are satisfied at a point in moduli space which is an extremum of V.

- The value of V at this point is proportional to the cosmological constant.

These assumptions are true if the effective action can be written in the form

$$S = \int \mathrm{d}^4x \sqrt{-\tilde{g}_4} \left\{ \tilde{R}_4 - V \right\}, \tag{3.2}$$

where $\tilde{R}_4$ is the only term in the Lagrangian that depends on the external metric, and V is the only term that depends on the moduli.

It is interesting to note that both assumptions are generically violated by higher order effects in the α'-expansion, unless the cosmological constant is zero. This follows from (3.1) which on-shell yields

$$\partial_\tau V \neq 0, \qquad V \not\propto \Lambda. \tag{3.3}$$

Hence, the equations of motion are in general satisfied at a point in moduli space which is not an extremum of V. Moreover, V is not proportional to the cosmological constant anymore. This effect is usually completely negligible when the cosmological constant is small. For inflation scenarios with a very high energy scale, these corrections might be more sizeable, but when they are, the validity of the perturbative α'-expansion would also be less obvious.

Acknowledgments

A Riemann tensor with warping

Let us compute the components of the Riemann tensor for the warped spacetime

$$\mathrm{d}s^2 = \frac{\mathrm{e}^{2A}}{\mathcal{V}\tau^2}\, \tilde{g}_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} + \mathrm{d}s_6^2. \tag{A.1}$$

In order to express the full Riemann tensor $R_{MRN}{}^P$ in terms of the Riemann tensor $\tilde{R}_{MRN}{}^P$ of the unwarped metric $\tilde{g}_{MN}$, we use the formula

$$R_{MRN}{}^P = -\tilde{\nabla}_M\Gamma^P_{RN} + \tilde{\nabla}_R\Gamma^P_{MN} + \Gamma^S_{NM}\Gamma^P_{RS} - \Gamma^S_{NR}\Gamma^P_{MS} + \tilde{R}_{MRN}{}^P, \tag{A.2}$$

where $M, N, \ldots = 0, \ldots, 9$ denote 10D spacetime indices, $\Gamma^M_{NP} = \frac{1}{2}g^{MR}(\tilde{\nabla}_P g_{RN} + \tilde{\nabla}_N g_{RP} - \tilde{\nabla}_R g_{NP})$ and $\tilde{\nabla}_M$ is the covariant derivative associated with the unwarped metric $\tilde{g}_{MN}$ (see e.g. [30]). This yields

$$R_{\mu\nu\lambda}{}^{\rho} = -2\frac{\mathrm{e}^{2A}}{\mathcal{V}\tau^2}\,\tilde{g}_{\lambda[\mu}\delta^{\rho}_{\nu]}(\partial A)^2 + \tilde{R}_{\mu\nu\lambda}{}^{\rho}, \qquad R_{ijk}{}^l = \tilde{R}_{ijk}{}^l,$$

$$R_{\mu j\lambda}{}^l = -\frac{\mathrm{e}^{2A}}{\mathcal{V}\tau^2}\,\tilde{g}_{\mu\lambda}\nabla_j\partial^l A - \frac{\mathrm{e}^{2A}}{\mathcal{V}\tau^2}\,\tilde{g}_{\mu\lambda}(\partial_j A)(\partial^l A). \tag{A.3}$$

Assuming that H has only internal components, it follows from (A.2) that introducing torsion with $\Gamma^M_{NL} \to \Gamma^M_{NL} - \frac{1}{2}H_{NL}{}^M$ modifies the internal components $R_{ijk}{}^l$ of the Riemann tensor and, in case of nontrivial warping, also some of the spacetime components

$$R^+_{\mu\nu\lambda}{}^{\rho} = R_{\mu\nu\lambda}{}^{\rho}, \qquad R^+_{\mu j\lambda}{}^l = R_{\mu j\lambda}{}^l - \frac{1}{2}\Gamma^m_{\mu\lambda}H_{jm}{}^l, \tag{A.4}$$

where $\Gamma^m_{\mu\lambda} = -\frac{\mathrm{e}^{2A}}{\mathcal{V}\tau^2}\,\tilde{g}_{\mu\lambda}\nabla^m A$. We thus find

$$R = \mathcal{V}\tau^2\mathrm{e}^{-2A}\tilde{R}_4 + \tilde{R}_6 - 20(\partial A)^2 - 8\nabla^2 A, \tag{A.5}$$

$$\mathrm{tr}|R^+|^2 = \mathcal{V}^2\tau^4\mathrm{e}^{-4A}|\tilde{R}_{\mu\nu\lambda}{}^{\rho}|^2 - 2\mathcal{V}\tau^2\mathrm{e}^{-2A}\tilde{R}_4(\partial A)^2 + |R_6^+|^2, \tag{A.6}$$

where $\mathrm{tr}|R^+|^2 = \frac{1}{2}R^+_{MNPQ}R^{+MNPQ}$ and $|\tilde{R}_{\mu\nu\lambda}{}^{\rho}|^2 = \frac{1}{2}\tilde{R}_{\mu\nu\lambda\rho}\tilde{R}^{\mu\nu\lambda\rho}$. For convenience, we also introduced the shortcut notation $|R_6^+|^2 = 12\left[(\partial A)^2\right]^2 + 4|R^+_{\mu j\lambda}{}^l|^2 + |R^+_{ikl}{}^m|^2$ to subsume all terms which in the tilded frame only depend on internal fields.

B Leading order constraints on heterotic supergravity

In [19], it was suggested that heterotic supergravity with leading α'-corrections could have solutions with a cosmological constant of the form

$$\Lambda = -\alpha' C + \mathcal{O}(\alpha'^2), \tag{B.1}$$

where C is a non-negative constant given by

$$C = \frac{1}{2\mathcal{V}'} \int \mathrm{d}^6 y \sqrt{\tilde{g}_6}\, \mathrm{e}^{6A-\frac{\phi}{2}} \left\{ 3 \left[(\partial\omega)^2\right]^2 + 2|(\partial_m\omega)(\partial_n\omega) - \tilde{\nabla}_m\partial_n\omega - \tilde{g}_{mn}(\partial\omega)^2|^2 \right.$$
$$\left. + \frac{1}{2}\mathrm{e}^{-4\omega}|H_{mn}{}^l\partial_l\omega|^2 \right\} \tag{B.2}$$

with $\mathcal{V}' = \int \mathrm{d}^6 y \sqrt{\tilde{g}_6}\, \mathrm{e}^{8A}$ and $\omega = A + \frac{\phi}{4}$. We will now show explicitly, using arguments similar to [31–33], that all terms in C vanish up to higher order α'-corrections due to the leading order equations of motion. The result of [19] is therefore not in conflict with the argument given in section 2.1.

To omit confusion, we will stick to the metric conventions of [19] in this appendix, which differ from those used in the main text of our paper. The unwarped metric is then defined as $\tilde{g}_{MN} = \mathrm{e}^{-2A} g_{MN}$, where g_{MN} is the usual ten-dimensional Einstein frame metric. In the following, terms are always contracted with the Einstein frame metric, except for tilded objects and all terms in (B.2), which are contracted with the unwarped metric $\tilde{g}_{MN}$.

The leading order dilaton equation in Einstein frame reads

$$\nabla_M \partial^M \phi + \frac{1}{2}\mathrm{e}^{-\phi}|H|^2 = \mathcal{O}(\alpha'). \tag{B.3}$$

Assuming that the dilaton only depends on the internal coordinates, we can write $\nabla_M \partial^M \phi = \mathrm{e}^{-10A}\tilde{\nabla}_m \mathrm{e}^{8A}\tilde{g}^{mn}\partial_n\phi$ and integrate over internal space to find

$$\frac{1}{2}\int \mathrm{d}^6 y \sqrt{\tilde{g}_6}\, \mathrm{e}^{10A-\phi}|H|^2 = \mathcal{O}(\alpha') \tag{B.4}$$

and hence

$$\mathrm{e}^{10A-\phi}|H|^2 = \mathcal{O}(\alpha'). \tag{B.5}$$

The traced internal and spacetime components of the leading order Einstein equation then read

$$-R_4 - 2R_6 + (\partial\phi)^2 = \mathcal{O}(\alpha'), \qquad -3R_4 - 2R_6 + (\partial\phi)^2 = \mathcal{O}(\alpha'). \tag{B.6}$$

Combining the two equations and rewriting R_4 in terms of the unwarped metric yields

$$R_4 = \mathrm{e}^{-2A}\tilde{R}_4 - \frac{1}{2}\mathrm{e}^{-10A}\tilde{\nabla}^2\mathrm{e}^{8A} = \mathcal{O}(\alpha'). \tag{B.7}$$

We can now integrate over internal space to find $\mathrm{e}^{8A}\tilde{R}_4 = \mathcal{O}(\alpha')$ which with (B.7) implies that $\tilde{\nabla}^2\mathrm{e}^{8A} = \mathcal{O}(\alpha')$. Hence the warp factor is a constant up to α'-corrections. The dilaton equation (B.3) then reduces to $\mathrm{e}^{-2A}\tilde{\nabla}^2\phi = \mathcal{O}(\alpha')$ and therefore also ϕ is a constant up to α'-corrections.

We have thus shown that two-derivative terms involving the warp factor or the dilaton are at least of order $\mathcal{O}(\alpha')$, which implies that the four-derivative terms appearing in (B.2) are of even higher order. It follows that $C = \mathcal{O}(\alpha')$, and hence (B.1) yields

$$\Lambda = \mathcal{O}(\alpha'^2). \tag{B.8}$$

C Ten-dimensional argument

The result $\Lambda = 0$ can also be derived directly from the ten-dimensional equations of motion. We write the ten-dimensional action (2.17) in the form

$$S = \int \mathrm{d}^{10}x \sqrt{-g}\, \mathrm{e}^{-2\phi} L, \tag{C.1}$$

where L includes all string theory α'-corrections to the ten-dimensional supergravity. We start by pulling out an overall warp factor $g_{MN} = e^{2\omega}\tilde{g}_{MN}$. We will later on relate ω to the warp factor A used in the main text. Writing the action in terms of the tilded metric $\tilde{g}$, we get

$$S = \int \mathrm{d}^{10}x \sqrt{-\tilde{g}}\, \mathrm{e}^{8\omega}\mathrm{e}^{-2\phi} \tilde{L}, \tag{C.2}$$

notice the warp factor dependence in the action. The leading order terms of $\tilde{L}$ are

$$\tilde{L} = \tilde{R} - 18(\tilde{\nabla}^2\omega + 4(\partial\omega)^2) + 4(\partial\phi)^2 - \frac{1}{2}e^{-4\omega}|H|^2 + \mathcal{O}(\alpha'). \tag{C.3}$$

Assuming that $\tilde{L}$ only depends on the derivatives of ϕ, the dilaton equation is easily derived up to a total derivative,

$$0 = \frac{1}{\sqrt{-\tilde{g}}}\frac{\delta S}{\delta\phi} = -2\mathrm{e}^{8\omega}\mathrm{e}^{-2\phi}\tilde{L} + \text{total derivative}. \tag{C.4}$$

Using this, we can simplify the Einstein equation

$$\begin{aligned}
0 = \frac{1}{\sqrt{-\tilde{g}}}\frac{\delta S}{\delta\tilde{g}^{MN}} &= \mathrm{e}^{8\omega}\mathrm{e}^{-2\phi}E_{MN} - \frac{1}{2}\tilde{g}_{MN}\mathrm{e}^{8\omega}\mathrm{e}^{-2\phi}\tilde{L} \\
&= \mathrm{e}^{8\omega}\mathrm{e}^{-2\phi}E_{MN} + \tilde{g}_{MN}(\text{total derivative}). \tag{C.5}
\end{aligned}$$

The tensor E_{MN} is simply the variation of the Lagrangian $\tilde{L}$.

Now take the ten-dimensional manifold to be a direct product of a six-dimensional compact space and maximally symmetric spacetime. We also let $\omega = \phi/4 + A$ to switch to the unwarped Einstein frame. Since spacetime is assumed maximally symmetric, all external covariant derivatives vanish and the total derivative in (C.5) is a total derivative in internal space. We therefore look at the integrated traced Einstein equation, where the total derivatives drop out. To complete our analysis it is then enough for us to show that, when both indices lie in spacetime, $E_{\mu\nu}$ is a sum of terms that contain a positive power of the external curvature tensor. The only covariant quantities with external indices are the metric, the curvature tensor and the epsilon tensor. Keeping this in mind, there are only three possibilities that give a non-vanishing contribution to $E_{\mu\nu}$:

- Terms where one or both of the free indices are that of a curvature tensor. These obviously carry a positive power of the Riemann tensor and we are done.

- Second are terms where the free indices are that of a metric, $E_{\mu\nu} \sim g_{\mu\nu}B$, coming from the ten-dimensional term $g_{MN}B$, where B is a ten-dimensional scalar. This

could be problematic, since B does not have to involve the Riemann tensor. Clearly, these terms cannot occur as a result of varying the determinant, we already got rid of those using the dilaton equation. However we could have such terms from varying curvature tensors or covariant derivatives or more generally the connection. But varying the connection always gives a total derivative because of the equation

$$\delta\Gamma^R_{MN} = \frac{1}{2}g^{RS}\left(\nabla_M \delta g_{NS} + \nabla_N \delta g_{MS} - \nabla_S \delta g_{MN}\right), \qquad (C.6)$$

and we see that B must be a total derivative. Again this reduces to a total derivative in internal space and upon integration drops out.

- The final possibility are terms where both external indices come from epsilon symbols. Clearly, an epsilon symbol must have four spacetime indices, and these must contract with something, the only possibility is a curvature tensor.

Other terms of the tensor E_{MN} will be those, where the free indices are that of derivatives or fluxes etc. These all vanish in the maximally symmetric external spacetime. We have thus shown that all terms in the Einstein equation, traced with the external metric and integrated over internal space, contain a positive power of the Riemann tensor. This eventually leads to (2.22), and our result follows.

References

[1] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [`hep-th/0502058`] [ɪɴSPIRE].

[2] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry breaking and alpha-prime corrections to flux induced potentials*, *JHEP* **06** (2002) 060 [`hep-th/0204254`] [ɪɴSPIRE].

[3] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [`hep-th/0105097`] [ɪɴSPIRE].

[4] L. Anguelova and C. Quigley, *Quantum corrections to heterotic moduli potentials*, *JHEP* **02** (2011) 113 [`arXiv:1007.5047`] [ɪɴSPIRE].

[5] L. Anguelova, C. Quigley and S. Sethi, *The leading quantum corrections to stringy Kähler potentials*, *JHEP* **10** (2010) 065 [`arXiv:1007.4793`] [ɪɴSPIRE].

[6] M. Dine and N. Seiberg, *Couplings and scales in superstring models*, *Phys. Rev. Lett.* **55** (1985) 366 [ɪɴSPIRE].

[7] M. Dine and N. Seiberg, *Is the superstring weakly coupled?*, *Phys. Lett.* **B 162** (1985) 299 [ɪɴSPIRE].

[8] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [`hep-th/0505160`] [ɪɴSPIRE].

[9] S.S. Haque, G. Shiu, B. Underwood and T. Van Riet, *Minimal simple de Sitter solutions*, *Phys. Rev.* **D 79** (2009) 086005 [`arXiv:0810.5328`] [ɪɴSPIRE].

[10] C. Caviezel et al., *On the cosmology of type IIA compactifications on* SU(3)-*structure manifolds*, *JHEP* **04** (2009) 010 [`arXiv:0812.3551`] [ɪɴSPIRE].

[11] R. Flauger, S. Paban, D. Robbins and T. Wrase, *Searching for slow-roll moduli inflation in massive type IIA supergravity with metric fluxes*, *Phys. Rev.* **D 79** (2009) 086011 [`arXiv:0812.3886`] [ɪɴSPIRE].

[12] M.P. Hertzberg, S. Kachru, W. Taylor and M. Tegmark, *Inflationary constraints on type IIA string theory*, *JHEP* **12** (2007) 095 [arXiv:0711.2512] [ɪɴSPIRE].

[13] E. Silverstein, *Simple de Sitter solutions*, *Phys. Rev.* **D 77** (2008) 106006 [arXiv:0712.1196] [ɪɴSPIRE].

[14] U.H. Danielsson, S.S. Haque, G. Shiu and T. Van Riet, *Towards classical de Sitter solutions in string theory*, *JHEP* **09** (2009) 114 [arXiv:0907.2041] [ɪɴSPIRE].

[15] B. de Carlos, A. Guarino and J.M. Moreno, *Flux moduli stabilisation, supergravity algebras and no-go theorems*, *JHEP* **01** (2010) 012 [arXiv:0907.5580] [ɪɴSPIRE].

[16] T. Wrase and M. Zagermann, *On classical de Sitter vacua in string theory*, *Fortsch. Phys.* **58** (2010) 906 [arXiv:1003.0029] [ɪɴSPIRE].

[17] O. Lechtenfeld, C. Nolle and A.D. Popov, *Heterotic compactifications on nearly Kähler manifolds*, *JHEP* **09** (2010) 074 [arXiv:1007.0236] [ɪɴSPIRE].

[18] A. Chatzistavrakidis, O. Lechtenfeld and A.D. Popov, *Nearly Kähler heterotic compactifications with fermion condensates*, *JHEP* **04** (2012) 114 [arXiv:1202.1278] [ɪɴSPIRE].

[19] S.R. Green, E.J. Martinec, C. Quigley and S. Sethi, *Constraints on string cosmology*, *Class. Quant. Grav.* **29** (2012) 075006 [arXiv:1110.0545] [ɪɴSPIRE].

[20] J. Held, D. Lüst, F. Marchesano and L. Martucci, *DWSB in heterotic flux compactifications*, *JHEP* **06** (2010) 090 [arXiv:1004.0867] [ɪɴSPIRE].

[21] B.A. Campbell, M.J. Duncan, N. Kaloper and K.A. Olive, *Gravitational dynamics with Lorentz Chern-Simons terms*, *Nucl. Phys.* **B 351** (1991) 778 [ɪɴSPIRE].

[22] B. Underwood, *A breathing mode for warped compactifications*, *Class. Quant. Grav.* **28** (2011) 195013 [arXiv:1009.4200] [ɪɴSPIRE].

[23] J. Derendinger, L.E. Ibáñez and H.P. Nilles, *On the low-energy $D = 4$, $N = 1$ supergravity theory extracted from the $D = 10$, $N = 1$ superstring*, *Phys. Lett.* **B 155** (1985) 65 [ɪɴSPIRE].

[24] M. Dine, R. Rohm, N. Seiberg and E. Witten, *Gluino condensation in superstring models*, *Phys. Lett.* **B 156** (1985) 55 [ɪɴSPIRE].

[25] S. Kachru, R. Kallosh, A.D. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240] [ɪɴSPIRE].

[26] H. Kunitomo and M. Ohta, *Supersymmetric AdS$_3$ solutions in heterotic supergravity*, *Prog. Theor. Phys.* **122** (2009) 631 [arXiv:0902.0655] [ɪɴSPIRE].

[27] M.R. Douglas and R. Kallosh, *Compactification on negatively curved manifolds*, *JHEP* **06** (2010) 004 [arXiv:1001.4008] [ɪɴSPIRE].

[28] J. Blaback et al., *Smeared versus localised sources in flux compactifications*, *JHEP* **12** (2010) 043 [arXiv:1009.1877] [ɪɴSPIRE].

[29] F. Denef, *Les Houches lectures on constructing string vacua*, arXiv:0803.1194 [ɪɴSPIRE].

[30] R.M. Wald, *General Relativity*, University of Chicago Press, Chicago U.S.A. (1984).

[31] A. Strominger, *Superstrings with torsion*, *Nucl. Phys.* **B 274** (1986) 253 [ɪɴSPIRE].

[32] B. de Wit, D. Smit and N. Hari Dass, *Residual supersymmetry of compactified $D = 10$ supergravity*, *Nucl. Phys.* **B 283** (1987) 165 [ɪɴSPIRE].

[33] J.M. Maldacena and C. Núñez, *Supergravity description of field theories on curved manifolds and a no go theorem*, *Int. J. Mod. Phys.* **A 16** (2001) 822 [hep-th/0007018] [ɪɴSPIRE].