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Height, growth and site index of jack pine (Pinus banksiana Lamb) in the Thunder Bay area : a system of site quality evaluation

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Height Growth And Site Index Of Jack Pine (*Pinus banksiana* Lamb.) In The Thunder Bay Area

- A System Of Site Quality Evaluation -

Daniel J. Lenthall ©

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ABSTRACT

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Key Words: height growth, height growth models, jack pine (*Pinus banksiana* Lamb.), nonlinear regression, site index, site index curves, site quality estimation, stem analyses

Height growth patterns and site index were studied using stem analyses taken from dominant and codominant trees growing on 109 plots located in mature, natural, fully stocked evenaged stands of jack pine (*Pinus banksiana* Lamb.) in the Thunder Bay area. The observed height/age data were modeled using several nonlinear biological growth models: Richards growth model (1959); a modified Weibull function; and an expansion of the Richards model proposed by Ek (1971).

Height growth patterns of jack pine varied with level of site index, being more curvilinear as level of site index increased. Height growth patterns were similar for jack pine growing on glacialfluvial sands, on moraines, on lacustrine soils and shallow to bedrock soils. Analyses showed that site index curves were more precise when based on breast height age instead of total age of the trees.

Height growth curves, site index curves and a site index prediction equation were calculated from the jack pine stem analyses data. A modification of the Chi-squared distribution was used for testing the accuracy of the site index curves and prediction equation. The accuracy of the computed curves was tested using independent stem analyses data from 32 additional confirmation plots. Comparisons with this independent data showed very close agreement; the 95% prediction intervals calculated for the site index curves and site index prediction equation using independent data are -0.17 ± 0.89 m and -0.20 ± 1.14 m respectively.

Comparison between Plonski's (1974) formulated site index curves for jack pine and the site index curves produced in this study indicate differences in predicted heights at ages greater than index age (50 years), but no differences younger than index age. Plonski's site index curves showed lower predicted heights for each level of site index at ages greater than 60 years.

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CHAPTER 1

INTRODUCTION

FOREST SITE QUALITY

"Forest site quality is concerned with the ability of forest land to grow trees; thus, site quality estimation corresponds to land capability estimation for various agricultural crops" (Carmean, 1975). Site quality is the sum total of many environmental factors: soil quality; topography; macroclimate and microclimate (Daniel *et al.,* 1979). Therefore, site quality is a relatively stable quantity in an undisturbed ecosystem. Site quality estimations for a given area vary by species. A site may provide a supply of nutrients and moisture that enables a given species of tree to grow vigorously, but may not provide all the necessary requirements for an alternate species of tree to achieve the same growth. The site does not change, but the site quality will depend on the species that occupies the site.

In a managed ecosystem, site quality may be increased by improving one or more of the factors limiting tree growth. Actions that can improve site quality are fertilization to correct nutrient deficiencies, drainage to remove excess moisture, and irrigation to correct moisture deficiencies. Likewise, site quality can be decreased by practices that damage factors critical to tree growth. Excess erosion that removes topsoil and nutrients, compaction that reduces soil aeration and water movement, and biomass harvest that may deplete the nutrient capital on some sites can degrade the site quality of an area. Proper forest management goals should attempt to improve site quality whenever possible and avoid practices that may degrade site quality.

FOREST SITE PRODUCTIVITY

Forest site productivity is the ability of the land to produce forest products under specific management regimes. Consequently, forest site productivity is the sum of the innate forest site quality, plus or minus any management inputs (Pritchett, 1979). Site productivity may be increased by altering one or more of the stand factors limiting tree growth. Site productivity can be improved by treatments that improve stocking, thinning to harvest natural stand mortality, species conversions that substitute fast growing tree species for slow growing species, or release of planted or natural regeneration.

Actual forest productivity is generally measured in terms of the gross volume or weight of bole wood per hectare per year over a normal rotation. Productivity data are often limited to specific localized areas, sometimes as local as individual stand values. The relationship between site quality and site productivity has not been fully defined over all conditions.

FOREST SITE PRODUCTIVITY RELATED TO FOREST SITE QUALITY

As both site quality and site productivity are species oriented, the relationship between site quality and site productivity must be defined in terms of a single species. Site productivity and site quality may be closely related for stands that are fully stocked for a given species. Understocked stands result in yields that are less than can be attained for a particular level of site quality. The relationship between site quality, site productivity and stocking level will be better understood as data from successive rotations are compiled and analyzed. In the interim, forest site quality evaluation attempts to accurately estimate the forest site quality over large forested areas for practical use. Intensive management practices such as site preparation, planting and cleaning can then be undertaken on the best sites to produce the highest yield of the most valued product in the shortest time.

IMPORTANCE TO MANAGEMENT

Forest site evaluation is one of the necessary tools for intensive forest management. Site evaluation is an important step in developing intensive management plans because the most productive sites should be managed most intensively. For any site, the intensity of management should be determined by: markets; labour supply; accessibility and site quality (McLintock and Bickford, 1957). That is, the productive potential of the site should dictate the level of forest management within limits set by markets, labour and accessibility.

Increased management activities are accompanied by increased costs that must be carried over the rotation of the stand. The value of the final forest product must cover the costs of management and return a profit. Yields in both agriculture and forestry are affected both quantitatively and qualitatively by the effects of site quality (Davis, 1966); thus site quality affects the value of the crop at rotation. The most intensive management practices should be applied to the highest quality sites, and to the most valuable products thus favorable economic returns are most likely to be realized.

STUDY SPECIES - JACK PINE (*Pinus banksiana* Lamb.)

Forest management in northwestern Ontario is becoming more intensive. Accurate identification of the most productive forest sites is needed to concentrate intensive management practices on the best sites. Other than Plonski's site class curves (1974), means for identifying forest site quality are generally lacking for the boreal forest species of northwestern Ontario.

Jack pine is one of the major species in Ontario's forest, ranking third in volume after black spruce (*Picea mariana* (Mill.) B.S.P.) and trembling aspen (*Populus tremuloides* Michx.). Fifteen per cent of the total wood volume of Ontario's boreal forest region is jack pine (Howse, 1984). However, on a commercial basis, jack pine is considered to be the second most important tree species in northwestern Ontario. Jack pine now accounts for 31% of all the wood harvested in the North Central Region of Ontario, 70% as pulpwood and 30% as sawlogs (Davison, 1984).

Jack pine can maintain growth on dry sandy or gravelly soils where other species can scarcely survive, but best height and diameter growth is usually found on moderately moist, sandy loam and clay loam soils where mid-summer water table is 1.5 to 2.0 m below the soil surface (Fowells, 1965; Moore, 1984). Extensive, pure, even-aged stands of jack pine are commonly associated with large outwash sand flats. Jack pine is a pioneer species that invades sites where mineral soil is exposed; hence, the role of fire in natural regeneration of jack pine (Moore, 1984). On moist sites, jack pine often grows in mixedwood stands in association with black spruce and aspen.

Jack pine in its native range is the fastest growing conifer other than tamarack (*Larix laricina* (Du Roi) K. Koch) (Fowells, 1965). Seedlings reach breast height in 5 to 8 years (Rudolf, 1950), and continue to grow rapidly in height for the next fifty years (Fowells, 1965). Height growth on average sites in Minnesota averaged approximately 30 cm per year for the initial 50 years, thereafter the height growth rate declines until age 100, where height growth nearly ceases (Fowells, 1965). These growth relations also apply to jack pine in the Thunder Bay area.

Jack pine is intensively managed for pulpwood and sawlogs in northwestern Ontario. Among the conifers of the boreal forest, jack pine is the most responsive to intensive management with regards to cost and success of regeneration and tending, rate of juvenile growth, and early returns on intensive management (Yeatman, 1984). Jack pine also has many advantages that make it a prime species for genetic improvement - early sexual maturity, high fecundity, and regular flowering (Yeatman, 1984). Tree improvement can provide trees with better form, insect and disease resistance, and ultimately modest volume gains (Brown, 1984). With accurate jack pine site quality information, forest managers can prescribe intensive silvicultural practices on the best sites to realize the highest returns from an intensively managed forest.

STUDY GOALS AND OBJECTIVES

The goals and objectives of this study are incorporated into a large project of forest site quality evaluation proposed by Dr. W. H. Carmean at Lakehead University - Thunder Bay, Ontario (Carmean, 1985). The specific goals of this study were as follows:

1) develop a system of site quality evaluation for jack pine sites based on previous height growth of natural jack pine determined by stem analyses;

2) quantify height growth of natural jack pine using mathematical formulae;

3) improve the methodology of constructing site index curves and accuracy of prediction;

4) test whether soil conditions associated with different landforms had an effect on height growth of natural jack pine;

5) test whether level of site index influenced the shape of height growth patterns of natural jack pine;

6) compare the calculated site index curves with Plonski's (1974) metric site class curves for jack pine presently used in forest management throughout Ontario.

CHAPTER 2

LITERATURE REVIEW

EARLY HISTORY OF SITE QUALITY EVALUATION

Traditionally, forestry in North America revolved around fire protection and efficient harvesting of the seemingly inexhaustible supply of virgin timber (Carmean, 1975). As forest management replaced exploitive forestry practices, the need to estimate future yields became necessary. And predicting future yields requires an understanding of growth and mortality as related to site quality (McKeever, 1946).

During the first two decades of the twentieth century, the need for a standard means of site evaluation was recognized. Intense controversy revolved about three separate methods of site evaluation. One faction favored volume as an expression of site quality, a system that was currently popular in Germany. "Current annual cubic foot increment is the only method of evaluating site quality, anything else is simply a makeshift method," (Bates, 1918). Although Bates (1918) stressed a volume expression of site evaluation, he did not offer any suggestions on how to measure or express the volume increment. A second group favored a system of "forest site types", (Zon, 1913). This system was based on plant indicator species that followed the early work of Cajander in Finland who later summarized his work (Cajander, 1926). This system classified land areas into similar ecological units based on vegetation, with associated height growth curves and yield tables for each unit.

The final group proposed an expression of height growth as an index of site quality (Roth, 1916, 1918; Watson, 1917; Frothingham, 1918). They recognized volume production as the ultimate standard of site productivity, but saw the practical difficulties of using volume production as a direct means of site evaluation (Carmean, 1975). Advocates of height growth state that: (1) height is a sensitive measure of site quality; (2) height growth is usually independent of species mixture and stocking; (3) measurements of height and age are easily determined and understood; and (4) site classifications determined by height growth are species specific, not permanent site types, so they are applicable to long or short lived species (Frothingham, 1918; Roth, 1916). Roth also states that a measure of site quality that a forester can use efficiently when he (she) is asked to survey a tract of timber land is desirable.

A committee of the Society of American Foresters recommended classifying sites on the basis of actual mean annual growth in cubic volume at approximate age of culmination of the mean volume growth for well stocked stands (Sparhawk *et al.*, 1923). They did not recommend any one method of determining site quality, but "were inclined to look with favor on the use of height growth of dominant trees in stands above juvenile stage, if neither too open nor too crowded." Following the recommendation of this committee, large numbers of normal yield tables based on dominant tree heights were constructed for major forest types (Monserud, 1984a).

THE CONSTRUCTION OF NORMAL YIELD TABLES

The yield table is essentially a German device (Spurr, 1952). The construction of yield tables involves two basic steps: (1) sorting data into site classes based on volume; and (2) the construction of growth or yield curves for each site class. European yield tables were generally constructed from several rotations of permanent plot data (Monserud, 1984a), but, this type of data was not available when most yield tables were constructed in North America. Yield tables were needed quickly for the extensive unmanaged forests of North America. The historical yield records used by the Europeans were not available, thus compromises were necessary.

Lacking any long term yield data, early yield tables of North America were constructed from temporary plot data. Although many mensurationists in the United States were involved, the methods used to construct site index curves and associated yield tables were almost entirely those specified by Bruce (Monserud, 1984a).

Early mensurationists faced several problems constructing normal yield tables using current temporary plot data. The first problem involved the concept of "normality". The natural unmanaged forest was assumed "normal", that is, fully stocked. When fully stocked stands were studied stocking was not considered as a variable that affected volumes of the sampled stands. Any difference in volume associated with a given site was then attributed to differences of age or site quality.

The second problem involved Bruce's (1926) assumptions of anamorphosis or proportional height growth. Forest stands growing on different sites were assumed to have height and volume growth proportional to the average of all stands. The rate of height growth and volume accretion was the same on all sites, but varied in magnitude according to estimated site quality. Bruce's (1926) anamorphic methods became the standard for constructing site index curves for site quality estimation and yield table stratification.

SITE INDEX AS SITE QUALITY ESTIMATION

Direct Measurements of Site Index

Today, forest managers within North America generally accept "site index", as an index of site quality. The Society of American Foresters defines site index as "a particular measure of site class based on the height of the dominant trees in a stand at an arbitrarily chosen age" (S.A.F., 1983). Generally height of dominant and codominant trees at 50 years of age is used as site index for eastern species, although Curtis and Post (1962) used 75 years as a base age for northern hardwoods in Vermont. Heights at 100 years are used for longer lived western species, but younger ages are sometimes used as index age for pine plantations in eastern and southern United States. The rationale for the use of a base age stems from the fact that height growth is strongly governed by two variables; site quality and age (Berglund, 1976). Therefore, by expressing site index in terms of height at a specific age, the effects of age on height growth is standardized, allowing tree height to be used as an expression of site quality.

Site index determination can be used for estimating site quality for a specific tree species if certain conditions are met; (1) reliable site trees are available for measurement, and (2) accurate site index curves have been developed. Accurate site index determination requires reliable site trees that are carefully selected from free-growing, uninjured, dominant trees growing in well stocked even-aged stands. Using these free-growing, uninjured trees minimizes external factors (suppression, wind damage, etc.) that disrupt the normal height growth of the selected site trees because such factors will affect the accuracy of site index estimation.

Accurate site index curves or equations must accompany height and age measurements from the reliable trees. Site index curves should accurately describe tree height growth patterns for a particular locality, soil type, or site class. Errors resulting from unreliable site trees or from inaccurate site index curves will adversely affect the reliability of site quality estimation hence the reliability of management decisions based on site index (Powers, 1972).

An assumption made when assessing site quality using site index is that trees now dominant have been dominant throughout their lives. In even-aged stands, especially those of shade intolerant species, this assumption is probably valid, thus the present dominants have always been free to attain their full height since establishment (Daniel, *et al.*, 1979). Accordingly, site index is the height growth projection of these free-growing dominant trees either forward or backward in time to a standardized index age.

In uneven-aged stands of shade-tolerant species, this assumption is not valid (McLintock and Bickford, 1957 ; Stage, 1963). Shade tolerant trees that are currently dominant may have existed for years as suppressed trees before release enabled them to attain their current status as dominants. Alternate methods of site evaluation are needed for uneven-aged stands.

Another assumption fundamental to the concept of site index is that height growth of trees in the main canopy is independent of stand density (Jones, 1969; Carmean, 1975). Ralston (1953) reported no difference in average height for dominant jack pine trees 25 years after planting at spacings of 1.22 x 1.22 m, 1.83 x 1.83 m, 2.44 x 2.44 m. This study indicates that site index for jack pine can be accurately estimated in stands with varying densities falling within the realm of "full or normal stocking".

Indirect Measurements of Site Index

Species Site Index Comparisons

Intensive forest management requires the ability to select the most productive and most desirable tree species for each area of land. A key question in intensive management is, "What is the most productive and desirable tree for each site?" Answering such a question requires site quality estimates for all alternative tree species that might be considered for management on each site. However, in most cases usable site trees of these various alternative species may be absent from a particular site, thus no suitable trees are available for direct determination of site index (Thurston, 1984). Site index comparisons enable a forester to use site index of the tree species present in the stand as a basis for estimating site index of species not present.

Site index comparison equations and graphs are based on site index measurements of two or more species found in mixed stands over a wide range of site quality. Linear regressions correlate the site indices of one species with other associated species. From the regression equations, average trend equations can be calculated for each pair of species that are commonly found in association. Curtis and Post (1962) developed site index comparisons for northern hardwoods in Vermont. Site index comparisons for northern hardwoods were also developed for the Lake States (Carmean, 1979a). McQuilkin (1974) and Carmean and Hahn (1983) developed site index comparison equations for white, black and scarlet oaks in the Central States. Thurston (1984) and Ortiz (1985) developed site index comparison equations and graphs for the major boreal species in northwestern Ontario.

Site index comparisons can aid the forester in selecting the most desirable species to manage on a particular site (Carmean, 1975). When selecting the most desirable species for management, site index comparisons are only the first necessary step; other factors also need to be considered. Height growth before and after index age, the volume produced at each level of site quality, and the economic value of the final product are important factors that also must be considered when selecting the best species for management on a particular site.

Growth Intercept Method

Ideally, site quality estimates should be available for all forest land regardless of the type, condition, or age of the stand. Most site index curves are constructed for stands 20 years and older. Thus other methods are needed for estimating site quality in young stands. The growth intercept method of site evaluation has been developed for young stands or plantations that have distinct internodes, marking the course of annual height growth (e.g. red pine (*Pinus resinosa* Ait.), white pine (*Pinus strobus* L.), white spruce (*Picea glauca* (Moench) Voss), and Douglas-fir (*Pseudotsuga menziesii* Mirb. Franco)).

The growth intercept method uses a selected period of early height growth as an index of site quality rather than long term height growth (Carmean, 1975). Wakeley and Marrero (1958) were the first to use the growth intercept approach. They found that for southern pines the total length of five internodes (the growth intercept) beginning at the first node below breast height was closely related to total tree height. When growth intercept and site index are related, then growth intercept is a useful means for estimating the site quality for young plantations (Alban, 1979).

The advantages of this method are: (1) it is used in stands too young for direct site index determination based on standard site index curves; (2) actual measurements of leader length are used rather than height growth projections thus reducing estimation errors; (3) it is quickly and easily applied; and (4) variation caused by early erratic height growth below breast height is reduced by measuring internodes above breast height (Alban, 1972; Carmean, 1975). The disadvantages of the growth intercept method are: (1) early height growth may not always characterize height growth in later years (Wilde, 1964); and (2) tree species that have distinct annual nodes are necessary for measurement, usually restricting the usefulness of the growth intercept method to conifers.

Soil-Site Studies

The methods of site evaluation previously discussed have relied upon dominant trees growing in well stocked even-aged stands. The need for accurate site evaluation also exists on land denuded of trees, or in uneven-aged or poorly stocked stands not suitable for site index determination. The soil-site method of site evaluation has been developed for areas where trees or stands are not suited for directly measuring site index.

Soil-site evaluation has received more emphasis in North America than any other indirect estimation of site index (Carmean, 1975). The soil-site method is based upon studies where a large number of site plots are located in fully stocked, even-aged mature forest stands on a wide range of soil and topographic features within the defined study area. Site index measured on each plot, preferably determined by stem analyses, is correlated using multiple regression techniques with soil and topographic data collected from each site plot.

Soil features found to be most important in soil-site studies are usually concerned with depth, texture and drainage; that is, properties that determine the quality and quantity of growing space for tree roots (Coile, 1952). Correlated site features need not be causative factors, but they must be consistently correlated with site index (Pritchett, 1979). For practical considerations, the correlated site features should be easily recognized and measured in the field thus resulting in rapid and easy estimates of site quality.

There is much room for advancement in soil-site research. Multiple regression methods are not designed to describe causative effects, therefore, future research should strive to identify the causative link between correlated site features and the growth of trees. Studies that better define these causative factors would result in a clearer understanding of the requirements for specific tree species, and thus can aid in properly matching species to site in intensively managed forests.

EARLY SITE INDEX WORK

Bruce's Anamorphic Site Index Curves

Bruce's (1926) anamorphic method of site index curve construction used total height and total age of many stands over a wide range of site quality. For each stand sampled, the height/age observation was plotted graphically. An average guiding curve was then drawn through the scattergram that combined all the plot height/age data. This average curve was then used to represent the growth of stands for the geographic area that was sampled in the study. Bruce used his proportional assumption to draw height growth curves for each level of site quality from this average guiding curve. A series of curves of height over age was drawn through each desired level of site index proportional to the average guiding curve. The resulting family of height growth curves had the same shape (i.e. harmonized) as the guiding curve, differing in magnitude only by a fixed percentage. This curve construction method produces site index curves that came to be known as "anamorphic" or "harmonized" site index curves.

Several assumptions are inherent in this anamorphic or harmonized site index curve technique. The first assumption stated by Bruce (1926) is that the sample plot data adequately represents the full range of site quality within each age class. Thus the scatter of the data when height is plotted against age adequately indicates the height growth of stands. The second assumption is that the effect of differences in site on height growth is relatively the same at all ages. Thus the scatter of data representing each age class is assumed to be normally distributed and that certain age classes are not represented by an abnormal distribution of good or poor sites. The final assumption is that growth curves are proportional to the site. Good sites and poor sites are assumed to have the same shape site index curve and that differences in soils, topography and climate have no effect on curve shape throughout the range of the species.

Usually none of these assumptions is true, certainly none is safe (Spurr, 1952). Despite these seemingly rash assumptions, Bruce's guidelines were used to create nearly all early site index curves. For almost two decades, these methods and assumptions received little criticism

(Monserud, 1984a).

Problems and Biases of Harmonized Curves

Many problems inherent with harmonized site index curves were the result of insufficient data, methods of construction and the assumptions upon which the curves were based (Curtis, 1964; King, 1966; Heger, 1968; Beck and Trousdell, 1973; Carmean, 1975).

Unfortunately, the period of vast forest exploitation preceded the age of forest management. This caused a major error in harmonized site index curves constructed by Bruce's (1926) method; the correlation of site index with age. Site index, the height the dominant trees actually attain at base age, was never validated for sample plots that were used for constructing harmonized site index curves. Therefore, it was assumed that each site class was fully represented within each age class. Older age classes were often represented by the lower site classes (Beck, 1971; Carmean, 1975; Monserud, 1984a).

The timber barons of the late nineteenth century may have been greedy, but they were not stupid (Monserud, 1984a). The best and most easily accessible stands were often harvested first, often leaving the poor quality stands. As the cutover lands regenerated, most of the young age classes were represented by the good quality sites, while most of the older age classes were represented by the poor quality sites. The guide curve and corresponding site index curves of harmonized construction were distorted by this correlation of site quality and age, resulting in inaccuracies.

Another criticism of harmonized site index curves is the interpretation of the guide curve. The individual site index curves created from the guide curve were supposed to represent the height growth of trees, yet how can this be? The guide curve was not a growth curve. It was merely an average curve based upon a set of one time observations. The only reliable information that could be derived from the height and age data was the average height of the stand at the time of measurement. Nothing can be inferred about the height growth patterns before or after the time of measurement.

Bruce (1926) states that the guide curve represents average growth tendencies for the entire stand over time, and different levels of site quality were represented by the proportional site index curves drawn from the guide curve. Spurr (1952) characterized this assumption as "an approximation that gives fairly good results in most instances without ever being true". Certainly, no biological evidence exists why both good and poor site curves should be proportional (Monserud, 1984a).

EVIDENCE OF POLYMORPHIC HEIGHT GROWTH

Criticism of Bruce's (1926) anamorphic method of site index curve construction came soon after the method was proposed. Bull (1931) was one of the first researchers to demonstrate "polymorphic" - (many shaped) patterns of height growth for red pine plantations, refuting the assumption of anamorphosis:

> Careful study of the actual progress of height growth on various sites, particularly the poorest and best, showed, however, that the anamorphic curves were not applicable, as the curves for the poorest sites do not have the same characteristics as curves for the best sites.

Unfortunately, most of Bull's contemporaries ignored Bull's observations, and anamorphism continued (McArdle *et al.*, 1930; Haig, 1932; Meyer, 1937, 1938; Schnur, 1937). The existence of polymorphic patterns of height growth has been shown by several methods (Bull, 1931; Beck, 1971; Carmean, 1968; 1975).

When harmonized curves were compared from different areas of the species's ranges, differences in height growth patterns existed (Carmean, 1956; 1972; 1975; 1979b; Powers, 1972; Trousdell, *et al.*, 1974). This contradicted the assumption of anamorphosis proposed by Bruce (1926). It also indicated the need for more localized curves other than the early site index curves that covered the entire range of a single species (Schnur, 1937; USDA Forest Service, 1929).

Remeasurement of permanent plots also indicated different height growth patterns associated with different levels of site quality. Lange (1951), Spurr (1956) and Watt (1960) all report that site index changed as the stand aged. Obviously, the site index of a tree or stand cannot change. It can only have one height at index age. In these studies, the estimate of site index changed over time, not site index. The apparent change in site index was the inability of harmonized site index curve to predict the actual height growth trends of the trees or stand.

SITE INDEX CURVES FROM STEM ANALYSES

By the early 1960's, dissatisfaction with site index curves based on the proportional guide curve method produced a second wave of growth and yield studies (Monserud, 1984a). These new studies had one thing in common; stem analyses of the site trees. Stem analysis brought many refinements for constructing site index curves. Internode measurements and height growth patterns from sectioned trees allowed investigators to observe the actual growth patterns and site index of the sample trees (Beck, 1971). Thus site index could be observed rather than estimated by proportional means. Ironically, Bruce (1926) considered stem analysis as an alternative to his anamorphic method, but rejected it. His fear was that today's site trees may not be the same site trees as in the past and stem analyses on individual trees may not indicate growth of stands (Spurr, 1952; Dahms, 1963; Monserud, 1984a).

Graphical methods of curve construction have gradually been replaced by mathematical expressions of height growth. Using stem analyses, the real growth series of the site trees could now be modeled, eliminating the need for a guide curve. Attempts to fit mathematical formulae to height growth of trees date back to the 19th century and possibly beyond (Beck, 1971). Curve forms having a definite mathematical formula have several advantages (Stage, 1963). First, they provide coefficients that can be related to competition, habitat type, soil features, or site index that may influence curve shape. Second, the computations are adapted to efficient analyses by digital computers. Third, mathematical formulae eliminate subjectivity, and reduce interpolation error of graphical methods.

Nonlinear regression has been widely used to formulate published harmonized site index curves for economically important species in Canada (Payandeh, 1974a,b) and the Lake States (Lundgren and Dolid, 1970; Hahn and Carmean, 1982). Wiant (1975), Hilt and Dale (1982) and Farrar (1985) all used different methods to formulate the original site index curves for upland oaks developed by Schnur (1937). The data for many of these formulations were taken from harmonized curves, so they apply to the readings made from the original site index curves, thus data deficiencies of the original curves still apply to the formulated curves.

Polymorphic site index curves based upon stem analyses or internode measurements have gained increasing popularity since the 1960's --- researchers finally arrived at the conclusions reached by Bull (1931). The earliest mathematical models for site index curve construction were entirely empirical models (Schumacher, 1939) hence, very restrictive in curve shape. Recent trends in biological modeling have produced more elegant and flexible growth curves able to model the true growth trends that vary with different levels of site quality, stocking or edaphic factors. Most models for polymorphic site index curves incorporate site index to provide for different curve shapes associated with different levels of site index (Lundgren and Dolid, 1970).

Site index has been incorporated into site index models by stratifying stem analyses data into similar classes of site index, with separate analyses for each class. Carmean (1972) stratified stem analyses data of several oak species into three metre site index classes for analysis. A family of polymorphic site index curves was constructed so that the shape of each curve was determined by the data from each class. Although the polymorphic growth was clearly expressed, this method has a major drawback. A single equation does not represent the entire family of site index curves because the model coefficients vary for each level of site index. Estimating site index from field data relies on the site index curves for a graphical determination of site index. In contrast, when a single mathematical formula is used site index can be quickly estimated free of personal judgement errors associated with graphical interpolations.

Site index can be directly incorporated into a model as another independent variable. This creates a single model that varies in shape and amplitude with level of site index. This is most commonly accomplished by expressing a coefficient of a generalized growth model as a function of site index (Lundgren and Dolid, 1970; Trousdell *et al.*, 1974; Burkhart and Tennent, 1977; Hahn and Carmean, 1982). This direct approach creates a more flexible and useful expression of height growth than an indirect incorporation of site index. Individual height growth curves are not restricted to the class used for calculation as in the indirect method,-but can be calculated for any level of site index represented within the data.

A height growth model also can incorporate other independent variables that may affect height growth. The possibility of various height growth patterns associated with stands having the same site index has been recognized (Heiberg and White, 1956; Stage, 1963; Carmean, 1975; Burkhart and Tennent, 1977; Grigal, 1984; Monserud, 1984a). Additional variables incorporated into a height growth model may eliminate some of the variation in pattern of height growth within a single site index class.

Stage (1963) included a variable for the number of rings in the first radial 1.5 inches (3.75 cm.) at breast height for grand fir (*Abies grandis* (Dougl.) Lindl.). He concluded that trees that have few rings in the first 1.5 inches radius had more rapid height growth in the years before index age (100 years) than trees that had a large number of rings in the first 1.5 radial inches. After index age, the growth trends reverse, trees with large radial values sustain height growth longer than trees with low radial values. He constructed site index curves for different values of annual rings in the first 1.5 inches of radius.

Monserud (1984b) used Daubenmire (1968) habitat types to describe different curve shapes of inland Douglas-fir. He found that, although mean site index was similar for most habitat types, height growth patterns at advanced ages varied with respect to habitat type.

Lynch (1958) and Alexander *et al.*, (1967) used estimates of stand density for calculating site index curves for ponderosa (*Pinus ponderosa* Laws.) and lodgepole pine (*Pinus contorta*·Dougl.). Lynch (1958) used basal area of the stand and Alexander *et al.*, (1967) used a crown competition factor. In both studies, height growth was less at high levels of stand density.

DESIRABLE PROPERTIES OF SITE INDEX CURVES

The methods for constructing site index curves have varied over time, within geographic areas, and for different species. Devan and Burkhart (1982) outlined specific criteria that they felt were desirable attributes of site index curves:

> Site index curves should be polymorphic in shape, and base-age invariant (no specific base age). For most species, the upper asymptote should be a function of site index, with trees on better sites reaching a higher ultimate height than those on poor sites. The equation should give a height of zero at age zero, and a height equal to site index at index age. The equation should also express an inflection point if the data used to estimate the parameters warrant it. The inflection point would be the age at which height increment is maximum. In order to be useful and readily accepted, any method for deriving site index equations should be easy to apply and the results easy to interpret.

Few site index curves will meet all these criteria, but the goal should be to meet as many as possible. The ability of site index curves to meet these criteria rests mainly on the model used to represent height growth and the means of model fitting.

MODEL SELECTION

The model used to simulate height growth of trees affects the construction of site index curves. Restrictive models may mask many differences in height growth patterns, and elaborate models may be too complicated for practical application. Numerous models have been presented in the literature, but the choice of the most appropriate model is, however, still highly problematic (Beck, 1971). Because forest growth processes are seldom linear many researchers have resorted to polynomial regression and logarithmic transformations to approximate forest growth. Though these techniques often provide reasonably adequate approximations, they occasionally fail to represent the true nature of the process being investigated (Schwandt, 1979).

Tree height in relation to age is one such example. As available data of height over age from stem analyses are related to growth, biological growth functions are more appropriate for modeling height growth than other mathematical models (Lundgren and Dolid, 1970).

Richards Growth Model

One of the most flexible and sophisticated growth functions commonly used today was derived by Richards (1959). The model is a generalized version of the von Bertalanffy function (von Bertalanffy, 1941). Although the model was originally developed to express the relationship during growth between an animal's metabolic rate and its weight, Pienaar and Turnbull (1973) established its adequacy for stand height, basal area and volume determination with respect to age.

The generalized Richards function, also known as the Chapman-Richards function (Chapman, 1961) can be expressed as:

$$HT = \beta_1 \left(1 - e^{-\beta_2 age}\right)^{\beta_3} + \epsilon \qquad [1]$$

where: HT= height, β_1, β_2, β_3 are model parameters to be estimated, and ϵ = error of the model. Each of the coefficients has a biological connotation important to biological growth functions (Cooper, 1961). The first coefficient, β_1, governs the upper asymptote of the growth function, the second, β_2, describes the monotonic growth rate after the point of inflection and the third, β_3, defines the allometric increase in growth before the point of inflection. The Richards model [eq. 1], as well as expansions of this model, have been used in recent years to construct many site index curves; this model has also been incorporated into several stand growth models (Monserud and Ek, 1977).

The biological interpretations of the coefficients allows the modeler to expand the generalized model to incorporate other variables affecting height growth. Graney and Burkhart (1973), Beck (1971), Burkhart and Tennent (1977), and Ek (1971) expanded one or more of the model parameters as a function of site index. These expansions created height growth models that have curves varying in shape and amplitude with site index. Hilt and Dale (1982), Payandeh (1974a,b) and Hahn and Carmean (1982) used an expansion of the Richards model to formulate existing graphical site index curves. Expansion of the Richards model offers a sound compromise between biological basis, empirical justification and extreme flexibility for application to site index curve construction. However, the validity of an expansion model has been questioned (Zeilde, 1978). He states that the dimensionality of many growth models can be reduced to two, because the parameters of many growth models are correlated; adding additional model parameters only increases the correlation between the model parameters.

Additional Procedures For Site Index Curve Construction

Although Richards's (1959) function or its expansions are widely used for constructing site index curves, other procedures have been proposed. Bailey and Clutter (1974) created base-age invariant site index curves by using height growth increment data. The height increment equation, depending on the model form, is either solved as a differential equation or integrated to obtain a total height growth equation. The resultant site index curves are base-age invariant because site index is introduced into the constant of integration, rather than used as a variable for parameter estimation.

Devan and Burkhart (1982) segmented polynomial differential models to produce site index curves for loblolly pine (*Pinus taeda* L.). They found that some site index models fit well at young ages while other models fit well at older tree ages. A different model should be used to describe height growth within given ranges of age. In another approach, site index curves for slash pine (*Pinus elliottii* Engelm.) were constructed by joining (splining) non-polynomial segments of two published site index curves with an algebraic method (Borders *et al.,* 1984). Although the latter two approaches created adequate site index curves for their respective species, no biological significance can be inferred from the results. Unless the parameters of a proposed growth equation can be shown to have some biological meaning, its application is little more than an exercise in mathematical curve fitting (Cooper, 1961).

CHOICE OF DEPENDENT VARIABLE

Site index curves may be constructed for the purpose of (1) estimating site quality of forest land, or (2) to show the height growth development of stands that reach a given height at a certain age. The dependent variable of the mathematical model used to construct the site index curves should dictate their uses. Strand (1964) and Curtis *et al.,* (1974) made the distinction between site index curves constructed with height as the dependent variable ,and site index curves with site index as the dependent variable.

Site index curves that use site index as the dependent variable are often called site index prediction curves (Curtis *et al.,* 1974; Herman *et al.,* 1978) or recently just site index curves (Clendenen, 1977; Barrett, 1978; Cochran, 1979), which creates confusion for the user. The term site index prediction curves shall be used for this type of curve in further discussion. Site index prediction curves do not show the true pattern of tree height growth; unrealistic heights are often shown for young and old ages. Instead, site index prediction equations give the most probable estimate of site index for a tree or stand of a given height and age. For classifying forest lands, site index prediction curves should be used so that the classification error is as small as possible (Strand, 1964).

Traditionally, site index curves have been developed with height as the dependent variable of a regression equation fit to stem analyses data. This procedure uses age and site index to show the expected height growth for stands of varying site quality. These site index curves are more clearly called height growth curves (Curtis *et al.,* 1974) because they show height growth over time, not site index. This conventional form of site index curve does not give optimum estimates of site index. Their proper application is for constructing yield tables (Curtis *et al.,* 1974) and estimation of future height growth of stands (Clutter *et al.,* 1983). The uses and limitations of site index curves and site index prediction curves must be fully understood when using one or the other for site quality estimation.

ACCURACY OF SITE INDEX CURVES

Information on the error associated with site index prediction, and the probability of misclassifying stands can help the forester to use site quality information wisely in management plans. In particular, error specifications offer an objective method of determining whether the site index tool is dependable for a particular use.

Errors in estimating site indices from a prediction equation or site index curves (assuming accurate equations and curves) can come from (1) the site index prediction of individual trees, (2) variations of sample tree heights and ages and, (3) measurement error (McQuilkin and Rogers, 1978). Monserud (1984a) added another source of possible error (4) improper use of the procedure. Statements of accuracy lessen the severity of the first two errors, only care and common sense will decrease errors associated with the latter two points.

Heger, (1971, 1973), McQuilkin (1974) and McQuilkin and Rogers (1978) produced statements of accuracy in an indirect manner through the use of confidence limits. The confidence limits were derived for a series of equations where site index was described as a function of height for a range of ages. The confidence intervals are interpreted as the minimum interval of site index that would be encountered in normal field use (McQuilkin, 1974).

Lloyd and Hafley (1977) estimated the probability of misclassification in site index determination. The probability of misclassifying site index decreases as (1) the width of site index class increases, (2) the number of trees measured for site index determination increases, or (3) the age of the stand nears index age. Given the probability of misclassifying a site of specific age, the forester can determine how many trees must be measured to meet the desired accuracy of site index prediction.

Confidence intervals for site index prediction and the probability of misclassifying site index apply to the range of the data used to define the intervals. These estimates of precision do not apply to future variation of the site index estimation. Reynolds (1984) describes procedures for estimating future prediction intervals of mathematical models.

CHAPTER 3

METHODS

FIELD METHODS

Sample jack pine stands were selected to cover a wide range of soil and topographic features located throughout northwestern Ontario (Figure 1). Although the site indices of the stands were unknown at the time of selection, attempts were made to sample a wide range of apparent site index. Attempts also were made to sample the major geologic landforms in the area (Table 1). Each stand selected was even-aged, fully stocked and undisturbed by previous cutting or fire. No specific measurement was used as an index of stand stocking, but very open or very dense stands were avoided.

Sample plots within the selected stands were located in an area that was fully stocked and that was apparently similar in soil and topography. A measurement of site index was needed for each plot, thus a minimum age of 50 years at breast height was an additional plot criterion. The

Table 1. Jack pine site plots sampled in each site index class and landform.

PLOTS PER SITE INDEX CLASS						
LANDFORM	< 11.5m	11.5 - 14.4m	14.5 - 17.4m	17.5 - 20.4m	≥ 20.5m	TOTAL
GLACIALFLUVIAL SANDS	0	5	15	30	1	51
MORAINE	0	3	20	17	3	43
LACUSTRINE	0	1	5	9	3	18
BEDROCK*	9	8	10	2	0	29
TOTAL	9	17	50	58	7	141

* Soil is 1.0 m. or less above bedrock

sample plots were approximately 0.08 ha to minimize variation in soil and topography that may affect site quality or height growth patterns of the sample trees.

For each plot, a map was drawn showing plot location and direction so that plots could be relocated in the future. The study area included most of the Thunder Bay Forest District and sections of the Nipigon and Ignace Districts. The study area was restricted primarily to areas of land managed by the forest products companies located in Thunder Bay (Figure 1).

Three to five dominant trees of each species present were selected for stem analysis from each plot. Site trees had no observable top damage, and had well developed, healthy crowns. In addition to the minimum age requirement of 50 years at breast height, total age of individual trees within a plot were within ± 10 years of each other. This lessens within plot variation of site index and variable tree height growth patterns caused by differences in tree age (McQuilkin, 1975), and reduces the chances of selecting site trees affected by early suppression. The criteria for plot and tree selection ensure that trees best representing site quality were selected for analyses.

The progression of height with age was determined using stem analysis methods. Each site tree was felled, limbed and total height recorded. Tree sections, three to five centimetres in thickness, were cut from specified intervals from the entire length of the bole. The first section was cut as close to ground level as possible. Additional sections were cut at 0.75, 1.3 and 2.0 m; sections were then cut at 1.0 m intervals to 13.0 m, and 0.5 m intervals thereafter.

Each section was labeled with plot number, tree number and bole height for future identification. The sections were then bagged and transported to the laboratory for analyses.

LABORATORY METHODS

Careful annual ring counts of each section determined accurate tree height growth curves for each tree. Each tree section was cleaned with a sharp knife before the annual rings were counted with magnification and illumination. After the age of each section was determined, recorded and checked for obvious counting errors, individual tree height-age curves were plotted.

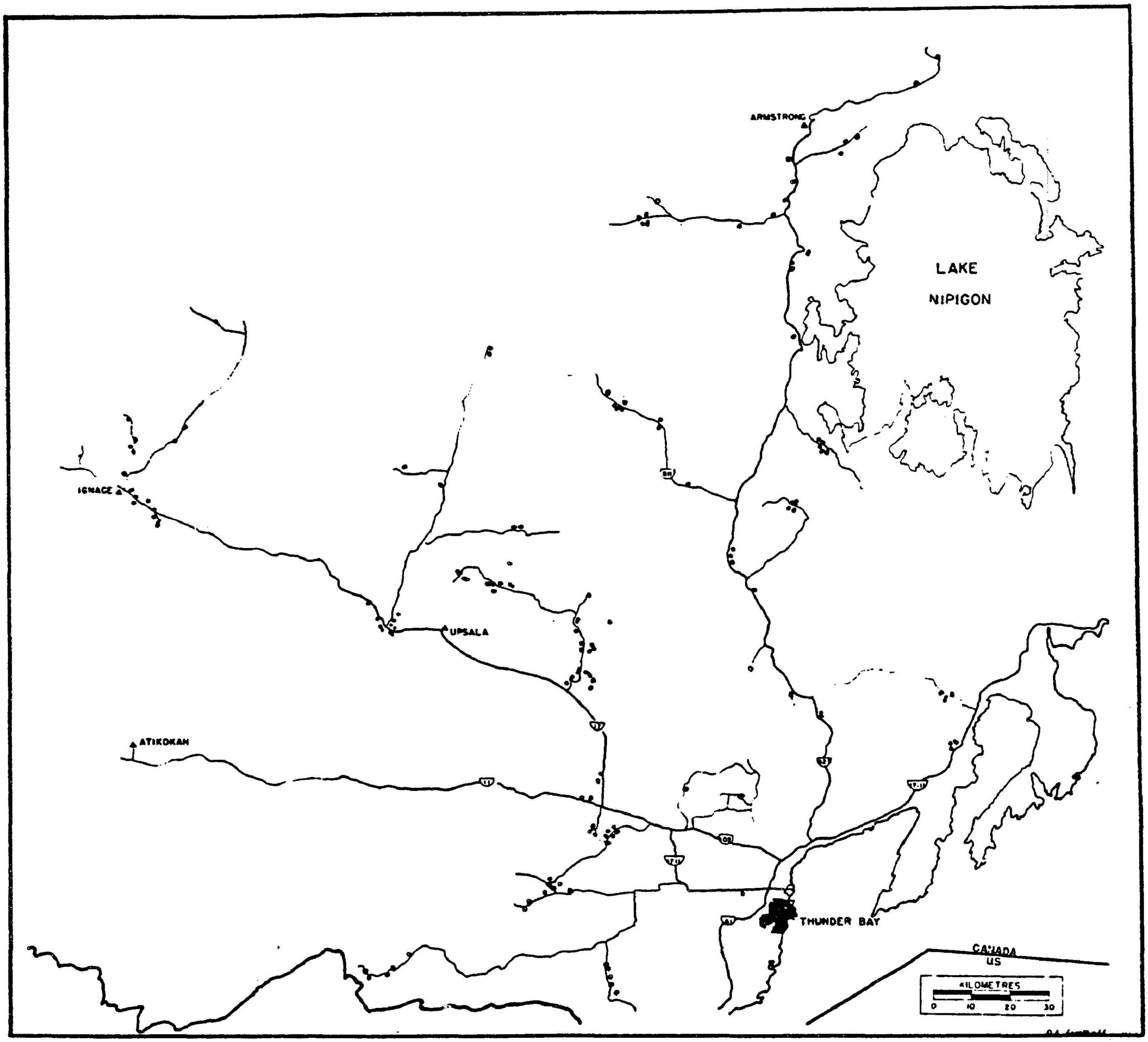

Figure 1. Location of jack pine site index plots in the Thunder Bay area.

The height-age curves for all trees sectioned on each plot were graphed and inspected for signs of early suppression or top damage that may result in abnormal tree height growth patterns. One plot that showed evidence of early suppression was eliminated from the data set, leaving 141 plots for analyses. Suitable trees from each plot were then used to calculate an average height growth curve for each plot.

Average plot height growth curves were calculated by averaging the age at each sectioning height. This procedure was the same as averaging the heights at a given age, but was averaged by a computer program for ease of calculation. The averaging routine also corrected for a small bias that results when the tree sections are cut from the bole of the tree. The actual height that an individual tree attained in a single growing season was underestimated at the point where tree sections were taken. The stem analyses data were adjusted to remove this bias in estimating tree height at each section point. Assumptions used for removing this bias were: on the average, the annual height growth was equal for each year lying between sectioning points; and that the sectioning point, on the average, occurred in the middle of the annual leader (Carmean, 1972; Lenhart, 1972). This adjustment procedure is expressed as:

$$\text{Adjusted Tree Height} = \text{Section Height} + (\text{bolt length}/\text{age difference})\ /2.$$

In essence, one half the average annual height growth between sectioning points was added to the sectioning point to account for sectioning bias.

By definition, site index is the average total height of the site trees at index age. In this study, the index age was 50 years measured at breast height (1.3 m). Plot site index was the height of the site trees 50 years after they have reached breast height. Breast height age was used rather than total tree age thus eliminating erratic early height growth usually unrelated to site quality (Curtis, 1964; King, 1966; Carmean, 1978; Monserud, 1984a).

The averaged, corrected curve based on breast height age of zero was plotted for each sample plot. Paired height-age observations for each plot were read from the curve at five year increments, height at breast height age of 50 being site index for that plot. The paired height-age and site index observations were used to construct height growth and site index curves for jack pine in the Thunder Bay area.

PRELIMINARY DATA ANALYSES

Preliminary and exploratory analyses were undertaken before the best model to express height growth of jack pine was determined. The data were stratified by landform and site index classes. Data subsets underwent model fitting procedures in an attempt to expand existing growth models; the objective was to develop models that accurately project height growth and accurately predict site index of natural jack pine.

Plot data were separated into two groups for further analyses. The majority of the plot data was used to calculate height growth and site index curves. The remaining data were reserved as an independent data set to validate the relationships developed from the calculation data set. Similar methods were used for southern pines (Graney and Burkhart ,1974; Devan and Burkhart ,1982).

One hundred and nine plots were used for model fitting and thirty-two plots were reserved for later testing procedures. Appendix I and II show the landform and site index classes of the calculation and reserve plots respectively. The reserve data set had the same characteristics as the data set used for computation (Table 2).

Table 2. Mean, variance, and standard deviation of the calculation and reserved plots.

	CALCULATION PLOTS	RESERVE PLOTS
NUMBER OF PLOTS	109	32
MEAN SITE INDEX (m)	16.62	16.97
VARIANCE	8.164	10.597
STANDARD DEVIATION	2.857	3.355

Data Grouped By Site Index Class

Data from the 109 plots chosen for analyses were grouped into 2 m site index classes. Using this stratification, the mean site index of the data for site index classes 10 and 12 did not have sufficient data for adequate representation of these classes; the data for these classes fell within the lower range of the specified class interval. Therefore, the class ranges for site index class 10 and 12 were redefined so that the mean value of the data more closely estimated the site index class specification. Thus site class 10 became site class 9 and site class 12 became site class 11. The Richards model [eq. 1] and a modified Weibull function [eq. 2] (Yang *et al.*, 1978) were separately fit to data for each site index class. The modified Weibull function [eq. 2] is a nonlinear, sigmoidal model similar to Richards model [eq. 1]. The biological connotations of the model parameters β_1, β_2, and β_3 are the same as in equation 1.

$$HT = \beta_1\left(1 - e^{-\beta_2 age^{\beta_3}}\right)+\epsilon \qquad [2]$$

The rationale for estimating the average height growth curve for each site index class was to determine whether height growth patterns varied in a definable way with level of site index. The estimate of each model parameter was examined for correlation with site index in an attempt to expand one or more model parameters as a function of site index. This procedure has been effectively used in past studies (Beck 1971; Graney and Burkhart, 1973; Trousdell *et al.*, 1974; Burkhart and Tennent, 1977; Griffin and Johnson, 1980).

Data Grouped By Landform

The data were regrouped into four landform types: (1) glacialfluvial sands; (2) moraines; (3) lacustrine; (4) bedrock (soil depth less than one metre). The plots within each landform type had a wide range of site index (Table 1). The expansion of the Richards model proposed by Ek (1971) [eq. 3] was fit to each landform so as to eliminate variations in curve form associated with differences in site index.

$$HT = \beta_1 SI^{\beta_2}\left(1 - e^{-\beta_3 age}\right)^{\beta_4 SI^{-\beta_5}}+\epsilon. \qquad [3]$$

The five coefficients of the model for each landform were compared for statistical differences using the 95% confidence limits of each coefficient. This procedure has been used by Alban and

Prettyman (1984) to test height growth curves of natural and planted red pine for differences. Height growth equations for the landforms that were statistically similar were combined **for** further analyses.

HEIGHT GROWTH AND SITE INDEX CURVE COMPUTATION

Parameters for the height growth model [eq. 3] were estimated from paired height-age observations and plot site indices using a FORTRAN curve fitting program. This program, NONLINWOOD, described in Daniel and Wood (1980), is an iterative, nonlinear least-squares regression program that uses Marquardt's Maximum Likelihood method (Marquardt, 1963) of minimization. Initial estimates of the coefficients were taken from Hahn and Carmean (1982) who formulated the Gevorkiantz (1956) site index curves for jack pine. By holding one or more of the model parameters constant while varying the others using NONLINWOOD, good initial estimates were obtained for further curve fitting. The program iterated until a minimum change in the residual sum of squares or model coefficients was reached. The models fit to the data were written into subroutines called by the main program.

The 95% confidence limits of each coefficient were used to determine whether a given coefficient was statistically different from zero. Model parameters estimated by coefficients not statistically different from zero were removed from the model. Estimations of the remaining model parameters continued until a minimum value of residual sum of squares was reached.

Height Growth Curves For Jack Pine

The 109 stem analyses plots, containing 1625 paired height-age observations, were fit with a variety of models; expansions of the Richards model [eq. 1] and modified Weibull model [eq. 2]. The best of the models tested was used to express height growth curves for jack pine. This fitted model was entered into a FORTRAN program that calculated average expected height for a given level of site index and age. Height growth curves for 3 m intervals of site index were plotted. The height growth curves show the average progression of height growth with age for each level of site index. They do not pass directly through the site index at index age; the model is not constrained for this purpose (Lundgren and Dolid, 1970).

Site Index Curves For Jack Pine

Site index curves, by definition *must* pass through a specified site index at index age. The height growth curves can be corrected graphically or mathematically to pass through the site index, although a small bias in predicted height is introduced. The mathematical approach was preferable because model coefficients were adjusted, therefore making the site index curves reproducible.

The mathematical procedure used to correct the height growth curves in this study was as follows:

1) For each level of site index, site index entered into the height growth equation was incremented until the height at age 50 was equal to the desired site index, e.g. (input SI=14.31 generates ht=14.00 at age 50).

2) The input site index calculated in step 1 was used to create a series of height-age observations for each level of site index.

3) These series of paired height-age observations and specified site index were grouped and fit with the Ek (1971) model [eq. 3], producing an equation that predicted the correct height at index age with minimal (rounding) error. The coefficients calculated from these data were used to construct site index curves for natural jack pine.

VALIDATING THE HEIGHT GROWTH AND SITE INDEX CURVES

The height growth curves, for breast height age and total age, were tested for goodness of fit using modifications of Chi-squared tests proposed by Freese (1960) and Reynolds (1984). These tests determined whether the mathematical model accurately described the underlying data, given allowable error limits set by the user. The confidence intervals and prediction limits for the mean residual were also defined for the height growth models.

The site index curves, created from the height growth curves, were tested with the stem analyses data reserved from calculation. As observed site index, total height and age was known for each reserved plot, predicted site index calculated from total height and age was compared with the observed site index as a validation procedure.

Site index graphically estimated from site index curves is often subject to interpolation error. Consistent site index estimates were made using an observation stated by Heger (1968). The relationship between height and site index was nearly linear, given that age was held constant. For each five year increment, ages 20 - 150, the equation, $SI = \beta_o + \beta_1(HT) + \epsilon$ was fit to the data computed from the site index curves (Appendix III). Site index was estimated for each reserve plot by using the appropriate linear equation for the plot age and height. Estimated site index and observed site index were compared, and tested for statistical differences.

SITE INDEX PREDICTION EQUATIONS

Site index prediction equations are superior to height growth or site index curves for classifying areas of land into similar units of site index (Curtis *et al.*, 1974). The Ek (1971) model [eq. 3] cannot be solved for site index, but Payandeh (1974a) proposed a model [eq. 4] that closely approximates the height growth model solved for site index.

$$SI = \beta_1 HT^{\beta_2}\left(1 - e^{-\beta_3 age}\right)^{-\beta_4 HT^{-\beta_5}} \qquad [4]$$

The site index prediction model was fit twice to the stem analyses data. Height-age observations for 5-20 years breast height age were included with the older aged data for model fitting. The second time, the young ages (5-20 years) were eliminated as unnecessary. The two fitted models were tested with the reserved data to determine whether one set of coefficients produced better estimates of site index than the other.

The same procedure of model fitting was used as with the height growth model [eq. 3]. Initial estimates of the coefficients were taken from Hahn and Carmean (1982) to begin the model fitting procedure.

Site Index Prediction Equation From Site Index Curves

A mathematical inversion of the site index curves for jack pine created a site index prediction equation. The site index prediction equation was formulated by incorporating the linear equations that express site index as a function of height at the same age into a single mathematical equation. The equation $SI = \beta_o + \beta_1(HT) + \epsilon$ for ages 20 - 150 (Appendix III) was fit to the data read from the site index curves for jack pine. The estimates of β_o and β_1 showed consistent trends when plotted with age (Figures 2,3). Expressing the estimates of β_o ($\hat{\beta_o}$) and β_1 ($\hat{\beta}_1$) as a function of age, the series of linear equations were incorporated into a single site index prediction equation.

Many mathematical models could be used to estimate β_o and β_1 with age. The shortest, simpliest equation was desired for practical use. The two equations used to express $\hat{\beta_o}$ and $\hat{\beta}_1$ as they vary with age are as follows.

$$\beta_o: \qquad \hat{\beta_o} = \beta_{oo}\left(50 - age\right)age^{-\beta_{o1}} \tag*{[5]}$$

$$\beta_1: \qquad \hat{\beta}_1 = \left(\beta_{1o} + \beta_{11}\frac{1}{age}\right) \tag*{[6]}$$

The equation to estimate β_o [eq. 5] was conditioned to pass through zero at age 50. When age is 50, $\left(50-age\right)$ becomes zero and the equation estimating β_o [eq. 5] becomes zero. The equation was then fit to the data shown in Figure 2 using nonlinear regression methods. The equation to estimate β_1 [eq. 6] was regressed on the data shown in Figure 3 using linear regression methods.

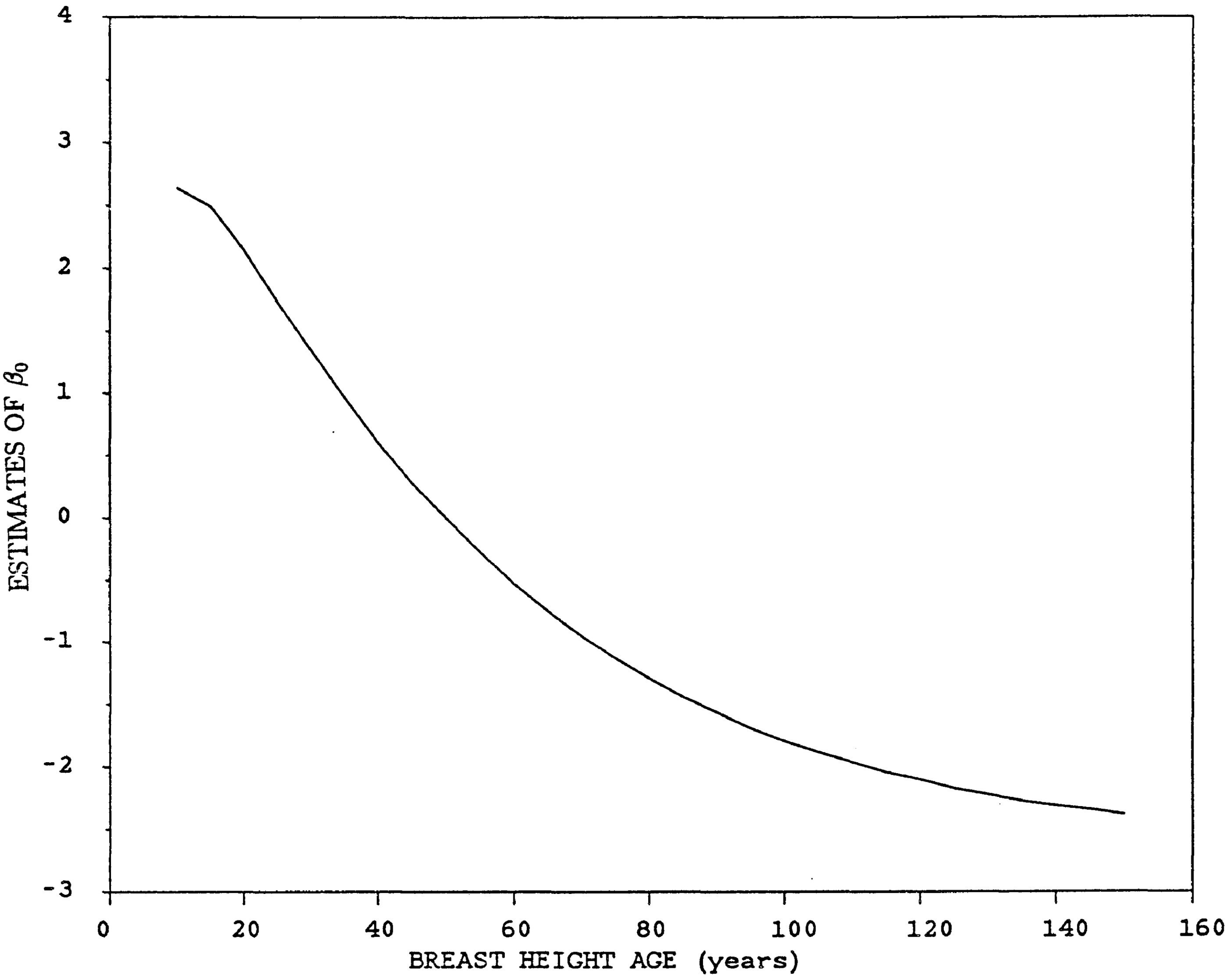

Figure 2. **Estimates of β_0 calculated from linear regression of site index on height (SI $= \beta_0 + \beta_1$ HT).**

A small correction factor was added to the constant of the regression equation [eq. 6] so that the equation would predict 1.0 at age 50. The site index prediction formulated from the site index curves is expressed as:

$$SI = \beta_1 \left(50 - age \right) age^{-\beta_2} + \left(\beta_3 + \beta_4 \frac{1}{age} \right) HT + \epsilon. \qquad [7]$$

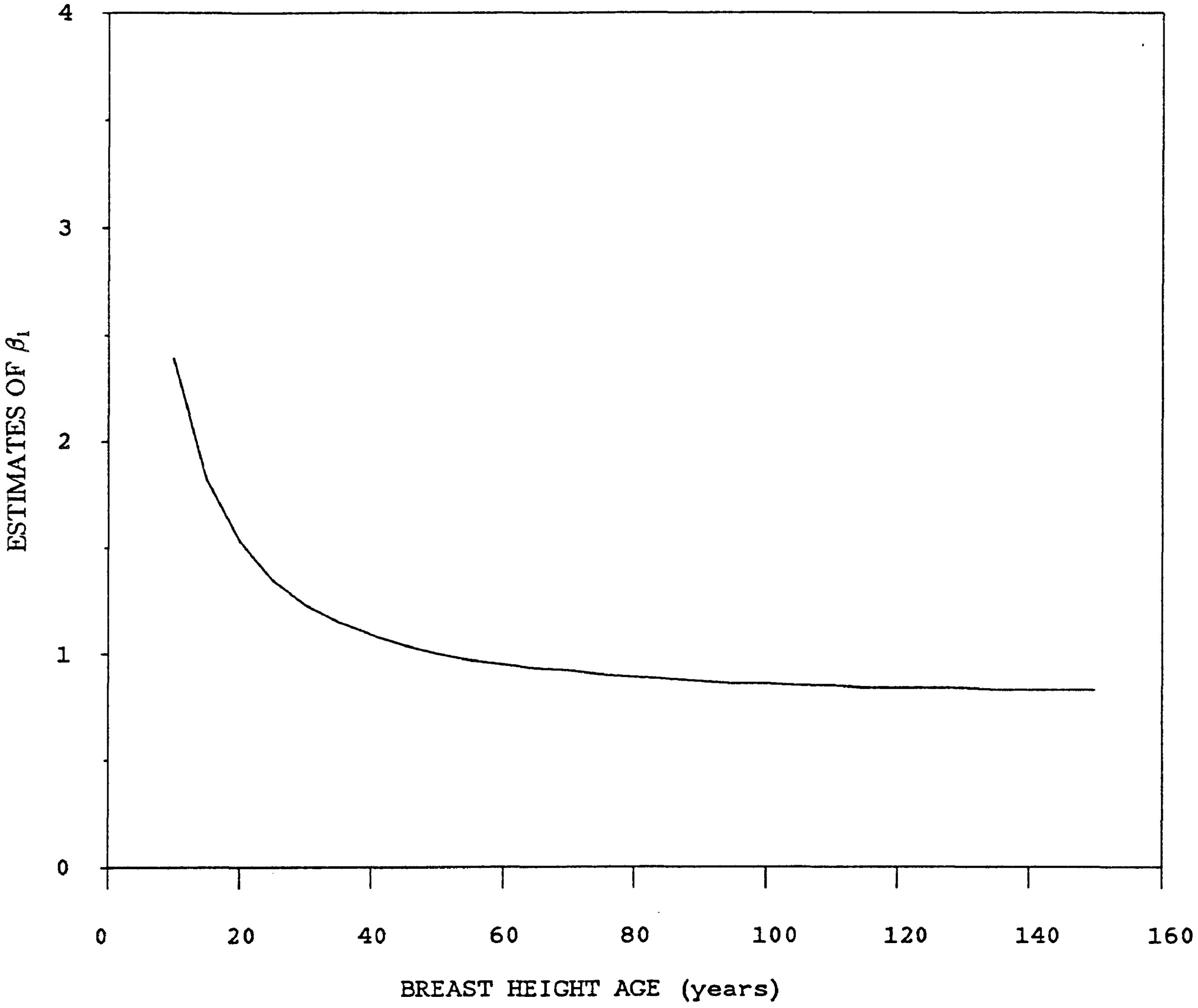

Figure 3. **Estimates of β_1 calculated from linear regression of site index on height ($SI = \beta_0 + \beta_1 HT$).**

COMPARING STUDY SITE INDEX CURVES WITH PLONSKI'S SITE INDEX CURVES

Site class curves developed in this study were compared with the site index curves for jack pine developed by Plonski (1974). Data taken from Plonski's graphical site index curves were formulated using the same model [eq. 3] used for the study site index curves based on total age. This allowed comparisons between corresponding coefficients of the fitted model for each set of site index curves to determine if the two methods of construction produced different results.

Payandeh (1974b) formulated Plonski's jack pine site index curves using Ek's (1971) expansion of the Richards model [eq. 3]. Payandeh's results could not be directly applied because data for formulation were derived from Plonski's site index curves, beginning at age 20, and the coefficients correspond to data in imperial rather than metric units. The lack of young ages, and early sigmoid height growth affects the estimation of the model parameters. Early height growth estimates were needed for each of Plonski's site classes to standardize Plonski's data with stem analyses data used here.

Richards (1959) generalized growth model [eq. 1] was used to estimate the early height growth for each site class defined by Plonski (1974). This fitted model provided estimates of the early height growth for jack pine not shown in Plonski's site index curves; the necessary sigmoid height growth patterns provided comparisons with stem analyses data used in this study. The coefficients and 95% confidence limits were used to test statistical differences between the corresponding coefficients of Plonski's site index curves and the study site index curves.

CHAPTER 4

RESULTS

ASSUMPTIONS FOR REGRESSION

Sample plots were selected to show a wide range of site quality and soil features. This sampling procedure was required to avoid oversampling the average conditions, but violates the requirements of random sampling for regression. The assumption that sample plots represent randomly selected stands was made to allow regression analyses.

The results of regression analyses apply *only* to the observed plots in strict statistical theory. A user of the height growth curves, site index curves and site index prediction equations can assume that the sample plots were randomly selected, for application within the study area. Tests with the independent data set indicate that this assumption may be justified.

DEFINITIONS - HEIGHT GROWTH CURVES, SITE INDEX CURVES, SITE INDEX PREDICTION EQUATIONS

Several terms that are frequently used throughout the RESULTS and DISCUSSION sections are defined for greater clarity.

Height growth curves:

> Height growth curves were calculated using height as the dependent variable for regression. Height growth curves were not constrained to predict site index at index age (50 years), but predicted heights at index age and site index were generally close. Height growth curves show average height growth trends for each level of site index.

Site index curves:

> Site index curves were calculated using height as the dependent variable for regression. Site index curves were corrected to predict site index at index age (50 years).

Site index prediction equations:

> Site index prediction equations were calculated using site index as the dependent variable for regression. Site index prediction equations estimate site index for an observed height and age.

HEIGHT GROWTH PATTERNS OF DIFFERENT SITE INDEX CLASSES

The height growth patterns were examined as they varied with level of site index. As described on p. 29 the data were stratified by site index class and were then fit with Richards (1959) model [eq. 1] and a modified Weibull model (Yang *et al.*, 1978) [eq. 2]. The coefficients for each model by site class are given in Table 3. All coefficients are statistically different from zero with 95% confidence. Figures 4 and 5 show the height growth patterns predicted by the Richards [eq. 1] and modified Weibull models [eq. 2] for each site index class. These two growth models produced nearly identical height growth curves for each site class.

Table 3. Coefficients for the Richards model [eq. 1] and modified Weibull model [eq. 2] fit to stem analyses data by site index class.

		RICHARDS MODEL			MODIFIED WEIBULL MODEL		
Site Class	No. Plots	β_1	β_2	β_3	β_1	β_2	β_3
9	4	25.123	0.00644	0.993	25.897	0.00657	0.988
11	8	21.816	0.01061	0.938	23.524	0.01338	0.933
14	13	22.075	0.01423	0.841	22.804	0.02627	0.872
16	24	20.715	0.02716	1.154	20.562	0.01741	1.094
18	33	22.011	0.02892	1.118	21.763	0.02172	1.081
20	27	23.488	0.03342	1.245	22.900	0.01629	1.169

The coefficients of the models were examined for consistent trends related to site index. No strong trends were observed by plotting the first coefficients (β_1 - asymptotic height) by site index class. However, a linear relationship was observed as the second coefficient (β_2 - growth rate) for the Richards model [eq. 1] increased as site class increased (Table 3.). The third coefficient (β_3 - initial height growth) also had a weak relationship to site index.

These coefficients were regressed on site index to express the second model parameter (β_2) as a function of site index, in an attempt to expand the generalized growth function.

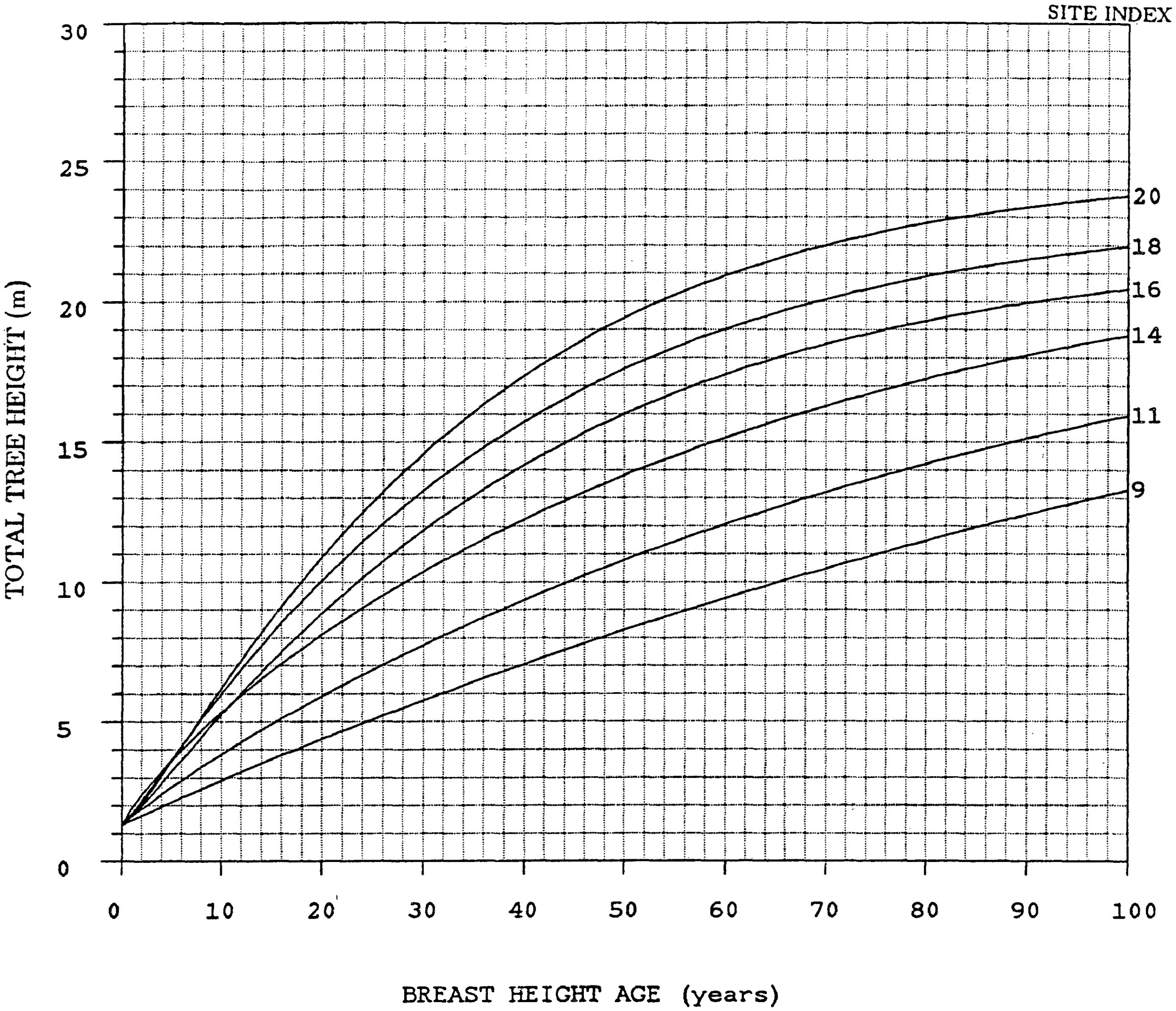

Figure 4. Average height growth curves for jack pine produced by fitting the Richards model [eq. 1] to data stratified by site index class.

Expressing (β_2) as a function of site index in the Richards model [eq. 1] produced a model that poorly predicted height growth of the total data set. The second and third model parameters of [eq. 1] and [eq. 2] were also expressed as linear and allometric equations of site index. Unfortunately, these original expansions of model [1] or model [2] attempted here suffered from overall lack of fit (high residuals) or systematic bias at either young or old ages.

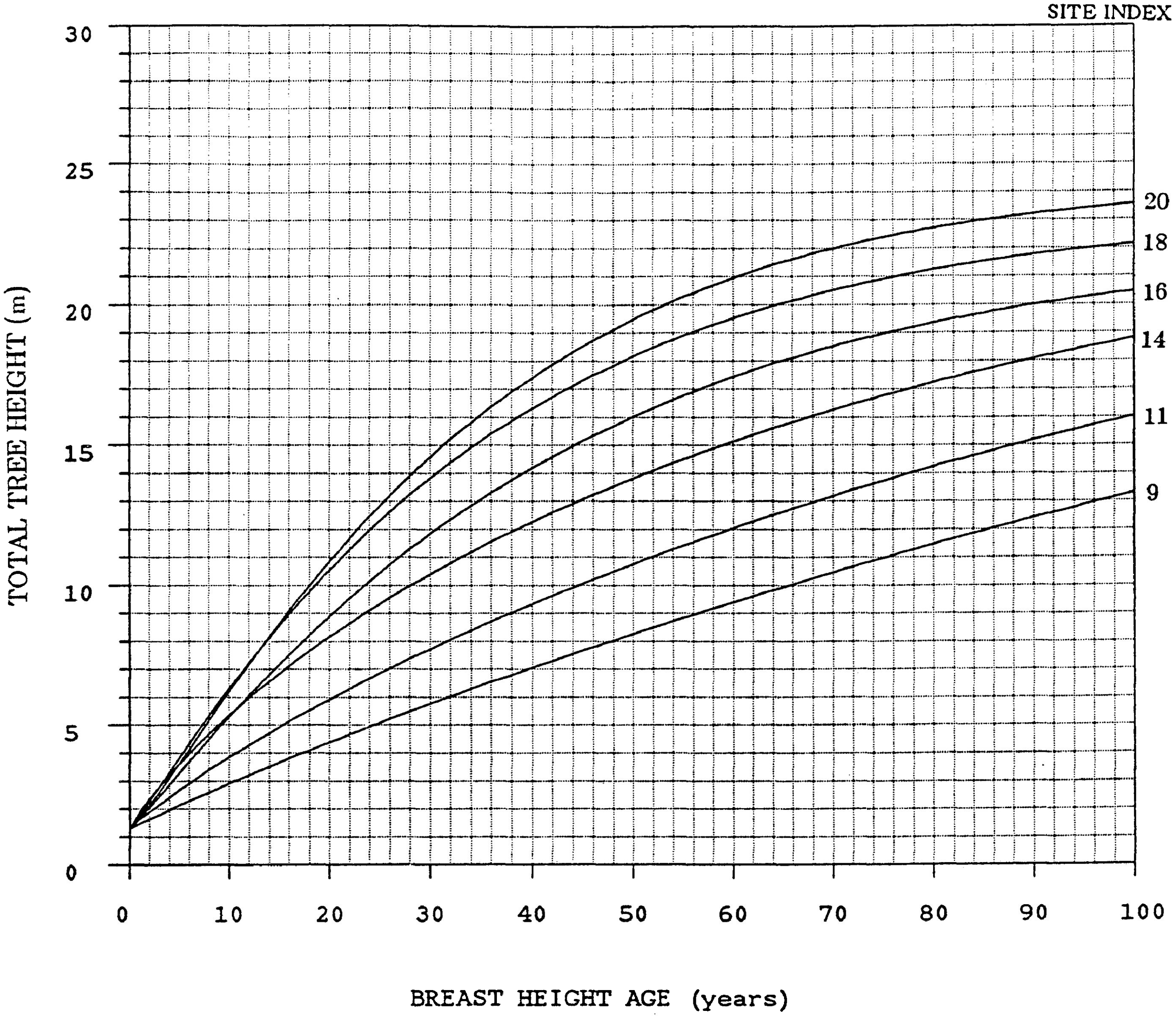

Figure 5. Average height growth curves for jack pine produced by fitting a modified Weibull model [eq. 2] to data stratified by site index class.

HEIGHT GROWTH PATTERNS ON DIFFERENT LANDFORMS

Height growth patterns were examined on the four basic landforms: glacialfluvial sands; moraines; lacustrine; and bedrock sites. Bedrock sites are generally shallow moraine soils that are less than one metre in depth, above a bedrock sub-strata. They were separated from the moraine soils because they represent sites with limited soil volume and rooting capacity that may alter height growth patterns of jack pine.

A model that could vary in shape with site index, and that was not influenced by total plot age was needed to show different height growth patterns that might be associated with different landforms. As described on p. 29, the expansion of the Richards height growth (1959) model proposed by Ek (1971) [eq. 3] was fit to the data for each soil type. The estimates of the model parameters for each landform are shown in Table 4.

Table 4. Coefficients of the Ek (1971) model [eq. 8] for different landforms - age is breast height age.

LANDFORM	PARM.	ESTIM. OF COEFFICIENTS	STD ERR. COEFF.	95% CONFID. LIMITS	
				UPPER	LOWER
GLACIALFLUVIAL SANDS	β_1	2.23926E+00	1.65E-01	1.92E+00	2.56E+00
	β_2	8.09961E-01	2.59E-02	7.59E-01	8.61E-01
	β_3	2.70616E-02	7.91E-04	2.55E-02	2.86E-02
	β_4	3.75978E+00	7.53E-01	2.28E+00	5.24E+00
	β_5	4.13915E-01	6.68E-02	2.83E-01	5.45E-01
MORAINES	β_1	2.26872E+00	2.30E-01	1.82E+00	2.72E+00
	β_2	8.10738E-01	3.76E-02	7.37E-01	8.84E-01
	β_3	2.37443E-02	9.43E-04	2.19E-02	2.56E-02
	β_4	3.70279E+00	9.87E-01	1.77E+00	5.64E+00
	β_5	4.55074E-01	8.87E-02	2.81E-01	6.29E-01
LACUSTRINE	β_1	2.90320E+00	3.22E-01	2.27E+00	3.53E+00
	β_2	7.59668E-01	3.49E-02	6.91E-01	8.28E-01
	β_3	2.17273E-02	2.15E-03	1.75E-02	2.59E-02
	β_4	1.06871E+00	4.22E-02	9.86E-01	1.15E+00
	β_5*	0.00000E+00	0.00E+00	0.00E+00	0.00E+00
BEDROCK	β_1	2.09628E+00	1.37E-01	1.83E+00	2.37E+00
	β_2	8.27268E-01	2.27E-02	7.83E-01	8.72E-01
	β_3	2.29516E-02	1.27E-03	2.05E-02	2.54E-02
	β_4	3.84685E+00	5.70E-01	2.73E+00	4.97E+00
	β_5	5.03441E-01	5.18E-02	4.02E-01	6.05E-01

* NOTE: The estimate of β_5 was statistically zero. Model [8] was fit to the lacustrine data without the fifth model parameter to obtain a better estimate of the remaining four model parameters.

The 95% confidence limits were used to test for statistical differences between the model coefficients for different landforms. The confidence limits for each coefficient shown are the most conservative confidence limits for the data. The coefficients are considered not statistically different if the confidence limits for a given coefficient overlap among the landforms. Little, if any statistical differences are evident between most of the five coefficients for the different landforms (Table 4).

A notable exception is the coefficients estimating β_4 and β_5 for the lacustrine soils. Analysis indicated that the fifth model coefficient (β_5) was not statistically different from zero, consequently, β_5 was removed from the model. The early height growth estimation is governed by β_4 in the lacustrine plots, rather than ${\beta_4}^{\beta_5}$ as in the remaining soil types. This reduction of model parameters had little effect in the shape of the height growth curve because the coefficients estimating the later height growth remained in the model.

The fitted model for each soil type was used to construct height growth curves for each soil type at the same level of site index (Figure 6). The height growth model for each soil type was solved for ages 0 - 100 and site index 16, the approximate mean site index for the data set. Visual inspection indicates that trees on lacustrine plots have sustained height growth at older ages; whereas trees located on till, sand and bedrock plots have a reduced rate of height growth at older ages. This observable difference cannot be statistically validated because a coherent, objective statistical method for testing differences in nonlinear models has not been developed (Chivenda, 1981). More likely, this observable difference is due to the limited number of plots (13) for jack pine growing on lacustrine soils. No hard evidence indicates that the models for each of the landforms are statistically different, thus data from the different landforms were combined.

An accurate model was desired to predict the height growth of jack pine with age in relation to site index. Original expansions of the generalized Richards (1959) [eq. 1] and a modified Weibull model [eq. 2] (Yang *et al.*, 1978) where site index was incorporated as a function of the model parameters did not produce an adequate height growth model. Likewise, exploratory analyses indicated that the height growth patterns associated with different landforms was similar so further model modifications that incorporated landform type were unnecessary.

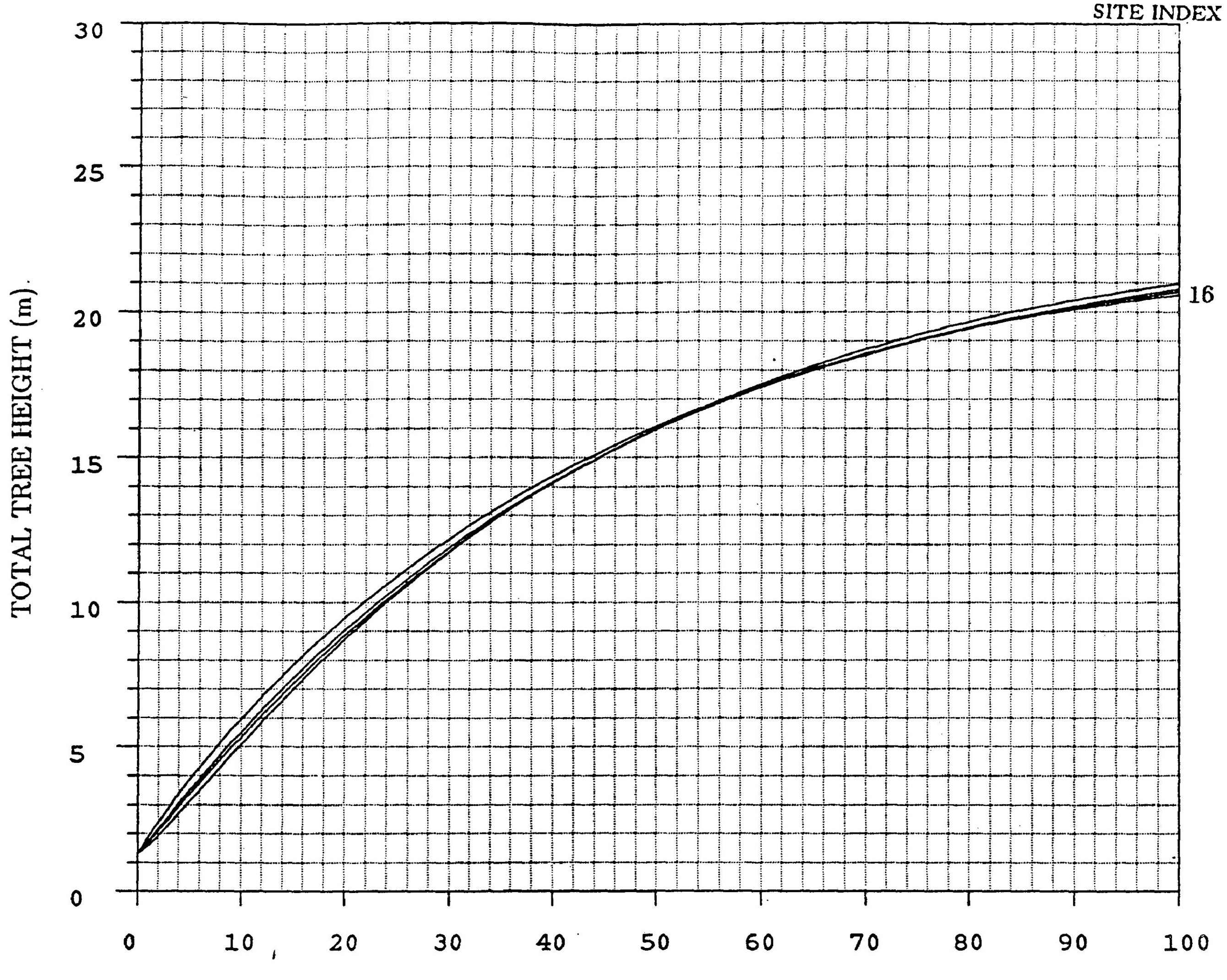

Figure 6. **Height growth curves for individual landforms calculated by Ek (1971) model [eq. 3]. Site index is 16.0 metres and age is breast height age.**

THE EK (1971) MODEL

The model proposed by Ek (1971) proved to be the best of all the models that were fit to the jack pine data. This model had the lowest residual mean square of the models tested and showed no indication of bias over the range of the data. Ek expanded the generalized Richards (1959) growth function [eq. 1] to formulate site index curves developed by Gevorkiantz (1956) for white spruce in the Lake States. This function has the desirable sigmoid shape with a height of zero at age zero. Site index incorporated into the equation varies the shape of the height growth curve with levels of observed site index.

Estimates of the model coefficients were obtained using a nonlinear regression program NONLINWOOD (Daniel and Wood, 1980). The five coefficients of the model were all statistically different from zero at a 95% probability level ($\alpha = 0.05$). The model was fit separately to the jack pine data based on total age and breast height age. For breast height age, the model was modified by adding a constant value, 1.3 m, as suggested by Ek (1971). This breast height model becomes:

$$HT = 1.3+\beta_1 SI^{\beta_2}\left(1-e^{-\beta_3 age}\right)^{\beta_4 SI^{-\beta_5}}+\epsilon. \qquad [8]$$

Tables 5 and 6 contain the estimated coefficients for the breast height age and total age models, respectively.

Table 5. Coefficients for jack pine height growth curves calculated using the Ek (1971) model [eq. 8] - age is breast height age.

PARM.	ESTIM. OF COEFFICIENT	STD ERR. COEFF.	95% CONFID. LIMITS LOWER	UPPER
β_1	2.13762E+00	8.21E–02	1.98E+00	2.30E+00
β_2	8.28004E–01	1.36E–02	8.01E–01	8.55E–01
β_3	2.52242E–02	5.14E–04	2.42E–02	2.62E–02
β_4	3.61558E+00	3.47E–01	2.94E+00	4.30E+00
β_5	4.25554E–01	3.19E–02	3.63E–01	4.88E–01

Table 6. Coefficients for jack pine height growth curves calculated using the Ek (1971) model [eq. 3] - age is total age.

PARM.	ESTIM. OF COEFFICIENT	STD. ERR. COEFF.	95% CONFID. LIMITS LOWER	UPPER
β_1	2.84598E+00	9.29E–02	2.66E+00	3.03E+00
β_2	7.54376E–01	1.19E–02	7.31E–01	7.78E–01
β_3	2.87387E–02	5.32E–04	2.77E–02	2.98E–02
β_4	3.89575E+00	3.48E–01	3.21E+00	4.58E+00
β_5	3.74920E–01	3.01E–02	3.16E–01	4.34E–01

The Ek (1971) model fit the jack pine height growth for total age [eq. 3] and breast height age [eq. 8] with acceptable results (Table 7). The residual sum of squares for the model fit to the breast height data is 18% lower than the residual sum of squares for the total age data.

The residual mean square is the mean estimate of the sample variance of the residuals. Therefore, an F-test can be used to test whether the model fits the breast height data better than the total age data. The null hypothesis that the sample variances are equal can be rejected if the ratio of the variances exceeds the tabulated F-value with 1620/1620 degrees of freedom. The ratio of the sample variances (residual mean square, 0.6396/0.5257) was 1.217. The calculated F-ratio exceeded the tabulated F-value of 1.085 with 1620/1620 degrees of freedom at the $\alpha = 0.05$ level of significance. Therefore, the null hypothesis was rejected in favor of the alternative hypothesis that the variances were not equal. The model explained height growth better for data based on breast height than when total age data were used.

The standardized residuals of the breast height age model [eq. 8] and total age model [eq. 3] showed no major bias when fit to their respective data sets (Figures 7 and 8). The standardized residuals clustered about the zero line in a random pattern. The greatest variation of the residuals occurred at the lower range of tree height. This variation can be explained by two factors: (1) the correlation of height with age; and (2) the presence of very poor growth for poor site plots within the data set.

Height growth at young ages may be influenced by factors other than site quality, suppression, competition, frost and animal damage. Some of this variation was eliminated when breast height was used as the base age thus the lower residuals for the breast height model.

Table 7. Squared residuals of the Ek (1971) model [eq. 3 and 8] for total and breast height age.

	TOTAL AGE	BREAST HEIGHT AGE
RESIDUAL ROOT MEAN SQUARE	0.79978045	0.72504837
RESIDUAL MEAN SQUARE	0.63964876	0.52569513
RESIDUAL SUM OF SQUARES	1036.23099685	851.62611689
NUMBER OF OBSERVATIONS	1625	1625

However, variation extends beyond breast height. The height growth models [eq. 3,8] cannot explain all variation in height growth at young ages, thus the slightly higher standard residuals for short tree heights.

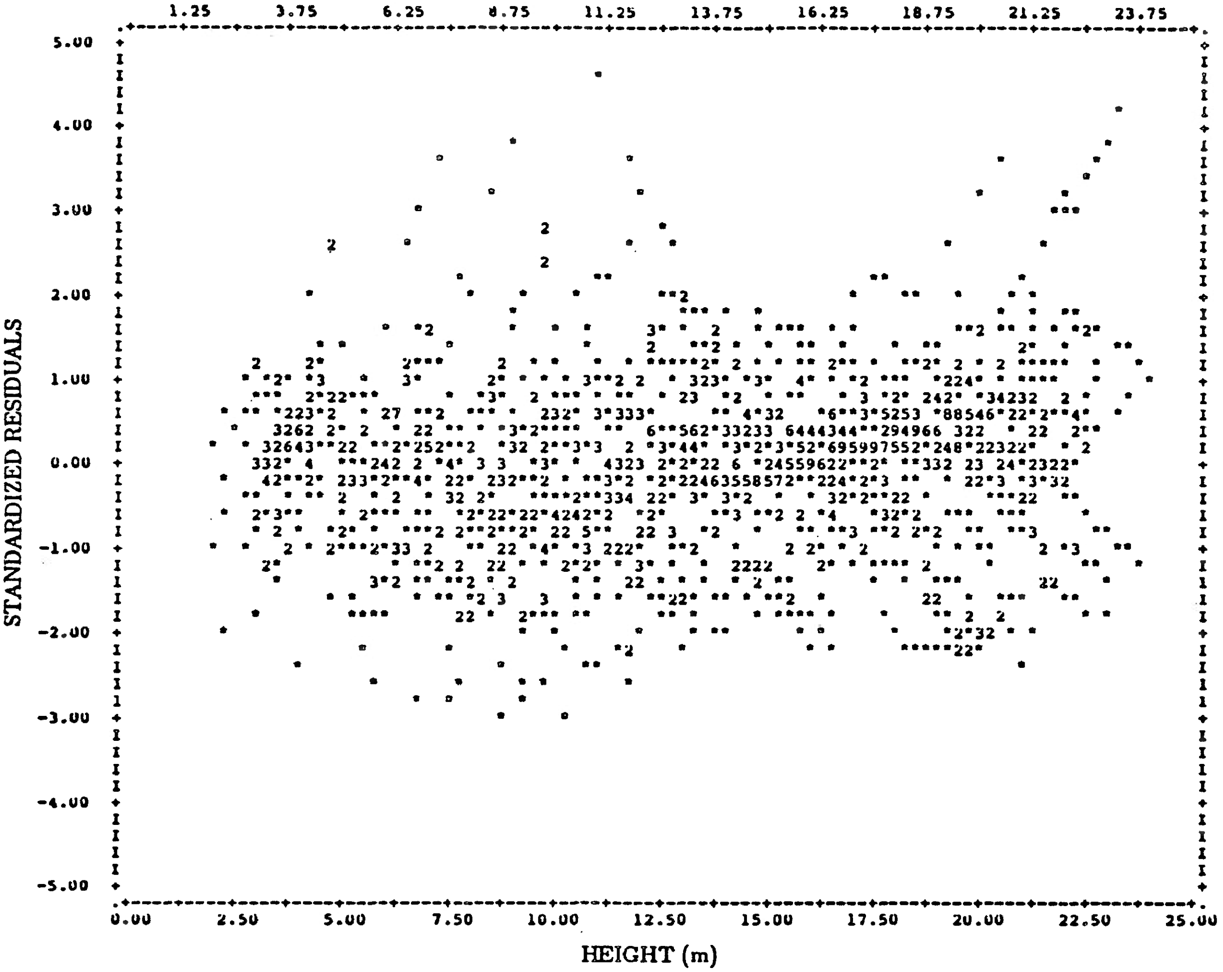

Figure 7. **Standardized residuals of the Ek (1971) model [eq. 8] fit to the breast height age data.**

Likewise, variation may increase at the opposite extreme, where height growth of old trees has ceased for many plots.

Plots with very low site index also add to the wider distribution of residuals at lower heights. Although the model accommodates most of this variation, not all variation in height growth can be explained. Trees on poor sites grow to a shorter height than trees on good sites.

On the good sites, trees may be in the rapid stage of height growth when they are 8 to 10 m tall whereas the trees on a poor site may be growing very slowly at the same height. This creates higher residuals at the lower range of the scattergram showing the standardized residuals.

The scattergrams of standardized residuals (Figure 7 and 8) showed no apparent heteroscedastic trends or systematic lack of fit. Several points at the extremes of the range lie outside the major body of standardized residuals. These points were a result of random variation and did not indicate lack of fit for the model.

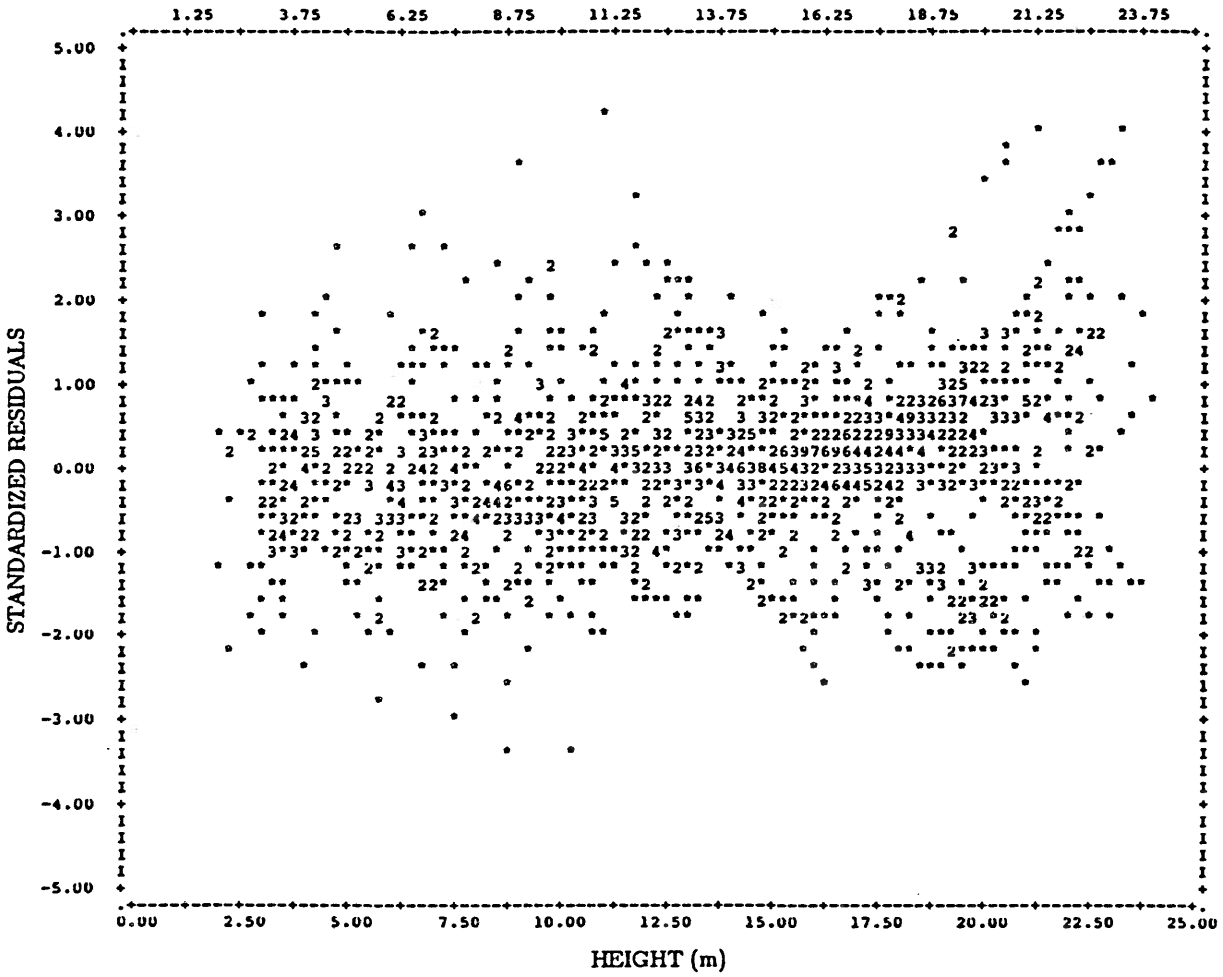

Figure 8. Standardized residuals of the Ek (1971) model [eq. 3] fit to the total age data.

TESTING GOODNESS OF FIT

Models for forest prediction and estimation are continually being developed and reported in the forestry literature (Reynolds, 1984). These models range from simple log rules to complicated distance-independent individual tree models used for forest growth projection. Many times new models were presented without adequate validation or assessment of possible estimation errors that result when the model is used. This lack of validation is especially true for most site index and height growth curves. Early harmonized curves (Bruce, 1926) were generally not validated at all. It was left to the users to decide whether the curves were applicable to their needs. The short-comings of these early site index curves were not realized for many years (Carmean, 1968). With the advent of electronic computers and more advanced statistical procedures, the ability to formulate, as well as validate site index curves increased.

Goodness of fit procedures for evaluating models are given by Freese (1960) and Reynolds (1984). One method of determining how well a model performs is to compare the predicted values with observations from the modeled system. Both Freese and Reynolds used a modification of the chi-squared probability function as a test for goodness of fit when comparing observed and predicted values.

Freese (1960) proposed a statistical procedure for determining whether the accuracy of an estimation technique is adequate to meet the requirements of the user. This goodness of fit test uses the chi-squared probability function to predict the accuracy of a new technique (the model), against a known standard (the data). In Freese's chi-squared procedure, the model is assumed adequate unless there is evidence to the contrary. For Freese's procedure, the null hypothesis (H_0) is stated as

$$H_o : E(D) = 0 \; ; \; VAR(D) < e^2/\chi^2_{(1-\alpha)(1)} \qquad [9]$$

where: $\alpha = 0.05$; E(D) = the inherent bias of the model; e = the acceptable error set by the user; $\chi^2_{(1-\alpha)(1)} = (1-\alpha)$ 100 percent quantile with (1) degree of freedom. The test statistic | D | is used as the measure of the accuracy of the model, so if E(D) = 0 and VAR(D) = $e^2/\chi^2_{(1-\alpha)(1)}$, then the test statistic

$$\hat{\chi}^2_{(1-\alpha)(n)} = \sum_{i=1}^{n} D_i^{\,2} \chi^2_{(1-\alpha)(1)} / e^2 \qquad [10]$$

has a chi-squared distribution with n degrees of freedom. The null hypothesis can be rejected at a significance of α if the test statistic exceeds the tabulated value of $\chi^2_{(1-\alpha)(n)}$. This test will tend to reject the null hypothesis if VAR(D) is large, E(D) is large, or both.

The height growth model proposed by Ek (1971) [eq. 8] was tested for goodness of fit using the null hypothesis [eq. 9] of Freese's procedure (1960). The calculated test statistic [eq. 10] with an allowable error of 1.5 m, 851.626(3.84146)/(1.5^2) = (1453.9945) does not exceed the tabulated $\chi^2_{(1-\alpha)}$ value (1714.771) with 1620 degrees of freedom, where $\sum_{i=1}^{n} D_i^{\,2}$ = residual sum of squares. The model adequately describes the data with probability of a type 1 error at 05%.

This procedure gives the "benefit of the doubt" to the model by the way that the null hypothesis is stated. Reynolds (1984) states that a more accurate and conservative procedure should be used to test the accuracy of models.

With Reynolds's procedure, the model is judged as adequate only if there is strong evidence that the model will predict results as accurately as specified by the user. Freese's (1960) procedure is modified so that the null hypothesis is formulated as:

$$H_o \ : \ VAR\,(D\,) > e^2 / \chi^2_{(1-\alpha)(1)}. \qquad [11]$$

This hypothesis is rejected at a significance α if the test statistic [eq. 10] is less than the tabulated value of $\chi^2_{\alpha(n)}$ that is, if the test statistic is in the lower tail of the chi-squared distribution (Reynolds, 1984). For the Ek (1971) model [eq. 8], the test statistic [eq. 10], 851.626(3.84146)/e^2 = 1453.9945 is less than the tabulated value of 1527.505. As the test statistic is less than the tabulated chi-squared value at five percent probability, the null hypothesis is rejected, with probability of making a type 1 error at 05%.

The conservative testing procedure proposed by Reynolds (1984) rejects the null hypothesis, indicating that the model adequately describes the height growth data for jack pine. In many cases, the user of a model does not wish to know how well a model meets certain standards, but rather what magnitude of errors will result when the model is used to predict future values

(Reynolds, 1984). In this case, prediction intervals are more important than hypothesis testing procedures.

Critical Errors Of Ek (1971) Model

One problem with Freese's general approach is that different users of a technique may have different accuracy requirements for the same technique. In the previous section, the acceptable error was stated as 1.5 m. This assumes that the height growth model for jack pine is used to estimate the site index (base age 50) for a stand. Setting the allowable error at 1.5, the model [eq. 8] accurately predicts site index within one 3 m class. Another user of the model may wish to estimate site index to a greater or lesser accuracy. The critical error for the model is the minimum allowable error where the model is statistically valid. If a user specifies an acceptable error (e) less than the critical error, the model will not be adequate for the user's needs.

Several authors, including Bell and Groman (1971), Rennie and Wiant (1978), and Ek and Monserud (1979) have calculated the critical error, e*, which is the smallest value of e that will lead to the acceptance of the null hypothesis using Freese's procedure, (Reynolds, 1984). The critical error, e*, is easily calculated by noting that the test statistic is just on the borderline of the critical region:

$$\sum_{i=1}^{n} D_i^2 \chi^2_{(1-\alpha)(1)} \;/\; e^2 = \chi^2_{(1-\alpha)(n)}. \qquad [12]$$

Solving for e, the critical value for e is calculated by

$$e^* = \sqrt{\sum_{i=1}^{n} D_i^2 \chi^2_{(1-\alpha)(1)} / \chi^2_{(1-\alpha)(n)}}. \qquad [13]$$

Thus, the critical error, e* is 1.381. Assuming that the model is unbiased, the critical error can be interpreted as the lower confidence bound for the upper 0.95 quantile of the distribution of $|D|$.

The critical error, e** , for the more conservative procedure recommended by Reynolds (1984) is given as:

$$e^{**} = \sqrt{\sum_{i=1}^{n} D_i^{2} \chi^{2}_{(1-\alpha)(1)} / \chi^{2}_{\alpha(n)}} \qquad [14]$$

so that $e^{**} = 1.463$, at the 95% confidence level.

Confidence Limits - Ek (1971) Model

Using both e^{*} and e^{**}, a two sided $(1 - 2\alpha)100$ percent confidence interval for e is obtained. Reynolds (1984) emphasizes that the critical errors cannot be directly translated into probability statements that future residuals $|D|$ will be below e^{*} or e^{**}. Instead, e^{*} and e^{**} are the confidence bounds for the upper 0.95 quantile of the distribution of $|D|$ under the assumption that the model is unbiased.

The confidence interval bound for the distribution of the residuals, $|D|$, resulting from regression lies between 1.381 and 1.463 m. This is the 90% confidence interval for the 95% quantile of the distribution of $|D|$. In more common terms, the point below which 95% of the absolute residuals lie is between 1.381 and 1.463 with 90% confidence. Examining the residuals of the Ek (1971) model, 95.08% fall within the range 0 ± 1.463, confirming the confidence interval.

Prediction Intervals - Ek (1971) Model

Estimation procedures for the distribution of errors is important to the user that employs a model for future prediction. Prediction intervals for future values of the dependent variable obtained from regression are usually given in most statistical textbooks, but Hahn and Nelson (1973) give a formula for the $(1 - \alpha)100$ percent interval for a given value in a non-regression setting

$$\bar{D} \pm \sqrt{1+\frac{1}{n}}\, S t_{(1-\alpha/2)}. \qquad [15]$$

In this interval, $S =$ the variance of the residuals, $t_{(1-\alpha/2)} =$ the student's t statistic with n-1 degrees of freedom. The interpretation of this interval is that the probability is $(1 - \alpha)$ that future values of the residuals (D), from randomly selected independent variables will fall within this random interval.

The future values of residuals from the Ek (1971) model used to predict site index for jack pine should fall within the interval 0.00441 ± 1.030680 with 95% confidence, assuming a normal population.

The same testing procedures are used for the Ek (1971) model fit to the total age data. The allowable errors (e) used to test the total age data must be greater than the critical errors (Table 8). The total age model accurately describes the data when an allowable error (e) of 1.6 is used for Freese's procedure and 1.7 for Reynolds's procedure.

The total age model has a higher residual sum of squares ($\sum_{i=1}^{n} D_i^2$) than the breast height age model, thus resulting in the higher critical errors and confidence intervals for the distribution of the absolute residuals. Future estimates of jack pine height growth using site index and total age should lie within the range 0.00141 ± 1.254098 m of the true height 95% of the time. Comparing the prediction intervals for the breast height age model [eq. 8] and total age model [eq. 3] indicate that the breast height age model can predict height growth more accurately than the total age model (Table 8).

Table 8. Critical errors, confidence limits and prediction intervals for the Ek (1971) model [eq. 8 and 3] for breast height age and total age.

	BREAST HT AGE MODEL	TOTAL AGE MODEL
CRITICAL e (e^*)	1.381	1.524
CRITICAL e (e^{**})	1.463	1.614
90% CONFIDENCE LIMITS FOR |D| LOWER	1.381	1.524
90% CONFIDENCE LIMIT FOR |D| UPPER	1.463	1.614
95% PREDICTION INTERVAL FOR $\bar{D}$	0.00441 ± 1.030680	0.00141 ± 1.254098

HEIGHT GROWTH AND SITE INDEX CURVES FOR JACK PINE

Height growth and site index curves were constructed with the Ek (1971) model fit to the jack pine height growth data. Coefficients for the height growth curves are given in Table 5 and coefficients for the site index curves are given in Table 9. Height growth and site index curves are graphical representations of the mathematical expressions of height growth.

Table 9. Coefficients for jack pine site index curves calculated using Ek (1979) model [eq. 8] - age is breast height age.

PARM.	ESTIM. OF COEFFICIENT	STD. ERR. COEFF.	95% CONFID. LIMITS LOWER	UPPER
β_1	1.84814E+00	1.59E-02	1.82E+00	1.88E+00
β_2	8.87197E-01	2.70E-03	8.82E-01	8.92E-01
β_3	2.37956E-02	1.96E-04	2.34E-02	2.42E-02
β_4	4.43721E+00	9.35E-02	4.25E+00	4.62E+00
β_5	5.11216E-01	6.80E-03	4.98E-01	5.25E-01

Height Growth Curves - Breast Height Age

The height growth model gives the best estimate of height growth trends for the jack pine data. The height growth curves represent the average expected pattern of height growth for each level of site index. Early height growth before the inflection point and upper asymptote of the height growth curves are determined by the level of site index. The height growth curves are not constrained to pass through site index at index age, but are generally close. The height growth curve calculated for site index 16, the approximate mean site index for the data, agrees closely with the specified site index of 16 (Figure 9). However, for curves greater or lesser than the mean the agreement is not as close. By definition, site index curves should pass through the specified site index at index age.

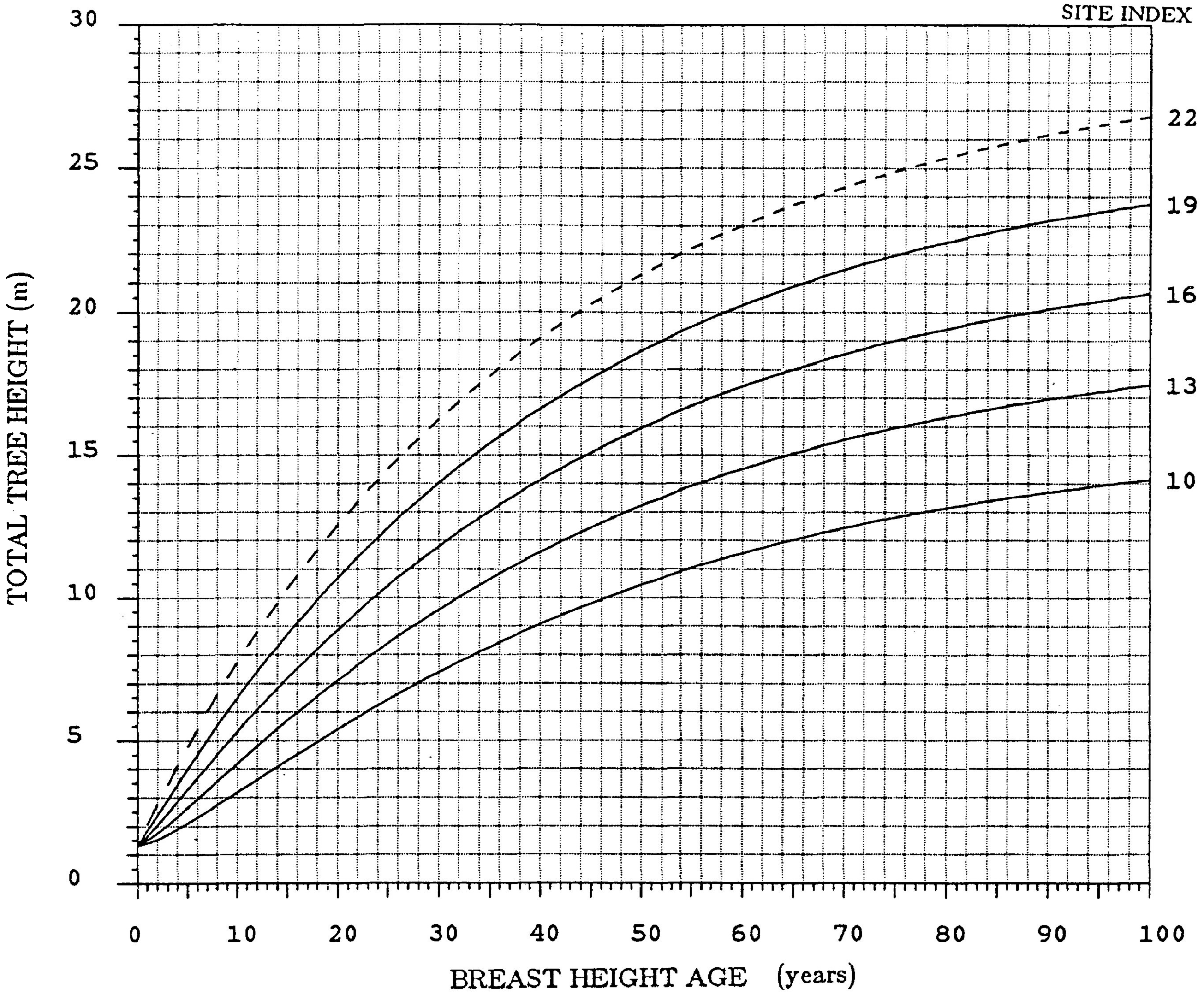

Figure 9. **Height growth curves calculated for jack pine using the Ek (1971) model [eq. 8].**

Site Index Curves - Breast Height Age

The height growth curves do not pass through site index at the index age (Figure 9). The fitted growth model can be adjusted to pass through index height at index age by graphical or mathematical methods, thus creating site index curves (Figure 10). Both methods introduce a bias, but the mathematical approach is more consistent and reproducible. The Ek (1971) model [eq. 8] was refit to estimate model parameters that create site index curves based upon breast height age (Table 9). The family of site index curves pass through the specified height (site index) at index age (Figure 10).

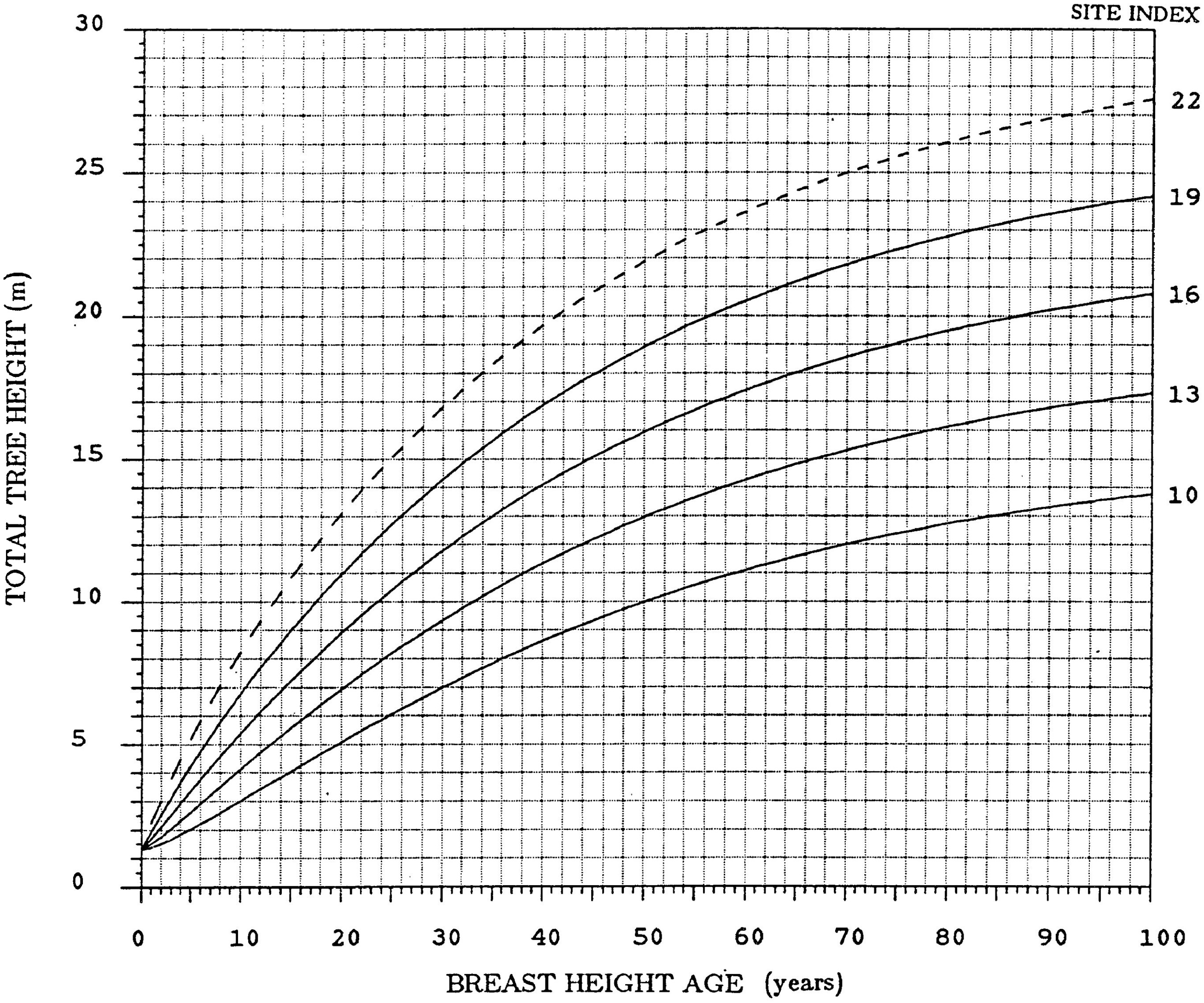

Figure 10. Site index curves calculated for jack pine using the Ek (1971) model [eq. 8].

The same procedures were used to calculate site index curves based upon total tree age (Table 10). Height growth curves and site index curves for jack pine based upon total tree age are given in Appendix IV and V.

Table 10. Coefficients for jack pine site index curves calculated using the Ek (1971) model [eq. 3] - age is total age.

PARM.	ESTIM. OF COEFFICIENT	STD. ERR. COEFF.	95% CONFID. LIMITS LOWER	UPPER
β_1	2.38292E+00	1.32E–02	2.36E+00	2.41E+00
β_2	8.23181E–01	1.80E–03	8.20E–01	8.27E–01
β_3	2.83291E–02	1.42E–04	2.81E–02	2.86E–02
β_4	4.64689E+00	7.03E–02	4.51E+00	4.78E+00
β_5	4.45181E–01	4.88E–03	4.36E–01	4.55E–01

VALIDATING THE SITE INDEX CURVES WITH INDEPENDENT DATA

Site index was estimated for each of the 32 independent site plots using the site index curves (Figure 10). To avoid interpolation error, the linear relationships shown in Appendix III were used to calculate the site index for each of the reserved plots. These estimated site indices were then compared to the actual site index derived from stem analyses (Appendix VII). A summary of the predicted residual values is given in Table 11.

The prediction error for site index is low on an individual basis, although a slight negative bias is present, $\bar{D} = -0.17$ m. This bias is not statistically different from zero for $\alpha = 0.05$, and poses no problem with accuracy of prediction. The critical error (e^{**}) for Reynolds's (1984) testing procedure is 1.625 m, indicating that an allowable error of 1.625 m is necessary to accept the site index curves as accurate. Several old, poor quality plots (85,99,100) have large prediction errors, creating this high critical error. These plots are older than the majority of the data used to calculate the site index curves, thus the estimation of site index is based on extrapolation of the model.

Table 11. Summary of residuals when site index was predicted for 32 reserved plots using Figure 10.

RESIDUAL SUM OF SQUARES	13.69
RESIDUAL MEAN SQUARE	0.4278
RESIDUAL ROOT MEAN SQUARE	0.6541
MEAN RESIDUAL	-0.17
NUMBER OF OBSERVATIONS	32

Future estimates of site index from the site index curves should lie within the prediction limits as defined by Reynolds (1984). The 95% prediction intervals calculated from the independent data points for the breast height age site index curves are: -0.17 m ± 0.886. The mean error of prediction should be no more than one metre 95% of the time for future estimates of site index using the site index curves based on breast height age (Figure 10). This prediction interval was calculated from the independent data that were not used to create the site index curves. The prediction interval calculated from the data that created the site index curves was similar (Table 8). The similar prediction intervals indicate that the site index curves will predict site index of jack pine in other portions of the study area as well as they predict site index of the data that created the site index curves.

SITE INDEX PREDICTION EQUATIONS

Site index curves used for classifying land into potential productivity units should use site index as the dependent variable of regression (Strand, 1964; Curtis *et al.*, 1974). The five coefficient Richards model proposed by Ek (1971) [eq. 3] cannot be solved for site index, but a variation used by Payandeh (1974a) and Hahn and Carmean (1982) gives a close approximation of the inverse [eq. 4]. The coefficients of this model [eq. 4] are given in Table 12.

Table 12. Coefficients for the site index prediction model [eq. 4].

PARM.	ESTIM. OF COEFFICIENT	STD ERR. COEFF.	95% CONFID. LIMITS LOWER	UPPER
β_1	6.71215E–01	6.14E–02	5.51E–01	7.92E–01
β_2	9.34538E–01	1.79E–02	8.99E–01	9.70E–01
β_3	8.48268E–03	1.24E–03	6.06E–03	1.09E–02
β_4	7.12502E–01	1.39E–02	6.85E–01	7.40E–01
β_5	7.88632E–02	1.36E–02	5.22E–02	1.60E–01

Examination of the residuals for the site index prediction model revealed a greater variation of residuals for young ages. The young age class and corresponding short heights showed greater residual site index values than data from older, taller trees. Juvenile height growth rates were often inconsistent with later growth rates and site index. Therefore, young stands often gave misleading estimates of site index. As site quality (site index) was the real item of interest, the data for young ages were eliminated to improve site index prediction. Thus the resulting site index prediction equations were applicable only to stands older than 20 years.

The site index prediction equation was fit to the stem analysis data for ages greater than 20 years. This was done because the heteroscedastic trends of the residuals were eliminated by restricting the range of the data to ages greater than 20 years. The estimates of the model parameters for the restricted data are given in Table 13.

Table 13. Coefficients for the site index prediction model [eq. 4] for data greater than 20 years of age.

PARM.	ESTIM. OF COEFFICIENT	STD ERR. COEFF.	95% CONFID. LIMITS LOWER	UPPER
β_1	6.01785E–01	4.43E–02	5.15E–01	6.89E–01
β_2	1.03482E+00	2.09E–02	9.94E–01	1.08E–00
β_3	1.70435E–02	1.24E–03	1.46E–02	1.95E–02
β_4	1.19870E+00	9.26E–02	1.02E–00	1.38E–00
β_5	1.66684E–01	3.49E–02	9.83E–02	2.35E–01

Restricting the data to ages greater than 20 years of age reduced the residual mean square, indicating increased precision of equation 4 (Table 14).

Table 14. Squared residuals for the site index prediction model [eq. 4] for all stem analyses data and for data greater than 20 years.

	ALL DATA	DATA > 20 YEARS
RESIDUAL ROOT MEAN SQUARE	1.22015929	0.77119168
RESIDUAL MEAN SQUARE	1.48878868	0.59473661
RESIDUAL SUM OF SQUARES	2411.83766895	704.16814882
NUMBER OF OBSERVATIONS	1625	1189

Table 15. contains estimates of the coefficients for the site index prediction equation formulated from the site index curves [eq. 7].

Table 15. Coefficients for site index prediction equation (7) developed by mathematically inverting the site index curves - age is breast height age.

PARAMETER	COEFFICIENT
β_1	0.414459
β_2	0.549633
β_3	0.679648
β_4	16.0176

Goodness Of Fit - Site Index Prediction Equations

The goodness of fit for the site index prediction equations was determined in the same manner as for the height growth and site index curves. Reynolds's (1984) Chi-squared test was used to determine critical errors and prediction intervals for the model. The site index prediction equation that resulted when equation [4] was fit to the stem analyses data greater than 20 years old had a lower prediction interval than the equation calculated when data of all ages were used (Table 16). The lower critical errors and prediction intervals indicated a more accurate and reliable equation for predicting site index. The calculated critical errors and prediction intervals have the same interpretation as previously given (Table 16).

Table 16. Critical errors and prediction limits for site index prediction equations [eq. 4] calculated using Reynolds (1984) chi-squared procedure.

	ALL DATA	DATA > 20 YEARS
CRITICAL ERROR (e^{**})	2.463	1.564
95% PREDICTION INTERVALS FOR $\bar{D}$	-0.00128±2.918923	-0.01325±1.166174

The three site index prediction equations (Tables 12, 13 and 15) predict site index equally well for each age group shown in Table 17. F-tests of the residual variances showed no statistical differences ($\alpha = 0.05$) between any pair of models within each data group. Similarly, the modified Richards models predicted site index equally well at age 30 and stand age, but the site index inversion model [eq. 7] predicted poorly at age 30. The ratio of the variances for the site index inversion model ($0.865/0.309 = 2.80$) exceeded the tabulated F-value (1.88) for $\alpha = 0.05$ with 28/28 degrees of freedom indicating a statistical difference in error of prediction between the two age groups. The best overall equation for predicting site index of jack pine is the Richards model [eq. 4] fit to data greater than 20 years of age (Table 13).

Table 17. Residual sum of squares, residual mean squares and critical errors for reserve data tested with site index prediction models.

AGE	MODEL	RESIDUAL SUM OF SQ.	RESIDUAL MEAN SQ.	CRIT. ERR. (e^{**})
Variable	[eq. 4] (all data)	15.593	0.487	1.72
Variable	[eq. 4] (data > 20 years)	17.521	0.549	1.83
Variable	Site Index Inversion Model	9.902	0.309	1.38
30 Years	[eq. 4] (all data)	23.246	0.726	2.11
30 Years	[eq. 4] (data > 20 years)	23.673	0.740	2.13
30 Years	Site index Inversion Model	27.684	0.865	2.30

PLONSKI'S SITE INDEX CURVES FOR JACK PINE FORMULATED

Data from Plonski's site index curves were formulated using Ek's (1971) model [eq. 4]. Each of the five coefficients was statistically different from zero with $\alpha = 0.05$ (Table 18).

Table 18. Coefficients for Plonski's formulated site index curves for jack pine calculated using Ek (1971) model [eq. 3] - age is total age.

PARM.	ESTIM. OF COEFFICIENT	STD ERR. COEFF.	95% CONFID. LIMITS LOWER	UPPER
β_1	1.63945E+00	8.54E–02	1.47E+00	1.81E+00
β_2	9.23549E–01	1.84E–02	8.87E–01	9.60E–01
β_3	3.66920E–02	9.94E–04	3.47E–02	3.87E–02
β_4	6.23072E+00	1.11E+00	4.01E+00	8.46E+00
β_5	4.83584E–01	6.13E–02	3.61E–01	6.06E–01

Plonski's formulated site index curves, when compared with the study height growth curves based on total age, showed distinct differences. The coefficients estimating β_4 and β_5 were statistically the same ($\alpha = 0.05$) for both sets of site index curves. These model parameters estimate early height growth, which was the same in each case. The remaining coefficients estimating β_1 , β_2 , β_3 are statistically different for both sets of curves (Figure 11). The trends shown are typical of most comparisons between site index curves constructed with height growth models and guide curves (Carmean, 1979b). Plonski's (1974) formulated curves are lower than curves resulting from this study at ages greater than index age (50 years total age).

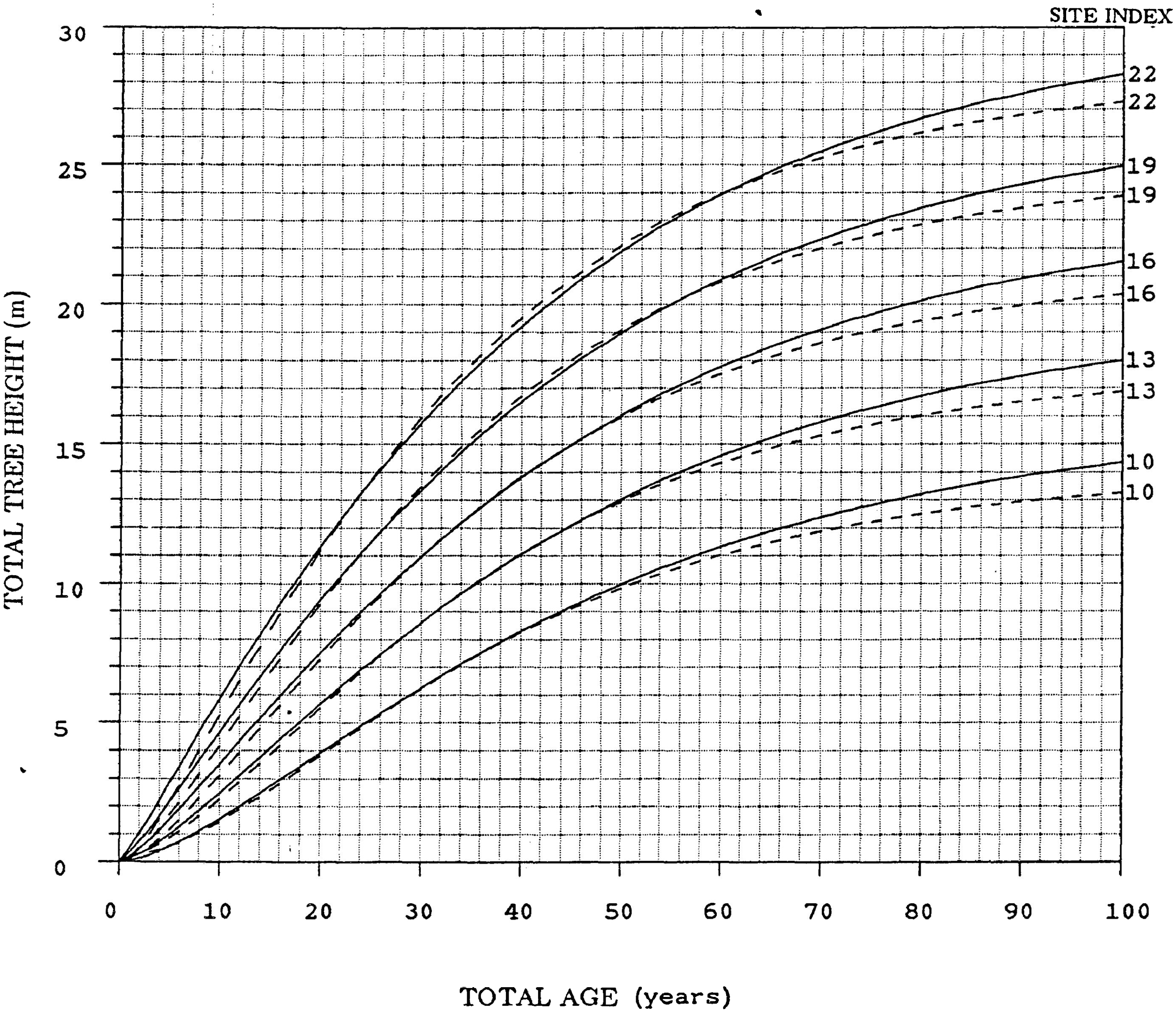

Figure 11. Site index curves compared to Plonski's site index curves for jack pine formulated using equation 3 - age is total age. Dashed curves are Plonski's formulated site index curves for jack pine.

CHAPTER 5

DISCUSSION

PATTERNS OF HEIGHT GROWTH BY LEVEL OF SITE INDEX

Average height growth patterns vary in shape and amplitude with different levels of site index (Figure 4). Each average height growth curve is based on data for the plots within each site index class and was computed using Richards (1959) growth model [eq. 1].

Richards model (1959) [eq. 1] estimated the average height growth curves for different levels of site index with very few anomalies within the range of the observed data. The height growth curves remain separate between ages 15 and 100 years at breast height. Extrapolating the height growth data beyond the range of the data will cause the estimated height growth curves to intersect. The projected upper asymptote is estimated by β_1 of the Richards model (1959), (Table 19).

The intersection of the average growth curves is a result of the data contained in each site class. The three poorest site classes, (si= 9,11,14), have poor predictions of the upper asymptote of the height growth curves. Height growth curves for plots on the low site classes show slow but steady, almost linear height growth trends without a pronounced slowing of height growth. These

Table 19. Predicted upper asymptote for jack pine height growth for different site classes using Equation 1.

SITE CLASS	PREDICTED UPPER ASYMPTOTE (m)	95% CONFIDENCE LIMITS (m)
9	25.123	± 12.40
11	21.816	± 6.25
14	22.075	± 1.35
16	20.715	± 0.40
18	22.011	± 0.40
20	23.488	± 0.55

nearly linear growth trends did not allow an accurate prediction of the upper asymptote (Table 19). The poor estimate of the upper asymptote is not a serious problem when illustrating height growth curves within the range of the data; extrapolation of the height growth curves beyond the range of the data should be avoided. In contrast, the good site classes (SI=16,18,20) show rapid early height growth with a pronounced inflection at 30 to 50 years of age. The pronounced reduction of height growth increment for the good site classes at older ages allow for accurate prediction of the upper asymptote (Table 19).

An inconsistency shown in Figure 4. is the intersection of the predicted height growth curve for site class 14 with the curve for site class 16 at age 11. The only explanation for this intersection is the variability of the early height growth data. The sample data for site class 14 are statistically ($\alpha = 0.05$) more variable at the early ages for site class 14 than is the data for site class 16. This variation creates a poor estimate of early height growth for site class 14, thus causing the intersection of the height growth curves. However, this inconsistency is unimportant because site index curves are usually not used for ages as young as 11 years.

Several problems arise when height growth and site index curves are produced from individual classes of stem analysis data. The average height growth curves for each site index class do not accurately represent the true height growth curves at advanced ages. This is particularly true for very good sites where only a few plots were available and these plots did not extend beyond 90 years. Fewer and fewer height observations are available for each age as age increases above 60 years. In this instance, the average height growth curve for site class 20 shows a rapid decrease of annual height growth similar to harmonized curves where site index was negatively correlated with age. The predicted upper asymptote for site class 20 (Table 19) has very narrow confidence limits, but is greatly influenced by a few, very old, plot height growth curves (Appendix VI). The predicted upper asymptote for site class 20 is really the upper asymptote predicted for the old plots within site class 20. Data from plots less than 70 years had no influence on the predicted upper asymptote for site class 20. Therefore, although the confidence limits for the predicted upper asymptotes for site class 20 may be narrow, the predicted upper asymptote is not necessarily an accurate estimate for the entire site class. This is also true for other site classes, but to a lesser degree.

This problem could be overcome if sample plots were all the same age when sampled. The data could have been adjusted to the same age by truncating the individual height growth curves at a common age, or by extrapolating the data to a common age (Curtis, 1964). As both methods introduce an unknown error for older ages, the data were not adjusted to a common age. Rather, the limitations of the predicted height growth curve to depict the "true" class height growth curve beyond 60 years was noted. The height growth curves for each site index class between the ages of 20 - 60 years is an accurate estimate of the data within the site index class. But after 60 years the height growth curves are based on fewer data points and thus are not as dependable for estimating height growth patterns.

Variation Among Predicted Growth Curves

Growth curve shapes vary from nearly linear for poor sites (SI=9) to highly curvilinear for the best sites (SI=20). Similar height growth curves related to site index have been shown for white pine (Beck, 1971) and oak species (Carmean, 1972). The poorest sites have very slow, but steady height growth. This pattern suggests a relatively constant, but low supply of nutrients and water in relation to the demand of the trees (Shea and Armson, 1972). Many of the poorest sites are bedrock overlain with shallow soil, restricting the soil volume available for tree growth. The volume of soil available to tree roots influences tree growth as it affects nutrient and moisture supplies, root development and anchorage against windthrow. Trees growing on shallow soils are generally less well supplied with water and nutrients than trees on deep soils (Pritchett, 1979). Trees on poor sites are able to grow, but do not attain their biological potential.

Trees growing on better sites show a typical sigmoid pattern of height growth. Average tree height growth on good sites exhibit three stages of growth common to living things: juvenile phase; phase of full vigor; phase of senescence (Assman, 1970). Sample plots with site indices greater than 16 m at 50 years breast height age were not found on very shallow soils (< 0.5 m depth), but occasionally were found on soils 0.5 - 1.0 m deep (Schmidt, 1985). The very shallow soils ($<$ 0.5 m depth) do not appear to have the capacity to produce stands of jack pine with site indices greater than 16 m. Shallow soils (0.5-1.0 m depth) will support stands with site indices of 16 m or more. For these better sites, growth rates increased as site index increases, but curve forms are relatively similar. Soil depth or other root restrictions (fragipans or high water level) on good sites may influence the later height growth of the stands. Site index is a function of growth rate and age, but does not explain why trees grow at varying rates on different soils. Better understanding of the physiological processes of height growth, detailed chemical analyses of the soil, and assessment of moisture availability are required to explain differences in rate of height growth on different sites.

Expanded Height Growth Models For Individual Site Class Curves

Commonly, the model parameters of the Richards model [eq. 1] governing the upper asymptote (β_1) and growth rate (β_2) are expanded as a function of site index. Problems identifying an estimate of the "true" upper asymptote for each site class eliminated β_1 from further consideration in this study. As previously discussed, the upper asymptotes showed no relation to observed site index; poor sites had higher predicted upper asymptotes than good sites.

In this study, consistent trends were not evident for any of the coefficients for either Richards model (1959) [eq. 1] or a modified Weibull function (Yang *et al.*, 1978) [eq. 2]. The expanded models showed an overall poor fit or severe bias at either young or old ages. The success of the previous authors may be due in part to the consistent fast growth of the species, age distribution or sample size. Variation within site class, unequal number of observations per site class, and wide range of soil conditions undoubtedly affected the expansion of the Richards model in this study.

PATTERNS OF HEIGHT GROWTH BY LANDFORM

The patterns of height growth by landform (mode of deposition) were statistically similar ($\alpha = 0.05$) for a given level of site index (Figure 6). Height growth curves calculated for the same level of site index eliminates the variation associated with different levels of site quality, allowing comparisons among the landforms.

Model [eq. 8] predictions of height growth (SI = 16) for the four landforms are nearly identical for ages younger than 50 years breast height age. This is contrary to site index curves constructed using soil-site equations (Carmean, 1956; Spurr, 1956; Zahner, 1962). They speculated that the early height growth was most variable and was influenced by drainage characteristics of the soil. The different height growth on soils with the same site index was explained as restricted root development caused by poor aeration of the soil.

In this study landform classes appear to be too broad to indicate any difference in height growth patterns. The predicted height growth curves for each landform type are nearly identical for site index 16 (Figure 6). They approach different upper asymptotes but the predicted upper asymptotes are not statistically different ($\alpha = 0.05$). Beck (1971) also found that soil and topographic variables did not statistically influence height growth curves for white pine in the southern Appalachians. Average jack pine height growth as determined by stem analyses was relatively uniform in shape among the landforms. The exception being very poor bedrock sites with less than 50 cm of soil.

Different height growth patterns associated with poor drainage are not represented within the data set due to the drainage conditions where jack pine usually occurs. Sites with very poor drainage typically do not support jack pine stands, thus poorly drained areas were not considered. for sampling. Depth to subsoil mottling was a statistically significant variable that reduces jack pine site index. However, for the same site index height growth patterns for soils with mottling are similar to patterns for well drained sites that do not have mottling (Schmidt, 1985).

Other soil and site attributes that influence height growth patterns and site index are obscured by the broad landform classes. Site quality was the main interest of this study; tree growth was merely a phytometer by which to examine the growth potential of the site. The glacialfluvial sands, moraines, and lacustrine sites sampled in this study have statistically the same mean site index value. The requirements for tree growth may be supplied equally well for each of the three landforms even though good, medium, and poor sites exist within each landform. Features in addition to landform should be used to separate soils into productivity units for jack pine growth. Soil-site studies can identify mappable soil attributes that are correlated with jack pine site index. A hierarchial system of soil classification based on features closely related to forest site quality should be developed for soil mapping in the Thunder Bay district.

ATTRIBUTES OF PREDICTED HEIGHT GROWTH CURVES

This study demonstrates that the five parameter Richards growth model proposed by Ek (1971) [eq. 8] offers a satisfactory fit to jack pine height growth data in northwestern Ontario. The model predicts height growth patterns as they vary with level of site index. Variation in height growth patterns for the same site index cannot be explained by the model. For example, trees growing on different sites with the same site index do not have the same patterns of height growth at ages older than the index age (Appendix VIII). Instead, an estimate of the average height growth for each level of site index is predicted by the model. The deviations from the average growth curve may be the result of different genetic structure of the trees, root competition, or a change in drainage affected by man, animals or stream migration. Future studies that compare the height growth patterns of trees with the same site index to the predicted height growth patterns may give insight to the causative factors of differential height growth.

Differences Among Height Growth Curves Related To Site Index

The sigmoid shape of the height growth curves is specified by the level of site index for each curve in the family of curves. The most striking difference in the curve shape is evident in the early height growth patterns for jack pine (Figure 9). The mean annual height growth increment for each of the height growth curves shows the polymorphic height growth patterns related to level of site index. (Figure 12). As level of site index increases, the culmination of mean annual height growth occurs at an earlier age. For example, for site index 10 the culmination of mean annual height growth occurs at age 33 while in contrast, for site index 22 the culmination of mean

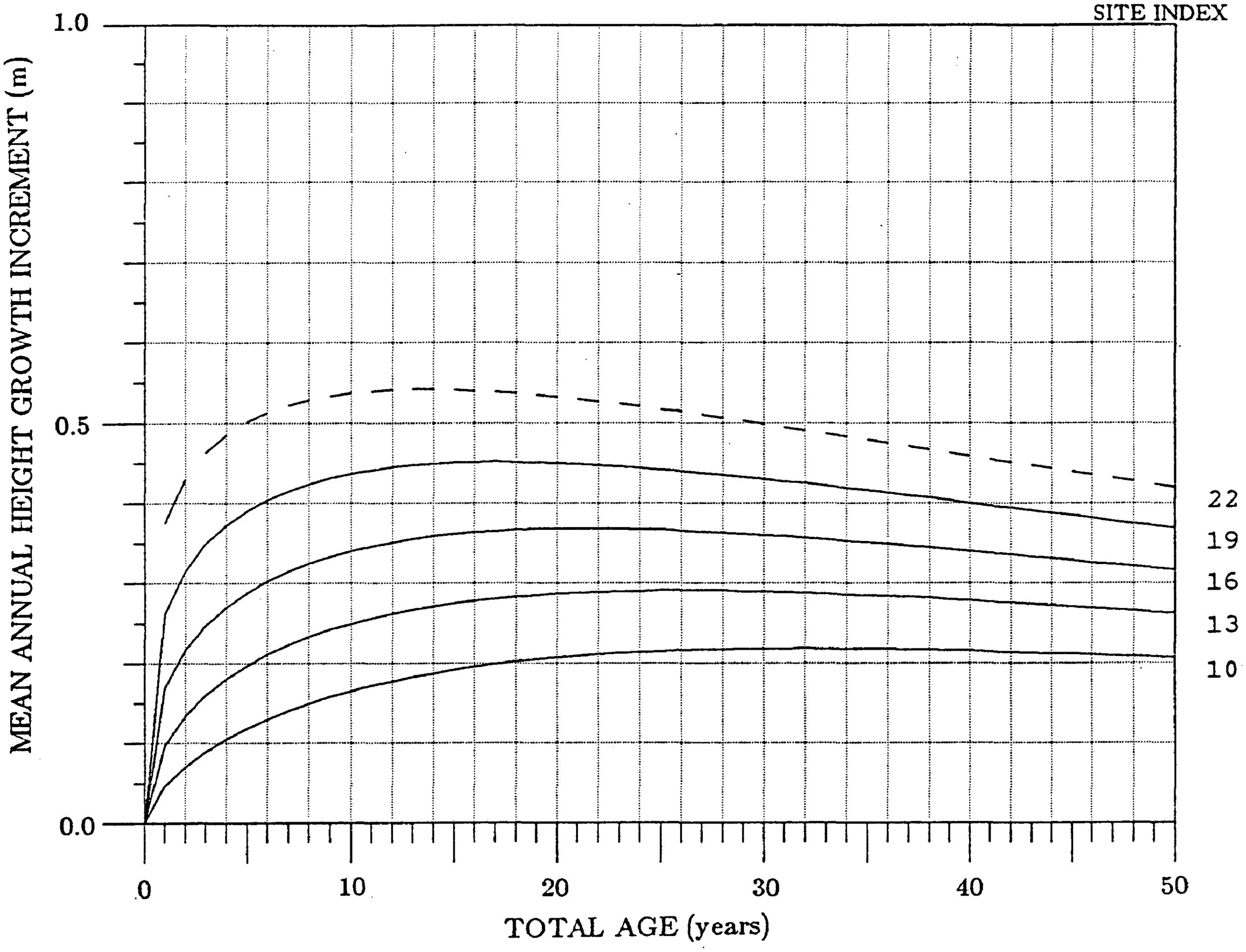

Figure 12. **Mean annual height growth predicted from height growth curves for each level of site index - age is total age.**

annual height growth occurs at age 12.

The culmination of mean annual height growth increment is the point of inflection for the height growth curve, that is, the point of most rapid height growth. Good sites reach a higher maximum growth rate at an earlier age. Model parameters (β_4 and β_5 in equation 3) in relation to site index determine the early height growth patterns of the predicted height growth curves. Early growth predictions are not an artifact of the model construction, as the underlying data influences the parameter estimation. The presence of non-zero value for β_5 confirms this observation. The coefficient estimating β_5 would be statistically equal to zero if the relationship between site index and early height growth of jack pine did not exist.

The predicted height growth for jack pine at ages 60 to 100 is affected by site quality. The model parameters (β_1 and β_2 in equation 8) in relation to site index determine the later growth and projected upper asymptote of the height growth curves (Figure 9). The estimated value of β_2, being statistically different from zero indicated that site index has an affect on later predicted height growth of jack pine. Trees growing on areas of high site index grow to a higher predicted upper asymptote than trees on areas of low site index (Figure 9) (Table 20). These are the predicted upper asymptotes for the height growth model (eq. 8) based on breast height age. The predicted upper asymptotes for Table 19 differ because the asymptotes were calculated for each site class separately using Richards model (eq. 1). In Table 20, the predicted upper asymptotes were estimated using Ek's expansion [eq. 8] for all the height/age data combined.

EVALUATION OF TECHNIQUES

A major strength of this study is the large data base collected for analyses. The size of the data base afforded certain luxuries not available to many previous studies. A subset of 32 plots was reserved as an independent set for testing. The independent data set represents additional "real-world" observations for testing the accuracy of the computed site index curves and prediction equations.

Table 20. Predicted upper asymptote for jack pine height growth curves using equation [8]. Age is breast height age.

SITE INDEX (m)	PREDICTED UPPER ASYMPTOTE (m) $\beta_1 \mathrm{SI}^{\beta_2}$
10	14.39
13	17.88
16	21.23
19	24.48
22	27.64

The large stem analysis data base consisted of height and age observations from trees growing on 141 plots; 109 plots were used for computation and 32 plots were reserved as confirmation plots. Stem analysis eliminates the assumption of proportional height growth necessary for harmonized site index curves. The height growth patterns and site indices of the trees were observed and thus were not assumed as in harmonized curves. Also individual tree height growth curves were examined for signs of suppression or damage that disrupt normal height growth patterns. The paired height and age observations were modeled to produce height growth and site index curves that varied in shape with observed site index.

The data was fit to biological growth models using nonlinear regression. The model chosen to express the height growth patterns and site index curves for jack pine was the model [eq. 8] that produced the lowest residual sum of squares when fit to the stem analyses data. A biological growth model produces an objective means of expressing the height growth patterns of jack pine. In addition, the model parameters are interpreted to have certain biological connotations that can lead to better understanding of the height growth of jack pine.

Unlike many previous studies, the height growth curves, site index curves, and site index prediction equations are accompanied with an estimate of prediction error. A modification of the Chi-squared distribution was used to test the accuracy and to calculate prediction error of the model (Reynolds, 1984). This procedure is more conservative than a commonly used procedure proposed by Freese (1960). The calculated critical error (e^{**}), confidence intervals, and prediction limits provide a statement of accuracy for the user. The stated testing procedures and accuracy statements are derived for 3 m site index classes. The formulae and methods are given to derive a statement of accuracy for other site index class intervals.

Weaknesses in this study are common to many previous studies. Sample plots were selected in a non-random manner to insure wide range of site quality and soil conditions. Complete random sampling was logistically impossible for such a large and remote study area thus sample plots were limited to accessible areas that met the plot selection criteria. Many plots chosen using a completely random method would not contain jack pine, would have been inaccessible, would not have met the plot or tree selection criteria for suitable site index determination, or would have grouped about the mean conditions of the study area. Plots were selectively chosen to avoid these problems although the regression requirement of random sampling was violated. As with other studies, the sample plots were assumed to represent randomly chosen observations from a specified population.

Comparisons between different sets of site index curves and between different nonlinear models are difficult because precise testing procedures are not defined for nonlinear regression. Corresponding model coefficients were tested for statistical differences using the 95% confidence intervals of the coefficients. If the confidence intervals overlapped, the coefficients were considered statistically the same. Two fitted models were considered the same if the coefficients of one model were statistically the same as the second model. The coefficients were tested for statistical differences in this manner, but the entire model was tested only indirectly. The estimates of the model parameters were correlated with each other, thus values of one estimated parameter may have been offset by differences in one or more of the remaining coefficients. A more accurate and comprehensive means of testing nonlinear regression equations is necessary to realize the full potential of height growth and site index curves.

Perhaps the greatest weakness of the methods and techniques is the lack of additional data to supplement the height/age data. Only height/age relationships of the dominant trees can be derived from the data. Nothing is known about stand density, volume, mortality or species composition of the sampled stands. This information was not collected because it was not needed to accomplish the specific objectives of the study. In hind-sight, this additional information could have enhanced the study with estimates of density or species composition related to site index. The height growth and site index curves can be used to estimate the future height growth of the dominant trees or site index, but inference as to appropriate rotation length or future wood yield cannot be made.

RECOMMENDATIONS FOR FUTURE RESEARCH

The site index curves for jack pine developed in this study can improve forest management by creating a foundation for future research in growth and yield of jack pine. These research projects are long-term studies that must begin in even-aged, mature, natural stands of jack pine. During pre-cut cruises of jack pine stands, careful estimates of jack pine site index should be recorded. Site index can then be used to delineate areas of similar site quality on inventory maps.

Silvicultural treatments should be prescribed for each of these areas so as to attain maximum efficiency of forest management. Productive areas having high site indices should be managed using the most intensive forestry practices, e.g. fertilization, site preparation, release, thinning, and stocking with genetically superior seedlings to produce the largest volume of pulpwood and sawtimber in the shortest period of time. Areas having poor site indices should be managed less intensively with emphasis on less expensive means of regeneration and site treatment over longer periods of time. Additional research is required to determine the best combinations of site quality and management intensity that produces the best management schemes.

Data from permanent sample plots of known site indices will provide the most opportunities of furthering site quality research in northwestern Ontario. As each permanent sample plot regenerates, more information will be gathered relating tree growth to an estimate of site quality. Several specific questions that can be answered with long-term site quality research are:

1) How does site index measured in natural stands compare to site index for planted jack pine?

2) Which silvicultural treatment will best regenerate land areas of each level of site index?

3) Does regeneration success correlate with estimates of site quality of the previous stand?

4) Can site index of the past stand accurately estimate site index, or growth and yield of the new stand? And what is the relationship between site index of the past stand with growth and yield of the new stand?

Although long-term research projects, these studies are only a link between the natural forest and the plantation culture that is developing in northwestern Ontario. As the "new forest" matures, reliance upon site quality and yield of the natural forest must give way to methods of site quality evaluation and yield prediction based upon the new plantation culture.

CHAPTER 6

CONCLUSIONS

Height growth patterns and site index were studied using stem analyses taken from dominant and codominant trees growing on 109 plots located in mature, natural, fully stocked, evenaged stands of jack pine in the Thunder Bay area. Analyses of data from these 109 plots plus an additional 32 independent confirmation plots led to the following conclusions:

1) Jack pine site index in the Thunder Bay area can be estimated from mature, even-aged stands of jack pine using either site index curves (Figure 10) or site index prediction equation [4], (Table 13).

2) Average height growth curves for jack pine showed polymorphic height growth patterns as level of site index increased (Figures 4 and 5). The average height growth curves were nearly linear for poor sites (SI = 9, 11) and were highly curvilinear for good sites (SI = 18, 20).

3) Annual height growth is clearly polymorphic at young ages (Figure 12). Good sites have very rapid initial height growth that reaches maximum annual height growth at earlier ages than do trees on poorer sites.

4) Ek's (1971) expansion of the Richards growth model (1959) fit the stem analyses data better for breast height age [eq. 8] than the stem analyses data for total age [eq. 3], (Table 7). The standardized residuals for each regression showed no apparent bias in either model (Breast height age - Figure 7), (Total age - Figure 8). Breast height age should be used as the base age to eliminate erratic early height growth, thus a more accurate estimate is obtained of the height growth patterns of jack pine.

5) Each landform had a wide range of observed site index. However, the mean observed site index for jack pine on glacialfluvial sands, moraines and lacustrine sites was statistically the same for each landform. Furthermore, no statistical difference was evident among the

height growth patterns of jack pine for the broad landform types (Table 6 and Figure 4).

6) Comparisons with formulated Plonski's (1974) jack pine site index curves showed that Plonski's site index curves were lower at ages greater than 60 years (Figure 11). This is partially due to the harmonizing techniques that Plonski used to develop his site index curves. Plonski also used tree of mean basal area which may be more variable in height than dominant or codominant trees. Plonski's site index curves will predict site index with the same accuracy as my site index curves based on total age if dominant and codominant trees are used in stands younger than 60 years of age. But site index curves based on breast height age are more reliable for estimating site quality (conclusion 4).

7) The site index prediction equation that is most reliable for estimating site index is the modified Richards model [eq. 4] proposed by Payandeh (1974a) fit to stem analyses data greater than 20 years old. This equation had a lower residual mean square than the equation calculated when data of all ages were used (Table 14). The site index prediction equation calculated by mathematically inverting the site index curves is not a suitable site index prediction model [eq. 7]. When equation [7] was tested using the reserve data, biased estimates of site index resulted from data at age 30. (Table 17).

8) Critical errors (e^{**}) and prediction intervals were calculated using a modification of the Chi-squared distribution proposed by Reynolds (1984). The prediction interval defines the range of probable errors associated with each model used to estimate site index. Site index can be accurately estimated within each 3 m site index class 95% of the time.

9) Jack pine stem analyses data from 32 confirmation plots were reserved for testing the site index curves and prediction equation. The site index curves and prediction equation [4] accurately predicted site index of the 32 reserve plots within 1.5 m 95% of the time.

The site index curves and prediction equation can be used to estimate site index of fully stocked, natural jack pine stands in the Thunder Bay area. Estimates of rotation length, wood yield and regeneration success of each level of site index will require additional studies coupled with sound professional judgement. The height growth patterns and site index estimates developed from this study must be continually tested using field observations. Only by further testing can the site index curves and prediction equations be accepted or rejected as an adequate means of jack pine site quality estimation.

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APPENDICES

APPENDIX I

JACK PINE SITE PLOTS FOR EACH LANDFORM AND SITE INDEX CLASS USED FOR CALCULATING HEIGHT GROWTH AND SITE INDEX CURVES

PLOTS PER SITE INDEX CLASS						
LANDFORM	< 11.5m	11.5 - 14.4m	14.5 - 17.4m	17.5 - 20.4m	≥ 20.5m	TOTAL
GLACIALFLUVIAL SANDS	0	2	14	21	1	38
MORAINE	0	3	16	13	1	33
LACUSTRINE	0	0	4	6	3	13
BEDROCK *	8	8	7	2	0	25
TOTAL	8	13	41	42	5	109

* Soil is 1.0 m. or less above bedrock

APPENDIX II

JACK PINE SITE CONFIRMATION PLOTS FOR EACH LANDFORM AND SITE INDEX CLASS USED TO INDEPENDENTLY TEST HEIGHT GROWTH AND SITE INDEX CURVES

PLOTS PER SITE INDEX CLASS						
LANDFORM	< 11.5m	11.5 - 14.4m	14.5 - 17.4m	17.5 - 20.4m	≥ 20.5m	TOTAL
GLACIALFLUVIAL SANDS	0	3	1	9	0	13
MORAINE	0	0	4	4	2	10
LACUSTRINE	0	1	1	3	0	5
BEDROCK *	1	0	3	0	0	4
TOTAL	1	4	9	16	2	32

* Soil is 1.0 m. or less above bedrock

APPENDIX III

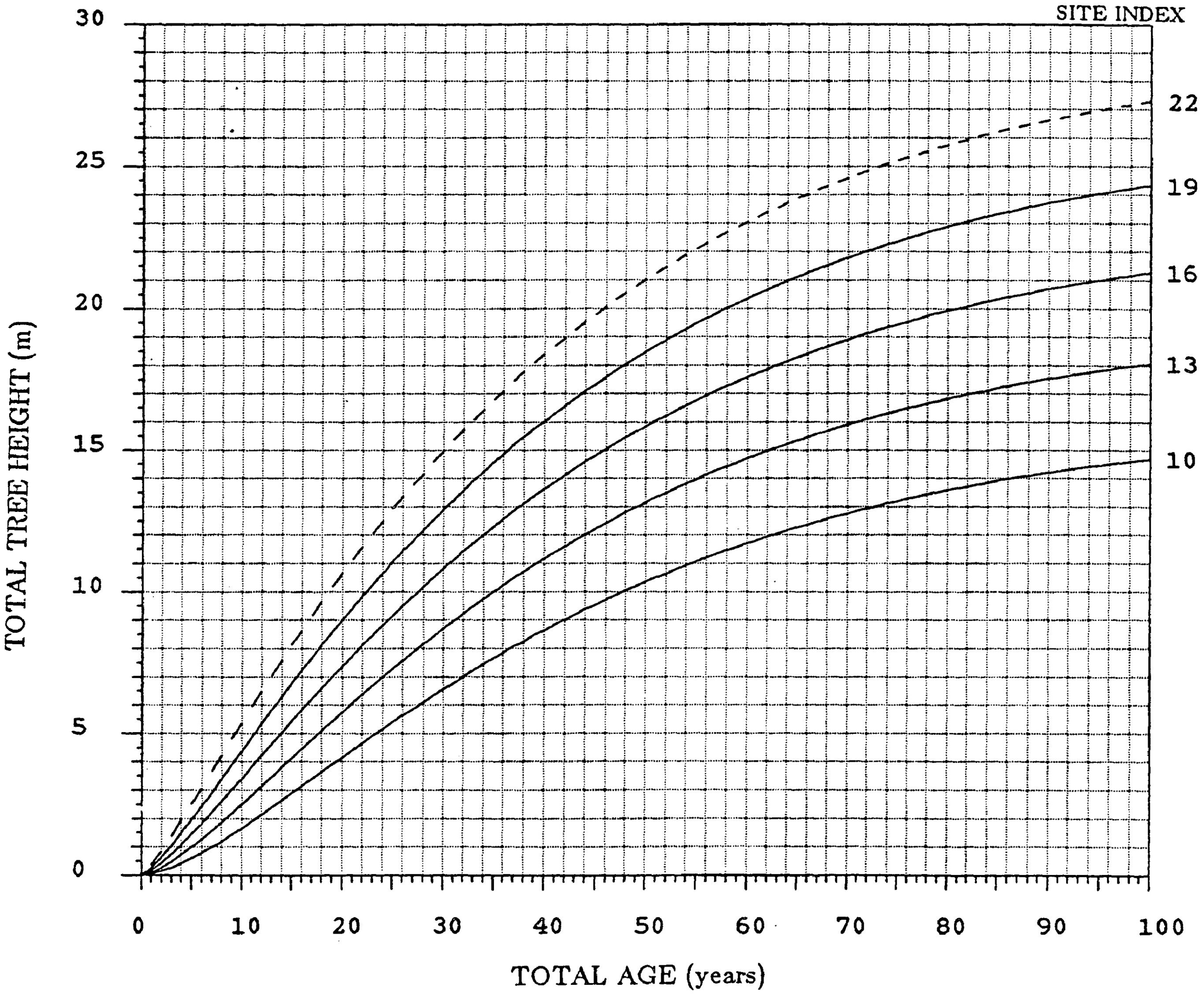
REGRESSION OF SITE INDEX ON HEIGHT FROM SITE INDEX CURVES FOR JACK PINE

$$SI = \beta_0 + \beta_1 HT + \epsilon.$$

BREAST HEIGHT AGE	β_0	β_1
10	2.6472	2.3981
15	2.4949	1.8235
20	2.1438	1.5288
25	1.7370	1.3515
30	1.3436	1.2327
35	0.9681	1.1486
40	0.6024	1.0874
45	0.2810	1.0401
50	0.0000	1.0000
55	-0.2882	0.9736
60	-0.5303	0.9497
65	-0.7501	0.9301
70	-0.9511	0.9220
75	-1.1311	0.9004
80	-1.2928	0.8890
85	-1.4444	0.8797
90	-1.5678	0.8716
95	-1.9614	0.8645
100	-1.7997	0.8586
105	-1.8864	0.8529
110	-1.9873	0.8488
115	-2.0483	0.8447
120	-2.1052	0.8409
125	-2.1719	0.8382
130	-2.2271	0.8356
135	-2.2758	0.8335
140	-2.3163	0.8314
145	-2.3480	0.8294
150	-2.3847	0.8280

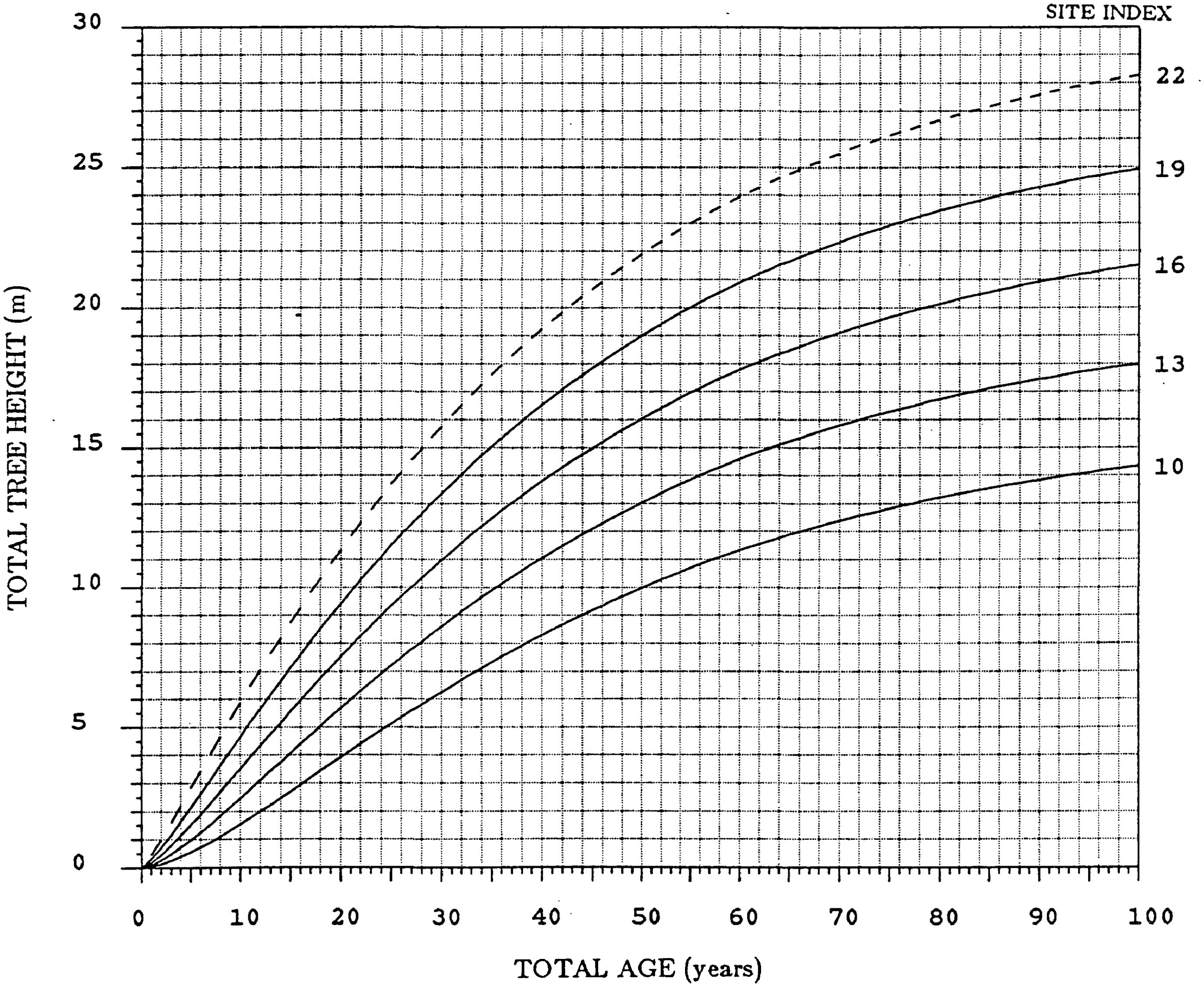
APPENDIX IV

HEIGHT GROWTH CURVES FOR JACK PINE
CALCULATED USING EK (1971) MODEL [eq. 3].



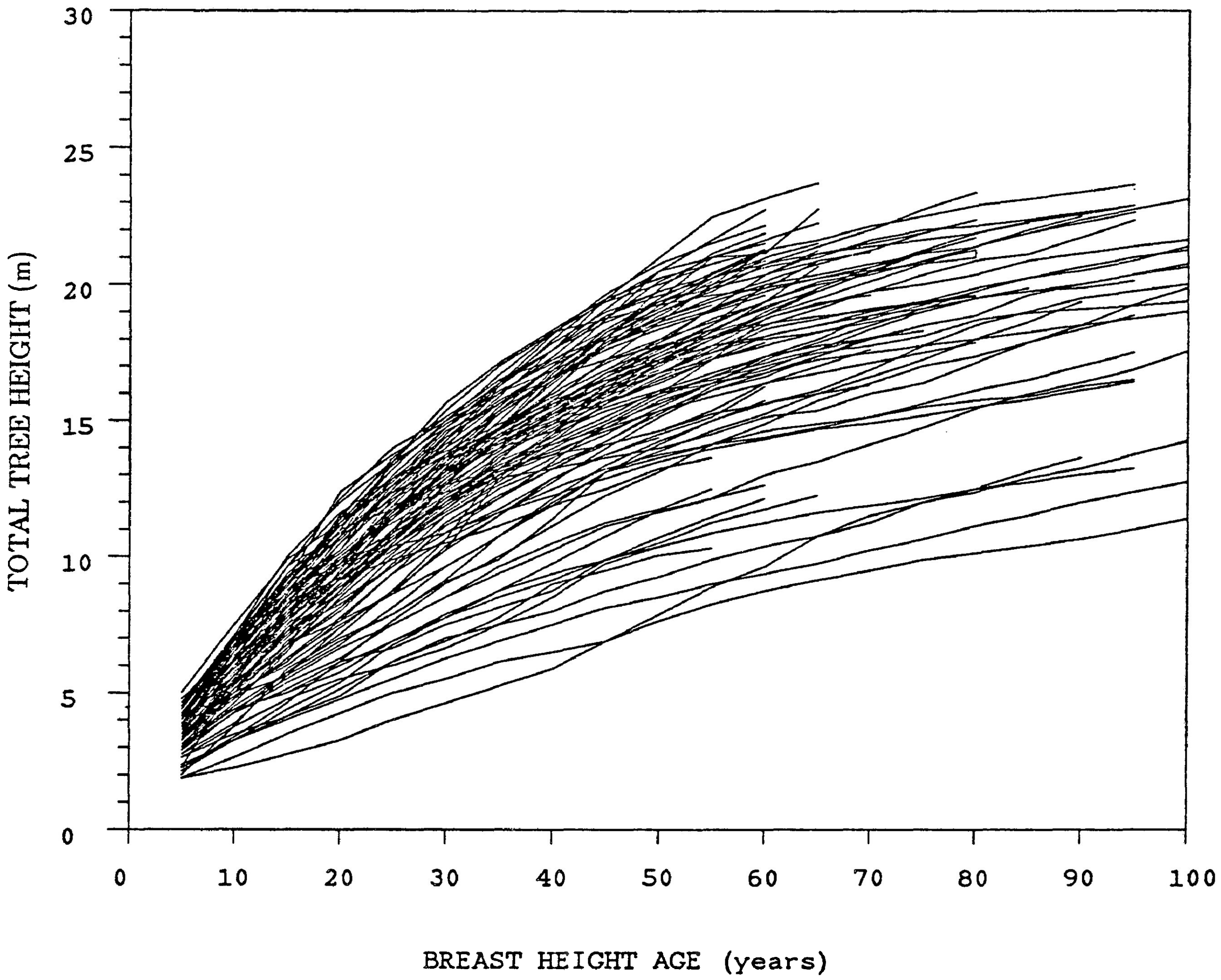
APPENDIX V

SITE INDEX CURVES FOR JACK PINE
CALCULATED USING EK (1971) MODEL [eq. 3].



APPENDIX VI

INDIVIDUAL PLOT HEIGHT GROWTH CURVES FOR JACK PINE SAMPLED IN THE THUNDER BAY AREA.



(Height growth curves were truncated at 100 years for display.)

APPENDIX VII

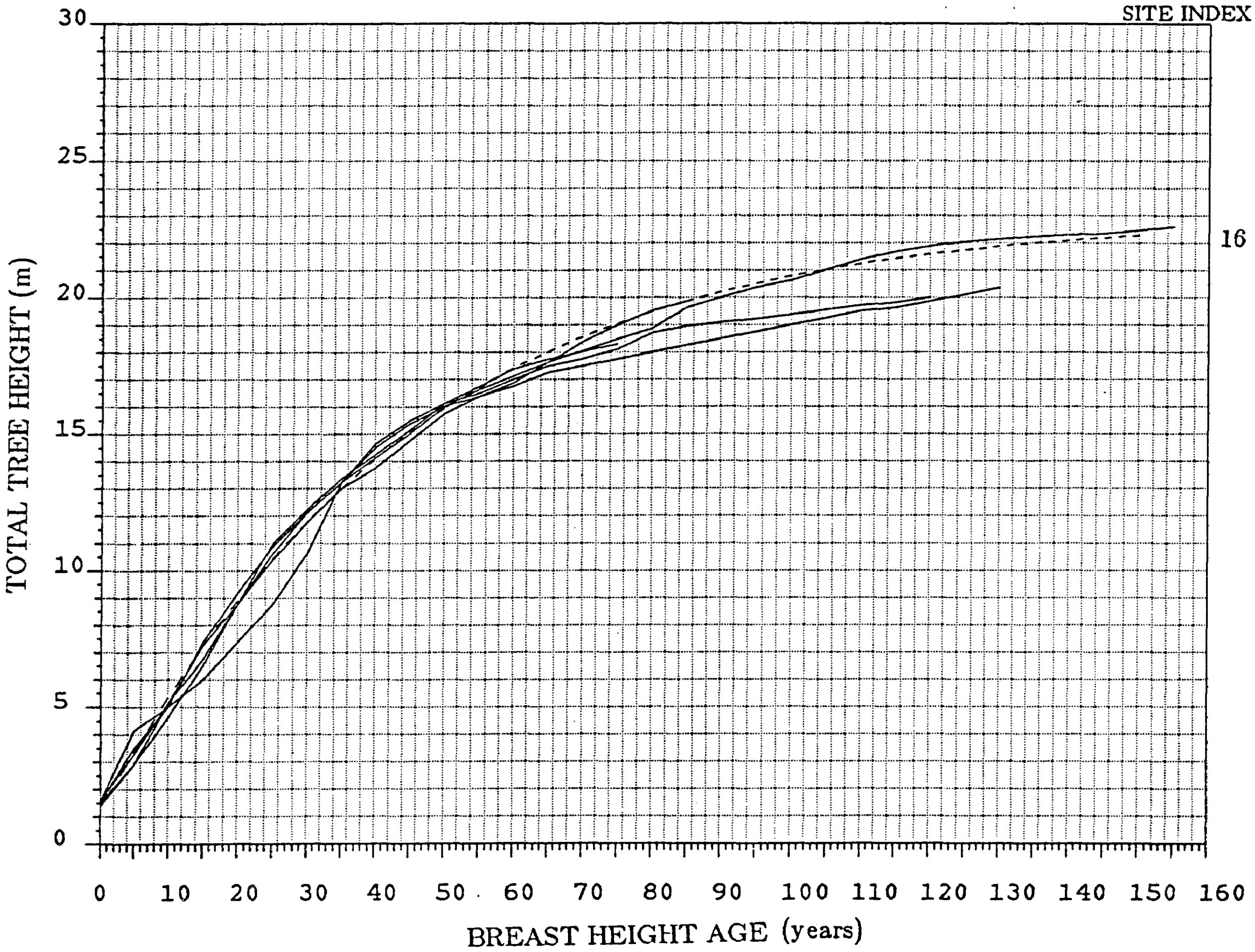
OBSERVED AND PREDICTED SITE INDICES FOR
32 JACK PINE SITE PLOTS INDEPENDENT OF
SITE INDEX CURVE CALCULATION.
SITE INDEX VALUES PREDICTED FROM SITE INDEX
CURVES BASED ON BREAST HEIGHT AGE.

PLOT	TOTAL HEIGHT(m)	BREAST HEIGHT AGE (YEARS)	OBSERVED SITE INDEX *	ESTIMATED SITE INDEX *	ERROR OF PREDICTION *
11	17.38	55	16.9	16.6	0.3
14	20.63	60	19.0	19.1	-0.1
19	21.38	70	19.4	18.6	0.8
20	21.25	85	17.1	17.2	-0.1
26	18.25	65	16.2	16.2	0.0
27	22.38	60	20.4	20.7	-0.3
33	19.25	60	18.0	17.8	0.2
35	18.13	85	15.1	14.5	0.6
38	25.38	105	19.1	19.8	-0.7
51	21.88	100	16.5	17.0	-0.5
59	20.88	60	18.9	19.3	-0.4
60	20.50	65	18.5	18.3	0.2
70	20.43	70	17.6	17.7	-0.1
73	22.59	80	18.8	18.8	0.0
74	25.18	75	21.6	21.5	0.1
80	18.50	105	13.6	13.9	-0.3
81	23.50	80	19.8	19.6	0.2
83	20.09	75	18.1	17.0	1.1
85	18.50	105	11.9	13.9	-2.0
89	20.19	70	17.8	17.5	0.3
90	24.19	65	22.4	22.3	0.1
95	24.38	80	19.4	20.4	-1.0
97	21.88	80	18.4	18.2	0.2
99	22.88	150	14.8	16.5	-1.7
100	21.38	150	13.0	15.3	-1.3
102	18.25	50	18.2	18.2	0.0
108	19.75	75	16.6	16.7	-0.1
115	18.00	55	17.1	17.2	-0.1
123	19.75	55	18.9	18.9	0.0
132	17.00	80	13.4	13.8	-0.4
141	22.38	120	17.0	16.7	0.3
154	11.63	80	8.6	9.0	-0.4

* Values in metres

APPENDIX VIII

JACK PINE HEIGHT GROWTH CURVES
FOR SITE INDEX 16 * COMPARED WITH
PREDICTED HEIGHT GROWTH CURVE FOR SITE INDEX 16
(DASHED LINE IS PREDICTED HEIGHT GROWTH CURVE)



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* Site index 16 = 15.75 - 16.25 m at 50 years breast height.