

# Global Sensitivity Analysis of a DSGE model of the Czech Economy

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**Abstract.** The paper shows some interesting results of Global Sensitivity Analysis on a particular dynamic stochastic general equilibrium model. The key behavior of the Czech economy is approximated by Lubik and Schorfheide model, which is a small-scale structural general equilibrium model of a small open economy. The sensitivity analysis class of methods presented in the paper consists of various individual analyses. Stability mapping analysis detects the parameters which are responsible for potential instability or indeterminacy in the model. Mapping the fit is a useful tool to learn about the linkages that drive the fit of trajectories of particular variables to data. Information provided by the results of mapping the fit can be used to unveil possible trade-offs between the fit of individual observables and maybe also to amend model structure or calibrate parameters properly in order to increase the fit of variables of researcher's interest. Other individual analyses are just mentioned.

## Introduction

This contribution introduces the reader to Global Sensitivity Analysis due to Saltelli et al. (2004 and 2008). The prototypical model for the exercise is the Lubik and Schorfheide (2003) model introduced in subsection 1.1. The same model is used by Ratto (2008a) with Canadian data set. This paper uses Czech data set and offers comparisons to Ratto's (2008a) paper.

The term "Global Sensitivity Analysis" is used in this contribution in the same sense that it is established and used in Saltelli et al. (2004 and 2008) or Ratto (2008a). The word "Global" means that the analysis is not "local", that is, it doesn't approximate solutions around one given point in space (like Taylor approximation does). Global methods use more points judiciously drawn from space and therefore overcome problems when the model is not linear.

A possible definition of sensitivity analysis is the following: "*The study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input*" (Saltelli et al. 2004). Sensitivity analysis is therefore not only the well-know ad-hoc exercise, when researcher arbitrarily changes inputs of the model and observes changes in output. It also encompasses methods/procedures that can somehow describe relations between inputs and output.

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Global Sensitivity Analysis is therefore an open group of individual analyses or methods, some of which are introduced in this paper (in subsections 2–4).

## 1 Preliminaries

### 1.1 The model

This paper presents some results of an analysis of a single model, which is a model of Lubik and Schorfheide (2003)<sup>1</sup>. It is a small-scale structural general equilibrium model of a small economy. This paper uses Czech data set so that model equations (1)–(8) describe elementary behavior of the Czech economy. Generally,  $\Delta$  denotes first difference so that e. g.  $\Delta\xi_t = \xi_t - \xi_{t-1}$ , star superscript (\*) relates to a foreign economy, subscript  $t$  denotes (relative) time and  $E_t$  denotes rational expectations made in time  $t$ . Also note that  $rr = -400 \cdot \log(\beta)$ .

$$y_t = E_t y_{t+1} - [\tau + \alpha(2 - \alpha)(1 - \tau)](R_t - E_t \pi_{t+1}) - \alpha[\tau + \alpha(2 - \alpha)(1 - \tau)]E_t \Delta q_{t+1} - \alpha(2 - \alpha) \frac{1 - \tau}{\tau} \Delta y_{t+1}^* - E_t z_{t+1} \quad (1)$$

$$\pi_t = \beta E_t \pi_{t+1} + \alpha \beta E_t \Delta y_{t+1} - \alpha \Delta q_t + \frac{k}{\tau + \alpha(2 - \alpha)(1 - \tau)} (y_t - \bar{y}_t) \quad (2)$$

$$\pi_t = \Delta e_t + (1 - \alpha) \Delta q_t + \pi_t^* \quad (3)$$

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\psi_1 \pi_t + \psi_2 (y_t - \bar{y}_t) + \psi_3 \Delta e_t) + e_{R,t} \quad (4)$$

$$\Delta q_t = \rho_q \Delta q_{t-1} + e_{q,t} \quad (5)$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + e_{y^*,t} \quad (6)$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + e_{\pi^*,t} \quad (7)$$

$$z_t = \rho_z z_{t-1} + e_{z,t} \quad (8)$$

Equation (1) is an open economy IS curve. If  $\alpha = 0$ , the equation becomes closed economy variant of IS equation. If  $\tau = 1$ , the world output shocks  $\Delta y_{t+1}^*$  drops out of IS equation and since it is not present in any other equation but AR1 process (6), it drops out of the system completely.

The open economy Phillips curve (2) also collapses to closed economy version if  $\alpha = 0$ . Consumer price index CPI is introduced in (3) with an assumption of a relative version of purchasing power parity.

Equation (4) is a monetary rule, or in another words, a nominal interest rate equation. It describes, how the monetary authority sets its instrument, when

<sup>1</sup> Cited from Ratto (2008a, p. 123); (hereafter LS model). The citation of the model equations is not literal, because sixth equation on page 123 of Ratto (2008a) is " $y_s = \rho_{y^*} y_{t-1} + e_{y^*,t}$ ", which is obviously an error. Also, in order to prevent confusion, an explanation to variables that have letter  $e$  in its notations follows here:  $e_t$  in equations (3) and (4) means nominal exchange rate, whereas  $e_{.,t}$  in equations (4)–(8) stands for exogenous shocks. Although the notation varies in the number of subscripts, the difference might not be obvious at first glance.

inflation or output depart from their targets or when the currency appreciates or depreciates.

Remaining model equations are just AR1 processes that describe the course of terms of trade, foreign output and inflation, and technological progress.

## 1.2 The data

The data span from the second quarter of 1996 to the fourth quarter of 2008. The source of all data is Czech Statistical Office and are per cent. There are five time series used, these are: output growth, inflation, interest rate, change in nominal exchange rate, and the change in terms of trade.

## 1.3 Used software

The main tool used in the analysis is the software package Dynare version 4.0.3. The analysis also requires Global Sensitivity Analysis (hereafter GSA) toolbox by Marco Ratto, which is – according to Dynare site – beginning to be added to Dynare version 4. This toolbox is downloadable from Euro-area Economy Modelling Centre web pages <http://eemc.jrc.ec.europa.eu/>. Documentation for these software packages is semifinished and is in Griffoli (2007) for Dynare version 4 and in Ratto (2008b) for GSA.

# 2 Stability mapping

## 2.1 Theory

Stability mapping helps to detect parameters  $X_i$  that are responsible for possible "bad behavior" of the model. First step of the computation is to define two subsets of a full domain: subset  $B$  produces behavior (= good behavior of the model), subset  $\bar{B}$  produces non-behavior (= bad behavior of the model).

$N$  Monte Carlo simulations are then run over the domain, which results in two subsets,  $(X_i|B)$  of size  $n$  and  $(X_i|\bar{B})$  of size  $\bar{n}$ , where  $n + \bar{n} = N$ . The two sub-samples may come from different probability density functions (PDFs)  $f_n(X_i|B)$  and  $f_{\bar{n}}(X_i|\bar{B})$ . Corresponding cumulative distribution functions (CDFs) are  $F_n(X_i|B)$  and  $F_{\bar{n}}(X_i|\bar{B})$ .

If  $F_n(X_i|B)$  and  $F_{\bar{n}}(X_i|\bar{B})$  differ for a given parameter  $X_i$ , the parameter may drive bad behavior of the model if its value falls within  $\bar{B}$  subset. The shape of  $F_{\bar{n}}(X_i|\bar{B})$  indicates, whether rather smaller or higher values of  $X_i$  drive the non-behavior. If the non-behavior CDF is to the left from behavior CDF, it indicates that rather smaller values of  $X_i$  are more likely to drive non-behavior. On the other hand, if the non-behavior CDF is to the right from the behavior CDF, it suggests that rather bigger values of  $X_i$  drive non-behavior.

In order to obtain also numerical results, a statistic that computes the greatest distance between behavior and non-behavior CDFs is computed. More formally, the (so-called) Smirnov  $d$  statistic is defined as

$$d_{n,\bar{n}}(X_i) = \sup ||F_n(X_i|B) - F_{\bar{n}}(X_i|\bar{B})||$$

The Smirnov  $d$  statistic has a domain  $[0, 1]$ , where 0 means that the two (behavior and non-behavior) CDFs perfectly overlap and 1 means that the two underlying subsets  $B$  and  $\bar{B}$  have no common elements. In other words,  $d = 1$  means that one of the CDFs reaches unity before the other increases from zero.

This analysis doesn't use data, so the results are just a matter of model relations (equations) and parameter calibration, not the data itself. The results are therefore the same as in Ratto (2008a).

### 3 Mapping the fit

#### 3.1 Theory

Since DSGE models consist of a number of observed variables, which should fit the data as well as possible, mapping the fit may be a useful tool to learn about the linkages that drive the fit of trajectories of particular variables to data. Information provided by the results of mapping the fit can be used to unveil possible trade-offs and maybe also amend model structure or calibrate parameters properly in order to increase the fit of variables of interest.

The procedure is carried out as follows:

1. Structural parameters are sampled from posterior distribution,
2. RMSE<sup>2</sup> of 1-step-ahead prediction is computed for each of observed series,
3. 10 % of lowest RMSE is defined as behavioral and  $B$  is defined as a subset of parameter values producing these behavioral results and
4. the calculations results in a number of distributions  $f_j(X_i|B)$  that represent the contribution of parameter  $X_i$  to best possible fit of  $j$ -th observed series.

Plotting the distributions (or better the CDFs) is one step further to trace possible trade-offs. A trade-off is present, when at least two distributions differ from posterior distribution (denoted in Figures as **base**) and differ from each other.

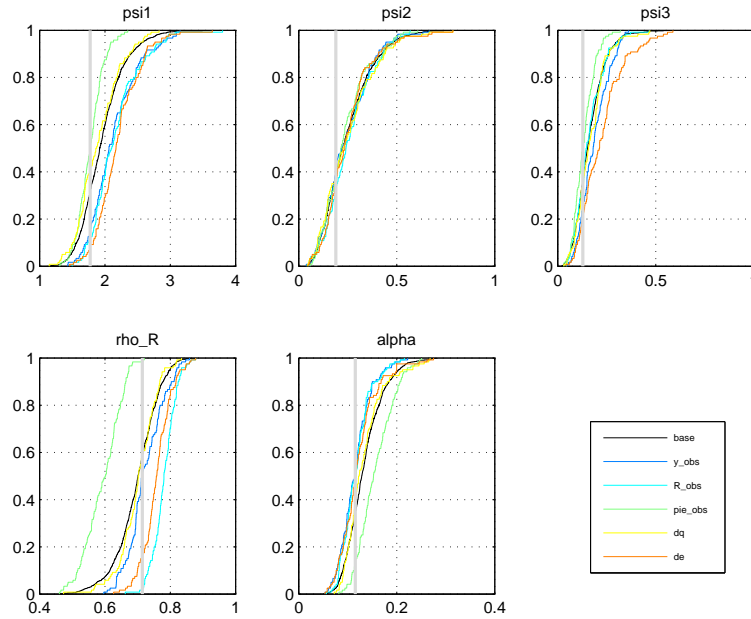
#### 3.2 Results for LS model

Ratto (2008a, p. 126) lists these parameters as the ones bearing biggest trade-offs:  $\psi_1, \psi_3, \rho_R, \alpha, k, \rho_q, \rho_{y^*}$ . This subsection compares and contrasts results obtained by Ratto and by this paper.

In Ratto (2008a), parameters  $\psi_1$  and  $\psi_3$  both represent similar trade-offs, albeit a bit smaller in volume in case of parameter  $\psi_3$ . Both parameters should be rather smaller in order to fit inflation  $\pi$  and rather larger in order to fit the change in nominal exchange rate  $\Delta e$ . Realization of the LS model on Czech data looks similarly – see figure 1, panel one and three.  $\psi_1$  and  $\psi_3$  should be smaller in order to fit inflation optimally and larger in order to fit the change in nominal exchange rate, as in Ratto's realization on Canadian data. The magnitude

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<sup>2</sup> root mean square error



**Fig. 1.** Cumulative posterior distributions (base) and the distributions of the filtered samples corresponding to the best fit for each singular observed series. Grey vertical lines denote posterior mode. (1 of 2)

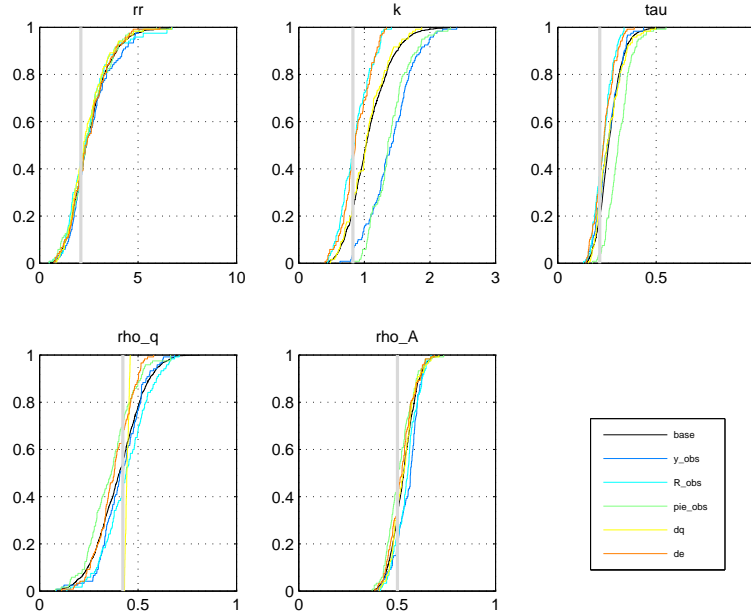
of these trade-offs is however visibly smaller. Also, larger value of  $\psi_1$  than its posterior distribution supports a better fit of output and interest rate.

Indications of trade-offs associated with parameter  $\rho_R$  are similar in Canadian and Czech realization of the LS model. Both realizations suggest that  $\rho_R$  should be lower in order to fit inflation better and higher in order to fit interest rate and the change in nominal exchange rate better. However, the magnitude is different. A deviation of the parameter from its posterior distribution is higher in the Czech model in case of interest rate and the change in nominal exchange rate: see Fig. 1, panel 4.

Parameter  $\alpha$  fits all variables rather well in both country realizations. Deviations are small and different in the two countries.

Parameter  $k$  is much more interesting. Ratto (2008a) states that all observed series have a preference for a larger value of  $k$ . The Czech model displays no preference for bigger  $k$  from all variables. Interest rate and the change in nominal exchange rate prefer lower  $k$  than posterior distribution. For details see Fig. 2, panel 2.

Parameter  $\rho_q$  has also interesting trade-offs.  $\Delta q$  alone would imply a given value of the parameter approximately 0.5, which is almost at posterior mode. Other variables fit data rather well with  $\rho_q$  at posterior distribution in both country realizations.



**Fig. 2.** Cumulative posterior distributions (base) and the distributions of the filtered samples corresponding to the best fit for each singular observed series. Grey vertical lines denote posterior mode. (2 of 2)

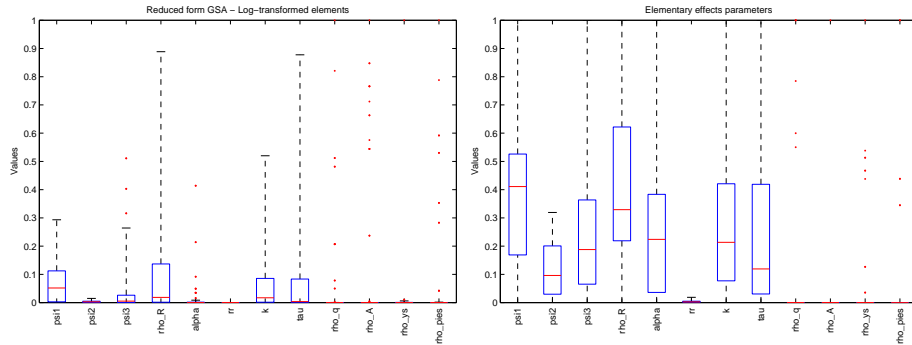
No other parameter causes major conflicts between fit of the variables and that holds for both Canadian and Czech data realization.

## 4 High dimensional model representation / Reduced form mapping

### 4.1 Theory

Due to the lack of space, the reader is referred e.g. to Ratto (2008a) for theoretical background.<sup>3</sup> We shall only define the most important measures here.

The first order sensitivity index can be defined as  $S_i = V_i/V$ , which is a scalar measure that shows the relative importance of structural parameter  $X_i$  on the variance of  $Y$  ( $V$  being the symbol for variance). A reduced form of a DSGE model can be written as  $y_t = Ty_{t-1} + Bu_t$  and generic output  $Y$  can be the entries of matrices  $T$  or  $B$ . These generic outputs  $Y$  are also called reduced form coefficients.



**Fig. 3.** Left panel: Boxplots of sensitivity indices. (All endogenous variables vs. all exogenous and all lagged endogenous with  $\log(-Y)$  transformation). Right panel: Elementary effects computed as a screening procedure with Morris sampling.

## 4.2 Results for LS model

According to the left panel of Fig. 3, the most influential parameters are  $\psi_1$ ,  $\rho_R$ ,  $k$  and  $\tau$ . The least influential parameters seem to be  $\psi_2$ ,  $rr$  and  $\rho_{y^*}$ . Parameter  $\psi_1$  has highest median, but upper quartile and upper whisker isn't much higher. Such characteristics could mean that  $\psi_1$  is important for many reduced form coefficients, but is rarely very important. Parameter  $\rho_R$  has highest upper quartile and upper whisker, but it has lower median than  $\psi_1$ . This backward-looking parameter of the monetary rule is therefore quite important for many reduced form coefficients and very important for some, too. On the other hand, lower median would suggest that the number of reduced form coefficients for which is  $\rho_R$  important is lower than in the case of  $\psi_1$ . Parameters  $k$  and  $\tau$  are even less important than the two just discussed. Both have lower values of median and upper quartile. Upper whisker is somewhat lower, too.

Both parameters  $\psi_2$  and  $rr$  have all sensitivity indices virtually zero, which should suggest that these parameters are unimportant for all possible reduced form coefficients. Boxplots of  $\alpha$ ,  $\rho_q$ ,  $\rho_A$ ,  $\rho_{y^*}$  and  $\rho_{\pi^*}$  represent rather peculiar results. All of these parameters have median and upper quartile virtually zero, but have some high outliers. Such results mean that these parameters are unimportant for most reduced form coefficients, but have important (in case of  $\alpha$ ) or very important (in case of the remaining four parameters) influence on some reduced form parameters.

Right panel of Fig. 3 shows similar results, but computed as a screening procedure with the so-called Morris sampling.<sup>4</sup> This screening can be computed as a preliminary check of importance of parameters. Its main advantage is that it takes about 200 times less time to compute in comparison with the full results (left panel of Fig. 3). Its understandable drawback is that the results are

<sup>3</sup> For primary literature, see e.g. Li et al. (2002 and 2006) and Sobol' (1993).

<sup>4</sup> See Morris (1991).

only approximate. The patterns (most influential parameters, noninfluential parameters) are similar in both panels, but the actual values of sensitivity indices/elementary effects differ. These details are not discussed here due to the lack of space.

## Conclusion

Global sensitivity analysis helps to better understand linkages that drive the behavior of a DSGE model. Various individual tools of GSA are used to illuminate dependencies in separate parts of DSGE models and together form a unified picture. This paper presented mainly Mapping the fit, which helps to detect trade-offs among parameter values in order to attain the best possible fit of an observable variable. Less space is devoted to the distribution of sensitivity indices, which can be used as a measure of the importance of individual parameters.

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