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Gela Kipiani [1], **Nika Botchorishvili** [2]

ANALYSIS OF LAMELLAR STRUCTURES WITH APPLICATION OF GENERALIZED FUNCTIONS

Abstract

Theory of differential equations in respect of the functional area is based on the basic concepts on generalized functions and splines. There are some basic concepts related to the theory of generalized functions and their properties are considered in relation to the rod systems and lamellar structures. The application of generalized functions gives the possibility to effectively calculate step–variable stiffness lamellar structures. There are also widely applied structures, in that several in which a number of parallel load bearing layers are interconnected by discrete-elastic links. For analysis of system under study, such as design diagrams, there are applied discrete and discrete-continual models.

Keywords

Plate, stability, generalized functions, spline-function, stiffness.

1 INTRODUCTION

Thin-walled lamellar shell structures are applied in various fields of engineering. Implementation in structural mechanics of modern large-span structures of having high strength characteristics low-modulus materials leads to necessity to taking into account of large deformations in comparison with thickness at analysis of thin-walled structures.

Improvements of strength of lamellar structures most naturally are carried out by arrangement of ribs. At acting along the walls with apertures of local loadings is advisable reinforcement of spatial structures by proper length ribs. The arrangements and type of attachments of these ribs make impact on behavior of structure at deformation.

Irregularity of geometrical and physical parameters of thin-walled structures causes the significant stress concentration and makes dangerous areas for propagation of cracks and plastic deformations. In most cases, their load bearing capability is determined due strength conditions or buckling in stress concentration areas.

In places of regularity break stress concentration zones makes essential influence on load bearing capability and stability of thin-walled structures. At this, known traditional analysis and numerical methods are less effective. Thus is necessary development of new effective methods for analysis of mentioned class of structures.

Currently theory of generalized, in particular, discontinuity impulse functions significantly extend possibilities of analysis of various having regularity breaks lamellar structures.

In structural mechanics the sandwich systems would be considered by having regularity breaks systems by thickness. The sandwich plate with lightweight filler and two external load bearing layer would be widely applied in structural mechanics as typical element, in that would be various structural singularities as additional links, breaks and so on.

[1] Gela Kipiani, Professor, Deputy Rector of Georgian Aviation University, 16, K. Tsamebuli str., Tbilisi, 0144, Georgia. e-mail: gelakip@gmail.com

[2] Nika Botchorishvili, Master of Georgian Aviation University, 16, K. Tsamebuli str., Tbilisi, 0144, Georgia. e-mail: zodeli@uahoo.com

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In addition by application of generalized functions analysis of including single layered as well as in composite materials lamellar structures is rather complex, topical and requires the development of special methods of analysis.

2 BASIC CONCEPTS OF THEORY OF GENERALIZED FUNCTIONS AND SPLINES

Theory of solution of inhomogeneous linear differential equations with constant coefficients and systems of similar equations containing discontinuous functions, was laid due the works of Heaviside and Dirac, and then substantially revised and developed by S.l. Sobolev [1].

The theory of differential equations applied to functional spaces is based on the fundamental concepts of generalized functions (distribution functions) and splines. Further are stated some of the basic concepts of the theory of generalized functions and are considered their properties in relation to the mechanics of rod systems [2-13].

To describe the continuously distributed values are used ordinary functions. As ordinary continuous function refers to such correspondence $f(x)$, in which each element x of the set E correspond an element y in the set F. At this the set E is called as the initial set, and the set F - as finite set of mapping. An element x is the independent value (argument) and the element $y = f(x)$ is the dependent value (function).

In some cases, instead of the function is applied the term of operator. Ordinary functions can be added and multiplied by real numbers, so they form a real linear space (linear mapping).

To overcome the mathematical difficulties in solving of problems containing concentrated inclusions (concentrated loads, distributed loads with discontinuities of the first kind, point masses, etc.), a class of ordinary functions is expanded through the application of discontinuous functions.

In mechanics form discontinuous functions are widespread unit Heaviside function $H(x - x_o)$H and delta function $\delta(x - x_o)$.

Determination of the delta function follows from the properties of the pulse function, which refers to continuous or piecewise continuous function s $\delta(x, \xi)$ of the argument x, depending on the parameter ξ, if they satisfy the conditions [7]:

$$1)\ \delta(x,\xi) = 0, |x| > \xi;\ 2)\ \delta(x,\xi) \geq 0, |x| \leq \xi;$$

$$3)\ \int_{-\infty}^{\infty} \delta(x,\xi)dx = \int_{-\xi}^{\xi} \delta(x,\xi)dx = 1.$$

Should be noted that

$$\lim_{\xi\to 0} \delta(x,\xi) = 0,$$

as for $x \neq 0$ $\delta(x,\xi) = 0$, if $|x| < \xi$.

The average height of the function $\delta(x,\xi)$ on the interval [-ξ, ξ] increases indefinitely, thus

$$\lim_{\xi\to 0} \frac{1}{2\xi} \int_{-\xi}^{\xi} \delta(x,\xi)dx = \lim_{\xi\to 0} \frac{1}{2\xi} \cdot 1 = \infty.$$

Let's consider the behavior of integral

$$\int_a^b f(x)\delta(x,\xi)dx$$

when ξ→0, if $f(x)$ – is the ordinary, continuous on [a, b]; $\delta(x,\xi)$ – is the impulse function.

Two cases are possible:

1. The interval [a, b] contains a point $x = 0$, i.e., $a < 0 < b$ and $\xi \leq min(|a|, b)$. From definition of an impulse function and generalized mean value theorem for definite integral it follows that

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$$\int_a^b f(x)\delta(x,\xi)dx = \int_{-1}^{1} f(x)\delta(x,\xi)dx = f(x_0)\int_{-\xi}^{\xi} \delta(x,\xi)dx = f(x_0),$$

where $x_0 = [-\xi, \xi]$.

If $\xi \to 0$, To $x_0 \to 0$, and due the continuality $f(x)$, $f(x_0) \to f(0)$. Thus at $0 \in (a,b)$

$$\lim_{\xi\to 0}\int_a^b f(x)\delta(x,\xi)dx = f(0).$$

2. The interval (a, b) does not contain the point x = 0. In this case it is obvious that

$$\lim_{\xi\to 0}\int_a^b f(x)\delta(x,\xi)dx = 0.$$

Is introduced the notation

$$\lim_{\xi\to 0}\int_a^b f(x)\delta(x,\xi)dx = \int_a^b f(x)\delta(x)dx\,,(1) \tag{1}$$

where the symbol $\delta(x)$ is the delta function. It characterize the limit behavior of the impulse function $\delta(x,\xi)$ at $\xi \to 0$, and the integral

$$\int_a^b f(x)\delta(x)dx$$

it should be understood only in the sense of equality (1), where firstly is necessary to calculate the integral

$$\int_a^b f(x)\delta(x)dx,$$

and then carry out the limiting passage at ξ → 0. At application of delta function is reduced the limiting passage operation, then from the above mentioned two cases, it follows that

$$\int_a^b f(x)\delta(x)dx \begin{cases} f(0), if\ 0 \in (a,b); \\ 0, if\ 0 \overline{\in} (a,b). \end{cases} \ (2) \tag{2}$$

Similarly is introduced the delta function with displacement to the point x_0:

$$\int_a^b f(x)\delta(x-x_0)dx = \begin{cases} f(0), if\ 0 \in (a,b); \\ 0, if\ 0 \overline{\in} (a,b). \end{cases} \ (3) \tag{3}$$

The above stated formulae (2), (3) illustrate the filtering properties of the delta function.

If $f(x) = 1$ we have

$$\int_a^b \delta(x)dx = \begin{cases} 1, if\ 0 \in (a,b); \\ 0, if\ 0 \overline{\in} (a,b). \end{cases}$$

$$\int_a^b \delta(x-x_0)dx = \begin{cases} 1, if\ x_0 \in (a,b); \\ 0, if\ x_0 \overline{\in} (a,b). \end{cases}$$

The right-hand parts of these equations are defined as unit Heaviside functions:

$$H(x) = \begin{cases} 1, if\ x \geq 0; \\ 0, if\ \mathrm{x} < 0. \end{cases}$$

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$$H(x-x_0)=\begin{cases}1, if\ x \geq x_0;\\ 0, if\ \mathrm{x} < x_0.\end{cases}$$

The relationship between the delta function and Heaviside unit function is expressed by the following ratios:

$$\int_{-\xi}^{x} \delta(x)dx = H(x); \quad \int_{-\xi}^{x} \delta(x-x_0)dx = H(x-x_0);$$

$$\frac{dH(x)}{dx} = \delta(x); \qquad \frac{dH(x-x_0)}{dx} = \delta(x-x_0).$$

At this are valid equalities

$$\delta(x) = \begin{cases}\infty, x = 0;\\ 0, x \neq 0,\end{cases} \quad \delta(x-x_0) = \begin{cases}\infty, x = x_0;\\ 0, x \neq x_0.\end{cases}$$

Graphic interpretation of a unit function, and δ-function is given in Fig. 1.

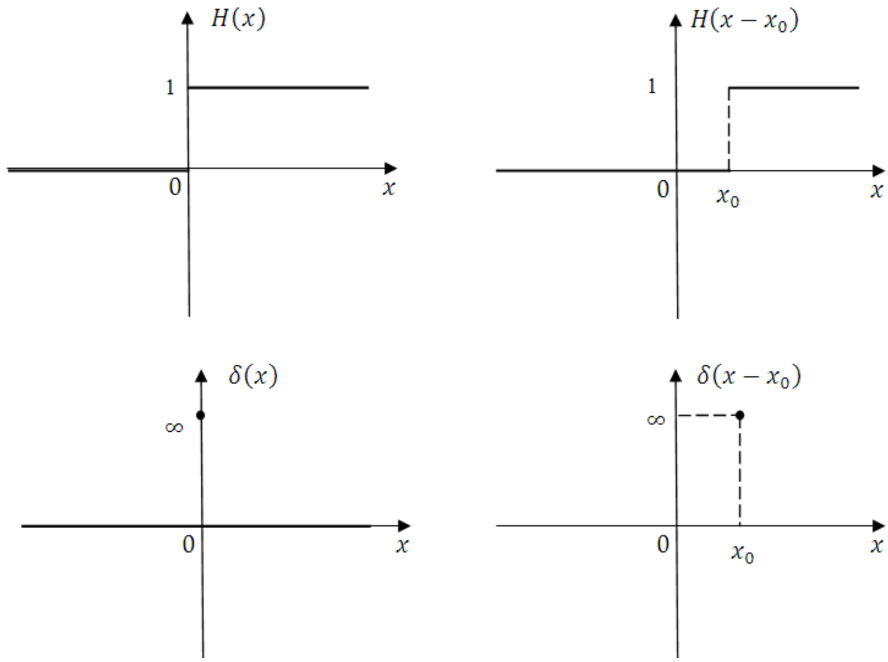

Fig. 1. Graphic interpretation of a single function, and δ-function

Then are stated the basic properties of a unit function, δ-function and its derivatives.

2.1 Spline Function

As spline functions commonly are called piecewise-polynomial functions having certain smoothness.

With regard to the theory of distributions as a spline function we will understand the function, composed from portions of various analytical functions that have derivatives up to the (n-1) order inclusive. Thus, the splines would include arbitrary continuous functions.

If you take the definite integral with variable upper limit of the unit Heaviside function, we obtain a simplest linear spline

$$s_1 = \int_{-\infty}^{x} H(x-a)dx = \int_{a}^{x} dx = (x-a) \text{ при } x \geq a, \tag{4}$$

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$$s_1 = (x-a)_{\lrcorner} = \begin{cases} 0, x < a; \\ x-a, x \geq x, \end{cases}$$

where the ⌋ symbol indicates the spline function.

2.2 Calculation of the plates on the action of local loads

In the examples of calculation obviously is demonstrated the efficiency of application of discontinuous functions for presentation of initial ratios, their algorithmic implementations and programming.

Even more effective is the application of discontinuous functions at consideration of two-dimensional problems. The group of tasks includes the problem of bending plates under concentrated loads.

Local loads acting on the plates, are causing mode of deformations, different from the beams. To research it in adjacent to the local loads, let's firstly consider the bending of circular plates, and then consider the rectangular plates.

2.3 The circular plate loaded symmetrically on the circumference

Let's consider a plate of radius a, in which the load is uniformly distributed over the circumference of radius b (Fig. 2).

In this particular case of symmetrical load bending is described by ordinary differential equation [11]

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{q}{D},$$

where r – is the radial coordinate;

w – is the desired deflection function;

D – is the cylindrical stiffness of the plate;

q – is the intensity of load.

In accordance with Chapter 1, the load would be represented as

$$q = \frac{P\delta(r-b)}{2\pi b}, \qquad (5)$$

where P – is the applied load.

Due consistently integrating (4) with taking into account (5) we have (3)

$$w = C_1 + C_2 r^2 + C_3 r^3 + C_4 t^4 + \frac{P}{8\pi D}\left[(b^2 + r^2)\ln\left(\frac{b}{r}\right) + (b^2 - r^2)\right] H(r-b),$$

where C_1 – are the integration constants determined from boundary conditions.

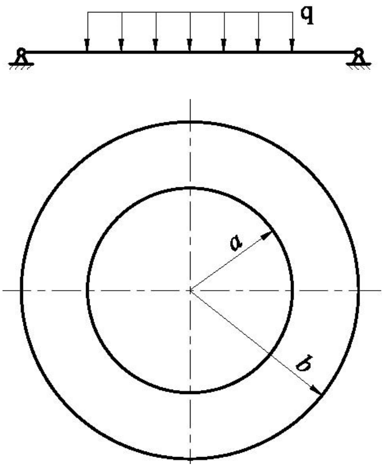

Fig. 2. Circular plate symmetrically loaded on circumference

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As the deflection at the center of the plate, and angle of rotation have limited value, is assumed $C_3 = C_4 = 0$ (as at $r \to 0$, $\ln r \to -\infty$).

The constants C_1, C_2 are determined depending on the conditions on the external contour. For example, at hinged supporting on the contour when at $r=a$, $w = 0$ and

$$M_r = -D\left(\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) = 0$$

$$C_2 = -\frac{P}{8\pi D}\left[\ln\left(\frac{b}{a}\right) + \frac{v-1}{2(v+1)}\right], \tag{6}$$

$$C_1 = -\frac{P}{8\pi D}\left[b^2 \ln\left(\frac{b}{a}\right) + \frac{(v+3)a^2 - (1-v)b^2}{2(v+1)}\right].$$

The bending of rectangular plate under the action of a linearly distributed load.

Let's consider rectangular plate under a normal load (Fig. 3). The equation of bending according to [11] will be as

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}. \tag{7}$$

If the load intensity $d(y)$ is distributed on line , accordingly of stated in Chapter 1

$$q(x, y) = q(y)\delta(x - x_1). \tag{8}$$

If on the edges of $y = 0$, $y = b$ occurs wivel supporting, i.e. $w = w'' = 0$ at $y = 0$, $y = b$, the desired function of load would be represented as a series

$$w = \sum_{k=1}^{\infty} w_k(x)\sin(\beta_k y);$$
$$q(y) = \sum_{k=1}^{\infty} q_k(x)\sin(\beta_k y), \tag{9}$$

where $\beta_k = \frac{k\pi}{b}$; $q_k = \frac{2}{b}\int_0^b q(y)\sin(\beta_k y)\,dy$.

Then the problem is reduced to the solution of independent ordinary differential equations as

$$\left(\frac{d^4}{dx^4} + 2\beta_k^2 + \beta_k^4\right) w_k = \frac{q_k}{D}\delta(x - x_1). \tag{10}$$

At a constant load $q_k = \dfrac{4q}{\beta_k b}$.

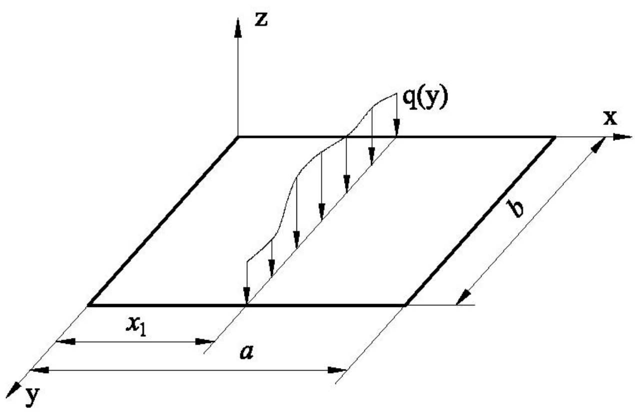

Fig. 3. Rectangular plate under action of normal load

As it is known from the theory of linear differential equations, the solution of equation (10) is a function of

$$w_k = C_{1k}e^{z_{1k}x} + C_{2k}e^{z_{2k}x} + C_{3k}e^{z_{3k}x} + C_{4k}e^{z_{4k}x} + w_k^*, \tag{11}$$

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where C_{1k} – are the integration constants determined from boundary conditions;

w_k^* – is a particular solution of equation (11);

z_{ik} – are the roots of the characteristic equation

$$z_k^4 - 2\beta_k z_k^2 + \beta_k^4 = 0. \tag{12}$$

Solving the equation (14), we obtain

$$z_{1k} = z_{3k} = \beta_k; z_{2k} = z_{4k} = -\beta_k. \tag{13}$$

As there are multiple roots, the solution (13) is represented by the formula

$$w_k = C_{1k}e^{\beta_k x} + C_{2k}e^{-\beta_k x} + C_{3k}e^{\beta_k x} + C_{4k}e^{-\beta_k x} + w_k^*, \tag{14}$$

or with the introduction of the hyperbolic functions

$$w^k = C'_{1k}ch(\beta_k x) + C'_{2k}sh(\beta_k x) + C'_{3k}ch(\beta_k x) + C'_{4k}sh(\beta_k x) + w_k^*. \tag{15}$$

To find a particular solution w_k^* let's use the method of integration constants variation.
As result we have

$$w_k = C'_{1k}ch(\beta_k x) + C'_{2k}sh(\beta_k x) + C'_{3k}xch(\beta_k x) + C'_{4k}xsh(\beta_k x) +$$
$$+\frac{q_k}{2D\beta^3}\left[\beta_k(x-x_1)ch\big(\beta_k(x-x_1)\big) - sh\big(\beta_k(x-x_1)\big)\right]H(x-x_1). \tag{16}$$

The integration constants C_{jk}(j=1, 2, 3, 4) are determined from the boundary conditions at the edges of x = 0 and x = a. For example, at simple support at x = 0 and x = a, $w_k = 0$ and $\frac{d^2 w_k}{dx^2} = 0$ that corresponds to the equality to zero of displacements and bending moments on the contours of the plate.

Under these conditions, and at $x_1 = \frac{a}{2}$

$$C'_{1k} = C'_{4k} = 0;$$
$$C'_{2k} = \frac{q_k}{D}\left(b_k a sh\left(\frac{\beta_k a}{2}\right)\Big/2 + ch\left(\frac{\beta_k a}{2}\right)\Big/\left(4\beta_k^3 ch\left(\frac{\beta_k a}{2}\right)\right)\right); \tag{17}$$
$$C'_{3k} = \frac{q_k}{D\left(4\beta_k^2 ch\left(\frac{\beta_k a}{2}\right)\right)}.$$

If one member of the series represents the load q, then the formula (16) gives an exact solution of the equation. In this case, for the moments and shear forces we obtain the following formulae

$$M_{xk} = D(w_k'' - \mu\beta_k^2 w_k) = D\{C_{2k}(1-\mu)\beta_k^2 sh(\beta b_k x) +$$
$$+C_{3k}\beta_k[2sh(\beta_k x) + (1-\mu)xb_k ch(\beta_k x)]\} +$$
$$+\frac{q_k}{2b_k}[(1-\mu)\beta_k(x-x_1)ch\big(\beta_k(x-x_1)\big) +$$
$$+(1+\mu)sh\big(\beta_k(x-x_1)\big)]H(x-x_1);$$
$$M_{yk} = D(w_k''\mu - \beta_k^2 w_k) = D\{C_{2k}(\mu-1)\beta_k^2 sh(\beta b_k x) +$$
$$+C_{3k}\beta_k[2\mu sh(\beta_k x) + (\mu-1)xb_k ch(\beta_k x)]\} +$$
$$+\frac{q_k}{2b_k}[(\mu-1)\beta_k(x-x_1)ch\big(\beta_k(x-x_1)\big) +$$
$$+(1+\mu)sh\big(\beta_k(x-x_1)\big)]H(x-x_1);$$
$$Q_1 = DC_3 2\beta_k^2 ch(\beta_k x) + q_k ch\big(\beta_k(x-x_1)\big)H(x-x_1);$$
$$Q_1 = DC_3 2\beta_k^2 ch(\beta_k x) + q_k sh\big(\beta_k(x-x_1)\big)H(x-x_1).$$

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$$
\begin{aligned}
M_{xk} &= D(w_k'' - \mu\beta_k^2 w_k) = D\{C_{2k}(1-\mu)\beta_k^2 sh(\beta b_k x) + \\
&+C_{3k}\beta_k[2sh(\beta_k x) + (1-\mu)x b_k ch(\beta_k x)]\} + \\
&+\tfrac{q_k}{2b_k}[(1-\mu)\beta_k(x-x_1)ch(\beta_k(x-x_1)) + \\
&+(1+\mu)sh(\beta_k(x-x_1))]H(x-x_1); \\
M_{yk} &= D(w_k''\mu - \beta_k^2 w_k) = D\{C_{2k}(\mu-1)\beta_k^2 sh(\beta b_k x) + \\
&+C_{3k}\beta_k[2\mu sh(\beta_k x) + (\mu-1)x b_k ch(\beta_k x)]\} + \\
&+\tfrac{q_k}{2b_k}[(\mu-1)\beta_k(x-x_1)ch(\beta_k(x-x_1)) + \\
&+(1+\mu)sh(\beta_k(x-x_1))]H(x-x_1); \\
Q_1 &= DC_3 2\beta_k^2 ch(\beta_k x) + q_k ch(\beta_k(x-x_1))H(x-x_1); \\
Q_1 &= DC_3 2\beta_k^2 ch(\beta_k x) + q_k sh(\beta_k(x-x_1))H(x-x_1).
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
M_{xk} &= D(w_k'' - \mu\beta_k^2 w_k) = D\{C_{2k}(1-\mu)\beta_k^2 sh(\beta b_k x) + \\
&+C_{3k}\beta_k[2sh(\beta_k x) + (1-\mu)x b_k ch(\beta_k x)]\} + \\
&+\tfrac{q_k}{2b_k}[(1-\mu)\beta_k(x-x_1)ch(\beta_k(x-x_1)) + \\
&+(1+\mu)sh(\beta_k(x-x_1))]H(x-x_1); \\
M_{yk} &= D(w_k''\mu - \beta_k^2 w_k) = D\{C_{2k}(\mu-1)\beta_k^2 sh(\beta b_k x) + \\
&+C_{3k}\beta_k[2\mu sh(\beta_k x) + (\mu-1)x b_k ch(\beta_k x)]\} + \\
&+\tfrac{q_k}{2b_k}[(\mu-1)\beta_k(x-x_1)ch(\beta_k(x-x_1)) + \\
&+(1+\mu)sh(\beta_k(x-x_1))]H(x-x_1); \\
Q_1 &= DC_3 2\beta_k^2 ch(\beta_k x) + q_k ch(\beta_k(x-x_1))H(x-x_1); \\
Q_1 &= DC_3 2\beta_k^2 ch(\beta_k x) + q_k sh(\beta_k(x-x_1))H(x-x_1).
\end{aligned}
$$

Charts of function w_k and its first derivative are smooth, the chart of its second derivative has a break, and the third - a jump that corresponds to the character of distribution of moment and shearing force. Due to the presence of discontinuous functions in the formulas (25) components of mode of deformation are defined in the zone of discontinuity with the same precision as in the area of soft load changes.

Charts of deflections ω_1, angles of rotation ω_1', moments $\omega_1'' - \mu\beta_k^2\omega_1$ and shear forces $\omega_1''' - \beta_1\omega'$, showing the distribution pattern of discontinuous functions ω_1 and its derivatives, are constructed on Fig. 4.

These charts are at $x = x_1$ have breaks and jumps, typical for moments and forces.

By similar transformations is obtained solution at a load distributed along the line $y-y_1$. In this case, all designed formulas are obtained from (10), (18) by replacing the $\beta\to\alpha$, $x\to y$, $y\to x$, $x_1\to y_1$.

With simultaneous application of loads concentrated on the lines x = x1 and y = y1, the total solution is obtained by the superposition of both solutions.

In the case of randomly distributed along the line load $x=x_1$, for each term of a series w_k and q_k (11) are valid all stated transformations.

If on the line $x=x_1$ is applied the moment load M, the equation (10) takes the form

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{M}{D}\delta'(x-x_1). \tag{19}$$

where $\delta(x-x_1)$ –is the derivative of the delta function.

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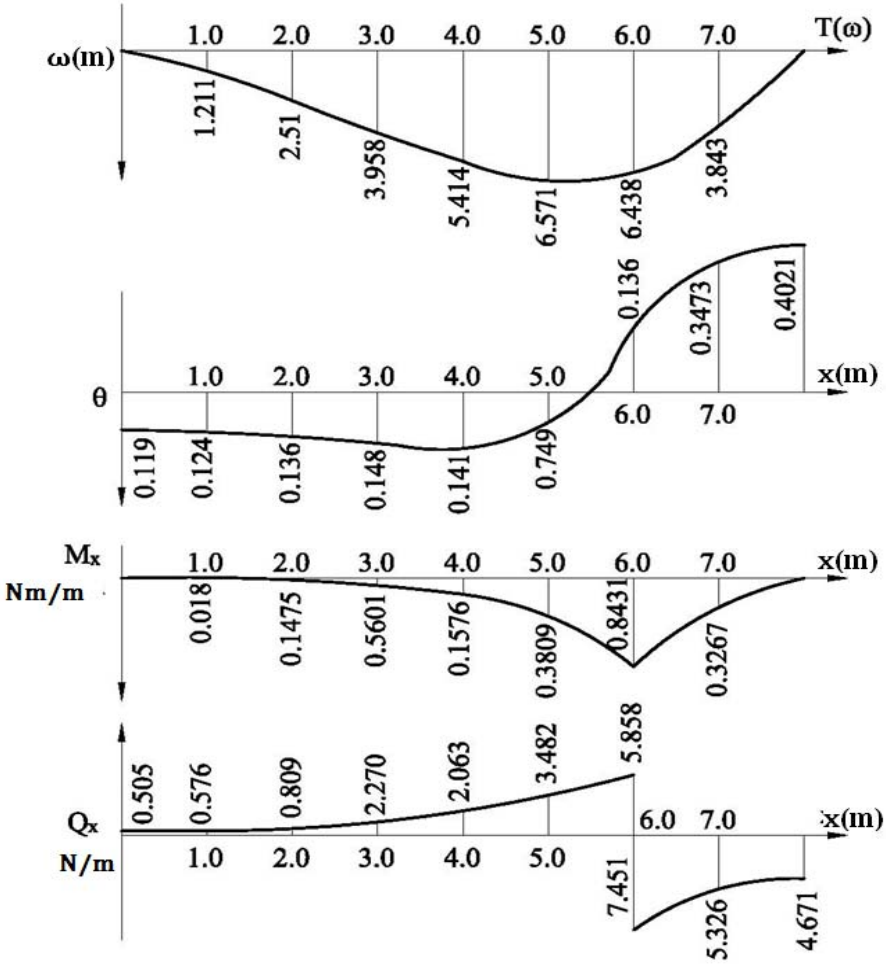

Fig. 4. Distributed discontinuous functions ω_1 and their derivatives

In this case, representing the desired function and the function of load in series similar to (11), we obtain the solution as (15), where a particular solution w^* is expressed by the formula

$$w_k^* = \frac{M}{2D\beta_k}\int_0^x \left[(x-\eta)sh\big(\beta_k(x-\eta)\big)\right]\delta(\eta - x_1)d\eta =$$

$$= \frac{M}{2D\beta_k}(x-x_1)sh\big(\beta_k(x-x_1)\big)H(x-x_1). \tag{20}$$

Due differentiating (15) with taking into account of (20), we obtain formulas for the forces and moments. Charts of moments constructed by these formulas, have jumps, diagrams of shear forces represents smooth curves.

Based on the decisive equation for the sandwich plate with a weak shear stiffness

$$2B\left(h+\frac{t}{2}\right)^2\nabla^4 w + \left(1-\frac{Bh}{G_3}\nabla^2\right)(2D\nabla^2 w + P) = 0, \tag{21}$$

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then, due comparing it with a biharmonic equation for the single-layer plate and presenting as a double trigonometric series function, is possible to obtain the value of the reduced cylindrical stiffness:

$$D_{cr} = \frac{(mn)^2\left[1+\frac{Bh}{G_3}(mn)^2\right]}{2B\left(h+\frac{t}{2}\right)^2(mn)^2+\left[1+\frac{Bh}{G_3}(mn)^2\right]2D(mn)^2}, \quad (22)$$

where

$$(mn) = \left(\frac{mn}{a}\right)^2 + \left(\frac{mn}{b}\right)^2;$$

where *a, b* – are the dimensions of the rectangular plate;

m=1, 3, 5, ...;

n=1, 3, 5, ... – are the an odd positive integers of natural sequence.

For taking into account the single cut parallel to, for example, of axis *y*, let's represented accordingly of [2] the angle of rotation of the tangent to the deformed surface as

$$\gamma_1^* = \gamma_1 - \Delta\gamma_1 H_x H_{yy}, \quad (23)$$

where $\Delta\gamma_1$—is the break angle of the deformed middle surface on the fracture line,

$$H_x = H(x - x_1);\ H_{yy} = H(y - y_1) - H(y - y_2)$$

of Heaviside function.

Substituting (23) in the equation of equilibrium of an infinitesimal element with taking into account the differential geometric relationships and elasticity ratios for bending plates leads to the following decisive equation

$$\nabla^4 w = \frac{P}{D_{cr}} + \left(\Delta\gamma_1\delta_x'' + \Delta\gamma_{1y}''\delta_x\right)H_{yy} + \Delta\gamma_{1y}'\delta_x\delta_{yy}, (24) \quad (24)$$

where $\delta_x = \delta(x - x_1)$ – is the delta function.

The equation of critical state in the conditions of longitudinal buckling would be obtained, if the load P represent as

$$P = -T_1 w_x'' - 2S w_{xy}'' - T_2 w_y'', \quad (25)$$

where T_1, T_2, S – are the contour compressive and shear loads.

The coefficient $\Delta\gamma_1$ is determined from the condition to equality to zero of bending moment on the cut edge

$$D_{cr}\left(w_x'' + \mu w_y''\right) = 0\ x = x_1;\ y < y < y_1. \quad (26)$$

As an example, let's consider the plate, compressed in a direction perpendicular to the cut line. Assuming hinged movable supporting on the contour, let's represent the required functions as

$$w = \sum w_m(x) \sin\beta_n y;\ \Delta\gamma_1 = \sum \Delta\gamma_{1n} \sin\bar{\beta}_n y, \quad (27)$$

where $\beta_n = \frac{\pi n}{b}$; $\bar{\beta}_n = \frac{\pi n}{b}$; $b' = y_2 - y_1$ – is the cut length.

Substituting (25) and (27) to (24) and using the procedure of the Bubnov-Galerkin method, we obtain for $n = 1$

$$\left(\frac{d^2}{dx^2} - \beta_1^2\right)^2 w_1(x) = \frac{T_1}{D_{cr}} w_1(x)\beta_1^2 + \Delta\gamma_{1n}\left(\bar{b}_1\delta_x'' + \delta_x\bar{B}_1\right) + \Delta\gamma_{1n}A_1, (28) \quad (28)$$

where $\bar{b}_1$, $\bar{B}_1$ and A_1 — are the constant coefficients.

The solution of equation (28) will be as

$$w_1(x) = w_{01}(x) + \Delta\gamma_1\bar{f}_1, \quad (29)$$

where $w_{01}(x)$ –eis the solution of equation

$$\left(\frac{d^2}{dx^2} - \beta_1^2\right)^2 w_1(x) = \frac{T}{D}. (30) \quad (30)$$

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and it can be represented as a double trigonometric series. The functions $f_{\ell k}$ and $\bar{f}_{\ell k}$ are solutions of the equations accordingly

$$\left(\frac{d^2}{dx^2}-\beta_1^2\right)^2 f_{\ell k}=\left(\delta_x'''-2\bar{\beta}_k^2\delta_x'\right)b_{k\ell}+\bar{B}_{k\ell}H_x, \quad (31)$$
$$\left(\frac{d^2}{dx^2}-\beta_1^2\right)^2 \bar{f}_{\ell k}=\left(b_{k\ell}\delta_x''-\delta_x\ B_{k\ell}\right)$$

and according to [13] are expressed by discontinuous functions.

Substituting (29) in the conditions on the cut edge (26) and using the procedure of the Bubnov-Galerkin method, we obtain together with (8) a system of homogeneous algebraic equations for the unknowns $w_{01\ell}$, Δw_ℓ, $\Delta\gamma_{1k}$. From the equality to zero of the determinant is obtained the expression for the critical load. By introducing discontinuous functions $f_{k\ell}$, $\bar{f}_{k\ell}$ the series in expression (27) converges so rapid that in practical calculations is sufficient of one term of series. Then, from the equality to zero of the determinant we obtain

$$T_{cr}=\frac{D_{cr}\pi^2}{b^2}\frac{\left(1+k_1^2\right)^2}{k_1^2}k_2, \quad (32)$$

where $k_1=\dfrac{b}{a}$ – is the ratio of the plate sides; k_2 – is the coefficient depending on the size and location of the cut.

Because the

$$T_{cr}^0=\frac{D_{cr}\pi^2}{b^2}\frac{\left(1+k_1^2\right)^2}{k_1^2} \quad (33)$$

is the value of T_{cr}^0 for continuous plate, coefficient $k_2=\dfrac{T_{cr}}{T_{cr}^0}$ characterize a reduction of critical load, caused by a cut.

3 CONCLUSION

Are compiled and studied the systems of differential equations that gives the possibility on a unified basis, in terms of non-linear deformations to investigate the mode of deformation for the class with having the ribs, breaks, concentrated supports structures. Are compiled various simplified versions of these equations with application of generalized functions. The design model reflects applied in the engineering structural elements.

Are developed the methods of calculation of have irregular lamellar structures in a conditions of linear and nonlinear deformation that provide the opportunity to identify with the same precision stresses and moments in the continuum area, as well as in adjacent of ribs.

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