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NUMERICAL MODEL OF AIR VALVE FOR COMPUTATION OF ONE-DIMENSIONAL FLOW

NUMERICKÝ MODEL ZAVZDUŠŇOVACÍHO VENTILU PRO VÝPOČET JEDNOROZMĚRNÉHO PROUDĚNÍ

Abstract

The paper is focused on a numerical simulation of unsteady flow in a pipeline. The special attention is paid to a numerical model of an air valve, which has to include all possible regimes: critical/subcritical inflow and critical/subcritical outflow of air. Thermodynamic equation of subcritical mass flow was simplified to get more friendly shape of relevant equations, which enables easier solution of the problem.

Abstrakt

Článek je zaměřen na numerickou simulaci nestacionárního proudění v potrubí. Zvláštní pozornost je věnována numerickému modelu zavzdušňovacího ventilu, který musí obsáhnout všechny možné režimy: kritické/podkritické sání a kritický/podkritický výfuk vzduchu. Termodynamická rovnice podkritického proudění plynu byla zjednodušena, takže její tvar je mnohem jednodušší pro další řešení.

Keywords

Air valve, numerical model, one-dimensional flow, water hammer, Lax-Wendroff.

1 INTRODUCTION

Flow in pipeline systems is a special case of a fluid motion. Axial component of the velocity dominates over the radial and tangential one. As various kinds of liquid are, usually, transported by a pipeline, it is necessary to pay an attention to this phenomenon. Jeopardy of a sudden velocity change has to be treated in each system as it can cause a rupture or a collapse of a pipe wall or can damage other components of the system [7].

Air valve is one way how to suppress a sudden drop of the pressure, which is connected with the transient flow. It is placed on a location, where this event is most likely to occur. It is, usually, on tops of the pipeline profile, downstream of an emergency valve and so on. When the pressure goes under the atmospheric value, the air valve opens and lets air get into the pipeline. When the pressure is greater than the atmospheric one, the air valve allows air to leave the pipeline, but keeps liquid inside.

When air valve is well designed, it makes pressure pulsations lower and protects the hydraulic system against impacts of the water hammer (e. g. [8]). To judge an effect of the air valve on the system behavior, a designer has to simulate the transient flow with one-dimensional version of continuity and momentum equations (1) and (2).

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$$\frac{\partial p}{\partial t}+\frac{K}{S}\cdot\frac{\partial Q}{\partial x}=0, \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{S}{\rho}\cdot\frac{\partial p}{\partial x}+\frac{\lambda}{2\cdot D\cdot S}\cdot|Q|\cdot Q=S\cdot g_p, \tag{2}$$

where:

D – diameter $[\mathrm{m}]$,

g_p – projection of gravity acceleration to pipe axis $\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$,

K – bulk modulus $[\mathrm{Pa}]$,

p – pressure $[\mathrm{Pa}]$,

Q – flow rate $\left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$,

S – pipe cross-section $[\mathrm{m}^2]$,

t – time $[\mathrm{s}]$,

x – space coordinate $[\mathrm{m}]$,

λ – friction coefficient $[-]$,

ρ – density $\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$.

These equations have to be solved together with boundary conditions. Mathematical description of the air valve is one of them.

2 NUMERICAL SOLUTION

Equations (1) and (2) are subject to a numerical solution. There are many possibilities. Probably, method of characteristic (or general method of characteristic) is the most popular [1], [5], [10], but Lax-Wendroff method is used in this paper. The numerical scheme is drawn in the figure 1.

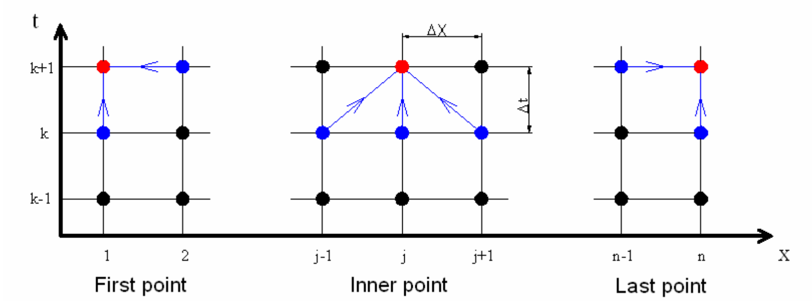

Fig. 1 Lax-Wendroff numerical scheme [3]

The method is based on the Taylor's expansion in the time direction and shows a numerical viscosity, which makes results more similar to real events than results by the method of characteristic. One can find a derivation of the method in [6]. The result is then given by initial conditions, which are important at the beginning of computation and then their influence disappears and by boundary condition, which control all computation.

Models of various hydraulic elements serve as boundary conditions and can be very simple (e. g. prescribed pressure, flow rate, resistance...) or more complex (surge tank, pump, turbine...), see [9]. Adaptation of boundary conditions for Lax-Wendroff method is described in [2].

2.1 Air valve

Model of the air valve can be understood as an air pocket with a variable mass. When the pressure is lower than the atmospheric pressure the mass increases, because air flows into the pipe. When the pressure exceeds the atmospheric value, air is being expelled and the mass decreases till no air remains in the pipe.

The state equation of a gas can be written in following form:

$$p \cdot V = m \cdot R \cdot T, \tag{3}$$

where:

m – mass of air in the pipe $[\mathrm{kg}]$,

R – gas constant $\left[\dfrac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}\right]$,

T – temperature in pipe $[\mathrm{K}]$,

V – volume of gas in the pipe $[\mathrm{m}^3]$.

Then, derivative of volume with respect to time is:

$$\frac{\partial V}{\partial t} = \frac{\partial m}{\partial t} \cdot \frac{R \cdot T}{p} - \frac{\partial p}{\partial t} \cdot \frac{m \cdot R \cdot T}{p^2}. \tag{4}$$

This volumetric change corresponds to a difference between water flowing out of the computational node representing the air valve and water flowing in. The time derivative of the mass obeys a thermodynamic law for flowing gases and has four possible shapes:

1. Ratio of the pressure in the pipe and atmospheric pressure is lower than critical value:

$$r_k = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}, \tag{5}$$

where:

r_k – critical ratio $[-]$,

n – polytrophic exponent (value in limits 1 to 1.4 for air) $[-]$,

then the air flow is critical and obeys following equation:

$$\frac{\partial m}{\partial t} = C_{in} \cdot S_{in} \cdot \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \cdot \sqrt{\frac{2 \cdot n}{n+1} \cdot \frac{p_a^2}{T_a \cdot R}}, \tag{6}$$

where:

C_{in} – inflow coefficient of the valve (value in limits 0 to 1) $[-]$,

p_a – atmospheric pressure $[\mathrm{Pa}]$,

S_{in} – inlet cross-section of the air valve $[\mathrm{m}^2]$,

T_a – temperature outside of pipe $[\mathrm{K}]$.

2. Subsonic inflow starts when ratio of pressures is lower than 1, but greater than r_k:

$$\frac{\partial m}{\partial t} = C_{in} \cdot S_{in} \cdot \sqrt{\left(\frac{p}{p_a}\right)^{\frac{2}{n}} \cdot \frac{n}{n-1} \cdot \left[1-\left(\frac{p}{p_a}\right)^{\frac{n-1}{n}}\right] \cdot \frac{2 \cdot p_a^2}{T_a \cdot R}}. \tag{7}$$

3. When the ratio of pressures is greater than 1 but lower than inverse value of r_k, subsonic outflow of air starts:

$$\frac{\partial m}{\partial t} = -C_{out} \cdot S_{out} \cdot \sqrt{\left(\frac{p_a}{p}\right)^{\frac{2}{n}} \cdot \frac{n}{n-1} \cdot \left[1-\left(\frac{p_a}{p}\right)^{\frac{n-1}{n}}\right] \cdot \frac{2 \cdot p^2}{T \cdot R}}, \tag{8}$$

where:

C_{out} – outflow coefficient of the valve (value in limits 0 to 1) $[-]$,

S_{out} – outlet cross-section of the air valve $\left[\mathrm{m}^2\right]$.

4. Critical outflow starts when the ratio of pressures is greater than inverse value of r_k:

$$\frac{\partial m}{\partial t} = -C_{out} \cdot S_{out} \cdot \left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \cdot \sqrt{\frac{2 \cdot n}{n+1} \cdot \frac{p^2}{T \cdot R}}. \tag{9}$$

Mass flow of air through the air valve can be, then, plotted as a function of pressure or pressure ratio, see fig. 2. Of course, when there is no air in the pipe, the flow equals zero.

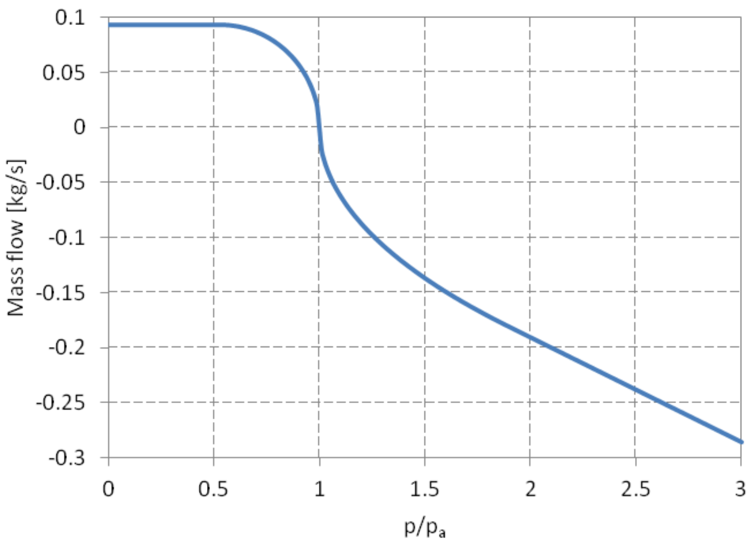

Fig. 2 Mass flow through the air valve

But there is one difficulty: subsonic mass flow is quite complicated function and makes problems in numerical model, because unknown pressure has the exponent, which is not a whole number. So, there is an effort to simplify it. Lee and Leow [4] split subsonic area into intervals and, in each interval, replaced the function with a parabola. The more intervals the better accuracy, but increased requirements on computational time, because it is necessary to solve all parabolas and look for solution lying at a right interval.

When one looks at the subsonic inflow (pressure ratio at interval 0.528 to 1.0), the function is similar to an ellipse, so it could be possible to use it at the entire interval. Thus, equation (7) is replaced by:

$$\frac{\partial m}{\partial t}=C_{in}\cdot S_{in}\cdot\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\cdot\sqrt{\frac{n}{n+1}\cdot\frac{2}{T_a\cdot R}\cdot\left[p_a^2-\left(\frac{p-p_a\cdot r_k}{1-r_k}\right)^2\right]}. \tag{10}$$

Equation for subsonic outflow (8) can be replaced in similar way by:

$$\frac{\partial m}{\partial t}=-C_{out}\cdot S_{out}\cdot\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\cdot\sqrt{\frac{n}{n+1}\cdot\frac{2}{T\cdot R}\cdot\left[p^2-\left(\frac{p_a-p\cdot r_k}{1-r_k}\right)^2\right]}. \tag{11}$$

Comparison of the original functions (7) and (8) with substituting functions (10) and (11) is shown in the fig. 3. Equations (10) and (11) are friendlier for further solution, because contains only first and second power of the pressure unlike equations (7) and (8).

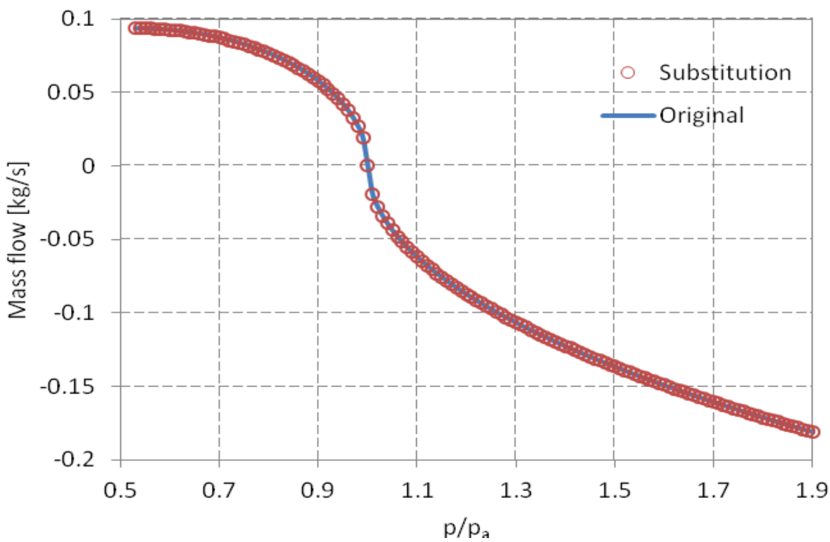

Fig. 3 Comparison of substitution with the original function

The error of substitution depends on the polytrophic exponent and is lower than 3.5% for values from 1 to 1.4. When the exponent has value 1.449, the substitution is the most accurate. See figure 4.

Now, the numerical model of the valve can be written in shape (12) using eqns. (4), (5), (6), (10) and (11). Function $m(t+\Delta t)=m(p(t+\Delta t))$.

$$Q_{out}(t+\Delta t)-Q_{in}(t+\Delta t)=\frac{\partial m(t+\Delta t)}{\partial t}\cdot\frac{R\cdot T}{p(t)}-\frac{p(t+\Delta t)-p(t)}{\Delta t}\cdot\frac{V(t)}{p(t)}, \tag{12}$$

where:

Q_{in} – water flow into the air valve node $\left[\frac{\mathrm{m}^2}{s}\right]$,

Q_{in} – water flow out of the air valve node $\left[\frac{\mathrm{m}^2}{s}\right]$,

Δt – time step of the computation $[s]$.

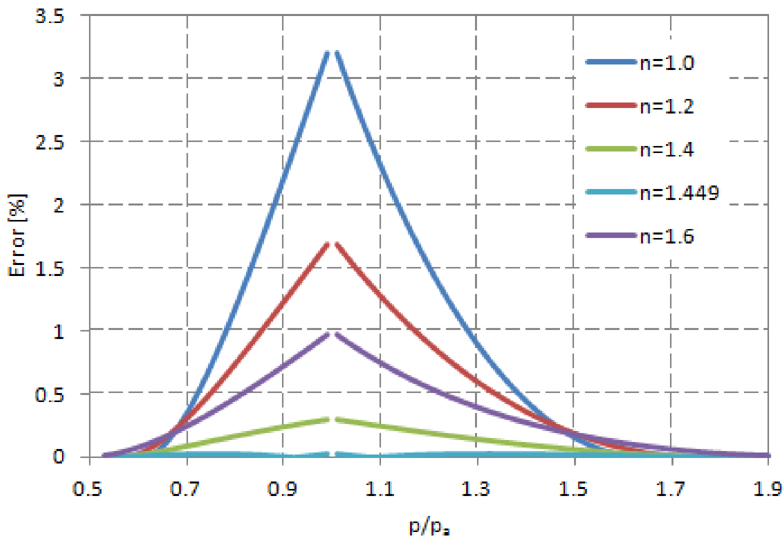

Fig. 4 Error of substitution for various values of the polytrophic exponent

Inflow and outflow come from equation (1) in form:

$$Q_{out}(t+\Delta t)=\frac{S\cdot\Delta x}{K\cdot\Delta t}\cdot[p(t+\Delta t)-p(t)]+Q_2(t+\Delta t), \tag{13}$$

$$Q_{in}(t+\Delta t)=\frac{S\cdot\Delta x}{K\cdot\Delta t}\cdot[p(t)-p(t+\Delta t)]+Q_{n-1}(t+\Delta t), \tag{14}$$

where:

Δx – space step of the computation $[m]$.

Variables Q_2 and Q_{n-1} are flow rates given by Lax-Wendroff method (one space step downstream and one space step upstream the air valve respectively), Q_1 and Q_n are the same as Q_{out} and Q_{in} respectively, see fig. 5. The only unknown is the pressure in the following time step $p(t+\Delta t)$, when equations (12) to (14) are being solved.

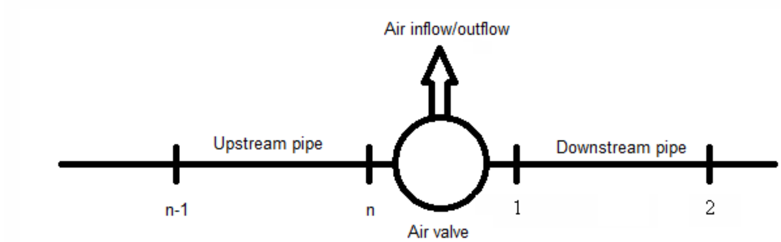

Fig. 5 Numerical scheme of the air valve node

2.2 Simulation

Simple task of a pipeline with an air valve was used to test the proposed numerical model. Figure 6 shows a pipeline profile. Air valve is placed five meters from the upstream end, where the pipeline becomes horizontal. This place is the most dangerous, because the column separation is most likely to appear here.

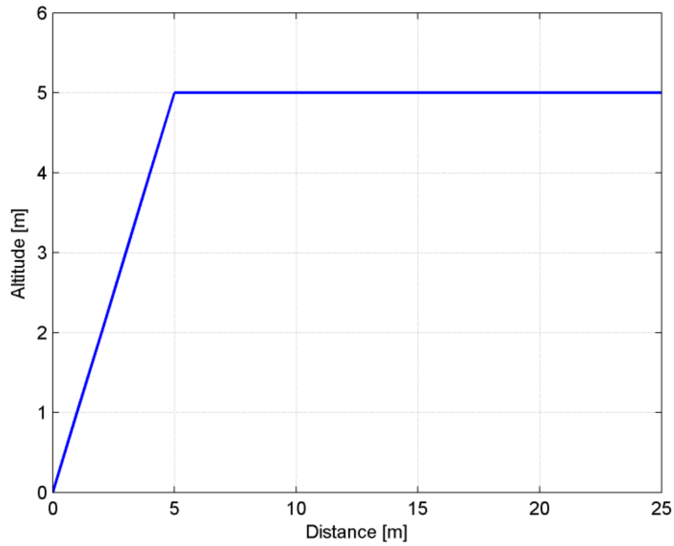

Fig. 6 Pipeline profile

The flow rate boundary condition at the beginning of pipeline is plotted in the fig. 7. Initial flow rate is 0.877 m^3/s, which is constant for one second. Then, it starts decreasing to zero following parabolic function. This is similar to closing a ball valve. Constant pressure 1.1 kPa is the outlet boundary condition. Parameters of the air valve were chosen as following: atmospheric pressure 10^5 Pa, gas constant 287 J/kg/K, temperature in pipeline 288 K, temperature outside of pipe 298 K, air inlet cross-section 10^{-3} m^2, air outlet cross-section $4.9 \cdot 10^{-5}$ m^2, polytrophic exponent of air 1.4, outflow and inflow coefficients of the valve are 1.

Diameter of pipeline is 0.5 m, roughness 1 mm, wave speed 1000 m/s, density of water 1000 kg/m^3 and viscosity 10^{-6} m/s^2. As the space step is 1 m the time step of computation is 0.001 s.

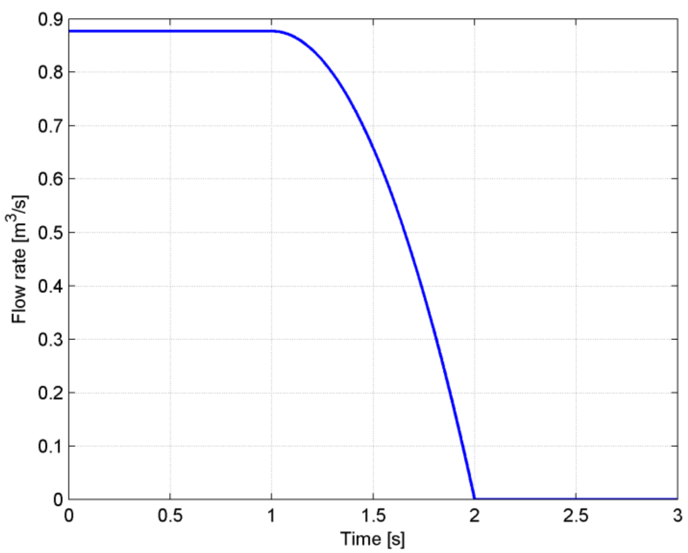

Fig. 7 Inlet boundary condition

Figure 8 shows pressure surge at the beginning of pipe and on the location of the air valve (which is not considered in this case). One can see that the top of the highest peak exceeds value 3.5 kPa and the lowest pressure is more than 0.5 kPa below the absolute zero, what means that a cavitation would appear there. (This task would deserve using an appropriate cavitation model, but this is not the goal of this paper).

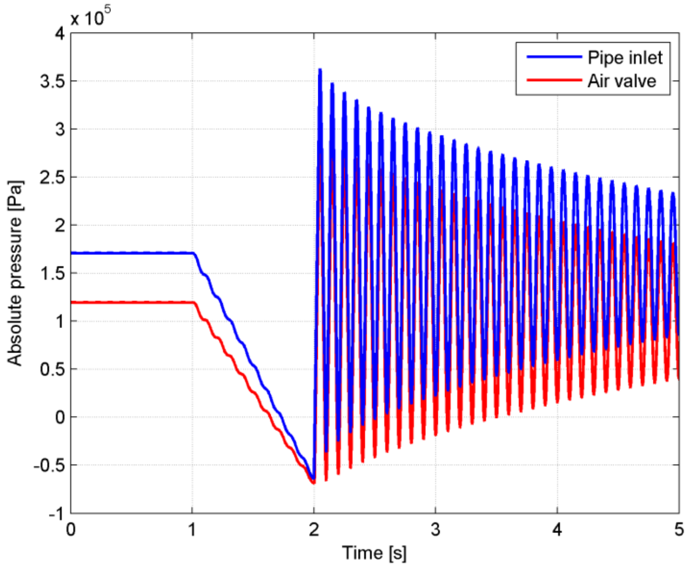

Fig. 8 Pressure pulsations without the air valve

When the air valve is considered on its location, the pressure pulsations are noticeably lower and minimal pressure is only 0.5 kPa below the atmospheric value, thus there is no risk of cavitation, see fig. 9. Figure 10 shows volume and mass of an air pocket, which originates when air is sucked into the pipe. The difference between frequencies of pulsations in figures 8 and 9 is given by the air pocket, which serves as an air vessel with variable capacity.

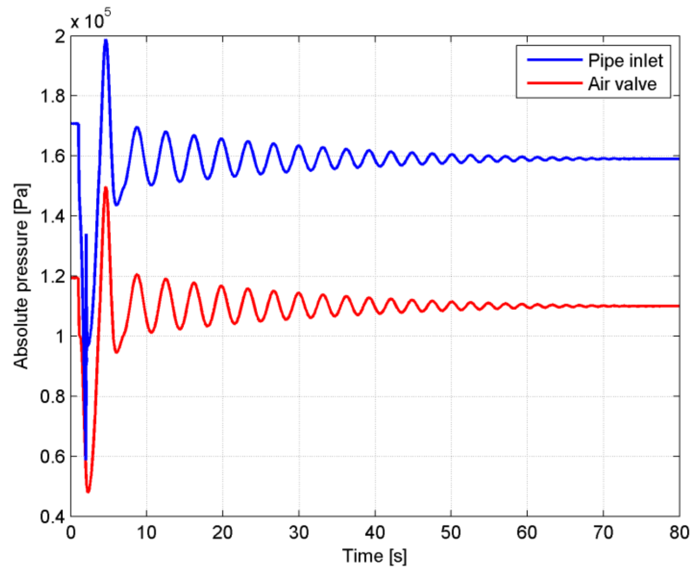

Fig. 9 Pressure pulsations with the air valve

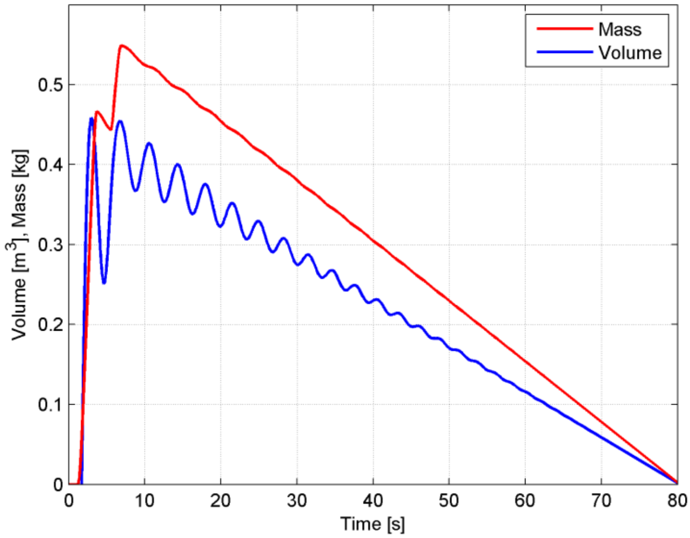

Fig. 10 Volume and mass of the air pocket during the transient event

3 CONCLUSIONS

Design of a numerical model of an air valve is described in this paper. Model comes from thermodynamic equation for flowing gas and state equation describing behavior of the air in a pipe. Since the subsonic flow of the gas is described by quite complicated equation, which makes further computation difficult, this function was substituted with a simpler one, which is similar to original one. The error of the substitution is low enough to justify this step.

Then, the final numerical model was tested as a boundary condition in a computation of a water hammer in a simple pipeline.

4 ACKNOWLEDGMENT

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