Development of a Forward/Backward Power Flow Algorithm in Distribution Systems Based on Probabilistic Technique Using Normal Distribution

Shahrokh SHOJAEIAN[1], *Ehsan SALLEALA-NAEENI*[1], *Ehsan TASLIMI RENANI*[2]

[1]Department of Electrical Engineerng, Faculty of Engineering, Khomeinishahr Branch, Islamic Azad University, No 159, 7th Boostan Street, Isfahan 84181-48499, Iran
[2]Department of Electrical Engineering, Faculty of Engineering, University of Malaya, Lingkungan Budi, Kuala Lumpur 50603, Malaysia

shojaeian@iaukhsh.ac.ir, ehsan.salleala@iaukhsh.ac.ir, taslimi.ehsan@siswa.um.edu.my

Abstract. *There are always some uncertainties in prediction and estimation of distribution systems loads. These uncertainties impose some undesirable impacts and deviations on power flow of the system which may cause reduction in accuracy of the results obtained by system analysis. Thus, probabilistic analysis of distribution system is very important. This paper proposes a new probabilistic load flow technique in presence of a 24 hours load changing regime in all seasons, by applying a normal probabilistic distribution in seven standard deviations for the loads and using this distribution function on Forward/backward algorithm. The losses and voltage of IEEE 33-bus test distribution network is investigated by our new algorithm and the results are compared with the conventional algorithm i.e., based on deterministic methods.*

Keywords

Distribution network, forward/backward power flow, normal distribution function, power losses, probabilistic power flow.

1. Introduction

Most of suggested methods to solve power flow in power systems consider deterministic in nature in which loads power have been taken into account constant (e.g., analysis is done in the worst system condition). In those methods, small change in the network requires resolving the power flow.

In addition, the system reliability may decline when the system designers utilize previous deterministic data. Nowadays uncertainties of the power systems are also their effects on the system operation are not undeniable and it is necessary to investigate the performance of the network over these uncertainties.

In previous studies output variation and system performance evaluation is determined by analyzing the input parameters based on probabilistic methods.

The classical deterministic load flow techniques are not able to cope with uncertainties and can only be used within constant power system parameters [1].

Probabilistic power flow was proposed in 1974 [2]. In [2], Borkowska used DC model of network and considered both substations load of different feeders and output information in form of density function. In [3], Dopazo used covariance matrix method however, these methods were developed in [4], [5]. In [6], Zhang and Lee combined the concept of Cumulants and Gram-Charlier expansion theory to obtain probabilistic distribution functions of transmission line flows. It has significantly reduced the computational time whit a high degree of accuracy. In [7], [8], Cumulants and Von Mises function and Monte Calro method have been used to solve probabilistic power flow.

Therefore, evaluating the system and addressing these uncertainties different methods such as probabilistic methods, fuzzy sets and interval analysis have been introduced [12]. PLF could be divided into numerical and analytical methods. Numerical method refers to Monte Carlo's simulation method [11], which is an alternative to the huge number of random variables for determination of PLF in numerical methods.

Due to the fact that implementation of this method could be very time consuming, this method cannot be used in some applications. On the other hand, implementation of the analytical method is faster however, there are many difficulties in implementing this method. The most significant methods in analytical method are Convolution [9] and cumulants methods [10]. Convolution method is based on probabilistic distribution of random variables.

The main contribution of this paper is introducing a simple and practical expansion approach for the well-known forward/backward power flow method. This approach can be used for modern distribution systems which may including several variable loads and renewable energy sources which in turn have non-deterministic power generation level.

After this introduction, in the second chapter using normal distribution function, a probabilistic model of load is presented. The third chapter is began with a review on the forward/backward power flow approach and is continued with introducing a probabilistic version of this approach. To simulate the proposed method, in the fourth chapter IEEE 33-bus test network is considered and results of probabilistic power flow of it is shown in the fifth chapter. The paper conclusion is stated in the sixth chapter.

2. Probabilistic Model of Load

It is well known fact that electrical loads in power systems are probabilistic in nature. There are two prime parameters in power flow which lead to load change in network, time (daily, weekly, seasonal and annual profiles) and climate conditions. It should be noted that in deterministic power flow these two parameters are ignored and typical sample of consumers' parameters can be obtained from stochastic analyzing.

Load during hour, week, month and season of the year is considered as a percentage of the peak value. Then load profile is created base on this information. A typical load profile is presented in [11]. This paper uses combination of daily load profile for each season and normal distribution function. Figure 1 presents daily load profile for total consumption four different seasons [13].

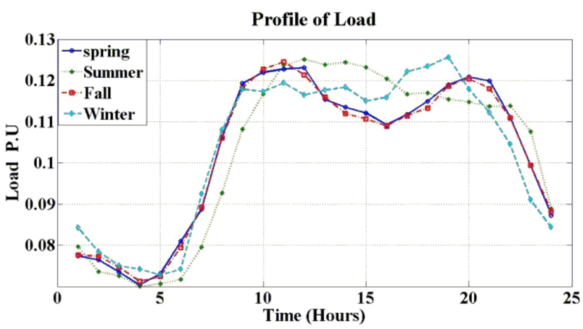

Fig. 1: Average daily load profile in different seasons.

If μ_{PLj} and μ_{QLj} are considered as the average active and reactive power of the jth load and m is a stochastic sample of time, average active and reactive power defined as follows:

$$\mu_{PLj} = \frac{1}{m}\sum_{\forall m} P_{Lj,m}, \tag{1}$$

$$\mu_{QLj} = \frac{1}{m}\sum_{\forall m} Q_{Lj,m}. \tag{2}$$

Also the active and reactive power's variance is calculated as follows:

$$\sigma^2_{PLj} = \frac{1}{m}\sum_{\forall m}(P_{Lj,m} - \mu_{PLj})^2, \tag{3}$$

$$\sigma^2_{QLj} = \frac{1}{m}\sum_{\forall m}(Q_{Lj,m} - \mu_{QLj})^2. \tag{4}$$

Considering the nature of the system loads, normal distribution function can be applied which is defined as:

$$f(x) = \frac{e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}}{\sigma\sqrt{2\pi}} \qquad -\infty < x < +\infty. \tag{5}$$

Figure 2 illustrates load profile of the 10^{th} bus of summer season with four-hour interval (6, 12, 18 and 24) using distribution function.

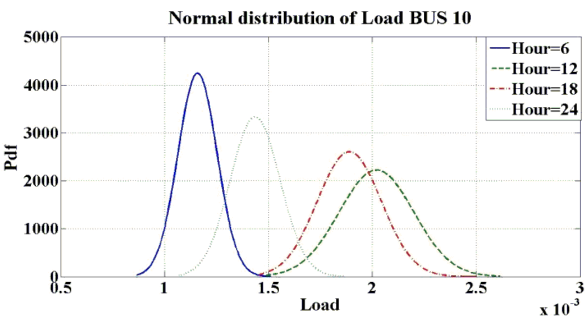

Fig. 2: Load profile of 10^{th} bus of summer season using normal distribution.

3. Power Flow in Distribution Networks

3.1. Deterministic Power Flow

In this paper Forward/backward method is used to solve power flow in distribution network. It is assumed that $V_s = 1 < 0$ is the source voltage, $S_{Lj} = P_{Lj} + jQ_{Lj} = V_j I_j*$ is the apparent power of the j^{th} load, S_j is the apparent output power of the j^{th} bus, and I_j is the current flowing through the j^{th} bus. In Forward/backward method, firstly the voltage of the last bus is assumed to be 1<0 then having the load powers and using sections impedances the following formula can be calculated [14]. A typical single line diagram of a distribution network is shown in Fig. 3.

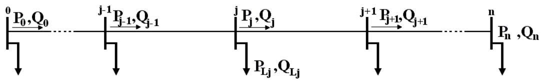

Fig. 3: Single line diagram of a distribution system.

$$P_{j-1} = P_j + r_j \frac{P'^2_j + Q'^2_j}{V_j^2} + P_{Lj}, \quad (6)$$

$$Q_{j-1} = Q_j + x_j \frac{P'^2_j + Q'^2_j}{V_j^2} + Q_{Lj}, \quad (7)$$

$$V_{j-1}^2 = V_j^2 + 2(r_j P'_j + x_j Q'_j) + (r_j^2 + x_j^2) \frac{P'^2_j + Q'^2_j}{V_j^2}, \quad (8)$$

where: $P'_j = P_j + P_{Lj}$, $Q'_j = Q_j + Q_{Lj}$.

Obtained power in backward path can be used to calculate the first bus output powers. For forward path, the following voltage equation can be used:

$$V_{j+1}^2 = V_j^2 - 2(r_j P_j + x_j Q_j) + (r_j^2 + x_j^2) \frac{P_j^2 + Q_j^2}{V_j^2}. \quad (9)$$

Forward/backward operation should be repeated to achieve convergence so that:

$$|V_j^{(k)} - V_j^{(k-1)}| < \varepsilon_v \ , \ |P_{loss}^{(k)} - P_{loss}^{(k-1)}| < \varepsilon_p. \quad (10)$$

Here ε_v and ε_p are the acceptable errors for bus voltages and the system power loss respectively and k is the number of the k^{th} iteration. In this paper, there is assumption that: $\varepsilon_v = \varepsilon_p = 10^{-6}$ pu. Following formula demonstrates how losses are calculated:

$$P_{Loss} = \sum_{j=0}^{n-1} r_j \left(\frac{P_j^2 + Q_j^2}{V_j^2} \right). \quad (11)$$

3.2. Probabilistic Power Flow

As mentioned in the previous section, in deterministic power flow, if there is an assumption that active and reactive powers of loads are probabilistic, by applying small changes in Eq. (6) and Eq. (7) and also using normal distribution function, PLF can be defined as follows:

$$P_{j-1} = P_j + r_j \frac{P'^2_j + Q'^2_j}{V_j^2} + P_{Lj} \sim N(\mu_{PLj}, \sigma_{PLj}), \quad (12)$$

$$Q_{j-1} = Q_j + x_j \frac{P'^2_j + Q'^2_j}{V_j^2} + Q_{Lj} \sim N(\mu_{QLj}, \sigma_{QLj}). \quad (13)$$

Flow chart of mentioned method is illustrated in Fig. 4, where S and T stand for season and hour respectively.

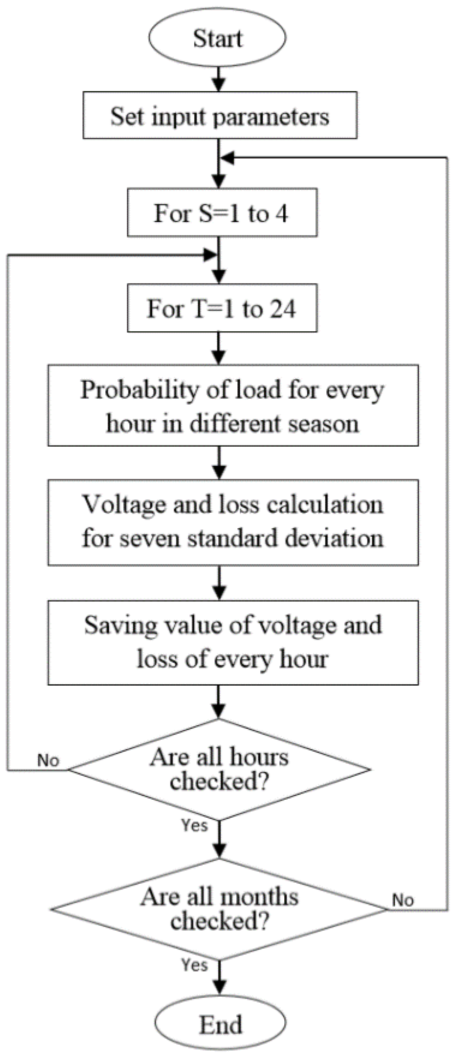

Fig. 4: Flow chart of Forward/backward method with probabilistic technique.

4. The Studied System

All information about the IEEE 33-bus test network is given in Fig. 5 [14], [15]. Base voltage and apparent power of the system are 12.66 kV and 10 MVA.

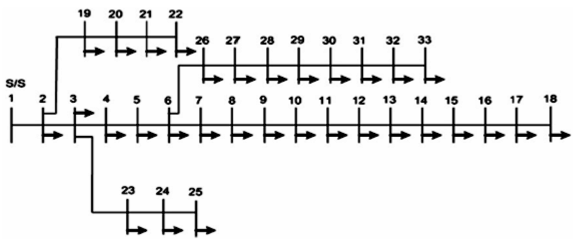

Fig. 5: IEEE 33-bus test system.

5. Result and Simulation

Change of the network loads in different hours of day affects the network losses, capacity of line, voltage and other parameters of network. By dividing normal distribution function to seven parts for every hours of different days in different seasons, the parameters of μ_{PLj}, μ_{QLj}, σ_{PLj}, σ_{QLj} are determined and the probabilistic power flow is solved. Figure 6 shows standard deviations of normal distribution which is assumed to be in the vicinity of the deterministic loads values.

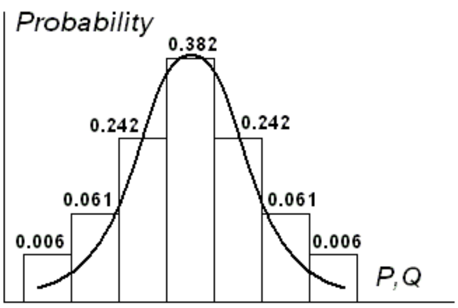

Fig. 6: Change of normal distribution to seven-deviation standard.

5.1. Probabilistic Losses of Network

Table 1 demonstrates P_{loss} in 24-hour and for each standard deviation. Table 2 presents both probabilistic and deterministic total power losses for a few sample hours in each season. Table 2 shows that the power losses obtained are remarkably different which testifies the values of errors in the deterministic method.

Figure 7 to Fig. 10 denotes power losses in all seasons, all standard deviations and 24-hour.

The experiments were carried out on a PC with a Intel Core i5, 2.66 GHz CPU, and 4 GB RAM with the Microsoft Windows XP operating system. The CPU time for deterministic and probabilistic load flow are 0.110696 and 2.957022 seconds respectively.

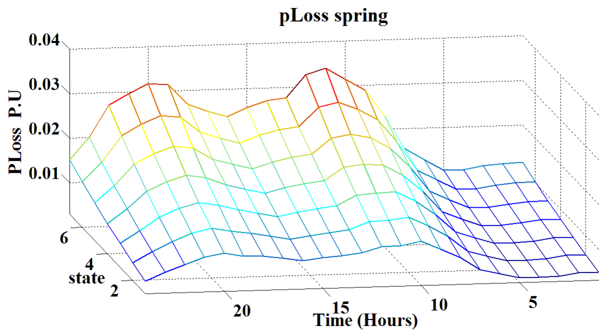

Fig. 7: Losses in spring.

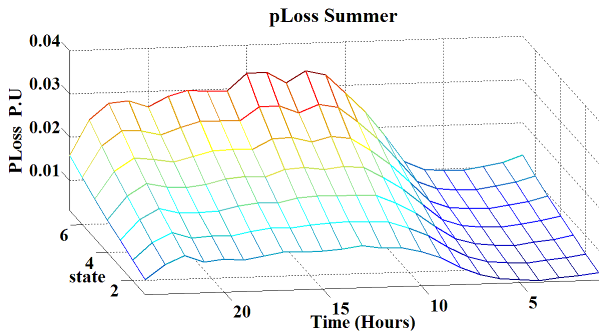

Fig. 8: Losses in summer.

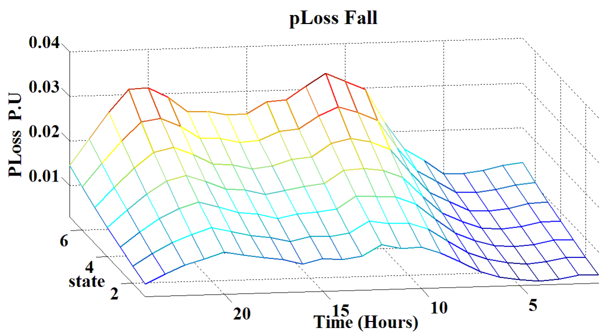

Fig. 9: Losses in fall.

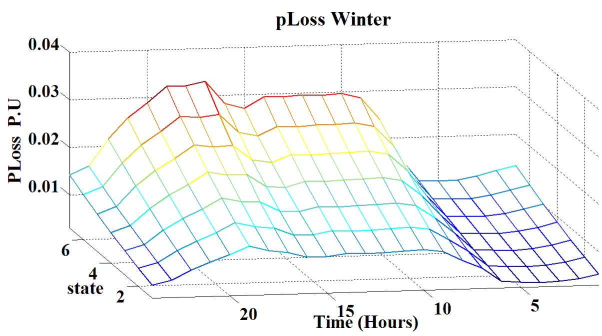

Fig. 10: Losses in winter.

Tab. 1: System loss at each hour for every season for seven standard deviation of normal distribution.

Sample hours	Season	State1	State2	State3	State4	State5	State6	State7
1	Spring	$4.586 \cdot 10^{-3}$	$5.552 \cdot 10^{-3}$	$6.621 \cdot 10^{-3}$	$7.795 \cdot 10^{-3}$	$9.076 \cdot 10^{-3}$	$1.046 \cdot 10^{-2}$	$1.197 \cdot 10^{-2}$
	Summer	$4.776 \cdot 10^{-3}$	$5.815 \cdot 10^{-3}$	$6.968 \cdot 10^{-3}$	$8.239 \cdot 10^{-3}$	$9.630 \cdot 10^{-3}$	$1.114 \cdot 10^{-2}$	$1.278 \cdot 10^{-2}$
	Fall	$4.592 \cdot 10^{-3}$	$5.572 \cdot 10^{-3}$	$6.659 \cdot 10^{-3}$	$7.854 \cdot 10^{-3}$	$9.160 \cdot 10^{-3}$	$1.058 \cdot 10^{-2}$	$1.211 \cdot 10^{-2}$
	Winter	$5.510 \cdot 10^{-3}$	$6.626 \cdot 10^{-3}$	$7.858 \cdot 10^{-3}$	$9.208 \cdot 10^{-3}$	$1.068 \cdot 10^{-2}$	$1.227 \cdot 10^{-2}$	$1.400 \cdot 10^{-2}$
2	Spring	$4.351 \cdot 10^{-3}$	$5.317 \cdot 10^{-3}$	$6.394 \cdot 10^{-3}$	$7.583 \cdot 10^{-3}$	$8.888 \cdot 10^{-3}$	$1.031 \cdot 10^{-2}$	$1.185 \cdot 10^{-2}$
	Summer	$4.319 \cdot 10^{-3}$	$5.149 \cdot 10^{-3}$	$6.058 \cdot 10^{-3}$	$7.050 \cdot 10^{-3}$	$8.125 \cdot 10^{-3}$	$9.285 \cdot 10^{-3}$	$1.053 \cdot 10^{-2}$
	Fall	$4.520 \cdot 10^{-3}$	$5.482 \cdot 10^{-3}$	$6.548 \cdot 10^{-3}$	$7.722 \cdot 10^{-3}$	$9.006 \cdot 10^{-3}$	$1.040 \cdot 10^{-2}$	$1.191 \cdot 10^{-2}$
	Winter	$4.890 \cdot 10^{-3}$	$5.814 \cdot 10^{-3}$	$6.826 \cdot 10^{-3}$	$7.928 \cdot 10^{-3}$	$9.122 \cdot 10^{-3}$	$1.041 \cdot 10^{-2}$	$1.179 \cdot 10^{-2}$
7	Spring	$6.027 \cdot 10^{-3}$	$7.322 \cdot 10^{-3}$	$8.762 \cdot 10^{-3}$	$1.034 \cdot 10^{-2}$	$1.209 \cdot 10^{-2}$	$1.398 \cdot 10^{-2}$	$1.604 \cdot 10^{-2}$
	Summer	$4.895 \cdot 10^{-3}$	$5.892 \cdot 10^{-3}$	$6.993 \cdot 10^{-3}$	$8.200 \cdot 10^{-3}$	$9.516 \cdot 10^{-3}$	$1.094 \cdot 10^{-2}$	$1.248 \cdot 10^{-2}$
	Fall	$6.222 \cdot 10^{-3}$	$7.502 \cdot 10^{-3}$	$8.916 \cdot 10^{-3}$	$1.046 \cdot 10^{-2}$	$1.216 \cdot 10^{-2}$	$1.400 \cdot 10^{-2}$	$1.599 \cdot 10^{-2}$
	Winter	$6.795 \cdot 10^{-3}$	$8.119 \cdot 10^{-3}$	$9.575 \cdot 10^{-3}$	$1.116 \cdot 10^{-2}$	$1.290 \cdot 10^{-2}$	$1.478 \cdot 10^{-2}$	$1.680 \cdot 10^{-2}$
8	Spring	$8.863 \cdot 10^{-3}$	$1.073 \cdot 10^{-2}$	$1.281 \cdot 10^{-2}$	$1.511 \cdot 10^{-2}$	$1.764 \cdot 10^{-2}$	$2.040 \cdot 10^{-2}$	$2.341 \cdot 10^{-2}$
	Summer	$6.849 \cdot 10^{-3}$	$8.243 \cdot 10^{-3}$	$9.786 \cdot 10^{-3}$	$1.148 \cdot 10^{-2}$	$1.333 \cdot 10^{-2}$	$1.534 \cdot 10^{-2}$	$1.751 \cdot 10^{-2}$
	Fall	$8.938 \cdot 10^{-3}$	$1.077 \cdot 10^{-2}$	$1.281 \cdot 10^{-2}$	$1.505 \cdot 10^{-2}$	$1.751 \cdot 10^{-2}$	$2.018 \cdot 10^{-2}$	$2.309 \cdot 10^{-2}$
	Winter	$8.973 \cdot 10^{-3}$	$1.095 \cdot 10^{-2}$	$1.317 \cdot 10^{-3}$	$1.563 \cdot 10^{-2}$	$1.834 \cdot 10^{-2}$	$2.130 \cdot 10^{-2}$	$2.453 \cdot 10^{-2}$
15	Spring	$9.850 \cdot 10^{-3}$	$1.197 \cdot 10^{-2}$	$1.433 \cdot 10^{-2}$	$1.695 \cdot 10^{-2}$	$1.983 \cdot 10^{-2}$	$2.298 \cdot 10^{-2}$	$2.641 \cdot 10^{-2}$
	Summer	$1.212 \cdot 10^{-2}$	$1.473 \cdot 10^{-2}$	$1.765 \cdot 10^{-2}$	$2.088 \cdot 10^{-2}$	$2.445 \cdot 10^{-2}$	$2.836 \cdot 10^{-2}$	$3.263 \cdot 10^{-2}$
	Fall	$9.381 \cdot 10^{-3}$	$1.153 \cdot 10^{-2}$	$1.395 \cdot 10^{-2}$	$1.663 \cdot 10^{-2}$	$1.960 \cdot 10^{-2}$	$2.286 \cdot 10^{-2}$	$2.643 \cdot 10^{-2}$
	Winter	$1.067 \cdot 10^{-2}$	$1.287 \cdot 10^{-2}$	$1.531 \cdot 10^{-3}$	$1.799 \cdot 10^{-2}$	$2.093 \cdot 10^{-2}$	$2.414 \cdot 10^{-2}$	$2.762 \cdot 10^{-2}$
16	Spring	$9.233 \cdot 10^{-3}$	$1.124 \cdot 10^{-2}$	$1.349 \cdot 10^{-2}$	$1.597 \cdot 10^{-2}$	$1.871 \cdot 10^{-2}$	$2.171 \cdot 10^{-2}$	$2.498 \cdot 10^{-2}$
	Summer	$1.149 \cdot 10^{-2}$	$1.392 \cdot 10^{-2}$	$1.662 \cdot 10^{-2}$	$1.960 \cdot 10^{-2}$	$2.288 \cdot 10^{-2}$	$2.647 \cdot 10^{-2}$	$3.038 \cdot 10^{-2}$
	Fall	$9.071 \cdot 10^{-3}$	$1.112 \cdot 10^{-2}$	$1.343 \cdot 10^{-2}$	$1.599 \cdot 10^{-2}$	$1.882 \cdot 10^{-2}$	$2.193 \cdot 10^{-2}$	$2.532 \cdot 10^{-2}$
	Winter	$1.048 \cdot 10^{-2}$	$1.280 \cdot 10^{-2}$	$1.539 \cdot 10^{-3}$	$1.828 \cdot 10^{-2}$	$2.148 \cdot 10^{-2}$	$2.499 \cdot 10^{-2}$	$2.883 \cdot 10^{-2}$
23	Spring	$7.785 \cdot 10^{-3}$	$9.359 \cdot 10^{-3}$	$1.109 \cdot 10^{-2}$	$1.300 \cdot 10^{-2}$	$1.509 \cdot 10^{-2}$	$1.735 \cdot 10^{-2}$	$1.980 \cdot 10^{-2}$
	Summer	$8.546 \cdot 10^{-3}$	$1.061 \cdot 10^{-2}$	$1.295 \cdot 10^{-2}$	$1.557 \cdot 10^{-2}$	$1.847 \cdot 10^{-2}$	$2.168 \cdot 10^{-2}$	$2.519 \cdot 10^{-2}$
	Fall	$8.158 \cdot 10^{-3}$	$9.691 \cdot 10^{-3}$	$1.137 \cdot 10^{-2}$	$1.321 \cdot 10^{-2}$	$1.520 \cdot 10^{-2}$	$1.736 \cdot 10^{-2}$	$1.969 \cdot 10^{-2}$
	Winter	$6.448 \cdot 10^{-3}$	$7.775 \cdot 10^{-3}$	$9.241 \cdot 10^{-3}$	$1.085 \cdot 10^{-2}$	$1.260 \cdot 10^{-2}$	$1.451 \cdot 10^{-2}$	$1.657 \cdot 10^{-2}$
24	Spring	$5.798 \cdot 10^{-3}$	$7.038 \cdot 10^{-3}$	$8.412 \cdot 10^{-3}$	$9.925 \cdot 10^{-3}$	$1.157 \cdot 10^{-2}$	$1.337 \cdot 10^{-2}$	$1.532 \cdot 10^{-2}$
	Summer	$6.101 \cdot 10^{-3}$	$7.391 \cdot 10^{-3}$	$8.822 \cdot 10^{-3}$	$1.039 \cdot 10^{-2}$	$1.212 \cdot 10^{-2}$	$1.399 \cdot 10^{-2}$	$1.602 \cdot 10^{-2}$
	Fall	$6.160 \cdot 10^{-3}$	$7.363 \cdot 10^{-3}$	$8.686 \cdot 10^{-3}$	$1.013 \cdot 10^{-2}$	$1.170 \cdot 10^{-2}$	$1.340 \cdot 10^{-2}$	$1.524 \cdot 10^{-2}$
	Winter	$5.542 \cdot 10^{-3}$	$6.613 \cdot 10^{-3}$	$7.901 \cdot 10^{-3}$	$9.319 \cdot 10^{-3}$	$1.087 \cdot 10^{-2}$	$1.256 \cdot 10^{-2}$	$1.439 \cdot 10^{-2}$

Tab. 2: Probabilistic and deterministic daily losses for each season.

Season	P_{loss} (Probabilistic)	P_{loss} (Deterministic)
Spring	0.35542	0.35277
Summer	0.35777	0.35489
Fall	0.35465	0.35190
Winter	0.35859	0.35589

Tab. 3: Comparison of probabilistic and deterministic value of cumulative 24-hour voltage deviation of each in the summer.

Hour	1	2	3	4	5	6	7	8
Probabilistic	0.0523	0.0459	0.0422	0.0383	0.0395	0.0421	0.0518	0.075
Deterministic	0.0519	0.0456	0.0419	0.038	0.0392	0.0418	0.0515	0.0744
Hour	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Probabilistic	0.0991	0.1175	0.133	0.1359	0.1319	0.136	0.1343	0.1237
Deterministic	0.0983	0.1166	0.1318	0.1346	0.131	0.1347	0.133	0.1227
Hour	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Probabilistic	0.1248	0.1223	0.1154	0.1112	0.1132	0.1188	0.1013	0.0670
Deterministic	0.1239	0.1212	0.1144	0.1104	0.1122	0.1181	0.1006	0.0666

5.2. Probabilistic Voltage Evaluation

Figure 11 to Fig. 13 illustrate voltage profile for all buses in summer and 24-hours for $(\mu_{PLj}-\sigma_{PLj}, \mu_{QLj}-\sigma_{QLj})$, (μ_{PLj}, μ_{QLj}), and $(\mu_{PLj}+\sigma_{PLj}, \mu_{QLj}+\sigma_{QLj})$. Voltage profile of the spring, fall and winter seasons are shown in Fig. 14 to Fig. 16 for all buses and 24-hour versus load average. Sum of voltage deviations squares in 24-hour for each bus in summer for the probabilistic and deterministic load flows are given in Tab. 3.

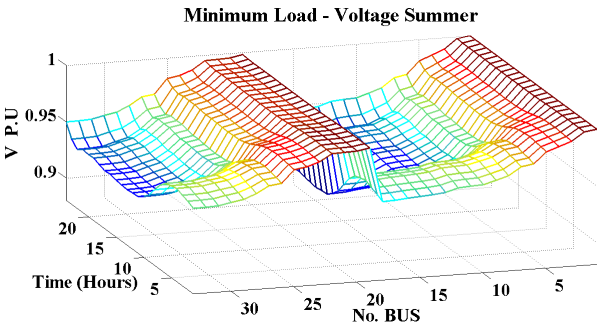

Fig. 11: Voltage profile of every hour in summer for $(\mu_{PLj}-\sigma_{PLj}, \mu_{QLj}-\sigma_{QLj})$.

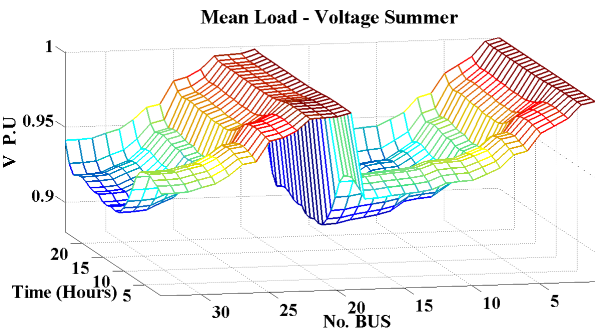

Fig. 12: Voltage profile of every hour in summer for (μ_{PLj}, μ_{QLj}).

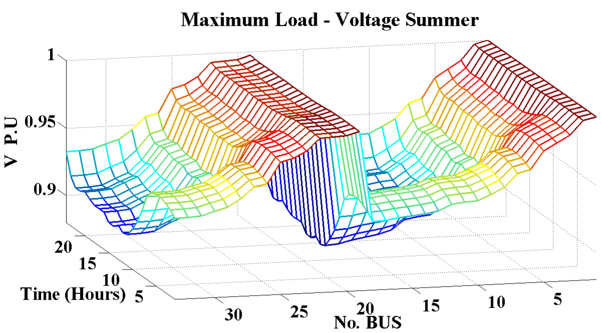

Fig. 13: Voltage profile of every hour in summer for $(\mu_{PLj}+\sigma_{PLj}, \mu_{QLj}+\sigma_{QLj})$.

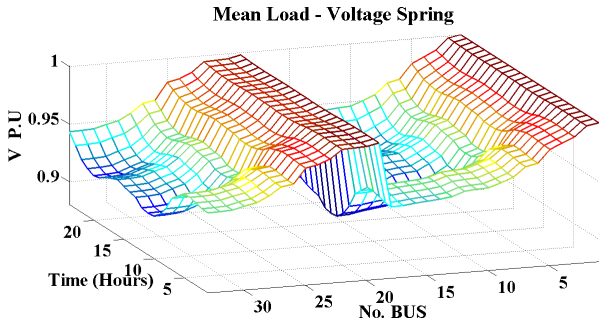

Fig. 14: Voltage profile for each hour in spring versus load average.

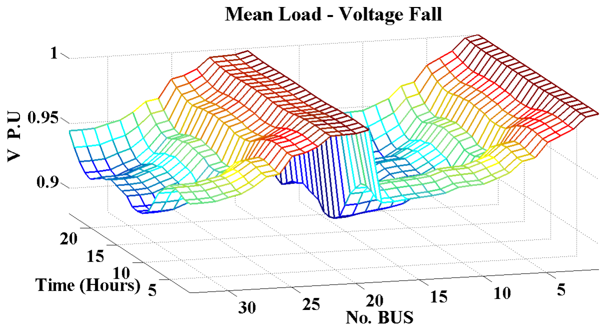

Fig. 15: Voltage profile for each hour in fall versus load average.

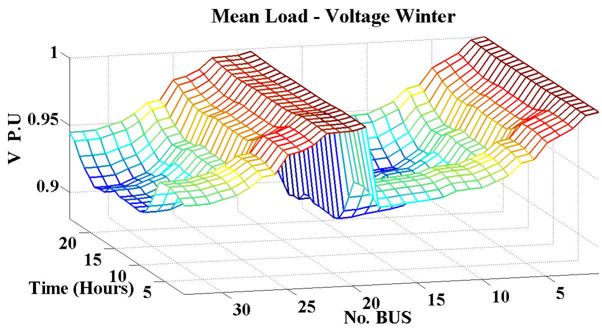

Fig. 16: Voltage profile for each hour in winter versus load average.

6. Conclusion

In this paper, probabilistic power flow with normal distribution function of loads has been proposed to deliver a deeper insight into the system performance. This method relies on the statistical data obtained in different times in 24-hour of different seasons. The obtained data was computed using normal distribution function with seven standard deviations. The power losses variation during 24-hour and all seasons are illustrated. The proposed approach tested on the IEEE 33-bus system. Comparison of loss and total voltage deviation for each bus between conventional and proposed method shows accuracy and superiority of the suggested method.

References

[1] MORALES, J. M., L. BARINGO, A. J. CONEJO and R. MINGUEZ. Probabilistic power flow with correlated wind sources. *IET Generation, Transmission & Distribution*. 2010, vol. 4, iss. 5, pp. 641–651. ISSN 1751-8695. DOI: 10.1049/iet-gtd.2009.0639.

[2] BORKOWSKA, B. Probabilistic load flow. *IEEE Transactions on Power Apparatus and Systems*. 1974, vol. 93, iss. 3, pp. 752–759. ISSN 0018-9510. DOI: 10.1109/TPAS.1974.293973.

[3] DOPAZO, J. F., O. A. KLITIN and A. M. SASSON. Stochastic load flows. *IEEE Transactions on Power Apparatus and Systems*. 1975, vol. 94, iss. 2, pp. 299–309. ISSN 0018-9510. DOI: 10.1109/T-PAS.1975.31855.

[4] ALLAN, R. N. and A. M. LEITE DA SILVA. Probabilistic load flow using multilinearisations. *IEE Proceedings C (Generation, Transmission and Distribution)*. 1981, vol. 128, iss. 5, pp. 280–287. ISSN 0143-7046. DOI: 10.1049/ip-c:19810047.

[5] LEITE DA SILVA, A. M., R. N. ALLAN, S. M. SOARES and V. L. ARIENTI. Probabilistic load flow considering network outages. *IEE Proceedings C (Generation, Transmission and Distribution)*. 1985, vol. 132, iss. 3, pp. 139–145. ISSN 0143-7046. DOI: 10.1049/ip-c:19850027.

[6] ZHANG, P. and S. T. LEE. Probabilistic load flow computation using the method of combined cumulants and Gram-Charlier expansion. *IEEE Transactions on Power Systems*. 2004, vol. 19, iss. 1, pp. 676–682. ISSN 0885-8950. DOI: 10.1109/TP-WRS.2003.818743.

[7] SANABRIA, L. A. and T. S. DILON. Stochastic power flow using cumulants and Von Mises functions. *International Journal of Electrical Power and Energy Systems*. 1986, vol. 8, iss. 1, pp. 47–60. ISSN 0142-0615. DOI: 10.1016/0142-0615(86)90025-6.

[8] LEITE DA SILVA, A. M., V. L. ARIENTI and R. N. ALLAN. Probabilistic load flow considering dependence between input nodal powers. *IEEE Transactions on Power Apparatus and Systems*. 1984, vol. 103, iss. 6, pp. 1524–1530. ISSN 0018-9510. DOI: 10.1109/TPAS.1984.318493.

[9] TENG, J. A Network Topology-based Three Phase Load Flow for Distribution Systems. *Proceedings of the National Science Council*. 2000, vol. 24, no. 4, pp. 259–264. ISSN 0255-6596.

[10] BILLINTON, R. and W. LI. *Reliability Assessment of Electrical Power Systems Using Monte Carlo Methods*. New York: Plenum Press, 1994. ISBN 978-0306447815.

[11] ALLAN, R. N, A. M. LEITE DA SILVA and R. C. BURCHETT. Evaluation methods and accuracy in probabilistic load flow solutions. *IEEE Transactions on Power Apparatus and Systems*. 1981, vol. 100, no. 5, pp. 2539–2546. ISSN 0018-9510. DOI: 10.1109/TPAS.1981.316721.

[12] ZHANG, P. and S. T. LEE. Probabilistic Load Flow Computation Using the Method of Combined Cumulants and Gram-Charlier Expansion. *IEEE Transactions on Power Systems*. 2004, vol. 19, no. 1, pp. 676–682. ISSN 0885-8950. DOI: 10.1109/TPWRS.2003.818743.

[13] SUBCOMMITTEE, P. M. IEEE reliability test system. *IEEE Transactions on Power Apparatus and Systems*. 1979, vol. 98, no. 6, pp. 2047–2054. ISSN 0018-9510. DOI: 10.1109/TPAS.1979.319398.

[14] ACHARYA, N., P. MAHAT and N. MITHULANANTHAN. An analytical approach for DG allocation in primary distribution network. *International Journal of Electrical Power and Energy Systems*. 2006, vol. 28, iss. 10, pp. 669–678. ISSN 0142-0615.

[15] KASHEM, M. A., V. GANAPATHY, G. B. JASMON and M. I. BUHARI. A novel method for loss minimization in distribution networks. In: *International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*. London: IEEE, 2000, pp. 251–256. ISBN 0-7803-5902-X. DOI: 10.1109/DRPT.2000.855672.

About Authors

Shahrokh SHOJAEIAN received his B.Sc. and M.Sc. degree from Isfahan University of Technology, Isfahan, Iran, in 1997 and 2000 respectively, and Ph.D. degree from Islamic Azad University, Science and Research Branch, Tehran, Iran, in 2012, all in electrical engineering. He is currently with the Faculty of Electrical Engineering, Islamic Azad University, Khomeinishahr Branch, Isfahan, Iran. His research interests are nonlinear control, power system control, stability, and reliability.

Ehsan SALLEALA-NAEENI received the B.Sc. and M.Sc. degree in electrical engineering from Islamic Azad University, Khomeinishahr Branch, Isfahan, Iran, in 2009 and 2010 respectively. His research

topics include power system simulation, renewable energy resources, and reliability, and evolutionary algorithms.

Ehsan TASLIMI RENANI received the B.Sc. degree in electrical engineering from Islamic Azad University, Khomeinishahr Branch, Isfahan, Iran, in 2008 and M.Sc. degree in industrial electronic from University of Malaya (UM), Kuala Lumpur, Malaysia, in 2012. He is currently working as research assistance in the UM Power Energy Dedicated Advanced Center (UMPEDAC). His research topics include demand side management and power electronics.