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The Classification and Identification of Asset Price Bubbles*

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Abstract
This article briefly summarizes approaches to and options for identifying bubbles in asset prices. Further, the article draws on bubble literature and it theoretically discusses classification of asset price bubbles according to features as differences in rationality of investors, their informational (a)symmetry, limits in arbitrage and heterogeneous beliefs. It also highlights the identification problems related with determining the fundamental value of an asset. We argue that disequilibrium asset price is a necessary but not sufficient condition for finding a bubble in a given asset. The market and country specifics have to be borne in mind. The article also specifies the procedure for monitoring and early identification of asset price bubbles. We recommend using all available spectrum of tools in the most effective arrangement ranging from charting methods (trends, filters, price-to--income ratios) via one-equation fundamentals based models to complex and structurally rich models.

1. Introduction

This article intends to briefly introduce wide scope of bubble related literature. In particular the article summarizes approaches to and options for identifying disequilibrium asset price movements, i.e. the situation where an asset price moves significantly away from its fundamental-based value. The article takes into account mainly stock prices, real estate prices and exchange rates but do not discus them separately into details. Further, it focuses on theories relating to the emergence, dynamics and persistence of asset price bubbles and it also theoretically discusses main stream methods for identifying such bubbles. It also highlights the problems related with determining the fundamental value of an asset and, subsequently, the difficulty of distinguishing deviations of the market price of an asset from its fundamental value. The article also specifies the effective procedure for monitoring and early identification of asset price bubbles.

We define in general terms an asset price bubble as an explosive and asymmetric deviation of the market price of an asset from its fundamental value, with the possibility of a sudden and significant reverse correction.[1] Developing countries

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[1] In the classification of asset price bubbles presented later, this definition comes close to the concept of a rational bubble.

are mostly liable to higher grow and also volatility of asset prices, which appear namely from underdeveloped segments of financial market. Therefore we argue that for final assessment of risks of presence asset price bubbles, we have to be borne in mind the market and country specifics. Furthermore, the theories of asset price bubbles have not been sufficiently investigated for the small open economies.

Although the theoretical decomposition of an asset price into components arising from fundamental factors and components affected by non-fundamental factors (e.g. euphoria or over-optimistic investment sentiment) seems straightforward, empirical application associated with explicit expression of the values of individual components is very limited. Where non-fundamental factors account for a major part of asset price growth, identifying a bubble is more complicated, since non-fundamental factors are not directly measurable. However, market sentiment or risk perception can be approximated using specific indices, for instance the Chicago Board Options Exchange Volatility Index (VIX).[2] From a fundamental point of view, the current price of an asset is formed primarily on the basis of economic agents' expectations, which are based, in turn, on currently available information regarding the discounted future cash flows. An asset price can be expressed as a discount factor multiplied by the flows of all future payments relating to the asset. It is affected by the following factors: (a) the expected flow of returns that the asset may generate (e.g. dividend), (b) the expected flow of returns on holdings of an alternative asset, (c) the expected price realised on the future sale of the asset, and (d) the relative risk and liquidity associated with holding the asset.

Since the fundamental value of an asset is not directly observable, it must be estimated. This means we have to rely on the discounted flow of all future payments, which is not observable either and must be estimated. This gives rise to the first pitfall of identifying asset price bubbles, namely estimating the fundamental value of the asset. DeMarzo, Kaniel and Kremer (2007) tighten up the definition of bubble by specifying three components: (1) the market price of an asset is higher than the discounted sum of its expected cash flows, with the discount factor being equal to the risk-free interest rate; (2) cash flows have a non-negative correlation with aggregate risk; (3) risk-averse investors rationally choose to hold the asset, despite their knowledge of (1) and (2). In an attempt to estimate the fundamental value more realistically, Ofek and Richardson (2003) define a range for the fundamental value of an asset, with the upper boundary of the range being of more interest. The upper boundary is formed on the basis of an estimate of the maximum achievable future cash flows of a company in a given sector and the minimum possible discount factor. Subsequently, if the market value of the asset is still higher than the fundamental value estimated in this way, a bubble is found in the price of the asset. Siegel (2003) proposes an operational definition of an asset price bubble as any time the realised asset return over given future period is more than two standard deviations from its expected return. He argues that bubble is not possible to identify immediately, but one has to wait a sufficient amount of time to determine whether the previous prices can be justified by subsequent cash flows.

Why are bubbles so popular and why are we interested in them? The first perspective is the interest of economic agents – in particular economic policy makers

[2] The indicator of implied volatility derived from option prices reflects traders' consensus forecast regarding short-term volatility and sentiment for the coming 30 days.

– in the effects of bubbles on the real economy, especially when those bubbles burst. The other perspective is investors' potential ability to use knowledge of the formation of bubbles to achieve excess returns – see, for example, Brunnermeir and Nagel (2004). These authors show the timing of riding the technology bubble in the case of some hedge funds. Although the two perspectives have different motivations, they share an intense effort to identify bubbles *ex ante*.

Seen from the first perspective, bubbles can affect the real economy either by distorting economic – and especially investment – decisions, or via the wealth channel and through banking sector balance sheets, i.e. specifically by affecting: (i) household consumption through the wealth channel (growth in property prices and financial assets held by households is perceived as growth in wealth and consumption financing sources), (ii) the banking sector balance sheet (property prices often serve as collateral in lending operations),[3] (iii) investment (according to Tobin's Q theory, the capital available for investment becomes cheaper as a result of growth in market equity prices, which in the case of a growing bubble implies an excessive decrease in the price of capital and distortion of investment decisions, hence inefficient investments with negative effects in the future may be made). These effects differ in strength over time and across economies, but they affect the real economy in the same direction.[4] The issue of whether the performance of the economy will be affected when a bubble bursts does not depend solely on asset prices; also important are the economic environment, the state of the financial sector, its ability to absorb shocks, its vulnerability and its fragility and the subsequent probability and strength of the monetary or fiscal policy response.

Empirical research confirms the economic intuition that more serious impacts on the real economy stem from the bursting of property market bubbles than that of stock market bubbles (see, for example, Helbling and Terrones, 2003a,b, and Bordo and Jeanne, 2002a,b). The effects stemming from the sudden bursting of property market bubbles generate higher output losses and last longer on average (about 4 years) than in the case of stock market bubbles (around 1.5 years). The bursting of property market bubbles also poses a greater threat to the financial stability of a country/region if the banking sector is more exposed through loans secured by property. On average it also generates greater social tension associated with subsequent housing-related problems.

The purely investor perspective differs from the economic policy maker perspective. The aim here is to capture the moment of reversal in the price of an asset and to profit from this. It is clear that this set of investors is in the minority, otherwise a bubble could not occur.

Some asset prices (property prices, certain share prices) are procyclical, i.e. they rise during the upward phase of the economic cycle. This is usually accompanied excessive lending, which feeds the rise in asset prices. Such procyclicality

[3] If property prices rise, the probable loss from selling the collateral on mortgage loans decreases, which, in turn, notionally increases the bank's capital and allows it to expand its investments and loans. However, a slump in property prices can lead to credit constraints, a credit crunch and a negative impact on economic activity.

[4] According to many studies (e.g. Bordo and Jeanne, 2002a,b, and Borio and Lowe, 2002), credit booms and asset price busts have had grave financial and economic consequences. For the discussion of asset prices and monetary policy from the perspectives of transitional economies see Frait and Komárek (2007).

may stem from a favourable macroeconomic environment and over-optimism among economic agents, which can destroy rational assessment of the situation and lead to excessive risk-taking, especially as regards credit risk. A correlation can thus be found between the cyclical behaviour of the real economy and cyclicality in risk perception and assessment. A cycle of financial risk thus arises as experience shows. This means that although perceived risk declines, the actual risk that might materialise in the future rises during boom times and financial imbalances increase at the same time. The risk materialises at times of falling economic growth, while the financial imbalances should be gradually eliminated.

The main ambition of this article is to present the main theories on the emergence of asset market bubbles and to theoretically outline how bubbles can be identified using traditional quantitative methods. The text is structured as follows. Section 2 outlines the issue of why bubbles emerge and why they could persist. Section 3 defines and discusses the classification of bubbles. Section 4 highlights the specific characteristic of each asset markets. Section 5 introduces the options for the empirical testing of bubbles. The final section concludes.

2. How do Bubbles Emerge?

A theoretical effort to understand bubbles was made by Keynes (1936, p. 138), who admitted the possible emergence of speculative bubbles. By contrast, Friedman (1953) and Fama (1965) tended towards the view that bubbles cannot form because rational speculators stabilise prices and sophisticated investors will not let bubbles develop.[5]

Experience from the world economy has shown that bubbles appear and disappear repeatedly in various asset categories; moreover, they tend to spill over between asset markets. What is generally known about bubbles is their dynamics, which tend to involve an exponential increase in the asset price with reverse correction afterward. However, the reasons for the emergence of bubbles are not as clear. Given the complexity of the environments in which the bubbles appear, it is likely that their emergence is conditional on the interplay of macroeconomic and microeconomic factors. Following recent developments in the global economy, excessively easy monetary policies of central banks, global liquidity surpluses and insufficient financial sector regulation and supervision have been blamed as contributors to the emergence of bubbles.[6]

A technology bubble developed in the USA as a result of growth in share prices due to rapid technological change. This led to exaggerated expectations among investors and imprudent assessment of the risks of individual share issues. Although the evaluation of these issues might initially have been based on careful analysis and rational consideration, the subsequent overvaluation was more a sign of irrationality, over-optimism and self-fulfilling expectations.[7] As a result of this episode, both

[5] Friedman (1953) distinguishes between stabilizing and destabilizing speculation. He argues that rational speculators stabilize prices although there are also speculators which drive market prices away from the fundamental value by (on average) buying assets when prices are high and selling them when prices are low. But such destabilizing speculators are excluded from the market relatively soon.

[6] Bubbles are examined here from the theoretical and microeconomic or financial point of view. Issues relating to economic policy and the global liquidity surplus have been deliberately left out.

[7] The debate about the technology bubble and its emergence is far from over. For example Pastor and Veronesi (2006) provide an interesting debate about the technology stock.

academics and practitioners started to think more carefully about the issues explaining the emergence of bubbles: Why did rational investors not enter the market and try to get market prices to converge to their fundamental values? What kind of irrationality can appear in trading? Were there too few rational investors to affect market prices? The following paragraph will outline – and section 3.2 will describe in more detail – the following key question: Why do bubbles persist even with rational investors? Can arbitrage[8] fail to have a converge effect on prices, i.e. can it fail to lead to convergence or postponed convergence of the market price to the fundamental price?

One possible way of clarifying the emergence of bubbles is to draw on behavioural finance theory.[9] This involves a belief that the residual value in the pricing of an asset is due not to the omission of a fundamentally important variable from the asset valuation equation, but to not fully rational behaviour by some economic agents. If the mispricing of an asset is recognised by rational investors, it should theoretically be removed by arbitrage. Arbitrageurs should take a position that will cause the mispricing to be eliminated and, as a result, bubbles should theoretically not emerge. But those investors could have difference motivation in taking positions and also they could have limitations to enter in "price stabilizing (converging)" position. Practical experience has shown that asset mispricing can persist for quite some time.[10] This raises the issue of arbitrage effectiveness and the speed of convergence between actual price and fundamental value of the assets. According to behavioural finance theory, the functioning (effectiveness) of arbitrage is affected by costs (e.g. transaction costs, holding costs, identification costs) and the risk (e.g. noise trade risk, synchronization risk and fundamental risk) assumed by arbitrageurs arising from such transactions. For these reasons, arbitrage may be limited. The issues are addressed, for example, by Barberis and Thaler (2003), Abreu and Brunnermeier (2002, 2003), De Long et al. (1990) and Shleifer and Vishny (1997). Other possible answers to the key question regarding the persistence of bubbles can be found in articles focusing on positive feedback bubbles (e.g. De Long et al., 1990b),[11] or heterogeneous beliefs combined with short-sale constraints (e.g. Ofek and Richardson, 2003).

[8] Arbitrage is according to Sharpe and Alexander (1990) defined as „the simultaneous purchase and sale of the same, or essentially similar, security in two different markets for advantageously different prices". Also as Schleifer and Vishny (1997) pointed out theoretically such arbitrage requires no capital and entails no risk.

[9] This theory helps explain market "inefficiency" primarily by considering psychological factors which influence the behaviour of economic agents (e.g. overconfidence, wishful thinking) and by considering so called limits to the arbitrage. Behavioural finance theory uses models in which some agents are not fully rational. According to this theory, rational and irrational agents trade with each other which can result in persistence of mispricing even for relatively long time. Moreover this mispricing also relates with various incentives of traders (e.g. different trading strategies) and limits to enter to position (e.g. implementation costs and identification costs).

[10] Experience over the centuries has shown the opposite outcome. The history of bubbles dates back, for example, to the 17^{th} century (tulip mania) and the 18^{th} century (the South Sea bubble).

[11] Positive feedback investors buy assets when prices rise and sell them when prices fall. Specifically, their strategies involve, for instance, following the trend or extrapolating asset price expectations as well as trading using stop-loss orders, i.e. liquidation of positions when the asset price reaches a certain limit. According to DeLong et al. (1990b) rational speculation can be destabilizing if positive feedback traders are present.

3. Classification of Bubbles

Individual views about the emergence of asset price bubbles[12] differ on the causes of the emergence of bubbles as well as on suitable methods for identifying and recognising them in real time. Four main categories of models – differing according to the conditions of the emergence of bubbles – are recorded in the literature (see, for example, Brunnermeier, 2007). The first category of models is based on the assumptions of rational investors and identical information. In this approach, bubbles follow an explosive path. The second category again involves rational investors, but this time they are asymmetrically informed. Bubbles form under more general conditions since their existence need not be generally known, unlike in the previous category of models. The third category is related to behavioural finance theory. In this case, bubbles can persist for a time, since the limits in arbitrage inhibit rational investors from the elimination of price impacts caused by behavioural traders. Unlike the two previous categories, this category does not rely solely on fully rational investors, but focuses on the interaction and trading between rational and irrational investors. The fourth category assumes that bubbles emerge under heterogeneous beliefs of investors about fundamental values, which can be based on psychological biases.

3.1 Rational Bubbles – Rational Investors and Symmetric Information

3.1.1 The Rational Bubble According to Blanchard and Watson

Rational bubbles are generated by extraneous events and the spreading of rumours. The emergence of such bubbles is due to investors' self-fulfilling expectations regarding future asset price growth which are not directly related to fundamentals, e.g. waves of over-optimism and sentiment. A rational bubble must grow at a rate that produces the expected rate of return. It gains in size as investors expect the asset to be sold at a profit in the future.

The emergence of rational bubbles, as specified in Blanchard and Watson (1982), is motivated by non-fundamental factors. The market price of asset can be decomposed into the following components:

$$P_t = P_t^f + B_t \tag{1}$$

where P_t is the market price of the asset, P^f_t denotes the fundamental value of the asset and B_t is the bubble component. Blanchard and Watson's contribution consists in specifying the dynamics of the bubble, which grows at a rate $\bar{r}$ with probability π and will burst with complementary probability $(1 - \pi)$; the bubble contains the forecast error u_t, i.e.:

$$B_{t+1} = \begin{cases} \dfrac{\left(1+\bar{r}\right)B_t}{\pi} + u_{t+1} & \text{with probability } (\pi) \\ 0 + u_{t+1} & \text{with probability } (1-\pi) \quad E\left(u_{t+1}\right) = 0\,, \end{cases} \tag{2}$$

where $\bar{r}$ is the long-term average return on the asset and $E\left(\bullet\right)$ denotes the expected variable.

[12] See, for example, Blanchard and Watson (1982) or Froot and Obsfeld (1991).

Before the bubble bursts, the present return on assets grows faster than the historical average return. This is due to investors' assumption that the asset price will continue rising and generate the required expected rate of return. If investors believe the bubble will burst in the future, it will indeed burst as a result of self-fulfilling expectations.

3.1.2 The Intrinsic Bubble According to Froot and Obsfeld

The emergence of a rational bubble described above is not related to fundamental factors (e.g. shares in relation to dividends) and the bubble component arises outside the framework of such factors, as a result of non-fundamental factors. Nevertheless, one can identify a bubble that is a specific example of a rational bubble – the so-called intrinsic bubble of Froot and Obsfeld (1991). A prerequisite for this bubble is the dependence of an asset on fundamentals represented by dividends in a non-linear deterministic approach. The bubble thus arises from overreactions to reports concerning stochastic dividends. It is specified as a non-linear power function of the product of an arbitrary constant and dividends raised to a higher power:

$$B\left(D_t\right) = cD_t^{\lambda} \tag{3}$$

where B denotes the bubble component and D_t dividends; c and λ are parameters satisfying the conditions $\lambda>1$ and $c>0$. It holds true that:

$$\hat{P}_t = P_t^f + B\left(D_t\right) \tag{4}$$

where $\hat{P}_t$ is the market price of the stock and $\hat{P}_t^f$ is the fundamental-based part of the price.

The bubble component $B(D_t)$ is determined by changes in dividends. Their growth rate is given by trend growth in dividends, the log of dividends and a random term with conditional mean zero and variance (a so-called geometric martingale of dividends). If the fundamentals remain unchanged, the bubble component remains constant. If the fundamentals show persistence, the bubble component and the asset price will be exposed to persistent deviations from the fundamental value.

The above-mentioned bubbles are equally driven by self-fulfilling expectations, but with the difference that the Blanchard and Watson bubble is affected by factors that do not relate to fundamentals, while the intrinsic bubble is affected by fundamental factors.

3.2 Bubbles Relating to Limited Arbitrage

As indicated in the introduction, there may be a category of investors whose trading is influenced by constraints and behavioural factors, i.e. investors who can make systematic mistakes and are thus not fully rational (behavioural traders). In this environment, bubbles form as a result of the interaction between fully rational, sophisticated, fully informed investors (rational arbitrageurs) and partially (ir)rational investors (behavioural traders). Given limited arbitrage, bubbles can persist for a time, since rational arbitrageurs cannot and/or do not want to fully correct the mispricing of assets for the reasons described below (see for example Barberis and Thaler, 2003).

3.2.1 Fundamental Risk

An arbitrageur is exposed to fundamental risk by being wrong about the arbitrage position he takes and suffering a loss as a result. The pre-requisite is that there are no perfect substitutes between assets in particular class of asset. For instance, the arbitrageur buys shares A and, at the same time, sells shares B, which are a substitute for shares A. The arbitrageur is aware of the possible downside risk for shares A; for that reason, he/she has created a hedging position in shares B. Nevertheless, shares A are not necessarily a perfect substitute for shares B and shares B may be mispriced, so it is very difficult to eliminate all fundamental risk (see Barberis and Thaler, 2003, p. 1056).

3.2.2 Noise Trade Risk

Let assume, that for the noise trade risk the pre-requisite of perfect substitutes holds. The interpretation of De Long et al. (1990a) is based on the idea that arbitrageurs cannot fully reverse the mispricing caused by noise traders. This is because arbitrageurs are active in the short run and are risk averse (i.e. they are exposed to noise trade risk). Noise traders do not made decision upon fundamental analysis and (falsely) believe that they have special information[13] about the future price of an asset that other traders are lacking. The interaction between fully and partially rational investors deflects asset prices from their fundamental values. Schleifer and Vishny (1997) explain the activity of arbitrageurs operating in the short term as described in De Long et al. (1990a). Arbitrageurs have short horizon because they often manage funds of various investors and lenders. Their performance is judged according to short term returns, thus it leads to focusing on short term horizons. Also the next reason is separation "brain and capital" which means that resources are separated from agents involved in arbitrage, i.e. relatively small group of arbitrageurs specialized at such trading manage money of other investors and it possibly creates agency relationship. Thus, the specialized arbitrageurs may avoid some extremely volatile positions which could be labelled as arbitrage due to risk of losses resulted from volatile and extreme positions regardless of attractive return profile of transaction.

3.2.3 Synchronisation Risk

Abreu and Brunnermeier (2002, 2003) answer the question regarding the persistence of bubbles by means of limited arbitrage caused by synchronisation risk. The synchronisation problem involves different views among rational arbitrageurs leading to (non-) synchronisation of the timing of the correction of the mispricing, that is, the rational arbitrageurs are not sure when their colleagues will take advantage of arbitrage and try to time this moment while riding the bubble. Thus, they are aware of the existence of the bubble and know that it will burst at some future point in time. By timing, they delay arbitrage and thus cause temporary persistence of the bubble. If they timed their position and jointly exerted pressure to sell, they would have sufficient strength to burst the bubble. Clearly, this is delicate timing game about when behavioural traders will be overwhelmed by rational arbitrageurs.

[13] Based, for instance, on technical analysis, the use of momentum, various recommendations, etc. See for example literature survey by Menkhoff and Taylor (2007) or Bask and Fidrmuc (2009), which are focused on technical analysis and noise traders and applied on the exchange rates.

4. Specific Characteristics of Asset Markets

The tools for identifying asset market bubbles are often based on the assumption that market prices deviate from the fundamental asset value. For identification purposes, it is necessary to bear in mind the specific attributes of those markets.[14] These include different trading liquidity, frequency and volumes, transaction costs, market rigidities, lags in data as well as lags caused by legal procedures, etc. To identify bubbles in a particular segment of the asset market, it can be helpful to consider developments in other segments of the market and other indicators such as monetary aggregates and credit and investment indicators. This can be helpful especially in case of real estate markets which are closely related with credit growth.

In the case of empirical analysis, it is necessary to take into account the specific characteristics of financial time series. Such series contain certain features that can affect the robustness of the empirical tests. Not only are exchange rates and stocks more volatile than macroeconomic variables, they can also show conditional heteroscedasticity, leptokurtic distributions, volatility clusters, stationarity and asymmetry in response to positive and negative movements in profit ratios.

The degree of misalignment or the presence of bubbles on asset markets can be determined by means of ratios which generally use very narrow set of fundamentals or by applying a model approach based on wider set of fundamentals. Price ratios (e.g. price to earnings, price to income) give an initial idea about an asset market, but they suffer from numerous shortcomings, in particular: (i) the alarm ratio indicating overvaluation or the existence of a bubble is ambiguous and volatile, i.e. comparing the ratio with its own historical values may be misleading, crucially dependent on the time horizon used; (ii) direct developments in interest rates are not taken into account; (iii) ratios have a narrower focus for assets for which returns can be achieved only through increases in their price (exchange rates, gold, commodities); (iv) time series have short histories, especially in the case of transition economies, so a long-term view of their evolution cannot be taken. In overall, price ratios are useful for getting a basic idea about a price development in respect of earnings or affordability at the particular asset market. Moreover, the modification of the ratios expressed as earnings to price can be compare with yield of other financial instruments (e.g. bond yields) to get rough picture about relative yield between financial instruments. Generally, though, it is appropriate – for both financial and non-financial assets – to combine a number of approaches to identifying bubbles in order to cover factors indirectly associated with asset prices (structural changes in the economy, demographic factors, etc.).

5. Options for the Empirical Identifying of Bubbles

The range of techniques suitable for identifying for the occurrence of bubbles is limited by the features of bubbles (asymmetrical dynamics, non-linearity, sudden and difficult-to-identify changes in investor sentiment, bad vs. good asset pricing, news and its incorporation into the price etc.). The success of econometric tests is ambiguous, since the tests results differ. Gurkaynak (2005) states in the conclusion of his empirical study that for every test that finds a bubble, there is another that disputes it. Moreover, he points out that we are still unable to distinguish bubbles from

[14] Real estate market, foreign exchange market and stock exchange market.

time-varying or regime-switching fundamentals. In our opinion, however, the primary question is whether the heterogeneity of the results of empirical testing is based on inadequately specified models or whether the bubbles are more a psychologically and behaviourally determined phenomenon that is very difficult to explain on the basis of fundamentals. This is consistent with a thorough knowledge of asset price determinants and should lead, in combination to expert judgment, to the reliable evaluation of forming bubbles.

Identification of the bubbles is not just about the confirmation of the existence of the bubble,[15] but also as mentioned in the part of classification bubbles, about specification of the formation process. Salge (1997) describes the specification of the formation bubble process as the direct tests (see, for instance, the intrinsic bubble of Froot and Obsfeld, 1991).[16] More general approach for bubble identifications, so-called indirect tests, differs from the direct test in sense of not directly specifying the bubble formation process (e.g. cointegration methods and specification test). Indirect test are designed to purely confirm or to refute the existence of a bubble. Further, this part of the article discuses the basic techniques for identifying bubbles also from the field of indirect tests, which consists mainly from (a) trend curves, statistical filters and price ratios, (b) empirical methods and models, and (c) specification tests.

5.1 Trend Curves, Statistical Filters and Price Ratios

The statistical approach to identifying misalignment of asset prices compares the time series trend with actual development. This approach can be used not only to get an initial idea of the degree of misalignment, but also determine asset price bubbles, which is forming through asset price booms. For the case of using trend approach one needs long[17] historical time series which have not been necessarily available especially in emerging or developing countries. Nevertheless, it is not necessarily the case *ex ante* that these results will be less successful than those obtained using much more sophisticated approaches. Univariate filters such as the Hodrick-Prescott (HP) filter or Band-Pass (BP) filter can be used to calculate the trend, which is broadly used in many studies.[18]

Goodhart and Hofmann (2008) follow the approach of Borio and Lowe (2002) and Adalid and Detken (2007) to define aggregate asset price booms. A house price--boom is defined as a persistent deviation of real house prices from a smooth trend, calculated on a one-sided HP filter with a (high) smoothing parameter of 100,000. A boom is defined as a positive deviation of house prices from this smooth trend of more than 5 percent lasting at least 12 quarters. Adalid and Detken (2007) follow a similar procedure to Goodhart and Hofmann (2008). A boom is defined as a persistent deviation from an HP trend (with smoothing parameter of 100,000) of more than 10 percent lasting at least 4 quarters. Bordo and Jeanne (2002) focus on the growth

[15] The effort to diagnose bubbles *ex ante* has even involved the use of tools taken from theories of complex systems (e.g. Sornette, 2003).

[16] Direct tests pertain mainly to the stock market (often testing the relationship between dividends and share prices) and are sensitive to misspecification of the model, e.g. the omission of an important determinant.

[17] Approximately 30–40 years of real estate market is suitable for the trend analysis.

[18] Having regard to the underdevelopment of some markets, especially in transition (developing) countries, which show strong growth, that is not possible to see as the bubble (base effect).

rate of asset price series to define booms.[19] A boom is defined where the 3-year (from t–2 to t) moving average growth rate of the (real) asset price exceeds the series average by a factor of 1.3 times the series standard deviation. When this condition is met, then a boom is declared in periods t–2, t–1 and t.

Also price ratios (price to earnings, price to income) could enter into statistical filtering. The divergence of actual ratio from the long-run development expressed by the trend may signal misalignment. The judgment about the existence of the bubble is based on distance between actual value of the ratio from the estimated trend by the number of standard deviations. Hume and Sentence (2009) investigate credit booms defined as deviations of credit to GDP ratios from an expanding HP filter as used in Gourinchas, Valdes and Landerretche (2001). An expanding HP filter extends the sample over which the filter is applied by one period as each successive period is added to the sample, such that for any point in time the trend is calculated only on data up to, and not beyond, that point in time. Hume and Sentence (2009) use a smoothing parameter of 100, a standard smoothing parameter for annual data. They define a boom (and so setting its threshold) as occurring when the data series deviated from its trend by more than one standard deviation, and set limit thresholds – determining when the boom begins and ends- as half a standard deviation.

5.2 Empirical Methods and Models

Diba and Grossman (1988) tested share prices in relation to dividends. By means of unit root tests and cointegration analysis, they observed the order of integration of these time series and whether there is an explosive element in the asset price time series. If the growth rate of the asset price is not more explosive than that of the key fundamental, a bubble is not present. If it were present, it would generate an explosive element in the relevant price. Classic unit root tests[20] and subsequent cointegration analysis are associated with methodological problems, i.e. standard linear econometric methods are not necessarily sufficient to determine the non-linear behaviour of an asset price component, particularly in the case of periodically collapsing bubbles (see, for example, Evans, 1991). Subsequently, the tests have been extended to cover the possibility of testing for a unit root in a model with a time-varying auto-regressive coefficient and in regime-switching models (for example van Norden, 1996, and van Norden and Vigfusson, 1998). This line of testing was chosen mainly because of a time-varying risk premium, which can be a source of excessive fluctuations, causing complications when testing for the presence of a bubble in traditional tests, i.e. those that do not take account of the time-varying variable. Proving the non-stationarity of a time series does not necessarily imply the presence of an asset price bubble.

Additionally, beyond of the narrow set of fundamentals as mentioned above one can applied an approach taking advantage of wide set of fundamentals linked to

[19] Detken and Smets (2004) argue that the use of an asset price gap (as in methods 1–3) is preferable to the growth rate for defining a boom as the price gap allows the concept of accumulated financial imbalances to be stressed – reducing the weight of periods of rapid asset price growth directly following an asset price collapse. It also allows sustained periods of only slightly above average growth to cumulate into a boom in levels space.

[20] See, for example, the Augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test or the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test.

particular market and using deeper structure of causalities. This approach seems to be applicable tool for identifying determinants especially for the real estate market on which is mainly focusing this special issue. Modelling both demand and supply side of the market brings valuable insight in to determination of value and thus indirectly judgment about potential misalignment. Methodologically there exists variety of possible estimation techniques, e.g. VAR models, classical and panel data regressions, etc. Subsequently, provided that the model is specified correctly, it is able to map non-standard movements on the market and partly or fully explain asset price movements using determinants.

Further, there exist broad spectrum of structurally-reach models, which can be used for deeper understanding of processes behind forming the bubbles. Based on the findings of the current financial crisis was introduced variety of theoretical models related to asset price bubbles. For example Barlevy (2008) offers a model in which credit plays an essential role in allowing for speculative bubbles that can be used to explore these claims. Dubecq, Mojon and Ragot (2009) set up a model where asset price bubbles due to risk shifting can be moderated by capital requirements, under imperfect information. Interesting empirical work was done by Alessi and Detken (2009), who find that global measures of liquidity are among the best performing indicators. They display forecasting records, which provide useful information for policy makers interested in timely reactions to growing financial imbalances.

5.3 Specification Tests

The basic idea of specification tests is that they compare estimates for discount factors that are taken from differently defined formulations of forward-looking asset price models and subsequent performance of a specification test. The following model formulations can be compared (see, for example, Taylor and Sarno, 2002[21]). The first one is based on the equation:

$$s_t = \lambda_0 E_t s_{t+1} + v_t + \varpi_t \quad (5)$$

where the current asset price s_t over time is an additive function of the expected discounted value of the asset in the subsequent period $\lambda_0 E_t s_{t+1}$ (containing the discount factor λ_0), the present fundamentals v_t and the error term ω_t.

An alternative estimate of the discount factor, derived from a purely fundamental asset model,[22] can be specified as follows:

$$\tilde{s}_t = \sum_{i=0}^{\infty} \lambda_1^i E_t v_{t+1} + \varpi_t \quad (6)$$

In the case of the alternative estimate, a simplifying assumption is made regarding the fundamentals, namely that they are determined by an AR(1)[23] process. The estimated discount factors are subsequently compared using the Hausman speci-

[21] The authors apply this approach to the exchange rate.

[22] The choice of specific fundamental model will greatly affect the bubble identification results. A bubble may be rejected either because it really does not appear in the data or, in the worse case, because the model is misspecified (an important variable has been omitted).

[23] AR(1): $v_t = \kappa v_{t-1} + u_t$, where coefficient $|\kappa| < 1$ and u_t denotes white noise.

fication test[24] (Hausman, 1978). If the discount factor estimates differ insignificantly, no bubble is found in the data.

6. Conclusion

Asset price bubbles have become a popular topic among both the public and experts and there is also wide discussion, whether they really exist or whether they constitute just an error in the valuation of assets. The ambition of the authors was not to contribute to a concrete debate about the existence of bubbles. The aim of the paper is to classify and distinguish different types of asset price bubbles and to point out the methods available for identifying asset overvaluation which can be potentially used for identifying bubbles. Identifying bubbles seems to be not easy task not only in time (especially *ex ante*), but also in terms of distinguishing fundamental and non-fundamental determinants and interpreting their value. It is therefore difficult to confirm with some degree of certainty the existence of a bubble, since it is no trivial task to determine the fundamental value of an asset or a bubble-generating deviation.

To conclude, we identify the following major issues related to identifying equilibrium asset price movements. Firstly, continuous monitoring of asset prices is necessary for early identification of unbalanced booms, which can later grow into asset price bubbles. Secondly, we recommend the use of the full spectrum of empirical tools in their optimum order ranking from simple to complex, i.e. from trend curves and statistical filters (such as Hodrick-Precsott or Band-Pass filters) through empirical methods and models (unit root tests, cointegration, specification tests) to deeply structured models (which are contextual in complex understanding of the economy). In other words, when we identify a possible unsustainable growth of assets by simple techniques such Hodrick-Prescott filter, it would be logical to pursue a deeper analysis based on structural models to identify crucial factors and determinants of the asset price movements. Thirdly, we argue that disequilibrium asset prices is a necessary but not sufficient condition for discerning a bubble in a given asset. Finally, we would remark that specifics across markets and countries have to be born in mind, since in underdeveloped markets sharp increases of monitored assets may not imply the creation of a bubble.

[24] Two discount factors are obtained from the estimates of equations (5) and (6), λ_0 from equation (5) and λ_1 from equation (6). The null hypothesis of the test states that both discount factor estimates are consistent (variance close to zero), but λ_1 is more efficient (has smaller asymptotic variance) than λ_0. The alternative hypothesis states that one or both discount factor estimates are inconsistent.

REFERENCES

Abreu D, Brunnermaier M (2002): Synchronization Risk and Delayed Arbitrage. *Journal of Financial Economics*, 66(2–3):341–360.

Abreu D, Brunnermaier M (2003): Bubbles and Crashes. *Econometrica*, 71(1):173–204.

Adalid R, Detken C (2007): Liquidity Shocks and Asset Price Boom/Bust Cycles. *ECB Working Paper Series*, no. 732.

Allesi L, Detken C (2009): "Real time" Early Warning Indicators for Costly Asset Price Boom/Bust Cycles: A Role for Global Liquidity. *ECB Working Paper Series*, no. 364.

Barberis N, Thaler R (2003): *Handbook of the Economics of Finance*. (Eds: G. M. Constantinides, M. Harris and R. Stulz). Elsevier Science B.V.

Barlevy G (2008): A Leverage-Based Model of Speculative Bubbles. *Federal Reserve Bank of Chicago Working Paper*, no. 2008-01.

Bask M, Fidrmuc J (2009): Fundamentals and Technical Trading: Behavior of Exchange Rates in the CEECs. *Open Econ Review*, 20:589–605.

Blanchard O, Watson M (1982): Bubbles, Rational Expectations and Financial Markets. In: Wachtel, P. (ed.): *Crises in the Economic and Financial Structure.* Lexington Books.

Bordo M, Jeanne O (2002a): Boom-busts in Asset Prices, Economic Instability, and Monetary Policy. *NBER Working Paper*, no. 8966.

Bordo M, Jeanne, O, (2002b): Monetary Policy and Asset Prices: Does Benign Neglect Make Sense? *International Finance*, 5(2):139–64.

Borio C, Lowe P (2002): Asset Prices, Financial and Monetary Stability: Exploring the Nexus. *BIS Working Papers*, no. 114.

Brunnermeir M (2007): Bubbles. In: Blume L, Durlauf S (Eds.): *The New Palgrave Dictionary of Economics.* New York, Oxford University Press.

Brunnermeir M, Nagel S (2004): Hedge Funds and the Technology Bubble. The *Journal of Finance*, LIX(5-October).

CNB (2007): *Financial Stability Report 2007*.

De Long JB, Schleifer A, Summers L, Waldmann R (1990a): Noise Trader Risk in Financial Markets. *Journal of Political Economy*, 98(4):703–738.

De Long JB, Schleifer A, Summers L, Waldmann R (1990b): Positive Feedback Investment Strategies and Destabilizing Rational Speculation. *Journal of Finance*, 45(2):379–395.

DeMarzo P, Kaniel R, Kremer I (2007): Relative Wealth Concerns and Financial Bubbles. *Review of Financial Studies*, 21(1):19–50.

Detken C, Smets F (2004): Asset price booms and monetary policy. *ECB Working Paper Series*, no. 364.

Diba B, Grossmann H. (1988): The Theory of Rational Bubbles in Stock Prices. The *Economic Journal*, September.

Dubecq S, Mojon B, Ragot X (2009): Fuzzy Capital Requirements, Risk-Shifting and the Risk Taking Channel of Monetary Policy. *Bank de France, Working Paper Series*, no. 254.

Evans GW (1991): Pitfalls in Testing for Explosive Bubbles in Asset Prices. *American Economic Review*, 4:922–930.

Fama EF (1965): The Behavior of Stock-Market Prices. *Journal of Business*, 38(1):34–105.

Frait J, Komárek L (2007): Monetary Policy and Asset Prices: What Role for Central Banks in New EU Member States? *Prague Economic Papers*, (1):3–23.

Friedman M (1953): *The Case for Flexible Exchange Rates. Essays in Positive Economics*. Chicago, Chicago University Press.

Froot K, Obstfeld M (1991): Intrinsic Bubbles: The Case of Stock Prices. The *American Economic Review*, December.

Goodhart C, Hofmann B (2008): House prices, money, credit, and the macroeconomy, *Oxf Rev Econ Policy*, 24:180–205.

Gourinchas PO, Valdes R, Landerretches O (2001): Lending Booms: Latin America and the World. *Economica*, 1(2):47–99.

Gurkaynak RS (2005): Econometric Tests of Asset Price Bubbles: Taking Stock. *Finance and Economics Discussion Series, Federal Reserve Board*, Washington, D.C.

Hausman JA (1978): Specification Tests in Econometrics. *Econometrica*, 46(6):1251–1271.

Helbling T, Terrones M (2003a): Real and Financial Effects of Bursting Asset Price Bubbles. (IMF) *World Economic Outlook*, April 2003.

Helbling T, Terrones M (2003b): *Asset Price Booms and Busts – Stylized Facts from the Last Three Decades of the 20th Century.* Paper presented at the European Central Bank workshop entitled Asset Prices and Monetary Policy, Frankfurt, 11–12 December.

http://www.bepress.com/snde/vol3/iss1/art1.

Hume M, Sentence A (2009): The Global Credit Boom: Challenges for Macroeconomics and Policy. *Bank of England External MPC Unit Discussion Paper*, no. 27.

Keynes JM (1936): The *General Theory of Employment, Interest, and Money*. Macmillan, London.

Menkhoff L, Taylor MP (2007): The Obstinate Passion of Foreign Exchange Professionals: Technical Analysis. *Journal of Economic Literature*, 45(4):936–972.

Norden S van (1996): Regime Switching as a Test for Exchange Rate Bubbles. *Journal of Applied Econometrics*, 11 (July):219–251.

Norden S van, Vigfusson R (1998): Avoiding the Pitfalls: Can Regime-Switching Tests Reliably Detect Bubbles? *Studies in Nonlinear Dynamics & Econometrics*, 3(1): Article 1

Ofek E, Richardson M (2003): DotCom Mania: The Rise and Fall of Internet Stock Prices. *NYU Stern School Working Paper*, FIN-01-037, 58(3):1113.

Pastor L, Veronesi P (2006): Was there a NASDAQ bubble in the late 1990s? *Journal of Financial Economics* (Elsevier), 81(1):61–100.

Salge M (1997): Rational Bubbles: Theoretical Basis, Economic Relevance, and Empirical Evidence with a Special Emphasis on the German Stock Market. *Lecture Notes in Economics and Mathematical Systems 451.* Springer Verlag, ISBN 3540626298.

Sharpe W, Alexander G (1990): *Investments.* Prentice Hall, Englewood Cliffs, N.J.. 4th edition.

Shleifer A, Vishny RW (1997): The Limits of Arbitrage. *Journal of Finance*, 52(1):35–55.

Siegel JJ (2003): What Is an Asset Price Bubble? An Operational Definition. *European Financial Management*, 9(1):11–24.

Sornette D (2003): *Why Stock Markets Cash (Critical Events in Complex Financial Systems)*. Princeton University Press, Princeton NJ, January 2003.

Taylor M, Sarno L (2002): The *Economics of Exchange Rates*. Cambridge, Cambridge University Press, 2002 - xii, 318, ISBN 0-521-48133.