

# DISTRIBUTION OF CELL IN MOBILE NETWORK

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**Abstract.** Femtocell concept has emerged as a cost-effective solution to manage indoor environment coverage and increasing capacity requirements. Compare to the conventional control macrocell deployment, femtocells are spread in the uncontrolled manner as they are deployed in network by customers themselves. This paper discusses multi-distance spatial analysis, Ripley's  $K$  function, to describe distribution of femtocells in a macrocell. In our study, we investigate various femtocell distributions and various numbers of femtocells in the macrocell.

## Keywords

*Femtocells, mobile networks, Ripley's  $K$  function, simulations.*

## 1. Introduction

Due to new range of applications, massive increase of connected devices and extended usage of video oriented applications, the traffic in mobile and wireless networks has significantly grown over the past years. Additionally, this trend is predicted to continue in the following years as well [1].

A possible cost-effective mean to deal with the traffic growth represents small cells such as femto, pico, or metro access points. The low-power access points enable to provide locally high capacity and data rates. This is applicable mainly in environments that are difficult to cover by traditionally macro base stations; for example houses, shopping malls, airports, railway stations, etc. Hereafter, we focus on the femtocell concept.

Femtocells can be seen as small cells covered by low-cost and low-power base stations typically deployed in network by customers themselves. These base stations are denoted as Femto Access Point, FAP, or us-

ing 3GPP notation as Home (evolved) NodeB, H(e)NB. The FAPs are connected to mobile operators' networks via subscribe lines that can be either wired or wireless; e.g. ADSLs, optics, WiFi, or nowadays even via satellites [2].

Compare to the conventional macro base stations, the number of operated FAPs within a mobile network can be very large. Therefore, to enable smooth and simple mass deployment of femtocells, a self-organizing concept has to be integrated in FAPs. Customers cannot be expected to have whatever knowledge how to install and how to configure FAPs; the whole configuration process has to be done automatically by FAPs themselves and/or under the network assistance.

The market status, the recent advances in algorithmic design and the ongoing standardization efforts for femtocells is presented in [3]. The technical impact and the business models of the various access control methods for femtocells are discussed in [4], while a wide range of radio resource allocation and power control techniques for cross-tier interference mitigation are included in [5], [6]. Various studies have addressed the issues such as energy saving [7], [8], interference mitigation [9], [10], location management [11], handover issues [12], [13], or performance analysis [14], [15].

In case of highly populated metropolises a quite high number of FAPs can operate under coverage of one macro base station. Due to customer self-deployment manner, a mobile operator has no real knowledge about placement or about at least sort of distribution of FAPs in its network (see Fig. 1). Having such sort of information would allow an operator to take proactive steps to manage better the "uncontrolled" deployment of FAPs by its customers, e.g. by allocating suitable predefined frequencies or physical cell identifier to FAPs.

In this paper, multi-distance spatial cluster analysis, Ripley's  $K$  function, to characterize the distribution of FAPs in the macrocell is studied. We discuss different possibilities of FAPs deployment such as random,

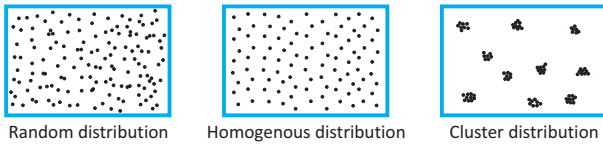


Fig. 1: Example of distributions of FAPs in a macrocell.

homogenous, in clusters and by taking into account different number of FAPs in our study.

The rest of paper is organized as follows. The next section provides brief overview the Ripley’s K function. Simulation model and obtained results are discussed in Section 3. . Finally, Section 4. concludes the paper.

## 2. Ripley’s K Function

The Ripley’s K function represents a tool to analyze completely mapped spatial point process data which consist of location events. These are usually recorded in two dimensions, but they may be locations along a line or in space. Here, we focus on two-dimensional spatial data scenario. Applications can include for example spatial patterns of trees [16], bird nests [17], disease cases [18], etc.

The  $K$  function is given as [19]:

$$K(r) = \lambda^{-1}E(r), \tag{1}$$

where  $K(r)$  describes characteristics of the point processes at many distance scales and  $\lambda$  is the event density (i.e. in our case FAPs per unit area) and can be written as:

$$\lambda = \frac{N}{A}, \tag{2}$$

where  $N$  represents the observed number of points and  $A$  is the area of study region, i.e. in our case one macrocell.

The  $E(r)$  is the expected number of points (FAPs) within a distance  $r$  (Ripley’s radius) and is calculated by using the following equation:

$$E(r) = \frac{1}{N} \sum_{k=0}^N I_k(r), \tag{3}$$

where  $I_k(r)$  is the number of points within a circle delaminated by radius  $r$  and centered at a point  $k$ .

Given the locations of all FAPs within a macrocell, how can  $K(r)$  be estimated?  $K(r)$  is a ratio of a numerator and the density of events,  $\lambda$ . The density,  $\lambda$  can be estimated using Eq. (2). It is customary to condition on  $N$ , so the uncertainty in  $\lambda$  can be ignored, although unconditionally unbiased estimators have been

suggested [20]. If edge effects are ignored, the numerator can be estimated as:

$$\frac{A}{N^2} \sum_{k=0}^N I_k(r). \tag{4}$$

However, the boundaries of the study area are usually arbitrary. Edge effects arise because points outside the boundary are not counted in the numerator, even if they are within distance  $r$  of the study area (see Fig. 2).

A variety of edge-corrected estimators have been proposed. The most commonly used one is [19]:

$$K(r) = \frac{A}{N^2} \sum_{k=0}^N \frac{I_k(r)}{w_k}, \tag{5}$$

where  $0 \leq r \leq 0.3\sqrt{A}$  (the restriction on  $r$  is necessary because the corrector factor  $w_k$  can become very small when  $r$  increases, which would characterize the distribution on a similar scale in the study area). The factor  $w_k$  is the proportion of the circumference of the circle included in the study area  $A$ . The effects of edge corrections are more important for large  $r$  because large circles are more likely to be outside the study area  $A$ . Other edge corrected estimators have been proposed for example in [21]. Although  $K(r)$  can be estimated for any  $r$ , it is common practice to consider  $r$  less than one-half of the shortest dimensions of the study area, if the study area is approximately rectangular.

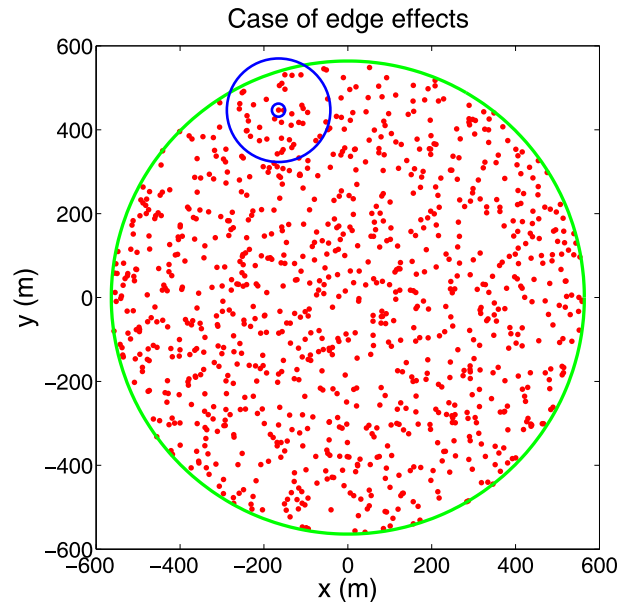


Fig. 2: Example of edge effects.

The correction of edge effects method proposed by Ripley consists to estimate the expected number of neighbors  $A_i$  which are outside of this area, for each distance  $r$ . The corrector factor can be calculated as:

$$w_k = \frac{2\pi - \alpha_{out}}{2\pi}, \tag{6}$$

where  $\lambda_{out}$  is given as:

$$\alpha_{out} = 2 \arccos \left( \frac{R_m^2 - d^2 - r^2}{2rd} \right), \quad (7)$$

where  $R_m$  is macrocell radius and  $d$  is distance of the studied point,  $k$ , from the macro base station.

### 3. Performance Evaluation

#### 3.1. Simulation Model

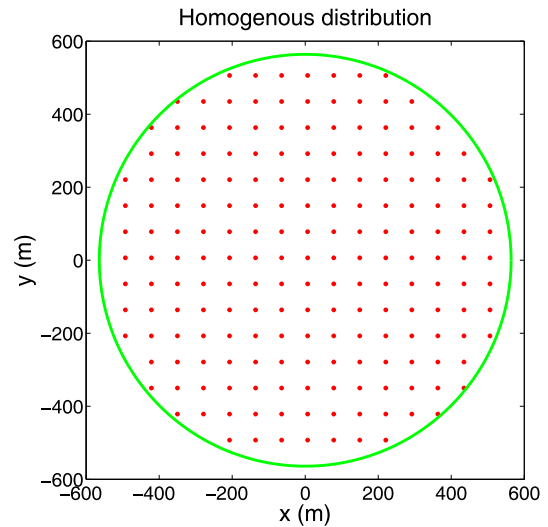
In our study, we assume a model consisting of one macrocell and set of FAPs,  $N$ . The femtocells may overlap to each other. The macrocell's radius,  $R_m$ , is set to 564 m (i.e. the covered area equals to 1 km<sup>2</sup>) and the femtocell's radius,  $R_F = 10$  m. The Ripley's  $K$  function is calculated using the Eq. (5).

Parameters used in simulations and their values are summarized in Tab. 1.

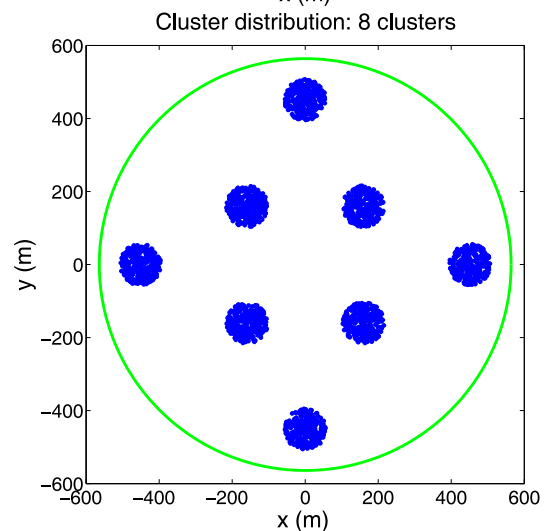
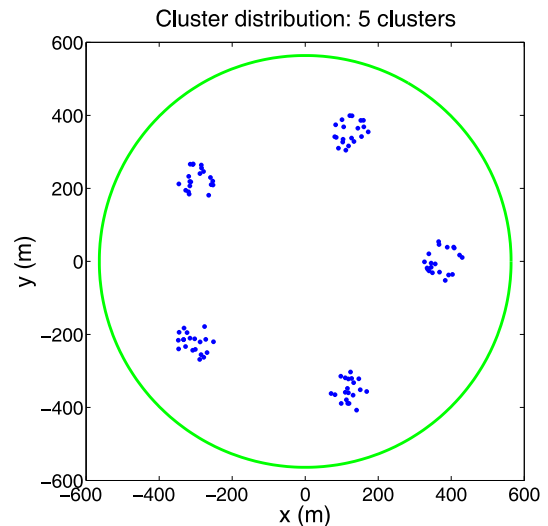
**Tab. 1:** Parameters and used values in simulations.

Parameters	Notations	Values
Number of FAPs	$N$	100–7000
Macrocell radius (macrocell surface)	$R_m$	564 m (1 km <sup>2</sup> )
Femtocell radius	$R_F$	10 m
Ripley's radius	$r$	30–350 m

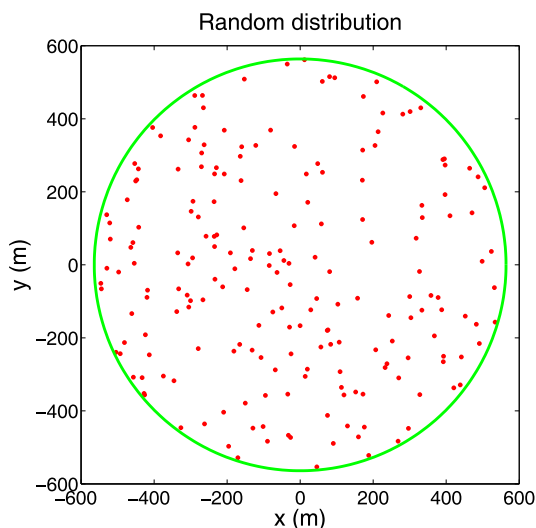
The FAPs are placed in the macrocell considering three distributions: a) random, b) homogenous, and c) cluster. In the analysis, different number of FAPs and clusters are taken into account. To simplify analysis, all clusters have same cluster sizes. An example of FAP distributions within the macrocell is shown in Fig. 3 and Fig. 4.



**Fig. 3:** Example of random and homogenous distribution of FAPs,  $N = 200$ .



**Fig. 4:** Example of distribution in clusters; a)  $N = 20$  per cluster, b)  $N = 300$  per cluster.



### 3.2. Results

Figure 5 shows the course of  $K$  function for all three distributions where  $N = 500$  and 3 clusters ( $N = 500$  per cluster). Notice that in case of homogeneous Poisson process (also known as complete spatial randomness) of FAPs, the  $K$  function equal to  $\pi r^2$  (see the green line in Fig. 5). This figure takes into account the edge effects that are corrected using Eq. (6). To illustrate the issue of edges, Fig. 6 shows how the results change when neglecting the edge effects. Mainly

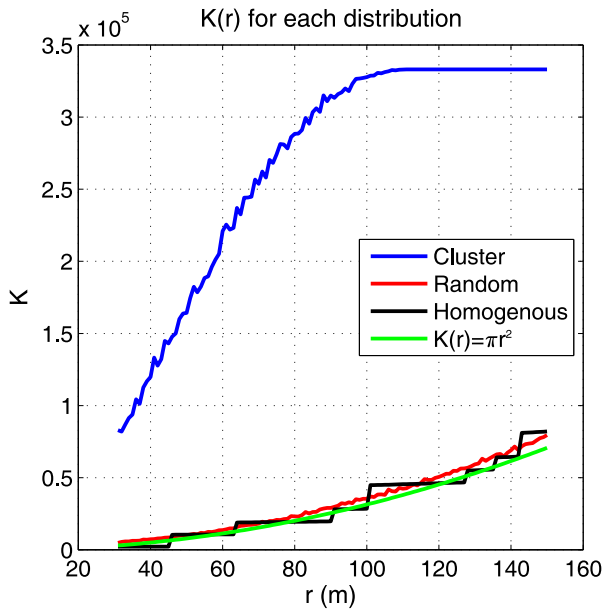


Fig. 5: K function for various distribution when correcting the edge effects,  $N = 500$ .

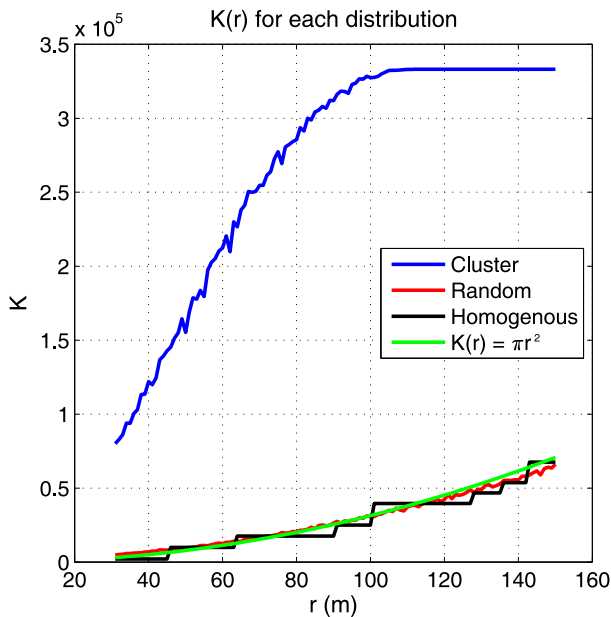


Fig. 6: K function for various distribution without correcting the edge effects,  $N = 500$ .

in case of random and homogenous distributions, we can observe the decrease of values.

Figure 7 illustrates the  $K$  function for the random distribution when considering various numbers of FAPs. We can observe that the curves become smoother and approaching more and more the curve  $K(r) = \pi r^2$ , as the number of FAPs in the macrocell grows.

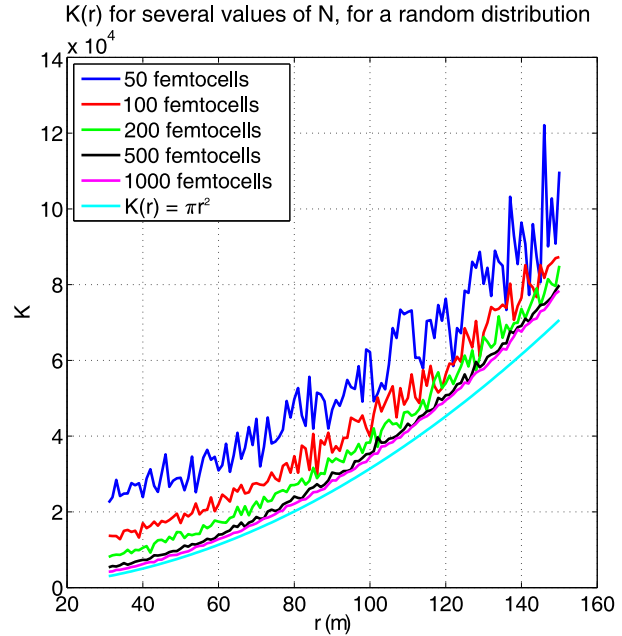


Fig. 7: K function for various number of FAPs and random distribution of FAPs.

Figure 8, resp. Fig. 9, illustrate the K function for different number of clusters when considering low and high number of FAPs per cluster ( $N = 10$ , resp.  $N = 300$ ); the FAPs are distributed randomly inside a cluster. For higher number of FAPs, the curves become smoother.

We can observe that the  $K$  function increases until reaches a certain value of  $r$ , i.e. until  $r$  reaches the cluster radius ( $r'$ ). For example, in case of 3 clusters,  $r' = 100$  m. Further increase of  $r$  do not results in any increase of  $K$  function until the size of  $r$  does not reach another cluster ( $r''$ ) and its FAPs, e.g. in case of 3 clusters,  $r'' = 360$  m. Knowing these two points, we can simply calculate distance between the clusters,  $d_{clus} = r'' - r' = 260$  m. This corresponds to the value used in our simulation when generating clusters within the macrocells. In the same way, we can find distances between clusters for the other curves. The distances between clusters based on Fig. 9 are summarized in Tab. 2.

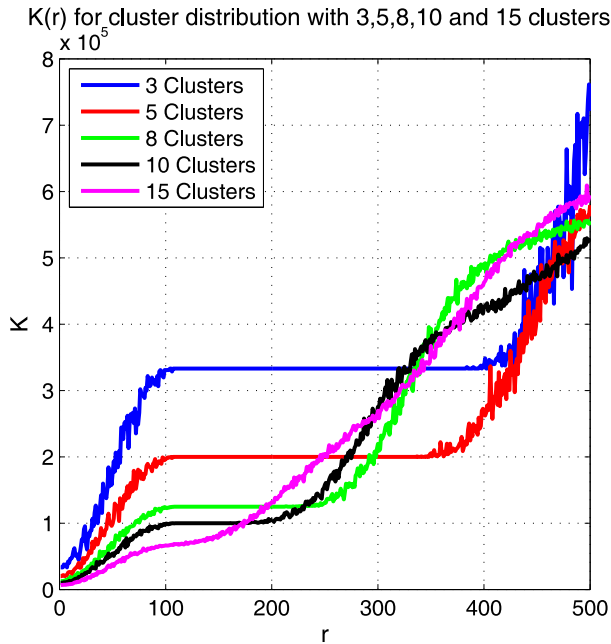


Fig. 8: K function for cluster distribution;  $N = 10$  per cluster.

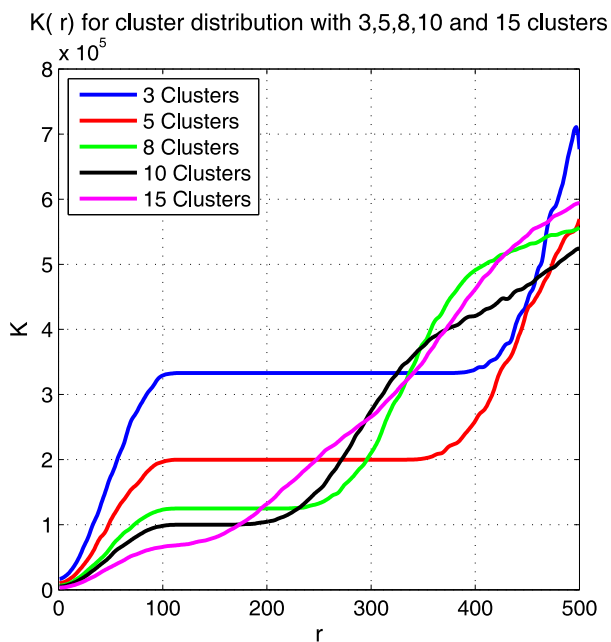


Fig. 9: K function for cluster distribution;  $N = 300$  per cluster.

Tab. 2: Distances among clusters based on Fig. 9.

Number of clusters	$r'$	$r''$	$d_{clus}$
3	100 m	390 m	290 m
5	100 m	360 m	260 m
8	100 m	250 m	150 m
10	100 m	200 m	100 m
15	90 m	150 m	60 m

As the number of clusters in the macrocell increases and thus the distance between them decreases, the cluster distribution slowly approaching the random distribution.

Notice that the growth of curves for  $r > r''$  can be faster than in the range  $r < r'$  as there can be more than one cluster in the range  $r'' > r$  (see for example the 10 clusters curve in Fig. 9).

## 4. Conclusion

In this paper, we discuss multi-distance spatial analysis, using Ripley's  $K$  function, to characterize distribution of FAPs in one macrocell. We analyze the variation of  $K$  function for random, homogenous and cluster distribution of FAPs and for different numbers of FAPs and clusters per macrocell. We also illustrate the edge effect on the  $K$  functions.

In the next work, we plan to characterize the distributions of FAPs using the linearized form of Ripley's  $K$ -function, called Besag's  $L$  function.

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