Adaptive Observer Based Actuator Faults Estimation

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Abstract. *An approach to fault estimation systems design, adjusted for linear continuous-time systems, is proposed in the paper. Based on LMI approach, the method exploits the state-space observer principle in an adaptive scheme intended for single actuator faults. A simulation example, subject to different type of failures, demonstrates the effectiveness of the proposed form of the fault estimation technique.*

Keywords

Adaptive state observer, fault estimation schemes, state-space system description.

1. Introduction

Operating conditions in modern engineering systems are still exposed to possibility of system failure. Any failure of sensors, actuators or other system components can drastically change the system behavior. Fault tolerant control (FTC) heavily relies on fault detection, identification and isolation schemes (FDI) allows a strategy to improve reliability of the system.

Estimation of an occurred actuator fault can be utilized in the processes of FDI. The final results from FDI should provide sufficient information for the second part of the remedial action, performed in the system after the fault occurrence, namely the control reconfiguration that is reconfiguring or adapting the nominal control to compensate undesired effects caused by a fault. The used principles of fault estimation cover schemes based on the reduced order observers [4], adaptive observers [1], [9], the unknown input observers [3] and sliding-mode observers [7]. This approach utilizing two step remedial process is typical for active FTC. Other option is to utilize passive FTC, but this however does not offer possibility to find solution that is optimal after the fault occurrence and is beneficial only in case of a priori considered faults.

An approach described in this paper utilizes the adaptive state observer and is usually denoted as the fast adaptive fault estimation [2], [8].

2. Problem Formulation

A linear dynamic multi-input, multi-output (MIMO) system in presence of an unknown fault can be described by the state-space equations in the following form:

$$\dot{q}(t) = \mathbf{A}\vec{q}(t) + \mathbf{B}\vec{u}(t) + \mathbf{E}\vec{f}(t), \tag{1}$$

$$\vec{y}(t) = \mathbf{C}\vec{q}(t), \tag{2}$$

where $\mathbf{A} \in \Re^{n\times n}$, $\mathbf{E} \in \Re^{n\times s}$, $\mathbf{B} \in \Re^{n\times r}$ and $\mathbf{C} \in \Re^{p\times n}$ are real matrices. To estimate faults, the following adaptive state estimator is proposed in the general form:

$$\dot{q}_e(t) = \mathbf{A}\vec{e}_e(t) + \mathbf{B}\vec{u}(t) + \mathbf{E}\vec{f}_e(t) + \mathbf{J}(\vec{y}(t) - \vec{y}_e(t)), \tag{3}$$

$$\vec{y}_e(t) = \mathbf{C}\vec{q}_e(t), \tag{4}$$

where $\mathbf{J} \in \Re^{n\times p}$ and $\vec{f}_e(t)$ is an estimate of the fault $\vec{f}(t)$. The above used vectors $\vec{q}(t), \vec{q}_e(t) \in \Re^n$, $\vec{u}(t) \in \Re^r$ and $\vec{y}(t), \vec{y}_e(t) \in \Re^p$ are vectors of the state, estimated state, input, output and estimated output variables, respectively.

The task is to design the matrix $\mathbf{J}$ in such a way that the observer system matrix $\mathbf{A}_e = \mathbf{A} - \mathbf{JC}$ be stable and the estimated fault $\vec{f}_e(t)$ approximates time properties of $\vec{f}(t)$. Considering single actuator faults, the matrix $\mathbf{E}$ takes form of the corresponding column of the input matrix $\mathbf{B}$, i.e. $\mathbf{E} = \mathbf{B}_i$ for $i \in \langle 1, 2, ...r\rangle$. Moreover, it is required that $rank(\mathbf{CE})$ is equal to $rank(\mathbf{E})$.

The state observer Eq. (3) and Eq. (4), is connected with the fault estimation updating law of the form [6], [8]:

$$\dot{f}_e(t) = \mathbf{G}\mathbf{H}^{\mathrm{T}}\vec{e}_y(t), \tag{5}$$

where $\mathbf{H} \in \Re^{p\times s}$ is the gain matrix and the matrix $\mathbf{G} = \mathbf{G}^{\mathrm{T}} > 0$, $\mathbf{G} \in \Re^{s\times s}$ is a learning weight matrix to be set interactively.

The design of the observer matrix parameters has to ensure asymptotic convergence of the estimation errors in Eq. (6) to zero values.

$$\begin{aligned} \vec{e}_f(t) &= \vec{f}(t) - \vec{f}_e(t), \\ \vec{e}_y(t) &= \vec{y}(t) - \vec{y}_e(t). \end{aligned} \tag{6}$$

Assumption 1 *The couple $(\mathbf{A}, \mathbf{B})$ is controllable and the couple $(\mathbf{A}, \mathbf{C})$ is observable.*

Assumption 2 *The unknown fault vector, changing unexpectedly when a fault occurs, is differentiable and bounded, i.e., $|\vec{f}(t)| < \bar{f}$, $\bar{f}$ is known, and the value of $\vec{f}(t)$ is set to zero until a fault occurs.*

Moreover, Ass. 2 implies that the derivative $\vec{e}_f(t)$ with respect to time can be considered as follows:

$$\vec{e}_f(t) = \vec{f}_0 - \vec{f}_e(t) \;\Rightarrow\; \dot{e}_f(t) = -\dot{f}_e(t). \tag{7}$$

From the point of the paper main results, the following lemma is needed to indicate the reasons for substantial modification of the actuator fault estimator design condition.

Lemma 1 [2] *The estimator system matrix is stable if there exist symmetric positive definite matrix $\mathbf{P} \in \Re^{n\times n}$ and matrices $\mathbf{H} \in \Re^{p\times s}$, $\mathbf{Y} \in \Re^{n\times p}$ such that:*

$$\mathbf{P} = \mathbf{P}^{\mathrm{T}} > 0, \tag{8}$$

$$\mathbf{P}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{P} - \mathbf{Y}\mathbf{C} - \mathbf{C}^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}} < 0, \tag{9}$$

$$\mathbf{P}\mathbf{E} = \mathbf{C}^{\mathrm{T}}\mathbf{H}. \tag{10}$$

When the above conditions hold, the observer gain matrix is given by:

$$\mathbf{J} = \mathbf{P}^{-1}\mathbf{Y} \tag{11}$$

and the adaptive fault estimation algorithm is:

$$\dot{f}_e(t) = \mathbf{G}\mathbf{H}^{\mathrm{T}}\mathbf{C}\vec{e}_q(t), \tag{12}$$

where

$$\vec{e}_q(t) = \vec{q}(t) - \vec{q}_e(t). \tag{13}$$

Since single actuator faults act on the system through different input vectors (columns of the matrix $\mathbf{B}$), it is possible to avoid design of different estimators with the tuning matrix parameter $\mathbf{G} > 0$ and formulate the task of the estimator design through the set of matrix equalities:

$$\mathbf{P}\mathbf{E}_{\mathrm{i}} = \mathbf{C}^{\mathrm{T}}\mathbf{H}. \tag{14}$$

Therefore solutions of Eq. (8), Eq. (9) and Eq. (14) are very conservative since the Lyapunov matrix $\mathbf{P}$ verifies the fault observer stability not only for $\mathbf{A}$, but also with respect to all polytops defined by $\mathbf{E}_{\mathrm{i}}$. To suppress this disadvantage it is proposed to decouple the Lyapunov matrix from all system parameters $\mathbf{A}$ and $\mathbf{E}_{\mathrm{i}}$ and so to obtain less conservative the enhanced design conditions. Another reason to apply this principle is if Eq. (10) is close to singular or singular.

3. Enhanced Design Conditions

In case of the enhanced design conditions the following theorem holds. It is evident that the theorem can be simply modified to respect Eq. (14).

Theorem 1 *The fault estimator is stable if for given positive tuning parameter $\delta \in \Re$ there exist a symmetric positive definite matrix $\mathbf{P} \in \Re^{n\times n}$, matrices $\mathbf{Q} \in \Re^{n\times n}$, $\mathbf{H} \in \Re^{p\times s}$, $\mathbf{Y} \in \Re^{n\times p}$, identity matrix $\mathbf{I}_{\mathrm{s}} \in \Re^{s\times s}$ and a positive scalar $\gamma \in \Re$ such that:*

$$\mathbf{P} = \mathbf{P}^{\mathrm{T}} > 0, \quad \gamma > 0, \tag{15}$$

$$\begin{bmatrix} \mathbf{P}^{\bullet}_{11} & * & * \\ \mathbf{P}^{\bullet}_{21} & -\delta(\mathbf{Q}+\mathbf{Q}^{\mathrm{T}}) & * \\ \mathbf{0} & \delta\mathbf{E}^{\mathrm{T}}\mathbf{Q} & -\gamma\mathbf{I}_{\mathrm{s}} \end{bmatrix} < 0, \tag{16}$$

where

$$\begin{aligned} \mathbf{P}^{\bullet}_{11} &= \mathbf{Q}^{\mathrm{T}}\mathbf{A} + \mathbf{A}^{\mathrm{T}}\mathbf{Q} - \mathbf{Y}\mathbf{C} - \mathbf{C}^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}} + \mathbf{C}^{\mathrm{T}}\mathbf{C}, \\ \mathbf{P}^{\bullet}_{21} &= \mathbf{P} - \mathbf{Q} + \delta\mathbf{Q}^{\mathrm{T}}\mathbf{A} - \delta\mathbf{Y}\mathbf{C} \end{aligned} \tag{17}$$

and

$$\mathbf{C}^{\mathrm{T}}\mathbf{H} = \mathbf{Q}\mathbf{E}. \tag{18}$$

When the above conditions hold, the estimator gain matrix is given by:

$$\mathbf{J} = \mathbf{P}^{-1}\mathbf{Y} \tag{19}$$

and the adaptive fault estimation algorithm is:

$$\dot{f}_e(t) = \mathbf{G}\mathbf{H}^{\mathrm{T}}\mathbf{C}\vec{e}_q(t). \tag{20}$$

Here and hereafter $*$ denotes the symmetric item in a symmetric matrix.

Proof: From system model Eq. (1), Eq. (2) and estimator model Eq. (3), Eq. (4) in this case can be seen that the time derivative of $\vec{e}_q(t)$ is:

$$\dot{e}_q(t) = (\mathbf{A}-\mathbf{JC})\vec{e}_q(t)+\mathbf{E}\vec{e}_f(t), \tag{21}$$

and, moreover, can be expressed in the following form:

$$(\mathbf{A}-\mathbf{JC})\vec{e}_q(t)+\mathbf{E}\vec{e}_f(t) - \dot{e}_q(t) = 0. \tag{22}$$

The Lyapunov function is considered as follows:

$$\begin{aligned} v(\vec{e}_q(t)) = \vec{e}_q^{\,T}(t)\mathbf{P}\vec{e}_q(t)+\vec{e}_f^{\,T}(t)\mathbf{G}^{-1}\vec{e}_f(t)+ \\ +\int_0^t(\vec{e}_y^{\,T}(r)\vec{e}_y(r)-\gamma\vec{e}_f^{\,T}(r)\vec{e}_f(r))\mathrm{d}r, \end{aligned} \tag{23}$$

where $\mathbf{P} = \mathbf{P}^{\mathrm{T}} > 0$, $\mathbf{G} = \mathbf{G}^{\mathrm{T}} > 0$ and γ is square of the H_∞ norm of the transfer matrix function between $\vec{e}_f$ and $\vec{e}_q$. Then the time derivative of $v(\vec{e}_q(t))$ is:

$$\begin{aligned} \dot{v}(\vec{e}_q(t)) = \vec{e}_q^{\,T}(t)\mathbf{P}\vec{e}_q(t) + \vec{e}_q^{\,T}(t)\mathbf{P}\dot{e}_q(t)+ \\ +\dot{e}_f^{\,T}(t)\mathbf{G}^{-1}\vec{e}_f(t) + \vec{e}_f^{\,T}(t)\mathbf{G}^{-1}\dot{e}_f(t)+ \\ +\vec{e}_y^{\,T}(t)\vec{e}_y(t) - \gamma\vec{e}_f^{\,T}(t)\vec{e}_f(t). \end{aligned} \tag{24}$$

If it is assumed, that the following statements Eq. (7) and

$$\dot{f}_e(t) = \mathbf{GH}^{\mathrm{T}}\vec{e}_y(t) = \mathbf{GH}^{\mathrm{T}}\mathbf{C}\vec{e}_q(t), \tag{25}$$

hold, then the substitution of Eq. (25) into Eq. (24) leads to

$$\begin{aligned} \dot{v}(\vec{e}_q(t)) = \dot{e}_q^{\,T}(t)\mathbf{P}\vec{e}_q(t) + \vec{e}_q^{\,T}(t)\mathbf{P}\dot{e}_q(t)- \\ -\vec{e}_q^{\,T}(t)\mathbf{C}^{\mathrm{T}}\mathbf{HGG}^{-1}\vec{e}_f(t)- \\ -\vec{e}_f^{\,T}(t)\mathbf{G}^{-1}\mathbf{GH}^{\mathrm{T}}\mathbf{C}\vec{e}_q(t)+ \\ +\vec{e}_y^{\,T}(t)\vec{e}_y(t) - \gamma\vec{e}_f^{\,T}(t)\vec{e}_f(t), \end{aligned} \tag{26}$$

that can be modified into the following form:

$$\begin{aligned} \dot{v}(\vec{e}_q(t)) = \dot{e}_q^{\,T}(t)\mathbf{P}\vec{e}_q(t) + \vec{e}_q^{\,T}(t)\mathbf{P}\dot{e}_q(t)- \\ -\vec{e}_q^{\,T}(t)\mathbf{C}^{\mathrm{T}}\mathbf{H}\vec{e}_f(t) - \vec{e}_f^{\,T}(t)\mathbf{H}^{\mathrm{T}}\mathbf{C}\vec{e}_q(t)+ \\ +\vec{e}_y^{\,T}(t)\vec{e}_y(t) - \gamma\vec{e}_f^{\,T}(t)\vec{e}_f(t). \end{aligned} \tag{27}$$

It is possible to define the condition based on Eq. (22) as:

$$\begin{aligned} (\vec{e}_q^{\,T}(t)\mathbf{S}_1^{\mathrm{T}} + \dot{e}_q^{\,T}(t)\mathbf{S}_2^{\mathrm{T}})\times \\ \times((\mathbf{A}-\mathbf{JC})\vec{e}_q(t)+\mathbf{E}\vec{e}_f(t) - \dot{e}_q(t)) = 0 \end{aligned} \tag{28}$$

and inserting into Eq. (27), the following expression is obtained:

$$\begin{aligned} \dot{v}(\vec{e}_q(t)) = \dot{e}_q^{\,T}(t)\mathbf{P}\vec{e}_q(t) + \vec{e}_q^{\,T}(t)\mathbf{P}\dot{e}_q(t)- \\ -\vec{e}_q^{\,T}(t)\mathbf{C}^{\mathrm{T}}\mathbf{H}\vec{e}_f(t) - \vec{e}_f^{\,T}(t)\mathbf{H}^{\mathrm{T}}\mathbf{C}\vec{e}_q(t)+ \\ +(\vec{e}_q^{\,T}(t)\mathbf{S}_1^{\mathrm{T}} + \dot{e}_q^{\,T}(t)\mathbf{S}_2^{\mathrm{T}})((\mathbf{A}-\mathbf{JC})\vec{e}_q(t)- \\ -\dot{e}_q(t)) + ((\mathbf{A}-\mathbf{JC})\vec{e}_q(t) - \dot{e}_q(t))^T \\ (\mathbf{S}_1\vec{e}_q(t) + \mathbf{S}_2\dot{e}_q(t)) + (\vec{e}_q^{\,T}(t)\mathbf{S}_1^{\mathrm{T}}+ \\ +\dot{e}_q^{\,T}(t)\mathbf{S}_2^{\mathrm{T}})\mathbf{E}\vec{e}_f(t) + \vec{e}_f^{\,T}(t)\mathbf{E}^{\mathrm{T}}(\mathbf{S}_1\vec{e}_q(t)+ \\ +\mathbf{S}_2\dot{e}_q(t)) + \vec{e}_y^{\,T}(t)\vec{e}_y(t) - \gamma\vec{e}_f^{\,T}(t)\vec{e}_f(t). \end{aligned} \tag{29}$$

If the following condition is defined:

$$\begin{aligned} 0 = \vec{e}_f^{\,T}(t)[\mathbf{E}^{\mathrm{T}}\mathbf{S}_1-\mathbf{H}^{\mathrm{T}}\mathbf{C}]\vec{e}_q(t)- \\ -\vec{e}_q^{\,T}(t)[\mathbf{S}_1^{\mathrm{T}}\mathbf{E}-\mathbf{C}^{\mathrm{T}}\mathbf{H}]\vec{e}_f(t), \end{aligned} \tag{30}$$

then the equality

$$\mathbf{S}_1^{\mathrm{T}}\mathbf{E} - \mathbf{C}^{\mathrm{T}}\mathbf{H} = 0 \tag{31}$$

implies

$$\mathbf{C}^{\mathrm{T}}\mathbf{H} = \mathbf{S}_1^{\mathrm{T}}\mathbf{E} \tag{32}$$

and this allows to transcribe Eq. (29) as:

$$\begin{aligned} \dot{v}(\vec{e}_q(t)) = \dot{e}_q^{\,T}(t)\mathbf{P}\vec{e}_q(t) + \vec{e}_q^{\,T}(t)\mathbf{P}\dot{e}_q(t)+ \\ +(\vec{e}_q^{\,T}(t)\mathbf{S}_1^{\mathrm{T}} + \dot{e}_q^{\,T}(t)\mathbf{S}_2^{\mathrm{T}})((\mathbf{A}-\mathbf{JC})\vec{e}_q(t)- \\ -\dot{e}_q(t)) + (\vec{e}_q^{\,T}(t)(\mathbf{A}-\mathbf{JC})^{\mathrm{T}} - \dot{e}_q^{\,T}(t)) \\ (\mathbf{S}_1\vec{e}_q(t) + \mathbf{S}_2\dot{e}_q(t)) + \dot{e}_q^{\,T}(t)\mathbf{S}_2^{\mathrm{T}}\mathbf{E}\vec{e}_f(t)+ \\ +\vec{e}_f^{\,T}(t)\mathbf{E}^{\mathrm{T}}\mathbf{S}_2\dot{e}_q(t) + \vec{e}_y^{\,T}(t)\vec{e}_y(t)- \\ -\gamma\vec{e}_f^{\,T}(t)\vec{e}_f(t) \end{aligned} \tag{33}$$

and with the notation:

$$\vec{e}_q^{\,\bullet T}(t) = \begin{bmatrix} \vec{e}_q^{\,T}(t) & \dot{e}_q^{\,T}(t) & \vec{e}_f^{\,T}(t) \end{bmatrix}, \tag{34}$$

it can be written as:

$$\dot{v}(\vec{e}_q(t)) = \vec{e}_q^{\,\bullet T}(t)\mathbf{P}^{\bullet}\vec{e}_q^{\,\bullet}(t) < 0, \tag{35}$$

where

$$\mathbf{P}^{\bullet} = \begin{bmatrix} \mathbf{P}_{11}^{\bullet} & * & * \\ \mathbf{P}_{21}^{\bullet} & -\mathbf{S}_2^{\mathrm{T}} - \mathbf{S}_2 & * \\ 0 & \mathbf{E}^{\mathrm{T}}\mathbf{S}_2 & -\gamma\mathbf{I}_s \end{bmatrix} < 0 \tag{36}$$

and

$$\begin{aligned} \mathbf{P}_{11}^{\bullet} = \mathbf{S}_1^{\mathrm{T}}(\mathbf{A}-\mathbf{JC}) + \mathbf{C}^{\mathrm{T}}\mathbf{C} + (\mathbf{A}-\mathbf{JC})^{\mathrm{T}}\mathbf{S}_1, \\ \mathbf{P}_{21}^{\bullet} = \mathbf{P} + \mathbf{S}_2^{\mathrm{T}}(\mathbf{A} - \mathbf{JC}) - \mathbf{S}_1. \end{aligned} \tag{37}$$

Inserting into Eq. (32), Eq. (36) and Eq. (37) the following substitutions:

$$\mathbf{S}_1 = \mathbf{Q}, \quad \mathbf{S}_2 = \delta\mathbf{S}_1, \quad \mathbf{S}_1^{\mathrm{T}}\mathbf{J} = \mathbf{Q}^{\mathrm{T}}\mathbf{J} = \mathbf{Y} \tag{38}$$

then Eq. (32), Eq. (36) and Eq. (37) implies Eq. (16), Eq. (17) and Eq. (18). This concludes the proof. ■

Note, Eq. (17) and Eq. (18) implies that the Lyapunov matrix **P** is decoupled from all system parameters.

4. Illustrative Example

To illustrate the proposed method, the system whose dynamics is described by the equations Eq. (1) and Eq. (2) is considered with the matrix parameters:

$$\mathbf{A} = \begin{bmatrix} 0.1650 & -0.0767 & 0.5173 & 2.0638 \\ 0.2151 & 0.1866 & -0.3077 & -1.5260 \\ -0.3851 & 0.4033 & 0.2887 & -0.3833 \\ -2.0829 & 1.5170 & 0.3093 & 0.1109 \end{bmatrix},$$

$$\mathbf{C}^{\mathrm{T}} = \begin{bmatrix} 2 & 2 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{bmatrix}, \quad \mathbf{B} = \mathbf{E} = \begin{bmatrix} -0.9042 & 0 \\ -1.6493 & 0.4361 \\ 0 & 1.8529 \\ 0 & -0.0355 \end{bmatrix}.$$

For simulation purposes only, the equilibrium of the system was stabilized by the state feedback controller:

$$\vec{u}(t) = -\mathbf{K}\vec{q}(t),$$

where the gain matrix $\mathbf{K}$ was designed using the pole-placement method as follows:

$$\mathbf{K} = \begin{bmatrix} -3.1315 & 1.6633 & 3.5641 & 11.3920 \\ -4.2235 & 3.1471 & 1.0377 & 0.9104 \end{bmatrix}.$$

Solving Eq. (8), Eq. (9) and Eq. (10) with respect to the LMI matrix variables $\mathbf{P}$, $\mathbf{H}$, and $\mathbf{Y}$ using Self-Dual-Minimization (SeDuMi) package [5] for Matlab, the estimator parameter design problem was solved as feasible and the LMI matrix variables were:

$$\mathbf{P} = \begin{bmatrix} 0.1770 & 0.2107 & 0.0321 & 0.0457 \\ 0.2107 & 0.6888 & 0.0678 & 0.2060 \\ 0.0321 & 0.0678 & 0.1586 & -0.0955 \\ 0.0457 & 0.2060 & -0.0955 & 0.6740 \end{bmatrix},$$

$$\mathbf{Y} = \begin{bmatrix} -0.3691 & 0.3090 \\ -0.7451 & -0.1267 \\ 0.0251 & 0.1647 \\ 0.2357 & -0.8632 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 0.4026 & -0.1207 \\ -0.2489 & 0.2774 \end{bmatrix},$$

so that the observer gain matrix were given as

$$\mathbf{J} = \begin{bmatrix} -1.2534 & 2.8937 \\ -1.1442 & -0.6872 \\ 1.5003 & -0.0182 \\ 0.9967 & -1.2693 \end{bmatrix}.$$

It can verify that the eigenvalue spectrum $\rho(\mathbf{A}_{\mathrm{e}})$ of the estimator system matrix $\mathbf{A}_{\mathrm{e}}$ is stable, where:

$$\rho(\mathbf{A}_{\mathrm{e}}) = \{-2.1745 \ \ -2.6067 \ \ -3.3377 \ \pm 0.1566\,\mathrm{i}\}.$$

Setting the tuning parameter $\mathbf{G}$ as:

$$\mathbf{G} = \begin{bmatrix} 1.7 & 1 \\ 1 & 5.5 \end{bmatrix},$$

the observer fault response is given in Fig. 2.

This figure presents the fault signal, as well as its estimation, reflecting a single actuator fault in the second actuator. Fault starts at the time instant $t = 30$ s and is applied for 40 s. The tuning parameter was experimentally set considering the maximal value of the fault signal amplitude. Then, at the time instant $t = 100$ s, the first actuator fault is applied for 40 s.

From the simulation results in Fig. 2 it can be observed that the differences between the signals reflecting a single actuator fault and the observer approximate ones tends to zero. Moreover, the states of the system converge to the equilibrium when the actuator fault disappeared, via the used controller.

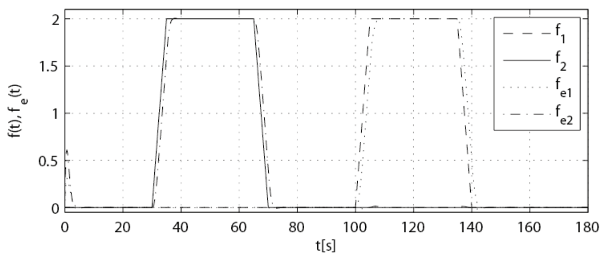

Fig. 1: The actuator faults and its estimation.

Applying the same toolbox to solve LMIs Eq. (15), Eq. (16), Eq. (17) and Eq. (18) conditioned by $\delta = 0.001$, the resulting set of the matrix variables was as follows:

$$\mathbf{P} = \begin{bmatrix} 23.2065 & -5.2282 & 6.5739 & 5.9199 \\ -5.2282 & 16.1633 & -5.5997 & 4.2321 \\ 6.5739 & -5.5997 & 11.1507 & 4.6793 \\ 5.9199 & 4.2321 & 4.6793 & 24.1004 \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} 23.2346 & -5.2370 & 6.5874 & 5.8961 \\ -5.2420 & 16.1435 & -5.6122 & 4.2479 \\ 6.5646 & -5.6171 & 11.1679 & 4.6516 \\ 5.9477 & 4.2403 & 4.7055 & 24.0935 \end{bmatrix},$$

$$\mathbf{Y} = \begin{bmatrix} -4.9513 & 3.7898 \\ -6.9495 & 11.8052 \\ 10.1032 & -1.6169 \\ 11.5372 & -3.4027 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 9.5143 & 13.2220 \\ -15.7000 & -8.3657 \end{bmatrix}$$

and the estimator gain matrix $\mathbf{J}$, obtained using Eq. (19), was:

$$\mathbf{J} = \begin{bmatrix} -0.7142 & 0.4462 \\ -0.5347 & 1.1482 \\ 0.8107 & 0.3902 \\ 0.5906 & -0.5282 \end{bmatrix},$$

which gives the following stable eigenvalue spectrum:

$$\rho(\mathbf{A}_{\mathrm{e}}) = \{-0.4091 \pm 3.2455\,\mathrm{i} - 0.8964 \pm 1.0815\,\mathrm{i}\}.$$

Finally, setting the tuning parameters $\mathbf{G}$ as follows

$$\mathbf{G} = \begin{bmatrix} 0.6 & 0.1 \\ 0.1 & 0.6 \end{bmatrix},$$

the observed actuator fault estimation on the output of the fault observer is given in Fig. 2.

This figure expresses the fault signal and its estimation, indicating a single actuator fault in the second actuator, starting at the time instant $t = 30$ s and applied for 40 s, as well as a single actuator fault in the first actuator occurring at the time instant $t = 100$ s and lasts for 40 s. The tuning parameters $\mathbf{G}$ and δ

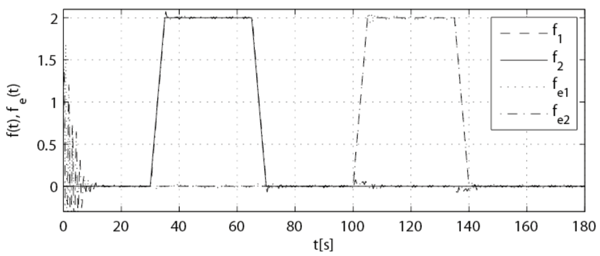

Fig. 2: The actuator faults and its estimation using the enhanced design condition.

were set experimentally considering the maximal value of fault signal amplitude. It is obvious that the obtained fault estimation dynamics is slightly faster then can be obtained using the standard design criteria.

Comparing the both methods, it is evident that the Frobenius norms of the observer gain matrices $\mathbf{J}$, $\mathbf{G}$, implying from the enhanced design criterion, are substantial less than the Frobenius norms of $\mathbf{J}$, $\mathbf{G}$ designed using the standard approach. This explains the higher sensitivity and faster dynamics of the proposed method.

Of course, the both fault observers which parameters were obtained using the solutions of the LMI problems specified by Lem. 1 and Thm. 1 can successfully provide for the observer steady-state properties and asymptotic dynamics.

5. Conclusion

Fault estimation for linear continuous-time systems based on the standard design conditions provides useful and in the process easily implementable fault detection, isolation and identification. The proposed approach to fault estimation for linear continuous-time systems, utilizing the enhanced design conditions, allows even better results, when the occurred actuator faults are estimated more precise then in previous case as it can seen in the simulation results.

Tuning parameters $\mathbf{G}$ and δ were set experimentally, incorrect values of the parameters would potentially result in unstable or noisy response of the fault estimation signal. It would be beneficial in the perspective to determine values of the parameters $\mathbf{G}$ and δ using any analytical or numerical solution.

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References

[1] DING, S. X. and P. M. Frank. An Adaptive Observer-Based Fault Detection Scheme for Nonlinear Dynamics Systems. In: *Preprints of the 12th IFAC World Congres.* Sydney: IEEE, 1993, pp. 307–310. ISBN 0-8493-0053-3.

[2] FILASOVA, A., D. GONTKOVIC and D. KROKAVEC. Observer-Based Fault Estimation for Linear Systems with Distributed Time-Delay. *Archives of Control Sciences.* 2013, vol. 23, no. 2, pp. 169–186. ISSN 1230-2384. DOI: 10.2478/acsc-2013-0010.

[3] KROKAVEC, D., A. FILASOVA and V. HLADKY. Takagi-Sugeno Model-Based State Estimation for One Class of Nonlinear Systems with Unknown Disturbances. In: *Proceedings of the 13th Internatonal Carpathian Control Conference ICCC'12.* Podbanske: IEEE, 2012, pp. 398–402. ISBN 978-1-4577-1866-3. DOI: 10.1109/CarpathianCC.2012.6228675.

[4] KROKAVEC, D. and A. FILASOVA. Actuator Faults Reconstruction Using Reduced-Order Fuzzy Observer Structures. In: *Proceedings of the 12th EUCA European Control Conference ECC'13.* Zurich: IEEE, 2013, pp. 4299–4304. ISBN 978-3-9524173-4-8.

[5] PEAUCELLE, D., D. HENRION, Y. LABIT and K. TAITZ. *User's Guide for SeDuMi Interface 1.04.* Toulouse: LAAS-CNRS, 2002.

[6] WANG, H. and S. DALEY. Actuator fault diagnosis: an adaptive observer-based technique. *IEEE Transactions on Automatic Control.* 1996, vol. 41, iss. 7, pp. 1073–1078. ISSN 0018-9286. DOI: 10.1109/9.508919.

[7] YAN, X.-G., C. EDWARDS and V. COCQUEMPOT. Nonlinear robust fault reconstruction and estimation using a sliding mode observer. *Automatica.* 2007, vol. 43, iss. 9, pp. 1605–1614. ISSN 0005-1095. DOI: 10.1016/j.automatica.2007.02.008.

[8] ZHANG, K., B. JIANG and V. COCQUEMPOT. Adaptive Observer-Based Fast Fault Estimation. *International Journal of Control, Automation and Systems.* 2008, vol. 6, no. 3, pp. 320–326. ISSN 1598-6446.

[9] ZHANG K. and J. BIN. Analysis and design of adaptive fault estimation for time-varying delay

systems. In: *27th Chinese Control Conference*. Kunming: IEEE, 2008, pp. 38–42. ISBN 978-7-900719-70-6. DOI: 10.1109/CHICC.2008.4605097.

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