Transactions of the VŠB – Technical University of Ostrava, Mechanical Series
No. 2, 2013, vol. LIX
article No. 1956

František FOJTÍK*

APPLICATION OF SELECTED MULTI-AXIAL FATIGUE CRITERIA ARE USING FIFTH-DIMENSIONAL SPACE ON THE RESULTS OF FATIGUE EXPERIMENTS

APLIKACE VYBRANÝCH KRITÉRII ÚNAVOVÉ PEVNOSTI VYUŽÍVAJÍCÍCH PĚTIROZMĚRNÝ PROSTOR NA VÝSLEDKY ÚNAVOVÝCH EXPERIMENTŮ

Abstract

The paper describes the experimental results obtained for the combined loading of the specimens in the region of high-cycle fatigue. The specimens were manufactured from common structural steel 11523.1.

There has been realized a set of proportional and non-proportional experiments in the area of high-cycle fatigue for basic methods of loading, namely: tension/compression, bending and torsion and its mutual combinations. By selected experiments, the specimens were additionally loaded by constant inner/outer pressure and axial prestress for a various loading levels. Experiments were realized on testing machines, situated at author´s workplace. The experimental results were undergone to application of selected and known criteria of fatigue strength and subsequently carried out its modification according to the results. For gaining necessary input values for criteria of fatigue strength, there was a need to realize a FEM simulation of test specimens.

Abstrakt

Článek popisuje výsledky experimentů při kombinovaném zatěžování zkušebních vzorků v oblasti vysokocyklové únavy. Použité zkušební vzorky byly vyrobeny z běžné konstrukční oceli 11523.1.

Byla provedena sada proporcionálních a neproporcionálních experimentů v oblasti vysokocyklové únavy pro základní způsoby zatěžování tahem/tlakem, ohybem a krutem a jejich vzájemnými kombinacemi. U vybraných experimentů byly zkušební vzorky navíc zatěžovány konstantním vnitřním, vnějším přetlakem a osovým předpětím pro různé hladiny zatížení. Experimenty byly realizovány na zkušebních zařízeních nacházejících se na pracovišti autora. Na výsledky experimentů byly aplikovány vybraná a známá kritéria únavové pevnosti, a provedena jejích modifikace. Pro získání potřebných vstupních hodnot kritérií pevnosti byla provedena napěťově deformační analýza zkušebních vzorků metodou konečných prvků.

Keywords

high-cycle fatigue, criteria of fatigue strength, multiaxial fatigue, experiment, combined loading

1 INTRODUCTION

Although the material failure phenomenon in the conditions of multiaxial fatigue is investigated for many years by world-known research institutes, a reliable mathematical description

* Ing., Ph.D., Department of Mechanics of Materials, VŠB-Technical University of Ostrava, av. 17. listopadu 15, CZ-708 33 Ostrava-Poruba, tel. (+420) 59 732 3292, e-mail: frantisek.fojtik@vsb.cz

making possible to describe this boundary state was not introduced yet. Hence, it is still necessary to perform expensive prototype verification. The number of laboratories especially in aircraft and automotive industry is evidence of this fact. We bring another build-stone into the mosaic of this interesting technical field in this contribution.

Number of fatigue experiments using both the reconstructed and new proposed devices, were performed at the VSB-TU Ostrava. The main aim was to verify a usability of selected criteria of fatigue strength and suggest its modification. There will be presented methods, which are using IDS method (Ilyushin Deviatoric Space) for analyzing amplitude of the stress tensor component. These methods are using fifth-dimensional space, whose coordinates are derived from deviator part of stress tensor for analysis of loading path.

The paper describes results of various types of fatigue experiments, realized by proportional and non-proportional loading with either in phase, or with particular phase shift and further with constant axial prestress in different directions. The experiments were performed on hollow specimens manufactured from the steel 11523.1. The experimental data obtained at the fatigue limit were evaluated primarily, i.e. for specimens which were damaged at 10^7 cycles. The below presented methodology is realized for this lifetime.

2 EXPERIMENTAL MATERIAL

The experiments were performed on hollow specimens (Fig. 1, Fig. 2) manufactured from low carbon steel CSN 411523.1 melt T31052. The specimens were polished on the outer surface. The chemical content and basic mechanical properties of this material are summarized in Tab. 1 and Tab. 2.

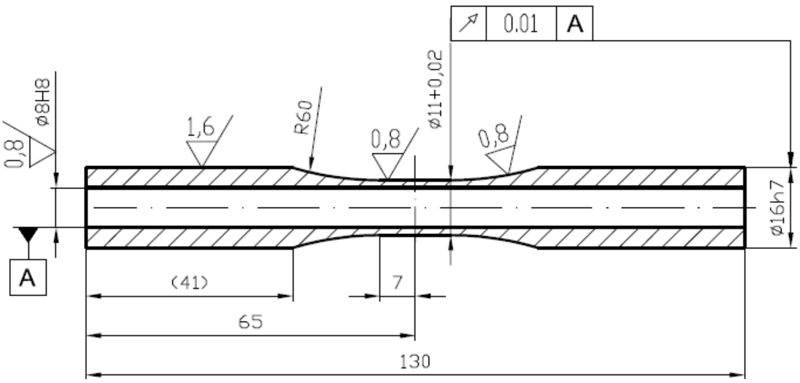

Fig. 1 Testing specimen

Tab. 1 Chemical properties of the material.

C [%]	Mn [%]	Si [%]	P [%]	S [%]	Cu [%]
0.18	1.38	0.4	0.018	0.006	0.05

Tab. 2 Mechanical properties of material.

Ultimate tensile strength [MPa]	Tensile yield stress [MPa]	Elongation at fracture [%]	Reduction of area at fracture [%]
560	400	31.1	74.0

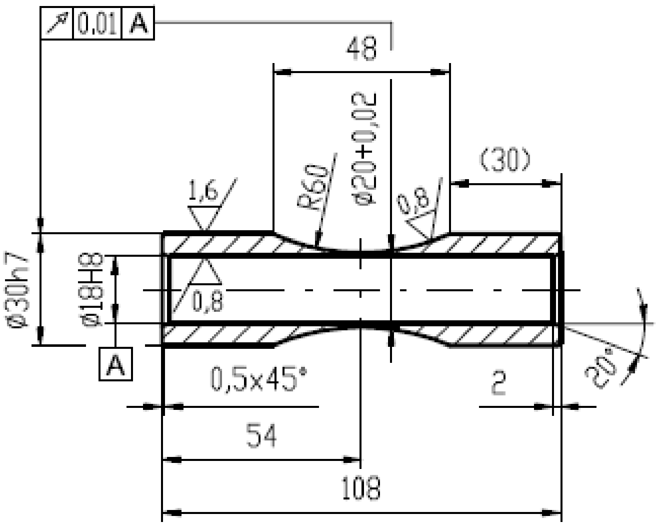

Fig. 2 Testing specimen

The following material parameters were experimentally set so that the fatigue criteria mentioned below could be evaluated.

Tensile modulus: $E = 2.06*10^5$ MPa,

Poisson's ratio: $\mu = 0.3$,

fatigue limit in fully reversed torsion: $t_{-1} = 162.0$ MPa,

fatigue limit in fully reversed tension: $f_{-1} = 239.7$ MPa,

fatigue limit in fully reversed bending: $b_{-1} = 310.91$ MPa,

fatigue limit in repeated tension: $f_0 = 377$ MPa,

torsion true fracture strength: $\tau_f = 516.6$ MPa.

Fatigue limits of the tests were obtained from experiments, referred in chapter 4 with procedure described in the standard ČSN 42 0363.

3 USED MULTIAXIAL FATIGUE METHODS

The generally used fatigue strength criteria were used for the analysis of realized experiments. The results obtained experimentally for the given loading combinations on the fatigue limit will be evaluated by them.

The complex loading is by used fatigue strength criteria taken into account duringcomputation of the second invariant of stress deviator. The second stress deviator is in this case determined with aim of Ilyushin deviatoric space, who Ilyushin defined the conversion from deviator tensor to five dimensional vectors. The process is based on transformation of tensor stress components to deviator and spherical tensor [8]. In this deviatoric space, we are searching formation, which characterizes loading path corresponding to smallest circumscribed hypersphere. Invariant of second stress deviator was calculated with aim of Pragtic software [8].

$$\boldsymbol{\sigma} = \boldsymbol{s} + \boldsymbol{K}_\sigma$$

$$\boldsymbol{s} = \begin{bmatrix} \frac{2}{3}\sigma_x - \frac{1}{3}\sigma_y - \frac{1}{3}\sigma_z & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & -\frac{1}{3}\sigma_x + \frac{2}{3}\sigma_y - \frac{1}{3}\sigma_z & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & -\frac{1}{3}\sigma_x - \frac{1}{3}\sigma_y + \frac{2}{3}\sigma_z \end{bmatrix} \quad (1)$$

$$\boldsymbol{K}_\sigma = \frac{1}{3}\begin{bmatrix} \sigma_x + \sigma_y + \sigma_z & 0 & 0 \\ 0 & \sigma_x + \sigma_y + \sigma_z & 0 \\ 0 & 0 & \sigma_x + \sigma_y + \sigma_z \end{bmatrix}$$

Hydrostatic stress:

$$\sigma_H = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right). \quad (2)$$

Ilyushin Deviatoric Space - fifth-dimensional space, in which the coordinates are derived from tensor stress deviator:

$$s_1 = \sqrt{\frac{3}{2}} s_{xx}; \qquad s_2 = \frac{1}{\sqrt{2}}\left(s_{yy} - s_{zz}\right);$$
$$s_3 = \sqrt{2} s_{xy}; \quad s_4 = \sqrt{2} s_{xz}; \quad s_5 = \sqrt{2} s_{yz}; \quad \left(\sqrt{J_2}\right)_a = \sqrt{\frac{1}{2}\boldsymbol{s}\cdot\boldsymbol{s}} = \sqrt{s \cdot s} \quad .(3)$$

3.1 Crossland method

Crossland published his results in the 50th of previous century. His criterion uses the square root from the second invariant of the stress tensor deviator. This invariant is determined from the stress amplitude. Another term added to the equation is the hydrostatic stress calculated from maximal stress values [3].

$$a_C \cdot \left(\sqrt{J_2}\right)_a + b_C \cdot \sigma_{H,\max} \le f_{-1}, \quad (4)$$

where coefficients a_C and b_C are defined as:

$$a_C = \frac{f_{-1}}{t_{-1}}$$
$$b_C = \left(3 - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}}\right),$$

other parameters in the equation are:

J_2 second invariant of stress tensor deviator, f_{-1} fatigue limit in fully reversed axial loading (in tension, in bending or in rotating bending), $\sigma_{H,\max}$ maximum hydrostatic stress during the loading period, t_{-1} fatigue limit in fully reversed torsion.

3.2 Sines method

Sines published his results in the same period as Crossland. The formulations of both criteria are similar. Sines calculate of hydrostatic stress from mean stress values [4].

$$a_S \cdot \left(\sqrt{J_2}\right)_a + b_S \cdot \sigma_{H,m} \leq f_{-1}, \tag{5}$$

where coefficients a_S and b_S are defined as:

$$a_S = \frac{f_{-1}}{t_{-1}}$$

$$b_S = 6 \cdot \frac{f_{-1}}{f_0} - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}},$$

where f_0 is fatigue limit in repeated tension or bending, $\sigma_{H,m}$ mean value of hydrostatic stress during load history, other parameters in the equation are defined as in the case of Crossland method.

3.3 Kakuno Kawada method (KK)

Kakuno and Kawada [9] proposed criterion taking into account the influence of mean stress by another way than it is by Crossland criterion. For the following method of evaluation of results and applicability of criterion (9), criterion has been transferred to the following form.

$$a_k \cdot \left(\sqrt{J_2}\right)_a + k \cdot \sigma_{H,a} + \lambda \cdot \sigma_{H,m} \leq f_{-1}, \tag{6}$$

where:

$$a_k = \frac{f_{-1}}{t_{-1}}$$

$$k = 3 - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}}$$

$$\lambda = 3 \cdot \frac{f_{-1}}{f_0} - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}},$$

where $\sigma_{H,a}$ amplitude value of hydrostatic stress during the load history, $\sigma_{H,m}$ mean value of hydrostatic stress during load history, other parameters in the equation are defined as in the case of Crossland and Sines method.

3.4 PDS Papuga method (PDS)

This criterion has been proposed by Papuga and it´s implemented in Pragtic software, version PragTic v.0.2betaX [9], of which he is the author. The criterion is based on Kakuno and Kawada criterion. Its exact formulation hasn´t been published yet.

3.5 New method (NEWI)

This method is based on Kakuno and Kawada criterion and has been proposed by the author of submitted article by the purpose of gaining better prediction results by particular material. There have been proposed relations for calculation of constants a_{nI} and c_{nI}, its form has been empirically derived from given initial material yield stress parameters. Criterion is stated in following form:

$$a_{nI} \cdot \left(\sqrt{J_2}\right)_a + b_{nI} \cdot \sigma_{H,a} + c_{nI} \cdot \sigma_{H,m} \leq f_{-1}, \tag{7}$$

where:

$$a_{nI} = \frac{f_{-1}}{t_{-1}}$$

$$a_{nI} \leq 1.7,\ \ is: b_{nI} = 3 - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}},\ \ c_{nI} = 3 - \sqrt{3} \cdot \frac{t_{-1}}{f_{-1}}$$

$$a_{nI} > 1.7,\ \ is: b_{nI} = \frac{3}{\sqrt{3}} - \frac{f_{-1}}{t_{-1}},\ \ c_{nI} = 3 - \sqrt{3} \cdot \frac{t_{-1}}{f_{-1}},$$

where $\sigma_{H,a}$ amplitude value of hydrostatic stress during the load history, $\sigma_{H,m}$ mean value of hydrostatic stress during load history, other parameters in the equation are defined as in the case of previous methods.

3.6 New method (NEWII)

Following method has similar configuration like mentioned Kakuno Kawada criterion and has been proposed by author of submitted article in order to get better predictive results for particular material. There have been proposed relations for calculation of constants a_{nII} and c_{nII}, its form has been empirically derived from given initial material yield stress parameters. Criterion is stated in following form:

$$a_{nII} \cdot \left(\sqrt{J_2}\right)_a + b_{nII} \cdot \sigma_{H,\max} + c_{nII} \cdot \sigma_{HMH,m} \leq f_{-1}, \tag{8}$$

where:

$$a_{nII} = \frac{f_{-1}}{t_{-1}}$$

$$a_{nII} \leq 1.7,\ \ is: b_{nII} = 3 - \sqrt{3} \cdot \frac{f_{-1}}{t_{-1}},\ \ c_{nII} = \frac{f_{-1}}{t_{-1}} \cdot \frac{\sqrt{3}}{\sqrt{(2.6 \cdot f_0)}}$$

$$a_{nII} > 1.7,\ \ is: b_{nII} = \frac{3}{\sqrt{3}} - \frac{f_{-1}}{t_{-1}},\ \ c_{nII} = \frac{f_{-1}}{t_{-1}} \cdot \frac{\sqrt{3}}{\sqrt{(2.6 \cdot f_0)}},$$

where $\sigma_{H,\max}$ maximum hydrostatic stress during the loading period, $\sigma_{HMH,m}$ is mean value of stress, determined according to von Mises hypothesis, other parameters in the equation are defined as in the case of previous methods.

3.7 Fatigue index error

All mentioned criteria according to the results from (4 – 8) evaluate if the component is able to transfer the infinite number of loading cycles. The fatigue index error ΔFI is used for evaluating those criteria. It shows the measure of a deviation from the ideal equilibrium of the left and right hand sides of mentioned criterion relations [8].

$$\Delta FI = \frac{LHS(load) - f_{-1}}{f_{-1}} \cdot 100\%\ , \tag{9}$$

where LHS is the left hand side of the equation. The relation $LHS(load) \leq f_{-1}$ has to be fulfilled. If LHS is greater, the component may fail.

4 EXPERIMENT

Totally, 22 types of experiments have been realized. The results of them are given in form of dominating component of stress tensor, gained with aim of FEM analysis for particular type of loading. The results are given for experiment, which has resisted 10^7 cycles at gradual reducing of loading amplitude, according to standard ČSN 42 0363 methodology.

4.1 tension - torsion

Results for this type of experiment are presented in Tab. 3, where $\sigma_{m,1}$ is axial prestress.

Tab. 3 Experimental results.

Nr.	σ_a [MPa]	$\sigma_{m,1}$ [MPa]	τ_a [MPa]	τ_m [MPa]	F_i [deg]
FF2	99.6	0	136.2	0	0
FF3	194.6	0	87.2	0	0
FF4	239.7	0	0	0	0
FF5	100.5	0	142.5	0	90
FF6	196.1	0	126.4	0	90
FF7	0	0	147.72	147.72	0
FF8	117.21	117.21	95	95	0
FF9	188.47	188.47	0	0	0
FF10	0	0	162	0	0

4.2 bending - torsion

Results for this type of experiment are presented in Tab. 4, where $\sigma_{m,1}$ is axial prestress, $\sigma_{m,2}$ prestress (circumferential) defined by inner pressure.

Tab. 4 Experimental results.

Nr.	σ_a [MPa]	$\sigma_{m,1}$ [MPa]	$\sigma_{m,2}$ [MPa]	τ_a [MPa]	τ_m [MPa]	F_i [deg]
FF11	176.24	0	0	118.17	0	0
FF12	269.48	0	0	60.231	0	0
FF13	310.91	0	0	0	0	0
FF14	286.66	0	202.02	0	0	0
FF15	274.4	0	312.88	0	0	0

4.3 torsion - prestress

Tab. 5 Experimental results.

Nr.	σ_a [MPa]	$\sigma_{m,1}$ [MPa]	$\sigma_{m,2}$ [MPa]	$\sigma_{m,3}$ [MPa]	τ_a [MPa]	τ_m [MPa]	F_i [deg]
FF17	0	77.07	141.25	0	147.38	0	0
FF18	0	53.4	97.832	0	156.1	0	0
FF19	0	107.19	196.39	0	124.51	0	0
FF20	0	247.27	18.66	0	136.45	0	0
FF21	0	177.13	13.35	0	177.13	0	0
FF22	0	101.57	-29.1	-40.03	157.36	0	0
FF24	0	29.97	124.62	0	142.67	0	0
FF25	0	62.25	258.83	0	91.72	0	0

Results for this type of experiment are presented in Tab. 5, where $\sigma_{m,1}$ is axial prestress, $\sigma_{m,2}$ prestress (circumferential) defined by inner pressure and $\sigma_{m,3}$ means prestress (radial), defined by inner and outer pressure

5 EXPERIMENTAL RESULTS ANALYSIS

In Tab. 6 are listed calculated values of fatigue index error ΔFI for all 22 types of realized experiments. Tab. 6 also contains mean value of fatigue index error ΔFI and standard deviation for all the six verified criteria.

Results of application of criteria in case of axial experiments FF4 and FF10 should be theoretically zero. The deviation from zero value is caused by inclusion of entire stress tensor into computations, which hasn´t got all other values equal to zero.

Criterion NEWII uses von Mises hypothesis for description of mean stress influence. According to definition, from square root do we obtain only positive values, which can be a problem for pressure loading.

Tab. 6 Experimental analysis results.

Nr.	ΔFI (%)					
	Crossland	Sines	Kakuno Kawada	PDS Papuga	NEWI	NEWII
FF2	-2.69	-8.86	-2.69	-2.84	-2.69	-2.69
FF3	-0.67	-12.73	-0.67	-0.83	-0.67	-0.67
FF4	-0.52	-15.37	-0.52	-0.41	-0.52	-0.52
FF5	-5.81	-12.03	-5.81	-5.75	-5.81	-5.81
FF6	-9.84	-21.99	-9.84	-8.79	-9.84	-9.84
FF7	-8.81	-8.81	-8.81	0.00	-8.81	-0.08
FF8	-13.83	-7.42	-31.84	-9.64	-11.13	-6.83
FF9	-10.31	0.00	-39.27	-7.85	-5.97	-3.73
FF10	0.00	0.00	0.00	0.00	0.00	0.00
FF11	-11.90	-5.28	-11.90	-12.98	-9.10	-9.10
FF12	-10.53	-0.40	-10.53	-11.80	-6.25	-6.25
FF13	-5.05	6.62	-5.05	-5.62	-0.12	-0.12
FF14	-20.09	36.21	-32.31	5.10	9.51	-5.65
FF15	-28.04	51.92	-46.75	4.20	14.63	-8.46
FF17	4.10	28.91	-28.86	-4.68	8.91	8.42
FF18	5.45	22.64	-17.38	-0.61	8.78	8.44
FF19	-4.89	29.61	-50.73	-17.19	1.79	1.11
FF20	0.26	30.56	-40.00	-7.55	6.13	8.54
FF21	2.05	23.75	-26.79	-3.46	6.25	7.98
FF22	7.45	26.93	-18.44	6.15	11.22	16.44
FF24	-2.64	14.91	-25.97	-7.93	0.76	1.30
FF25	-24.09	12.36	-72.54	-35.44	-17.03	-15.91
diameter	-6.38	8.71	-22.1	-5.8	-0.5	-1.1
Standard deviation	9.1	19.6	18.8	8.7	8.1	7.4

6 CONCLUSIONS

In submitted article, there are presented totally six methods (two of them are newly proposed), which are during the square root computation of second invariant of stress amplitude tensor using Iyushin´s deviatoric space for analysis of loading path.

Presented criteria have been tested on own experimental data. It has been performed twenty two sets of various types of proportional and non-proportional experiments, which are in more detail described in chapter 4.

According to mean fatigue index error ΔFI value and standard deviation values from table 6, the best results can be found in case of newly named criteria NEWI and NEWII. It is necessary to mention, that both criteria and its constants were appropriately chosen for particular tested material and its material constants. For general verification of mentioned criteria, it is necessary to have wider group of experimental results, like in Pragtic software database.

ACKNOWLEDGEMENT

The paper was created under support of TAČR, project no: TA01011274. Mary thanks to MSc. Jan Papuga, Ph.D. for the help in solving. This work was supported by the European Regional Development Fund in the IT4Innovations Centre of Excellence project (CZ.1.05/1.1.00/02.0070).

REFERENCES

[1] FUXA. J. & KUBALA. R. & FOJTÍK. F. Idea of Conjugated Strength Criterion. Acta Mechanica Slovaca. vol. 1. 2006. p. 125-130. ISSN 1335-2393.

[2] FOJTIK. F. & FUXA. J. The multiaxial material fatigue under the combined loading with mean stress in three dimension. Applied and Computational Mechanics. Vol. 3. 2009. p. 267-274. ISSN: 1802-680X.

[3] CROSSLAND. B. Effect of large hydrostatic pressure on the torsional fatigue strength of an alloy steel. In: Proc. Int. Conf. on Fatigue of Metals. Institution of Mechanical Engineers. London. 1956. p. 138-149.

[4] SINES. G. Behavior of metals under complex static and alternating stresses. In: Metal Fatigue. Red. G. Sines a J.L. Waisman. New York. McGraw Hill 1959. p. 145-469.

[5] McDIARMID. D. L. A general criterion for high cycle multiaxial fatigue failure. Fatigue Fract. Engng. Mater. Struct.. 14. 1991. No. 4. p. 429-453.

[6] PAPADOPOULOS. I. V. & DAVOLI. P. & GORLA. C. & FILIPPINI. M. & BERNASCONI. A. A comparative study of multiaxial high-cycle fatigue criteria for metals. Int. J. Fatigue 19. 1997. No. 3. p. 219-235.

[7] PAPUGA. J. & RŮŽIČKA. M. Two new multiaxial criteria for high cycle fatigue computation. Int. J. Fatigue 30. 2008. No. 1. p. 58-66.

[8] PAPUGA. J. & ŠPANIEL. M. PragTic Fatigue Freeware and FatLim Databaze. MECCA. Vol. 5 (2007). No. 3.

[9] PAPUGA. J. WCFA2011 & PUM04, Workshop on Computational Fatigue Analysis 2011 & the 4th PragTic Users' Meeting, Břežany, October 2011, (CD)