Journal of Hydrodynamics, Ser. B, 4 (2001), 24—27
China Ocean Press, Beijing—Printed in China

# A LARGE EDDY SIMULATION TURBULENCE MODEL FOR COASTAL SEAS AND SHALLOW WATER PROBLEMS*

Zhan Jie-min,
Department of Applied Mechanics and Engineering, Zhongshan University, Guangzhou 510275, China
Li Yok-sheung
Department of Civil & Structural Engineering, The Hong Kong Polytechnic University, Hong Kong, China

(Received Nov. 1, 2000)

**ABSTRACT:** In large scale motions of circulations in coastal seas and shallow-water problems, different characteristics of flow in the horizontal plane and in the vertical direction are expected. In this paper, a new large eddy simulation model was proposed. There are some differences between the present method and the other LES models. The philosophy of the large eddy simulation and the directional eddy viscosity method were applied in the horizontal plane and in the vertical direction, respectively. Comparied with the other LES models in which there is no difference between horizontal viscosity and vertical viscosity, the proposed method is resonable.

**KEY WORDS:** Large Eddy Simulation (LES), directional eddy viscosity method, coastal sea, shallow water

## 1. INTRODUCTION

The LES approach originated from the needs of meteorologists to predict or simulate the global weather. Generally Smagorinsky (1963) was credited with the initial development of LES. The first application of the LES approach to problems of engineerig interest was made by Deardorff (1970), who treated the channel flow with reasonable success. The development and applications of LES have also been made by many other researchers, for example, Aldama (1982), Ferziger (1977) and Su (1990). Many applications of large eddy simulation have succeeded in studies of the dynamics of the atmosphereic circulation, but there is a lack of those in hydraulics. However, the LES as a viable tool for oceanic turbulence research has been recognized only recently and its oceanic applications are just beginning (McWilliams et al. 1993, Skylingstad and Denbo, 1995). Because of the use of the Smagorinsky-Lilly approach, the viscosity in the Large Eddy Simulation (LES) not only varies with grid sizes, but also varies with the trace of the strain rate tensor $S_{ij}$, Because the value of the nondimensional constant $C_s$ in the Smagorinsky-Lilly viscosity formula can be predicted (Lilly 1967) using the Kolmogorov $-\frac{5}{3}$ power law, the variable viscosity is more reasonable than a constant one. In a few previous models in hydraulics (Bedford and Babajimopoulos 1980, Babajimopoulos and Bedford 1980, and Denbo and Skylingstad 1996), there is no difference between horizontal viscosity and vertical viscosity. In the present study, the philosophy of the large eddy simulation and the directional eddy viscosity method are applied in the horizontal plane and in the vertical direction, respectively.

## 2. LARGE EDDY SIMULATION AND PARAMETERIZATION OF VERTICAL VISCOSITY

Using the Gaussian filter and filtering hydrodynamics equations, we obtain

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0 \tag{1}$$

$$\frac{\partial \bar{u}}{\partial t}+\frac{\partial(\overline{uu})}{\partial x}+\frac{\partial(\overline{vu})}{\partial y}+\frac{\partial(\overline{wu})}{\partial z}-f\bar{v}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} \tag{2}$$

$$\frac{\partial \bar{v}}{\partial t}+\frac{\partial(\overline{uv})}{\partial x}+\frac{\partial(\overline{vv})}{\partial y}+\frac{\partial(\overline{wv})}{\partial z}+f\bar{u}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} \tag{3}$$

$$\frac{\partial \bar{w}}{\partial t}+\frac{\partial(\overline{uw})}{\partial x}+\frac{\partial(\overline{vw})}{\partial y}+\frac{\partial(\overline{ww})}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial z}-g \tag{4}$$

In shallow water problems, because of the different characteristics of flow, and grid sizes, in the hori-

* Project supported by the National Natural Science Foundation of China. (No: 19802023)

zontal plane and in the vertical direction, different values of viscosity in the horizontal plane and in the vertical direction are expected. Using the Taylor expansion, the filtered value of the nonlinear terms in the horizontal momentum equation can be approximated by

$$\overline{u_i u_j} = \overline{u_i} \cdot \overline{u_j} + L(\overline{u_i}, \overline{u_j}) + \overline{u_i' u_j'} \quad (5)$$

in which $i, j = 1, 2$ and

$$L(\overline{u_i}, \overline{u_j}) = \frac{\Delta_1^2}{2\gamma}\frac{\partial \overline{u_i}}{\partial x}\frac{\partial \overline{u_j}}{\partial x} + \frac{\Delta_2^2}{2\gamma}\frac{\partial \overline{u_i}}{\partial y}\frac{\partial \overline{u_j}}{\partial y} + \frac{\Delta_3^2}{2\gamma}\frac{\partial \overline{u_i}}{\partial z}\frac{\partial \overline{u_j}}{\partial z}$$

where $\gamma$ is called the filter coefficient and equals 6.0 and $\Delta_i$ is the filter size in the ith direction.

If the scale of the grid cell is small enough, Eq. (5) is available for $i, j = 1, 2, 3$. Generally, limilted by the capability of computers, the scale of the grid cell (especially on the horizontal plane of the shallow-water flow field) cannot be small enough. So Eq. (5) is only used for the terms $\overline{uu}$, $\overline{uv}$ $\overline{vv}$. In the horizontal plane, using the smagorinsky-Lilly approach, the term $\overline{u_i' u_j'}$ can be modeled as

$$\overline{u_i' u_j'} - \frac{2}{3}\delta_{ij} E = -\frac{1}{2}\mu\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) \quad (6)$$

in which

$$\mu = (C_s \Delta)^2\left[\left(\frac{\partial \overline{u}}{\partial x}\right)^2 + \left(\frac{\partial \overline{v}}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x}\right)^2\right]^{1/2},$$

$$\overline{E} = \frac{1}{2}\overline{u_i' u_j'} \quad (7)$$

and $\delta_{ij}$ is the Kronecker delta; $\Delta$ is a length related to the grid cell by $\Delta = (\Delta_1 \Delta_2)^{1/2}$; $C_s$ is a dimensionless constgant (=0.2 in this paper), which can be predicted using the Kolmogorov $-\frac{5}{3}$ power law; $E$ is the SGS turbulent kinetic energy per unit mass. $2E$ acts like pressure forces and can be absorbed by the pressure-gradient term so that in fact the pressure can be replaced as an unknown quantity by the modified pressure

$$\boldsymbol{P} = \overline{p} + \frac{2}{3}E \quad (8)$$

Because of the different characteristics of flow in the horizontal plane and in the vertical direction, the authors suggest that the nonlinear terms $\overline{uw}$ and $\overline{vw}$ are treated using the directional eddy viscosity method, e.g.,

$$\rho\overline{uw} = \rho\overline{u} \cdot \overline{w} - \tau_{xz} = \rho\left(\overline{u} \cdot \overline{w} - \varepsilon\frac{\partial u}{\partial z}\right) \quad (9)$$

$$\rho\overline{vw} = \rho\overline{v} \cdot \overline{w} - \tau_{xz} = \rho\left(\overline{v} \cdot \overline{w} - \varepsilon\frac{\partial v}{\partial z}\right) \quad (10)$$

For different problems, the formula of $\varepsilon$ has different forms. For the simulation of the South China Sea, the following vertical viscosity has been used, which is based upon Irish Sea observations (Bowden et al. 1959, Davies and Lawrence 1995).

$$\varepsilon = 0.0025(\hat{u}^2 + \hat{v}^2)^{\frac{1}{2}} h \quad (11)$$

where $\hat{u}$, $\hat{v}$ = vertically-averaged velocity components and $h$ is water depth.

By assuming hydrostatic pressure distribution in the vertical direction, the equations of $\overline{u}$, $\overline{v}$, $\overline{w}$ and $\eta$ can be obtatined, where $\eta$ is the water surface elevation.

## 3. NUMERICAL METHOD

To reduce the effects of numerical errors on the physical process, we need very stable and high order numerical schemes. Therefore, based on the staggered mesh, the spline interpolation and Lagrangrian interpolation are used in the spatial discretization in the horizontal plane and in the vertical direction, respectively. In the time discretization, the following fourth-order accurate scheme is used:

$$(f^{n+\frac{1}{2}})^k = \frac{f^n + (f^{n+1})^k}{2} - \frac{\Delta t}{8}\left\{\left[\left(\frac{\partial f}{\partial t}\right)^{n+1}\right]^k - \left(\frac{\partial f}{\partial t}\right)^n\right\} \quad (12)$$

$$(f^{n+1})^{k+1} = f^n + \frac{2\Delta t}{3}\left[\left(\frac{\partial f}{\partial t}\right)^{n+\frac{1}{2}}\right]^k + \frac{\Delta t}{6} \cdot \left\{\left[\left(\frac{\partial f}{\partial t}\right)^{n+1}\right]^k + \left(\frac{\partial f}{\partial t}\right)^n\right\} \quad (13)$$

and $(f^{n+1})^0 = f^n$, $\left[\left(\frac{\partial f}{\partial t}\right)^{n+1}\right]^0 = \left(\frac{\partial f}{\partial t}\right)^n$ when $k = 0$ with $f$ representing $\overline{u}$, $\overline{v}$ or $\eta$. Knowing $(f^{n+\frac{1}{2}})^k$, $\left[\left(\frac{\partial f}{\partial t}\right)^{n+\frac{1}{2}}\right]^k$ can be obtained from the equations of $\overline{u}$, $\overline{v}$

or $\eta$. The iterative procedure is continued until the absolute values of the differences in $\bar{u}$, $\bar{v}$ and $\eta$ between two iterations are less than $10^{-5}$. When the iterative procedure at each time step is completed, $(f^{n+1})^{k+1}$ is denoted by $f^{n+1}$.

Numerical experiments show that, the proposed model has good numerical stability and high-order accuracy. The effect of numerical errors on the physical progress is very small.

## 4. EXAMPLE AND DISCUSSION

A second-order accurate version of the proposed LES model has been verified (Zhan and Li, 1993, 1998). In this paper, the high order accurate LES model is applied to simulate the flow pattern of the South China Sea (SCS) driven by seasonal wind stresses and the Kuroshio.

The SCS is one of the largest marginal seas of the western Pacific Ocean (Xu, Su and Qiu 1995, Wyrki 1961, Metzger and Hurlburt 1996). The computational domain is between 98°E and 126°E and between 3°S and 26°N. The grid Size is 0.25°×0.25°. From surface to bottom, the thicknesses of the five water layers used in the simulation are 60m, 150m, 300m, 500m and $h_1$, where

$$h_1 = \begin{cases} 0, & h \leqslant 1010\text{m} \\ h - 1010, & h > 1010\text{m} \end{cases} \quad (14)$$

For $h_1 > 0$, the bottom stress components are used. When $h_1 = 0$, $\bar{u}_1 = \bar{v}_1 = 0$. The time step size is 150s. The division of water body in the vertical driection is based on the hydrographic analysis of the SCS by Xu, Su and Qiu (1995).

To simulate correctly the flow pattern of the SCS driven by seasonal wind stresses and the Kuroshio, Computation was run for two years using data interpolated from monthly mean wind stress fields. The open boundary conditions on 126°E and 26°N are interpolated from the results obtained by Cai and Li (1997), and Li and Cai (1999). The first annual cycle was used to warm up the model from a motionless state. The simulated flow patterns in the surface layer in January is shown in Fig. 1. It can be seen that there are cyclonic eddies near the Natuna Island. In the gulf of Thailand, eddy circulation can be observed. The same phenomena are also reported in Naga report 2 (Wyrki 1961). In the Beibu Gulf, there is also eddy circulation, because of the geometric shape of the Gulf. When the Kuroshio flows near the Bashi Channel, the main part of the Kuroshio will turn right and flow northeastward back into the Pacific. In the northwest of the Luzon and the southeast of the Taiwan, there are strong eddies. It seems that the eddies mix the water from the SCS and the Kuroshio. In the SCS, many meso-scale eddies can be observed. This phenomenon is a common feature of oceanic circulations.

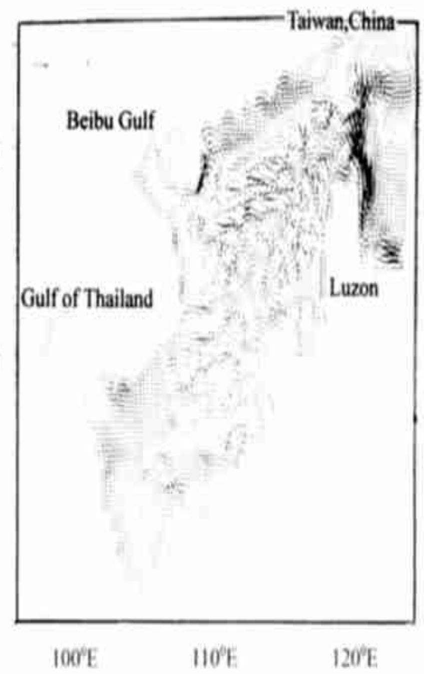


Fig. 1 The simulated flow pattern in the surface layer in January using the LES model

## ACKNOWLEDGMENT

The Scientific Research Foundation for the Returned Oversea Chinese Scholars, State Education Ministry.

## REFERENCES

1. Aldama A. A., 1982: Filtering Techniques for Turbulent Flow Simulation, Springer-Verlag.
2. Babajimopoulos C. and K. W. Bedford, 1980: Formulating Lake Models which Preserve Spectral Statistics, Journal of the Hydraulics Division, ASCE, 106(HY1), 1~19.
3. Beford K. and C. Babajimopoulos, 1980: Verifying Lake Models with Spectral Statistics, Journal of the Hydraulics Division, ASCE, 106(Hy1), 21~38.
4. Bowden K. F., L. A. Fairbairn and P. Hughes 1959: The Distribution of Shearing Stresses in a Tidal Current, Geophys. J. Roy. Astron. Soc., 2, 288~305.
5. Cai Y. and Li Y. S., 1997: A 3-dimensional Baroclinic Circulation Model of the Tropical and Northern Pacific, Chinese Journal of Oceanology and Limnology, 15(1), 70~79.
6. Davies A. M. and Lawrence J., 1995: Modeling the Effect of Wave-Current Interaction on the Three-Dimensional Wind Driven Circulation of the Eastern Irish Sea, Journal of

Physical Oceanography, 25, 29～45.

7. Deardorff J. W. 1970: A Numerical Study of Three-Dimensional Turbulent Channel Flow at Large Reynolds Numbers, J. Fluid Mech., 41, 453～480.

8. Denbo D. W. and E. D. Skyllingstad, 1996: An Ocean Large-Eddy Simulation Model with Application to Deep Convection in the Greenland Sea, Journal of Geophysical Research, 101(C1), 1095～1110.

9. Ferziger J. H., 1977: Large Eddy Numerical Simulations of Turbulent Flows, AIAA Journal, 15(9), 1261～1267.

10. Lilly D. K., 1967: The Representation of Small-Scale Turbulence in Numerical Simulation Experiments, Proceedings of the Tenth IBM Scientific Computing Symposium on Environmental Sciences, 195～210, Thomas J. Watson Research Genter, Yorktown Heights.

11. Li Y. S. and Cai Y., 1999: Numerical Simulation of the Kuroshio Using Nested Model of Three-Dimensional Baroclinic Circulation, Chinese Journal of Oceanology and Limnology, 17(1), 19～27.

12. Mcwilliams J. C., Gallacher P. C., Moeng C. H. and Wyngaard J. C., 1993: Modeling the Oceanic Planetary Boundary Layer, In: Large Eddy Simulation of Complex Engineering and Geophysical Flows, Edited By Boris Galperin and Steven A. orzag, 441～454, Cambridge Univ. Press, New York.

13. Metzger E. J. and H. E. Hurburt 1996: Coupled Dynamics of the South China Sea, the Sulu Sea, and the Pacific Ocean, Journal of Geophysical Research, 101(C5), 12331～12352.

14. Skyllingstad E. D. and Denbo D. W., 1995: An Ocean Large-Eddy Simulation of Langmuir Circulations and Convection in the Surface Mixed Layer, Journal of Geophysical Research, 100(C5), 8501～8522.

15. Smagorinsky J., 1963: General Circulation Experiments with the Primitive Equations, Monthly Weather Review, 91(3), 99～165.

16. Su M. D., 1990: Algebraic Modeling of Large Eddy Simulation and its Application, Science in China (Series A), 33 (2), 185～195.

17. Wyrtki K., 1961: Scientific Results of Marine Investigations of the South China Sea and Gulf of Thailand, Naga Report Vol 2, Scripps Institution of Oceanography, The University of California.

18. Xu J. P., Su J. L. and Qiu D. Z., 1995: Hydrographic Analysis on the Intruding of Kuroshio Water into the South China Sea, Proceedings of Symposium of Marine Sciences in Taiwan Strait and Its Adjacent Waters, China Ocean Press, 30～44.

19. Zhan J. M. and Li Y. S., 1993: A 3-Dimensional Turbulent Shallow-Water Model Using LES and Directional Eddy Viscosity Method, Proc. of XXV Congress of International Association for Hydraulic Research (Tokyo, Japan), 4, 32～39.

20. Zhan J. M. and Li Y. S., 1998: A Hydrodynamic Large-Eddy Simulation-Model Using Directional Eddy Viscosity Method in Vertical Direction, Journal of Hydrodynamics Ser. B, 10(1), 36～43.