

**EFFICIENCY ANALYSIS OF SEVERAL EU STOCK MARKETS:
MEAN-RISK EFFICIENT PORTFOLIOS**

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ABSTRACT

Mean-risk problems are ones of the most important tools in financial risk management and decision making. They jointly focus on a portfolio mean maximization and a portfolio risk minimization. A portfolio risk can be expressed using various dependency/concordance measures. In order to analyze the performance of mean-risk portfolios, a stochastic dominance approach can be employed. In particular, the notion of the second-order stochastic dominance (SSD) is the most popular one. It is based on comparisons of expected utilities for all concave utility functions. In this paper, we focus specifically on selected EU stock markets and analyze the SSD efficiency of portfolios on the mean-risk efficient frontier if the risk is represented by standard deviations and concordance matrices set up on the basis of either Pearson linear correlation or Spearman rho or Kendall tau. It is empirically documented which measures and which portfolios from the mean-risk efficiency frontiers should be of interest for at least one risk averse investor under given conditions

KEYWORDS

Concordance measure; portfolio efficiency; stochastic dominance; stock index.

JEL Classification: G11, C44

1. INTRODUCTION

Identification of attractive investments is in the spotlight of all market participants, including companies and households. Actually, starting from Markowitz (1952), a portfolio selection problem is one of the most important issues of financial decision making as concerns both, the mathematical treatment and empirical analysis. In last 60 years, several portfolio selection problems were introduced and analyzed. As a generalization of the Markowitz model, mean-risk models proved to be the most popular ones. They jointly minimize a portfolio risk and maximize the portfolio mean return. The optimal portfolio composition heavily depends on the dependency among the evolution of particular risk factors. Therefore, there is a need for a suitable measure of dependency. A standard assumption of multivariate normal distribution of large portfolios and the

assumption of dependency described well by a linear measure of correlation (Pearson coefficient of correlation) may not be fulfilled in real applications and the Pearson correlation may not be sufficiently robust to describe the dependency of market returns (see e.g. Rachev et al., 2008).

One of the reasons probably is the increasing speed of market innovations and deep interconnection of particular market segments. A possible way, how to deal with these market aspects is to consider a proper measure for the dependency. In theory, the best measure should be the copula function (see Nelsen, 2006). However, copula functions are often very computationally demanding, especially within portfolio selection problems.

Instead, one can consider some of the well-known concordance measures, such as Kendall's tau, Spearman's rho, Blomquist's beta or Ginni's gamma, see Ortobelli and Tichý (2009). All of them can be regarded as the more advanced candidates for a suitable dependency measure. Simultaneously, their implementation is relatively easy. By minimizing portfolio's risk based on such measures, subject to constraints on minimal required mean return, one can obviously construct an alternative to the standard mean-risk efficiency frontier. The question is how to analyze the performance of these mean-risk efficient portfolios and compare them among each other.

In this context one can consider a stochastic dominance approach since it offers an attractive portfolio evaluation approach (see e.g. Hanoch and Levy (1969) or Hardy et al. (1934)). The definition of second-order stochastic dominance (SSD) relation is based on comparisons of twice cumulative distribution functions lower partial moments, second quantile functions, conditional Value-at-Risks or expected utilities of returns employing concave utility functions (see for example Levy, 2006 and Kopa, 2010). The SSD relation between two portfolios is observed if all risk averse decision makers prefer one portfolio to the other one.

Following the mean-variance efficiency analysis, the second-order stochastic dominance criterion can be used in portfolio efficiency as well. In this case, a given portfolio is defined as SSD efficient (relative to all portfolios created from a set of assets) if no other portfolio SSD dominates the portfolio. (see for example Branda and Kopa(2012) and references therein). Since it is empirically impossible to identify directly the whole set of SSD efficient portfolios, the tests of the SSD efficiency of a given portfolio proved to be very useful. In particular, Post (2003), Kuosmanen (2004), Kopa and Chovanec (2008), Kopa (2010) and Dupačová and Kopa (2012) proposed several mathematical programming algorithms for testing SSD efficiency.

In this paper, inspired by Kopa and Tichý (2012), we analyze several portfolios constructed from stock indices of European stock markets, so that these portfolios are mean-risk efficient (as usually, we assume risk averse decision makers). Contrary to Kopa and Tichý (2012), we analyze all portfolios from efficiency frontier and not only the portfolios with minimal risk. Moreover, we turn our attention to Europeans stock markets. In particular, we combine five reference currencies, three measures of dependency/concordance (Pearson, Spearman and Kendall), two restriction conditions on short sales and two data periods (before and after September 2008), we obtain 60 mean-risk efficient frontiers for which we apply the Kuosmanen SSD efficiency test in order to

analyze the SSD (in) efficiency of portfolios from all mean-risk frontiers obtained under given conditions.

We proceed as follows. First, in Section 2, we summarize the basic theoretical concepts of concordance measures and portfolio selection problem. Next, in Section 3, stochastic dominance approach with a special focus on portfolio efficiency with respect to SSD is presented. In Section 4, we introduce and analyze data for an empirical study. Section 5 presents results of the empirical study: 60 mean-risk efficiency frontiers and SSD (in) efficiency measure calculations of portfolios from these frontiers. The results enable comparing the SSD performance of considered mean-risk efficient portfolios among each other. In Section 6 the most important conclusions and remarks are stated.

2. CONCORDANCE MEASURES AND PORTFOLIO SELECTION

Let us consider n assets with random vector $r = (r_1, r_2, \dots, r_n)'$ of returns having discrete probability distribution. The distribution is given by T scenarios that are taken with the same probabilities. For various scenarios, the returns are collected in matrix

$$X = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^T \end{pmatrix}$$

where $x^t = (x_1^t, x_2^t, \dots, x_n^t)$ is the t -th row of matrix X , that is, $x^t = (x_1^t, x_2^t, \dots, x_n^t)$ express the returns of the assets along t -th scenario. Moreover, we use $w = (w_1, w_2, \dots, w_n)$ for the vector of portfolio weights. Throughout the paper, we will consider two special sets of portfolio weights:

$$W_M = \left\{ w \in \mathbf{R}^n : \sum_{i=1}^n w_i = 1, w_i \geq 0, i = 1, 2, \dots, n \right\}$$

$$W_B = \left\{ w \in \mathbf{R}^n : \sum_{i=1}^n w_i = 1, 1 \geq w_i \geq -1, i = 1, 2, \dots, n \right\}.$$

Besides that, we use the following notation: mean returns $m = (\mu_1, \mu_2, \dots, \mu_n)'$, standard deviations of returns $s = (\sigma_1, \sigma_2, \dots, \sigma_n)$, and correlation matrix $R = [\rho_{ij}]$, i.e. it consists of all combinations of ρ_{ij} – the Pearson linear coefficient of correlation, where $i, j = 1, \dots, n$.

Following the standard portfolio selection problem of Markowitz (1952, 1959) no riskless investment is allowed and only the mean return and the risk measure of standard deviation matter, mainly since the Gaussian distribution of price returns is assumed. In such a setting, the efficient frontier of portfolios, i.e., the only combination of particular assets that should be considered for risky investments is bounded by minimal variance

portfolio, Π_A , (with mean return m_A) from the left and maximal return portfolio, Π_K , (with mean return m_K) from the right. We can construct the efficient frontier by solving the following problem:

Task 1 Mean-risk efficient portfolio Π :

$$\min \text{var}(\Pi) = w' \Sigma w$$

$$\Sigma = [\sigma_i \sigma_j \rho_{ij}]$$

$$\text{s.t. } w'm \geq m_r$$

$$w \in W_M$$

for each minimal required mean return $m_r \in [m_A, m_K]$.

Alternatively, Task 1 can be solved subject to $w \in W_B$, i.e., limited short positions in any of the assets are allowed with additional restriction on long position. The optimal portfolio under both models depends on preferences of a particular investor. Alternatively, it can be detected on the basis of a given utility function, a performance measure (Sharpe ratio, information ratio, etc.), a risk measure (VaR, CVaR) or their combinations.

Obviously, the composition of any portfolio, except the maximal return one, will depend on the correlation matrix. The elements of the correlation matrix R , i.e., a crucial factor to determine optimal weights for Π , describe the linear dependency among two variables. The main drawback is that it can be zero even if the variables are dependent and it does not take into account tail dependency. It follows that the correlation is suitable mainly for problems with elliptically distributed random variables. Since the assumption about the Gaussianity of financial returns is unjustifiable, there is a need for alternative measures, which should allow us to obtain better performance, diversification or both.

A general family of measures that is not restricted to the case of linear dependency consists of *concordance measures*. A measure of concordance is any measure that is normalized to the interval $[-1, 1]$ and pays attention not only to the dependency but also to the co-monotonicity and anti-monotonicity. For more details on all properties of concordance measures and their proofs see e.g. Nelsen (2006).

Following Nelsen (2006), two random variables (X, Y) with independent replications (x_1, y_1) and (x_2, y_2) are concordant if $x_1 < x_2$ ($x_1 > x_2$) implies $y_1 < y_2$ ($y_1 > y_2$). Similarly, the two variables are discordant if $x_1 > x_2$ ($x_1 < x_2$) implies $y_1 < y_2$ ($y_1 > y_2$). The concordance measures are easily definable by copula functions, since they rely only on the "joint" features, having no relations to the marginal characteristics. In statistics, the basic measures of concordance are Kendall's tau and Spearman's rho, which can be accompanied by some further measures, such as Gini's gamma or Blomqvist's beta.

The first measure of concordance in mind is thus Kendall's tau, τ_K , since it is given by the probability of concordance which is reduced by the probability of discordance:

$$\tau_K(X, Y) = \mathbf{P}((x_1 - x_2)(y_1 - y_2) > 0) - \mathbf{P}((x_1 - x_2)(y_1 - y_2) < 0). \quad (1)$$

In order to define the second popular measure of concordance, Spearman's rho, ρ_S , the third replication of both random variables, (x_3, y_3) , should be considered:

$$\rho_S = 3\mathbf{P}((x_1 - x_2)(y_1 - y_3) > 0) - \mathbf{P}((x_1 - x_2)(y_1 - y_3) < 0). \quad (2)$$

It means that the Spearman's rho is given as the probability of concordance reduced by the probability of discordance, in contrast to the Kendall's tau, for the pairs (x_1, y_1) and (x_2, y_3) . The proof that both measures introduced in this section are really measures of concordance can be found e.g. in Nelsen (2006).

Thus, being equipped with formulas to calculate (estimate) alternative dependency measures, we can replace the elements of the covariance matrix Σ from Task 1:

$$\text{cov}(X, Y) = \sqrt{\text{var}(X)}\sqrt{\text{var}(Y)}\rho(X, Y)$$

by the elements of a concordance matrix e.g. by terms of Spearman's rho (2):

$$\text{cov}_S(X, Y) = \sqrt{\text{var}(X)}\sqrt{\text{var}(Y)}\rho_S(X, Y),$$

or Kendall's tau (1):

$$\text{cov}_K(X, Y) = \sqrt{\text{var}(X)}\sqrt{\text{var}(Y)}\tau_K(X, Y).$$

3. SECOND ORDER STOCHASTIC DOMINANCE AND PORTFOLIO EFFICIENCY

Stochastic dominance relation was firstly introduced in statistics for comparison of two random variables. Later on, it was applied in portfolio theory for comparison random portfolio returns (see Levy, 2006 and reference therein). For portfolio with weights \mathbf{w} , denoted by $F_{r^w}(x)$, the cumulative probability distribution function of its returns. Since each portfolio is uniquely given by its weight vector we will shortly denote this portfolio by \mathbf{w} , too. The twice cumulative (integrated) probability distribution function of returns of portfolio \mathbf{w} , known also as the first lower partial moment, is defined as:

$$F_{r^w}^{(2)}(t) = \int_{-\infty}^t F_{r^w}(x) dx$$

and portfolio \mathbf{v} dominates portfolio \mathbf{w} by the second-order stochastic dominance ($r^v \succ_{SSD} r^w$) if and only if

$$F_{r^v}^{(2)}(t) \leq F_{r^w}^{(2)}(t) \quad \forall t \in R$$

where strict inequality is fulfilled for at least one $t \in R$. This relation is sometimes called strict second-order stochastic dominance because of the at least one strict inequality. Alternatively, one may use several different ways of defining the second-order stochastic dominance criterion— see Kopa (2010) or Branda and Kopa (2012) for a review.

Since the decision maker may form infinitely many portfolios, the criteria for pairwise comparisons have only limited use in portfolio efficiency testing. To test whether a given portfolio \mathbf{w} is SSD efficient, three linear programming tests were developed, see e.g. Kopa and Chovanec (2008), Kuosmanen (2004) and Post (2003). Recently, Dupačová and Kopa (2012) derived a new test for SSD efficiency expressed in terms of CVARs.

In our empirical section, we apply the Kuosmanen SSD efficiency test. We formulate it for SSD efficiency with respect to W_M . However, one can easily rewrite it for W_B . The test consists of solving two linear programs, in order to identify a dominating portfolio (if it exists) which is already SSD efficient. The test is based on majorization principle (see Hardy et al. 1934) and double stochastic matrix P . Let

$$\varphi^*(\mathbf{w}) = \min_{P, \mathbf{v} \in W_M} \sum_{t=1}^T (x^t \mathbf{v} - x^t \mathbf{w}) \quad (3)$$

$$\text{s.t. } PX \mathbf{w} \leq X \mathbf{v}$$

$$\sum_{j=1}^T p_{ij} = 1, \quad \sum_{i=1}^T p_{ij} = 1, \quad p_{ij} \geq 0 \quad i, j = 1, 2, \dots, T$$

and

$$\varphi^{**}(\mathbf{w}) = \min_{P, S^+, S^-, \mathbf{v} \in W_M} \sum_{j=1}^T \sum_{i=1}^T (s_{ij}^+ + s_{ij}^-) \quad (4)$$

$$\text{s.t. } X \mathbf{v} = PX \mathbf{w}$$

$$s_{ij}^+ - s_{ij}^- = p_{ij} - \frac{1}{2} \quad i, j = 1, 2, \dots, T$$

$$s_{ij}^+, s_{ij}^-, p_{ij} \geq 0 \quad i, j = 1, 2, \dots, T$$

$$\sum_{j=1}^T p_{ij} = 1, \quad \sum_{i=1}^T p_{ij} = 1 \quad i, j = 1, 2, \dots, T,$$

where $S^+ = \{s_{ij}^+\}_{i,j=1}^T$, $S^- = \{s_{ij}^-\}_{i,j=1}^T$ and $P = \{p_{ij}\}_{i,j=1}^T$. Then portfolio \mathbf{w} is SSD efficient with respect to W_M if and only if

$$\varphi^*(\mathbf{w}) = 0 \wedge \varphi^{**}(\mathbf{w}) = \frac{T^2}{2} - \sum_{k=1}^T k \varepsilon_k$$

Where ε_k express the number of k -way ties in the vector of returns $X\mathbf{w}$. If $\varphi^*(\mathbf{w}) > 0$ then a dominating portfolio \mathbf{v}^* (an optimal solution of (3)) exists and, hence, portfolio \mathbf{w} is SSD inefficient. Moreover, the portfolio \mathbf{v}^* is already SSD efficient. Finally, $\varphi^*(\mathbf{w})$ can serve as a measure of SSD inefficiency of portfolio \mathbf{w} . The higher the measure is, the greater improvement (in terms of mean returns) can be done by moving from \mathbf{w} to \mathbf{v}^* , see Kuosmanen (2004). If portfolio \mathbf{w} is classified as SSD efficient then the measure equals zero.

4. DATA

In our analysis we consider 9 stock market indices throughout the Europe. In particular, we selected stock indices from countries with relatively developed and liquid stock markets, such as Austria (ATX), Belgium (BFX), France (FCHI), Germany (GDAXI), Netherland (AEX), Greece (GDAT), Spain (IBEX), Italy (FTSEMIB) and Ireland (ISEQ). Although one may argue that some of them are more or less close to bankruptcy, their inclusion into the admissible set might increase the investment opportunities. Since all these indices are denominated in European single currency EUR, the data set also covers FX rates for reference currencies of selected investors, i.e. Central European CZK, HUF and PLN, and Swiss franc (CHF).

Since the recent sub-prime crises, which started in late summer 2008, undermined the confidence into the single European currency, and on the other hand increased the sensitivity of investors to several risk factors (emerging markets, liquidity constraints), the results of the analysis might differ due to the reference currency, as well as the period over which the data are collected. We therefore decided to analyze the efficiency of particular portfolios under five different reference currencies (EUR, CHF, CZK, HUF, PLN) and in two separate periods, distinguished by September 2008 (the assumed beginning of the sub-prime crises). Since the last available data were collected in September 2012, and we prefer to have the same number of observations in both subsets, the first data were taken in September 2004. For simplicity and due to the data properties, we use index values and FX rates as reported for the first business day on each month – thus we obtain 49 monthly observations for each series.

Average log-returns and their standard deviations,¹ both per annum, of each index are depicted in Table 1 – in the local currency (first part), CZK (second part) and CHF (last part). If the number is depicted in bold, the index can be regarded as efficient in the mean-variance sense over given period (B and A states *before* and *after* September 2008, respectively).

We can easily see that assuming only the local currency, i.e. EUR, higher returns with slightly lower risk – formulated by standard deviation – were recorded for most of the indices before the start of the crises. The next two columns of Table 1 depict average returns and their standard deviations recalculated to CZK, i.e. the currency which was significantly appreciating with respect to EUR during the first period. It implies that the returns are significantly lower during the before crises period, though the risk is mostly similar. Similar results can be observed for the other currencies we consider in this study.

5. RESULTS

Traditional equity markets in the US and EU were badly evolved by the impacts of the crises at financial markets. The recent events at the financial markets bring changes in the way how investors perceive risk, including potential dependencies among particular investments. It might also be interesting to examine if alternative risk minimizing portfolios based on *Spearman* or *Kendal rank correlation* instead of *Pearson measure* of

¹ The same calculations were made for discrete turns. Obviously, there was no unexpected impact on the result – there turns were only slightly lower in absolute terms.

linear correlation could be regarded as efficient in the SSD portfolio efficiency sense and, if inefficient, how enormous the inefficiency score is and what was the impact of the financial crises on the (in) efficiency of particular portfolios. The inefficiency score (optimal objective value of (3)) gives an information about maximal possible increase of mean return when moving from the tested portfolio to some dominating one.

Since investing into the risk minimizing portfolio, which we denote as Π_A , can lead to very low and sometimes even negative expected return, we examine also portfolios with pre-specified level of return. For simplicity, we assume 9 portfolios as follows – knowing the weights of particular indices in Π_A , we find a structure of the portfolio with maximal return, Π_K , which allows us to calculate the equidistant expected return for inner portfolios $\Pi_B, \Pi_C, \dots, \Pi_J$.

The risk minimizing portfolios are generally obtained as the optimal solutions of the quadratic programming problem:

$$\begin{aligned} \text{var}(\Pi_A) &\rightarrow \min, \text{ with } \text{var}(\Pi_A) = w^T \Sigma w \\ \Sigma &= [\sigma_i \sigma_j \rho_{ij}] \\ \text{s.t. } w^T \mathbf{1} &= 1 \\ w &\in W, \end{aligned}$$

where d_{ij} is an element of either Pearson, Spearman, or Kendall matrix of dependence/concordance and W states the constraint on the portfolio weights. Here, we consider either $W = W_M$ (short selling is not allowed) or $W = W_B$, when short selling is allowed up to the original wealth (100%) for each asset, but we cannot hold more than 100% in any asset. This constrain is related to finite volume of the markets, liquidity concerns and diversification requirements.

Similarly, for portfolio Π_K we replace risk minimization by expected return maximization, $E(R(\Pi_K)) = w^T E(r) \rightarrow \max$, while the weights for inner portfolios are obtained by adding the condition on preset return to the program above (Task 1).

Pearson linear correlation

We start with the composition of the min-risk portfolio that is presented in Table 2. Once again, we depict the results obtained when (i) the reference currency is EUR or (ii) CZK and (iii) CHF, respectively, before (B) and after (A) September 2008. For all cases, shortselling is either prohibited, $w \in W_M$ (NoSS) or restricted by the initial funds, $w \in W_B$ (ReSS). As concerns the SSD efficiency of the MV efficient frontier with shortselling possibilities, we can clearly see from Figure 1 that (i) all portfolios with low risk are inefficient, but (ii) the inefficiency increases after September 2008 and could reach even more than 40%.

Table 1: Basic Descriptive Statistics of Considered Index Returns

Country	EUR-B		EUR-A		CZK-B		CZK-A		CHF-B		CHF-A	
	mean	st.dev.	mean	st.dev.	mean	st.dev.	mean	st.dev.	mean	st.dev.	mean	st.dev.
Austria	8%	24%	-8%	29%	1%	24%	-8%	29%	0%	25%	-2%	28%
Belgie	0%	17%	-3%	21%	-6%	17%	-3%	21%	-7%	18%	3%	20%
Francie	3%	15%	-4%	21%	-4%	15%	-4%	21%	-5%	16%	2%	20%
Germany	10%	16%	5%	24%	4%	15%	5%	23%	3%	17%	10%	23%
Netherland	1%	19%	0%	21%	-6%	19%	0%	22%	-7%	20%	6%	21%
Greece	5%	22%	-37%	40%	-1%	21%	-37%	39%	-2%	23%	-32%	40%
Spain	8%	15%	-10%	26%	2%	15%	-10%	27%	1%	16%	-4%	28%
Italy	-2%	15%	-13%	27%	-8%	15%	-13%	27%	-9%	16%	-7%	26%
Ireland	-11%	22%	-3%	25%	-18%	22%	-3%	25%	-19%	23%	3%	23%

Expected log returns and their standard deviations are reported in (i) EUR before September 2008 (EUR-B) and after September 2008 (EUR-A) and in a similar manner for (ii) Czech koruna and (iii) Swiss franc. Bold numbers show indices, which can be regarded as mean-variance efficient.

Table 2: Weights and Basic Descriptive Statistics of Risk Minimizing Portfolios

Country	EUR-B		EUR-A		CZK-B		CZK-A		CHF-B		CHF-A	
	mean	st.dev.	mean	st.dev.	mean	st.dev.	mean	st.dev.	mean	st.dev.	mean	st.dev.
Austria	0%	-44%	0%	-67%	0%	-41%	0%	-70%	0%	-41%	0%	-47%
Belgie	0%	7%	41%	91%	0%	20%	35%	87%	0%	22%	43%	83%
Francie	24%	5%	32%	91%	26%	-12%	40%	100%	10%	-28%	35%	100%
Germany	0%	34%	0%	10%	0%	41%	1%	14%	0%	47%	0%	-24%
Netherland	0%	8%	26%	28%	0%	12%	17%	20%	0%	8%	22%	34%
Greece	0%	-38%	0%	5%	0%	-36%	0%	9%	0%	-52%	0%	-2%
Spain	40%	60%	0%	23%	35%	54%	0%	16%	45%	70%	0%	41%
Italy	35%	64%	0%	-100%	39%	73%	0%	-100%	45%	80%	0%	-100%
Ireland	0%	3%	2%	19%	0%	-11%	8%	24%	0%	-5%	0%	14%
$E(R_A)$	3%	1%	-3%	8%	-4%	-3%	-3%	7%	4%	4%	-10%	-2%
$\sigma(R_A)$	14%	12%	20%	15%	14%	12%	21%	16%	15%	12%	24%	19%
$E(R_K)$	10%	46%	5%	63%	4%	40%	5%	63%	11%	48%	-3%	56%
$\sigma(R_K)$	16%	35%	24%	49%	15%	35%	24%	49%	17%	36%	29%	53%

The first panel shows weights of particular index in the minimal risk portfolio Π_A before and after September 2008, if either EUR (*EUR-B*, *EUR-A*) or other reference currency is considered (*CZK* and *CHF*, respectively) assuming no shortselling allowed (*NoSS*) and shortselling restricted by the initial funds (*ReSS*). If the number is in bold, the asset is used in long position for maximal return portfolio Π_K . By contrast, if the number is in italic, such asset is not used for long-position investment in any portfolio at the

efficient frontier. The second panel shows expected return and its standard deviation of these portfolios.

We can easily see that maximal return portfolio, regardless the shortselling possibility, is always efficient in the SSD portfolio sense. It is natural since there is no other asset or their combination at the market that can lead to higher return at a given level of risk. However, it is not true for minimal risk portfolio except the case *NoSS* after September 2008. The reason is that concordance measures are generally not consistent with SSD relation unless the distribution of returns is elliptical. It means that a SSD dominating portfolio need not exhibit the smallest concordance measure. Hence a minimal risk portfolio can be SSD inefficient because a dominating portfolio with higher concordance measure exists.

Interesting question is how big the rate of the inefficiency of particular portfolios on the efficient frontier is. We plot the rate of the inefficiency into the charts at Figure 1 (top right *NoSS*, top left *ReSS*). First, without shortselling opportunities, most of the portfolios can be regarded as efficient. Some level of inefficiency can be observed for low-risk portfolios, and it mostly vanishes after September 2008. It is also interesting to see that the inefficiency in terms of CZK, PLN, and CHF is higher than the inefficiency in terms of EUR and HUF.

Allowing short positions, the number of SSD inefficient portfolios significantly increases, especially after September 2008.

We also calculated the inefficiency measure of the whole efficient frontier under assumption of discrete returns, but we did not find any noticeable difference.

Spearman and Kendal measure of concordance

Spearman and Kendal measure of concordance provide an alternative to classic measure of linear correlation, but theoretical foundations of these two measures suggests, that portfolios selected by replacing ρ by ρ_S or τ_K in the “covariance” matrix of Task 1 might not be interested for risk averse investors.

We apply the same procedure as described above to find the efficient frontier due to the Markowitz approach without shortselling (*NoSS*) and also with restricted shortselling (*ReSS*). As concerns the composition of particular portfolios, it is obvious that there cannot be any impact of these two measures on max-return portfolio, since the risk does not play any role there. By contrast, as we move along the efficient frontier to the left, the differences are more evident. This is stressed when some shortselling is allowed.

It also results into quite different levels of inefficiency, as it is apparent from Figure 1 (Spearman in the middle, Kendal at the bottom). Starting with the results due to Spearman measure of concordance without shortselling possibilities, almost all portfolios before crises can be regarded as SSD inefficient, except the max-return one. Here, it is relatively surprising that the highest inefficiency was mostly recorded for CHF. By contrast, almost all portfolios calculated after September 2008 must be regarded as SSD efficient.

Furthermore, if we replace Spearman rho by Kendal tau the inefficiency before the crises is almost the same, but the inefficiency in terms of CHF after September 2008 almost reaches the values before September 2008 in terms of other currencies.

If we consider shortselling opportunities, the inefficiency can rise even above 50%. Moreover, it is significantly higher after September 2008 – it is in contradiction to the observations without shortselling constraints.

6. CONCLUSION

In this paper we studied the SSD (in) efficiency of selected portfolios created from the 9 EU stock indices via Kuosmanen test. These portfolios were found as mean-risk efficient ones, when the dependency matrix is build up on the basis of concordance measures (namely, Pearson, Spearman and Kendall measures of dependency). Moreover, the analysis was executed for five different currency bases, two different time periods (before September 2008 and after September 2008) and two different models: with and without short selling.

We observed that almost all min-var portfolios (no restrictions on the mean return) were classified as SSD inefficient. When conditions on minimal mean return are imposed the SSD inefficiency measure of mean-risk efficient portfolios is decreasing as minimal required mean return increases. Obviously, portfolios with the highest mean return were always SSD efficient. This behavior was detected in all 60 considered mean-risk frontiers with only few exceptions for small minimal required returns. We also found that allowing short sales dramatically decrease the fraction of portfolios from the mean-risk portfolios that can be classified as SSD efficient. In all cases we have found that Pearson frontiers contain more (or equally) SSD efficient portfolios than Spearman ones, no matter what currency or data period are considered. Finally, we analyzed the crises effect and we compared efficient frontiers before September 2008 and after. Having no short sales, we found that the efficiency frontiers for all concordance measures before crises always contains less SSD efficient portfolios than during crises frontiers, thought, this is not the case when restricted short sales are allowed.

All these results can be of great value for portfolio managers in banks and other financial institutions, thought, before making a final conclusion about the suitability of particular risk and dependency measures in portfolio theory also other measures of dependency should be examined assuming wider series of data.

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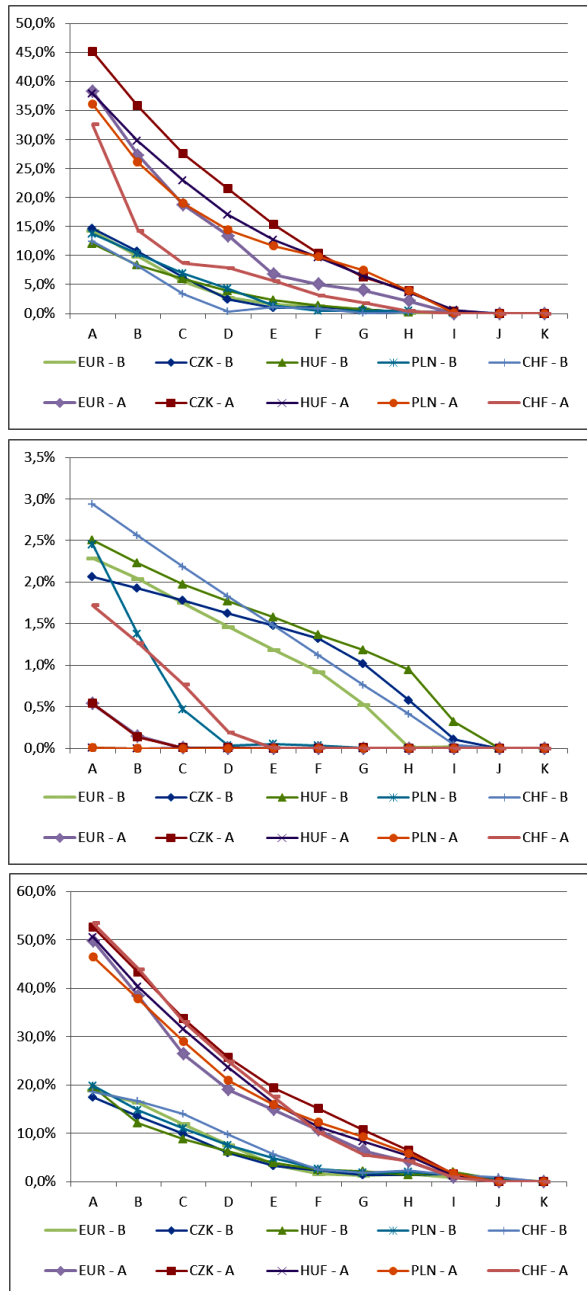


Fig. 1: SSD Portfolio (in) Efficiency of MV Efficient Frontier
 Efficient frontier calculated due to Pearson, Spearman and Kendall dependency matrix (from top to bottom) assuming no shortselling (left) and shortselling restricted by initially available funds (right).