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EFFECTIVENESS OF ECONOMIC POLICY: ASSESSMENT BASED ON
NONNORMALITIES

ACADEMIC DISSERTATION

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Päivi Puonti

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Chapter 1

Introduction

Research questions that are important both academically and for practical policy-making were not difficult to find after the global financial crisis of the early 2000s. At the same time interesting new methods for empirical macroeconomic research were developed. In this thesis I show how applying these novel time series econometric methods can broaden our understanding of highly topical policy questions.

Structural vector autoregressive (SVAR) models are an important tool in the empirical analysis of monetary and fiscal policy. The difficulty with conventional SVARs is the identification of structural shocks of interest needed for meaningful impulse response analysis. The conventional setting requires restrictions derived from economic theory or based on institutional knowledge. Typically either zero restrictions are imposed to exclude instantaneous or permanent effects on some of the variables, or a set of impulse response functions satisfying certain inequality constraints are admitted in the analysis.

In this thesis I opt for a fairly novel approach to identify economically interpretable shocks which are then used to assess the effects of various economic policies on the macroeconomy. The so-called statistical identification methods are particularly attractive in analyzing economic policies that are not so firmly based on theory to obtain theoretically justified restrictions or when economic theory provides a range of predictions. This is the case with the three research chapters of this thesis.

Characterized by a long period of steady growth and low and stable inflation, the period before the global financial crisis was known as the Great Moderation. Central banks conducted monetary policy within the framework of flexible inflation targeting. Already prior to the crisis Borio and Lowe (2002) had expressed concerns about monetary policy neglecting financial stability and Taylor (2007) soon showed that preceding the crisis the Federal reserve's monetary policy had been excessively loose compared to earlier times.

Conventional monetary policy transmission channels could not entirely explain the role of monetary policy in the lead-up to the crisis (Bean et al. 2010) but Borio and

Zhu (2008) presented the idea that low interest rates particularly encouraged financial intermediaries to excessive risk taking. Empirical analysis of the risk taking channel of monetary policy surged before the phenomenon was theoretically well understood, and this makes the statistical identification technique that I adopt in Chapter 2 particularly attractive.

As a response to the global financial turmoil and the resulting drop in economic activity, major central banks lowered interest rates to or near the effective zero lower bounds. Nonetheless economic recovery remained sluggish and many governments turned to fiscal policy, which is the topic of Chapter 3. In the third chapter I show that when the empirical literature does not seem to reach a conclusion (in this case with respect to the sign or size of the government fiscal multiplier), the identification strategy could play a role. In this case the strength of the statistical identification method is being able to discriminate between existing identification strategies. Even when based on theory, not all of the identifying restrictions are necessarily supported by the data or there might be various theoretically grounded identification schemes that may lead to different results.

The lack of consensus about the effectiveness of fiscal policy in stimulating the economy, its optimal design and eventually the high levels of government debt in many countries either limited or made the use of the fiscal policy instrument less attractive. The eyes again turned to central banks. But central banks had exhausted their traditional armory of methods to stimulate the economy and had to come up with something new. In Chapter 4 I assess the effectiveness of these unconventional operations that were mostly ad hoc measures based on central banks' own judgement.

Similarly to the previous chapters, in Chapter 4 I show the virtue of being able to combine statistical, data-based information with information from other sources to identify the structural model when economic theory is not conclusive or lags behind empirical analysis. When the data lends support for the restrictions coming from other sources, the statistically identified structural shocks and economic shocks are aligned. The assessment of economic policy can then be based on impulse response functions that are both economically meaningful and compatible with the sample data.

1.1 Methodology

This section introduces the empirical methodology that I use in the thesis. I present the identification problem of conventional structural vector autoregressive models and explain at a general level how the issue can be solved based on nonnormalities.

1.1.1 From Reduced Form to Structural Vector Autoregressions

Consider first a standard K -dimensional reduced form vector autoregressive (VAR) model with p lags (see e.g. Lütkepohl 2007, Ch 9):

$$\mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \dots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t \quad (1.1)$$

where \mathbf{y}_t is a $K \times 1$ vector of observable time series variables, the \mathbf{A}_j 's, $j = 1, \dots, p$ are $K \times K$ coefficient matrices and the $K \times 1$ error term $\mathbf{u}_t \sim (\mathbf{0}, \Sigma_u)$ is uncorrelated in time. In the presentation of this section deterministic terms are excluded since they don't affect structural modelling and impulse response functions. A stationary process \mathbf{y}_t satisfies the stability condition

$$\det(\mathbf{I}_n - \mathbf{A}_1 z - \dots - \mathbf{A}_p z^p) \neq 0, |z| \leq 1 (z \in \mathbb{C}),$$

and has a moving average (MA) representation

$$\mathbf{y}_t = \mathbf{u}_t + \Phi_1 \mathbf{u}_{t-1} + \Phi_2 \mathbf{u}_{t-2} + \dots \quad (1.2)$$

with $\Phi_0 = \mathbf{I}_K$ and the Φ_s , $s = 1, 2, \dots$ matrices are obtained by the recursion $\Phi_s = \sum_{j=1}^s \Phi_{s-j} \mathbf{A}_j$. The elements of the MA-matrices Φ_s contain the impulse responses of the system so that the jk 'th element of Φ_s captures the effect on variable j of a unit shock to variable k that occurred s periods ago.

The reduced-form VAR model describes the joint dynamics of a multivariate time series process and is useful for forecasting. In a system of simultaneous equations like the VAR, all variables are endogenous and the error terms in different equations are likely to be correlated, i.e. Σ_u is not a diagonal matrix. Because impulse response analysis involves tracing out the effect of a single shock at a time on the other variables in the system, the reduced form impulse response functions may not correctly reflect the relations between the variables in the VAR, which is essential for policy analysis. In contrast, if the error

terms of different equations are uncorrelated then it is reasonable to assume that a shock occurs in one variable at a time. Therefore we are after structural shocks $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \mathbf{I}_K)$ that are some linear combinations of the reduced form errors $\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$. The relations

$$\boldsymbol{\varepsilon}_t = \mathbf{B}^{-1}\mathbf{u}_t \quad (1.3)$$

and

$$\mathbf{E}(\mathbf{u}_t \mathbf{u}_t') = \boldsymbol{\Sigma}_u = \mathbf{B}\boldsymbol{\Sigma}_\varepsilon \mathbf{B}' = \mathbf{B}\mathbf{B}' \quad (1.4)$$

illustrate that to obtain the structural shocks $\boldsymbol{\varepsilon}_t$ we have to find a suitable matrix \mathbf{B} and suggest using the estimated covariance matrix $\widehat{\boldsymbol{\Sigma}}_u$ to recover \mathbf{B} .

However, the fact that the covariance matrix is symmetric means that these relations are not enough to identify the elements in \mathbf{B} . For example, with $K = 2$, (1.4) becomes

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} b_{11}^2 + b_{12}^2 & b_{11}b_{21} + b_{12}b_{22} \\ b_{11}b_{21} + b_{12}b_{22} & b_{21}^2 + b_{22}^2 \end{bmatrix}$$

which yields

$$\begin{aligned} \sigma_1^2 &= b_{11}^2 + b_{12}^2 \\ \sigma_{12} &= b_{11}b_{21} + b_{12}b_{22} \\ \sigma_2^2 &= b_{21}^2 + b_{22}^2 \end{aligned} \quad (1.5)$$

The identification problem essentially means that the four parameters on the right hand side of (1.5) cannot be solved based on the three equations. In general terms, the estimated covariance matrix contains $\frac{K(K+1)}{2}$ distinct elements, while \mathbf{B} has K^2 unknowns.

Typically the identification problem is solved by restricting some of the elements of the \mathbf{B} matrix, for example. Specifically, $K^2 - \frac{K(K+1)}{2} = \frac{K(K-1)}{2}$ such restrictions are needed.

Substituting $\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$ into (1.1) gives the corresponding structural VAR (SVAR) model, and makes clear that \mathbf{B} contains the impact effects of the structural shocks on the variables. Predetermining some of the elements of the \mathbf{B} matrix hence mean imposing restrictions on the impact effects.

Similarly to the reduced form VAR, a stable SVAR model has a MA-representation

$$\mathbf{y}_t = \boldsymbol{\varepsilon}_t + \boldsymbol{\Phi}_1 \mathbf{B}\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\Phi}_2 \mathbf{B}\boldsymbol{\varepsilon}_{t-2} + \dots \quad (1.6)$$

in which $\boldsymbol{\Phi}_0 = \mathbf{I}_K$ and the matrices $\Theta_i = \boldsymbol{\Phi}_i \mathbf{B}$, $i = 0, 1, \dots$ contain the structural impulse response functions. Therefore not only the \mathbf{B} matrix contains the contemporaneous re-

lations of the structural shocks but the choice of \mathbf{B} affects the whole impulse response analysis.

1.1.2 Nonstationarity and SVARs

In Chapter 3 I specify a model closely related to the SVAR model. When \mathbf{y}_t contains unit root variables, the structural vector error correction (SVEC) model allows us to distinguish between shocks that have transitory or permanent effects. The SVEC(p) model is

$$\Delta \mathbf{y}_t = \alpha \beta' \mathbf{y}_{t-1} + \Gamma_1 \Delta \mathbf{y}_{t-1} + \cdots + \Gamma_{p-1} \Delta \mathbf{y}_{t-p+1} + \mathbf{B} \boldsymbol{\varepsilon}_t$$

where Δ is the first difference operator such that $\Delta \mathbf{y}_t = \mathbf{y}_t - \mathbf{y}_{t-1}$, the $K \times 1$ vector of time series \mathbf{y}_t may contain unit roots, $\boldsymbol{\alpha}$ is a $K \times r$ matrix of loading coefficients, $\boldsymbol{\beta}$ is a $K \times r$ cointegration matrix, Γ_j is a $K \times K$ short run coefficient matrix for $j = 1, \dots, p-1$, and the $K \times 1$ vector $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \mathbf{I}_K)$ contains the structural shocks. The long-run effects of the shocks are captured by the common trends term (for details see e.g. Lütkepohl 2007, Chapter 9)

$$\boldsymbol{\Xi} \mathbf{B} \Sigma_{i=1}^t \boldsymbol{\varepsilon}_t \tag{1.7}$$

with $\boldsymbol{\Xi} = \boldsymbol{\beta}_\perp [\boldsymbol{\alpha}'_\perp (I_K - \Sigma_{i=1}^{p-1} \Gamma_i) \boldsymbol{\beta}_\perp]^{-1} \boldsymbol{\alpha}'_\perp$. Here the symbols $\boldsymbol{\alpha}_\perp$ and $\boldsymbol{\beta}_\perp$ denote the orthogonal complements of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ respectively. In a SVEC model with a cointegration rank of $r < K$, at most r of the shocks can have transitory effects only, and they are associated with zero columns in the long run matrix $\boldsymbol{\Xi} \mathbf{B}$. Therefore in the SVEC-model the long run restrictions can be based on our knowledge or statistical evidence of the cointegrating rank of the system.

1.1.3 Identification via Nonnormalities

In the last chapter I make use of sign restrictions used in the literature. These restrictions are somewhat different from the ones presented above. Instead of fixing or excluding some of the effects beforehand, this identification strategy consists of admitting a whole range of impulse responses with a predetermined sign. However what all of these identification strategies have in common is that any restrictions imposed by the researcher matter for the subsequent impulse response analysis used for answering economic questions of interest.

On the other hand, whenever it is reasonable to assume or there is statistical evidence of non-normal error distributions, modeling a more general or otherwise more appropriate

distribution explicitly yields additional information for identification. I now briefly illustrate with the Lanne and Lütkepohl (2010) method, that I use in the first two research chapters of the thesis, how nonnormalities can be exploited to identify the model and structural shocks without restrictions.

Lanne and Lütkepohl (2010) assume the K -dimensional error term \mathbf{u}_t to be a mixture of two serially independent normal random vectors

$$\mathbf{u}_t = \begin{cases} \mathbf{e}_{1t} \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma}_1) & \text{with probability } \gamma \\ \mathbf{e}_{2t} \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma}_2) & \text{with probability } 1 - \gamma \end{cases} \quad (1.8)$$

where $\mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma})$ denotes a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}$. In the model $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ are $K \times K$ covariance matrices that are assumed to be distinct ($\boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}_2$) and γ is the mixture probability, $0 < \gamma < 1$, a parameter of the model. The covariance matrices can be decomposed as $\boldsymbol{\Sigma}_1 = \mathbf{W}\mathbf{W}'$ and $\boldsymbol{\Sigma}_2 = \mathbf{W}\boldsymbol{\Psi}\mathbf{W}'$ with a diagonal matrix $\boldsymbol{\Psi} = \text{diag}(\psi_1, \dots, \psi_K)$, $\psi_i > 0$, $i = 1, \dots, K$ and a $K \times K$ matrix \mathbf{W} which is locally unique except for a change in sign of a column, as long as all ψ_i 's are distinct. The covariance matrix of the reduced form error vector \mathbf{u}_t now becomes

$$\boldsymbol{\Sigma}_u = \gamma\mathbf{W}\mathbf{W}' + (1 - \gamma)\mathbf{W}\boldsymbol{\Psi}\mathbf{W}' = \mathbf{W}(\gamma\mathbf{I}_K + (1 - \gamma)\boldsymbol{\Psi})\mathbf{W}' \quad (1.9)$$

and following equation (1.4), a locally unique \mathbf{B} is given by

$$\mathbf{B} = \mathbf{W}(\gamma\mathbf{I}_K + (1 - \gamma)\boldsymbol{\Psi})^{1/2} \quad (1.10)$$

This \mathbf{B} matrix diagonalizes the covariance of the reduced form errors and hence delivers structural shocks that are contemporaneously uncorrelated as required. Given that the equations

$$\begin{aligned} \mathbf{B}^{-1}\boldsymbol{\Sigma}_u\mathbf{B}^{-1'} &= \mathbf{I}_K \\ \mathbf{B}^{-1}\boldsymbol{\Sigma}_1\mathbf{B}^{-1'} &= (\gamma\mathbf{I}_K + (1 - \gamma)\boldsymbol{\Psi})^{-1} \\ \mathbf{B}^{-1}\boldsymbol{\Sigma}_2\mathbf{B}^{-1'} &= (\gamma\mathbf{I}_K + (1 - \gamma)\boldsymbol{\Psi})^{-1}\boldsymbol{\Psi} \end{aligned}$$

are all diagonal matrices, this choice of \mathbf{B} yields shocks that are orthogonal regardless of the regime they come from. To see how this solves the identification problem, with $K = 2$ we now have the following equations

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} \sigma_{1,1}^2 & \sigma_{12,1} \\ \sigma_{12,1} & \sigma_{2,1}^2 \end{bmatrix} = \mathbf{W}\mathbf{W}' = \begin{bmatrix} w_{11}^2 + w_{12}^2 & w_{11}w_{21} + w_{12}w_{22} \\ w_{11}w_{21} + w_{12}w_{22} & w_{21}^2 + w_{22}^2 \end{bmatrix}$$

and

$$\boldsymbol{\Sigma}_2 = \begin{bmatrix} \sigma_{1,2}^2 & \sigma_{12,2} \\ \sigma_{12,2} & \sigma_{2,2}^2 \end{bmatrix} = \mathbf{W}\boldsymbol{\Psi}\mathbf{W}' = \begin{bmatrix} \psi_1 w_{11}^2 + \psi_2 w_{12}^2 & \psi_1 w_{11}w_{21} + \psi_2 w_{12}w_{22} \\ \psi_1 w_{11}w_{21} + \psi_2 w_{12}w_{22} & \psi_1 w_{21}^2 + \psi_2 w_{22}^2 \end{bmatrix}$$

which yield the following six equations

$$\begin{aligned}
\sigma_{1,1}^2 &= w_{11}^2 + w_{12}^2 \\
\sigma_{12,1} &= w_{11}w_{21} + w_{12}w_{22} \\
\sigma_{2,1}^2 &= w_{21}^2 + w_{22}^2 \\
\sigma_{1,2}^2 &= \psi_1 w_{11}^2 + \psi_2 w_{12}^2 \\
\sigma_{12,2} &= \psi_1 w_{11}w_{21} + \psi_2 w_{12}w_{22} \\
\sigma_{2,2}^2 &= \psi_1 w_{21}^2 + \psi_2 w_{22}^2
\end{aligned} \tag{1.11}$$

From these we can solve for the six unknown structural parameters $(w_{11}, w_{12}, w_{21}, w_{22}, \psi_1, \psi_2)$ and finally recover the elements of \mathbf{B} in (1.10).

More recently, Lanne et al. (2017) have introduced a yet more general approach that allows more wide-ranging specifications for the error distribution, and encompasses the mixed normal distribution as a special case. The authors show that identification obtains by strengthening the assumptions typically imposed on the error term ε_t . Specifically, they assume that the error process $\varepsilon_t = (\varepsilon_{1,t}, \dots, \varepsilon_{K,t})$ has at least $K - 1$ non-Gaussian components that are independent both contemporaneously and temporally. In the conventional Gaussian case, the mutual independence of the components $\varepsilon_{i,t}, i = 1, \dots, K$ is not explicitly imposed but nonetheless obtains, because ε_t is assumed to be independent and identically normally distributed with mean zero and a diagonal covariance matrix. Under non-Gaussianity, the independence requirement is stronger than mere uncorrelatedness.

While statistical identification methods such as the ones presented above have facilitated statistical testing of exactly identifying short-run or long-run restrictions, they have been less suitable to formally assess the plausibility of sign restrictions. The method put forth by Lanne and Luoto (2016), which I apply in Chapter 4, is an exception to this, as it allows the formal assessment of given sign restrictions. Also in this case identification is achieved based on non-Gaussianity as in Lanne et al. (2017). The procedure then allows to compute the conditional probabilities that given sign restrictions are compatible with the data.

1.2 Summary of the Chapters

In this section I briefly summarize my main research questions, contributions and findings in the following three chapters of the thesis.

1.2.1 Chapter 2: Macro-Level Evidence of the Risk-Taking Channel from SVAR with Nonnormal Errors

In this chapter I reconsider the SVAR model of Adrian et al. (2010) to study the macroeconomic effects of the risk-taking channel in the US. According to the mechanism analyzed by Adrian et al. (2010), when loose monetary policy boosts asset prices, risk perceptions and the pricing of risk in the economy change. This in turn encourages financial intermediaries to extend loans to riskier borrowers.

I apply the SVAR model proposed by Lanne and Lütkepohl (2010) in which identification is achieved by means of nonnormal errors. The previously used identifying restrictions then become over-identifying and statistically testable. The methodological improvement allows us to learn about the impact effects between the variables from the data instead of ruling out some of the effects beforehand.

I find that the data supports a recursive identification strategy different from the benchmark paper's. The resulting impulse response functions confirm previous empirical findings that during the sample period monetary policy affected the balance sheet management of financial institutions, determination of risk premiums and consequently the level of real activity in the US.

1.2.2 Chapter 3: Fiscal Multipliers in a Structural VEC Model with Mixed Normal Errors

The third chapter addresses the question whether increasing government spending stimulates real economic activity in the US. Unlike previous empirical research using SVARs I estimate a vector error correction (VEC) model that takes into account cointegration between the variables. Fiscal policy shocks are identified with the data driven Lanne and Lütkepohl (2010) method. This paper is the first one to apply statistical identification methods to fiscal policy.

The impulse response functions are quite different from those typically obtained from SVAR models. The results show that a deficit financed government spending shock has a weak negative effect on output, whereas a tax increase to finance government spending has a positive impact on GDP.

1.2.3 Chapter 4: Data-Driven Structural BVAR Analysis of Unconventional Monetary Policy

In the last chapter of the thesis I study the macroeconomic effects of the Bank of Japan's, Federal Reserve's and European Central Bank's balance sheet policies. I use a novel Bayesian vector autoregressive method due to Lanne and Luoto (2016) which allows me to base the whole analysis on the data and to obtain structural shocks using nonnormal error distributions. Importantly, the Lanne and Luoto (2016) method provides a formal way to assess the plausibility of given sign restrictions against the data.

I find statistical support for the sign restrictions used in the literature. In contrast to previous empirical research using SVARs, my data-based impulse response analysis reveals differences in the output and price effects of the three central banks' balance sheet operations.

References

- Adrian T., Moench E., and Shin H. S. (2010). Macro Risk Premium and Intermediary Balance Sheet Quantities. *IMF Economic Review* **58**(1).
- Borio, C. and Lowe, P. (2002). Asset Prices, Financial and Monetary Stability: Exploring the Nexus. Working paper 114, Bank for International Settlements.
- Lanne, M., and Luoto, J. (2016). Data-driven inference on sign restrictions in Bayesian structural vector autoregression. CREATES Research paper 2016-4.
- Lanne M., and Lütkepohl H. (2010). Structural Vector Autoregressions with Non-Normal Residuals. *Journal of Business & Economic Statistics* **28**: 159-168.
- Lanne M., Meitz M., and Saikkonen P. (2017). Identification and Estimation of Non-Gaussian Structural Vector Autoregressions. *Journal of Econometrics* **196**(2), 288-304.
- Lütkepohl H. (2007). *New Introduction to Multiple Time Series Analysis*, Springer-Verlag, Berlin.
- Taylor, J. B. (2007). Housing and Monetary Policy in Housing, Housing Finance and Monetary Policy, 463-476. Federal Reserve Bank of Kansas City: Kansas City.

Chapter 2

Macro-Level Evidence of the Risk-Taking Channel from SVAR with Nonnormal Errors

Abstract¹

The identifying restrictions of a previously used SVAR model are validated to assess the macroeconomic impact of the risk-taking channel of monetary policy in the US. Structural shocks are obtained by exploiting the nonnormality of errors. The data is found to object to the previously imposed recursive ordering while a different recursive ordering is supported. Based on the resulting impulse responses, there is statistically significant evidence in favor of the risk-taking channel during the sample period. The main results are in line with the predictions of the underlying theoretical model and confirm previous empirical findings.

¹This chapter is based on *HECER Discussion Paper No. 394 (2015)*.

2.1 Introduction

The financial crisis of 2007-09 raised the question whether low levels of interest rates induce excessive risk-taking in the financial sector. If the so-called risk-taking channel of monetary policy (Borio and Zhu 2012) exists but is ignored, unsustainable economic expansions may show up first in the form of financial imbalances rather than in the form of rising inflation. According to Brunnermeier and Sannikov (2012) monetary policy has direct effects on financial stability by affecting financial institutions' balance sheets through asset prices, but if central banks are forced to stabilize the financial sector then long-run price stability may be compromised. As the future of central banking, monetary policy and financial stability is widely debated at the moment, this paper contributes to a discussion of direct practical relevance.

Even though the literature on monetary policy and banks' risk taking has evolved rapidly in recent years², few studies (Adrian and Shin 2010, Adrian et al. 2010, Buch et al. 2011) analyze the macro-level effects. Therefore this paper takes the empirical analysis of Adrian et al. (2010) as a benchmark to assess empirically the macroeconomic impact of the link between monetary policy, banks' balance sheet management and measures of risk.

The structural VAR (SVAR) and impulse response analysis in Adrian et al. (2010) indicates that there is a connection between rapid growth of financial intermediary balance sheets, lower risk premiums and higher real activity. For methodological reasons however, as acknowledged by the authors, their results cannot be taken as conclusive. As is commonly done, Choleski decomposition is used to identify the economic shocks of interest. Since there is not enough theory to determine a correct ordering for the variables, the ordering is essentially arbitrary. This is of concern because in a recursively identified model the ordering of the variables in the VAR matters for the results. Without further identifying restrictions one cannot be sure that the shocks and impulse responses tell us about the underlying economic processes we are essentially interested in. This gives reason for further research.

Lanne and Lütkepohl (2008, 2010) and Rigobon (2003) among others have pointed out that sometimes statistical properties of the data can yield further information for identification in a SVAR framework. Examples of such statistical properties are residual

²See Section 2.2 for a literature review.

distribution and structural breaks.³ Even when economic theory suffices to identify the shocks of interest, often there is no over-identifying information to test theories against data. Since theories on the risk-taking channel are relatively scarce, there is not much additional theory to put structure in the empirical model. Therefore statistical identification strategies are clearly invoked.

We apply the Lanne and Lütkepohl (2010) approach in which shock identification is based on nonnormality of the errors. The errors are assumed to follow a mixture of two normal distributions where the regimes cannot be determined beforehand but are assigned endogenously. In addition to being relatively simple, the chosen method allows us to exploit the fact that in applied work VAR residuals are often found to be nonnormal (Lanne and Lütkepohl 2010). For the data at hand, normality of errors was strongly rejected by statistical tests.⁴ This supports the proposed identification strategy and allows us to test whether the just-identifying restrictions of the benchmark paper are consistent with the data.

The methodological improvement enables us to learn about the impact effects between the variables from the data instead of ruling out some of the effects *ex ante*. Even though the method only guarantees a statistical identification, meaning that it delivers orthogonalized shocks but does not give an economic interpretation, we find a recursive ordering that is not rejected by the data. This facilitates attaching economic labels to the statistically identified shocks. Our impulse response analysis provides statistical evidence in favor of the risk-taking channel during the sample period.

The rest of the paper is organized as follows. Section 2.2 briefly presents the risk-taking channel of monetary policy and the relevant literature. Technical details of the empirical method are provided in Section 2.3. Section 2.4 covers the empirical analysis and Section 2.5 concludes.

³For example, Rigobon (2003), Lanne and Lütkepohl (2008), Lanne, Lütkepohl and Maciejowska (2010) and Lütkepohl and Netšunajev (2013) have exploited residual heteroskedasticity to extract further identifying information from the data. Rigobon (2003) and Lanne and Lütkepohl (2008) assume that changes in the volatility of shocks are determined exogeneously and partition the sample period accordingly, while Lanne et al. (2010) as well as Lütkepohl and Netšunajev (2013) model the changes in volatility endogeneously as Markov switching (MS) regimes.

⁴See Section 2.4 for details.

2.2 The Risk-Taking Channel of Monetary Policy

During normal times, expansionary monetary policy is expected to raise asset prices through the conventional monetary transmission channel, as lower policy rates usually lead to lower long-term interest rates. The risk-taking channel of monetary policy (Borio and Zhu 2012) is based on the idea that when loose monetary policy boosts asset prices, risk perceptions and hence risk premia change. Adrian and Shin (2010) and Adrian et al. (2010) argue that this encourages financial intermediaries to extend loans to riskier borrowers and so to further expand their balance sheets. It is when this risk-taking turns excessive that risk accumulates and financial imbalances build up (Borio and Zhu 2012).

Literature on monetary policy transmission through the banking sector can be subdivided into two broad categories. The line of research following Bernanke and Getler (1995) emphasizes the channel through *demand* for credit and borrowers' balance sheet, while the bank lending channel studied by Bernanke and Blinder (1992) focuses on the impact of policy rate on credit *supply*. The risk-taking channel has common features with the latter branch of research: interest lies in the passage of the policy rate through the asset side of the banks' balance sheets.

What makes the channel distinct however is the mechanism that links the policy rate to banks' balance sheets. In Bernanke and Blinder (1992) it is binding reserve requirements of commercial banks. According to Adrian and Shin (2010), this approach is not applicable to the 2007-09 financial crisis because reserve requirements were not binding and because credit contraction did not originate from the commercial banking sector. In fact, Adrian and Shin (2009) show that among all financial intermediaries it was credit supply by market-based intermediaries, not traditional commercial banks that saw the most rapid growth before the crisis – as well as the most dramatic contraction afterwards.⁵

The business of these institutions is to borrow short term and lend long term and therefore the spread between the short and long term interest rates is indicative of their expected profits. This is in contrast to the traditional view, where a bank is thought to intermediate between depositors and borrowers and the effectiveness of monetary policy is assessed by its impact on long rates only. Furthermore, since the supply of credit in the US. has shifted from the traditional banking sector to market-based institutions,

⁵Market-based intermediaries include broker-dealers, issuers of asset-backed securities, finance companies and funding corporations, the last three of which are called shadow banks.

this distinction has to be taken into account in the empirical analysis of monetary policy transmission.

The empirical analysis in Adrian et al. (2010) is based on a theoretical model due to Shin (2010), which illustrates that the financial intermediary sector has an active role in the business cycle through the pricing of risk and suggests the following mechanism: monetary policy induced balance sheet adjustment by financial intermediaries leads to a lower price of risk and higher real activity in the economy. When asset prices change e.g. due to monetary policy changes, in addition to the normal valuation effect there is an additional quantity adjustment of balance sheets. This sets in motion the amplifying effect of financial intermediaries on the boom-bust cycle.

A common finding of the empirical studies at the micro level is that lax monetary policy increases the riskiness of new loans by commercial banks. Using an extremely large, confidential micro-level data set for Spain Jimenez et al. (2014) find, that in an environment of low interest rates, the riskiness of bank portfolios is affected by both higher collateral values and search for yield. In the short run, the default probability of bank loans decreases, while it is found to increase in the long run when the search for yield effect prevails.

Building on this Altunbas et al. (2010) construct various proxies for bank default risk and analyze a panel dataset that covers banks operating in 16 OECD countries. They find that interest rates below the Taylor rule increase the default probability of banks.

Maddaloni and Peydró (2011) use the European Central Bank's Bank Lending Survey to explore the determinants of bank lending standards in the Euro Area. According to their panel regression a monetary expansion leads to lower credit standards for both corporate and personal loans. De Santis and Surico (2013) study heterogeneity of bank lending across euro area countries using BankScope data in panel regressions. The results indicate that the bank lending channel in the eurozone is highly heterogeneous across the four countries and bank typologies studied.

Finally, Buch et al. (2011) use a factor-augmented vector autoregressive (FAVAR) model for macro-level data for the US. and find that small domestic banks respond to expansionary monetary shock by increasing the amount of risky loans, but there is no evidence of increased risk-taking for the banking system as a whole.

2.3 SVAR Model with Nonnormal Errors

Consider first a standard K -dimensional reduced form stable VAR with p lags (see e.g. Lütkepohl 2007, Ch 9):

$$\mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \dots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t \quad (2.12)$$

where \mathbf{y}_t is a $K \times 1$ vector of observable time series variables, the \mathbf{A}_j 's ($j = 1, \dots, p$) are $K \times K$ coefficient matrices and the error term \mathbf{u}_t is K -dimensional white noise with $\mathbf{u}_t \sim (\mathbf{0}, \boldsymbol{\Sigma}_u)$. In the presentation of this section deterministic terms are excluded since they don't affect structural modelling and impulse response functions. Being a system of simultaneous equations, all variables in the VAR are endogenous and the error terms in different equations are likely to be correlated. Usually the purpose is to conduct impulse response analysis, which means representing a stationary VAR -process in the following Wold moving average (MA) form:

$$\mathbf{y}_t = \mathbf{u}_t + \boldsymbol{\Phi}_1 \mathbf{u}_{t-1} + \boldsymbol{\Phi}_2 \mathbf{u}_{t-2} + \dots \quad (2.13)$$

where $\boldsymbol{\Phi}_s = \sum_{j=1}^s \boldsymbol{\Phi}_{s-j} \mathbf{A}_j$, $s = 1, 2, \dots$ and $\boldsymbol{\Phi}_0 = \mathbf{I}_k$.

Interest then lies in the elements of the $\boldsymbol{\Phi}_j$, the MA coefficient matrices, which contain the impulse responses of the system; responses of a variable to an impulse in another. If the error terms are contemporaneously correlated – $\boldsymbol{\Sigma}_u$ is not a diagonal matrix – it means that shocks come in a bunch. In this case setting all other error terms to zero to trace out single impulses can be misleading. Impulses may not correctly reflect the relations between the variables in the VAR. On the other hand if the error terms of different equations are uncorrelated then it is reasonable to assume that a shock occurs in one variable at a time. Therefore orthogonalizing the error terms implies identifying single shocks and impulses.

In a so-called B-model (see Lütkepohl 2007, Ch 9), to orthogonalize the error term of the reduced form model means deriving shocks $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \mathbf{I}_K)$ such that $\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$. In other words we want to find a matrix \mathbf{B} such that

$$\boldsymbol{\varepsilon}_t = \mathbf{B}^{-1} \mathbf{u}_t \quad (2.14)$$

and

$$\mathbf{E}(\mathbf{u}_t \mathbf{u}_t') = \boldsymbol{\Sigma}_u = \mathbf{B} \boldsymbol{\Sigma}_\varepsilon \mathbf{B}' = \mathbf{B} \mathbf{B}' \quad (2.15)$$

As the covariance matrix is symmetric, these equations only define $\frac{K(K+1)}{2}$ equations, while \mathbf{B} contains K^2 elements. Hence $K^2 - \frac{K(K+1)}{2} = \frac{K(K-1)}{2}$ additional restrictions on \mathbf{B} are needed to identify all of its K^2 elements. A common choice of \mathbf{B} is a lower triangular matrix obtained from a Choleski decomposition of Σ_u because it yields exactly the right number of restrictions. This is done by decomposing the covariance matrix Σ_u as $\Sigma_u = \mathbf{P}\mathbf{P}'$ where \mathbf{P} is a lower triangular matrix. Then by defining $\mathbf{P} = \mathbf{B}$ and $\Theta_i = \Phi_i \mathbf{P}$ ($i = 0, 1, 2, \dots$) one obtains shocks $\varepsilon_t = \mathbf{P}^{-1} \mathbf{u}_t$ and the corresponding vector moving average (VMA) representation

$$\mathbf{y}_t = \Theta_0 \varepsilon_0 + \Theta_1 \varepsilon_1 + \Theta_2 \varepsilon_2 + \dots \quad (2.16)$$

Since the components of ε_t are uncorrelated with unit variance, it is possible to interpret the jk -th element of the matrix Θ_i as capturing the effect on variable j of a unit shock in variable k that occurred i periods ago. This identification strategy based on Choleski decomposition is easily and often used. However the \mathbf{B} matrix obtained with Choleski decomposition depends on the order of the variables in the vector \mathbf{y}_t . This implies that there can be several triangular matrices that do the orthogonalization equally well. Moreover as the \mathbf{B} matrix contains instantaneous effects of the shocks on the variables ($\Theta_0 = \mathbf{B}$), different choices of \mathbf{B} can yield different results in terms of impulse responses.

The fact that the choice of \mathbf{B} has an impact on results means that non-statistical information is needed to impose restrictions. This requires economic theory that describes the relationships of interest. In the case of Choleski decomposition this means determining, which variables do not have an instantaneous impact on some others and then ordering the variables in the vector \mathbf{y}_t accordingly. Other popular identification methods include the use of inequality or sign restrictions (Canova and De Nicolò 2002, Uhlig 2005), where a whole variety of shocks of a predetermined sign are admitted, or the exclusion of instantaneous or long-run effects of variables (Blanchard and Quah 1989, Lütkepohl 2005), where zero effects of some variables are assumed. The resulting VARs with restrictions on the transformation matrix obtained from economic theory are called structural VARs. In the \mathbf{B} -model the error terms \mathbf{u}_t of the estimable reduced form VAR are seen as linear functions of some meaningful economic disturbances, ε_t , called structural shocks. In other words the information content of reduced form dynamics is transformed into behavioral ones.

A common feature of all these identification strategies is that they identify the structural shocks but do not allow the identification to be statistically tested. Without further identifying restrictions one cannot be sure that the shocks and impulse responses tell us about the underlying economic processes we are essentially interested in. Furthermore sometimes there is not enough economic theory to obtain a full set of restrictions in which case arbitrary restrictions are imposed.

Instead, if there is reason to believe, or there is evidence from a VAR analysis that errors might not be normally distributed, then this information may be useful for identification. The error distribution might have heavy tails and produce “outliers”, which can be thought to be generated by a different distribution – from a different stochastically generated regime. As Lanne and Lütkepohl 2010 suggest, by modelling a more general distribution explicitly, further identifying information can be extracted.

Consider again the reduced form VAR reported above. As in the model proposed by Lanne and Lütkepohl (2010), now assume the K -dimensional error term \mathbf{u}_t to be a mixture of two serially independent normal random vectors

$$\mathbf{u}_t = \begin{cases} \mathbf{e}_{1t} \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma}_1) & \text{with probability } \gamma \\ \mathbf{e}_{2t} \sim \mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma}_2) & \text{with probability } 1 - \gamma \end{cases} \quad (2.17)$$

where $\mathbf{N}(\mathbf{0}, \boldsymbol{\Sigma})$ denotes a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}$. In the model $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ are $K \times K$ covariance matrices that are assumed to be distinct, γ is the mixture probability, $0 < \gamma < 1$, a parameter of the model. The parameter γ is only identified if $\boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}_2$ hence this is assumed to hold. If some parts of $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ are identical then some components of \mathbf{u}_t may be normally distributed. In any case there only needs to be one nonnormal component in \mathbf{u}_t . The distribution of the reduced form error term now becomes

$$\mathbf{u}_t \sim (\mathbf{0}, \gamma \boldsymbol{\Sigma}_1 + (1 - \gamma) \boldsymbol{\Sigma}_2) \quad (2.18)$$

The distributional assumption for \mathbf{u}_t allows to define a locally unique matrix \mathbf{B} in the following way. As shown in the Appendix A by Lanne and Lütkepohl (2010), a diagonal matrix $\boldsymbol{\Psi} = \text{diag}(\psi_1, \dots, \psi_K)$, $\psi_i > 0$, $i = 1, \dots, K$ and a $K \times K$ matrix \mathbf{W} exist such that $\boldsymbol{\Sigma}_1 = \mathbf{W}\mathbf{W}'$ and $\boldsymbol{\Sigma}_2 = \mathbf{W}\boldsymbol{\Psi}\mathbf{W}'$ and \mathbf{W} is locally unique except for a change in sign of a column, as long as all ψ_i 's are distinct. Now we can rewrite the covariance matrix of the reduced form error vector \mathbf{u}_t as

$$\Sigma_u = \gamma \mathbf{W}\mathbf{W}' + (1 - \gamma) \mathbf{W}\Psi\mathbf{W}' = \mathbf{W}(\gamma \mathbf{I}_K + (1 - \gamma)\Psi)\mathbf{W}' \quad (2.19)$$

Then following equation (2.15) a locally unique \mathbf{B} is given by

$$\mathbf{B} = \mathbf{W}(\gamma \mathbf{I}_K + (1 - \gamma)\Psi)^{1/2} \quad (2.20)$$

This choice of \mathbf{B} means that the orthogonality of shocks is independent of regimes. This can be seen by applying (2.15) to the covariance matrices as

$$\begin{aligned} \mathbf{B}^{-1}\Sigma_u\mathbf{B}^{-1'} &= \mathbf{I}_K \\ \mathbf{B}^{-1}\Sigma_1\mathbf{B}^{-1'} &= (\gamma \mathbf{I}_K + (1 - \gamma)\Psi)^{-1} \\ \mathbf{B}^{-1}\Sigma_2\mathbf{B}^{-1'} &= (\gamma \mathbf{I}_K + (1 - \gamma)\Psi)^{-1}\Psi \end{aligned} \quad (2.21)$$

As the equations in (2.21) are all diagonal matrices, the choice of \mathbf{B} as in (2.20) yields shocks that are orthogonal in both regimes.

The model is estimated with maximum likelihood (ML) method. Rewriting (2.12) in lag operator form

$$\mathbf{A}(\mathbf{L})\mathbf{y}_t = \mathbf{u}_t \quad (2.22)$$

where $\mathbf{A}(\mathbf{L}) = \mathbf{I}_n - \mathbf{A}_1(\mathbf{L}) - \dots - \mathbf{A}_p\mathbf{L}^p$ is a matrix polynomial in the lag operator \mathbf{L} then the conditional distribution of \mathbf{y}_t given $\mathbf{Y}_{t-1} = (\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots, \mathbf{y}_{t-p+1})$ can be written as

$$\begin{aligned} f(\mathbf{y}_t | \mathbf{Y}_{t-1}) &= \gamma \det(\mathbf{W})^{-1} \exp\left\{-\frac{1}{2}(\mathbf{A}(\mathbf{L})\mathbf{y}_t)'(\mathbf{W}\mathbf{W}'^{-1})(\mathbf{A}(\mathbf{L})\mathbf{y}_t)\right\} \\ &+ (1 - \gamma) \det(\Psi)^{-\frac{1}{2}} \det(\mathbf{W})^{-1} \exp\left\{-\frac{1}{2}(\mathbf{A}(\mathbf{L})\mathbf{y}_t)'(\mathbf{W}\Psi\mathbf{W}'^{-1})(\mathbf{A}(\mathbf{L})\mathbf{y}_t)\right\} \end{aligned} \quad (2.23)$$

Collecting all the parameters into the vector θ , the log-likelihood is

$$l_T(\theta) = \sum_{t=1}^T \log f(\mathbf{y}_t | \mathbf{Y}_{t-1}) \quad (2.24)$$

The log-likelihood function (2.24) can be maximized with standard nonlinear optimization algorithms.

2.4 Empirical Analysis of Macro Dynamics

2.4.1 The Data and the VAR Model

There are two important variables in the theoretical model due to Shin (2010) that are difficult to quantify: the price of risk in the economy and financial intermediaries' risk

taking capacity. Adrian et al. (2010) manage to overcome the problem by constructing two proxy variables.⁶ This enables empirical analysis of the mechanism of interest.

The first one, called *Macro Risk Premium*, measures the hurdle rate of return for new projects financed in the economy.⁷ It reflects the ease of credit conditions and is measured from yield spreads of fixed income securities. The second proxy variable is labelled *Financial Intermediary Risk Appetite Factor* as it measures the looseness of financial intermediary capital constraints. This variable is important as it enables to circumvent the problem of measuring marginal loan supply.

Since there is a variety of institutions that provide credit to the real economy, the authors first choose the institutions that are most important in determining risk premiums. In the US those turn out to be broker-dealers and shadow banks, whose liabilities are short term and marked to market so that funding conditions in the economy are more promptly reflected in the balance sheets. This is mostly not the case with traditional banks. Therefore balance sheet measures of these institutions were used in the analysis.

We use the same variables and dataset as Adrian et al. (2010) who consider a five variable VAR including quarterly GDP growth (Δgdp_t), inflation π_t , Federal Funds target rate (FFR_t), macro risk premium (MRP_t) and the financial intermediary risk appetite factor (FI_t). The data consists of quarterly US data for the period of 1985:1 - 2010:4 and it was provided by the authors.

2.4.2 Previous Identification Restrictions

Identification in the benchmark paper is obtained with the following exclusion restrictions on the transformation matrix \mathbf{B} .

$$\mathbf{B} = \begin{bmatrix} * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \\ * & * & * & * & * \end{bmatrix} \quad (2.25)$$

The asterisks denote unrestricted elements and the zeros are imposed so that \mathbf{B} is lower triangular. The variable ordering

$$\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, MRP_t, FI_t)' \quad (2.26)$$

⁶See Adrian et al. (2010) for details.

⁷Hurdle rate of return = minimum acceptable rate of return to accept a new project.

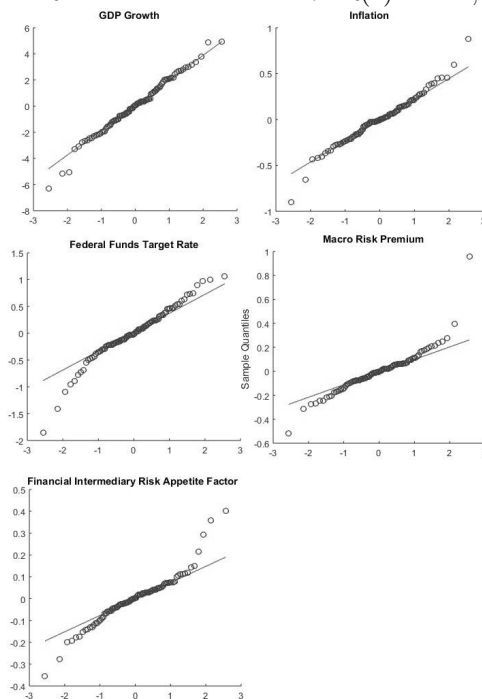
implies that a shock to GDP growth is allowed to have a contemporaneous effect on all other variables, whereas there is no instantaneous feedback effect from an impulse on financial intermediary risk appetite to any of the variables. Is there a plausible economic interpretation for the exclusion restrictions required by the identification scheme? The theoretical model due to Shin (2010) illustrates how a positive shock to asset values, say a decrease in short rates, that increases the capital buffer (equity) of banks, leads to a lower risk premium and induces banks to take on additional debt to purchase more risky securities, or to supply new loans. In the model, the amount of risky assets on the balance sheets increases more than in the case of a mere valuation effect. An empirical hypothesis of interest could then be formulated as the impact of monetary policy interest rate to the risk premium and financial intermediaries' risk taking capacity. Accordingly in (2.25) a shock to federal funds target rate is allowed to affect contemporaneously both the macro risk premium and financial intermediary risk appetite factor, and a shock to macro risk premium is allowed to have a contemporaneous effect on risk appetite.

Lütkepohl and Netšunajev (2013) point out that even in those cases where restrictions are derived from generally accepted economic models, the empirical and theoretical models do not necessarily coincide. As potential reasons they name measurement errors, trend and/or seasonal adjustment, and observation frequency for the data that is different from that of the theoretical model. Moreover the variables in the empirical and theoretical models might not perfectly coincide. In the present case the main challenge arises from the frequency of the data. Is it likely that there is no feedback effect from the right to the left of (2.26) within the same quarter? Another source of gap between the economic and empirical models stems from the fundamental differences between the two modelling approaches. A theoretical model is bound to abstract from some effects in order to describe relations within a set of variables only. To avoid problems with omitted variables, an empirical model on the other hand often requires the inclusion of variables outside of the theoretical model that are known to be important in practice (Lütkepohl and Netšunajev 2013). From this point of view, the inclusion of the first two variables in (2.26) is easy to justify.

Even without an appealing, justifiable theoretical reasoning a recursive identification scheme is convenient whenever there is only one shock of interest, which can be ordered at the bottom of the variable list (2.26). In all other cases identification via recursive

ordering as in (2.25) necessarily implies that one is excluding certain impact effects ex-ante rather than learning about it from the data. As explained in Section 2.3, if it is reasonable to assume that the vector of reduced form errors \mathbf{u}_t follows a mixed normal distribution with covariance matrix as in (2.19), and if the elements of the Ψ matrix are all distinct, then these concerns become irrelevant since the validity of the restrictions in (2.25) can be statistically tested as proposed by Lanne and Lütkepohl 2010.

Figure 2.1: Residuals of the linear VAR(1) model, QQ plots



QQ-plots of the residuals of the linear VAR(1) model are shown in Figure 2.1. The plots feature a mostly linear pattern in the center of the data, while the tails show departures from the fitted line. Compared to a normal distribution, a slightly more S-shaped curve emerges. This kind of distribution with heavy tails and outliers can be captured by a mixture of normal distributions. The outliers can be thought to be generated by a different distribution than the rest of the observations. Then identification of the shocks is obtained from heteroskedasticity across regimes.

The results of normality tests are reported in Table 2.1. The Jarque-Bera test rejects the null hypothesis of normality for each of the estimated residuals. The high overall kurtosis of the Doornik-Hansen test for multivariate normality (p -value of < 0.001) yields

Table 2.1: Tests for normality of residuals

	Skewness	Kurtosis	Jarque-Bera	<i>p</i> -value
u_1	-0.34	4.02	5.97	0.0506
u_2	-0.19	4.89	14.75	0.0006
u_3	-0.86	4.98	27.23	0.0000
u_4	1.9	13.7	510.08	0.0000
u_5	0.49	6.24	45.24	0.0000

further support for the mixture distribution. Hence formal tests support the proposed identification strategy, and the exclusion restrictions in (2.25) can be statistically tested.

2.4.3 Statistical Analysis

To answer the main question of interest, i.e. whether the initial effects matrix \mathbf{B} as in (19) is supported by the data, we proceed as follows. Following the benchmark paper, lag length of one is selected according to the Bayesian Information Criterion (BIC). We first estimate an *unrestricted* SVAR(1) model with variable ordering (2.26) assuming that the error term \mathbf{u}_t follows a mixture of normal distributions as in (2.17). The estimation results

Table 2.2: Estimation results for the SVAR(1) model with nonnormal errors

Parameter	Model 1		Model 2	
	Unrestricted \mathbf{B}	Restricted \mathbf{B}	Unrestricted \mathbf{B}	Restricted \mathbf{B}
$\hat{\gamma}$	0.80 (0.04)	0.87 (0.04)	0.44 (0.06)	0.41 (0.07)
$\hat{\psi}_1 \times 10$	3.10 (1.18)	3.06 (2.05)	20.43 (7.34)	12.33 (5.07)
$\hat{\psi}_2 \times 10$	0.44 (0.18)	0.57 (0.28)	0.78 (0.25)	0.83 (0.32)
$\hat{\psi}_3 \times 10$	0.03 (0.01)	0.02 (0.01)	1.40 (0.54)	2.98 (1.02)
$\hat{\psi}_4 \times 10$	0.06 (0.02)	0.03 (0.02)	0.49 (0.17)	0.75 (0.23)
$\hat{\psi}_5 \times 10$	20.94 (8.11)	19.76 (11.8)	3.07 (1.02)	1.98 (1.01)
$\max l_T(\theta)$	255.07	228.08	248.90	241.01
LR		53.97		15.77
<i>p</i> -value		4.919×10^{-8}		0.1065

NOTES: Models 1 and 2 correspond to $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, MRP_t, FI_t)$ and $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)$, respectively. Standard errors in parenthesis are obtained from the inverse Hessian of the log-likelihood function. $LR = 2(\log L_T - \log L_T^r)$ where L_T^r denotes the maximum likelihood under H_0 : restricted \mathbf{B} and L_T denotes the maximum likelihood for the model under H_1 : unrestricted \mathbf{B} . *p*-values were computed assuming asymptotic $\chi^2(10)$ distribution for the *LR* test statistic. The estimated ψ_i 's are multiplied by 10 for reporting purposes.

are reported in Table 2.2.⁹ In this case identification is obtained with a distributional assumption, and the restrictions in (2.25) become over-identifying if the ψ_i 's are distinct.

⁹The computations were done with GAUSS programs. To compute the ML estimates, the BHHH procedure of the Gauss CMLMT library was used. In a first step, VAR coefficients were estimated from

Therefore we first need to ensure that a statistical identification of the shocks has been obtained.

Although the standard errors in Table 2.2 indicate a fairly good estimation precision, pairwise equality of the ψ_i 's has been tested with Wald tests. Since the estimators have the usual normal limiting distributions, the Wald tests have asymptotic χ^2 -distributions. The null hypotheses and the resulting p -values are listed in Table 2.3. The first column shows that except for ψ_3 and ψ_4 , the equality of all ψ_i 's can be rejected at the 10% significance level and hence statistical identification of shocks has been obtained.

Table 2.3: p -values of Wald tests for equality of psi 's for models from Table 1

H_0	Model 1		Model 2	
	Unrestricted \mathbf{B}	Restricted \mathbf{B}	Unrestricted \mathbf{B}	Restricted \mathbf{B}
$\hat{\psi}_1 = \hat{\psi}_2$	0.03	0.16	0.01	0.02
$\hat{\psi}_1 = \hat{\psi}_3$	0.01	0.09	0.01	0.07
$\hat{\psi}_1 = \hat{\psi}_4$	0.01	0.09	0.01	0.02
$\hat{\psi}_1 = \hat{\psi}_5$	0.03	0.18	0.02	0.05
$\hat{\psi}_2 = \hat{\psi}_3$	0.02	0.05	0.29	0.05
$\hat{\psi}_2 = \hat{\psi}_4$	0.03	0.05	0.35	0.87
$\hat{\psi}_2 = \hat{\psi}_5$	0.01	0.10	0.03	0.27
$\hat{\psi}_3 = \hat{\psi}_4$	0.32	0.59	0.11	0.03
$\hat{\psi}_3 = \hat{\psi}_5$	0.01	0.09	0.15	0.51
$\hat{\psi}_4 = \hat{\psi}_5$	0.01	0.09	0.01	0.25

NOTES: Models 1 and 2 correspond to $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, mrp_t, FI_t)'$ and $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)'$, respectively, where Δgdp_t denotes quarterly GDP growth, π_t inflation, FFR_t the Federal Funds target rate, MRP_t is the macro risk premium and FI_t the financial intermediary risk appetite factor.

Now a statistical test of the exclusion restrictions (2.20) can be performed. To this end we next estimate a *restricted* model by imposing the recursive ordering (2.25).¹¹ The statistical test then takes the form of a simple LR test, which has an asymptotic $\chi^2(N)$ distribution, where N is the number of restrictions. The hypotheses are H_0 : restricted \mathbf{B} and H_1 : unrestricted \mathbf{B} . The estimation results together with the LR test value (computed assuming $N = 10$) and the associated p -value are also reported in Table 2.2.

a linear model. In a second step, these estimates were used as starting values to estimate the parameters of the *unrestricted* model with a mixed normal distribution. Finally, the parameter estimates of the unrestricted model were used as starting values of the *restricted* model. To ensure nonsingularity of the covariance matrices, their determinants are bounded away from zero. Also the diagonal elements of the Ψ matrix are bounded away from zero.

¹¹In practice this is done with restrictions on the \mathbf{W} matrix in $\mathbf{B} = \mathbf{W}(\gamma \mathbf{I}_n + (1 - \gamma)\Psi)^{1/2}$.

As the LR-test rejects the H_0 at all significance levels, we conclude that the restrictions are not compatible with the data. Note that the Wald tests for the restricted model in the second column of Table 2.3 reveal that the pairwise equality of ψ_1 and ψ_5 or ψ_3 and ψ_4 cannot be rejected, which implies that the LR statistic has less than 10 degrees of freedom. Given the high value for the LR, it still leads to rejection.

As pointed out in Section 2.4, a challenge that arises from the variable ordering (2.26) is that no feedback effect from the right to the left within the same quarter is allowed. Our method essentially allows us to test whether the statistically identified shocks satisfy any recursive ordering. Therefore we can order the variables as

$$\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)' \quad (2.27)$$

In (2.27) the ordering of FFR_t (Federal Funds target rate), MRP_t (macro risk premium) and FI_t (financial intermediary risk appetite) still conforms with the theory, while the inclusion of Δgdp_t (quarterly GDP growth) and π_t (inflation) is again justified to avoid omitted variable bias. The main difference with (2.26) is that now changes in the price of risk and financial intermediaries' risk appetite are allowed to affect economic fluctuations within the same quarter already. The generally accepted view that changes in monetary policy are reflected in GDP growth earlier than in inflation holds here as well.

The estimation results for this model are shown on the right hand side of Table 2.2. Again, p -values of pairwise equality tests of the ψ_i 's are shown in Table 2.3. The LR test indicates that the H_0 : restricted \mathbf{B} cannot be rejected even at the 10 % significance level. At this time, taking into account that the equality of ψ_2 and ψ_3 , ψ_2 and ψ_4 , and ψ_3 and ψ_5 cannot be rejected, there is still no strong evidence against the imposed restrictions.

Therefore we conclude that the data at hand does not strongly object to a recursive ordering implied by (2.27). Inability to reject (2.27) simply tells us that during the sample period monetary policy has been promptly transmitted from the financial sector to the real economy. As the columns of a triangular matrix cannot be permuted, the ordering of the shocks corresponds to the lower-triangular \mathbf{B} -matrix so that the statistically identified shocks can be economically labelled in line with the ordering in equation (2.27).

2.4.4 Robustness Analysis

To analyze the sensitivity of the results with respect to the proxy variables being used, the models were additionally estimated with an alternative risk premium measure, the

Table 2.4: Robustness of the estimation results for the SVAR(1) model with nonnormal errors

Parameter	Model 1		Model 2	
	Unrestricted \mathbf{B}	Restricted \mathbf{B}	Unrestricted \mathbf{B}	Restricted \mathbf{B}
$\hat{\gamma}$	0.55 (0.06)	0.44 (0.15)	0.55 (0.06)	0.51 (0.06)
$\hat{\psi}_1 \times 10$	5.11 (1.79)	5.30 (3.44)	13.35 (4.26)	9.10 (3.15)
$\hat{\psi}_2 \times 10$	0.89 (0.30)	3.81 (1.97)	0.89 (0.30)	0.78 (0.28)
$\hat{\psi}_3 \times 10$	0.26 (0.09)	1.55 (0.00)	0.79 (0.25)	0.68 (0.24)
$\hat{\psi}_4 \times 10$	0.79 (0.24)	0.70 (0.44)	0.26 (0.09)	0.78 (0.25)
$\hat{\psi}_5 \times 10$	13.35 (4.26)	12.64 (21.97)	5.11 (1.85)	6.39 (2.53)
$\max l_T(\theta)$	191.49	165.53	191.49	184.65
LR		51.92		13.68
p -value		1.181×10^{-7}		0.1881

NOTES: Models 1 and 2 correspond to $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, MRP_t, FI_t)$ and $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)$, respectively. Standard errors in parenthesis are obtained from the inverse Hessian of the log-likelihood function. $LR = 2(\log L_T - \log L_T^r)$ where L_T^r denotes the maximum likelihood under H_0 : restricted \mathbf{B} and L_T denotes the maximum likelihood for the model under H_1 : unrestricted \mathbf{B} . p -values were computed assuming asymptotic $\chi^2(10)$ distribution for the LR test statistic. The estimated ψ_i 's are multiplied by 10 for reporting purposes.

Excess Bond Premium (EBP) of Gilchrist and Zakrajsek (2012). The EBP variable has been constructed to capture cyclical changes in the relationship between measured default risk and credit changes, and an increase in the excess bond premium reflects a reduction in the effective risk-bearing capacity of the financial sector (Gilchrist and Zakrajsek 2012, 2), and is therefore suitable for our purposes. The estimation procedure is as in Section 2.4.3. The model with the EBP variable was first estimated with variable ordering as in Adrian et al. (2010), or (2.26), and then according to (2.27). The estimation results are reported in Table 2.4.

Given the high value of the LR in the first case, the test rejects the imposed restrictions at all significance levels even if some of the ψ_i 's were identical. Also in the second case some of the ψ_i 's may not be distinct (see Table 2.5), which would decrease the p -value of LR-test. One would still not be able to reject the restrictions at usual significance levels. As these results conform perfectly with those of the baseline case, we conclude that the results are robust to the alternative proxy variable.

Table 2.5: p-values of Wald tests for equality of psi 's for models from Table 2

H_0	Model 1		Model 2	
	Unrestricted \mathbf{B}	Restricted \mathbf{B}	Unrestricted \mathbf{B}	Restricted \mathbf{B}
$\hat{\psi}_1 = \hat{\psi}_2$	0.02	0.77	0.00	0.01
$\hat{\psi}_1 = \hat{\psi}_3$	0.01	0.22	0.00	0.01
$\hat{\psi}_1 = \hat{\psi}_4$	0.02	0.22	0.00	0.01
$\hat{\psi}_1 = \hat{\psi}_5$	0.07	0.70	0.75	0.51
$\hat{\psi}_2 = \hat{\psi}_3$	0.04	0.05	0.80	0.78
$\hat{\psi}_2 = \hat{\psi}_4$	0.80	0.07	0.05	0.10
$\hat{\psi}_2 = \hat{\psi}_5$	0.00	0.70	0.02	0.03
$\hat{\psi}_3 = \hat{\psi}_4$	0.04	0.22	0.04	0.77
$\hat{\psi}_3 = \hat{\psi}_5$	0.00	0.62	0.02	0.02
$\hat{\psi}_4 = \hat{\psi}_5$	0.00	0.59	0.01	0.03

NOTES: Models 1 and 2 correspond to $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, mrp_t, FI_t)'$ and $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)'$, respectively, where Δgdp_t denotes quarterly GDP growth, π_t inflation, FFR_t the Federal Funds target rate, MRP_t is the macro risk premium and FI_t the financial intermediary risk appetite factor.

2.4.5 Model Diagnostic

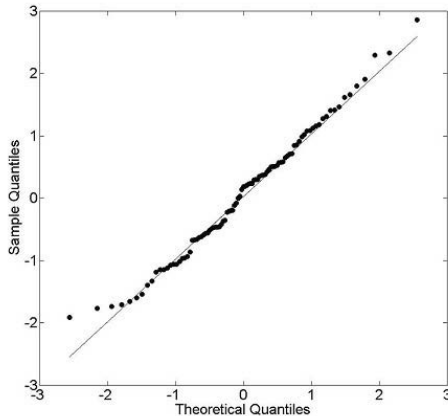
In models based on mixtures of distributions, statistical tests based on conventional residuals cannot be used to check the model specification. In these cases, Kalliovirta (2012) proposes a test based on quantile residuals, which are obtained by two transformations of the estimated residuals. First, the estimated cumulative distribution function (CDF) implied by the model is used to transform the observations into approximately independent, uniformly distributed random variables. Second, the inverse of the CDF of the standard normal distribution is used to get variables that are approximately independent with standard normal distribution.

These results assume that the model is correctly specified and parameters consistently estimated. Therefore quantile residuals that exhibit departures from these properties provide evidence of model misspecification. This approach has been generalized to multivariate models in Kalliovirta and Saikkonen (2010), where tests based on univariate joint quantile residuals are developed. Model misspecification can then be detected with normality, autocorrelation and conditional heteroskedasticity tests of the joint quantile residuals.

Figure 2.2 shows the QQ-plot of the joint quantile residuals obtained from the SVAR with mixed normal errors. Apart from a few outliers at both tails, the normality as-

sumption seems to hold reasonably well. A formal test of normality yields a p-value of 0.38, while autocorrelation and heteroskedasticity tests for different lags range from 0.23 to 0.90 and from 0.18 to 0.99, respectively. As a conclusion, the diagnostic tests provide clear support for our model specification, where a mixed-normal distribution is assumed.

Figure 2.2: Joint quantile residuals, QQ plot

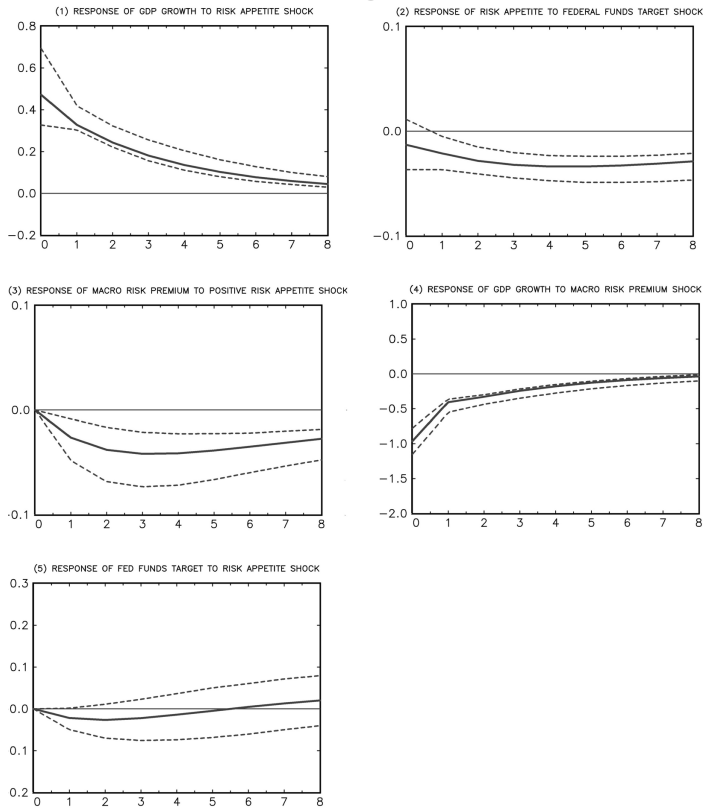


2.4.6 Impulse Response Analysis

Given that economically meaningful shocks have been identified, impulse response functions based on the SVAR(1) model with nonnormal errors can be computed. Because of the difficulties with the optimization of the likelihood function, confidence intervals for the impulse response functions cannot be easily computed with classical residual based bootstrap methods. Herwartz and Lütkepohl (2014) note that one has to ensure that only bootstrap replications in the area of the parameter space of the original estimation step are considered, and the same sign and ordering of the shocks is preserved. To this end, the diagonal elements of Ψ and the transition probability γ are not subjected to resampling. Bootstrap impulse response functions are obtained by nonlinear optimization of the log-likelihood with ML estimates as starting values. The bootstrap confidence intervals are the 16th and 84th quantiles of 1000 bootstrap replications.

Finally we are ready to analyze the macroeconomic effects of changing risk perceptions and risk tolerance by financial intermediaries. The impulse responses most important from the point of view of the mechanism of interest are displayed in Figure 2.3 together with

Figure 2.3: Selected impulse responses based on the VAR(1) model with mixed normal residuals and restricted B with 68% bootstrap confidence bands.



68 % bootstrap confidence intervals.¹⁴ The 68% confidence bands are common in the literature, and the bootstrapped confidence bands tend to give a more precise picture of the estimation uncertainty of the coefficients in a small sample.

The first picture in Figure 2.3 shows that a unit shock to financial intermediaries' risk appetite has a positive impact on GDP growth and the effect lasts for several quarters. Based on the theory, a way to interpret this is that when financial intermediaries more easily obtain funding, they increase the supply of credit, which contributes to higher GDP growth.

The second picture in the first row plots the response of risk appetite to a positive Federal Funds target shock. The impulse response suggests that a sudden monetary policy tightening decreases intermediaries' risk appetite for several periods, and the effect becomes significant after one quarter.

¹⁴The rest of the impulse response functions are reported in the Appendix.

The first picture in the middle row displays the response of macro risk premium to a positive risk appetite shock. Per construction, the response is negative.

Plotted in the second picture of the middle row is the negative effect on GDP growth of a higher macro risk premium. As the macro risk premium measures the hurdle rate of return required to finance new projects in the economy, this can be interpreted as tighter credit conditions having an adverse effect on GDP growth.

Finally, the fifth picture displays the response of Fed Funds target rate to a risk appetite shock. Contrary to the previous four impulse response functions, which are in line with the findings of Adrian et al. (2010), higher risk appetite is not followed by a monetary tightening. Instead the response is negative and insignificant along the whole horizon.

To sum up, changes in either financial intermediaries' risk appetite or macro risk premium are found to affect economic activity measured by quarterly GDP growth. There is also evidence of a positive and significant reaction of financial intermediaries' risk appetite to lax monetary policy during the sample period. Macro risk premiums appear to be driven by financial intermediaries' balance sheet adjustment as measured by the risk appetite factor.

These observations are in line with the predictions of the underlying theory on the risk-taking channel and confirm the results of the previous empirical study. Specifically, the balance sheet adjustment by financial intermediaries and fluctuations in the price of risk have both contributed to economic fluctuations during the sample period, and there is evidence of a link between the two and monetary policy.

2.5 Conclusions

This paper analyzed empirically the macroeconomic effects of the risk-taking channel of monetary policy by reconsidering the SVAR study of Adrian et al. (2010). We applied the method due to Lanne and Lütkepohl (2010) and exploited statistical properties of the data to identify the model and structural shocks without imposing any restrictions. Although the Lanne and Lütkepohl (2010) method only guarantees a statistical identification in that it delivers orthogonalized shocks without attaching economic labels to them, we were able to find a recursive ordering not rejected by the data and to label the shocks accordingly.

The resulting impulse responses were very similar to those reported by Adrian et al.

(2010) and, judging by the confidence bands, provided empirical evidence in support of the risk-taking channel. Specifically, we confirmed that monetary policy can affect the balance-sheet management of financial intermediaries, the determination of risk premiums, and eventually the level of real activity in the US.

We computed the 68% confidence bounds from bootstrap estimates of a more complex empirical model based on nonnormality of the errors. Although the downside of the complexity was that it made estimation computationally intensive, our empirical model had two advantages. First, because the nonnormality of errors was a feature encountered in the data, estimation was based on a more realistic assumption. Second, the bootstrap method should improve the precision of the confidence intervals in a small sample like the one analyzed here.

Our impulse response analysis provided evidence in favor of a positive and significant reaction of financial intermediaries' risk appetite to lax monetary policy during the sample period. Also risk premiums in the economy appeared to be significantly driven by financial intermediaries' balance sheet adjustment as measured by the risk appetite factor. These observations are in line with the predictions of the underlying theory on the risk-taking channel and confirm the results of the benchmark empirical study.

References

- Adrian T., and Shin H. (2010). Financial Intermediaries and Monetary Economics in Friedman, B., Woodford, M. (eds.), *Handbook of Monetary Economics*. New York, N.Y.: Elsevier, 601-650.
- Adrian T., and Shin H. (2009). Money, Liquidity and Monetary policy. *American Economic Review* **99**(2): 600-605.
- Adrian T., Moench E., and Shin H. S. (2010). Macro Risk Premium and Intermediary Balance Sheet Quantities. *IMF Economic Review* **58**(1).
- Altunbas, Gambacorta, and Marques-Ibanes (2010). Bank Risk and Monetary Policy. *Journal of Financial Stability* **6**(3): 121-129.
- Bernanke B., and Blinder A. (1992). The Federal Funds Rate and the Channel of Monetary Transmission. *American Economic Review* **79**: 14-31.

- Bernanke B., and Getler M. (1995). Inside the Black Box: the Credit Channel of Monetary Policy Transmission. *Journal of Economic Perspectives* **9**: 27-48.
- Blanchard O., and Quah D. (1989). The Dynamic Effects of Aggregate Demand and Supply Disturbances. *American Economic Review* **79**: 655-673.
- Borio C., and Zhu H. (2012). Capital Regulation, Risk-Taking and Monetary Policy: A Missing Link in the Transmission Mechanism? *Journal of Financial Stability* **8**: 236-251.
- Brunnermeier M.K., and Sannikov Y. (2012). Redistributive Monetary Policy. Policy Forum of the Federal Reserve Bank of Kansas City in Jackson Hole 2012.
- Buch C., Eickmeier S., and Prieto E. (2011) In Search for Yield? Survey-Based Evidence of Bank Risk Taking. Deutsche Bundesbank Discussion Paper 10/2011
- Canova F., and De Nicoló G. (2002). Monetary Disturbances Matter for Business Fluctuations in the G-7. *Journal of Monetary Economics* **49**: 1131-1159.
- De Santis R., and Surico P. (2013). Bank Lending and Monetary Transmission in the Euro Area. *Economic Policy* **28**(75): 423-457.
- Gilchrist S., and Zakrajšek E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review* **102**(4): 1692-1720.
- Goncalves, S., and Kilian, L. (2004). Bootstrapping Autoregressions with Conditional Heteroskedasticity of Unknown Form. *Journal of Econometrics* **123**: 89-120.
- Herwartz H., and Lütkepohl H. (2014). Structural Vector Autoregressions with Markov Switching: Combining Conventional with Statistical Identification of Shocks. *Journal of Econometrics* **183**(1): 104-116.
- Jimenez G., Ongena S., Peydro J., and Saurina J. (2014). Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking? *Econometrica* **82**(2): 463-505.
- Kalliovirta, L. (2012). Misspecification Tests Based on Quantile Residuals. *Econometrics Journal* **15**(2): 358-93.

- Kalliovirta, L., and Saikkonen P. (2010). Reliable Residuals for Multivariate Nonlinear Time Series Models. <http://blogs.helsinki.fi/lkvaisan/files/2010/08/ReliableResiduals.pdf>.
- Lanne M., and Lütkepohl H. (2008). Identifying Monetary Policy Shocks via Changes in Volatility. *Journal of Money, Credit and Banking* **40**: 1131-1149.
- Lanne M., and Lütkepohl H. (2010). Structural Vector Autoregressions with Non-Normal Residuals. *Journal of Business & Economic Statistics* **28**: 159-168
- Lanne M., and Lütkepohl H. and Maciejowska K. (2010). Structural Vector Autoregressions with Markov Switching. *Journal of Economic Dynamics and Control* **34**: 121-131.
- Lütkepohl H. (2005). *New Introduction to Multiple Time Series Analysis*, Springer-Verlag, Berlin.
- Maddaloni A., and Peydró J.L. (2011). Bank Risk-taking, Securitization, Supervision, and Low Interest Rates: Evidence from the Euro-area and the U.S. Lending Standards. *Review of Financial Studies* **24**: 2121-2165.
- Rigobon R. (2003). Identification Through Heteroskedasticity. *Review of Economics and Statistics* **85**: 777-792.
- Shin H.S. (2010). *Risk and Liquidity*. Clarendon Lectures in Finance. Oxford University Press, London.
- Uhlig H. (2005). What Are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure. *Journal of Monetary Economics* **52**: 381-419.

Appendix: Additional Results

Figure 2.4: Impulse responses based on the SVAR model with mixed normal errors and restricted B with 68% bootstrap confidence bands. The columns contain responses of all variables to shocks in inflation, federal funds rate and macro risk premium.

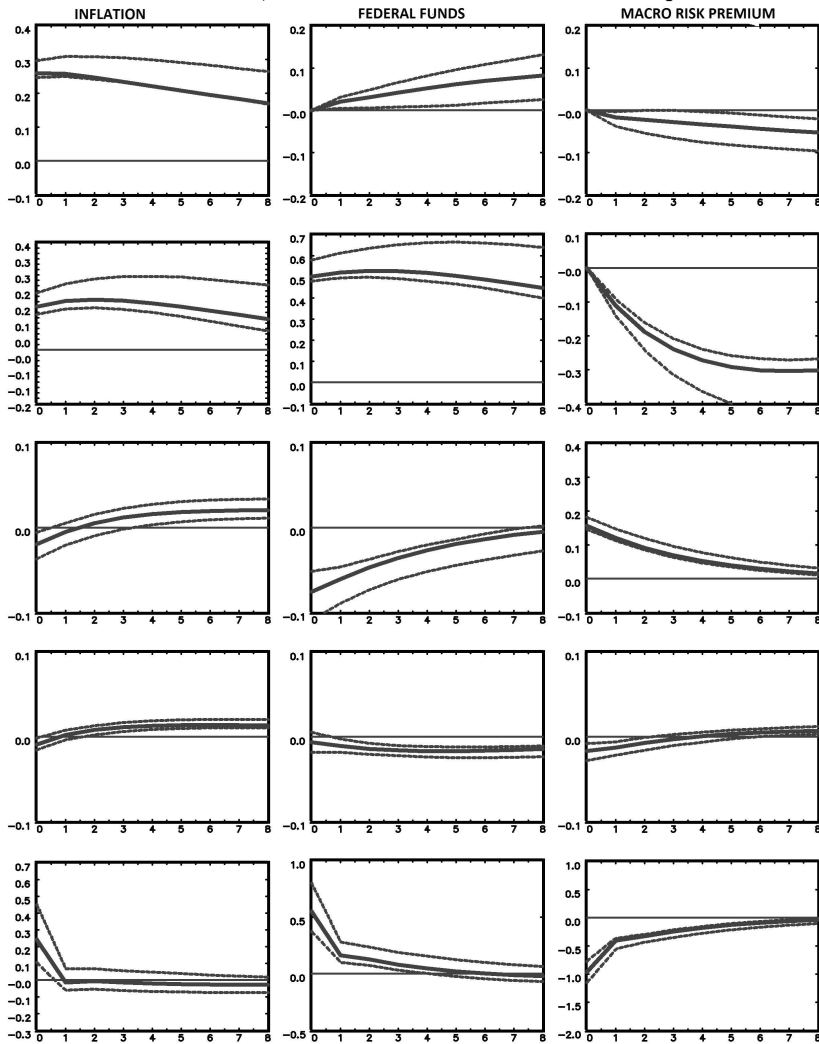


Figure 2.5: Impulse responses based on the SVAR model with mixed normal errors and restricted B with 68% bootstrap confidence bands. The columns contain responses of all variables to shocks in financial intermediary risk appetite and gdp growth.

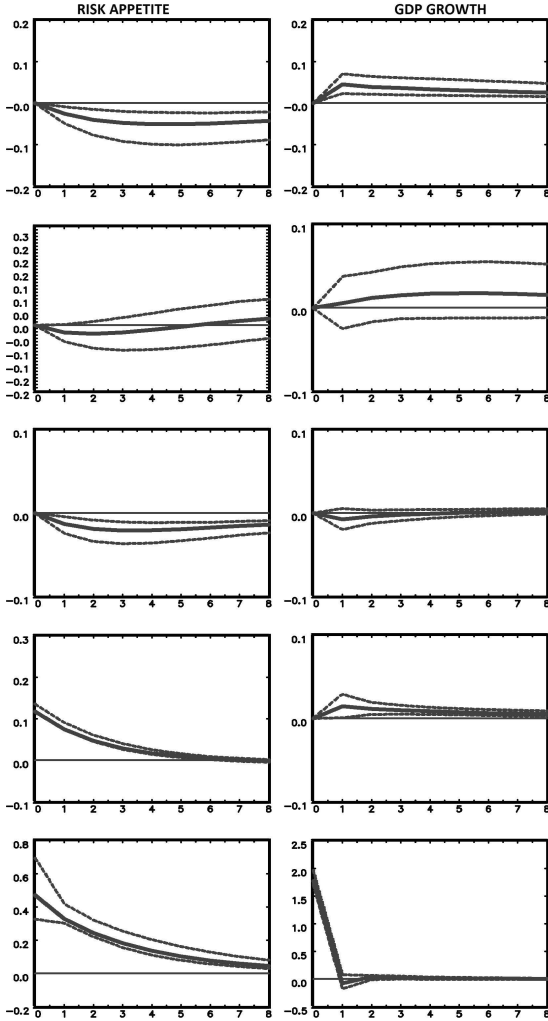


Table 2.6: Estimation results with variable ordering (14) and unrestricted W

	Elements of each vector				
	1	2	3	4	5
intercept	7.31 (1.71)	-0.05 (0.24)	1.24 (0.21)	0.31 (0.07)	-0.06 (0.05)
A [1,·]	0.11 (0.11)	0.02 (0.01)	-0.00 (0.01)	-0.01 (0.01)	0.01 (0.00)
A [2,·]	0.39 (0.31)	0.96 (0.04)	-0.03 (0.04)	0.05 (0.01)	0.05 (0.01)
A [3,·]	0.05 (0.13)	0.01 (0.02)	0.95 (0.02)	-0.01 (0.01)	-0.03 (0.00)
A [4,·]	-2.45 (0.97)	-0.03 (0.13)	-0.64 (0.12)	0.74 (0.04)	-0.00 (0.03)
A [5,·]	3.31 (1.27)	-0.28 (0.16)	-0.16 (0.15)	-0.19 (0.05)	0.69 (0.04)
W [1,·]	2.13 (0.20)	-0.68 (0.39)	0.09 (0.31)	-0.21 (0.30)	0.29 (0.27)
W [2,·]	0.01 (0.05)	0.01 (0.03)	-0.09 (0.04)	-0.04 (0.05)	0.22 (0.02)
W [3,·]	0.12 (0.04)	0.03 (0.06)	0.40 (0.17)	-0.35 (0.18)	0.16 (0.02)
W [4,·]	-0.07 (0.01)	0.08 (0.02)	0.03 (0.08)	0.17 (0.02)	0.00 (0.01)
W [5,·]	-0.01 (0.01)	-0.12 (0.01)	0.03 (0.03)	0.04 (0.03)	-0.00 (0.01)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (14) is $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, MRP_t, FI_t)'$. Standard errors in parenthesis.

Table 2.7: Estimation results with variable ordering (14) and W restricted to lower triangular

	Elements of each vector				
	1	2	3	4	5
intercept	7.26 (1.94)	-0.06 (0.17)	1.36 (0.18)	0.39 (0.08)	-0.13 (0.11)
A [1,·]	0.16 (0.12)	0.02 (0.01)	-0.02 (0.01)	-0.01 (0.01)	0.01 (0.01)
A [2,·]	-0.32 (0.31)	1.03 (0.03)	0.01 (0.03)	0.03 (0.01)	0.04 (0.02)
A [3,·]	-0.03 (0.14)	-0.02 (0.01)	0.91 (0.01)	-0.01 (0.01)	-0.02 (0.01)
A [4,·]	-2.37 (1.06)	-0.08 (0.08)	-0.67 (0.09)	0.70 (0.04)	0.02 (0.06)
A [5,·]	3.41 (1.37)	-0.40 (0.12)	-0.32 (0.12)	-0.21 (0.05)	0.66 (0.08)
W [1,·]	2.18 (0.17)
W [2,·]	0.04 (0.02)	0.28 (0.02)	.	.	.
W [3,·]	0.13 (0.02)	0.13 (0.03)	0.55 (0.04)	.	.
W [4,·]	-0.08 (0.01)	-0.02 (0.01)	-0.05 (0.02)	0.16 (0.01)	.
W [5,·]	0.03 (0.01)	-0.03 (0.01)	-0.02 (0.01)	0.01 (0.01)	0.11 (0.01)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (14) is $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, MRP_t, FI_t)'$. Standard errors in parenthesis.

Table 2.8: Estimation results with variable ordering (15) and unrestricted W

	Elements of each vector				
	1	2	3	4	5
intercept	-0.1 (0.23)	1.4 (0.30)	0.29 (0.14)	-0.09 (0.06)	6.87 (1.58)
A [1,·]	0.99 (0.04)	0.07 (0.05)	0.05 (0.02)	0.04 (0.01)	0.06 (0.29)
A [2,·]	-0.01 (0.02)	0.91 (0.02)	-0.01 (0.01)	-0.02 (0.01)	-0.01 (0.13)
A [3,·]	-0.01 (0.13)	-0.78 (0.17)	0.74 (0.08)	0.01 (0.03)	-2.51 (0.87)
A [4,·]	-0.22 (0.18)	-0.21 (0.21)	-0.12 (0.1)	0.59 (0.05)	3.26 (1.14)
A [5,·]	0.03 (0.02)	-0.01 (0.02)	-0.01 (0.01)	0.01 (0.01)	-0.04 (0.1)
W [1,·]	0.18 (0.03)	-0.01 (0.08)	0.05 (0.07)	-0.11 (0.05)	0.12 (0.05)
W [2,·]	0.16 (0.03)	0.51 (0.24)	0.06 (0.24)	0.47 (0.31)	0.04 (0.08)
W [3,·]	0.01 (0.02)	-0.13 (0.06)	0.04 (0.10)	-0.06 (0.9)	-0.19 (0.03)
W [4,·]	-0.01 (0.01)	-0.11 (0.07)	-0.02 (0.05)	0.12 (0.07)	0.04 (0.02)
W [5,·]	-0.13 (0.18)	0.33 (1.4)	2.23 (0.8)	0.98 (0.82)	1.7 (0.61)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (15) is $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)'$. Standard errors in parenthesis.

Table 2.9: Estimation results with variable ordering (15) and W restricted to lower triangular

	Elements of each vector				
	1	2	3	4	5
intercept	-0.10 (0.25)	1.20 (0.32)	0.31 (0.14)	-0.1 (0.07)	7.22 (1.65)
A [1,·]	0.95 (0.04)	0.06 (0.06)	0.04 (0.02)	0.04 (0.01)	-0.12 (0.32)
A [2,·]	0.01 (0.02)	0.93 (0.02)	-0.01 (0.01)	-0.02 (0.01)	0.03 (0.13)
A [3,·]	0.02 (0.14)	-0.71 (0.18)	0.73 (0.08)	0.01 (0.04)	-2.55 (0.90)
A [4,·]	-0.32 (0.19)	-0.2 (0.23)	-0.1 (0.1)	0.62 (0.05)	2.98 (1.18)
A [5,·]	0.03 (0.02)	0.01 (0.02)	-0.01 (0.01)	0.00 (0.00)	-0.05 (0.10)
W [1,·]	0.24 (0.04)
W [2,·]	0.15 (0.04)	0.74 (0.09)	.	.	.
W [3,·]	-0.02 (0.01)	-0.11 (0.03)	0.21 (0.3)	.	.
W [4,·]	-0.01 (0.01)	-0.01 (0.02)	-0.02 (0.01)	0.18 (0.02)	.
W [5,·]	0.23 (0.17)	0.82 (0.4)	-1.26 (0.28)	0.70 (0.36)	2.44 (0.33)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (15) is $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)'$. Standard errors in parenthesis.

Table 2.10: Estimation results with variable ordering (14) and unrestricted W. Robustness analysis

	Elements of each vector				
	1	2	3	4	5
intercept	3.43 (0.74)	-0.1 (0.1)	0.1 (0.13)	-0.22 (0.06)	-0.04 (0.03)
A [1,·]	0.13 (0.10)	0.02 (0.01)	0.044 (0.02)	0.01 (0.01)	0.01 (0.00)
A [2,·]	-0.58 (0.29)	0.98 (0.04)	-0.02 (0.05)	0.03 (0.02)	0.03 (0.01)
A [3,·]	0.18 (0.12)	0.01 (0.02)	0.95 (0.02)	0.01 (0.01)	-0.02 (0.01)
A [4,·]	-1.56 (0.49)	-0.05 (0.06)	-0.35 (0.09)	0.89 (0.04)	-0.04 (0.02)
A [5,·]	2.65 (1.53)	-0.20 (0.20)	-0.02 (0.25)	-0.02 (0.12)	0.68 (0.05)
W [1,·]	2.02 (0.32)	0.14 (1.05)	0.78 (0.38)	-0.52 (0.49)	-0.71 (0.42)
W [2,·]	0.16 (0.07)	0.001 (0.1)	-0.08 (0.04)	-0.05 (0.05)	0.2 (0.04)
W [3,·]	0.13 (0.06)	0.55 (0.13)	0.3 (0.14)	-0.06 (1.03)	0.1 (0.04)
W [4,·]	-0.03 (0.03)	0.09 (0.45)	-0.30 (0.08)	0.25 (0.18)	-0.02 (0.02)
W [5,·]	0.02 (0.01)	-0.06 (0.21)	0.08 (0.04)	0.11 (0.12)	-0.01 (0.01)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (14) is $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, MRP_t, FI_t)'$. Standard errors in parenthesis.

Table 2.11: Estimation results with variable ordering (14) and W restricted to lower triangular. Robustness analysis

	Elements of each vector				
	1	2	3	4	5
intercept	3.42 (0.76)	-0.08 (0.1)	0.01 (0.22)	-0.13 (0.08)	-0.1 (0.07)
A [1,·]	0.12 (0.12)	-0.13 (0.08)	-0.1 (0.07)	0.12 (0.12)	0.005 (0.01)
A [2,·]	-0.56 (0.31)	0.97 (0.04)	-0.01 (0.08)	0.01 (0.03)	0.05 (0.02)
A [3,·]	0.16 (0.13)	-0.01 (0.02)	0.96 (0.03)	0.01 (0.02)	-0.02 (0.01)
A [4,·]	-1.52 (0.52)	-0.05 (0.06)	-0.36 (0.11)	0.93 (0.13)	-0.06 (0.03)
A [5,·]	2.66 (1.59)	-0.43 (0.28)	-0.04 (0.30)	-0.03 (0.14)	0.63 (0.09)
W [1,·]	2.32 (0.42)
W [2,·]	0.02 (0.03)	0.32 (0.03)	.	.	.
W [3,·]	0.10 (0.05)	0.10 (0.06)	0.65 (0.11)	.	.
W [4,·]	-0.05 (0.03)	-0.02 (0.03)	-0.06 (0.07)	0.42 (0.06)	.
W [5,·]	0.02 (0.01)	-0.03 (0.02)	-0.03 (0.03)	0.01 (0.02)	0.099 (0.05)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (14) is $\mathbf{y}_t = (\Delta gdp_t, \pi_t, FFR_t, MRP_t, FI_t)'$. Standard errors in parenthesis.

Table 2.12: Estimation results with variable ordering (15) and unrestricted W. Robustness check

	Elements of each vector				
	1	2	3	4	5
intercept	-0.09 (0.10)	0.01 (0.15)	-0.22 (0.06)	-0.04 (0.03)	3.43 (0.74)
A [1,·]	0.98 (0.04)	-0.02 (0.05)	0.03 (0.02)	0.03 (0.01)	-0.58 (0.295)
A [2,·]	0.01 (0.02)	0.95 (0.02)	0.01 (0.01)	-0.02 (0.00)	0.18 (0.12)
A [3,·]	-0.05 (0.06)	-0.35 (0.09)	0.89 (0.04)	-0.04 (0.02)	-1.56 (0.49)
A [4,·]	-0.20 (0.21)	-0.02 (0.26)	-0.02 (0.13)	0.67 (0.05)	2.65 (1.54)
A [5,·]	0.02 (0.01)	0.04 (0.02)	0.01 (0.01)	0.00 (0.00)	0.13 (0.10)
W [1,·]	0.2 (0.04)	-0.00 (0.53)	-0.05 (0.05)	-0.08 (0.04)	0.16 (0.07)
W [2,·]	0.09 (0.04)	0.55 (0.57)	-0.06 (5.6)	0.3 (0.15)	0.13 (0.08)
W [3,·]	-0.02 (0.02)	0.09 (2.44)	0.25 (0.89)	-0.30 (0.08)	-0.03 (0.03)
W [4,·]	-0.01 (0.01)	-0.06 (1.11)	0.11 (0.62)	0.07 (0.04)	0.02 (0.01)
W [5,·]	-0.71 (0.42)	0.14 (5.43)	-0.52 (1.43)	0.78 (0.38)	2.02 (0.32)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (15) is $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)'$. Standard errors in parenthesis.

Table 2.13: Estimation results with variable ordering (15) and W restricted to lower triangular. Robustness analysis

	Elements of each vector				
	1	2	3	4	5
intercept	-0.06 (0.10)	0.07 (0.13)	-0.15 (0.07)	-0.06 (0.03)	3.48 (0.73)
A [1,·]	0.96 (0.04)	-0.02 (0.05)	0.01 (0.03)	0.04 (0.01)	-0.62 (0.29)
A [2,·]	0.01 (0.02)	0.96 (0.02)	0.01 (0.01)	-0.02 (0.00)	0.20 (0.12)
A [3,·]	-0.06 (0.06)	-0.36 (0.08)	0.93 (0.06)	-0.05 (0.02)	-1.59 (0.51)
A [4,·]	-0.29 (0.20)	-0.09 (0.23)	-0.06 (0.12)	0.70 (0.05)	2.68 (1.54)
A [5,·]	0.02 (0.01)	0.05 (0.02)	0.01 (0.01)	0.00 (0.00)	0.11 (0.11)
W [1,·]	0.26 (0.03)
W [2,·]	0.12 (0.03)	0.66 (0.07)	.	.	.
W [3,·]	-0.02 (0.02)	-0.06 (0.05)	0.40 (0.04)	.	.
W [4,·]	-0.01 (0.01)	-0.03 (0.02)	0.00 (0.02)	0.15 (0.02)	.
W [5,·]	0.25 (0.21)	0.71 (0.31)	-0.81 (0.31)	0.53 (0.29)	2.02 (0.25)

Notes: **A**[i,·] and **W**[i,·] indicate the ith row of matrices **A** and **W**, respectively.

The variable ordering (15) is $\mathbf{y}_t = (\pi_t, FFR_t, MRP_t, FI_t, \Delta gdp_t)'$.

Chapter 3

Fiscal Multipliers in a Structural VEC Model with Mixed Normal Errors

Abstract¹

This paper estimates the effects of fiscal policy shocks on GDP in the United States with a vector error correction (VEC) model in which shocks are identified by exploiting the non-normal distribution of the model residuals. Unlike previous research, the model used here takes into account cointegration between the variables, and applies a data driven method to identify fiscal policy shocks. The approach also allows statistical testing of previous identification strategies, which may help discriminate between them and hence also explain differences between various fiscal multiplier estimates. Our results show that a deficit financed government spending shock has a weak negative effect on output, whereas a tax increase to finance government spending has a positive impact on GDP.

¹This chapter is based on an article published in the *Journal of Macroeconomics* (Puonti, 2016)

3.1 Introduction

After the recent financial crisis many central banks have had to come to terms with the limits of conventional monetary policy. Because of the zero lower bound on one hand and the prolongation of the economic downturn on the other, policymakers and economists alike have again turned their attention to fiscal policy. Common monetary policy, which is not necessarily optimal from the point of view of any one member country, emphasizes the role of fiscal policy in the euro area.

Compared to monetary policy, fiscal policy has been viewed as a less agile policy instrument mainly because of implementation lags, but also because of its multi-faceted nature. Fiscal policy consists of the allocation of government expenditure between different categories of consumption and investment as well as decisions about its finance with a particular tax-debt mix. These political decisions are taken at different levels of government administration (e.g. federal, state, provincial, or municipal). Unlike monetary policy, the stance of which can be summarized by an interest rate announced by the central bank, fiscal policy regime cannot be described by a single variable.

Nonetheless, there has been an upsurge of academic research in the macroeconomic effects of government expenditure and tax changes in recent years. Broadly speaking, the key question of interest is whether government spending can stimulate the economy, and what the size (and sign!) of this fiscal or government spending multiplier is. Ramey (2011a) provides a review of both theoretical and empirical research on the government spending multiplier. Theoretically defined multipliers provide a wide range of values depending on the type of model used, the assumptions about the behavior of monetary policy, the type and persistence of government spending, and how it is financed (Ramey 2011a). Consequently, the size of the multiplier is first and foremost an empirical issue.

Given the variety of theoretical and empirical results, many researchers have recently asked whether the multiplier depends on the state of the economy, i.e. whether government fiscal stimulus is more effective when it is used to supplement scant private demand in an economic downturn than in an upturn (Auerbach & Gorodnichenko 2012, Caggiano et al. 2015). Interestingly, Caggiano *et al.* (2015) show that this is indeed the case with *deep* recessions and *extreme* economic peaks in the US, while no statistically significant differences between normal times, i.e. normal economic downturns and upturns are found. Owyang et al. (2013), and Ramey and Zubairy (2014) also find no evidence of larger

fiscal multipliers during downturns. This means that research based on linear models is informative about the effectiveness of the fiscal policy instrument in normal times. Given the relative rarity of events like the recent Great Recession,² knowledge about the effectiveness of fiscal stimulus during an ordinary business cycle is admittedly valuable. This paper thus focuses on linear models.³

Vector autoregressive (VAR) models seem to have become the main econometric tool for determining the macroeconomic effects of both monetary and fiscal policy (Ramey 2011a, Caldara & Kamps 2008). Both strands of the empirical literature need to tackle the inherent shock identification problem. Fiscal policy research has relied on four identification strategies: 1) the recursive approach of Sims (1980) applied to fiscal policy by e.g. Auerbach and Gorodnichenko (2012), 2) the frequently applied structural VAR proposed by Blanchard and Perotti (2002), 3) the sign restrictions developed by Uhlig (2005) and applied by Mountford and Uhlig (2009) and 4) the narrative approach introduced by Ramey and Shapiro (1998), which exploits unexpected increases in military spending.

Studies using different VAR model specifications and identification schemes have come to diverging conclusions about the size and sometimes even the sign of the multiplier. Unlike with monetary policy, the fifth available strategy, statistical identification methods, has not yet been applied to the study of fiscal policy. Statistical methods that yield additional data based information may be helpful in shock identification, and/or possibly help choose the most suitable among the proposed identification strategies.

This paper thus applies the statistical method introduced by Lanne and Lütkepohl (2010), in which the non-normality of the errors is exploited to identify the structural shocks. More precisely, the errors are assumed to follow a mixture of two normal distributions. The identification strategy of Lanne and Lütkepohl (2010) allows not only identification of the statistical model without any identifying restrictions, but also statistically testing of whether any of the previously used identification strategies are compatible

²In the 32-year period studied by Caggiano *et al.* (2015), they identified two deep recessions in the U.S., whereas according to the NBER Recession Indicator the total number of recessions amounted to five.

³The inclusion of the Great Recession in the small sample considered may lead to a distorted picture of the effects of government spending shocks in normal times. However, excluding the financial crisis from the sample would significantly reduce the sample size, leading to less accurate estimates and poorer identification. Moreover, from the point of view of nonlinearities, the Great Recession is not a unique event since another such recession was identified by Caggiano *et al.* (2015) in the sample.

with the properties of the data. If statistical identification of shocks (see Section 3.3) is obtained following Lanne and Lütkepohl (2010), then the restrictions on the contemporaneous relationships between the variables imposed in the previous identification schemes can be statistically tested. This may also be helpful in labeling the statistically identified shocks, which is always based on outside information (Lanne et al. 2015, Lütkepohl & Netšunajev 2014). Although additional information is needed to interpret the shocks identified, being able to test their compatibility with the data is an advantage over traditional approaches. Obtaining results that are not dependent on the identification strategy chosen may be seen as a robustness check of previous empirical research.

Unlike any of the previous studies using VARs – linear and non-linear – the vector error correction (VEC) model used in this paper also takes into account the cointegration properties of the variables. The usual practice in the literature is to include the log levels of variables such as GDP, government spending and taxes (Ramey & Zubairy 2014), even though they are likely to contain a unit root. Phillips (1998) demonstrates that impulse responses are not consistently estimated in structural VARs (SVARs) with variables in levels in the case of unit roots, whereas the VEC specification significantly improves them even for short horizons when the cointegration relations are either known or consistently estimated. Phillips (1998) points out that differing treatments of nonstationarity in models such as unrestricted VAR, Bayesian VAR with unit root priors and reduced rank regression has substantial effects on policy analysis. An additional advantage of the VEC specification is that the cointegration relations provide identification restrictions and allow us to distinguish shocks that have either permanent or transitory effects.

As it has not yet been done for fiscal VARs, this paper 1) expands the set of identification strategies with increasingly popular statistical methods and 2) takes into consideration the cointegration properties of the time series. Both extensions – dealing with the nonstationarity of the data, and combining statistical and theoretical information for identification – are expected to increase the accuracy of the results (Phillips 1998, Herwartz & Lütkepohl 2014).

Quarterly data for the United States are used. The data cover the period 1981Q3 to 2012Q4 and were previously used by Caggiano *et al.* (2015), as well as Auerbach and Gorodnichenko (2012). Similarly to Caggiano *et al.* (2015), fiscal policy anticipation effects, or foresight are addressed by including the fiscal news variable proposed by Gam-

betti (2012). A drawback of using this variable is the relatively short sample. While we recognize that this is one limitation of the analysis, there are advantages in following this approach (see Section 3.3.1).

The analysis highlights differences between the different VAR specifications used to analyze the effects of fiscal policy. The impulse responses based on the VEC model with mixed normal errors are quite different from those typically obtained from SVAR models, as the latter mostly coincide with theoretical models in the Keynesian tradition. Our results show that a government spending shock has a weak but negative effect on GDP, while the response of taxes is not statistically different from zero even if no restrictions are imposed on taxes. As government revenue does not change, this can be interpreted as a fiscal policy shock financed by a deficit as in Mountford and Uhlig (2009). Also quite surprisingly, a government revenue shock triggers a positive response in both government expenditure and GDP. In line with the interpretation of the spending shock, this can be interpreted as a tax increase to finance government spending, which has a positive impact on GDP. The fiscal multiplier for the horizons $h = 1, 4, 8, 12, 20$ after the initial shock ranges from -1.27 to -1.61 and achieves its maximum at $h = 1$.

The rest of the paper is organized as follows. Technical details of the empirical method are given in Section 3.2. Section 3.3 covers the empirical analysis and Section 3.4 concludes the paper.

3.2 Vector Error Correction (VEC) Model with Non-normal Error Distribution

Unlike what is typically done in the existing fiscal policy literature, this paper specifies a vector error correction model (VECM) and estimates it to take into account the cointegration properties of the variables. If some or all of the variables are $I(1)$ and some of the variables are cointegrated, there are advantages in using the VEC representation of the process instead of the vector autoregressive (VAR) representation. Utilizing the cointegration properties of the variables provides identification restrictions, allowing us to distinguish between permanent and transitory shocks.

A reduced form $VEC(p)$ model with a cointegration rank $r < K$ (deterministic terms omitted for simplicity) is

$$\Delta \mathbf{y}_t = \alpha \beta' \mathbf{y}_{t-1} + \Gamma_1 \Delta \mathbf{y}_{t-1} + \cdots + \Gamma_{p-1} \Delta \mathbf{y}_{t-p+1} + \mathbf{u}_t$$

where \mathbf{y}_t is a $K \times 1$ vector of a time series, α is a $K \times r$ matrix of loading coefficients, β is a $K \times r$ cointegration matrix, Γ_j is a $K \times K$ short run coefficient matrix for $j = 1, \dots, p-1$, and $\mathbf{u}_t \sim (\mathbf{0}, \Sigma_u)$ is a white noise error vector. The process has the vector moving average (VMA) representation

$$\mathbf{y}_t = \Xi \sum_{i=1}^t \mathbf{u}_i + \Xi_j^* \sum_{j=0}^{\infty} \mathbf{u}_{t-j} + \mathbf{y}_0^*$$

where the Ξ_j^* are absolutely summable and \mathbf{y}_0^* contains the initial values (see e.g. Lütkepohl 2007, Chapter 9).

The long-run effects of the shocks are therefore captured by the common trends term

$$\Xi \sum_{i=1}^t \mathbf{u}_i \tag{3.28}$$

and the matrix

$$\Xi = \beta_{\perp} [\alpha'_{\perp} (I_K - \sum_{i=1}^{p-1} \Gamma_i) \beta_{\perp}]^{-1} \alpha'_{\perp}$$

has a rank of $K - r$. The symbols α_{\perp} and β_{\perp} denote the orthogonal complements of α and β respectively. Substituting the relation $\mathbf{u}_t = \mathbf{B} \boldsymbol{\varepsilon}_t$ in the common trends term (3.28) gives $\Xi \mathbf{B} \sum_{i=1}^t \boldsymbol{\varepsilon}_i$. The term $\Xi \mathbf{B}$ contains the long-run effects of the structural shocks and has a rank $K - r$. At most r of the shocks can have transitory effects only, and they are associated with zero columns in the long run matrix $\Xi \mathbf{B}$.

To obtain additional information for identification, Lanne and Lütkepohl (2010) assume that the K -dimensional error term \mathbf{u}_t is a mixture of two serially independent normal random vectors

$$\mathbf{u}_t = \begin{cases} \mathbf{e}_{1t} \sim \mathbf{N}(\mathbf{0}, \Sigma_1) & \text{with probability } \gamma \\ \mathbf{e}_{2t} \sim \mathbf{N}(\mathbf{0}, \Sigma_2) & \text{with probability } 1 - \gamma \end{cases} \tag{3.29}$$

where $\mathbf{N}(\mathbf{0}, \Sigma)$ denotes a multivariate normal distribution with a mean of $\mathbf{0}$ and a covariance matrix Σ . In the model Σ_1 and Σ_2 are $K \times K$ covariance matrices that are assumed to be distinct, γ is the mixture probability, and $0 < \gamma < 1$, a parameter of the model. Since the term γ is only identified if $\Sigma_1 \neq \Sigma_2$, this is assumed to hold. If some parts of Σ_1 and Σ_2 are identical then some components of \mathbf{u}_t may be normally distributed. In

any case there only needs to be one non-normal component in \mathbf{u}_t . The distribution of the reduced form error term now becomes

$$\mathbf{u}_t \sim (\mathbf{0}, \gamma \boldsymbol{\Sigma}_1 + (1 - \gamma) \boldsymbol{\Sigma}_2)$$

The distributional assumption for \mathbf{u}_t allows us to define a locally unique \mathbf{B} matrix in the following way. As shown in Appendix A by Lanne and Lütkepohl (2010), a diagonal matrix $\boldsymbol{\Psi} = \text{diag}(\psi_1, \dots, \psi_k)$, $\psi_i > 0$ ($i = 1, \dots, K$) and a $K \times K$ matrix \mathbf{W} exist such that $\boldsymbol{\Sigma}_1 = \mathbf{W}\mathbf{W}'$ and $\boldsymbol{\Sigma}_2 = \mathbf{W}\boldsymbol{\Psi}\mathbf{W}'$ and \mathbf{W} is locally unique for some ordering of ψ_i 's except for a change in the sign of a column, as long as all ψ_i 's are distinct. Now we can rewrite the covariance matrix of the reduced form error vector \mathbf{u}_t as

$$\boldsymbol{\Sigma}_u = \gamma \mathbf{W}\mathbf{W}' + (1 - \gamma) \mathbf{W}\boldsymbol{\Psi}\mathbf{W}' = \mathbf{W}(\gamma \mathbf{I}_k + (1 - \gamma) \boldsymbol{\Psi})\mathbf{W}' \quad (3.30)$$

Given that the structural shocks $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \mathbf{I}_K)$ are related to the reduced form errors as

$$\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$$

and

$$E(\mathbf{u}_t \mathbf{u}_t') = \boldsymbol{\Sigma}_u = \mathbf{B}\boldsymbol{\Sigma}_\varepsilon \mathbf{B}' = \mathbf{B}\mathbf{B}' \quad (3.31)$$

it follows that a locally unique \mathbf{B} matrix is given by

$$\mathbf{B} = \mathbf{W}(\gamma \mathbf{I}_n + (1 - \gamma) \boldsymbol{\Psi})^{1/2} \quad (3.32)$$

This is sufficient for identification.

This choice of \mathbf{B} also means that the orthogonality of shocks is independent of regimes.

This can be seen by applying (3.31) to the covariance matrices as

$$\begin{aligned} \mathbf{B}^{-1} \boldsymbol{\Sigma}_u \mathbf{B}^{-1} &= \mathbf{I}_k \\ \mathbf{B}^{-1} \boldsymbol{\Sigma}_1 \mathbf{B}^{-1} &= (\gamma \mathbf{I}_k + (1 - \gamma) \boldsymbol{\Psi})^{-1} \\ \mathbf{B}^{-1} \boldsymbol{\Sigma}_2 \mathbf{B}^{-1} &= (\gamma \mathbf{I}_k + (1 - \gamma) \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi} \end{aligned} \quad (3.33)$$

As the equations in (3.33) are all diagonal matrices, the choice of \mathbf{B} as in (3.32) yields shocks that are orthogonal in both regimes. The model is estimated by the maximum likelihood (ML) method.

A number of other statistical identification procedures for SVAR models have been proposed in the literature recently, and have already been applied to monetary policy (see

e.g. Lanne and Lütkepohl 2014). Rigobon (2003) and Lanne and Lütkepohl (2008) have developed methods based on regimes with different covariance structures. Heteroskedasticity may arise as a result of financial crises, for example. These methods further assume that changes in the covariance occur at fixed points during the sample period. This may be a problematic assumption if no such break points are known to exist.

In contrast, both Lanne et al. (2010) and Lütkepohl and Netšunajev (2014) model the volatility shifts as a Markov regime switching process, in which changes in volatility are endogenously determined.

All of these methods are based on either conditional or unconditional heteroskedasticity. More recently Lanne et al. (2015) have introduced a yet more general approach that encompasses most of the methods previously introduced. Similarly to the method employed in this paper, identification in their approach is based on non-Gaussianity of the error terms but more wide-ranging specifications for the error distribution are allowed.

The choice of the identification method based on mixed normality used in this paper is largely dictated by the data. There is no known break in the sample as required by Rigobon (20013) and Lanne and Lütkepohl (2008). On the other hand, modeling volatility regimes as a Markov switching process as in Lanne et al. (2010) is numerically demanding, especially if short time series are used. Finally, Lanne *et al.* (2015) only discuss a stationary VAR process, the use of which is not feasible given that our data appear cointegrated. Further evidence in support of the specific distributional assumption is presented in Section 3.3.2. Normality is rejected by formal tests and an investigation of the residuals speaks in favor of a mixed normal specification, which can encompass a wide variety of distributions with the characteristics observed in the residuals. The VEC specification is justified by statistical analysis of the data.

3.3 Empirical Analysis of the Fiscal Multiplier in the United States

3.3.1 Data

In the analysis quarterly US data in a four variable VECM $y_t = (G_t, T_t, Y_t, \eta_{13}^g)'$ is used, in which G_t is log real government (federal, state, local) expenditure on consumption and investment, T_t is log real government receipts of direct and indirect taxes net of transfers

to businesses and individuals, and Y_t is log real gross domestic product (GDP) in chained 2009 dollars. The variables are constructed using the Bureau of Economic Analysis' NIPA Tables.⁴ These data have been available since 1947Q1 and were previously used by Auerbach and Gorodnichenko (2012), Mountford and Uhlig (2009) and Caggiano *et al.* (2015), among others.

Fiscal foresight creates problems with structural VAR analysis. If economic agents adjust their behavior based on anticipated future shocks, or news shocks, while standard VARs take into account current and past shocks only, analysis based on these may be misleading. Leeper *et al.* (2013) show that foresight about changes in future variables leads to non-invertible moving average representations. Instead of the standard (causal) VAR representation, the process has a noncausal representation in this case.

Using data for the United States, Lanne and Saikkonen (2009) provide evidence of noncausality in a VAR model with fiscal foresight. This finding invalidates analyses based on conventional causal VARs, as the errors from a standard VAR cannot be used to reveal the true fiscal shocks precisely.

Even if noncausality is detected, methods for such things as impulse response analysis from noncausal VAR models are unfortunately not yet readily available (Lanne & Saikkonen 2013). As the foresight problem arises because the econometrician does not have all the information that economic agents may have, an alternative approach is to solve the inherent missing variable problem by adding variables to the VAR (see Lütkepohl 2014 and the references therein).

To deal with fiscal foresight, we follow Caggiano *et al.* (2015) who apply the expectations revisions, or news variable approach proposed by Gambetti (2012). A news variable η_{1J}^g is constructed from forecast revisions of the growth rate of real government expenditure and added to the VAR. In other words, the VAR is augmented by information about the anticipated fiscal spending shock, which should bring the econometrician's information set closer to that of economic agents. As the forecast revisions used to construct the news variable have been collected by the Survey of Professional Forecasters (SPF) since

⁴Government expenditure is the sum of consumption expenditure and gross investment minus the consumption of fixed capital. Government revenue is computed as the difference between current receipts and government social benefits. The implicit GDP deflator is used to transform nominal series into real terms.

1981Q3, the whole sample is restricted to the 1981Q3-2013Q1 period.⁵

As already pointed out by Caggiano et al. (2015), who are the first to use the fiscal news variable, the relatively short sample is one limitation of the analysis. To avoid potential small sample issues, an alternative would be to use Ramey's military news variable (2011b). There are two reasons why the military spending variable does not constitute a solution in this case. According to Ramey (2011b) and Christiano (2013), the military shock variable is a relevant instrument as long as WWII or the Korean War is included. However, during the two wars, fiscal spending was accompanied by considerable increases in taxes and, especially during the Korean War, the increase in spending was permanent. Therefore, the resulting multiplier is not necessarily applicable to a situation in which government spending is financed differently (Ramey 2011b, Christiano 2013). Caggiano et al. (2015) also point out that rationing was in place during WWII, which restrained public spending from increasing further.

Christiano (2013) and Caggiano et al. (2015) conclude, that all these elements are likely to contaminate the computation of the fiscal multiplier based on Ramey's military spending variable. Moreover, given the limited applicability of Ramey's variable, using it would prevent us from drawing conclusions on the effects of government spending in the current situation, in which fiscal stimulus packages have been financed by debt. We choose to follow Caggiano et al. (2015) because theirs was also the most recent approach to tackling the issue of fiscal foresight.

The cumulated fiscal news variable is constructed by adding up revisions of expectations as follows (Caggiano *et al.* 2015, Gambetti 2012):

$$\eta_{1J}^g = \sum_{j=1}^J (E_t g_{t+j} - E_{t-1} g_{t+j})$$

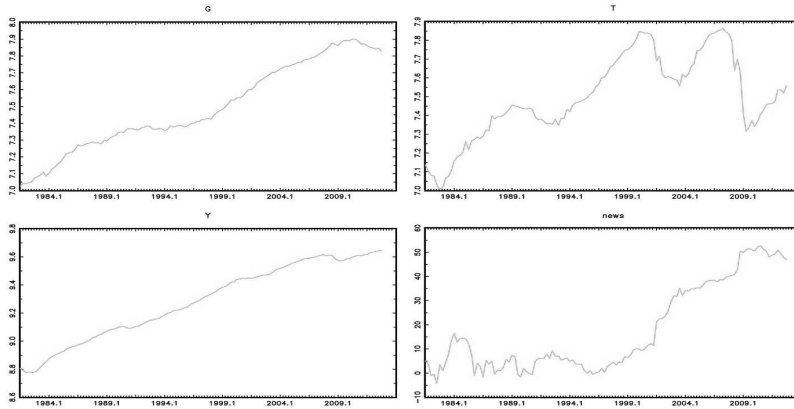
where $E_t g_{t+j}$ is the forecast of the growth rate in real federal government expenditure from period $t + j - 1$ to period $t + j$ based on the information available at time t . Therefore $E_t g_{t+j} - E_{t-1} g_{t+j}$ represents the news that becomes available to private agents between times $t - 1$ and t about the growth rate of government expenditure j periods ahead. As the SPF collects forecasts conditional on time $t - 1$ up to time $t + 3$, to exploit the largest amount of news available, $J = 3$ has been selected (Caggiano *et al.* 2015).

⁵The public expenditure news variable was provided by Giovanni Caggiano. All other variables were constructed by the author.

Caggiano *et al.* (2015) show that residuals typically employed in a standard trivariate VAR are partly predictable by the components of η_{13}^g and cannot be interpreted as fiscal shocks - the authors claiming that the forecast revisions included in the variable η_{13}^g , which they interpret as a measure of anticipated fiscal shocks, can augment the information content of the VAR system. Therefore, by adding the cumulated fiscal news variable in the VAR, one obtains a shock that is not predictable and can be interpreted as a fiscal shock.

3.3.2 Model Setup

Figure 3.6: Plot of logarithmic time series 1981Q3-2012Q4. G = government expenditure, T = government revenue, Y = GDP, news = cumulated fiscal news



The empirical analysis starts with checking the orders of integration of the four time series, which are depicted in Figure 3.6. A trend was included in the augmented Dickey-Fuller (ADF) unit root test for all series and autoregressive lags were chosen according to the Akaike information criterion. The tests show that all the variables included in the analysis are $I(1)$, although T is only at the 5% significance level, not at the 10%.

The next step is to investigate the cointegration rank of the four dimensional VECM for $y_t = (G_t, T_t, Y_t, \eta_{13}^g)'$. This requires determining the number of lagged differences in the system first. Here we use the fact that if a VAR(p) process contains cointegrated variables, the process has a VEC(p-1) representation. In other words the order p is chosen so that no residual autocorrelation is left in the corresponding VAR model. For a reduced form Gaussian VAR, AIC, HQ and BIC select VAR(6), VAR(2) and VAR(1) models, respectively. According to the adjusted portmanteau test there is autocorrelation

Table 3.14: Cointegration tests

Johansen Trace test					
Included lags (levels)	H_0	Test value	Critical values		p -value
			10%	5%	
2	0	90.17	50.50	53.94	0.00
	1	32.31	32.25	35.07	0.099
	2	15.40	17.98	20.16	0.21
	3	2.51	7.60	9.14	0.68
Saikkonen and Lütkepohl test					
Included lags (levels)	H_0	Test value	Critical values		p -value
			10%	5%	
2	0	53.97	37.04	40.07	0.00
	1	17.85	21.76	24.16	0.26
	2	4.09	10.47	12.26	0.70
	3	2.49	2.98	4.13	0.14

left in the VAR(1) model (p -value < 0.001), while a p -value of 0.082 for VAR(2) suggests that a second order model is sufficient.

Table 3.14 reports the results of the Johansen Trace test with an unrestricted constant. The cointegration rank $r = 0$ is rejected at all significance levels, while $r = 1$ clearly cannot be rejected at the 5% level and is barely rejected at the 10 % level.⁶ The Saikkonen and Lütkepohl (2000) cointegration test – also reported in Table 3.14 – provides further support for $r = 1$.⁷

To conclude the initial analysis, diagnostic tests have been performed to assess the suitability of the VEC(1) model with $r = 1$. There appears to be no remaining autocorrelation (adjusted portmanteau test p -value 0.18). There is however evidence of non-normality in the errors, as is evident from the quantile-quantile (QQ) plots of the model residuals, plotted in Figure 3.7.

Normality is also rejected by formal normality tests, of which the Doornik and Hansen test for joint normality yields a p -value of < 0.001 , and the p -values of univariate Jarque-Bera tests are reported in Table 3.15.

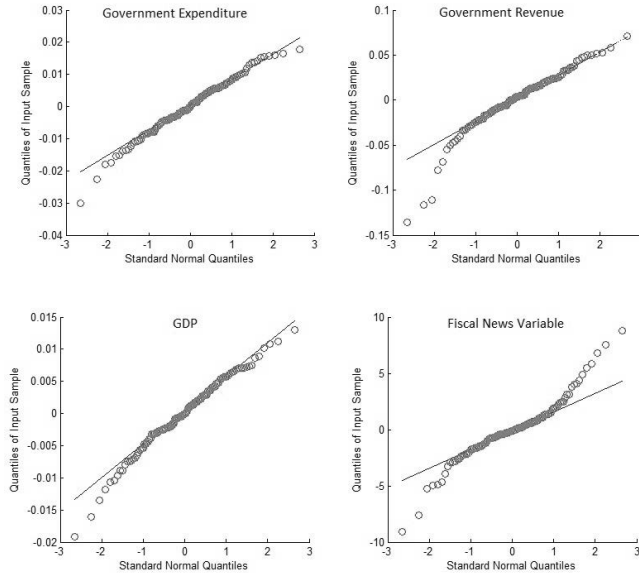
Table 3.15: Summary Statistics

	Skewness	Kurtosis	Jarque-Bera	p -value
Government Expenditure	-0.6092	4.0185	12.7127	0.0017
Government Revenue	-1.1325	5.8725	67.4635	0.0000
GDP	-0.4198	3.6481	5.6724	0.0586
Fiscal News	0.1002	6.0327	46.5728	0.0000

⁶The low power of the test has meant that the rank is often selected according to the 10 % significance level (Brüggemann & Lütkepohl 2005).

⁷As a robustness check, the mixture VECM was estimated with $r = 2$ as well and the test results are qualitatively the same as those reported in Section 3.3.

Figure 3.7: Residuals of VEC(1) -model with cointegration rank $r=1$, QQ plots



The QQ plots illustrate that most discrepancies from a normal distribution occur at the tails. The curved pattern of the QQ plots for government expenditure, government revenue and GDP can arise because of a left skewed data distribution compared to the normal, while the QQ plot of the fiscal news variable shows heavy tails at both ends of the distribution. These observations are confirmed by the figures in Table 3.15. In fact, government expenditure, government revenue and GDP feature negative/left skewness, whereas the fiscal news variable is positively/right skewed. Moreover, the kurtosis shows values greater than 3 for all variables, indicating heavier tails and higher peaks than in a normal distribution.

Heavy tails and skewness are typical features of financial time series such as asset returns. To accommodate these characteristics, mixtures of normal distributions have been used to analyze financial data. According to Tsay (2005), studies of stock returns have started to use a mixed normal distribution because it can capture the skewness and excess kurtosis of the time series. By using a mixture distribution, one can obtain densities with higher peaks and heavier tails than in the normal distribution. Kon (1984), for example used a mixed normal model to explain the observed significant kurtosis and significant positive skewness in the distribution of daily rates of stock returns. Overall, because of their flexibility, mixture models are increasingly exploited to model unknown

distributions (McLachlan & Peel 2000).

In the present VECM setup with mixed normal errors, normal distribution is obtained if $\Sigma_1 = \Sigma_2$ in (3.29). Therefore the normality tests may be seen as a test of $H_0 : \Sigma_1 = \Sigma_2$, the rejection of which supports the assumption that $\Sigma_1 \neq \Sigma_2$, and hence a mixed normal error distribution (Lanne and Lütkepohl 2010).

Given these properties of the data, explicitly modeling the error distribution as a mixed normal distribution is well grounded. The considerable advantage of the specific distributional assumption is that it yields additional databased information, which allows us to identify the model without restrictions. As a result, identification restrictions derived from other sources become over-identifying and their validity can be statistically tested.⁸

3.3.3 Estimation Results and Structural Identification

The estimation of the mixture VEC model proceeds in two steps (Lanne and Lütkepohl 2010). As the cointegration relations are not known beforehand, they are first estimated with the Johansen reduced rank regression, which yields $\beta = (1, -0.447, -0.171, -0.007)$.⁹ In the second step the log-likelihood function is maximized with respect to the other parameters, conditional on the estimated cointegration relations.¹⁰

In the ML estimation, VECM coefficients from a linear model are used as starting values to estimate the parameters of an unrestricted VEC model with a mixed normal distribution. The estimation results of the unrestricted model appear in the left column of Table 3.16.¹¹

The model has been identified if the ψ'_i s are distinct. As shown in Table 3.16, the estimation results are quite precise and the ψ'_i s yield approximate values of 0.11, 0.26, 0.06 and 0.76, while the mixture probability γ is estimated to be 0.24.

Statistical identification delivers orthogonal shocks but their labeling has to be based on outside information (Lanne et al. 2015, Lütkepohl & Netšunajev 2014). One option

⁸We follow previous literature and test normality in a first step and, conditional on rejecting it, test standard SVAR identification schemes in a second step. This is the common procedure in the literature employing statistical identification in SVAR models because a joint test would be nonstandard and probably difficult to perform in practice. We thank the editor for pointing out this potential problem.

⁹The first step computations were performed with JMulTi.

¹⁰These computations were done with GAUSS programs using the CMLMT library. To avoid numerical problems in estimation, the fiscal news variable is scaled to match the magnitude of the other variables.

¹¹The rest of the parameter estimates are reported in the Appendix.

Table 3.16: Estimation results of the VECM with mixture distribution, restricted and unrestricted B matrix. Standard errors in parenthesis.

$\mathbf{y}_t = (G_t, T_t, Y_t, \eta_{13}^g)'$			
Parameter	Unrestricted B	Restricted B	Restricted B and $\Xi\mathbf{B}$
$\hat{\gamma}$	0.239 (0.086)	0.124 (0.025)	0.232 (0.065)
$\hat{\psi}_1$	0.115 (0.042)	0.164 (0.069)	0.097 (0.037)
$\hat{\psi}_2$	0.255 (0.110)	0.062 (0.027)	0.081 (0.030)
$\hat{\psi}_3$	0.061 (0.024)	0.142 (0.058)	0.189 (0.083)
$\hat{\psi}_4$	0.762 (0.307)	0.725 (0.000)	0.748 (0.277)
$\max l_T(\theta)$	1620.56	1605.15	1618.64
LR		30.82	3.84
p -value		2.743×10^{-5}	0.698

is to test the validity of a recursive identification scheme that has been used before. If the previously used restrictions cannot be rejected, the recursive structure provides a straightforward interpretation of the resulting impulse response functions. Statistical testing of a recursive identification scheme is therefore an important part of the economic interpretation of the results.

To this end, another VEC model is estimated in which lower triangularity is imposed on the \mathbf{B} matrix as in Caggiano *et al.* (2015)¹². In estimating the restricted model, the ML estimates of the unrestricted model are used as starting values. In both cases, their determinants are bounded away from zero to ensure nonsingularity of the covariance matrices. The diagonal elements of the Ψ matrix are also bounded away from zero, as required.

The results of the key parameters are reported in the middle column of Table 3.16 together with the outcome of the likelihood ratio test¹³. The LR test has the asymptotic χ^2 -distribution with 6 degrees of freedom given by the number of restrictions. The recursive structure is clearly rejected (p -value < 0.001) and hence is not helpful in labelling the shocks.

The VECM specification allows another option based on long run relations between the variables, as shown in Lütkepohl (2007, Chapter 9). Suppose the cointegration rank is known to be r . Then, as in Section 3.2, there are at most r transitory shocks, ε_t^r and at least $K - r$ permanent shocks, ε_t^p . Arranging them such that $\varepsilon_t' = (\varepsilon_t^{p'}, \varepsilon_t^{r'})$, it follows that $\Xi\mathbf{B} = [\Phi_{K \times (K-r)} : \mathbf{0}_{K \times r}]$ where $\Phi_{K \times (K-r)}$ is an $K \times (K - r)$ matrix. In a VEC model

¹²In the present mixture model this is done in practice by restricting the \mathbf{W} matrix in $\mathbf{B} = \mathbf{W}(\gamma\mathbf{I}_n + (1 - \gamma)\Psi)^{1/2}$ to be lower triangular.

¹³The rest of the results are reported in the Appendix.

with $r < K$, all shocks can in principle be permanent shocks and $\Xi\mathbf{B}$ may not have zero columns even if it has reduced rank.

In Section 3.3.2., $r = 1$ was found for the data to hand. This translates into the following set of long run restrictions¹⁴

$$\Xi\mathbf{B} = \begin{bmatrix} * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \\ * & * & * & 0 \end{bmatrix} \quad (3.34)$$

which can also be tested using a LR-test.¹⁵ Therefore another restricted VEC model with mixed normal errors is estimated. The following matrix of impact effects is assumed

$$\mathbf{B} = \begin{bmatrix} * & 0 & 0 & * \\ * & * & 0 & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \quad (3.35)$$

in addition to the long run restriction in (3.34). In other words, the often used recursive structure for the key variables $\mathbf{y}_t = (G_t, T_t, Y_t)$ is imposed as well. This implies that government expenditure does not respond contemporaneously to shocks to other variables, while government revenue does not react contemporaneously to output shocks.

Note that the restrictions imposed here (3.34 and 3.35) differ from those required for identification in a standard VECM framework (see e.g. Lütkepohl 2007, Chapter 9). Because the matrix $\Xi\mathbf{B}$ has a reduced rank of $K - r$, each column of zeros stands for $K - r$ independent restrictions only. In other words the r transitory shocks represent $r(K - r)$ independent restrictions, i.e. 3 in the present case. As just-identification in the standard VECM requires a total of $\frac{K(K-1)}{2}$ restrictions, additional restrictions based on theoretical considerations are needed. To identify both transitory and permanent shocks, it is not sufficient to impose arbitrary restrictions on \mathbf{B} and $\Xi\mathbf{B}$, however. The advantage of the VEC specification in the standard setting is that the $r(K - r)$ restrictions are based on the cointegration rank, which can be determined by statistical tests.

In the current framework, assuming that structural shocks are in fact identified by the mixed normality of errors, any restrictions become over-identifying and can be statistically tested. Testing the exclusion restrictions in (3.35) is of interest because they are commonly

¹⁴Again the asterisks denote unrestricted elements and zeros indicate the elements that are restricted to be zero.

¹⁵The assumption that government spending, revenue and output shocks have permanent effects is in line with the literature using the same data (see e.g. Mountford & Uhlig 2009, and the results from both linear and nonlinear models by Auerbach & Gorodnichenko 2012).

assumed to obtain just-identification with standard three variable VARs. Obviously, the three restrictions in (3.35) alone are not enough to identify a four variable VAR.

The estimation results of the second restricted VEC model appear in the right column of Table 3.16.¹⁶ The p -value of the LR test (0.698) based on the $\chi^2(6)$ -distribution indicates that restrictions (3.34) and (3.35) are well supported by the data (see the right column of Table 3.16).

Finally, an examination of the standard errors suggests that some of the ψ'_i s may not be distinct. This means that the \mathbf{B} matrix may not be unique. The nonuniqueness of \mathbf{B} may imply that the actual number of degrees of freedom of the χ^2 -distribution in the LR-test is less than 6 (see Lütkepohl and Velinov 2014). Given the rejection of the first restricted model at 6 degrees of freedom, the same test statistic leads to rejection with fewer degrees of freedom as well. Therefore, even though the \mathbf{B} matrix may not be unique, by assuming mixed normality of the errors, the restrictions imposed are sufficient to reject the recursive identification scheme. On the other hand, given the small value of the LR test statistic related to the second restricted model, even with less than 6 degrees of freedom there is still no strong evidence against the restrictions imposed.

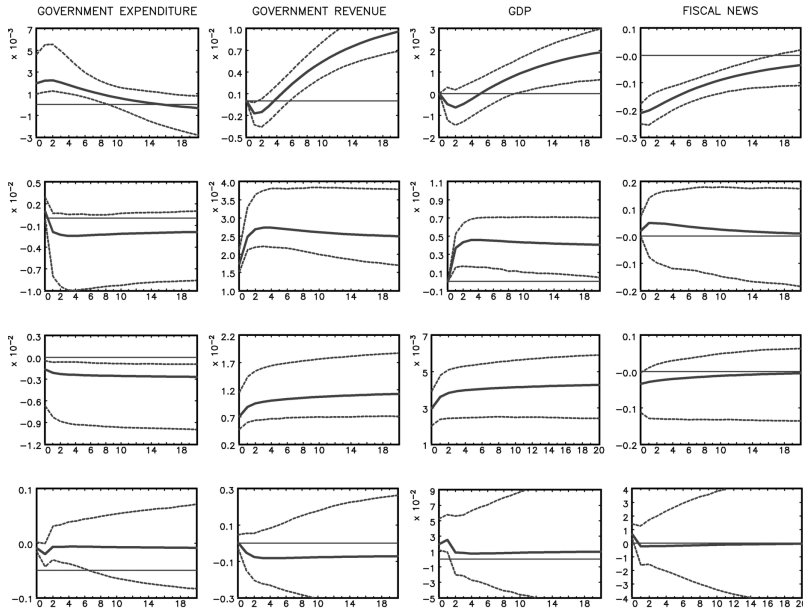
3.3.4 Impulse Response Analysis

Given the previous results, and assuming that the ψ_i 's are distinct and the model is in fact fully identified, we compute impulse responses based on the restricted mixture VEC model, which imposes both contemporaneous and long-run restrictions (3.34 and 3.35) not rejected by the data. We report the 90% Hall's percentile confidence bands, which are obtained from 1000 replications of bootstrap impulse responses. Following Herwartz and Lütkepohl (2014), to ensure that only bootstrap replications around the parameter space of the original estimation step are considered, bootstrap parameter estimates of \mathbf{c} , \mathbf{W} , $\boldsymbol{\alpha}$ and $\boldsymbol{\Gamma}_1$ are determined conditionally on the initially estimated $\boldsymbol{\Psi}$ and γ . Bootstrap estimates are obtained by nonlinear optimization of the log-likelihood with ML estimates as starting values.

The impulse responses are shown in Figure 3.8. Each column contains the responses of all variables to one shock, the size of each shock being set to unity. In this case the long and short run restrictions provide interpretation. As the impulse responses are

¹⁶The rest of the results are reported in the Appendix.

Figure 3.8: Impulse response functions with 90% Hall's percentile confidence bands of the restricted VEC model with mixed normal errors.



computed by restricting the impact effects as in (3.35), the following contemporaneous effects are ruled out: a government revenue shock has no contemporaneous impact on government expenditure (G_t), and an output shock cannot have a contemporaneous effect on government expenditure (G_t) and revenue (T_t). From the long run restriction (3.34) we also know that the effect of the last shock – fiscal news (η_{13}^g) – is transitory. These permit us to uniquely label the shocks as a government spending, government revenue, output and fiscal news shock. In other words they appear in the same order as the variables in the vector $y_t = (G_t, T_t, Y_t, \eta_{13}^g)'$.

The first column of Figure 3.8 depicts impulse responses to a positive government spending shock. Interestingly, the response of output is negative although very weak, while the response of taxes is not statistically different from zero, even if no restrictions on taxes are imposed. As government revenue does not change, this can be interpreted as a fiscal policy shock financed by a deficit as in Mountford and Uhlig (2009). From a practical point of view, this is of great interest since fiscal stimulus packages are mostly financed by deficits.

The second column reports impulse responses to a positive government revenue shock. The impact response of government expenditure is restricted to zero, but it becomes

positive and significant after 6 quarters, and so follows the shape of GDP. In other words, surprisingly, a government revenue shock is found to trigger a positive response in both government expenditure and GDP. In line with the interpretation of the spending shock, one could interpret government spending financed by a tax increase as having a positive impact on GDP. In the literature a positive tax shock is typically found to have a negative effect on output (see e.g. Ramey 2011a, Favero & Giavazzi 2012, Mountford & Uhlig 2009, Auerbach & Gorodnichenko 2012). For example, in the linear framework of Auerbach and Gorodnichenko (2012), output responds negatively and government spending positively to a positive tax shock, while Mountford and Uhlig (2009) report a negative impact on both output and spending. According to the latter authors their finding is also intuitive. To investigate what happens to the response of government spending to a tax shock at longer horizons, we computed impulse responses for such horizons as well. The response of government spending does not seem to stabilize.

The third column displays impulse responses to a positive output shock. Although the impact response of government revenue is restricted to zero here, the output shock behaves like a business cycle shock in Mountford and Uhlig (2009) in that both output and government revenue increase, whereas the response of government expenditure is not countercyclical, also increasing although with a lag. The reason given by Mountford and Uhlig (2009) also applies here, namely the government expenditure variable is defined as consumption plus investment but does not include transfer payments, which automatically vary counter cyclically.

Finally, the last column shows impulse responses to a positive fiscal news shock, which Caggiano *et al.* (2015) interpret as an anticipated fiscal expenditure shock. The shapes of the impulse responses are similar to Caggiano *et al.*'s (2015) but there are differences in the impact effects. This is not unexpected given their identification strategy, which imposes zero impact effects of the fiscal news shock on all variables. When the responses to the fiscal news shock are left unrestricted in the mixture VEC model, we see that the impact responses of government expenditure and output are negative but increasing, while the government revenue reacts positively at first and then starts to decrease. The response of the news shock itself is very short-lived. Of these, the responses of government revenue and output are insignificant, however. Apparently, the breadth of the confidence bands reflects the fact there is great uncertainty around the behavior of the cumulated

fiscal news variable, which is constructed by adding up forecast revisions of the growth rate of real government expenditure.

Fiscal multipliers are computed according to the usual practice in the literature (see e.g. Auerbach & Gorodnichenko 2012, Caggiano et al. 2015, Ramey and Zubairy 2013, Mountford & Uhlig 2009). Specifically, impact multipliers are calculated as the response of output at a given horizon divided by the initial fiscal shock. As the log of variables is used in estimation, we scale the impulse response functions by the sample average of the output to government spending ratio, Y/G (taken in levels) to convert percentage changes into dollar changes. Impact multipliers are computed for the $h = 1, 4, 8, 12, 20$ horizons. The multiplier ranges from -1.27 to -1.61 and achieves its maximum at $h = 1$. As already pointed out, these results rest on the assumption that the mixture VEC model is fully identified.

A comparison with previous empirical studies reveals that the effects of fiscal policy obtained from SVARs are typically of the opposite sign, in accordance with theoretical models in the Keynesian tradition (e.g. Blanchard & Perotti 2002, Ramey 2011b, Favero & Giavazzi 2012). There is however a lot of variation in the size of the multiplier, both within and across studies (see Ramey 2011a and references therein).

Similarities also exist. Perotti (2005) finds evidence of large differences in the effects of fiscal policy in the pre- and post-1980 periods. His results for the whole US sample (1960Q1-2001Q4) are similar to those obtained by others using the same sample, whereas a negative spending multiplier emerges for the post-1980 period. He concludes that there has been a drastic reduction in the effects of government spending shocks on GDP since 1980. His results are therefore in line with the ones obtained in this paper, which also considers the post-1980 period.

Mountford and Uhlig (2009) analyze a government spending shock financed by a deficit by not allowing taxes to change for 4 quarters. They find that deficit spending only stimulates the economy weakly on impact and has a negative effect on output in the long run. Their basic government spending shock resembles the deficit spending shock in that although no restrictions on government revenue are imposed, it does not change significantly. Since the same result is obtained here, we interpret our government spending shock as deficit financed.

Negative fiscal multipliers also emerge in studies using nonlinear model specifications,

for example during periods of high public debt (Ilzetzki *et al.* 2013), and during expansions in the post-1980 period (Auerbach & Gorodnichenko 2012).

3.4 Conclusions

In the fiscal policy literature using structural vector autoregressions (SVARs), fiscal policy shocks are identified in several ways. Fiscal multipliers, i.e. estimates of the impact of fiscal stimulus on output, are then defined either as the peak of the impulse response or as an accumulated response. As is well known, the VAR identification strategy matters for the impulse responses, and hence may be one reason for the differing results.

Moreover, as the usual practice in the literature is to use the log of variables, the estimated elasticities are converted to dollar equivalents with an ex post conversion factor, a practice that has also been criticized (Ramey & Zubairy 2014). Using log levels of variables such as real GDP, government revenue and expenditure also introduces another potential source of uncertainty in the analysis, namely nonstationarity. Phillips (1998) demonstrates that impulse responses are not consistently estimated in the SVARs with variables in levels in the case of unit roots, whereas the vector error correction (VEC) specification significantly improves them even for short horizons. Phillips (1998) found that differential treatment of nonstationarity in various models has substantial effects on policy analysis.

This paper contributes to the existing fiscal policy literature in two ways. First, unlike any of the studies using VARs – linear and non-linear – the vector error correction (VEC) model used in this paper takes into account the cointegration properties of the variables as well. Second, statistical properties of the data are exploited to identify the model, and to test the validity of two popular identification strategies in the fiscal VAR literature.

As proposed by Lanne and Lütkepohl (2010), the non-normality found in the VAR residuals is explicitly modelled, which yields additional data based information. In the Lanne and Lütkepohl (2010) method a mixed normal error distribution is used because of its suitability for the features often found in the residuals. Any restrictions from other sources used for identification then become over-identifying and can be statistically tested.

The test results indicate that the commonly used recursive structure for all four variables is too restrictive from a statistical point of view. However, a long run restriction together with a recursive structure for the key variables government expenditure (G_t)

government revenue (T_t) and GDP (Y_t) is not rejected by the data. As Caggiano *et al.* (2015) point out, ordering the fiscal news variable last in a recursive model may be seen as inconsistent with expectational effects.

In the next step, fiscal policy shocks are analyzed using a model with restrictions not rejected by statistical tests. The resulting impulse responses are quite different from those typically obtained from SVAR models. The latter mostly coincide with theoretical models in the Keynesian tradition. According to our results, government spending shock has a weak but negative effect on GDP, while the response of taxes is not statistically different from zero even if no restrictions are imposed on taxes. As government revenue does not change, this can be interpreted as a fiscal policy shock financed by a deficit as in Mountford and Uhlig (2009). Also quite surprisingly, a government revenue shock triggers a positive response in both government expenditure and GDP. In line with the interpretation of the spending shock, this can be interpreted as a tax raise to finance government spending, which has a positive impact on GDP. The fiscal multiplier for horizons $h = 1, 4, 8, 12, 20$ after the initial shock ranges from -1.27 to -1.61 and achieves its maximum at $h = 1$.

References

- Auerbach A.J., and Gorodnichenko Y. (2012). Measuring Output Responses to Fiscal Policy. *American Economic Journal: Economic Policy* **4**(2): 1-27.
- Blanchard O., and Perotti R. (2002). An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. *Quarterly Journal of Economics* **117**(4):1329-68.
- Brüggemann, R. and Lütkepohl, H. (2005). Uncovered Interest Rate Parity and the Expectations Hypothesis of the Term Structure: Empirical Results for the US and Europe. *Applied Economics Quarterly* **51**, 143-154.
- Caggiano, G., Castelnuovo, E., Colombo, V., and Nodari, G. (2015). Estimating Fiscal Multipliers: News From a Nonlinear World!. *The Economic Journal* **125**(584): 746-776.
- Caldara D., and Kamps C. (2008). What Are the Effects of Fiscal Policy Shocks? A VAR-based Comparative Analysis. *Working Paper Series 877. European Central Bank.*

- Christiano L.J. (2013). Comment on 'The Household Effects of Government Spending'. In Alesina A., Giavazzi F. (Eds.): *Fiscal Policy After the Financial Crisis*, NBER, University of Chicago Press, 141-149.
- Favero, C., and Giavazzi, F. (2012). Measuring tax multipliers: The narrative method in fiscal VARs. *American Economic Journal: Economic Policy* **4**(2): 69-94
- Gambetti L. (2012). Fiscal Foresight, Forecast Revisions and the Effects of Government Spending in the Open Economy. Universitat Autònoma de Barcelona, mimeo.
- Herwartz H., and Lütkepohl H. (2014). Structural Vector Autoregressions with Markov Switching: Combining Conventional with Statistical Identification of Shocks. *Journal of Econometrics* **183**: 104-116.
- Ilzetzki E., Mendoza E.G., and Végh C.A. (2013). How Big (Small?) Are Fiscal Multipliers? *Journal of Monetary Economics* **60**: 239-254.
- Kon S.J. (1984). Models of Stock Returns - a comparison. *The Journal of Finance* **39**(1): 147-165.
- Lanne M., and Lütkepohl H. (2008). Identifying Monetary Policy Shocks via Changes in Volatility. *Journal of Money, Credit and Banking* **40**: 1131-1149.
- Lanne M., and Lütkepohl H. (2010). Structural Vector Autoregressions With non-normal Residuals. *Journal of Business & Economic Statistics* **28**(1):159-168.
- Lanne, M. and Lütkepohl, H. (2014). *A Statistical Comparison of Alternative Identification Schemes for Monetary Policy Shocks*. In: Knif, J., Pape, B. (Eds.), *Contributions to Mathematics, Statistics, Econometrics, and Finance: Essays in Honour of Professor Seppo Pynnönen*. University of Vaasa, Vaasa.
- Lanne M., Lütkepohl H., and Maciejowska K. (2010) Structural Vector autoregressions with Markov Switching. *Journal of Economic Dynamics and Control* **34**: 121-131.
- Lanne M., Meitz M., and Saikkonen P. (2015). Identification and Estimation of Non-Gaussian Structural Vector Autoregressions. School of Economics and Management, University of Aarhus. *CREATES Research Paper No. 2015-16*.

- Lanne M., and Saikkonen P. (2009). Noncausal Vector Autoregression. *Research Discussion Papers 18. Bank of Finland*.
- Lanne M., and Saikkonen P. (2013). Noncausal Vector Autoregression. *Econometric Theory* **29**:447-481.
- Leeper E.M., Walker T.B., and Yang S.-C. S. (2013). Fiscal Foresight and Information Flows. *Econometrica* **81**(3): 1115-1145.
- Lütkepohl H. (2007). *New Introduction to Multiple Time Series Analysis*, Springer-Verlag, Berlin.
- Lütkepohl H. (2014). Fundamental Problems with Nonfundamental Shocks. In: Haldrup N., Meitz M. and Saikkonen P. (Eds.), *Essays in Nonlinear Time Series Econometrics*, Oxford University Press, pp. 198-214.
- Lütkepohl H. and Netšunajev A. (2014). Disentangling Demand and Supply Shocks in the Crude Oil Market: How to Check Sign Restrictions in Structural VARs. *Journal of Applied Econometrics*. **29**(3):479-496.
- Lütkepohl H. and Velinov A. (2014). Structural Vector Autoregressions: Checking Identifying Long-run Restrictions via Heteroskedasticity. *Journal of Economic Surveys* 2014.
- McLachlan G., and Peel D. (2000). *Finite Mixture Models*. John Wiley & Sons, NY.
- Mountford A., and Uhlig H. (2009). What Are the Effects of Fiscal Policy Shocks? *Journal of Applied Econometrics* **24**: 960-992.
- Owyang M. T., Ramey V. A. and Zubairy S. (2013). Are government spending multipliers greater during periods of slack? Evidence from 20th century historical data. *The American Economic Review* **103**(3): 129-134.
- Perotti R. (2005). Estimating the Effects of Fiscal Policy in OECD Countries. *CEPR Discussion Paper 4842*. C.P.E.R. Discussion Papers.
- Phillips P.C.B (1998). Impulse Response and Forecast Error Variance Asymptotics in Nonstationary VARs. *Journal of Econometrics* **83**: 21-56.

- Puonti, P. (2016). Fiscal Multipliers in a Structural VEC Model with Mixed Normal Errors. *Journal of Macroeconomics* **48**: 144-154.
- Ramey V.A. (2011a). Can Government Purchases Stimulate the Economy? *Journal of Economic Literature* **49**(3): 673-685.
- Ramey V.A. (2011b). Identifying Government Spending Shocks: It's All in the Timing. *Quarterly Journal of Economics* **126**(1):1-50.
- Ramey V.A., and Shapiro M.D. (1998). Costly Capital Reallocation and the Effects of Government Spending. *Carnegie-Rochester Conference on Public Policy* **48**: 145-94.
- Ramey V. A. and Zubairy S. (2014). Government spending multipliers in good times and in bad: evidence from US historical data. *NBER Working Paper 20719*. National Bureau of Economic Research, Inc.
- Rigobon, R. (2003). Identification through heteroskedasticity. *Review of Economics and Statistics* **85**: 777-792.
- Saikkonen P. and Lütkepohl H. (2000). Testing for the Cointegrating Rank of a VAR process with an intercept. *Econometric Theory* **16**:373-406.
- Sims C. (1980). Macroeconomics and Reality. *Econometrica* **48**(1):1-48.
- Tsay R.S. (2005). *Analysis of Financial Time Series*. John Wiley & Sons, Inc., Hoboken, New Jersey.
- Uhlig H. (2005). What Are the Effects of Monetary Policy? Results from an Agnostic Identification Procedure. *Journal of Monetary Economics* **52**:381-419.

Appendix: Additional Results

Table 3.17: Estimated parameters of the unrestricted VEC model with mixed normal errors. Standard errors in parenthesis.

	Elements of each vector			
	1	2	3	4
constant $\times 100$	19.29 (3.27)	-8.72 (9.55)	2.99 (2.10)	-18.41 (91.92)
$\alpha \times 100$	-7.64 (1.34)	3.72 (3.91)	-0.97 (0.86)	9.01 (38.58)
$\mathbf{\Gamma}_1[1,\cdot] \times 100$	5.72 (7.85)	-6.54 (2.50)	-20.05 (16.25)	0.10 (0.30)
$\mathbf{\Gamma}_1[2,\cdot] \times 100$	-32.92 (23.95)	-5.45 (10.43)	119.38 (52.67)	0.42 (0.80)
$\mathbf{\Gamma}_1[3,\cdot] \times 100$	-4.23 (5.11)	0.93 (1.74)	19.86 (11.32)	0.01 (0.19)
$\mathbf{\Gamma}_1[4,\cdot] \times 100$	25.17 (183.80)	-36.74 (74.59)	29.43 (66.47)	-17.13 (6.60)
$\mathbf{W}[1,\cdot] \times 100$	0.07 (0.26)	-1.75 (1.61)	0.11 (0.18)	-18.17 (9.45)
$\mathbf{W}[2,\cdot] \times 100$	0.40 (0.20)	-0.99 (0.63)	-0.43 (0.11)	-3.44 (6.15)
$\mathbf{W}[3,\cdot] \times 100$	0.28 (0.15)	3.00 (1.13)	0.20 (0.15)	-15.53 (12.05)
$\mathbf{W}[4,\cdot] \times 100$	0.72 (0.11)	0.03 (0.40)	0.33 (0.10)	7.76 (2.53)

Notes: $\mathbf{\Gamma}[i,\cdot]$ and $\mathbf{W}[i,\cdot]$ indicate the i th row of matrices $\mathbf{\Gamma}$ and \mathbf{W} , respectively. The parameter estimates are multiplied by 100 for reporting purposes.

Table 3.18: Estimated parameters of the VEC model with mixed normal errors, B restricted to lower triangular. Standard errors in parenthesis.

	Elements of each vector			
	1	2	3	4
constant $\times 100$	20.80 (3.02)	-7.19 (8.70)	3.70 (1.89)	-18.70 (93.46)
$\alpha \times 100$	-8.26 (1.23)	3.09 (3.55)	-1.27 (0.77)	9.24 (38.14)
$\mathbf{\Gamma}_1[1,\cdot] \times 100$	6.92 (7.81)	-6.59 (2.70)	-18.67 (14.36)	0.12 (0.29)
$\mathbf{\Gamma}_1[2,\cdot] \times 100$	-32.32 (22.07)	-7.07 (7.51)	121.20 (42.42)	0.23 (0.80)
$\mathbf{\Gamma}_1[3,\cdot] \times 100$	-4.64 (4.85)	1.46 (1.54)	20.25 (8.81)	0.06 (0.18)
$\mathbf{\Gamma}_1[4,\cdot] \times 100$	25.17 (173.78)	-37.56 (66.24)	29.33 (399.88)	-14.87 (8.55)
$\mathbf{W}[1,\cdot] \times 100$	0.91 (0.10)	.	.	.
$\mathbf{W}[2,\cdot] \times 100$	-0.12 (0.11)	3.23 (0.50)	.	.
$\mathbf{W}[3,\cdot] \times 100$	0.32 (0.10)	1.16 (0.29)	0.62 (0.07)	.
$\mathbf{W}[4,\cdot] \times 100$	0.05 (0.08)	0.02 (0.23)	0.01 (0.05)	24.91 (1.42)

Notes: $\mathbf{\Gamma}[i,\cdot]$ and $\mathbf{W}[i,\cdot]$ indicate the i th row of matrices $\mathbf{\Gamma}$ and \mathbf{W} , respectively. The parameter estimates are multiplied by 100 for reporting purposes.

Table 3.19: Estimated parameters of the VEC model with mixed normal errors, contemporaneous and long run restrictions. Standard errors in parenthesis.

	Elements of each vector			
	1	2	3	4
constant $\times 100$	19.35 (3.19)	-1.44 (2.43)	3.58 (1.62)	-1.84 (3.66)
$\alpha \times 100$	-7.67 (1.30)	0.72 (0.99)	-1.21 (0.66)	2.38 (0.86)
$\mathbf{\Gamma}_1[1,\cdot] \times 100$	5.78 (7.80)	-6.51 (2.47)	-17.98 (14.87)	0.09 (0.30)
$\mathbf{\Gamma}_1[2,\cdot] \times 100$	-34.26 (23.33)	-7.62 (8.68)	123.55 (44.14)	0.26 (0.80)
$\mathbf{\Gamma}_1[3,\cdot] \times 100$	-4.37 (5.07)	0.70 (1.66)	20.82 (10.24)	-0.02 (0.19)
$\mathbf{\Gamma}_1[4,\cdot] \times 100$	25.64 (184.05)	-41.77 (66.57)	29.54 (439.39)	-17.49 (6.54)
$\mathbf{W}[1,\cdot] \times 100$	0.35 (0.12)	.	.	-23.57 (2.86)
$\mathbf{W}[2,\cdot] \times 100$	0.16 (0.14)	3.22 (0.42)	.	2.23 (2.99)
$\mathbf{W}[3,\cdot] \times 100$	-0.30 (0.17)	1.31 (0.37)	0.49 (0.08)	-3.73 (1.83)
$\mathbf{W}[4,\cdot] \times 100$	0.73 (0.08)	0.04 (0.34)	0.34 (0.08)	7.30 (2.19)

Notes: $\mathbf{\Gamma}[i,\cdot]$ and $\mathbf{W}[i,\cdot]$ indicate the i th row of matrices $\mathbf{\Gamma}$ and \mathbf{W} , respectively. The parameter estimates are multiplied by 100 for reporting purposes.

Chapter 4

Data-Driven Structural BVAR Analysis of Unconventional Monetary Policy

Abstract¹

This paper applies a novel Bayesian structural vector autoregressive method to analyze the macroeconomic effects of unconventional monetary policy in Japan, the US and the euro area. The method exploits statistical properties of the data to uniquely identify the model without restrictions, and enables to formally assess the plausibility of given sign restrictions. Unlike previous research, the data-based analysis reveals differences in the output and price effects of the Bank of Japan's, Federal Reserve's and European Central Bank's balance sheet operations.

¹This chapter is based on *HECER Discussion Paper No. 406 (2016)*.

4.1 Introduction

Many central banks undertook unconventional monetary policy (UMP) measures in the aftermath of the 2007-09 financial crisis to restore the normal functioning of the monetary transmission mechanism when the policy rates reached the zero lower bound of interest rates (ZLB), or to provide further stimulus to the economy. Each central bank adopted measures deemed most suitable to the circumstances of its currency area (See Fawley and Neely (2013) and Ugai (2007) for reviews). This means that country-specific results can be thought to reflect the effectiveness of various measures (Gambacorta et al. 2014) but also that the experience of Japan, which has the longest history of UMP at the ZLB, cannot necessarily be generalized to other countries.

While conventional monetary policy targets low and stable inflation with a short-term interest rate as an instrument, UMP commonly consists of massive expansion of central banks' balance sheets and/or aims to influence longer term interest rates. In addition to the adoption of new monetary policy tools, utilizing standard tools more frequently, intensely or for non-standard purposes can be classified as UMP. In this paper UMP refers to the use of the central bank's balance sheet as a monetary policy instrument, also called 'balance sheet policies' by Borio and Disyatat (2010).²

Although there is some empirical evidence that unconventional measures have been effective in influencing financial and macroeconomic variables (Cecioni et al. 2011), there is still considerable uncertainty around the quantification of those effects (Joyce et al. 2012). The relatively limited literature analyzing the macroeconomic effects of central banks' balance sheet policies mostly uses structural vector autoregressions.³ In the few

²This deliberate choice thus rules out those central bank's operations that leave the size of its balance sheet unaffected, for example the Federal Reserve's (Fed) maturity extension program known as 'Operation Twist', and the central bank's use of communication about future policy decisions. However the choice is not necessarily restrictive. According to Cecioni et al. (2011), the communication of future interest rates belongs to the toolkit of some central banks even in normal times so that it is not clear whether communication can be regarded as an unconventional monetary policy measure at all.

³The biggest strand of empirical UMP literature consists of event studies based on policy announcements. The limitation of the event-study literature is the narrow focus on high-frequency financial data. Event studies assume an immediate response of the variables of interest although the exact timing and duration of a policy intervention cannot be known (Martin et al. 2012), while macroeconomic variables such as output and inflation generally respond with a lag. Therefore this line of research is not appropriate to analyze macroeconomic effects (Joyce et al. 2012) and mostly concerns UMP's impact on the

studies (Meinusch and Tillmann 2016, Weale and Wieladek 2016, Boeckx et al. 2016, Gambacorta et al. 2014, Schenkelberg and Watzka 2013) focusing on the macroeconomic effects over a sample period during which central banks actually targeted macroeconomic conditions, no major differences between the countries arise. Specifically, an expansionary UMP shock is found to lead to a delayed significant temporary rise in output and prices in all countries, and the results are robust to alternative variables.

Structural vector autoregressions (SVARs) identified by sign restrictions are common in the literature analyzing conventional or unconventional monetary policy. In the UMP literature, sign restrictions are often combined with short-run zero restrictions in order to reduce the set of admissible impulse responses and hence to sharpen identification. In some cases the additional zero restrictions are also needed to disentangle the UMP-shock from the business cycle or financial shocks (e.g. Gambacorta et al. 2014, Schenkelberg and Watzka 2013). As the theoretical foundations of UMP are not well established, both the signs and their restriction horizons are inevitably arbitrary. Obviously, if we are interested in the macroeconomic effects of certain policy, it is particularly desirable to leave the responses of macrovariables unrestricted.

To the best of our knowledge, the so-called statistical identification methods have not yet been employed in the UMP literature.⁴ These methods facilitate statistical testing of exactly identifying short-run or long-run restrictions in SVAR models (see e.g. Lanne et al. 2017), whereas methods to assess the plausibility of sign restrictions have been either informal or difficult to generalize (see Lanne and Luoto 2016, and the references therein). In this paper we employ the method recently put forth by Lanne and Luoto (2016) that exploits the statistical properties of the data to uniquely identify a SVAR model and enables the evaluation of the plausibility of sign restrictions by their probabilities of being compatible with the data. This is helpful in either labeling the statistically identified shocks, which do not carry any economic meaning as such, or in concluding that the sign restrictions imposed in the previous literature are not supported.

Apart from being able to assess the plausibility of sign restrictions, our approach has a number of additional benefits compared to the conventional approach to sign restrictions.

financial market (Meinusch and Tillmann 2016).

⁴As examples of using statistical information to identify conventional monetary policy shocks, see Bacchiocchi et al. (forthcoming), Lanne, Meitz and Saikkonen (2016), Lanne and Lütkepohl (2014), Normandin and Phaneuf (2004).

First, it should yield more accurate impulse response functions. This follows from the fact that our impulse response analysis relies only on economic shocks that are found to plausibly satisfy the given restrictions. Second, since our model is uniquely identified, the uncertainty surrounding the impulse responses of sign and other set identified models – the so-called model identification problem (see e.g. Fry and Pagan 2011) – disappears and reporting the results of impulse response analysis is straightforward. Furthermore, a genuinely uninformative prior can be used, allowing us to learn about the impulse responses from the data.

We find statistical support for the sign restrictions used in a number of previous studies in all three currency areas. This allows us to interpret the statistically identified shocks and impulse responses along the lines of our reference studies (Schenkelberg and Watzka 2013, Gambacorta et al. 2014, Boeckx et al. 2016). However, our impulse responses of these shocks differ in interesting ways from those reported in these studies.

Importantly, unlike previous research, our analysis reveals differences in the macroeconomic impact of the three central banks' actions. Our unrestricted impulse response functions indicate that a UMP shock did not have a statistically significant impact on the consumer price index (CPI) in Japan, while there is weak evidence of a lagged, positive impact on prices in the US and in the euro area, depending on the specification. Our results also point to an immediate positive output response in the euro area, to a more delayed and persistent impact in the US than previously found, and that the positive output effect in Japan was unlikely due to lower long-term interest rates. The differences in the effectiveness of the balance sheet operations can be explained by the differences in the unconventional measures adopted by the three central banks.

The rest of the paper is organized as follows. Technical details of the econometric method are given in Section 4.2. Section 4.3 covers the empirical analysis and Section 4.4 concludes the paper.

4.2 Methodology

Structural vector autoregressions (SVARs) are a common tool to analyze conventional monetary policy. Lanne et al. (2017) have shown that the SVAR model can be uniquely identified by statistical properties of the data. However, their model is only statistically, as opposed to economically, identified, and additional information is needed to give the

shocks an economic interpretation. This information may come in the form of short-run on long-run restrictions that can also easily be tested in the framework of Lanne et al. (2017), and if not rejected, used for interpretation. However, as discussed in the Introduction, in the UMP literature, identifying restrictions are typically sign restrictions that are not approached in a straightforward manner by classical methods, and to that end, we employ the Bayesian procedures recently devised by Lanne and Luoto (2016). In particular, they show how to assess the plausibility of a set of sign restrictions by their posterior probability, and we apply their approach to check the sign restrictions used in a number of previous empirical UMP studies.

Our empirical results are based on the following n -variate SVAR(p) model

$$\mathbf{y}_t = \mathbf{a} + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{B} \boldsymbol{\varepsilon}_t, \quad (4.36)$$

where \mathbf{y}_t is an $(n \times 1)$ vector of time series of interest, \mathbf{a} ($n \times 1$) is an intercept term, $\mathbf{A}_1, \dots, \mathbf{A}_p$ are $(n \times n)$ coefficient matrices and the $(n \times n)$ impact matrix \mathbf{B} , containing the contemporaneous relations of the structural errors $\boldsymbol{\varepsilon}_t$, is assumed nonsingular. The $(n \times 1)$ error term $\boldsymbol{\varepsilon}_t$ is a sequence of stationary random vectors such that each component $\varepsilon_{it}, i = 1, \dots, n$ is independent in time with zero mean and finite positive variance. It is also assumed that the components ε_{it} are mutually independent, and at most one of them has a Gaussian marginal distribution.

Lanne et al. (2017) show that under the non-Gaussianity and independence assumptions of the structural error term $\boldsymbol{\varepsilon}_t$, the matrix \mathbf{B} is uniquely identified up to permutation and scaling of its columns. Changing the order of the columns of \mathbf{B} means a different ordering of the structural shocks ε_{it} .

If the process \mathbf{y}_t satisfies the stability condition

$$\det(\mathbf{I}_n - \mathbf{A}_1 z - \cdots - \mathbf{A}_p z^p) \neq 0, |z| \leq 1 (z \in \mathbb{C}),$$

then the SVAR(p) model (4.36) has a moving average representation

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j \mathbf{B} \boldsymbol{\varepsilon}_{t-j}, \quad (4.37)$$

where $\boldsymbol{\mu}$ is the unconditional expectation of \mathbf{y}_t , $\boldsymbol{\Psi}_0$ is the identity matrix and $\boldsymbol{\Psi}_j, j = 1, 2, \dots$ are obtained recursively as $\boldsymbol{\Psi}_j = \sum_{l=1}^j \boldsymbol{\Psi}_{j-l} \mathbf{A}_l$. Interest then lies in the matrices

$\Psi_j \mathbf{B} \equiv \Theta_j, j = 0, 1, \dots$, the k th column of which contains the impulse responses of the k th structural shock $\varepsilon_{it}, i = 1, \dots, n$.⁵

In this paper, we are only interested in the unconventional monetary policy shock. In other words, our goal is to find out whether there is a single shock among the n statistically identified ones that satisfies the sign restrictions imposed in each of the previous studies that we consider. If such a shock can indeed be found, we compare its impulse responses to those of the original study. To that end, we employ the Bayesian procedure of Lanne and Luoto (2016).

We start out by estimating the joint posterior distribution of the parameters of the unrestricted SVAR model (4.36), and then compute the posterior distribution of the reduced-form impulse response matrices $\Psi_j, j \in L$, where L consists of indices of the restricted impulse responses. For instance, if the sign restrictions are imposed on the first $q + 1$ impulse responses, $L = \{0, 1, \dots, q\}$.⁶ Because any or none of the n components of ε_t can satisfy the restrictions and hence be the structural shock of interest, we next compute the conditional probability of each shock $\varepsilon_{it}, i = 1, \dots, n$ satisfying the restrictions, conditional on none of the others satisfying them. In practice this is done using the posterior distribution of the identified structural impulse responses $\Theta_j = \Psi_j \mathbf{B}, j \in L$. A more detailed description of the computation of the posterior probabilities is deferred to an appendix.

For each $i \in \{1, \dots, n\}$, this probability can be interpreted as the posterior probability of the restricted SVAR model where the sign restrictions are imposed on the i th column of the $\Theta_j, j \in L$ matrices only. Among the n models, those satisfying the sign restrictions in the (true) data-generating process (DGP) are expected to have high posterior probabilities. Therefore, one can rank the SVAR models satisfying the restrictions by their posterior probabilities, and so find a shock that is most likely the shock of interest.⁷ The economic shocks with the greatest probability can be given the economic interpretation

⁵Although the MA-representation (4.37) does not exist for integrated VAR(p) processes, their impulse responses are given by the same recursion. A similar decomposition exists for I(1) variables and is known as the Beveridge-Nelson decomposition (see Lütkepohl 2006, Section 6.1).

⁶Because different permutations of \mathbf{B} produce the same shocks and impulse responses, the choice of the permutation does not matter. Just to ensure that the whole analysis is based on the same ordering of the shocks, the permutation of the columns of \mathbf{B} is fixed (for details, see Lanne and Luoto 2016).

⁷The procedure described here can be generalized to the case of multiple structural shocks, see Lanne and Luoto (2016).

related to the corresponding restrictions. On the other hand, if the sum of the posterior probabilities is small, i.e. all of the models take a negligible probability, we can conclude that the data does not lend support to the restrictions.

It is important to realize that apart from facilitating the assessment of the plausibility of the restrictions, our non-Gaussian SVAR framework has a number of other benefits compared to the conventional approach to sign restrictions. In the standard setting the matrix \mathbf{B} cannot be identified without restrictions such as sign restrictions which are popular in both conventional and unconventional monetary policy literature. The drawback of sign-identified SVAR models is that they are only set-identified, which means that the posterior of the structural parameters is proportional to the prior and hence an uninformative prior cannot be used. In fact, Baumeister and Hamilton (2015) have recently shown that the results from sign-identified SVARs are driven by the (implicit) priors. In contrast, under our assumptions the impulse responses are point-identified so that their posterior distributions need not be driven by the priors. Because of point-identification an uninformative prior can be used, and this facilitates learning about the impulse responses from the data.

4.3 Empirical Analysis of Unconventional Monetary Policy

The Bank of Japan's (BoJ), the Federal Reserve's (Fed) and the European Central Bank's (ECB) actions mainly differ because of differences in the structures of the economies and financial markets in particular. While the euro area and Japan are bank-centric economies, bond markets play an important role in the United States. The respective central banks therefore provided liquidity and support to different segments of the financial sector: the Fed concentrated on bond purchases, the ECB on lending directly to banks, and the BoJ's strategy involved both.

Most UMP measures consist of an active use of the central banks' balance sheet (Borio and Disyatat 2010), which is therefore a natural gauge for UMP although other measures have also been used in the literature. In line with our reference studies, the policy instruments are the reserves for the BoJ and central bank assets for the Fed and the ECB. The reason is that we analyze the Japanese monetary policy of the early 2000s, when the BoJ had an explicit target for reserves, whereas the Fed's and the ECB's actions focus on the asset side of the balance sheet.

Although the major central banks' unconventional measures were only undertaken after the financial crisis, a few studies are based on longer samples (e.g. Lenza et al. 2010, Peersman 2011). These also include nonlinear model specifications and policy instruments different from those discussed above (Darracq-Paries and De Santis 2015, Baumeister and Benati 2013, Kapetanios et al. 2012). Since UMP measures are only undertaken when the economy faces particularly difficult times (Martin and Milas 2012), utilizing data far beyond such a period may not be adequate to assess the effects of those measures (Boeckx et al. 2016, Gambacorta 2014). Therefore our samples cover periods over which UMP was in use and the central banks had macroeconomic goals. A detailed description of the data is deferred to an appendix.

We now provide a few details concerning the practical implementation, and then present the results of the formal assessment of previously used identification schemes and analyze impulse response functions in each geographical area in turn.

4.3.1 The set-up

We first identify structural shocks statistically and, following Lanne and Luoto (2016) then proceed to formally assess the validity of the sign restrictions used by Schenkelberg and Watzka (2013) for Japan, Gambacorta et al. (2014) for the US and Boeckx et al. (2016) for the euro area. As the data turns out to lend support to the restrictions, we then move on to impulse response analysis of the economic shocks.

We assume that the i th independent component of the error vector ε_{it} follows a univariate Student's t distribution with λ_i degrees of freedom. Non-Gaussianity is required for identification, as discussed in Section 4.2, and we provide evidence that the fat-tailed t distribution is in fact a suitable assumption for the errors.

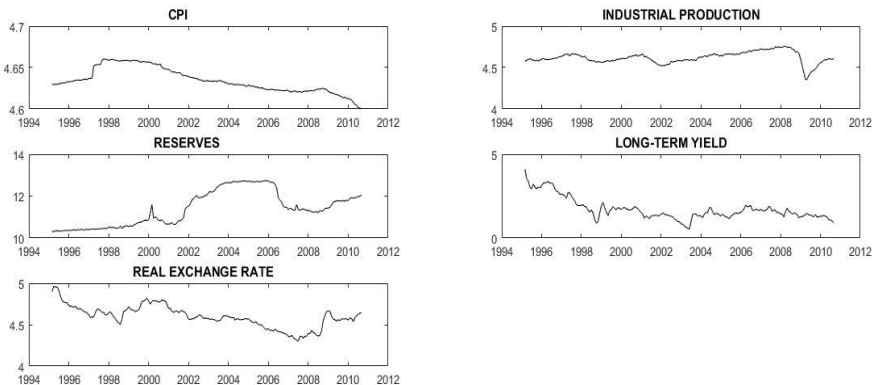
Point identification facilitates incorporating any prior information in Bayesian estimation. However, in order to learn as much as possible about the impulse responses from the data, we use non-informative priors. We assume an exponential prior distribution with mean 5 and variance 25 for each degree of freedom parameter λ_i and a Gaussian prior for the inverse of the error impact matrix $vec(\mathbf{B}^{-1}) \equiv \mathbf{b}$, $\mathbf{b} \sim \mathbf{N}(\underline{\mathbf{b}}, \underline{\mathbf{V}}_{\mathbf{b}})$ where $\underline{\mathbf{V}}_{\mathbf{b}}^{-1} = c_{\mathbf{b}} \mathbf{I}_{n^2}$ and $c_{\mathbf{b}} = 0$, which results in an uninformative (improper) prior for \mathbf{B}^{-1} , $p(\mathbf{B}^{-1}) \propto 1$. For the deterministic terms and coefficient matrices, collected in matrix $\mathbf{A} = [a, A'_1, \dots, A'_p]'$, $vec(\mathbf{A}) \equiv \mathbf{a}$, we assume a normal prior distribution, i.e. $\mathbf{a} \sim \mathbf{N}(\underline{\mathbf{a}}, \underline{\mathbf{V}}_{\mathbf{a}})$ with $\underline{\mathbf{a}} = 0$ and

$\mathbf{V}_a = 10000^2 \mathbf{I}_{pn^2+n}$. For the US and the euro area we also present results based on a relatively more informative prior for $vec(\mathbf{A})$, which corresponds to the standard Minnesota/Litterman prior.

4.3.2 Japan

The burst of the asset price bubble in the early 1990s in Japan led the Bank of Japan (BoJ) to be the first central bank to adopt the zero-interest rate policy. In March 2001 the BoJ changed its main operating target from the overnight call rate to the outstanding current account balances (CABs) held at the BoJ (Honda et al. 2013).⁸ In contrast to most central banks the operating target of the BoJ was on the *liability* side of its balance sheet. The BoJ set explicit targets for bank reserves, committed to maintain high reserves levels in the future and increased the outright purchases of long-term government bonds in order to attain the target on bank reserves (Ugai 2007, Borio and Disyatat 2010).

Figure 4.9: Plot of logarithmic (excl. long-term yield) time series 1995M3–2010M9 for Japan.



We adopt the specification in Schenkelberg and Watzka (2013) who have analyzed the real effects of the Japanese unconventional monetary policy at the ZLB using post-1995 data in a sign-restricted BVAR. The Japanese data, plotted in Figure 4.9, are analyzed with a five-variable structural BVAR model with an intercept and a trend.⁹ Monthly data for Japan spans from March 1995 until September 2010.¹⁰ The variables included are

⁸Current account balances is the technical term for the part of the monetary base that consists of the bank reserves held at the BoJ.

⁹The results are qualitatively the same with linearly detrended data and no trend in the model.

¹⁰The BoJ reintroduced QE measures – money market operations to increase the monetary base – in

the core consumer price index (CPI), the Japanese industrial production index (IP), the bank reserves held at the Bank of Japan (RES), the 10-year yield of Japanese government bonds (LTY) and the real effective exchange rate of Yen against other currencies (EXR). Except for the long-term yield, all variables are expressed in logs. Given that we analyze the same variables and sample period as Schenkelberg and Watzka (2013), we follow them and include six lags in the VAR model.

In the present setup, the impact matrix \mathbf{B} in (4.36) is uniquely identified under non-Gaussianity of at least four components of the error vector. The strength of the identification can easily be checked because a t -distributed random variable converges to a Gaussian as the number of degrees of freedom goes to infinity. Hence, small values indicate (strong) identification. The posterior means of the degree-of-freedom parameters of the univariate t distributions specified for the components of the error term lying between 2.2 and 4.6 thus provide evidence of successful identification.

To study the effects of unconventional monetary policy on output and price level, we need to pin down the right structural shock among the statistically identified ones. For that purpose we exploit the sign restrictions used by Schenkelberg and Watzka (2013) who assume that an expansionary UMP shock has a positive effect on the reserves held at the BoJ and a non-negative effect on consumer prices for 12 months.¹¹ Given the arbitrariness of the 12-month restriction horizon of Schenkelberg and Watzka (2013), we first compute the posterior probability of each structural shock satisfying the restrictions on impact only ($h = 0$), and then for the cases $h = 0, 1$ and $h = 0, \dots, 12$. The results are reported in the left panel of Table 4.20. The sums of the posterior probabilities for these different cases range between 0.14 and 0.41, lending overall support to the restrictions irrespective of the horizon although the evidence is clearly weaker when the restrictions are required to hold for an entire year. Moreover, there is only one shock (ε_{3t}) with a high posterior probability when only the impact effect is restricted. It is found the likeliest candidate for the UMP shock also when the first two impulse responses are restricted although ε_{1t}

2013 as part of the 'Abenomics' strategy. Since a linear model is not suitable to study a sample period which includes a change in the monetary policy regime, the sample cannot be extended to include the 'Abenomics' period.

¹¹The identification scheme in Schenkelberg and Watzka (2013) contains an additional contemporaneous zero restriction on consumer prices to disentangle the UMP-shock from demand and supply shocks. This is not required in our setup because identification is based on statistical properties of the data.

seems to be almost equally likely. Only in the case $h = 0, \dots, 12$ the restrictions fail to pin down the shock. These results altogether speak in favor of a unique labeling of the UMP shock so that impulse responses can be analyzed. This labeling turns out to be robust to two alternative specifications, which we consider next.

Table 4.20: Formal assessment of sign restrictions: Japan

Shock	Benchmark model			Shorter sample		
	$h = 0$	$h = 0, 1$	$h = 0, \dots, 12$	$h = 0$	$h = 0, 1$	$h = 0, \dots, 12$
ε_{1t}	0.04	0.12	0.05	0.01	0.08	0.02
ε_{2t}	0.02	0.04	0.01	0.03	0.12	0.05
ε_{3t}	0.20	0.13	0.04	0.15	0.16	0.24
ε_{4t}	0.02	0.05	0.03	0.00	0.02	0.01
ε_{5t}	0.09	0.07	0.01	0.01	0.07	0.03
Sum	0.37	0.41	0.14	0.20	0.45	0.35

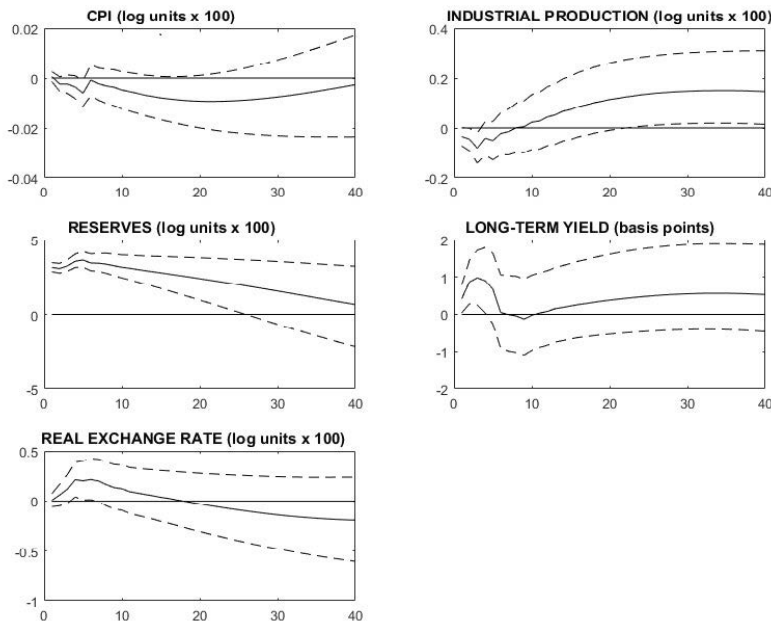
Notes: The figures in the top panel are the posterior probabilities of shock $\varepsilon_{it}, i = 1, \dots, 5$ satisfying the sign restrictions that the reserves be positive and consumer prices be non-negative for various time horizons, and hence being the structural shock of interest. Benchmark model: reserves as the policy instrument. Shorter sample: sample period 2000M3-2007M3.

Figure 4.10 depicts the median impulse responses to a unit UMP shock along with the 16% and 84% percentiles of the posterior distribution.¹³ The UMP shock raises reserves approximately 3%, industrial production at most 0.15% after about two years but the impact on the price level and long-term government yield are insignificant. The effect on the real exchange rate is positive but barely significant.

In contrast to previous studies, these impulse response functions are obtained without restricting the effects on any of the variables and are solely based on the data. Therefore it is interesting to compare the results with those of Schenkelberg and Watzka (2013). It is worth noting that their response of reserves is of the same shape and persistence as ours, and they also find a virtually insignificant effect of the UMP shock on the real exchange rate. On the other hand, their price response is weakly positive and temporary, while we find it to be insignificant also during the first year, when they restricted it non-negative. There is also a small difference in the negative impact response of industrial production, which only we find significant, but it is temporarily positive after 20 months in both studies. However the main difference is in the reaction of the long-term government bond

¹³Unlike with the impulse responses based on conventional sign restrictions, because of point identification, we are able to set the size of the shock. Furthermore, as unique impulse response functions are produced the conventional pointwise posterior median impulse responses and error bands can be reported.

Figure 4.10: Impulse responses to an expansionary UMP shock: Japan 1995M3–2010M9. Median responses (solid lines) together with 68% Bayesian credible sets (dashed lines).

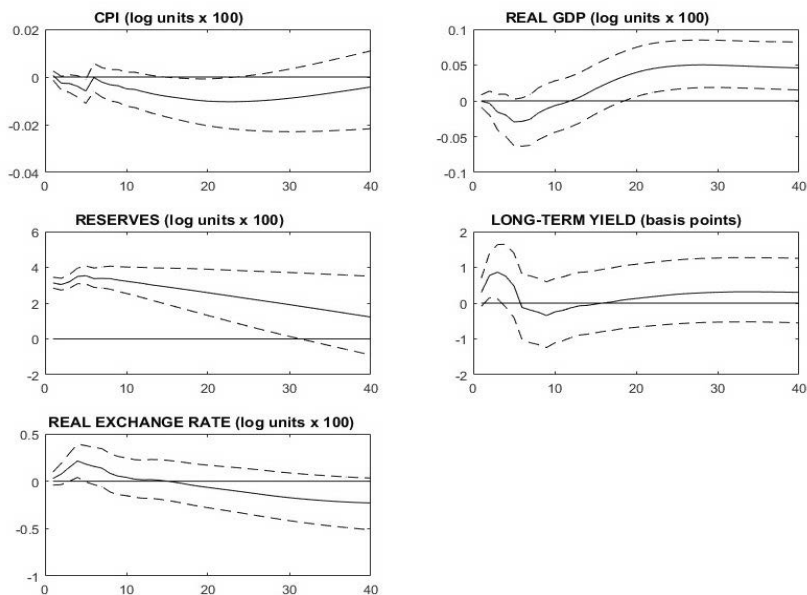


yield, which Schenkelberg and Watzka (2013) report to be significantly *negative* for two years, whereas we observe a significantly *positive*, although very weak (one basis point), transient response of approximately six months. This finding is particularly interesting because asset purchases, which the BoJ engaged in to attain its target on reserves, are typically thought to work by lowering long-term rates.

As a robustness check we analyze a model with interpolated real GDP (instead of the industrial production), which has been used as a measure of aggregate output in Gambacorta et al. (2014) and Boeckx et al. (2016). A monthly measure of real GDP was constructed using the Chow-Lin interpolation method with monthly industrial production as a reference series. We observe that a similar pattern of probabilities emerges as in the previous specification: requiring reserves and the CPI to be non-negative on impact only uniquely identifies the UMP shock, while there are other shocks with positive probabilities in the case $h = 0, 1$, and no labeling is clearly supported for twelve months (posterior probabilities range from 0.02 to 0.05). There are also no major differences in the impulse responses reported in Figure 4.11 compared to the benchmark case.

As another robustness check, we follow Schenkelberg and Watzka (2013) and consider

Figure 4.11: Impulse responses to an expansionary UMP shock: Japan 1995M3–2010M9. Real GDP as a measure of output. Median responses (solid lines) together with 68% Bayesian credible sets (dashed lines).

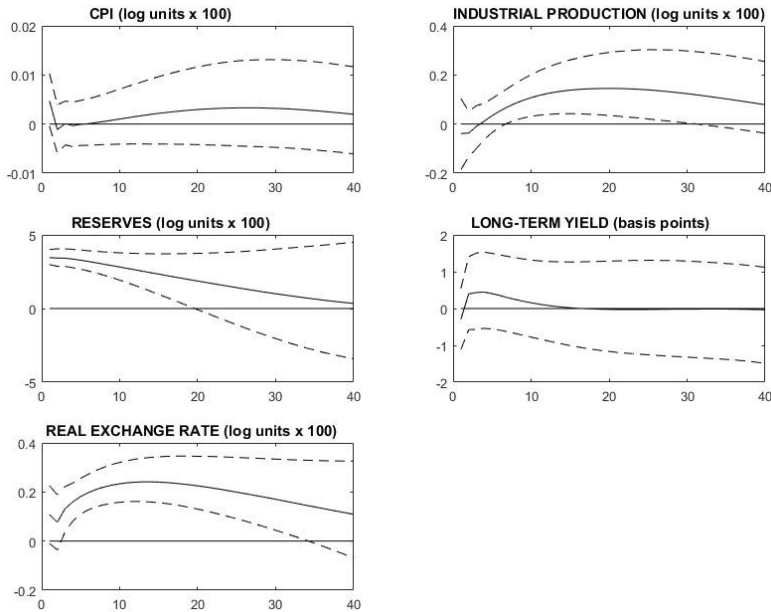


a shorter sample period ranging from March 2000 to March 2007.¹⁴ The sample period covers approximately a year before and after the BoJ targeted current account balances. In fact, one could argue that although the BoJ’s target rate was very close to zero since 1995, starting to target reserves marks the beginning of a different monetary policy regime. The posterior probabilities reported in the right panel of Table 4.20 show that, interestingly, the same shock (ε_{3t}) is uniquely identified as the UMP shock for all restrictions horizons.

The impulse response functions, shown in Figure 4.12, are aligned with the short sample results in Schenkelberg and Watzka (2013). Their price response became insignificant as well, their response of real exchange rate turned from insignificant to positive, and in both studies the significant output effect occurs earlier than in the benchmark case. Interestingly, the main difference remains: we observe an insignificant effect on the long-term rate, while they documented an initial negative effect which then turns positive. We therefore conclude that our results are robust to the alternative output measure but shortening the sample period triggers sharper responses in output and real exchange rate,

¹⁴With the shorter sample lag length is set to $p = 2$.

Figure 4.12: Impulse responses to an expansionary UMP shock: Japan. Shorter sample period 2000M3–2007M3. Median responses (solid lines) together with 68% Bayesian credible sets (dashed lines).



while the effect on the long term yield can be considered negligible in both cases.

To summarize, the sign restrictions in Schenkelberg and Watzka (2013) are supported by the data on impact and after the first month following the shock, but they are not able to uniquely identify the UMP shock when imposed for an entire year, except for the shorter sample period. This allows us to pin down the right structural shock among the statistically identified ones and to conduct impulse response analysis.

Importantly, because we do not impose a positive price response, we are able to conclude that a UMP shock has no effect on the price level. This is in contrast to Schenkelberg and Watzka (2013) who forced the shock to have a positive effect for twelve months. They also documented a negative effect on the long-term government bond yield, whereas in our case positive, although very small values (one basis point) are included in the 68% posterior error bands. Our findings are robust to a different output measure but not entirely to a shorter sample period. The results indicate that the Japanese monetary policy with an explicit target for reserves had no effect on the core consumer price index. The policy managed to stimulate real economic activity with a delay but there is no strong

evidence that it operated by lowering long-term interest rates.

4.3.3 United States

In the aftermath of the 2007-09 financial crisis, when short interest rates were approaching their effective zero lower bound, the Fed, the ECB and other major central banks started to pursue less conventional monetary policies to restore financial and macroeconomic stability. Initially both central banks' actions focused on dysfunctional financial markets, while broader macroeconomic conditions soon became the targets.

Due to the collapse of the housing price bubble and the related subprime crisis in the US, the Fed prioritized housing credit markets within its large scale asset purchase (LSAP) programs. In the first phase it pursued outright asset purchases of government-sponsored enterprise (GSE) debt, mortgage-backed securities (MBS) and long-term Treasury securities. Fears of disinflation and sluggish economic recovery led the Fed to increase its purchases of US Treasuries at several stages during the sample period. Although some of the operations were sterilized, i.e. left the monetary base unaffected, most of them were unsterilized (for details, see Fawley and Neely 2013).

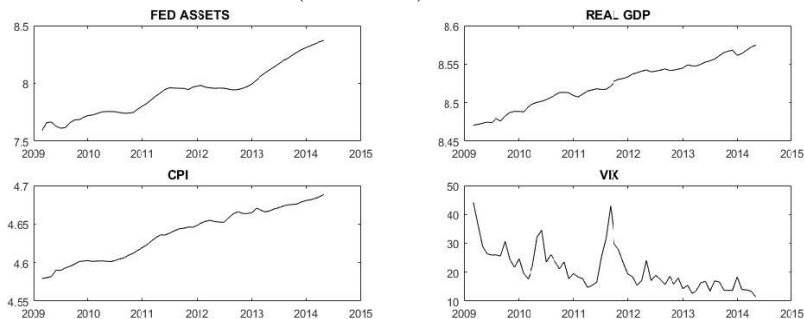
The existing literature on the macroeconomic effects of the Fed's balance sheet operations (Gambacorta et al. 2014, Meinusch and Tillmann 2016, Weale and Wieladek 2016) uses different Bayesian VAR specifications (panel VAR, Qual VAR and SVAR, respectively), but obtains the same result for the key macroeconomic variables; an expansionary UMP shock leads to a temporary significant rise in output and prices.

The Fed's first large scale asset purchase program (LSAP) was only expanded from \$600 billion to \$1.75 trillion in March 2009 (Martin and Milas 2012), and therefore our monthly four-variable dataset for the US, plotted in Figure 4.13, covers the period 2009M3-2014M5. Although with a different set of variables, Weale and Wieladek (2016) were the first to analyze this sample, which does not span beyond the UMP period and is, hence, less susceptible to the Lucas Critique. To capture the main features of the crisis (Gambacorta et al. 2014) the variables included in the BVAR are the log of seasonally adjusted real GDP (GDP)¹⁵, the log of seasonally adjusted consumer price index (CPI), the log of seasonally adjusted central bank assets (CBA) and the level of implied stock

¹⁵A monthly measure of real GDP is constructed using the Chow-Lin interpolation procedure with industrial production and retail sales as reference series.

market volatility (VIX) to control for the central bank’s balance sheet expansion resulting from financial market disturbances.

Figure 4.13: Plot of logarithmic (excl. VIX) time series 2009M3–2014M5 for the US.



We specify a BVAR(2) with a constant consisting of the four variables.¹⁶ With four variables, non-Gaussianity of at least three components of the error vector is crucial for identification. The posterior means of the degree-of-freedom parameters of the t distributions of the error terms turned out to range from 2.8 to 4.2, lending support to fat-tailed error distributions and, hence, successful identification.

Table 4.21: Formal assessment of sign restrictions: United States

Shock	Benchmark model		Industrial production		Monetary base	
	$h = 0$	$h = 0, 1$	$h = 0$	$h = 0, 1$	$h = 0$	$h = 0, 1$
ε_{1t}	0.00	0.00	0.00	0.00	0.01	0.03
ε_{2t}	0.00	0.00	0.00	0.01	0.01	0.02
ε_{3t}	0.01	0.03	0.01	0.04	0.02	0.03
ε_{4t}	0.11	0.16	0.05	0.11	0.39	0.42
Sum	0.12	0.19	0.06	0.16	0.43	0.50

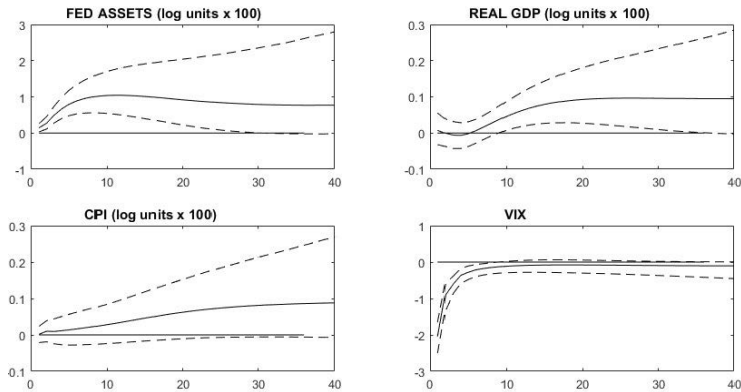
Notes: The figures in the table are the posterior probabilities of shock $\varepsilon_{it}, i = 1, \dots, 4$ satisfying the sign restrictions that the central bank assets be nonnegative and the VIX be nonpositive for various time horizons, and hence being the structural shock of interest. The figures on the bottom line are the sums of the posterior probabilities. Benchmark model: central bank assets as policy instrument. Industrial production: Industrial production as a measure of aggregate output. Monetary base: monetary base as policy instrument.

In order to find out whether any of the statistically identified shocks can be labeled as the monetary policy shock, we proceed with a formal assessment of the sign restrictions in Gambacorta et al. (2014) whereby an expansionary UMP shock increases central bank

¹⁶Also Weale and Wiedalek (2016) use $p = 2$ for this sample period.

assets but does not increase stock market volatility on impact and one month after the shock.¹⁸ Again in the absence of a clear justification for the restriction horizon we check the validity of the signs on impact only ($h = 0$) as well as for the case $h = 0, 1$. The results reported in the left panel of Table 4.21 show that there is not much difference between the posterior probabilities in the two cases. The sums of the posterior probabilities (0.12 and 0.16) lend overall support to the restrictions. Moreover, there is in both cases only one shock (ε_{4t}) with a high posterior probability, with the probability of the other shocks virtually zero, so that a UMP shock can be regarded as uniquely identified in probability.

Figure 4.14: Impulse responses to an expansionary UMP shock: US 2009M3–2014M5. Median responses (solid lines) together with 68% Bayesian credible sets (dashed line).

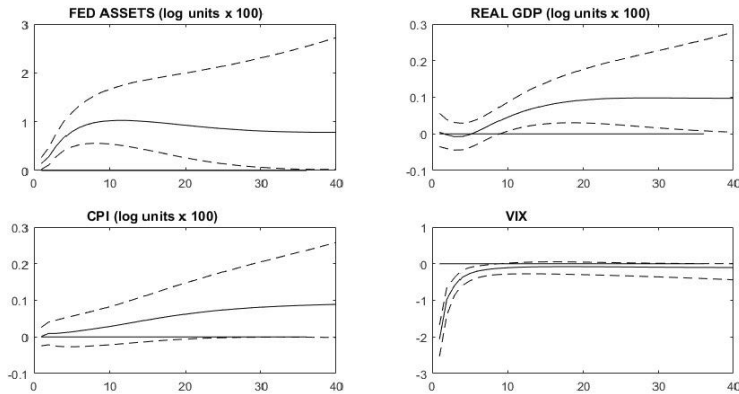


The impulse responses, plotted in Figure 4.14, show that a unit UMP shock increases the central bank assets on impact but the median peak response of 1% occurs after approximately eight months. While Gambacorta et al. (2014) forced output and prices to respond with a lag and documented peak responses after six months, and Weale and Wiedalek (2016) found output and prices to rise for 20–40 months after a UMP shock regardless of the identification scheme, our unrestricted impulse response functions indicate that the output response turns significantly positive only after ten months. We also observe a more persistent output response, lasting up to 35 months. In contrast, the evidence for a positive CPI response is weaker, as the 68% Bayesian credible sets just include the zero

¹⁸Gambacorta et al. (2014) and Boeckx et al. (2016) impose additional contemporaneous zero restrictions on output and consumer prices to reduce the number of admissible impulse responses and so to sharpen identification. These are not required in our setup because the model is uniquely identified based on statistical properties of the data.

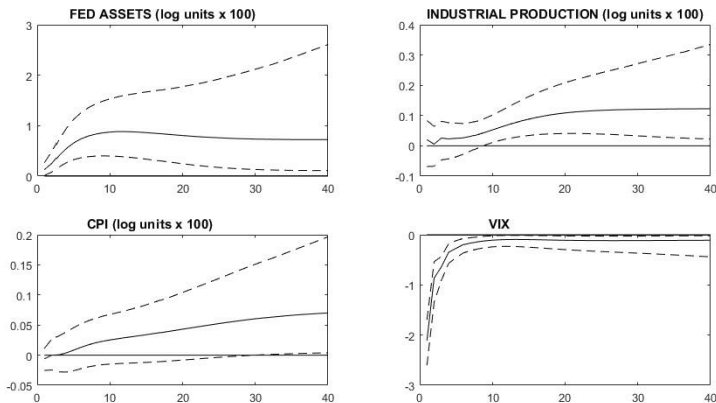
line.

Figure 4.15: Impulse responses to an expansionary UMP shock: US 2009M3–2014M5. Informative prior. Median responses (solid lines) together with 68% Bayesian credible sets (dashed line).



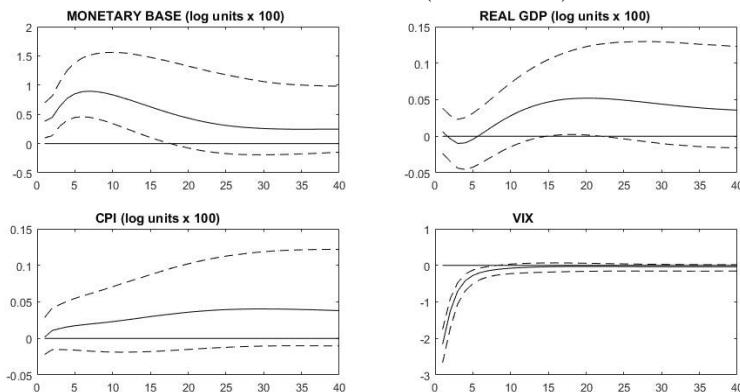
Taking into account the very small sample size, we also considered a more informative prior distribution, corresponding to the standard Minnesota/Litterman prior. Interestingly, Figure 4.15 shows that the relatively more informative prior results in a positive price response after 30 months, with the rest of the responses unaltered. Moreover, further tightening the prior made the positive price response to occur even earlier, but still much later than previously found.

Figure 4.16: Impulse responses to an expansionary UMP shock: US 2009M3–2014M5. Industrial production as a measure of output. Median responses (solid lines) together with 68% Bayesian credible sets (dashed line).



To check the robustness of our results, we considered industrial production as a measure of output and monetary base as the quantitative policy instrument. The middle and right panels of Table 4.21 show that the labeling is robust both variables and the same shock (ε_{4t}) is uniquely identified in probability. There are, however, differences in the impulse response functions compared to the benchmark specification. Interestingly, when industrial production is used (Figure 4.16), the positive CPI response becomes significant after 30 months even when a non-informative prior is used, while the rest of the responses remain the same. Again, tightening the prior has the same effect in that the CPI response becomes significantly positive earlier.

Figure 4.17: Impulse responses to an expansionary UMP shock: US 2009M3–2014M5. Monetary base as a policy instrument. Uninformative prior. Median responses (solid lines) together with 68% Bayesian credible sets (dashed line).



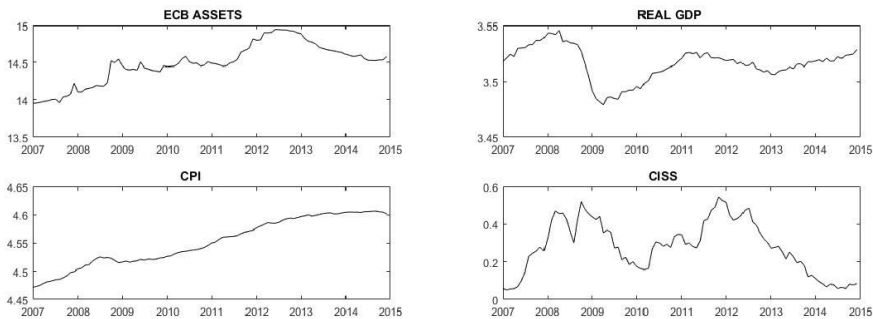
On the other hand, unlike documented by Gambacorta et al. (2014) and what we found for Japan and the euro area (see Section 4.3.4), the results from the impulse response analysis for the US are not robust to an alternative quantitative policy instrument (monetary base). Although the posterior probabilities in Table 4.21 indicate that the sign restrictions are supported by the data, the impulse responses of the two macrovariables of interest are statistically insignificant. Furthermore, only a very tight prior triggers a significant positive output response similar to the previous specifications, while the price response remains insignificant (Figure 4.17). This finding is consistent with the fact that the effectiveness of balance sheet policies does not hinge on an accompanying change in the monetary base (Borio and Disyatat 2010), and as already noted by Gambacorta et al. (2014), monetary base expanded less than central bank assets in the US over part of the

sample period. It also indicates that differences between countries make panel methods less suitable to study the country-specific impact of unconventional monetary policies.

4.3.4 Euro area

Similarly to the Fed, the ECB’s asset purchase programs aimed to improve the functioning of specific markets. The covered bond purchase program (CBPP) stimulated the issuance of covered bonds, and therefore eased funding conditions for banks (Beirne et al. 2011), whereas the objective of the Securities Markets Program (SMP) – later replaced by Outright Monetary Transactions (OMT) – was to address the malfunctioning of the securities markets caused by the sovereign debt crisis.¹⁹ Apart from the SMP and its follower OMT, the majority of the ECB’s operations during the sample period consisted of providing funding for banks. The ECB expanded both the availability and maturity of bank loans as well as eased the conditions for receiving funding on several occasions. Its asset purchases were modest in size and mostly sterilized, reversing their effects on the monetary base.

Figure 4.18: Plot of logarithmic (excl. CISS) time series 2007M1–2014M12 for the euro area.



To investigate the effectiveness of the policy measures that expand the ECB’s balance sheet, we adopt the VAR model specification of Boeckx et al. (2016)²⁰. The monthly ECB data, plotted in Figure 4.18, spans from January 2007 until December 2014. Although the ECB has continued its unconventional policies beyond this date, we follow Boeckx et al.

¹⁹See the 5.10.2010 ECB press release www.ecb.eu/press/pr/date/2010/html/pr100510.en.html

²⁰As Boeckx et al. (2016) build on Gambacorta et al. (2014), also our study is related to theirs, with the difference of a longer sample period and the use of the CISS variable to measure overall financial stress in the euro area.

(2016) and end the sample period before the beginning of the Expanded Asset Purchase Program (EAPP).

The vector of endogenous variables comprises the log of seasonally adjusted real GDP (GDP), the log of seasonally adjusted consumer price index (CPI), the log of seasonally adjusted central bank assets (CBA) and the level of the Composite Indicator of Systemic Stress (CISS). Boeckx et al. (2016) also included in their model the main refinancing operations (MRO) policy rate and the spread between the EONIA and the MRO-rate. However with six variables the number of parameters to estimate increases considerably when no restrictions are imposed, and because of the short sample period this obviously creates problems in estimation.²¹

Table 4.22: Formal assessment of sign restrictions: Euro area

Shock	Benchmark model		Monetary base	
	$h = 0$	$h = 0, 1$	$h = 0$	$h = 0, 1$
ε_{1t}	0.17	0.19	0.07	0.08
ε_{2t}	0.25	0.28	0.34	0.38
ε_{3t}	0.06	0.05	0.04	0.05
ε_{4t}	0.06	0.04	0.16	0.16
Sum	0.54	0.56	0.61	0.67

Notes: The figures in the table are the posterior probabilities of shock $\varepsilon_{it}, i = 1, \dots, 4$ satisfying the sign restrictions that the central bank assets be nonnegative and the CISS be nonpositive for various time horizons, and hence being the structural shock of interest. The figures on the bottom line are the sums of the posterior probabilities. Benchmark model: central bank assets as policy instrument. Monetary base: monetary base as policy instrument.

We include a constant and two lags in the VAR model.²³ The posterior means of the degree-of-freedom parameters of the t distributions specified for the components of the error term between 2.3 and 5.6 suggest that identification based on non-Gaussianity of the errors has once again been achieved. We therefore proceed with the formal assessment

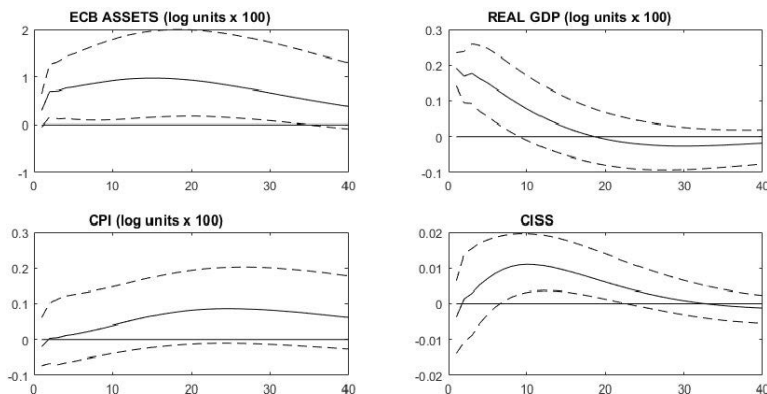
²¹In fact, with six variables the method adopted in this paper yielded results that did not allow us to make any conclusions even when using a very tight prior. Because one of the advantages of the method is the ability to check the compatibility with the data of the restrictions imposed in the conventional approach, we choose to stick to the 4-variable specification. Moreover, our conclusions turn out to be similar to those obtained by Boeckx et al. (2016) and most differences can be seen to follow from (the absence of) restrictions.

²³Our results are robust to $p = 3$ used in Boeckx et al. (2016) although the IRFs are somewhat smoother with $p = 2$.

of the sign restrictions in Boeckx et al. (2016), who assume that a UMP shock increases the balance sheet of the ECB but does not increase financial stress. The restrictions are imposed on impact and in the first month after the shock.

The results reported in the left panel of Table 4.22 show that the restrictions are supported by the data and two of the shocks (ε_{1t} and ε_{2t}) receive a relatively high probability (0.17 and 0.25, respectively). The results do not depend on the horizon over which the restrictions are imposed, and we regard ε_{2t} maximizing the posterior probability as our UMP shock of interest.

Figure 4.19: Impulse responses to an expansionary UMP shock: Euro area 2007M1–2014M12. Median responses (solid lines) together with 68% Bayesian credible sets (dashed lines).

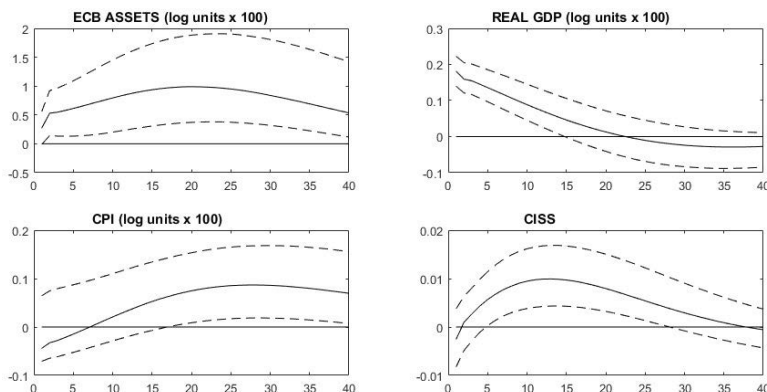


An inspection of the impulse responses in Figure 4.19 reveals that a unit UMP shock results in an increase in the ECB assets of approximately 0.4% on impact, leads to a significant increase in output and an (insignificant) initial decline in the CISS indicator.²⁴ The main difference with Boeckx et al. (2016) or the country-level results in Gambacorta et al. (2014) is the response of prices, which they found to be significantly positive persistently, while we find no significant effect. In contrast, the size of the output effect is similar to theirs, lasting less than a year. Given that our results are obtained without restrictions, it is interesting to note that also the timing of the output response differs

²⁴The normalization rule used to compute the posterior probabilities reported in Table 4.22 generates bimodal posterior distributions for the impulse response functions, resulting in error bands that do not properly reflect parameter uncertainty (see Waggoner and Zha 2003). For the error bands to be informative about the reliability of the estimates, we report impulse responses computed with a different normalization rule which, however, does not affect the posterior probabilities.

from Gambacorta et al. (2014) and Boeckx et al. (2016). Specifically, when the impact response is not ruled out *ex ante*, a positive output response is found to occur earlier than reported in these previous studies. While Boeckx et al. (2016) found output to peak after eight months and Gambacorta et al. (2014) after three months, according to our results output peaks immediately.

Figure 4.20: Impulse responses to an expansionary UMP shock: Euro area 2007M1–2014M12. Informative prior. Median responses (solid lines) together with 68% Bayesian credible sets (dashed lines).

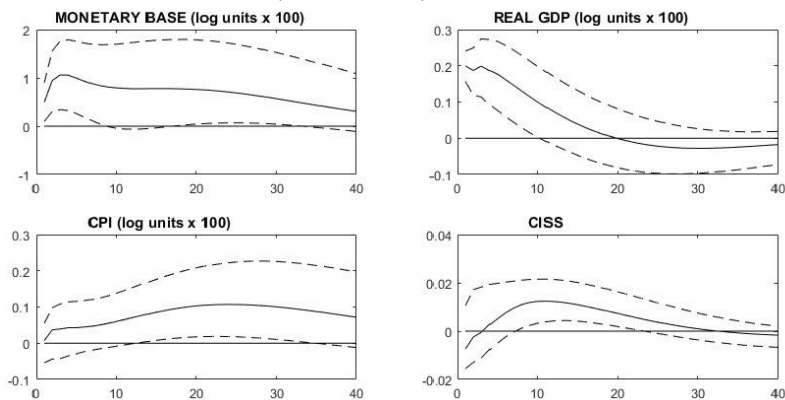


Taking into account the relatively small sample size and the implicit tight priors of conventional sign-identified SVARs, we also considered a more informative prior distribution. The impulse response functions reported in Figure 4.20 show that the relatively more informative prior results in a positive transient price response after 18 months, whereas the rest of the responses remain unaltered.

We checked the robustness of our results with respect to the monetary base instead of central bank assets as the monetary policy instrument. The right panel of Table 4.22 shows that the UMP shock is more sharply identified in that the posterior probability of the likeliest shock (ε_{2t}) is greater when monetary base is used instead of central bank assets as the quantitative policy instrument, confirming that this shock indeed is our UMP shock of interest. The results from the impulse response analysis (see Figure 4.21) are robust with respect to the alternative instrument save one interesting exception: a positive price response occurs already after one year even when a non-informative prior is used, i.e. the analysis is solely based on the data.

The latter finding is in contrast to Boeckx et al. (2016), whose price response proved

Figure 4.21: Impulse responses to an expansionary UMP shock: Euro area 2007M1–2014M12. Monetary base as a policy instrument. Median responses (solid lines) together with 68% Bayesian credible sets (dashed lines).



robust to the alternative policy instrument. Nonetheless, the authors point out an important difference between the two variables: the ECB’s asset purchases were mostly sterilized and hence are not included in the monetary base. As a consequence the evolution of the European Monetary Union’s monetary base reflects extensions of the long term refinancing operations (LTROs) only (Fawley and Neely 2013). This can explain our finding that central bank assets and monetary base had a different impact on the price level and suggests that extending the maturity of the longer bank loans showed up sooner in the euro area consumer prices than purchases of private assets or government bonds.

4.4 Conclusions

We have applied a novel Bayesian SVAR identification method due to Lanne and Luoto (2016) to estimate the macroeconomic effects of the Bank of Japan’s, the Federal Reserve’s and the European Central Bank’s balance sheet operations. The procedure exploits non-Gaussianity and independence of the structural error terms to uniquely identify the shocks as in Lanne et al. (2017). In contrast to the SVAR models identified by sign restrictions, our model and the impulse responses are point-identified. This entails a number of advantages over the conventional approach to sign restrictions. Importantly, instead of being forced to impose the set of sign restrictions used in the previous literature, we are able to formally assess their plausibility against the data.

According to our results, the sign restrictions used in the previous literature were

mostly supported by the data. However, unlike previous literature, we found an expansionary unconventional monetary policy shock to have different macroeconomic effects in the three geographical areas. Not only the timing, persistence and statistical significance of the output and price responses varied from country to country but also the robustness of the results to alternative variables used in the literature. Although we looked at policies that expand each central bank's balance sheet, the policy instrument encompasses different operations for each central bank, which therefore turned out to have different economy-wide effects.

References

- Bacchiocchi, E., Castelnuovo, E. and Fanelli, L. (forthcoming). Give me a break! Identification and estimation of the macroeconomic effects of monetary policy shocks in the U.S. *Macroeconomic Dynamics*.
- Baumeister, C. and Benati, L. (2012). Unconventional monetary policy and the great recession - Estimating the macroeconomic effects of a spread compression at the zero lower bound. Working Paper 2012-21, Bank of Canada.
- Baumeister, C. and Hamilton J.D. (2015). Sign restrictions, structural vector autoregressions and useful prior information. *Econometrica* **83**: 1963-1999.
- Beirne, J., Dalitz, L., Ejsing, J., Grothe, M., Manganelli, S., Monar, F., Sahel, B., Sušec, M., Tapking, J., and Vong, T. (2011) The impact of the Eurosystem's covered bond purchase programme on the primary and secondary markets. ECB Occasional paper series No. 122, European Central Bank, January 2011.
- Boeckx, J., Dossche, M., and Peersman, G. (2016). Effectiveness and Transmission of the ECB's Balance Sheet Policies. Forthcoming in *International Journal of Central Banking*.
- Borio, C. and Disyatat P. (2010). Unconventional monetary policies: an appraisal. *The Manchester School*, **78**(s1): 53-89.
- Cecioni, M., Ferrero, G., and Secchi, A. (2011). Unconventional monetary policy in theory and practice. Questioni di economia and finanza (Occasional papers) 102, Bank of Italy, Economic research and international relations area.

Darracq-Paries, M., and De Santis R.A. (2015). A non-standard monetary policy shock: The ECB's 3-year LTROs and the shift in credit supply. *Journal of International Money and Finance* **54**: 1-34.

European Central Bank press release 5.10.2010
www.ecb.eu/press/pr/date/2010/html/pr100510.en.html

Fawley, B.W., and Neely, C.J. (2013). Four stories of quantitative easing. *Federal Reserve Bank of St. Louis Review*, **95**(1): 51-88.

Fry, R., and Pagan, A. (2011). Sign restrictions in structural vector autoregressions: A critical review. *Journal of Economic Literature* **49**(4): 938-960.

Gambacorta, L., Hofmann, B., and Peersman, G. (2014). The effectiveness of unconventional monetary policy at the zero lower bound: a cross-country analysis. *Journal of Money, Credit and Banking* **46**(4): 615-642.

Honda, Y., Kuroki, Y., Tachibana, M. (2013). An injection of base money at zero interest rates: Empirical evidence from the Japanese experience 2001-2006. *Japanese Journal of Monetary and Financial Economics* **1**(1): 1-24.

Joyce, M., Miles D., Scott A., and Vayanos D. (2012). Quantitative easing and unconventional monetary policy - an introduction. *The Economic Journal* **122**: 271-288.

Kapetanios, G., Mumtaz, H., Stevens I., and Theodoridis, K. (2012). Assessing the economy-wide effects of quantitative easing. *The Economic Journal* **122**: 319-347.

Lanne, M., and Luoto, J. (2016). Data-driven inference on sign restrictions in bayesian structural vector autoregression. CREATES Research paper 2016-4.

Lanne, M. and Lütkepohl, H. (2014). *A Statistical Comparison of Alternative Identification Schemes for Monetary Policy Shocks*. Knif J. & Pape B. (Eds.): Contributions to Mathematics, Statistics, Econometrics, and Finance: Essays in Honour of Professor Seppo Pynnönen. University of Vaasa, Vaasa.

Lanne, M., Meitz, M., Saikkonen P. (2017). Identification and estimation of non-Gaussian structural vector autoregressions. *Journal of Econometrics* **196**(2), 288-304.

- Lenza, M., Pill H., and Reichlin L. (2010). Monetary policy in exceptional times. *Economic Policy* **25**: 295-339.
- Lütkepohl, H. (2007) *New introduction to multiple time series analysis*. Springer-Verlag: Berlin.
- Martin , C., and Milas, C. (2012) Quantitative easing: a sceptical survey. *Oxford Review of Economic Policy* **28**(4): 750-764.
- Meinusch, A., and Tillmann, P. (2016). The macroeconomic impact of unconventional monetary policy shocks. *Journal of Macroeconomics* **47**: Part A, 58-67.
- Normandin, M. and Phaneuf, L. (2004). Monetary policy shocks: Testing identification conditions under time-varying conditional volatility. *Journal of Monetary Economics* **51**: 1217-1243.
- Peersman (2011). Macroeconomic effects of unconventional monetary policy in the euro area. CESifo Working Paper Series 3589. CESifo Group, Munich.
- Schenkelberg, H., and Watzka, S. (2013). Real effects of quantitative easing at the zero lower bound: Structural VAR-based evidence from Japan. *Journal of International Money and Finance* **33**: 327-357.
- Ugai, H. (2007). Effects of the quantitative easing policy: A survey of empirical analyses. Bank of Japan Monetary and Economic Studies **25**: 1-47.
- Waggoner, D.F, and Zha, T. (2003). Likelihood preserving normalization in multiple equation models. *Journal of Econometrics* **114**, 329-347.
- Weale, M. and Wieladek, T. (2016). What are the macroeconomic effects of asset purchases? *Journal of Monetary Economics* **79**: 81-93.

Appendix A: Data

The data have been retrieved from the FRED database provided by the Federal Reserve Bank of St Louis (<https://research.stlouisfed.org/fred2/>), from the Bank for International Settlements' (BIS) website (www.bis.org), Bank of Japan's statistics (BOJ) website (<http://www.boj.or.jp/en/statistics/index.htm/>), CBOE (www.cboe.com) and the ECB Statistical Data Warehouse (ECB) (<http://sdw.ecb.europa.eu/>).

Series employed in the empirical analysis for Japan:

- Real effective exchange rate (RNJP), BIS
- Core consumer price index (JPNCPICORMINMEI), FRED
- Industrial production (JPNPROINDMISMEI), FRED
- Average outstanding current account balances (BJ'MABS1AN113), BOJ
- 10-year government bond yield (IRLTLT01JPM156N), FRED
- Real GDP (NAEXKP01JPQ661S), FRED

Series for the USA:

- Total Federal Reserve bank's assets (WALCL), FRED
- Consumer price index (CPALTT01USM661S), FRED
- CBOE volatility index (VIX), CBOE
- Industrial production (INDPRO), FRED
- Retail sales (RSXFS), FRED
- Monetary base (AMBSL), FRED

Series for the euro area:

- Central bank assets for the euro area (ECBASSETS), ECB
- Composite indicator of sovereign stress (CISS.M.U2.Z0Z.4F.EC.SOV_GDPW.IDX), ECB

- Harmonized index of consumer prices (ICP.M.U2.Y.000000.3.INX), ECB
- Real GDP (NAEXKP01EZQ661S), FRED

Appendix B: Computation of the conditional probabilities

In this appendix I provide details about the computation of the conditional probabilities for finding one structural shock of interest with the Lanne and Luoto (2016) method.

Suppose the shock is characterized by having non-positive and/or non-negative impact effects on J of the variables in y_t . We start out by collecting these sign restrictions in a $J \times n$ matrix \mathbf{R} , the elements of which equal 1, -1 or 0. We then define a set Q such that $Q = \{\theta_{0k} : R\theta_{0k} \geq 0_{J \times 1}\}$, where θ_{0k} is the k th column of Θ_0 , i.e. of the impact matrix \mathbf{B} , corresponding to shock ε_{kt} . In other words, the set Q consists of all the columns of \mathbf{B} that satisfy the sign restrictions, and if none of the shocks satisfy them, the set is empty.

As explained in Section 4.2, because the procedure identifies \mathbf{B} up to permutation of its columns, any or none of the n components of ε_t can satisfy the restrictions and hence be the structural shock of interest. We therefore proceed to compute the conditional probability of satisfying the sign restrictions for each shock ε_{kt} , $k = 1, \dots, n$,

$$\Pr(\theta_{0k} \in Q, \theta_{0,m/k} \in Q^c | \mathbf{y}),$$

where \mathbf{y} is the vector of data, obtained by stacking y_t for $t = 1, \dots, T$, Q^c denotes the complement of Q , and $m \in \{1, \dots, n\}$. For each $k \in \{1, \dots, n\}$, this probability can be interpreted as the posterior probability of the restricted SVAR model, where the sign restrictions contained in \mathbf{R} are imposed on the k th column of \mathbf{B} only. Given that these are posterior probabilities of disjoint events that only occur separately, we can calculate the overall probability of the sign restrictions being satisfied by simply summing up the probabilities over $k \in \{1, \dots, n\}$.