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**Economics of boreal conifer species in continuous cover and
clearcut forestry**

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Tiivistelmä - Referat - Abstract <p>This study optimizes the management regime of boreal Norway spruce (<i>Picea abies</i> (L.) Karst.) and Scots pine (<i>Pinus sylvestric</i> L.) stands. The aim is to compare the economic profitability of continuous cover management and clearcut management and to study the hypothesis that continuous cover forestry is more favorable in the case of Norway spruce, compared to Scots pine. Additionally, the study analyses the outcomes of two different growth models for these tree species and compares the results with the requirements of the Finnish Forest Act of 2014.</p> <p>Earlier studies comparing the suitability of Norway spruce and Scots pine to continuous cover forestry have applied unclear model specifications and unnecessary limitations in the optimization methods. In this study, the optimization is carried out using a theoretically sound economic optimization model that determines the choice of the management regime as an outcome of the optimization. The model uses empirically estimated ecological growth models and includes both fixed and variable harvesting costs. Two different empirically estimated ecological growth models are used and compared. The optimization model is solved as a bi-level problem where harvest timing is the upper-level problem and harvesting intensity the lower-level problem. The optimization is solved using gradient-based methods for the lower-level problems and genetic and hill-climbing algorithms for the upper-level problems. This is the first study using this method to solve optimal continuous cover solutions for Scots pine.</p> <p>The results show that the main differences in optimal solutions between the two species are independent of the ecological two growth models used. According to both ecological models, continuous cover forestry is less favorable for Scots pine compared to Norway spruce, in both low and average fertility sites. However, the magnitude of this favorability and the characteristics of the optimal solutions strongly depend on the ecological model. Optimal continuous cover solutions for Scots pine are also found to have very low stand densities. Almost all economically optimal solutions are illegal because of their low number of trees or basal area per hectare.</p>			
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Tiivistelmä - Referat - Abstract <p>Tämä tutkimus optimoi boreaalisten alueiden havupuiden metsänhoitomenetelmän valintaa yhden puulajin metsissä. Tutkimuksessa mukana olevat lajit ovat kuusi (<i>Picea abies</i> (L.) Karst.) ja mänty (<i>Pinus sylvestric</i> L.). Tutkimuskysymyksenä on ”onko eri-ikäisrakenteinen metsänhoito edullisempaa kuuselle, kuin männylle?”. Tutkimuksessa myös tarkastellaan optimiratkaisuja Suomen vuoden 2014 metsälain valossa.</p> <p>Suurimmassa osassa aihepiirin aiempia tutkimuksia on käytetty taloustieteellisesti ongelmallisia malleja, sekä tarpeettomia rajoituksia. Tämän tutkimuksen optimoinnissa käytetään teoreettisesti vankkaa mallia, jossa metsänhoitotavan valinta tapahtuu osana mallin optimiratkaisua. Mallissa käytetään empiirisesti estimoitua kasvumallia, sekä yksityiskohtaista kuvausta hakkuukustannuksista. Tutkimuksessa käytetään ja vertaillaan kahta eri kasvumallia. Optimointi muodostuu kaksitasoiseksi ongelmaksi, jossa harvennusten ajankohta on ylemmän ja intensiteetti alemman tason ongelma. Optimointi tehdään hyödyntäen sisäpistemenetelmiä alemman tason ongelmaan ja geneettistä sekä hill-climbing algoritmia ylemmän tason ongelmaan.</p> <p>Tulokset osoittavat huomattavia eroja männyn ja kuusen välillä, joiden suuruus riippuu käytetystä kasvumallista. Tuloksista käy ilmi että heikohkossa ja heikossa kasvupaikassa eri-ikäisrakenteinen metsänhoito on edullisempaa kuuselle kuin männylle. Tämän edullisuuden suuruus ja optimiratkaisujen yksityiskohdat riippuvat oleellisesti käytetystä kasvumallista. Optimaalinen eri-ikäisrakenteinen metsänhoito johtaa männyllä alhaisiin metsän tiheyksiin. Lähes kaikki taloudellisesti optimaaliset ratkaisut ovat lisäksi laittomia niiden alhaisen hehtaarikohtaisen puumäärän tai pohjapinta-alan vuoksi.</p>			
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1 Introduction

The optimal use of forest resources has been debated for centuries. Before the modern forest economic thinking, the ideas on the use of forest resources in the mid-18th century Europe were influenced by national self-sufficiency and government intervention. Thus, early forest regulations were designed to guarantee sustainable and regular timber flow, resulting in fully regulated or normal forest. (Viitala 2016.) Today the forestry methods in the Nordic countries can be divided into clearcut and continuous cover forestry. This division is also often referred to as even- vs. uneven-aged forestry. In clearcut management, the forest is simply clearcut at the end of some rotation and then replanted artificially. Continuous cover management relies solely on selection cuttings and natural regeneration. Thus, the stand under continuous cover management is never harvested completely.

In the Fennoscandia region, forest policy has promoted the clearcut regime since WWII or earlier. This orientation has been motivated by the perceived need to ensure and increase the flow of material supply for the economically significant forest industries (Kuuluvainen et al. 2012). For further discussion about the historical background of the clearcut regime in Sweden and Finland, see Lundmark et al. (2013) and Siiskonen (2007).

The ideas of optimal clearcut and continuous cover management have both been developed for centuries. Today those ideas can be studied using the progress of computational capabilities. Using advanced computational calculations, we can try to get a more clear understanding of the differences between these two forest management regimes.

1.1 Earlier research

Faustmann (1849) determined the bare land value of a forest in clearcut management by summing up the present values of all future net revenues. This is done by assuming an infinite amount of identical rotations and by summing them up using the theorem of geometric series. Faustmann (1849) however, did not solve the optimal rotation of his model. The Faustmann (1849) model remained almost forgotten until Samuelsson (1976), when it was brought into the general knowledge of economists. The original model by Faustmann (1849) included thinning which were left out by Samuelsson (1976). The classic rotation model is still very much the basics of forestry related literature of resource economics (Tahvonen 2015b).

The expansions of the classic rotation model cover a wide array of ideas. For example, Hartman (1976) expands the classic rotation model by adding amenity services into the forest stand and shows them to have a great impact on the harvest timing. Clark (1976) presents a simplified model for optimal thinning. Van Kooten et al. (1995) use the classic rotation model to analyse the effect of carbon taxes and subsidies on optimal rotation and supply of carbon services, and show that it is optimal in some cases to never harvest the stand. However, they exclude thinning.

An alternative to the clearcut management is continuous cover (or even-aged) management. One of the earliest contributions to continuous cover management can be dated back to the studies of a French forester de Liocourt (1898). He wrote that when using a size class setup to describe the state of the forest, the ideal number of trees is larger in the smaller diameter classes than in the larger classes. Thus, the distribution of trees per size class forms a reversed J-shape, often referred as the law of de Liocourt. (Pommering and Murphy 2004.)

The economic models for clearcut management are currently much more established than the ones developed for continuous cover management. Overall the research on continuous cover management has been dominated by the use of simplified static models and economically unclear model specifications. Still today the understanding of continuous cover optimization is not fully completed. (Tahvonen and Rämö 2016.) The literature on continuous cover management is varied and often has no firm economic foundation (Tahvonen 2015b).

Today there are two different approaches to continuous cover infinite time horizon problems. The first one is to use such a long time horizon, that the discounted revenues from the end of horizon do not change the optimal solution toward some steady state. The second one is to somehow “optimize” a steady state solution and then solve an optimal path to this state. (Wikström 2000.) The second approach is problematic since it is not mathematically correct to solve the steady state with a positive interest rate if the transition path is not optimized simultaneously. The steady state is optimal only if deviating towards another steady state does not become beneficial. Using a positive interest rate, such a deviation becomes always optimal if the steady state is specified independently of the optimal approach. This is rather basic knowledge in the optimal control theory (Tahvonen and Rämö 2016), but often neglected by forest economics.

Duerr and Bond (1952) analyse optimal stocking in continuous cover management, assuming separate age classes that are not connected with each other nor in economic or

ecologic ways. Their results state that the optimal forest stock level is reached when the marginal value growth equals the interest rate. This optimum is known as the marginal value model. Following their work, Adams and Ek (1974) study optimal continuous cover management, using numerical nonlinear optimization. They use a size-structured model in a two-phase optimization by first determining the steady state and then solving the optimal path to it. The computational capabilities were limited in the 1970s, forcing the steady state to be reached within a 10-year transition period, with only three harvests. To determine the steady state they use the investment-efficient model, which is closely related to the marginal value model. However, the investment-efficient model is shown to be economically flawed and has been criticized for example by Haight (1985) and Tahvonen and Rämö (2014). Most of the other older studies on continuous cover management such as Buongiorno and Michie (1980) and Chang (1981) have also used the method of determining the steady state and then solving the problem using static models (Wikström 2000). After Chang (1981) it was common in many studies to assume that the steady state must be achieved with just one harvest. Using one harvest to reach the steady state however limits the optimal solution seriously and calls for generalizations. (Tahvonen and Rämö 2016.)

Continuous cover management studies using economic models that are also theoretically sound have in the past had limitations in the form of either including fixed harvesting intervals or applying harvesting in each period (Rämö and Tahvonen 2016). Haight and Monserud (1990) apply the approach of lengthening the time horizon until further lengthening no longer changes the optimal path towards the steady state. However, they do not take into consideration any fixed costs of harvesting, which leads to the thinning of the stand in every period. Wikström (2000) includes fixed harvesting costs but uses otherwise simplified models and unnecessary restrictions in optimization. However, Wikström (2000) is the first study in which the harvesting interval is allowed to vary (Rämö and Tahvonen 2016).

The Finnish Forest Act was reformed in the beginning of 2014. The biggest reform was the removal restrictions that had practically banned continuous cover management for 70 years. This allows the forest owner to apply alternative management methods for the even-aged clearcut. The Forest Act has also given space for business concentrating solely on the continuous cover management. However, the number of studies on continuous cover forestry is quite low (Rämö and Tahvonen 2015). Hence, the optimal choice between continuous cover management and clearcut management has remained quite unclear.

Hanewinkel (2002) offers a discussion on the problems faced in the profitability comparisons between continuous cover and clearcut management. Tahvonen (2009), is the first “new wave” study on the idea on choosing the management regime optimally. In Pukkala et al. (2010, 2011) continuous cover management is economically optimal for both Norway spruce (*Picea abies* (L.) Karst.) and Scots pine (*Pinus sylvestric* L.), even with very low interest rates. Medium fertile Scots pine stands in Northern Finland and fertile Norway spruce stand in Southern Finland, with low interest rate are the only exceptions in Pukkala et al. (2010). However, the results were acquired using the criticized static investment-efficient model. Also, they determine the postharvest tree distributions using the Weibull distribution, instead of optimizing them freely. Andreassen and Øyen (2002) on the other hand present that clearcut is economically superior to any other type of forestry in Norway spruce stands. In contrast Tahvonen et al. (2010) use dynamic optimization with a transition matrix model and show that continuous cover management can be economically optimal in Norway spruce stands.

Tahvonen and Rämö (2016) present a bi-level optimization problem for both continuous cover management and clearcut management, where the timing of harvests and the intensity of harvests are solved sequentially. They show that continuous cover management can be optimal in Norway spruce stands. This method developed by Tahvonen (2015a and b) presents a theoretically sound economic optimization model that uses empirically detailed, size-structured ecological growth model with both fixed and variable harvesting costs, and solves the optimal management regime endogenously. This questions the previously dichotomous division between the management regimes as a self-evident truth. The bi-level optimization requires computation time, thus alternative methods like genetic algorithms can help to speed up the computation (Rämö and Tahvonen 2016). Sinha et al. (2017) present a genetic algorithm to solve this bi-level optimization of harvest timing and harvest intensity.

Earlier economic studies on forestry in Nordic conditions have mainly been based on three ecological models of (Pukkala et al. (2009), Bollandsås et al. (2008), Kolström (1993)). However, the optimization results may vary depending on the ecological model used (Ramo and Tahvonen 2014). Thus, using and comparing different ecological models would be interesting.

1.2 Purpose of this study

There exists a gap in the research that uses economically correct models to determine the optimal forest management regime: they do not include Scots pine stands. Continuous

cover management depends on natural regeneration instead of artificial regeneration. For this reason, the differences in natural regeneration between Norway spruce and Scots pine may have important implications for the viability of continuous cover forestry. While Norway spruce is a shade-tolerant species that regenerates even under a canopy, Scots pine is a shade-intolerant species that requires light and therefore cannot manage well as an understory.

The first research question of this study is whether continuous cover management performs less favorably for Scots pine compared to Norway spruce when the comparison of the management regimes is based on a theoretically sound economic optimization model. Using a size-structured transition matrix model, this question is analysed using two different empirically estimated ecological growth models. The second research question is to analyse whether the economic optimization results are dependent on the ecological growth models used (Bollandsås et al. (2008), Pukkala et al. (2013)). The economic optimization model applied is the model developed in Tahvonen and Rämö (2016). This model includes fully flexible optimization of harvest timing. My study is the first using this model to determine optimal solutions for Scots pine and first that fully applies this model with the growth model by Pukkala et al. (2013). Additionally, the optimal solutions will be analysed with respect to the Finnish 2014 forest legislation.

2 Size-structured optimization problem and economic parameters

This study uses the ecological models in a transition matrix or stage-structured setup, which are well tractable in numerical analyses. The transition matrix describes growth, ingrowth (number of naturally regenerated trees entering the smallest size class), and mortality of trees, in classified discrete size classes. Each size class has a characteristic average tree height and tree volume. The growth, ingrowth, and mortality describe the transition of trees from a given size class to the next and give each size class its own characteristics of development. (Getz and Haight 1989, p. 230–239.)

The optimization problem is constructed as in Tahvonen and Rämö (2016). Let the number of trees of a specific species in size class s , at the beginning of period t , be denoted by x_{st} , $s = 1, \dots, n, t = t_0, t_0 + 1, \dots, T$. Further assume that $T \in [t_0, \infty]$ and that stand state is denoted by $\mathbf{x}_t = x_{1t} + x_{2t}, \dots, x_{nt}$. The fraction of trees that move to the next size class in the end of each period t is denoted by $0 \leq \alpha_s(\mathbf{x}_t) \leq 1, s = 1, \dots, n-1$. The fraction of trees that die during period t is given as $0 \leq \mu_s(\mathbf{x}_t) \leq 1, s = 1, \dots, n$. Thus, the fraction of trees that stay in the same size class during period t can be given as $1 - \alpha_s(\mathbf{x}_t) - \mu_s(\mathbf{x}_t) \geq 0$. Let

the model also include natural regeneration of trees by function ϕ , that depends on the stand state \mathbf{x}_t . Let $h_{st}, s = 1, \dots, n$, denote trees of size class s that are harvested at the end of each time period and let $\mathbf{h}_t = (h_{1t}, h_{2t}, \dots, h_{nt})$.

Costs from an artificial regeneration are denoted by $w \geq 0$. Revenues and costs are given separately for thinning and clearcut, both depending on the size and the amount of trees that are harvested. Thinning revenues and costs are denoted by $R(\mathbf{h}_t)$ and $C_{th}(\mathbf{h}_t)$ respectively. Similarly clearcut revenues and costs are denoted by $R(\mathbf{x}_T)$ and $C_{cc}(\mathbf{x}_T)$ respectively. The fixed harvesting costs are denoted by C_f . The discrete time discount factor is denoted by $b^\Delta = 1/(1+r)^\Delta$, where r is the interest rate and Δ is the length of the period. Because of the fixed harvesting costs, harvesting may not be optimal in every period t , unlike for example in classic harvesting model by Clark (1976). This is noted by including binary variables $\delta_t : Z \in \{0,1\}, t = t_0, t_0 + 1, \dots$ and the Boolean operator $h_t = \delta_t h_t$. Thus, when $\delta_t = 1$, the harvesting $h_{st} \geq 0, s = 1, \dots, n$ can be freely optimized and when $\delta_t = 0$, it leads to $\delta_t h_t = 0$ and no harvesting can occur. In sum the optimizing problem over the infinite time horizon can be given as:

$$J(\mathbf{x}_{t_0}, T) = \max_{\{h_{st}, \delta_t, t=t_0, \dots, T, T\}} \frac{-w + \sum_{t=t_0}^{T-1} [R(\mathbf{h}_t) - C_{th}(\mathbf{h}_t) - \delta_t C_f] b^{\Delta(t+1)} + [R(\mathbf{x}_T) - C_{cc}(\mathbf{x}_T) - \delta_T C_f] b^{\Delta(T+1)}}{1 - b^{\Delta(T+1)}} \quad (1)$$

subject to

$$x_{1,t+1} = \phi(\mathbf{x}_t) + [1 - \alpha_1(\mathbf{x}_t) - \mu_1(\mathbf{x}_t)]x_{1t} - h_{1t} \quad t = t_0, \dots, T \quad (2)$$

$$x_{s+1,t+1} = \alpha_s(\mathbf{x}_t)x_{st} + [1 - \alpha_{s+1}(\mathbf{x}_t) - \mu_{s+1}(\mathbf{x}_t)]x_{s+1,t} - h_{s+1,t} \quad s = 1, \dots, n-1, t = t_0, \dots, T \quad (3)$$

$$h_{st} = \delta_t h_{st} \quad s = 1, \dots, n, t = t_0, \dots, T \quad \delta_t : Z \in \{0,1\} \quad (4)$$

$$x_{t_0}, \text{ given} \quad (5)$$

where $T \in [t_0, \infty]$ and the non-negativity conditions $x_{st} \geq 0, h_{st} \geq 0, t = t_0, \dots, T, s = 1, \dots, n$ must hold.

The objective function (1) resembles the classic optimal rotation model, but it takes into account the optimal harvest timing and the possibility that no clearcut is applied. This occurs when T becomes infinitely long. Choosing T hence defines the optimal choice between clearcut and continuous cover solutions.

As the classic rotation model, the economic problem (1)–(5) is valid under a set of assumptions (Amacher et al. 2009).

- 1 All future prices and costs are constant and known
- 2 Future interest rates are constant and known
- 3 The growth function of stand is known
- 4 Perfect markets for forestland
- 5 Perfect capital markets

Volumes of trees are calculated as described by Heinonen (1994). This study uses 10 different size classes, ranging according to their mean diameter breast height (d_s , where $s = 1, \dots, 10$ is the given size class), from 7.5 cm to 52.5cm with a 5 cm intervals. In Finland timber has been traditionally divided into sawlog and pulpwood. For Scots pine, the minimum mean diameter for sawlog is 17.5–22.5cm depending on site productivity. Scots pine logs contain sawlog at $d_s = 17.5$ cm at average site productivity (SI_{11}) and at $d_s = 22.5$ cm on poor site productivity (SI_6). Norway spruce contains sawlog at $d_s = 22.5$ cm independently of site productivity. The increase of sawlog content leads into decreased pulpwood-ratios. For example, with an average site productivity (SI_{11}), with $d_s = 32.5$ cm, the pulpwood-ratios for Scots pine and Norway spruce are 4.7% and 7.6% respectively.

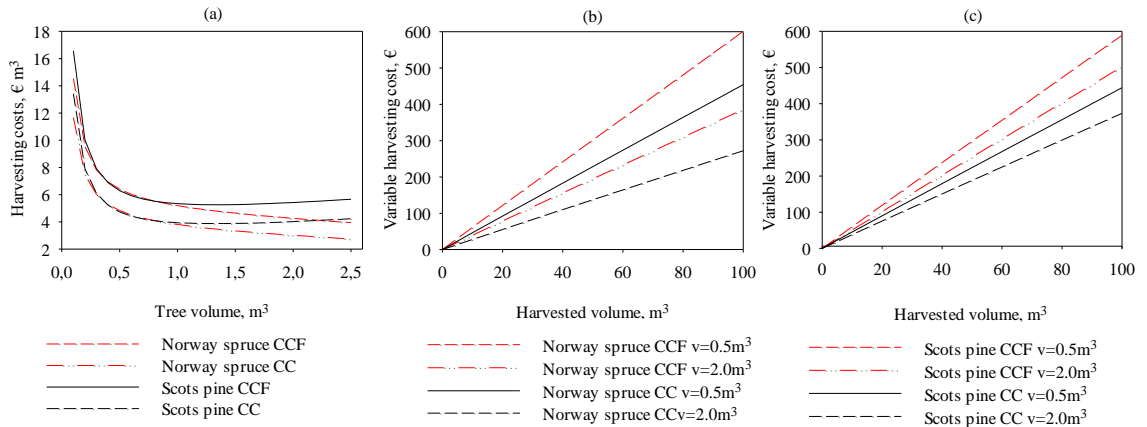
Table 1: Parameter values for the harvesting cost functions.

Norway spruce								
i	C_{i0}	C_{i1}	C_{i2}	C_{i3}	C_{i4}	C_{i5}	C_{i6}	C_{i7}
th	2.100	1.150	0.412	0.758	0.180	1.000	2.272	0.535
cc	2.100	1.000	0.397	0.758	0.180	1.000	1.376	0.393
Scots pine								
i	C_{i0}	C_{i1}	C_{i2}	C_{i3}	C_{i4}	C_{i5}	C_{i6}	C_{i7}
th	2.100	1.150	0.547	0.196	-0.308	1.000	2.272	0.535
cc	2.100	1.000	0.532	0.196	-0.308	1.000	1.376	0.393

The harvesting and hauling costs for both clearcut and thinning, are from a detailed empirical model by Nurminen et al. (2006). The model was developed using modern medium-sized single grip harvesters, forwarders and their professional operators, and can be given as:

$$C_i = C_{i0} C_{i1} \sum_{s=1}^n h_{st} (C_{i2} + C_{i3} v_s - C_{i4} v_s^2) + C_{i5} \left[C_{i6} \sum_{s=1}^n h_{st} v_s + C_{i7} \left(\sum_{s=1}^n h_{st} v_s \right)^{0.7} \right] \quad i = th, cc \quad (6)$$

Table 1 presents the parameter values for (6). The costs depend on the amount and the size of harvested trees. The parameters are given separately for C_{ij} , $i = th$ (thinning), CC (clearcut) and $j = 0, \dots, 7$. C_{i0} is the cost per minute spent cutting down a tree and C_{i1} the time (minutes) spent cutting down one tree and moving to a next one. C_{i5} and C_{i6} are costs per minute and time hauling respectively. The parameter $C_{th1} = 1.150$ is larger than $C_{cc1} = 1.000$, describing that it is more costly to move from one tree to another during thinning than in clearcut (Tahvonen and Rämö, 2016).



Figures 1a–c: Harvesting and hauling costs per tree and per volume. Note: (a) per tree and (b and c) per volume.

Figures 1a–c show some main properties of the harvesting cost function (7). Harvesting and hauling costs are smaller in clearcut management than in continuous cover management for both species. However, there are differences between the species. The costs of harvesting one m^3 of Scots pine start rising with large trees. This does not occur when harvesting Norway spruce. The larger the trees, the more remarkable the cost differences between the species.

Two different timber quality pricings are used (sawlog and pulpwood). For Scots pine, the roadside prices for sawtimber and pulp are set to $€58.65 m^3$ and $€30.51 m^3$ respectively. Similarly, for Norway spruce, the roadside prices for sawlog and pulpwood are set to $€58.44 m^3$ and $€34.07 m^3$ respectively. These prices are same in both thinning and clearcut. Subtracting harvesting and hauling costs from the roadside prices forms the stumpage price.

A more productive site leads into more valuable trees and the value of a single tree rises with mean tree diameter d_s . This follows since when a tree matures, it starts containing higher amounts of valuable sawlog. Tree value differences between site productivities are

species dependent. With large d_s , a single Scots pine tree is more valuable than a single Norway spruce tree independently of site productivity.

3 Ecological growth models

Many early ecological models had a basic unwarranted feature that the number of trees in each size class rockets by exponential growth, if not harvested (Buongiorno and Michie 1980). This study uses two different ecological models with appropriate density dependence and do not have this kind of problem (See Figures 2a–d). The ecological models used are a Finnish model by Pukkala et al. (2013) and a Norwegian model by Bollandsås et al. (2008). The tree species used are Scots pine and Norway spruce, the two most common conifer species in Scandinavia.

3.1 Bollandsås et al. (2008) model for Norway spruce and Scots pine

The data for the ecological model by Bollandsås et al. (2008) comes from the National Forest Inventory of Norway and consists of a total of 7241 plots. For comparison, the number of plots is 3.78 times higher than the number of plots used in developing the Finnish model by Pukkala et al. (2013). The functions for trees moving to the next size class, a probability of a tree dying, a probability of ingrowth and the number of ingrowth by Bollandsås et al. (2008) model are presented in (7)–(9). The single tree diameter increment models ($I_s(\mathbf{x}_t)$) of both of the ecological models used can be modified to fit the transition matrix model by dividing them with the width of the size class q , i.e. $\alpha_s(\mathbf{x}_t) = q^{-1}(I_s(\mathbf{x}_t))$, $s = 1, \dots, n$, $t = 0, 5, \dots$, where s denotes the size class and t time (in 5-year periods). Now the fraction of trees that transfers to the next size class during the 5-year period can be given as:

$$\alpha_s(\mathbf{x}_t) = q^{-1}(a_0 + a_1 d_s - a_2 \times 10^{-5} d_s^2 - a_3 B_s(\mathbf{x}_t) + a_4 SI - a_5 B(\mathbf{x}_t) - a_6 L) \quad (7)$$

The parameters $a_0 - a_{18}$ in (7)–(9) are species-specific regression coefficients and they are presented in Appendix IV. Function $B(\mathbf{x}_t)$ is the basal area of the plot ($\text{m}^2 \text{ha}^{-1}$), defined as $B(\mathbf{x}_t) = \sum_{s=1}^n \gamma_s x_{st}$ and $B_s(\mathbf{x}_t) = \sum_{i=s+1}^n \gamma_i x_{it}$, $s = 1, \dots, n-1$, where γ_s is the basal area per tree in a size class s . Symbol L is latitude and represents the climate conditions and is set to 61.9°N , to represent the climate of Central Finland and to make the results comparable with the earlier studies using the same model and Pukkala et al. (2013) model. The probability of a tree in size class s dying in the next 5-year period is given as:

$$\mu_s(\mathbf{x}_t) = \left[1 + e^{(a_7 + a_8 d_s + a_9 d_s^2 + a_{10} B(\mathbf{x}_t))} \right]^{-1} \quad (8)$$

The number of ingrowth in the next 5-year period is given as:

$$\phi(\mathbf{x}_t) = \frac{a_{11} B(\mathbf{x}_t)^{a_{12}} SI^{a_{13}} Y^{a_{14}}}{1 + e^{-(a_{15} + a_{16} B(\mathbf{x}_t) + a_{17} SI + a_{18} Y)}} \quad (9)$$

Variable Y in (9) is the percentage of the basal area of the referred species in the stand. However, this study only focuses to single-species stands. The number of ingrowth in (9) is obtained by multiplying the number of ingrowth (when the probability of ingrowth would be 100%), with the probability of ingrowth.

3.2 Pukkala et al. (2013) model for Norway spruce and Scots pine

The data for Pukkala et al. (2013) ecological model is from four different data sets from Finland, consisting of a total of 1914 plots. The diameter increment function of Pukkala et al. (2013) model can be modified to fit the transition matrix similarly as Bollandsås et al. (2008) model. Thus, the fraction of trees that transfer to the next size class during the 5-year period can be given as:

$$\alpha_s(\mathbf{x}_t) = q^{-1} e^{\left(b_0 + b_1 \sqrt{d_s} + b_2 d_s + b_3 \ln(TS) + b_4 OMT + b_5 VT + b_6 CT + b_7 \ln(B(\mathbf{x}_t)) + b_8 \frac{B_{s,pine}(\mathbf{x}_t)}{\sqrt{d_s+1}} + b_9 \frac{B_{s,spruce}(\mathbf{x}_t)}{\sqrt{d_s+1}} \right)} \quad (10)$$

In (10)–(12) parameters $b_0 - b_{25}$ are the species-specific regression coefficients and they are presented in Appendix IV. Parameter TS is the temperature sum of the stand. We set temperature sum to 1100, to represent the climate in Central Finland. OMT , VT and CT are Finnish forest types which indicate the site productivity. Site MT is the reference site when all other forest type indicators are set as zero.

The estimated mortality in size class s within the 5-year period is given as:

$$\mu_s(\mathbf{x}_t) = 1 - \left[1 + e^{-(b_{10} + b_{11} \sqrt{d_s} + b_{12} d_s + b_{13} \sqrt{B_{s,pine}(\mathbf{x}_t)} + b_{14} \sqrt{B_{s,spruce}(\mathbf{x}_t)} + b_{15} Period)} \right]^{-1} \quad (11)$$

The $Period$ in (11) is the length between two measurements collecting the data which is only significant to the survival of Norway spruce. We set $Period$ to 5 which is the most common time period between the measurements in the data.

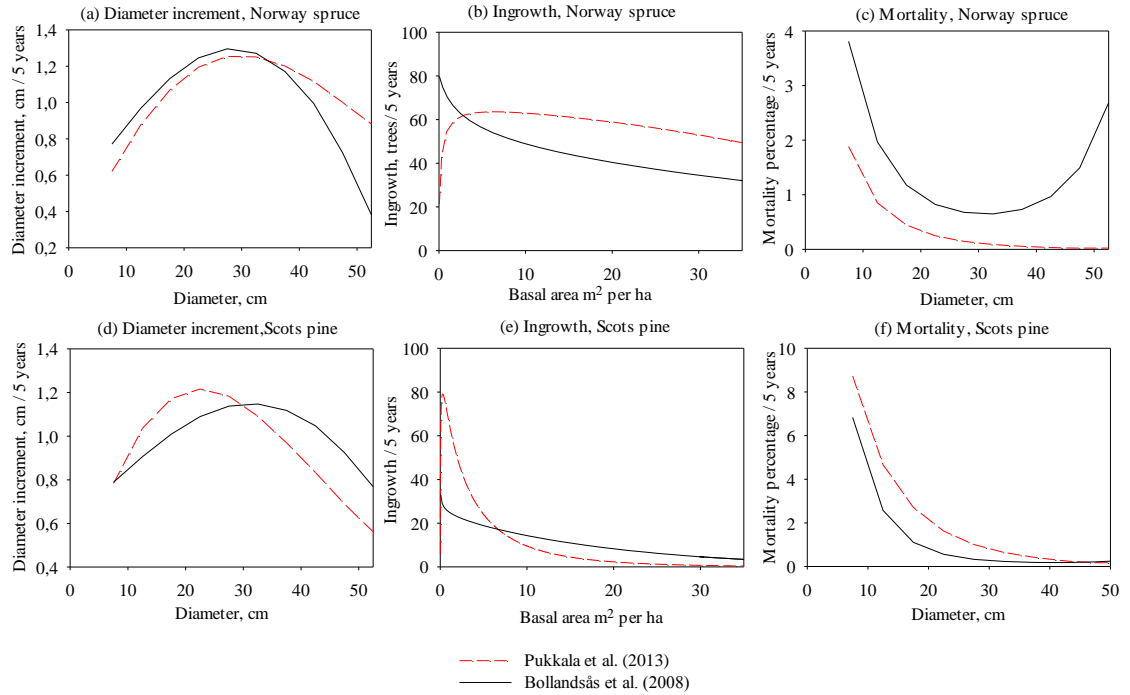
The number of ingrowth in the next 5-year period is given as:

$$\phi(\mathbf{x}_t) = \frac{e^{(b_{16}+b_{17}\sqrt{B(\mathbf{x}_t)})}}{1 + e^{-(b_{18}+b_{19}\ln(B(\mathbf{x}_s))+b_{20}\sqrt{B_{pine}(\mathbf{x}_t)}+b_{21}B_{pine}(\mathbf{x}_t)+b_{22}B(\mathbf{x}_t)+b_{23}OMT+b_{24}VT+b_{25}CT)}} \quad (12)$$

The numerator of (12) is the number of ingrowth (when the probability of ingrowth is 100%) and the denominator is the probability of ingrowth. The basal area of Scots pine ($B_{pine}(\mathbf{x}_t)$) is used as an additional predictor of ingrowth because in the mixed stands it influences the other species ingrowth.

3.3 Comparison of the ecological models

Both ecological models mostly comprise of very similar pieces. Geographical location is taken into consideration in Pukkala et al. (2013) model by using temperature sum, while Bollandsås et al. (2008) model uses latitude instead. However, comparing Bollandsås et al. (2008) and Pukkala et al. (2013) models is not totally problem-free. Site productivity with Bollandsås et al. (2008) is determined by site indices which are influenced mostly by the temperature and humidity. Whereas Pukkala et al. (2013) model uses the Finnish forest types Calluna (*CT*), Vaccinium (*VT*), Myrtillus (*MT*) and Oaxlis-Myrtillus (*OMT*), by Cajander (1949), where the limiting factor for site productivity is mainly the aridness of the soil. However, Pukkala et al. (2013) and Bollandsås et al. (2008) models can be considered comparable based on their productivity. To do this, the site indices are set to equal Finnish forest types in following way: $SI_6 = CT$, $SI_{11} = VT$, $SI_{15} = MT$, and $SI_{17} = OMT$. The subindex of *SI* is the height of dominant trees at the age of 40 years in meters. These site indices and forest types are referred to as SI_6 , SI_{11} , SI_{15} , and SI_{17} from now on. After artificial regeneration, trees are set to reach the smallest size class depending on the *SI* in following way: $SI_{11} = 25$ years, $SI_{15} = 20$ years and $SI_{17} = 15$ years as in Tahvonnen and Rämö (2016), with the addition of $SI_6 = 30$ years.



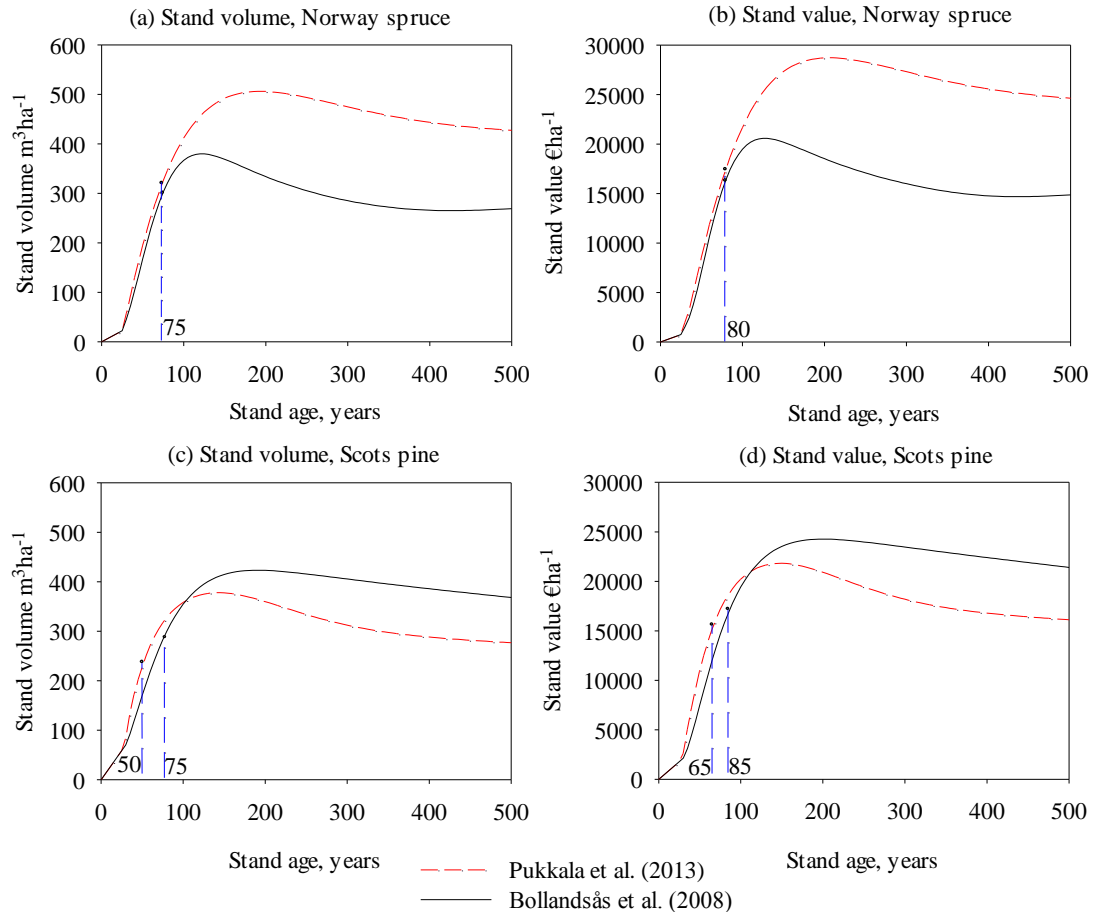
Figures 2a–f: Diameter increment, ingrowth, and mortality for both ecological models. Note: SI_{11} , For diameter increment and mortality the stand state is $\mathbf{x} = [300, 180, 120, 80, 50, 30, 18, 10, 2, 0]$.

To present an uneven-aged structure, the tree distribution in Figures 2a,c,d, and f is $\mathbf{x} = [300, 180, 120, 80, 50, 30, 18, 10, 2, 0]$, as in Rämö and Tahvonon (2014). As can be observed for Norway spruce, Pukkala et al. (2013) model predicts smaller diameter increment with small trees than Bollandsås et al. (2008) model. For Scots pine, the situation is reversed, with higher diameter increment achieved with small trees using Pukkala et al. (2013) model. For both ecological models and tree species, the diameter increment increases with tree diameter until maximum increment is reached, and then begins to decrease due to the maturing of trees.

The Bollandsås et al. (2008) model predicts a higher probability of mortality for Norway spruce than Pukkala et al. (2013) model. For Scots pine, the situation is reversed, with Pukkala et al. (2013) model predicting a higher possibility of mortality.

For Norway spruce, the dependence of ingrowth from the basal area is concave when using Pukkala et al. (2013) model and convex using Bollandsås et al. (2008) model. Considering a basal areas larger than $5\text{m}^2\text{ha}^{-1}$, the Pukkala et al. (2013) model predicts much higher numbers of ingrowth than Bollandsås et al. (2008) model. For Scots pine, the Pukkala et al. (2013) model predicts an increase of ingrowth with extremely low basal areas, until quickly reaching a maximum and starting to shape convexly. Bollandsås et al.

(2008) model predicts a convex dependence of ingrowth from the basal area, even with extremely low basal areas. The predictions of ingrowth are the most distinctive differences between the two models and species.



Figures 3a–d: Developments of stand volume and value of standing trees without thinning. Note: SI_{11} , harvesting costs are zero, $w = \text{€}0\text{ha}^{-1}$, $\mathbf{x}_0 = [1750, \dots, 0]$.

Figures 3a–d present the developments of stand volume and value of standing trees when thinning is excluded for Norway spruce and Scots pine. The blue dotted lines present the maximum sustained yield solutions for Figures 3a and 3c and forest rent solutions for Figures 3b and 3d. With both species and ecological models, to achieve maximum sustained yield, the stand should be clearcut before the forest rent cutting age is reached. This is caused by the increased amount of valuable sawlog in the older/larger trees.

Both species and ecological models seem to behave similarly by first increasing in volume until reaching a maximum and beginning to decrease due to increased mortality and decreased diameter increment and ingrowth. It can be seen from Figures 3a–b that the increase of stand volume and value of standing trees for Norway spruce is fairly similarly shaped with both models, but the maximum volumes are higher with Pukkala et al. (2013).

The differences in mortality displayed in Figures 3a–f, carry over to Figures 3a–d. For example, Norway spruce reaches maximum stand volume earlier with Bollandsås et al. (2008) model, compared to Pukkala et al. (2013) model where mortality is higher.

For Scots pine, Pukkala et al. (2013) model predicts a much steeper early growth for both volume and value than Bollandsås et al. (2008) model. This steeper growth leads into shorter maximum sustained yield rotations. Steep growth follows since Pukkala et al. (2013) has maximum growth with smaller trees, and higher ingrowth (see Figures 3d and 3e). The model by Pukkala et al. (2013) also reaches the maximum in both volume and value earlier than Bollandsås et al. (2008) model. This derives from the combination of higher mortality, ingrowth and earlier peak growth of Pukkala et al. (2013) model. However, the differences in maximum volume and value between the two models are not as high as with Norway spruce.

4 Computational methods

The optimization problem is solved as a mixed-integer nonlinear programming problem. This structure follows due to the fixed harvesting costs that make it necessary to optimize harvest timing. The variables $\delta_t : Z \in \{0,1\}, t = t_0, t_0 + 1, \dots$ for harvest timing are integers, while $h_{st}, s = 1, \dots, n, t = t_0, t_0 + 1, \dots$ for harvest intensity are continuous. Bi-level optimization described by Colson et al. (2007) is used, where the binary variables δ_t - that represent harvest timing are taken as an upper-level problem, and the continuous variables defining the harvest intensity as a lower-level problem. The hierarchical relationship between the autonomous variables is a prominent feature of bi-level optimization (Colson et al. 2007). Our optimization also optimizes the rotation length, so the problem is actually a “tri-level” optimization.

As in Tahvonen and Rämö (2016), if the bare land value reaches maximum value with some $T \in [40, 180]$, the rotation is considered finite. The continuous cover steady state harvesting intervals are set to be reached at least within 7 harvests. This is enough to imply that the steady state interval is actually reached earlier, i.e. this upper bound is not binding. The time horizon is set to 500 years, which is long enough that further lengthening no longer changes the transition path towards the steady state. The discrete time computation is carried out by Knitro optimization software versions 9.1 and 10.1, using gradient-based methods for the continuous variables and genetic and hill-climbing algorithms for the integers. Because of the potential nonconvexities, several different randomly chosen initial points are used for the search algorithm.

5 Results

Because of the two different ecological models and the complexities of the optimization problem, the setup is first analysed under simplifications. This allows us to see further how the two ecological models differ and how a change in the economic and/or ecologic factors influence the solutions.

We first present the classic optimal rotation solutions without thinning. Then we present continuous cover solutions that maximize sustained yield and next the continuous cover solutions given the harvesting interval is fixed to 20 years. Finally, we consider solutions with optimized harvest timing, which are then used to define the optimal management regime. Interest rates of 1% and 3% are used, which can be considered typical in Finland.

5.1 Classic rotation model solutions

Obtaining the classic rotation solutions, δ_t from (1)–(5) is set to zero for $t \in [t_0, T - 1]$ and harvesting is based on clearcuts only. As shown in table Table 2, increasing site productivity decreases the rotation length. Faster growth increases the bare land value and this increases the opportunity costs of lengthening the rotation. This effect of site productivity is independent of the ecological model used.

Table 2: Classic rotation model solutions without thinning for Norway spruce.

Norway spruce	Pukkala et al. (2013)		Bollandsås et al. (2008)	
	SI_6	SI_{11}	SI_6	SI_{11}
$r = 0.01$				
Rotation (years)	100	80	90	75
Average yield m^3ha^{-1}	2.8	4.4	2.1	4.0
Bare land value €ha^{-1}	6 099	11 403	5 085	11 705
$r = 0.03$				
Rotation (years)	70	60	70	60
Average yield m^3ha^{-1}	2.6	4.1	2.0	3.8
Bare land value €ha^{-1}	759	1 719	651	1 746

Note: Regeneration cost (w) = $\text{€}0\text{ha}^{-1}$ and natural regeneration maintained. Initial size distribution of $\mathbf{x}_0 = [1750, \dots, 0]$.

An increase in the interest rate leads to shorter rotations since the opportunity costs of delaying harvest revenues are increased, and forest cannot continue producing sufficient capital productivity. Increasing the interest rate decreases the average annual yield. This is a consequence of operating below the maximum sustained yield rotation level (Note the shape of the volume development curve in Figures 3a and c). Ecological models have

a significant effect on the optimal solutions. Using Bollandsås et al. (2008) model, bare land values obtained with SI_6 are lower than with Pukkala et al. (2013) model. When site productivity is increased to SI_{11} , the differences in the bare land values change and Bollandsås et al. (2008) produces higher bare land values. However, the rotation lengths are very similar between the models

Table 3 presents classic rotation results for Scots pine stands. Interest rate and site productivity both have a clear effect on the rotation length and the average yield. Increasing the interest rate results to shorter rotations, as does increasing the site productivity. The explanations for these effects are the same as for Norway spruce. The higher interest rate decreases the mean annual yields, with one exception. Using Pukkala et al. (2013) model, when increasing the interest rate from 1% to 3% with SI_{11} , the mean annual yield increased. The reason for this is that the 50 years rotation with 3% interest rate is, in fact, the maximum sustained yield rotation length (visible in Figure 3c). Thus, interest rate effect on the mean annual yield depends on whether the rotation length is longer or shorter than the maximum sustained yield rotation.

Table 3: Classic rotation model solutions without thinning for Scots pine.

Scots pine	Pukkala et al. (2013)		Bollandsås et al. (2008)	
	SI_6	SI_{11}	SI_6	SI_{11}
$r = 0.01$				
Rotation (years)	85	60	95	75
Average yield m^3ha^{-1}	2.4	4.5	1.9	3.8
Bare land value $€ha^{-1}$	6 097	13 951	4 471	11 421
$r = 0.03$				
Rotation (years)	70	50	75	60
Average yield m^3ha^{-1}	2.5	4.6	1.9	3.7
Bare land value $€ha^{-1}$	835	2 456	539	1 747

Note: Regeneration cost (w) = 0 $€ha^{-1}$ and natural regeneration maintained. Initial size distribution of $\mathbf{x}_0 = [1750, \dots, 0]$.

Interest rate and site productivity also determine the diameter of harvested trees. This shows as a change in the harvesting and hauling costs and in the sawlog-ratios. Table 4 shows that increasing the interest rate increases the total harvesting costs per tree and per m^3 . This is due to the decreased rotation length that increases the number of small trees. This implies that more trees are needed to harvest per one cubic meter of wood. More productive sites leads into decreased harvesting costs both per tree and per m^3 . This occurs since trees move to the next size class faster, thus in the harvest, the trees are larger and cheaper to harvest on average.

Table 4: The effects of interest rate and site productivity on harvesting and hauling costs and sawlog-ratios of harvested trees in classic rotation model solutions.

Scots pine	Pukkala et al. (2013)		Bollandsås et al. (2008)	
	SI_6	SI_{11}	SI_6	SI_{11}
$r = 0.01$				
Total harvesting costs per tree	0.64	0.63	0.68	0.61
Total harvesting costs per m^3	7.82	7.37	9.15	6.97
Sawlog-ratio (%)	70	73	64	76
$r = 0.03$				
Total harvesting costs per tree	0.71	0.69	0.74	0.68
Total harvesting costs per m^3	10.25	9.30	12.48	9.16
Sawlog-ratios (%)	56	61	47	63
Norway spruce	Pukkala et al. (2013)		Bollandsås et al. (2008)	
$r = 0.01$				
Total harvesting costs per tree	0.83	0.78	0.80	0.70
Total harvesting costs per m^3	9.79	8.39	9.91	7.52
Sawlog-ratios (%)	60	57	53	67
$r = 0.03$				
Total harvesting costs per tree	0.91	0.84	0.87	0.78
Total harvesting costs per m^3	12.77	10.19	12.87	9.42
Sawlog-ratio (%)	42	49	42	52

Note: Total harvesting costs consists of harvesting and hauling costs. Costs in €. Initial size distribution of $\mathbf{x}_0 = [1750, \dots, 0]$.

The increased number of harvested small trees with higher interest rate implies changes in the average sawlog-ratios. Increasing interest rate shortens the optimal rotation, leading to harvesting of younger/smaller trees, thus containing less sawlog. The effect of site productivity on the sawlog-ratio is, however, more complex. Shorter rotation leads to decreases in sawlog, while increased site fertility increases the sawlog content. The direction of the sawlog-ratio change depends on the relative forces of these two effects.

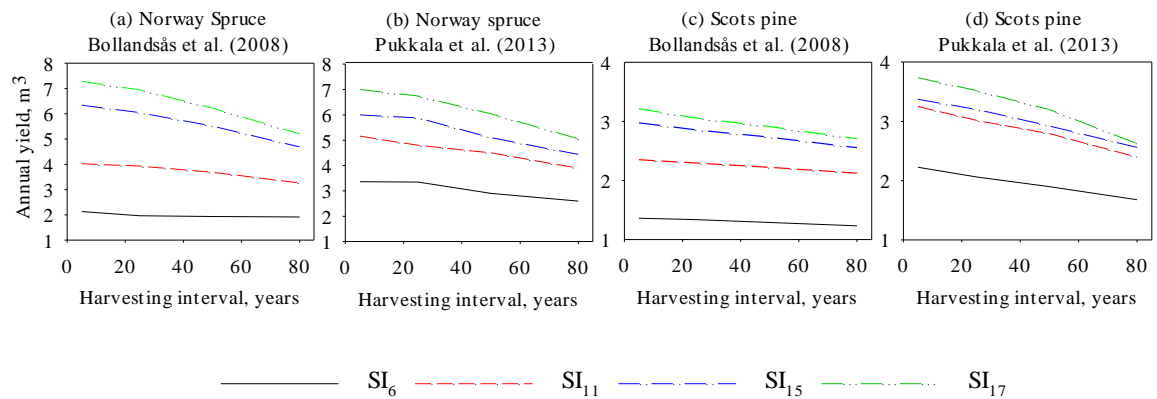
5.2 Continuous cover solutions under maximum sustained yield objective

Under continuous cover management the stand is never harvested completely. This means that all revenues from the stand come from thinning. Finding out the optimal continuous cover solutions under the maximum sustained yield objective, all cutting costs plus interest rate are set to zero and the price for pulpwood and sawlog is set to unity. Thus, the sawlog percentage of a tree harvested does not influence the solutions and the objective becomes the maximization of biological production. This makes it optimal to harvest at every period t (every 5 years in optimization), almost without exceptions. The

optimal continuous cover solutions under maximum sustained yield objective can be used to gain more understanding about the characteristics and differences of the two ecological models.

Figures 4a–d display how the steady state annual yield (m^3ha^{-1}) depends on the site productivity and harvesting interval. Increasing site productivity increases the annual yield with both species and growth models. Shortening the harvesting interval increases the annual yield with both species and models. This points out that the 5-year harvesting interval, which is the minimum in the optimization model, gives us the best possible solutions for the continuous cover under maximum sustained yield objective.

The largest differences between the ecological models are found in the low productive sites. The Pukkala et al. (2013) ecological model predicts much higher volume outputs with SI_6 and SI_{11} than the Bollandsås et al. (2008) model. This difference mostly follows from ingrowth. The annual yields produced by the two models can be considered realistic and thus they increase the confidence towards the validity of the results.



Figures 4a–d: Average annual yields varying harvesting interval in continuous cover management under maximum sustained yield objective.

It should be kept in mind that the Figures 4a–d do not include the option of applying clearcut and artificially planted new trees. This means that the steady states rely solely on natural regeneration. This explains why the annual yield is a decreasing function of harvesting interval. In reality, the best possible maximum sustained yield solution is most likely to be achieved with clearcut and artificial regeneration, as shown by Tahvonen (2009). An example of this is in Table 3, where the optimal clearcut solution with Pukkala et al. (2013) model, with interest rate of 3% and SI_{11} , is, in fact, the maximum sustained

yield solution. The average annual yield in this Table 3 solution is $4.6\text{m}^3\text{ha}^{-1}$, while in Figure 5b the maximum annual yield is only $3.2\text{m}^3\text{ha}^{-1}$.

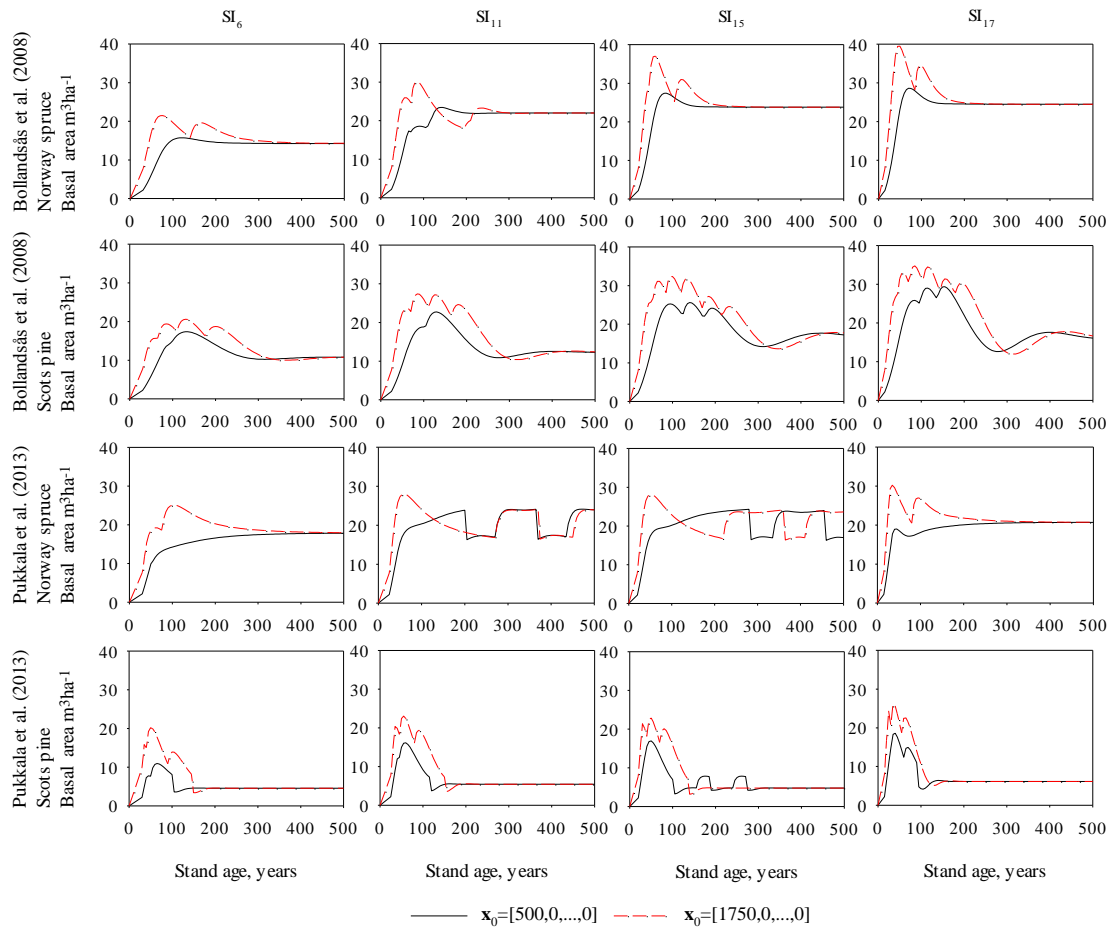
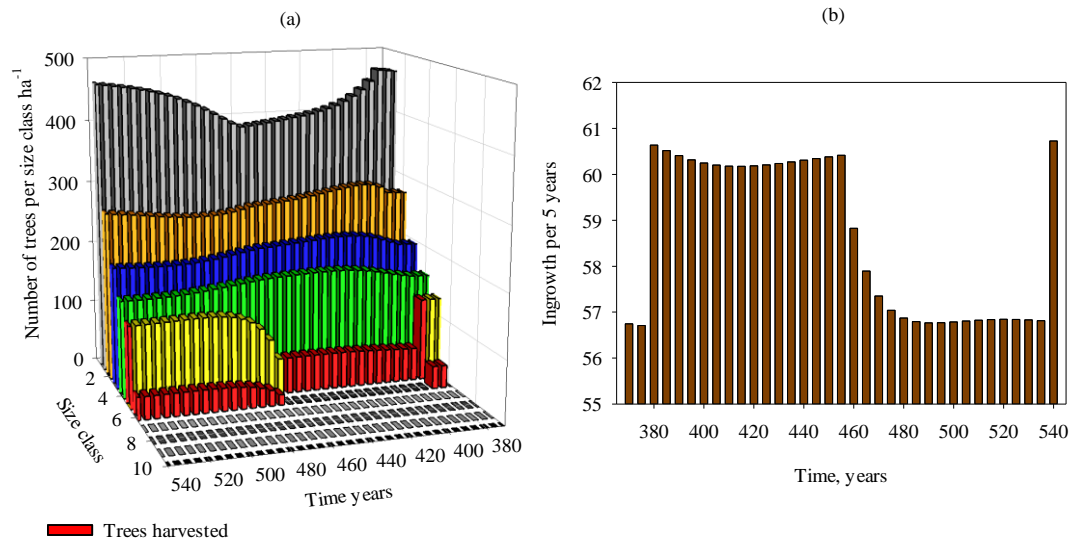


Figure 5: Development of stand basal areas from two different initial states in continuous cover management under maximum sustained yield objective. Note: Initial states of $\mathbf{x}_0 = [500, \dots, 0]$ and $\mathbf{x}_0 = [1750, \dots, 0]$ used.

Figure 5 suggests that the optimal steady states are independent of the initial state of the stand. Two different initial states are used in Figure 5, $\mathbf{x}_0 = [500, \dots, 0]$ and $\mathbf{x}_0 = [1750, \dots, 0]$. The initial state of the stand only changes the transition time and toward the optimal steady state. For example using Pukkala et al. (2013) model on site SI_6 , Scots pine takes 110 years to reach the steady state when the initial amount of trees is 500ha^{-1} . When the initial amount of trees is increased to 1750ha^{-1} , it takes 190 years to reach the steady state.

Using Pukkala et al. (2013) model, some steady states have a cyclical pattern. Norway spruce with SI_{11} and SI_{15} does not settle in a steady state where the solution would remain constant. For example with SI_{11} , when the basal area reaches $24.0\text{m}^2\text{ha}^{-1}$ it becomes

optimal to reduce it to $16.3\text{m}^2\text{ha}^{-1}$. These non-constant steady state solutions of SI_{11} and SI_{15} remained even when generating 1000 different starting points in the optimization. The cyclical patterns are caused by the increase in ingrowth and diameter increment that the removal of the largest trees produces.



Figures 6a–b: Analysis of the cyclical steady state solution for Norway spruce. Note: Pukkala et al. (2013), SI_{11} , $\mathbf{x}_0 = [1750, \dots, 0]$.

Figures 6a–b display in more detail the cyclical steady state with SI_{11} . In the beginning, all trees entering size class 6 ($d_s = 32.5\text{cm}$) are harvested but after a switch, all trees from both size class 6 and 5 are harvested. The harvesting then continues with harvests from size class 5. When harvests switch to size class 5, there is an increase in the diameter growth in the smaller size classes. This causes small trees to rapidly move into larger size classes. This shows as a sharp decrease of the trees in the smallest size class and as a sequential increase of trees in larger size classes. When trees are again allowed to grow into size class 6, the stand starts developing into a state identical to the beginning of the cycle. When this state is fully achieved, another similar cycle begins. The removal of the trees in the biggest size class also increases the ingrowth (from 56.7 to 60.7 per 5 years). During the cycle, these new trees have time to grow into harvesting size. This solution allows higher average yield than any constant steady state. This suggests that producing trees in cohorts have potentially some advantages compared to smooth continuous cover solution and that our optimization method is capable of revealing these phenomena.

The steady states in Figure 5 depend strongly on the ecological model. Using Pukkala et al. (2013) model, the Scots pine steady state basal areas are low (ranging between $4.6\text{m}^2\text{ha}^{-1}$ and $6.1\text{m}^2\text{ha}^{-1}$). For comparison, using Bollandsås et al. (2008) model, the Scots

pine steady state basal areas vary between $10.0\text{m}^2\text{ha}^{-1}$ and $16.3\text{m}^2\text{ha}^{-1}$. This is due to the differences between the models, mainly in the ingrowth and diameter increment. The Pukkala et al. (2013) model allows high numbers of ingrowth with low basal areas and the peak diameter increment occurs with smaller trees than with Bollandsås et al. (2008) model.

Table 5 presents the diameter of harvested trees in the steady states of optimal continuous cover solutions under maximum sustained yield objective. The size of harvested trees depends strongly on the ecologic model and species. Using Pukkala et al. (2013) model, harvested trees are smaller independently of species used. This is a consequence of ingrowth differences. Using Bollandsås et al. (2008) model, the low ingrowth means that trees are larger, since volume maximization cannot be based on a high number of small trees. With Pukkala et al. (2013) model harvested trees are smaller with Scots pine than with Norway spruce (except in SI_{17}). This occurs because, in order to gain a sufficient number of ingrowth, the stand density must be kept low. Using Bollandsås et al. (2008) model the solution is reversed and the harvested trees are larger with Scots pine than with Norway spruce. The reason for this is the low ingrowth of Scots pine when using Bollandsås et al. (2008) model. This is one example of differences between the ecological models and species that can be observed in the optimization outcomes.

Table 5: Size of harvested trees in optimal continuous cover solutions under the maximum sustained yield objective.

	Pukkala et al. (2013)				Bollandsås et al. (2008)			
	SI_6	SI_{11}	SI_{15}	SI_{17}	SI_6	SI_{11}	SI_{15}	SI_{17}
Norway spruce								
Diameter of	25–	25–	25–	30–	30–	35–	35–	35–
harvested trees, cm	29.9	34.9	34.9	34.9	34.9	39.9	39.9	39.9
Scots pine								
Diameter of	20–	25–	25–	30–	40–	40–	50–	50–
harvested trees, cm	24.9	29.9	29.9	34.9	44.9	44.9	54.9	54.9

In reality, maximization of wood production is not a rational objective from the economic point of view. This is due to harvesting costs, interest rate and different pricing of different timber assortments. Still, as pointed out already by Samuelsson (1976), the yield maximizing objective (applied to a rotation model) has been popular and defended by foresters. While the yield maximizing continuous cover solutions are purely theoretical in the sense that they do not produce any economically useful guidelines, they do offer insight on the characteristics of the ecological models.

5.3 Continuous cover solutions with fixed harvesting interval

When trees are harvested at a fixed interval of 20 years (4 periods in the optimization), the only optimized variable is the harvested number of trees in each size class. All harvesting costs and prices are included in the optimization. Regeneration costs are set to zero since they do not change the optimal path towards the steady state. The fixed harvesting interval solutions are used to gain more information about the ecological models and to study the effect of optimizing the harvest timing.

Table 6: Bare land values in continuous cover solutions with fixed harvesting interval.

	Norway spruce		Scots pine	
Bollandsås et al.(2008)	SI_6	SI_{11}	SI_6	SI_{11}
$r = 0.01$	5 653	13 627	4 351	10 603
$r = 0.03$	714	2 103	583	1 791
Pukkala et al. (2013)				
$r = 0.01$	8 320	15 427	7 236	14 046
$r = 0.03$	995	2 326	1 128	2 880

Note: Bare land value in €ha⁻¹, Harvesting interval set to 20 years, regeneration costs of (w) = €0ha⁻¹ maintained and initial size distribution of $\mathbf{x}_0 = [1750, \dots, 0]$.

Table 6 presents the maximized bare land values of continuous cover solutions when the harvesting interval is fixed to 20 years. The results again show that the growth models have a strong effect on the solutions. Increasing the site productivity increases bare land values while increasing the interest rate decreased bare land value. The increase of the interest rate also leads to decreased stand densities (not displayed), implying a need to liquidate capital earlier. With both species, Pukkala et al. (2013) model yields higher bare land values, which is caused by the models superior ingrowth.

5.4 Optimal solutions with optimized harvest timing

The comparison of the two species in tri-level optimization is carried out for site productivities SI_6 and SI_{11} . More productive sites are excluded since pure continuous cover stands for Scots pine may be unrealistic in them, due to much higher ingrowth of Norway spruce. This is clearly as suggested by the model by Bollandsås et al. (2008).

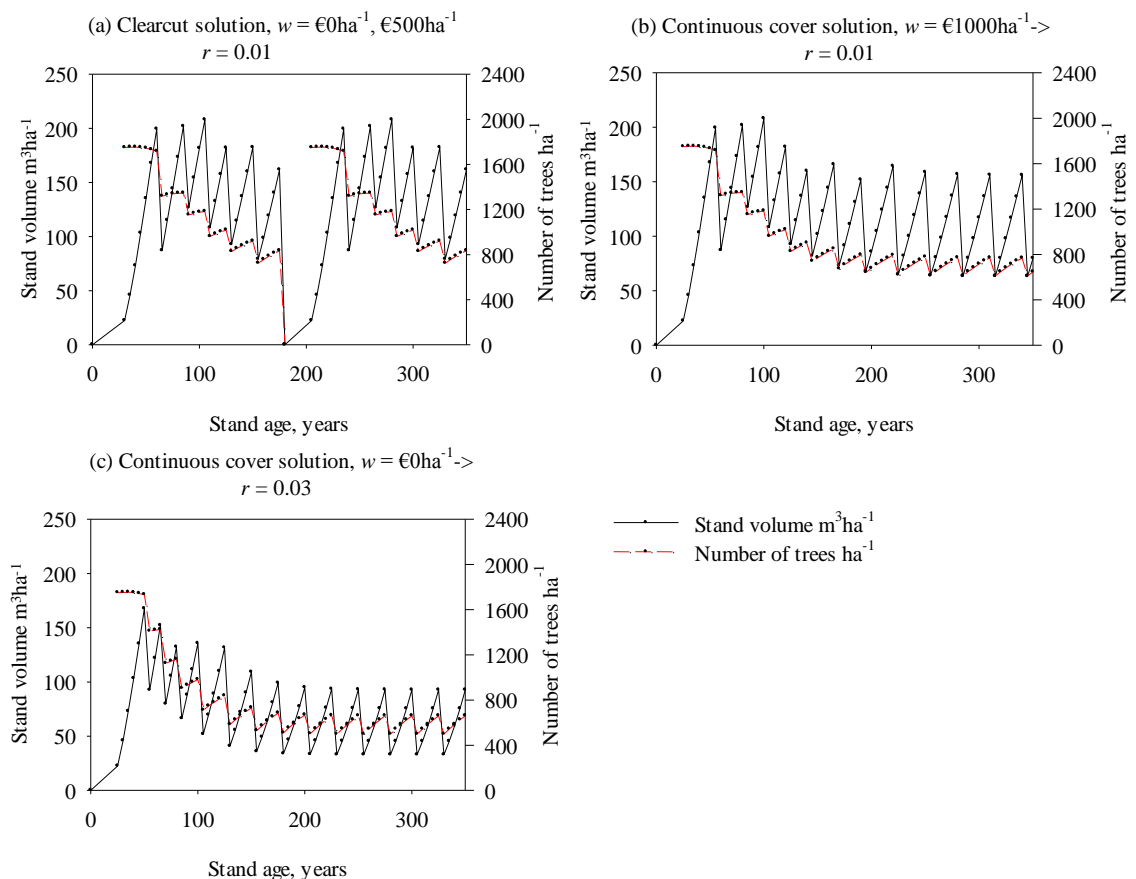
Applying tri-level optimization, the rotation period with highest bare land value is considered optimal (finite or infinite). Artificial regeneration costs are varied between €0ha⁻¹ and €2000ha⁻¹, with €500ha⁻¹ intervals. There are differences between the species independently of the ecological models used, as already shown in chapters 5.1–5.3.

However, the ecological models have a great impact on the optimal solutions. The results will show that Pukkala et al. (2013) model is systematically more favorable towards continuous cover management than Bollandsås et al. (2008) model, for both Norway spruce and Scots pine. Additionally, Norway spruce is more favorable towards continuous cover management compared to Scots pine with both ecological models.

5.4.1 Norway spruce

Bollandsås et al. (2008) growth model

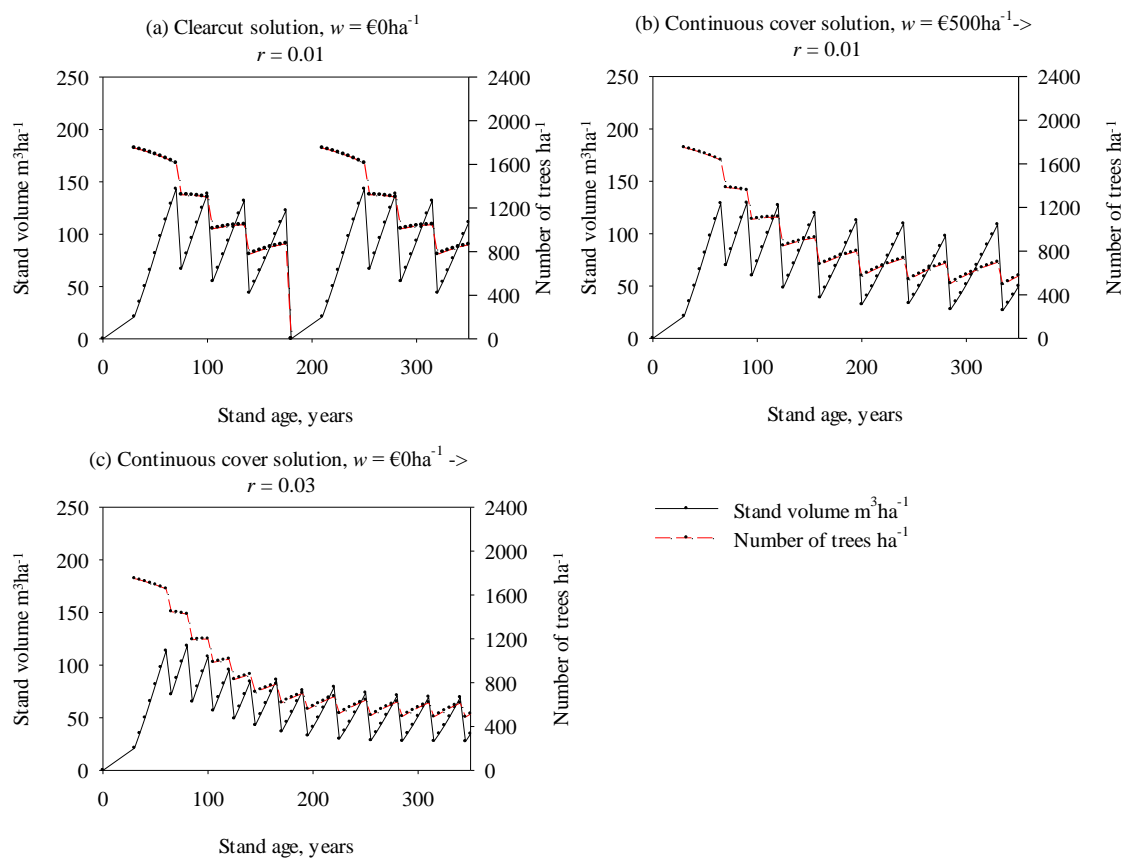
In Figures 7a–b, using Bollandsås et al. (2008) model, SI_{11} , and interest rate of 1%, the optimal solution is clearcut forestry up to €500ha⁻¹ regeneration costs. When the regeneration costs are increased from €0ha⁻¹ to €500ha⁻¹, the optimal rotation stays the same at 175 years, with a mean annual yield of 4.5m³ha⁻¹.



Figures 7a–c: Optimal Norway spruce solutions with optimal harvest timing and average site productivity, Bollandsås et al. (2008) growth model. Note: SI_{11} , $\mathbf{x}_0 = [1750, \dots, 0]$.

Increasing the regeneration costs to €1000ha⁻¹ switches the optimal solution to continuous cover forestry (Figure 7b). In the steady state, the harvesting interval is 30 years, the mean annual yield is 3.7m³ha⁻¹ and ingrowth per 5 years varies between 39 before and 49 after thinning. The steady state mean annual yield of 3.7m³ha⁻¹ is clearly lower than the mean annual yields of the clearcut solutions. However, since the natural regeneration is costless, the continuous cover represents the economically optimal solution.

In Figure 7c, the 3% interest rate makes continuous cover solution optimal even with zero regeneration costs. In the steady state, harvesting interval is 25 years, the mean annual yield is 3.1m³ha⁻¹, and the steady state ingrowth per 5 years varies between 45 and 54.



Figures 8a–c: Optimal Norway spruce solutions with optimal harvest timing and poor site productivity, Bollandsås et al. (2008) growth model. Note: SI_6 , $\mathbf{x}_0 = [1750, \dots, 0]$.

In Figures 8a–c, site productivity is decreased from SI_{11} to SI_6 . Now continuous cover solutions becomes optimal with lower regeneration costs, mean annual yields decrease and steady state harvesting intervals increase. With 1% interest rate, optimal solution is clearcut with zero regeneration costs, but not with €500ha⁻¹. With 3% interest rate, continuous cover solution becomes optimal even with zero regeneration costs.

Pukkala et al. (2013) growth model

In Figure 9a, using Pukkala et al. (2013) model, SI_{11} , and 1% interest rate, optimal solution is continuous cover forestry, independently on regeneration costs. In Figure 9a, the steady state ingrowth per 5 years varies between 57 and 63, the steady state harvesting interval is 25 years, and the steady state mean annual yield is $4.8\text{m}^3\text{ha}^{-1}$. The optimal solution in Figure 9a differs from the one in Figures 7a–b where Bollandsås et al. (20087) model is used. The main reason for the Pukkala et al. (2013) model to be more favorable toward continuous cover management is its higher ingrowth. The higher ingrowth also leads to shorter harvesting interval and higher steady state mean annual yield.

In Figure 9b, increasing interest rate from 1% to 3% shortens the steady state harvesting interval to just 15 years, decreases mean annual yield to $4.5\text{m}^3\text{ha}^{-1}$. At the same time, the ingrowth per 5 years in the steady state increases to vary between 61 and 63.

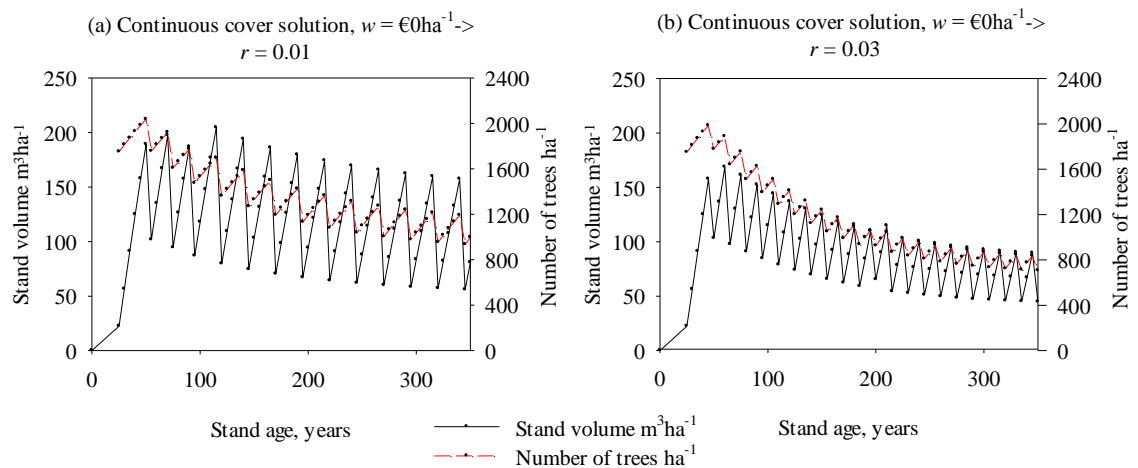
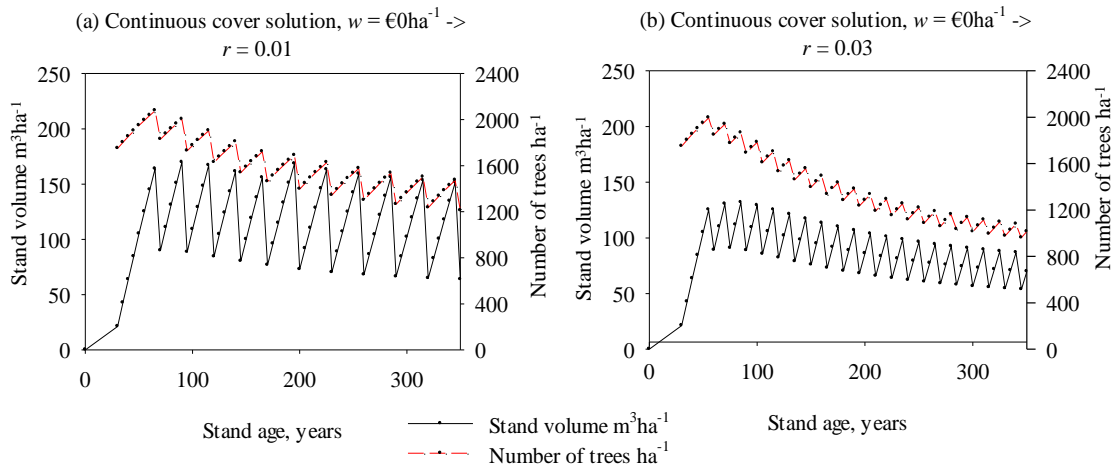


Figure 9a–b: Optimal Norway spruce solution with optimal harvest timing and average site productivity, Pukkala et al. (2013) growth model. Note: SI_{11} , $\mathbf{x}_0 = [1750, \dots, 0]$.

In Figures 10a–b, using Pukkala et al. (2013) model, decreasing site productivity from SI_{11} to SI_6 , optimal solution is continuous cover forestry, even with zero regeneration costs. The mean annual yields decreases, and with 1% interest rate the steady state harvesting interval increases to 30 years.



Figures 10a–b: Optimal Norway spruce solution with optimal harvest timing and poor site productivity, Pukkala et al. (2013) growth model. Note: SI_6 , $\mathbf{x}_0 = [1750, \dots, 0]$.

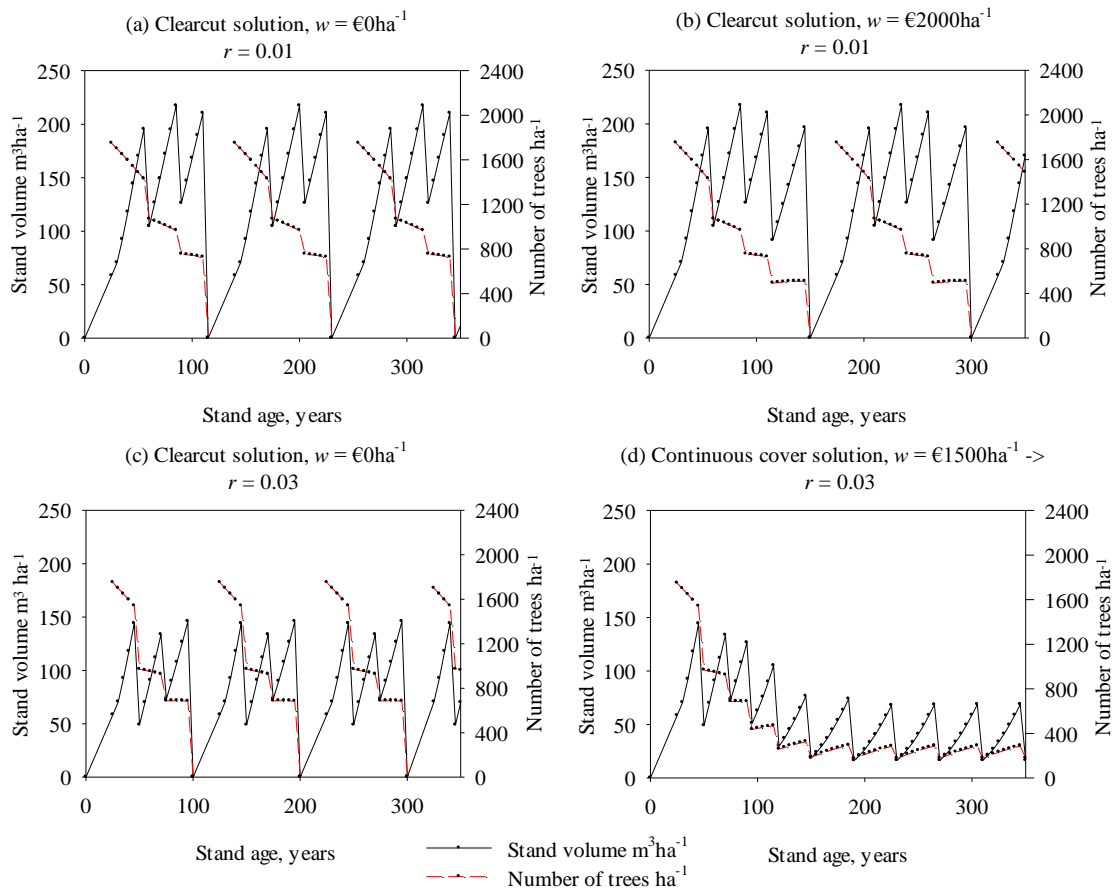
Compared to Bollandsås et al. (2008) model, Pukkala et al. (2013) model is clearly more favorable towards continuous cover forestry. In fact, using Pukkala et al. (2013) model, the optimal solution is continuous cover forestry, independently of site productivity, interest rate, and regeneration costs used. The different outcomes of the ecological models are mainly caused by the higher ingrowth of Pukkala et al. (2013) model (26% to 51% higher in the steady state compared to Bollandsås et al. (2008) model). Also, unlike with Bollandsås et al. (2008) model, the number of trees in optimal solutions using Pukkala et al. (2013) model, does not decrease until the stand is thinned. This is caused by the higher ingrowth and the lower mortality of the Pukkala et al. (2013) model (see Figures 2b–c).

5.4.2 Scots pine

Bollandsås et al. (2008) growth model

Using Bollandsås et al. (2008) model in Figures 11a–b with SI_{11} and 1% interest rate, the optimal solution is clearcut forestry, even with €2000ha⁻¹ regeneration costs. This is a very different outcome than in Figures 7a–b with Norway spruce. The reason for this is the weak ingrowth of Scots pine, which makes artificial regeneration economically optimal, even if the costs are high. In Figures 11a–b, increasing the regenerations costs from €0ha⁻¹ to €2000ha⁻¹ increases the optimal rotation from 115 years to 150 years and decreases the mean annual yield from 4.0m³ha⁻¹ to 3.9m³ha⁻¹. This decrease occurs since when rotation is lengthened, the slow growing valuable large trees form a higher share of the mean annual yield.

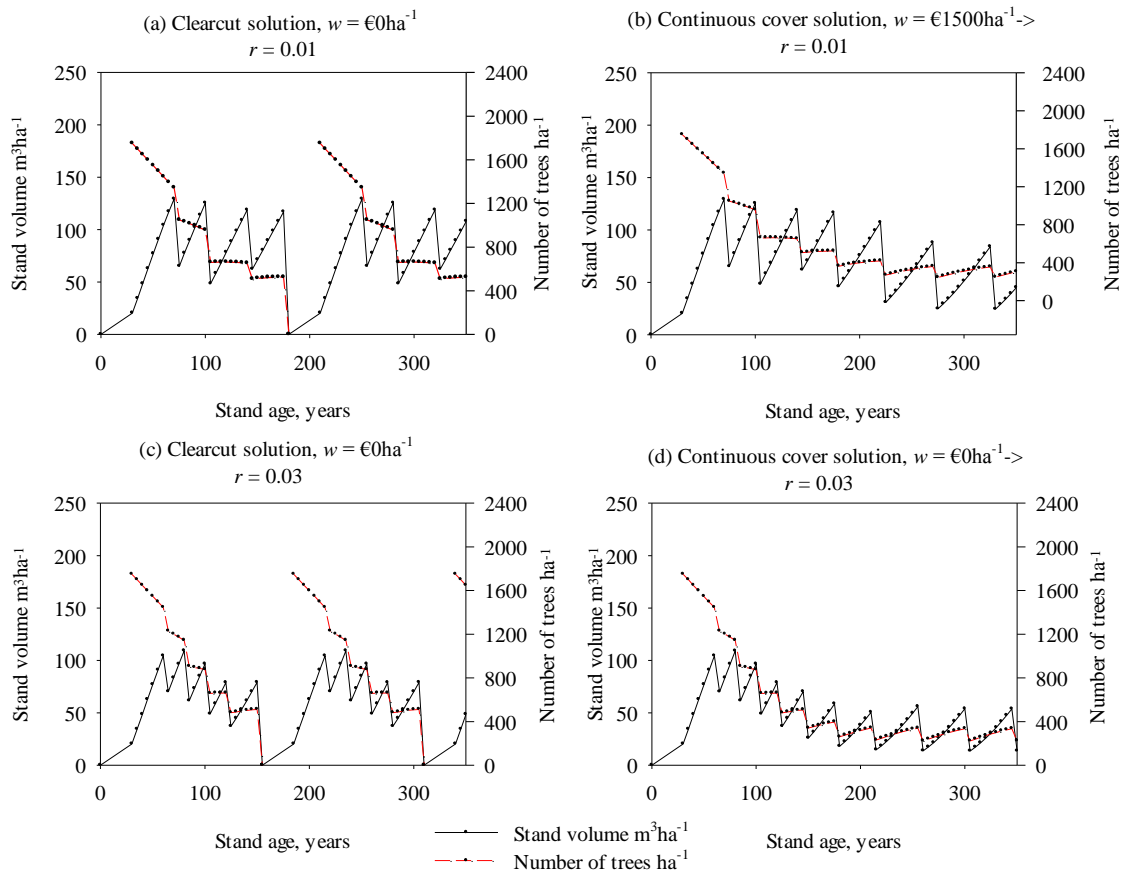
In Figures 11c–d, when the interest rate is increased from 1% to 3%, clearcut forestry remains optimal solution only up to €1000ha⁻¹ regeneration costs. When the regeneration costs are increased from €0ha⁻¹ to €1000ha⁻¹ the optimal rotation increases from 100 years to 120 years. The mean annual yield again simultaneously decreases. When the regeneration costs reach €1500ha⁻¹, the optimal solution switches to continuous cover forestry and the steady state harvesting interval settles to 40 years. The mean annual yield in the steady state is 2.0m³ha⁻¹ and the ingrowth per 5 years varies between 15 and 22. Compared to optimal Norway spruce steady state in Figure 8c, these numbers of ingrowth and mean annual yield are much lower.



Figures 11a–d: Optimal Scots pine solutions with optimal harvest timing and average site productivity, Bollandsås et al. (2008) growth model. Note: SI_{11} , $\mathbf{x}_0 = [1750, \dots, 0]$.

In Figures 12a–d, using Bollandsås et al. (2008) model, decreasing the site productivity from SI_{11} to SI_6 makes continuous cover solution optimal with lower regeneration costs. It also decreases the mean annual yields and lengthens the steady state harvesting intervals. With 1% interest rate, optimal solution is clearcut forestry up to €1000ha⁻¹ regeneration costs. With 3% interest rate, both continuous cover and clearcut solutions

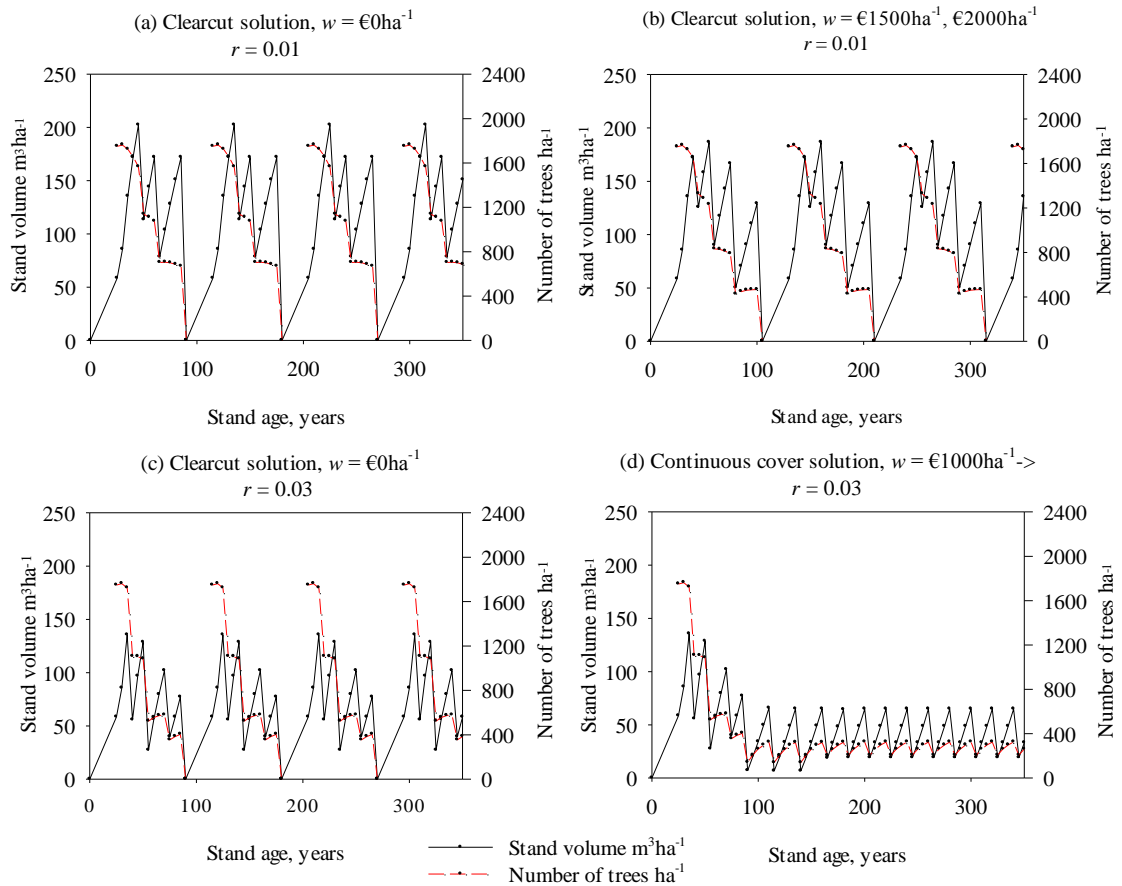
with zero regeneration costs produce exactly the same bare land values. Thus, with any positive regeneration costs, continuous cover solution becomes optimal.



Figures 12a–d: Optimal Scots pine solutions with optimal harvest timing and poor site productivity, Bollandsås et al. (2008) growth model. Note: SI_6 , $\mathbf{x}_0 = [1750, \dots, 0]$.

Pukkala et al. (2013) growth model

Using Pukkala et al. (2013) model, optimal solution for Scots pine in Figures 13a–b with 1% interest rate and SI_{11} is clearcut forestry, even with $€2000\text{ha}^{-1}$ regeneration costs (same result as with Bollandsås et al. (2008) model). When the regeneration costs are increased from $€0\text{ha}^{-1}$ to $€2000\text{ha}^{-1}$ the optimal rotation length systematically increases from 90 years to 105 years. Similarly, at the same time, the mean annual yield decreases from $4.7\text{m}^3\text{ha}^{-1}$ to $4.0\text{m}^3\text{ha}^{-1}$. Due to low ingrowth of Scots pine, these results clearly differ from the optimal solution of Norway spruce in Figure 9a, where the continuous cover solution is optimal, even with zero regeneration costs.

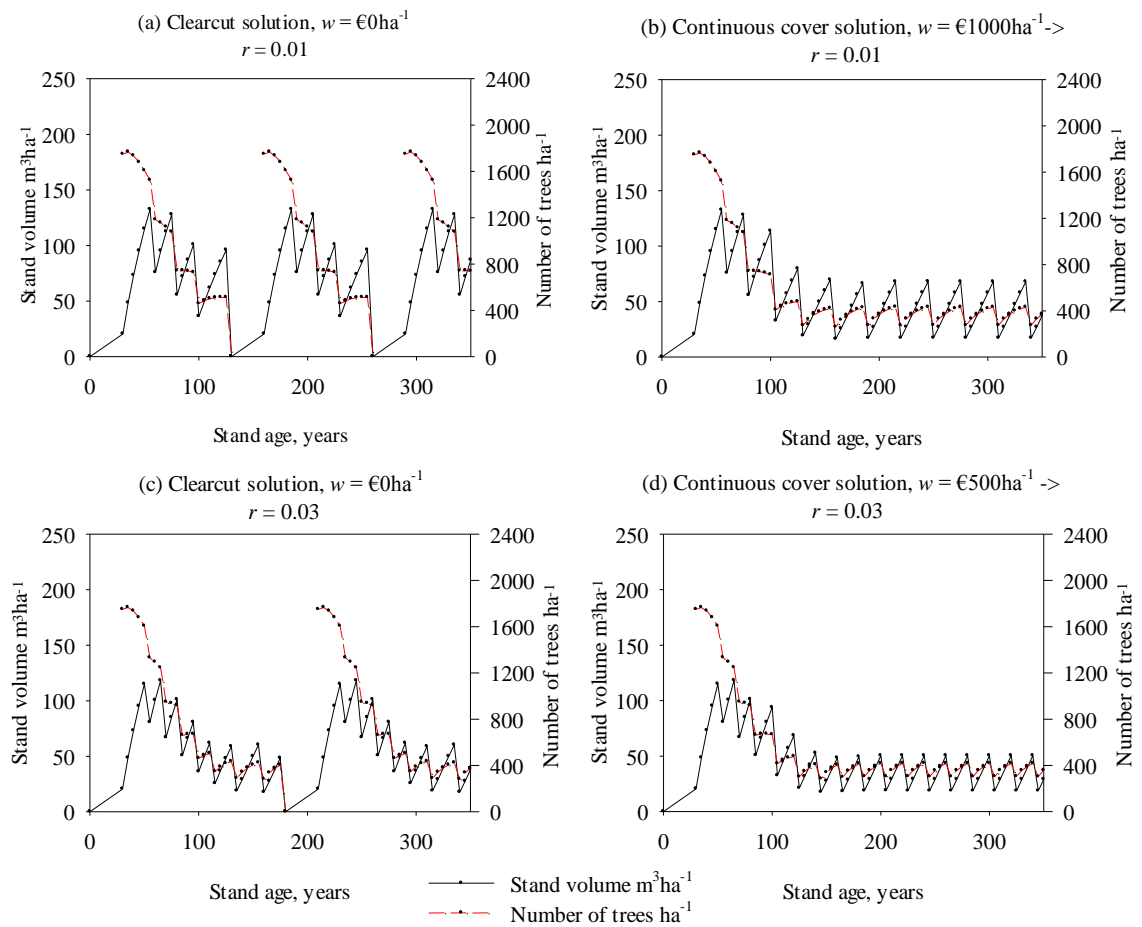


Figures 13a–d: Optimal Scots pine solutions with optimal harvest timing and average site productivity, Pukkala et al. (2013) growth model. Note: SI_{11} , $\mathbf{x}_0 = [1750, \dots, 0]$.

In Figures 13c–d with 3% interest rate, the clearcut solutions are optimal only up to $€500\text{ha}^{-1}$ regeneration costs. Increasing the regeneration costs from $€0\text{ha}^{-1}$ to $€500\text{ha}^{-1}$ increases the optimal rotation from 90 years to 175 years and decreases the average annual yield from $4.7\text{m}^3\text{ha}^{-1}$ to $3.9\text{m}^3\text{ha}^{-1}$. With regeneration costs of $€1000\text{ha}^{-1}$ (lower than with Bollandsås et al. (2008)) or higher, the continuous cover solution becomes optimal. In the steady state, the average annual yield is only $3.0\text{m}^3\text{ha}^{-1}$. The ingrowth per 5 years varies between 24 and 57, and the harvesting interval is 20 years. Compared to the steady state solution of Norway spruce in Figure 9b, ingrowth and mean annual yields are much lower.

Using Pukkala et al. (2013) model in Figures 14a–d and decreasing the site productivity from SI_{11} to SI_6 , the continuous cover solution becomes optimal with lower regeneration costs and the mean annual yield decreases. This is a similar reaction as in with Bollandsås et al. (2008) model. In Figures 14a–b, using 1% interest rate, the optimal solution is clearcut forestry up to $€500\text{ha}^{-1}$ regeneration costs. With 3% interest rate in Figures 14c–d, the clearcut forestry solution under zero regeneration costs produces only $€1$ higher

bare land value than continuous cover solution. Thus, with €500ha⁻¹ regeneration costs, continuous cover solution becomes clearly optimal.



Figures 14a–d: Optimal Scots pine solutions with optimal harvest timing and poor site productivity, Pukkala et al. (2013) growth model. Note: SI_6 , $\mathbf{x}_0 = [1750, \dots, 0]$.

The higher ingrowth of Pukkala et al. (2013) model causes continuous cover solutions with Scots pine to become optimal with lower regeneration costs than with Bollandsås et al. (2008) model (39% to 48% higher in the steady state compared to Bollandsås et al. (2008) model). This outcome between the ecological models is the same as with Norway spruce. Also, using Pukkala et al. (2013) model, the number of trees in the optimal solutions do not decrease as rapidly in the beginning when no thinning is applied, as with Bollandsås et al. (2008) model. This occurs because of the superior ingrowth of Pukkala et al. (2013) model, despite its higher mortality (see Figures 2a–f).

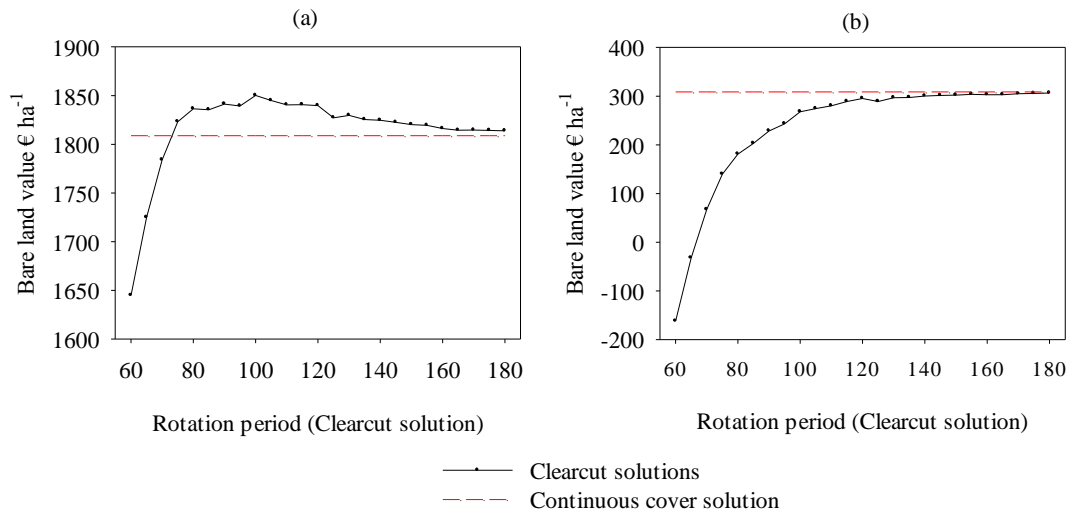
5.4.3 Comparison of the optimal solutions

The characteristics of economically optimal solutions depend strongly on the ecological model and species used. However, the basic behavior of the two models is mainly similar. When harvest timing is optimized, the optimal solution reacts to changes in ecological and economic parameters by changing the time period between harvests, in addition to changing the intensity of harvests and rotation length.

The optimal solutions in Figures 7–14 show that low artificial regeneration costs, low interest rate, and more productive site favor clearcut management. When the interest rate is increased, the periods without harvests after clearcut become more costly, which causes the interest rate to favor continuous cover management. Increasing the interest rate from 1% to 3% decreases the stand density of the optimal solution and shortens the harvesting intervals. These effects show clearly in Figures 7–14 and derive from the need to liquidate capital earlier when capital productivity is higher elsewhere. Furthermore, the decreased stand densities lead to increased ingrowth, as shown in Figures 2b and e.

Less productive site causes trees to grow slower, increasing the opportunity costs of waiting for the artificially regenerated trees, which favors continuous cover management. A less productive site also naturally decreases the mean annual yield of the stand by decreasing growth. Thus, in a less productive site, it takes longer for the trees to reach the optimal harvesting age, which lengthens the steady state harvesting intervals. These effects of site productivity show clearly in Figures 7–14.

The effect of regeneration costs on the optimal rotation by bare land value is displayed in Figures 15a–b. If the regeneration costs are low, there exists an optimal finite rotation period, as in Figure 15a. If the regeneration costs are high enough, as in Figure 15b, the clearcut solution approaches the continuous cover solution from below and no finite optimum exists. In Figures 15a–b continuous cover solution becomes optimal when regeneration costs equal $\text{€}1500\text{ha}^{-1}$.



Figures 15a–b: Scots pine bare land values with different rotation periods. Note: Bollandsås et al. (2008), SI_{11} , $r = 0.03$, (a) $w = €0\text{ha}^{-1}$ and (b) $w = €1500\text{ha}^{-1}$, $\mathbf{x}_0 = [1750, \dots, 0]$.

An explanation for this effect is that when the regeneration costs are increased, the optimal rotation lengthens similarly as in the classic rotation model. In the classic rotation model, the increase of regeneration costs decreases the bare land value and thus the opportunity costs of lengthening the rotation period. This leads to longer rotations. (Johansson and Löfgren 1985, p. 82.) However, the economic optimization model used in this study has the possibility of yielding continuous cover forestry as optimal solution. By lengthening the rotation, the regeneration costs are postponed further into the future, decreasing their current value. With large enough regeneration costs, it becomes optimal to avoid future regeneration costs totally (as in Figure 15b), which is possible since natural regeneration is included.

Figure 16 presents the Norway spruce steady state tree distributions of the optimal continuous cover solutions, revealing differences between the ecological models. The number of trees is clearly larger in almost every size class with Pukkala et al. (2013) model, deriving from its superior ingrowth. Since ingrowth is weaker with Bollandsås et al. (2008) model, the optimal steady state harvesting intervals are longer and trees become larger than with Pukkala et al. (2013) model. This also shows as differences in the steady state harvests mean sawlog-ratios, which are systematically higher with Bollandsås et al. (2008) model. With interest rate of 1%, increasing site productivity from SI_6 to SI_{11} increases the mean sawlog-ratio from 87% to 91% with Bollandsås et al. (2008) model. Using Pukkala et al. (2013) model instead, the same change in the site productivity increases the mean sawlog-ratio from 83% to 84%.

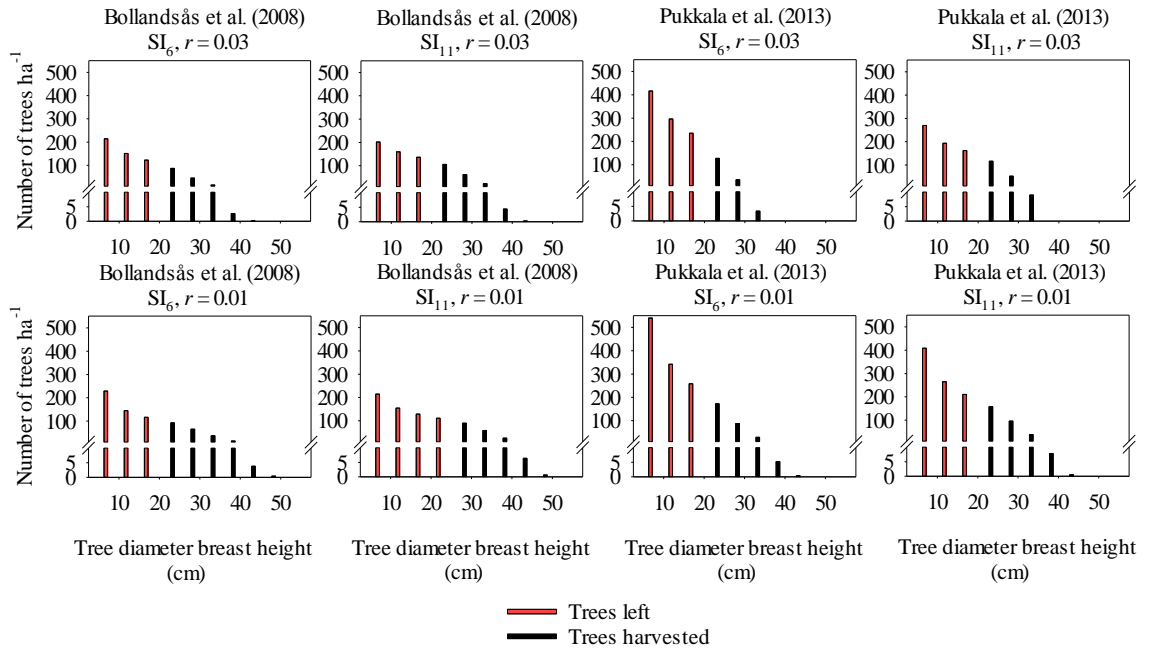


Figure 16: Norway spruce tree distributions in the optimal continuous cover solution steady states. Note: $\mathbf{x}_0 = [1750, \dots, 0]$.

Increasing site productivity allows trees to transfer to the next size class faster. Using Pukkala et al. (2013) model, increasing site productivity from SI_6 to SI_{11} decreases the number of trees in the small size classes and increases it in the large size classes (Figure 16). With Bollandsås et al. (2008) model this effect is smaller due to lower ingrowth. Increasing the interest rate shortens the steady state harvesting interval, implying smaller size of harvested trees. Since the large trees decrease the growth of the smaller trees, increasing interest rate in Figure 16 decreased the number of small trees with both ecological models.

Figure 17 presents the Scots pine tree distributions in the optimal continuous cover solution steady states. The higher ingrowth of Pukkala et al. (2013) model produces a higher amount of trees in the small size classes, when compared to Bollandsås et al. (2008) model. Higher ingrowth also leads to shorter harvesting intervals, which leads to harvesting of smaller trees on average. This shows in the sawlog-ratio of harvested trees. Using Bollandsås et al. (2008) model and 1% interest rate, increasing site productivity from SI_6 to SI_{11} increases the mean sawlog-ratio of harvested trees from 95% to 98%. The same change of site productivity using Pukkala et al. (2013) model increases the sawlog-ratio from 91% to 96%. These are similar results as with Norway spruce.

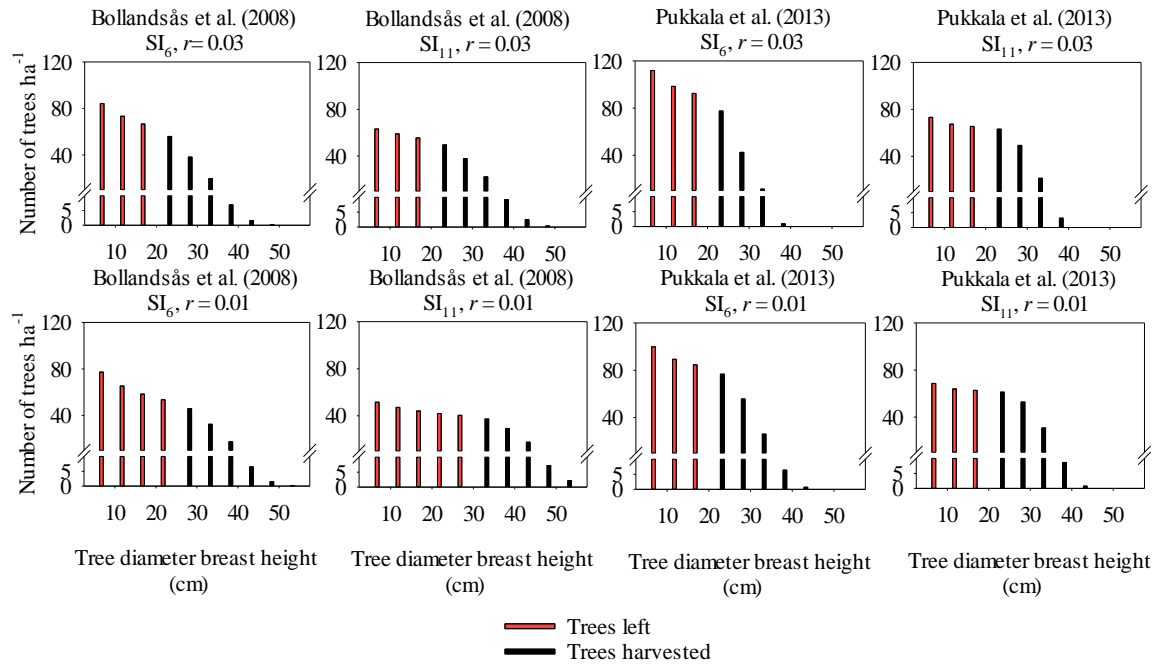


Figure 17: Scots pine tree distributions in the optimal continuous cover solution steady states. Note: $\mathbf{x}_0 = [1750, \dots, 0]$.

As with Norway spruce, Increasing site productivity with Scots pine allows a larger share of trees to transfer into the next size class each period and decreases the harvesting interval. This decreases the number of trees in small size classes in Figure 17, independently of the ecological model used. The ingrowth of Scots pine is very sensitive to changes in stand density. Thus, when increasing interest rate in Figure 17 the number of small trees increases. This is a reverse reaction compared to Norway spruce.

Comparing Figures 16 and 17 shows that the number of trees is significantly lower with Scots pine with both ecological models, site productivities, and interest rates. Similarly, in both Figure 16 and 17, the number of trees is higher with Pukkala et al. (2013) model than with Bollandsås et al. (2008) model. This demonstrates the high ingrowth of both Norway spruce and Pukkala et al. (2013) model.

With both ecological models, the optimal steady state harvests are intense with 31% to 74% of the stand basal area removed. The decrease of basal area is generally larger with Scots pine, since ingrowth is possible only if the basal area is low. This is shown in Table 7 that combines the steady states of both ecological models and species. In Table 7, Scots pine has significantly lower post-harvest basal areas than Norway spruce, with both ecological models. The lowest Scots pine post-harvest basal area in Table 7 is just $2.3\text{m}^2\text{ha}^{-1}$, while the lowest post-harvest basal area with Norway spruce is $5.6\text{m}^2\text{ha}^{-1}$.

Table 7: Optimal continuous cover steady state solutions with optimized thinning.

	Ecological model	SI_i/r	Mean annual yield, m ³ ha ⁻¹	Revenues per harvest, €ha ⁻¹	No. of trees before/after harvest, ha ⁻¹	Basal area before/after harvests, m ² ha ⁻¹	Average annual ingrowth per year	Diameter breast height of harvested trees, cm	Harvesting interval, years
Scots pine	B	$SI_{11}/0.03$	2.0	2699	299/187	8.6/2.3	3.7	20.0–49.9	40
Scots pine	B	$SI_{11}/0.01$	2.2	4665	318/225	15.2/5.9	2.8	20.0–54.9	45
Scots pine	B	$SI_6/0.03$	1.0	1791	346/224	9.1/2.9	3.6	20.0–44.9	45
Scots pine	B	$SI_6/0.01$	1.2	2584	362/256	12.6/4.7	3.1	25.0–49.9	50
Scots pine	P	$SI_{11}/0.03$	3.0	2598	343/206	8.5/2.7	7.7	20.0–39.9	20
Scots pine	P	$SI_{11}/0.01$	3.1	3371	349/195	10.0/2.6	7.1	20.0–44.9	25
Scots pine	P	$SI_6/0.03$	2.1	1511	435/303	9.0/3.9	7.9	20.0–39.9	20
Scots pine	P	$SI_6/0.01$	2.0	2463	440/274	11.5/3.6	7.0	20.0–44.9	30
Norway spruce	B	$SI_{11}/0.03$	3.1	3239	694/498	14.2/6.1	9.9	20.0–39.9	25
Norway spruce	B	$SI_{11}/0.01$	3.7	5180	792/610	22.1/10.4	8.7	25.0–49.9	30
Norway spruce	B	$SI_6/0.03$	1.7	1906	763/614	10.2/5.7	6.9	20.0–39.9	30
Norway spruce	B	$SI_6/0.01$	1.8	4828	707/491	17.3/5.6	6.5	20.0–49.9	50
Norway spruce	P	$SI_{11}/0.03$	4.5	2665	811/597	14.2/8.5	12.5	20.0–34.9	15
Norway spruce	P	$SI_{11}/0.01$	4.8	5355	1184/884	23.2/10.1	12.1	20.0–44.9	25
Norway spruce	P	$SI_6/0.03$	3.2	1726	1115/948	16.2/11.1	10.4	20.0–34.9	15
Norway spruce	P	$SI_6/0.01$	3.3	4271	1434/1140	25.6/12.8	9.8	20.0–44.9	30

Note: B = Bollandsås et al. (2008), P = Pukkala et al. (2013), $SI_i, i \in \{6, 11\}$

Table 7 shows that the Pukkala et al. (2013) model has a much higher ingrowth than Bollandsås et al. (2008) model with both Norway spruce and Scots pine. Thus, Pukkala et al. (2013) model has shorter harvesting intervals and higher mean annual yields than Bollandsås et al. (2008) model, independently of species used. Norway spruce on the other hand has higher ingrowth than Scots pine with both ecological models. Thus, Norway spruce has shorter harvesting intervals and higher mean annual yields than Scots pine. These differences of ingrowth cause Pukkala et al. (2013) model and Norway spruce to favor continuous cover management, compared to Bollandsås et al. (2008) model and Scots pine, respectively.

5.5 Optimal choice of the management regime for Norway spruce and Scots pine

The optimal management regime is chosen by comparing the bare land values of optimal clearcut and continuous cover solutions. Tables 8 and 9 present the optimal management regimes with SI_{11} and SI_6 respectively when interest rate and regeneration costs are varied. The first figure in Tables 8 and 9 is the bare land value for the optimal clearcut forestry solution (if it exists) and the second for the optimal continuous cover solution (that always exists). The black line in Tables 8 and 9 presents the break-even point of the management regime. The asterisk sign denotes that the bare land value becomes negative, implying that artificial regeneration is not optimal.

It should be noted that the optimal continuous cover solutions in Tables 8 and 9 all produce higher bare land value than the bare land values in Table 6, where the harvesting interval is fixed. Bare land values in Tables 8 and 9 are also higher than those in Tables 2 and 3, where no thinning is applied. These are natural results since the elimination of unnecessary management restrictions can only increase the bare land value.

Both Tables 8 and 9 show Norway spruce being more favorable towards continuous cover management than Scots pine, independently of the ecological model used. However, as already shown in chapters 5.1-5.4, the ecological models have a remarkable effect on the nature of the optimal solutions. Using Pukkala et al. (2013) model, the solutions for both species are systematically more favorable towards continuous cover management than with Bollandsås et al. (2008) model. Pukkala et al. (2013) model also has systematically shorter harvesting intervals in the continuous cover steady states. These differences mainly derive from the ingrowth differences, as explained in the earlier chapters.

Table 8: Bare land value comparison of optimal clearcut and continuous cover solutions: average site productivity.

$w =$	€0	€500	€1000	€1500	€2000
Scots pine					
Bollandsås et al. (2008)					
$r = 3\%$	1850 /1809	1325 /1309	810 /809	- /309	- /*
$r = 1\%$	12320 /10975	11624 /10475	10947 /9975	10275 /9475	9626 /8975
Pukkala et al. (2013)					
$r = 3\%$	3044 /3014	2515 /2514	- /2014	- /1514	- /1014
$r = 1\%$	15921 /14197	15076 /13697	14279 /13197	13489 /12697	12718 /12197
Norway spruce					
Bollandsås et al. (2008)					
$r = 3\%$	- 2127	- /1627	- /1127	- /627	- /127
$r = 1\%$	14046 /13863	13440 /13363	- /12863	- /12363	- /11863
Pukkala et al. (2013)					
$r = 3\%$	- /2441	- /1941	- /1441	- /941	- /441
$r = 1\%$	- /16308	- /15808	- /15308	- /14808	- /14308

Note: SI_{11} , bare land value in €ha⁻¹, above (below) black line continuous cover (clearcut) forestry is optimal, * = artificial regeneration is not optimal. $\mathbf{x}_0 = [1750, \dots, 0]$.

The optimal Scots pine solutions clearly differ between ecological models. Using Pukkala et al. (2013) model, a high ingrowth is possible with low enough basal areas. On the other hand, Bollandsås et al. (2008) model cannot produce large amounts of ingrowth with any reasonable basal area. With Bollandsås et al. (2008) model, the harvesting intervals become long, in pursuit of high sawlog-ratios. In contrast, with Pukkala et al. (2013) model, harvesting intervals are shorter and the average trees harvested are smaller. Under low interest rate, the low ingrowth of Bollandsås et al. (2008) model causes large differences in the bare land values between optimal clearcut and continuous cover solutions. The low ingrowth causes clearcut forestry to be the superior solution, even with very large regeneration costs. This follows also with Pukkala et al. (2013) model, but on a different scale, since high ingrowth is possible with low basal areas. This means the stand has to be harvested intensively, leading to short harvesting intervals. Thus, under low interest rate, it is still optimal to clearcut, rather than to maintain low stand volume. However, when interest rate is increased, Pukkala et al. (2013) model has a regime shift to continuous cover management with lower regeneration costs than Bollandsås et al. (2008) model.

The optimal Norway spruce solutions also differ between the ecological models. As with Scots pine, the main reason for this is also ingrowth. Using Pukkala et al. (2013) model,

basal areas larger than $5\text{m}^2\text{ha}^{-1}$ produce up to 50% larger amounts of ingrowth than Bolladsås et al. (2008) model. Using Pukkala et al. (2013) model, ingrowth produced is sufficient for continuous cover solutions to be optimal even under low interest rate and zero regeneration costs. In contrast, the low ingrowth of Bolladsås et al. (2008) model causes clearcut forestry to be optimal solution under a low interest rate and low regeneration costs.

Table 9: Bare land value comparison of optimal clearcut and continuous cover solutions: poor site productivity.

$w =$	€0	€500	€1000	€1500	€2000
Scots pine					
Bolladsås et al. (2008)					
$r = 3\%$	622	- /122	- /*	- /*	- /*
$r = 1\%$	5183	4583 /4409	3983 /3909	- /3409	- /2909
Pukkala et al. (2013)					
$r = 3\%$	1155	- /654	- /154	- /*	- /*
$r = 1\%$	7598	6921 /6863	- /6 363	- /5863	- /5363
Norway spruce					
Bolladsås et al. (2008)					
$r = 3\%$	- /746	- /246	- /*	- /*	- /*
$r = 1\%$	6098	- /5579	- /5079	- /4579	- /4079
Pukkala et al. (2013)					
$r = 3\%$	- /1038	- /538	- /38	- /*	- /*
$r = 1\%$	- /8935	- /8435	- /7935	- /6935	- /6435

Note: SI_6 , bare land values in €ha^{-1} , above (below) black line continuous cover (clearcut) forestry is optimal, * = artificial regeneration is not optimal. $\mathbf{x}_0 = [1750, \dots, 0]$.

The optimal management regimes for SI_6 are presented in Table 9. The same conclusions about the differences caused by species and ecological models apply as in Table 8. Norway spruce, Pukkala et al. (2013) model, and SI_6 favor continuous cover solution, compared to Scots pine, Bolladsås et al. (2008) model, and SI_6 .

The bare land values in both optimal clearcut and continuous cover solutions are strongly dependent on the ecological model. For example, using Bolladsås et al. (2008) model, the optimal Scots pine continuous cover solution with SI_{11} , 3% interest rate, and zero regeneration costs has a bare land value of $\text{€}1809$. Using Pukkala et al. (2013) model and the same parameters instead, the bare land value is $\text{€}3014$. The 66% increase in bare land value is caused mainly by ingrowth. This can be seen from Table 7, where Pukkala et al. (2013) model has almost twice as high ingrowth. The higher ingrowth leads to 20 years

shorter harvesting interval, with only 4% smaller revenues per harvests than Bollandsås et al. (2008) model.

6 Discussion

In this study, the optimal choice between clearcut and continuous cover management is studied using a theoretically sound economic optimization model that extends the classic rotation model by including optimized thinning and natural regeneration. The optimal management regime is determined endogenously within the model. Both Norway spruce and Scots pine have their own empirically estimated ecological growth models and economic parameters (prices and harvesting costs).

The ecological models were first analysed using simple optimization (classical economic rotation model, continuous cover management under the objective of maximum sustained yield, and continuous cover management with fixed harvesting interval), before the tri-level optimization. The results show that ecological models and species have major impacts on the optimal solutions. These differences can, however, be explained by the properties of the ecological models. The Norway spruce solutions for continuous cover management under maximum sustained yield objective reveal that in some cases, Pukkala et al. (2013) model has a peculiar steady state with cyclical pattern, which is caused by the increase of diameter increment and ingrowth.

One of the main results of this study is the importance of the ecological model. The models estimated from Finnish and Norwegian data produced systematically different outcomes. Andreassen and Øyen (2011) write that in general the growth models from Finland, Sweden, and Norway produce similar levels of growth. The results of this study, however, point out that even moderate differences in the ecological models may change the economically optimal solutions significantly.

The largest differences between ecological models and species follow from ingrowth. Scots pine produces similar levels of ingrowth with both ecologic models, except for very low basal areas where Pukkala et al. (2013) model starts producing high levels of ingrowth. With Norway spruce, the ingrowth also differs between the ecological models. In Pukkala et al. (2013) model Norway spruce ingrowth is a concave function of basal area, whereas in Bollandsås et al. (2008) model the function is convex instead. With basal areas larger than $5\text{m}^2\text{ha}^{-1}$, Pukkala et al. (2013) model starts producing a much higher ingrowth than Bollandsås et al. (2008) model. Ingrowth also influences the steady state harvesting intervals. Harvesting intervals with Scots pine are systematically shorter than

with Norway spruce and Pukkala et al. (2013) model has shorter harvesting intervals than Bollandsås et al. (2008) model, independently of species used. The results raise the question of why a model estimated from a Finnish empirical data favors continuous cover management systematically when compared to a model estimated from a Norwegian data.

Comparing the results of this study to the earlier studies on the choice between clearcut management and continuous cover management that use different economic optimization models, there are similarities in the effects of interest rate, regeneration costs, and site productivity. For example in studies like Hyytiäinen and Haight (2009) or Chang (1981), higher interest rate favors continuous cover solution. In Sánchez Orois et al. (2004), low site productivity favors continuous cover solution, and in Hotvedt et al. (1989), high interest rate leads to low steady state basal areas. However, compared to these studies, the results of my study are based on very different and more advanced economic optimization model.

The main results of the present study differ from those of Pukkala et al. (2010), which uses different economic optimization model and includes Scots pine. The most obvious difference is the optimal management regime of Scots pine. In Pukkala et al. (2010), continuous cover solution is always optimal for Scots pine, with the exception of medium fertile stands in Northern Finland with low interest rate. However, the results of my study show that high site productivity favors clearcut solution. Thus, according to our economic optimization model, clearcut management should be less favorable in Northern Finland than in Southern Finland, since the site productivity in Northern Finland is lower.

Rämö and Tahvonen (2014) study continuous cover forestry using the Bollandsås et al. (2008) model and include Scots pine. They, however, apply a fixed harvesting interval of 15 years. Thus, comparing the tri-level optimization results of this study to Rämö and Tahvonen (2014), the optimal solutions are quite different. For example, in my study, the steady state harvesting interval for Scots pine under 3% interest rate with SI_{11} is 40 years. This is a considerably longer harvesting interval than 15 years. My study demonstrates that bare land values with fixed harvesting intervals are smaller than with tri-level optimization. During the transition towards the steady state, the harvesting interval varies, and the majority of the bare land value is obtained during this period. This underlies the importance of the tri-level optimization.

Only one initial state (bare land) is used in this study. However, the optimal choice between management regimes is sensitive on the initial state, as shown in Tahvonen and Rämö (2016). The optimization of harvest timing allows optimal transition from any

initial state towards the steady state. The initial state of the forest might disfavor continuous cover solution, although site productivity, regeneration costs, species and interest rate would favor it. In this kind of situation, it may become optimal to apply clearcut and after artificial regeneration, continue with continuous cover management. Thus, the results presented in this study are dependent on the initial state used.

There is an ongoing debate about alternative forest management methods, covering all aspects of forestry, including ecological and cultural values (for discussion, see Puettmann et al. 2015). There are many positive values associated with continuous cover management. For example, continuous cover management might have greater resilience than clearcut management to risks such as storms, landslides, and insects (O'Hara 2013). Continuous cover forestry can also provide greater biodiversity of species than clearcut forestry (Calladine et al. 2015). However, probably the largest single reason for the public to promote continuous cover management is its aesthetic qualities (O'Hara 2014, p. 169).

The optimal Scots pine steady state stand densities in this study are very low with both ecological models. This is caused by low ingrowth, but also by not considering other forest values than timber production, for example, ecological or cultural. Thus, in addition to poor economic performance in continuous cover management, economically optimal continuous cover solutions for Scots pine may also perform poorly considering these other values. Optimal Norway spruce continuous cover solutions did not have low stand densities. Thus, it is possible that for Norway spruce, continuous cover management contributes more than clearcut management to these non-timber values of forests.

The Finnish Forest Act of 2014 does not restrict the management type. However, the stand must reach minimum requirements set for basal area and number of trees per hectare. (L 1308/2013.) If the stand state does not reach these requirements, it must be artificially regenerated. These minimum requirements are displayed in Appendix II. All of the optimal solutions in Table 8 are illegal, except the optimal Norway spruce continuous cover solution with Pukkala et al. (2013) model and 1% interest rate. In Table 9 all solutions are also illegal, except the optimal Norway spruce continuous cover solutions using Pukkala et al. (2013) model. The lower boundaries of the Finnish Forest Act of 2014 can be thought as a way to conserve the non-timber values of forests. Therefore the solutions point out an existence of trade-off between optimal solutions considering only timber production and optimal solutions considering other values also.

7 Conclusions

This is the first study that produces optimal solutions for Scots pine using the economic optimization model previously used in Tahvonen and Rämö (2016). The results of this study show that when using a theoretically sound economic optimization model, continuous cover management performs less favorable for Scots pine compared to Norway spruce, with both average and poor sites productivities, independently of the ecological model used. In addition, high interest rate, high regeneration costs, and low site productivity favor the optimality of continuous cover forestry. Also, the optimal continuous cover solutions for Scots pine have very low stand densities. Additionally, the results show that the Finnish Forest Act of 2014 is restricting the economically optimal solutions, with its strict post-harvest lower boundaries on basal area and number of trees per hectare

The results show that the ecological models have a strong effect on the solutions, mainly caused by their differences in ingrowth. Thus, The Pukkala et al. (2013) model is significantly more positive towards continuous cover management than the Bollandsås et al. (2008) model. Since the most significant difference between the ecological models is ingrowth, ecological models that could predict ingrowth more accurately would be beneficial considering future research.

With €1500ha⁻¹ or higher regeneration costs, average site productivity, and 3% interest rate, both ecological models and species yield continuous cover forestry as the optimal solution. With average site productivity and low interest rate, the optimal solution for Scots pine is clearcut forestry, even with high regeneration costs. Typical regeneration costs in Finland range between €1500ha⁻¹ and €2000ha⁻¹, which suggests that with 3% interest rate, continuous cover forestry is optimal in many cases for both Norway spruce and Scots pine.

Since the optimal choice of the management regime clearly depends on species, the results of this study suggest that including species variation into the debate on continuous cover management is crucial.

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Appendices

Appendix I. Size class specific parameter values for Scots pine and Norway spruce.

Size classes	Scots pine					Norway spruce				
	γ_s	d_s	v_{1s}	v_{2s}	SI_{11}	γ_s	d_s	v_{1s}	v_{2s}	SI_{11}
1	0.0044	7.5	0.0117	0	0.0334	0	0.0121	0	0.0129	0
2	0.0123	12.5	0.0552	0	0.0637	0	0.0549	0	0.0606	0
3	0.0241	17.5	0.1302	0	0.0969	0.0924	0.1349	0	0.1506	0
4	0.0398	22.5	0.0423	0.2003	0.0374	0.2562	0.0613	0.1915	0.0686	0.2144
5	0.0594	27.5	0.0288	0.3543	0.0292	0.4609	0.0529	0.3532	0.0605	0.3955
6	0.0830	32.5	0.0252	0.5409	0.0269	0.7098	0.0437	0.5462	0.0487	0.6168
7	0.1106	37.5	0.0213	0.7600	0.0260	0.9937	0.0392	0.761	0.0459	0.8564
8	0.1419	42.5	0.0197	1.0116	0.0255	1.3217	0.0342	0.9951	0.0437	1.1175
9	0.1772	47.5	0.0156	1.2958	0.0252	1.6907	0.0322	1.2431	0.0379	1.4022
10	0.2165	52.5	0.0129	1.6125	0.0251	2.1008	0.0300	1.4943	0.0357	1.6884

Note: γ_s , $s=1, \dots, 10$ is the basal area of a tree (m^2), v_{1s} , $s=1, \dots, 10$ is the pulpwood volume (m^3) per tree and v_{2s} , $s=1, \dots, 10$ is the sawlog volume (m^3) per tree.

Appendix II. Minimum number of trees per hectare and basal area after harvest in Central Finland (L 1308/2013, § 19).

	SI_6	SI_{11}	SI_{15}	SI_{17}
No. of trees after harvest, ha^{-1}	700	800	700	700
Basal area after harvest, m^2ha^{-1}	8	8	9	9

Appendix III. Species-specific regression coefficients for ecological models.

Bollandsås et al. (2008)			Pukkala et al. (2013)		
a_i	Norway spruce	Scots pine	b_i	Norway spruce	Scots pine
0	17.839	25.543	0	-9.6448	5.9901
1	0.0476	0.0251	1	0.455	0.5057
2	-11.585	-5.66	2	-0.05741	-0.07699
3	-0.3412	-0.216	3	1.4551	0.987
4	0.906	0.698	4	0.2908	0.1791
5	-0.024	-0.123	5	-0.0491	-0.07558
6	-0.268	-0.336	6	-0.4037	-0.3945
7	-2.492	-1.808	7	-0.3081	-0.3593
8	-0.02	-0.027	8	-0.02915	-0.141
9	3.2	3.3	9	-0.1473	-0.1399
10	0.031	0.055	10	5.871	2.333
11	43.142	67.152	11	1.536	1.518
12	-0.157	-0.076	12	-0.122	-0.083
13	0.368	0	13	-0.106	-0.602
14	-2.291	-3.552	14	-0.69	-686
15	-0.018	-0.062	15	-0.465	0
16	0.066	0	16	4.378	6.109
-			17	-0.0265	-0.844
-			18	1.001	-0.375
-			19	0.641	1.045
-			20	0	-0.556
-			21	0.046	0
-			22	-0.0658	0
-			23	0	-0.162
-			24	0	0.277
-			25	-0.814	0.755

$a_i, s = 0, \dots, 16$ are the species-specific regression coefficients for Bollandsås et al. (2008) model. $b_i, s = 0, \dots, 25$ are the species-specific regression coefficients for Pukkala et al. (2013) model.