Mathematical Programming Applications in Agroforestry Planning

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MATHEMATICAL PROGRAMMING APPLICATIONS IN AGROFORESTRY PLANNING

by

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ABSTRACT

Mathematical Programming Applications in
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Agroforestry as a sustainable production system has been recognized as a land use system with the potential to slow encroachment of agriculture onto forested lands in developing countries. However, the acceptance of nontraditional agroforestry systems has been hampered in some areas due to the risk-averse nature of rural agriculturalists. By explicitly recognizing risk in agroforestry planning, a wider acceptance of agroforestry is possible. This thesis consists of a collection of three papers that explore the potential of modern stock portfolio theory to reduce financial risk in agroforestry planning.

The first paper presents a theoretical framework that incorporates modern stock portfolio theory through mathematical programming. This framework allows for the explicit recognition of financial risk by using a knowledge

of past net revenue trends and fluctuations for various cropping systems, with the assumption that past trend behavior is indicative of future behavior. The paper demonstrates how financial risk can be reduced by selecting cropping systems with stable and/or negatively correlated net revenues, thereby reducing the variance of future net revenues.

Agroforestry systems generally entail growing simultaneously some combination of plant and/or animal species. As a result, interactions between crops usually cause crop yields within systems to deviate from what would be observed under monocultural conditions, thus requiring some means of incorporating these interactions into mathematical models.

The second paper presents two approaches to modeling such interactions, depending on the nature of the interaction. The *continuous system* approach is appropriate under conditions where yield interactions are linear between crops and allows for a continuous range of crop mixtures. The *discrete system* approach should be used where nonlinear interactions occur. Under this second approach, decision variables are defined as fixed crop mixtures with known yields.

In the third paper, the techniques presented above were applied to a case study site in Costa Rica. Using MOTAD programming and a discrete system approach, a set of

minimum-risk farm plans were derived for a hypothetical farm. For the region studied, results indicate that reductions in risk require substantial reductions in expected net revenue. (49 pages)

CHAPTER I

INCORPORATING ECONOMIC RISK AVERSION IN AGROFORESTRY PLANNING

Abstract. The ability to use a knowledge of past market price fluctuations to reduce the risk of future financial returns is explored in the context of planning an agroforestry system with a cash crop component. It is demonstrated that if past crop price behavior is indicative of future price behavior, planting crops with stable and/or negatively correlated net revenues can reduce the variance of future net revenues and hence decrease the financial risks of agroforestry systems.

Introduction

The use of linear programming in guiding the planning of agroforestry systems has been demonstrated [3,6,9,16,17]. But while linear programming is typically carried out in a deterministic framework, empirical studies have demonstrated that agriculturalists in developing countries face great uncertainties and are strongly risk-averse [2,4,8].

Portfolio theory, as operationalized through quadratic programming, offers an alternative to deterministic modeling that allows for the explicit recognition of risk and may lead to modeling decisions that more closely resemble those made by rural agriculturalists [13].

Blandon [5] discussed the role of portfolio theory in reducing the financial risk of agroforestry systems with a cash crop component. This paper extends Blandon's work to demonstrate the application of portfolio theory through quadratic programming. A hypothetical agroforestry system is used as an example.

Portfolio theory and agroforestry planning under risk

Consider the planning of an agroforestry system where J crops are sold for revenue. Each crop's per-hectare net revenue, NR_j, can be defined as:

$$NR_j = P_j Y_j - C_j \qquad \text{for all } j \tag{1}$$

where:

j = 1,...,J crops,
P_j = the expected price of crop j per bushel at harvest,
Yj = the yield of crop j in bushels per hectare, and
C_j = the per-hectare cost of growing crop j.

Since the actual costs and revenues associated with different crops at planting time are unknown, NR_j is really a "best guess" of future per-hectare net returns. Time series data of historical net revenues can serve as a guide for estimating NR_j:

$$NR_j = \left[\sum_{t=1}^{T} (NR_{jt}) \right] (1/T) \qquad \text{for all } j \tag{2}$$

where:

t = 1,...,T years of time series data, and

NR_{jt} = the per-hectare net revenue of crop j received in year t.

The fluctuation of NR_{jt} across time for a particular crop indicates the probability of NR_j actually occurring at harvest. For example, if the net revenue associated with a particular crop has exhibited wide fluctuations over the past 10 years, it is likely that such fluctuations will continue in the future. As a result, there would be considerable uncertainty over receiving NR_j at harvest. On the other hand, if a crop's net revenue has been relatively stable in the past, there is good reason to believe that stable net returns will continue and that NR_j is a likely estimate of the actual per-hectare net return of crop j received at harvest.

Expanding the discussion to an agroforestry system with several cash crops, the total expected net revenues (NR) received for the J crops is calculated as:

$$NR = \sum_{j=1}^{J} (X_j)(P_jY_j - C_j) \qquad (3)$$

where:

j = 1,...,J crops,
X_j = the number of hectares planted to crop j, where:

$\sum_{j=1}^{J} (X_j)$ = the total number of hectares to be planted,

P_j = the expected price of crop j per bushel at harvest,
Y_j = the yield of crop j in bushels per hectare, and
C_j = the per-hectare cost of growing crop j.

By utilizing equation (1), equation (3) reduces to:

$$NR = \sum_{j=1}^{J} (X_j)(NR_j) \tag{4}$$

The variance of total expected net revenues received at harvest, V_{NR}, indicates the risk or probability of actually receiving NR at harvest:

$$V_{NR} = \sum_{j=1}^{J} \sum_{k=1}^{K} (X_j)(X_k)(\sigma_{jk}) \tag{5}$$

where:

X_j = the number of hectares planted to crop j, and
σ_{jk} = the variance of per-hectare net revenues for crop j when j=k, and the covariance of per-hectare net revenues for crops j and k when j≠k.

Assuming that past net returns are indicative of future net returns, the higher V_{NR} is for a particular crop mixture, the less likely it is that NR will be received at harvest. Manipulating the representation of crops in an agroforestry system to reduce V_{NR} increases the probability of actually receiving NR at harvest, although expected net returns may decrease. Such actions rely on the same principles used to reduce the risk of stock portfolios [14].

The variance of NR can be reduced two ways--by favoring crops with low net revenue variances, and by exploiting patterns in the fluctuations of past crop net revenues. The first approach is intuitive. The second relies on the covariance of past net revenues between crops.

To illustrate, equation (5) can be rewritten as:

$$V_R = \sum_{j=1}^{J} (X_j^2)(\sigma_j^2) + \sum_{k \neq j}^{K} \sum_{j=1}^{J} (X_j)(X_k)(\Gamma_{jk})(\sigma_j)(\sigma_k) \qquad (6)$$

where:

X_j = the number of hectares planted to crop j,
σ_j^2 = the variance of per-hectare net revenues for crop j,
Γ_{jk} = the correlation coefficient of per-hectare net revenues between crops j and k,
σ_j = the standard deviation of per-hectare net revenues for crop j, and
σ_k = the standard deviation of per-hectare net revenues for crop k.

Equation (6) demonstrates that the variance of total net revenues, V_{NR}, is the sum of two components--a variance term and a covariance term. Since $\sigma_j^2 \geq 0$, the first term is necessarily non-negative. The second term, however, may be positive or negative since $-1 \leq \Gamma_{jk} \leq 1$. In other words, the net returns of two crops may be negatively or positively correlated depending on how their returns have historically varied over time. Two crops are positively correlated (i.e., $0 < \Gamma_{jk} \leq 1$) if their net revenues increase or decrease simultaneously, and negatively correlated (i.e., $-1 \leq \Gamma_{jk} < 0$) if their net revenues are inversely related.

When an agroforestry system is composed of cash crops whose net returns are positively correlated, both terms in equation (6) are positive and thus increase the variance of expected net returns. But when systems are composed of crops with negatively correlated net returns, the second term in equation (6) is negative and the variance of future net revenues, V_{NR}, is reduced.

Mathematical programming and agroforestry planning

The expected value-income variance (E-V) criterion recognizes risk averse behavior in the selection of stock portfolios and serves as the foundation of modern stock portfolio analysis. In an agroforestry context, a farmer behaving under the E-V criterion would evaluate the desirability of an agroforestry system based on its expected net return *and the variance of past returns*.[1] Simply stated, a risk averse farmer acting under the E-V criterion would only consider agroforestry plans that have the lowest risk (i.e., lowest net revenue variance) for a given level of expected net return.

Quadratic programming (QP) can determine an agroforestry system's set of minimum-risk plans by sequentially minimizing the variance of expected net revenue (V_{NR}) for different levels of total expected net revenue (NR). As an example, consider a hypothetical agroforestry system consisting of four crops: crops 1, 2, 3 and 4. Time series data of per-hectare net revenues and the resulting net revenue variance-covariance matrix for the crops are shown in Table 1.

[1] A quadratic utility function for income, U(NR), is sufficient for an individual's risk preferences to be completely described by the mean and variance of expected net returns [10].

Four constraints apply to the system:

1) total area available for planting is 15 hectares,
2) a total of 1,200 hours of labor are available,
3) at least 1 hectare must be planted to crop 1, and
4) at least 1 hectare must be planted to crop 2.

By defining four decision variables to represent the number of hectares planted to each crop (i.e., X_1, X_2, X_3 and X_4 for crops 1, 2, 3, and 4, respectively), the corresponding deterministic LP model that maximizes net returns is (see Betters [3]):

Maximize:

$146X_1 + 195X_2 + 240X_3 + 395X_4$ (total net revenue,pesos)

Subject to:

$X_1 + X_2 + X_3 + X_4 = 15$ (hectares available)

$45X_1 + 25X_2 + 35X_3 + 40X_4 < 1{,}200$ (labor availability)

$X_1 > 1$ (crop 1 requirement)

$X_2 > 1$ (crop 2 requirement)

The optimal solution to this deterministic model is 5,476 pesos (Plan A in Table 2). This profit maximizing, risk-neutral plan allocates all additional acres beyond those needed for the last two constraints to crop 4 (Table 2). This result is intuitive since crop 4 has the highest expected return, but it also has the highest net revenue variance, making it the riskiest crop as well (Table 1).

Hence, while Plan A has the highest expected net return, it may be unacceptable to a risk-averse agroforester.

To incorporate risk aversion into the modeling process, the model is reformulated as a quadratic programming problem where the variance of expected net revenue is minimized subject to meeting a parametrically-altered scalar of required net revenue, δ [10]:

Minimize:

$$\begin{aligned}\text{Variance} = {} & 849X_1^2 + 5{,}250X_2^2 + 12{,}355X_3^2 + 149{,}848X_4^2 \\ & - 2(185)X_1X_2 + 2(1{,}476)X_1X_3 - 2(2{,}043)X_1X_4 \\ & - 2(7{,}265)X_2X_3 + 2(27{,}641)X_2X_4 - 2(38{,}893)X_3X_4\end{aligned}$$

Subject to:

$146X_1 + 195X_2 + 240X_3 + 395X_4 \geq \delta$ (total net revenue,pesos)

$X_1 + X_2 + X_3 + X_4 = 15$ (hectares available)

$45X_1 + 25X_2 + 35X_3 + 40X_4 \leq 1{,}200$ (labor availability)

$X_1 \geq 1$ (crop 1 requirement)

$X_2 \geq 1$ (crop 2 requirement)

The QP model was solved by GINO, a nonlinear programming optimization computer package available for use on personal computers and mainframe systems [12]. The net revenue constraint was sequentially-altered between runs such that δ equalled 1%, 5%, 10% and 20% reductions from the maximum net return of 5,476 pesos found by the deterministic model.

Sequentially solving the QP model for different levels of required net revenue resulted in the set of minimum-risk agroforestry plans shown as arc BE in Figure 1 and Plans B through E in Table 2 (Plan A in the figure and table represents the deterministic solution described earlier).

Arc AE represents an upper-bound to the set of all feasible agroforestry plans. For a risk-averse agroforester, only the minimum-risk plans on arc AE are relevant since any plan below the arc has a lower expected return for a given level of risk. For example, a risk-averse farmer would never prefer Plan F to B since it has a lower expected net return for the same level of risk (Figure 1). Further note that any plan above arc AE violates the problem's constraints and is infeasible.

As expected net revenue was reduced to minimize risk, optimal agroforestry plans shifted from allocating all excess hectares to crop 4 (i.e., Plan A in Table 2), to allocations that included both crops 3 and 4 (i.e., Plans B through E in Table 2). This occurs since these two crops have the highest expected net returns and are negatively correlated (Table 1). Hence, growing crops 3 and 4 in combination can lower the financial risk of an agroforestry plan while providing relatively high expected net returns (Table 2).

While the plans on arc AE are the only ones relevant to a risk-averse agroforester, the preferred plan on the arc is determined by the agroforester's trade-off between expected net return and risk. For example, Plan C in

Figure 1 might be preferred to the deterministic plan that maximizes net returns (Plan A) since the relatively small reduction in expected net return for Plan C (5%) is associated with a substantial reduction in risk (31%) (Table 2).

Application and extension

Biophysical interactions between crops are an important characteristic of agroforestry systems [1]. Competitive relationships and yields between cropping components vary as a function of management practices and resource sharing characteristics associated with spatial (both vertical and horizontal) and temporal factors [7].

One limitation of linear and quadratic programming formulations like those presented here is that interactions between crops that affect yields are difficult if not impossible to model. Several remedies exist, however. The first is to consider only monocultures. Unfortunately, this strategy is of limited use since a basic characteristic of agroforestry systems is mixed cropping and the maximization of any beneficial yield interactions that may result [16].

A second approach, and one commonly taken in the agroforestry literature, is to assume interactions are negligible or linear, and use single-crop planning models like those developed in this paper, but allow for crop mixtures in implementation. This method is undesirable for several reasons. First, it essentially assumes that each crop's Land Equivalent Ratio (LER) is 1.0 for all possible crop mixtures (see Mead and Willey [15] for a discussion of the LER). A second shortfall of this approach is that it defines a continuous range of cropping systems when, most likely, only a few discrete crop mixtures have been examined. In essence, the method allows the modeler to extrapolate beyond available data. Given the many complex interactions common to agroforestry systems, such extrapolations are undesirable.

A third and preferred approach is to define decision variables as complete systems, each with a set of expected yields for each component crop. The mathematical programming model then allocates hectares to systems of crops rather than individual crops. The advantage of this approach is that any between-crop interactions that affect crop yields are reflected in each system's yield coefficients. Since each system must have a set of yield estimates for each cropping component, this third modeling approach discourages extrapolating limited field data into new cropping systems with unknown actual yields.

For cropping systems with components that provide yields over more than a single period, multi-stage models are easily constructed [6]. Crops with seasonal yields can be modeled by multi-stage models where periods are defined to be less than a full year. In longer-term models, where the timing of future costs and benefits are an important consideration in planning, returns can be discounted by an appropriate discount rate [11].

Conclusion

Empirical studies have demonstrated the important role that uncertainty plays in determining optimal farming strategies in developing countries. The application of portfolio theory through quadratic programming is one method of recognizing uncertainty to reduce the risk associated with agroforestry plans. Such considerations of risk should play a prominent role in planning agroforestry systems.

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Table 1. Time series net revenue data and covariances for four hypothetical agroforestry crops.

	Crop 1	Crop 2	Crop 3	Crop 4
		net revenue (pesos/hectare)		
Year 1	153	97	390	-150
Year 2	123	230	126	532
Year 3	109	223	175	653
Year 4	165	145	323	148
Year 5	179	278	187	790
Average	146	195	240	395
		covariance (pesos)		
Crop 1	849	-	-	-
Crop 2	-185	5,250	-	-
Crop 3	1,476	-7,265	12,355	-
Crop 4	-2,043	27,641	-38,893	149,848

Table 2. Set of minimum-risk agroforestry plans

Plan	Total net return	Percent reduction	Net return variance	Percent reduction	Land allocation X1	X2	X3	X4
	(pesos)		(1×10^6 pesos)		hectares			
A	5,476	0%	26.2	0%	1.0	1.0	0.0	13.0
B	5,421	1%	24.4	7%	1.0	1.0	0.3	12.7
C	5,202	5%	18.1	31%	1.0	1.0	1.7	11.3
D	4,928	10%	11.5	56%	1.0	1.0	3.5	9.5
E	4,381	20%	2.9	89%	1.0	1.0	7.1	5.9

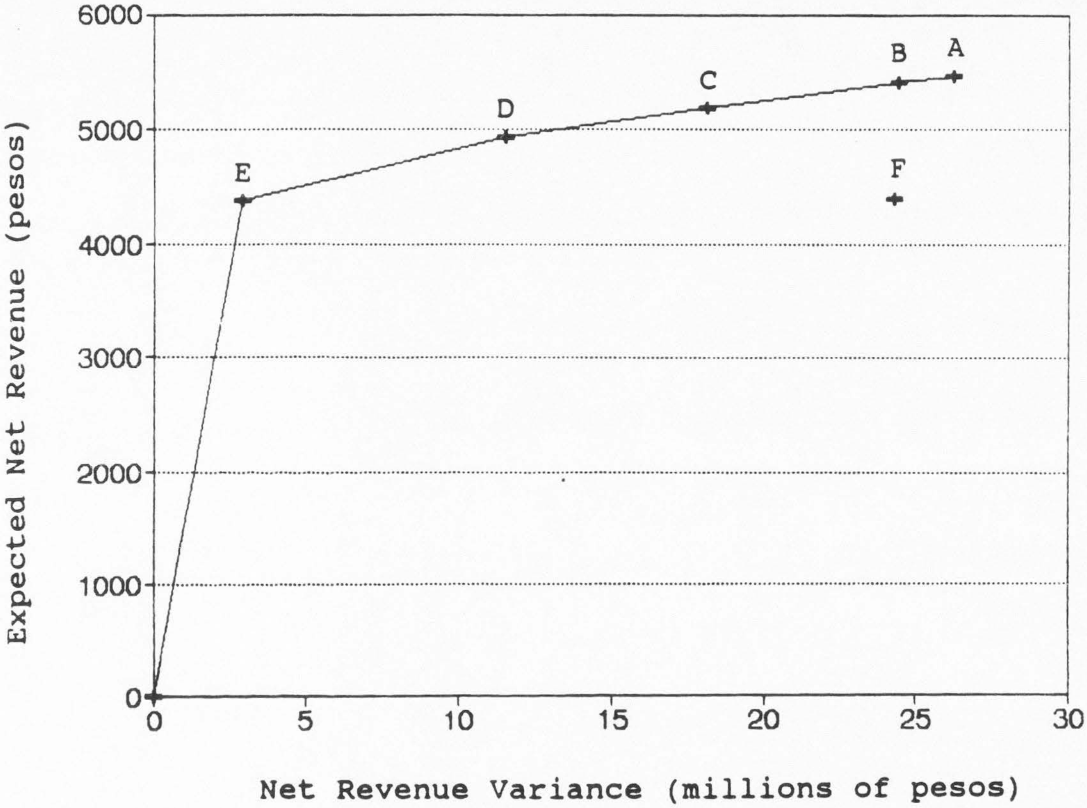

Figure 1. Set of minimum-risk agroforestry plans (arc AE).

CHAPTER II

CROP YIELD INTERACTIONS IN MATHEMATICAL PROGRAMMING MODELS OF AGROFORESTRY SYSTEMS: A METHODOLOGICAL REVIEW

Abstract. Two methods frequently used to model interactions between agroforestry crops are discussed. The continuous system approach assumes linear interactions between crop yields and a continuous range of possible crop mixtures. The discrete system approach defines decision variables as fixed crop mixtures with yields reflecting any interactions between component crops. The continuous system approach is only suitable when crop interactions are negligible, whereas the discrete system approach is better suited when crop interactions occur.

Introduction

Agroforestry systems are commonly designed to take advantage of any interactions between crops that result in increased yields over those obtained in monocultural production. Such interactions may cause yields to fluctuate widely depending on crop composition and density, warranting a standardized method for comparing yield advantages. The Land Equivalent Ratio (LER) measures crop yield interactions between systems using the quantity of land required to obtain the same yield as a base [6,10].

The LER for J crops is defined as:

$$LER = \sum_{j=1}^{J}(L_j) = \sum_{j=1}^{J}(Y_j/S_j)$$

where:

j = 1,...,J crops,
L_j = LER for crop j,
Y_j = crop j's yield when intercropped, and
S_j = crop j's yield in a monoculture.

For example, an LER_j of 1.5 indicates that a monoculture of 1.5 hectares of crop j will produce the same yield as one hectare of crop j intercropped. The ability to compare the relative efficiencies of monocultures and other cropping systems is useful when planning intensive agricultural strategies.

Due to the complex nature of plant interactions in agroforestry systems, identifying, quantifying, and incorporating yield advantages into mathematical programming models is paramount. This paper compares two frequently used methods for describing crop yield interactions in linear programming models of agroforestry systems and discusses their relative advantages and disadvantages.

Continuous vs. discrete system optimization

Continuous system

Modeling crop yields with a *continuous system* approach assumes that each crop's per-hectare yield is directly related to the fractional area allocated to each crop. For example, in the following linear program, each decision variable, x_j, is defined as the fraction of land allocated to crop j, where x_j can vary from zero to one. The model maximizes the objective function by selecting the total number of hectares to be allocated to each crop.

$$\text{Maximize } \sum_{j=1}^{J} (\alpha_j)(x_j)$$

Subject to:

$$\sum_{j=1}^{J} (x_j) \leq \delta \qquad \text{(land constraint)}$$

$$(c_{jk})(x_j) \ \{ \leq, =, \geq \} \ b_k \qquad \text{(for all j given k resource constraints)}$$

$$x_j \geq 0 \text{ for all j} \qquad \text{(non-negativity constraints)}$$

where:

j = 1,...,J crops,
α_j = per-hectare contribution of crop j to the objective function,
x_j = number of hectares allocated to crop j,
δ = available hectares,
b_k = right hand side resource constraint k, and
c_{jk} = technical coefficient for crop j in constraint k.

Under the continuous system, the model may assign any number of the total hectares available to each crop species using the same yield coefficient for each respective crop, thereby assuming an LER of 1.0 for all possible crop combinations.

Figure 2 demonstrates the concept of the linear interactions expressed by a LER of 1.0. For example, if 75% of the land area is occupied by crop A and 25% with crop B, then the yields for crops A and B will be 75% and 25%, respectively, of what would be expected under a monoculture on the same land area. Hence, there exists a one-to-one relationship between the fraction of hectares devoted to each crop and the fraction of each per-hectare yield obtained.

The continuous system is frequently used in conceptual papers to illustrate modeling techniques because of the simplicity it lends to model formulation (see Blandon [1], Betters [2], Dykstra [3], Lilieholm and Reeves [5]). This approach also provides planners with a myriad of options since the number of potential systems
is immense since $0 \leq X_j \leq 1$ for all j such that $\sum_{j=1}^{J} (X_j) = 1$.

The wide range of possible crop combinations, however, renders the continuous system approach inappropriate for modeling agroforestry systems with crops having dynamic LERs, a dynamic LER being any LER which has a value other than one over some range of the crop mixture. In this case, unique yield coefficients must be assigned to each crop as the abundance of that crop fluctuates within the crop mixture. This may require dubious extrapolation, since the yields of only a few crop combinations are typically known.

Discrete System

The *discrete system* approach defines each decision variable as a complete cropping system with a fixed crop ratio. The yield coefficient for each component crop is based on the unique crop mixture, thereby reflecting any between-crop interactions ignored by the continuous system approach. Extrapolating limited data is discouraged under this system since each system requires a knowledge of actual yields.

While the continuous system approach allocates land to individual crops, thus defining the crop mixture of the system, the discrete system allocates land to each pre-determined crop mixture based on the system's yield coefficient. For example, in the following linear program, each decision variable, X_i, defines a predetermined crop mixture. The model selects the optimal number of hectares to be allocated to each discrete crop combination, thereby defining the overall system.

$$\text{Maximize } \sum_{i=1}^{I} (\alpha_i)(x_i)$$

Subject to:

$$\sum_{i=1}^{I} (x_i) \leq \delta \qquad \text{(land constraint)}$$

$$(c_{ik})(x_i) \; \{ \leq, =, \geq \} \; b_k \qquad \text{(for all i given k resource constraint)}$$

$$x_i \geq 0 \text{ for all i} \qquad \text{(non-negativity constraints)}$$

where:

αi = per-hectare contribution of crop mixture i to the objective function,
x_i = fraction of land allocated to crop mixture i,
δ = available hectares,
bk = right hand side resource constraint k, and
$c_i k$ = technical coefficient for crop i in constraint k.

Figure 3 graphically illustrates this same concept using three discrete decision variables in a two-crop system. The decision variables represent fixed ratios of the land area covered by the two crops, and the sum of the individual crop yields defines the total yield for the system. For example, X_2 is defined by an area evenly split between crops A and B, with the yield of each crop known for that mixture. Notice that the LER is not constant as the crop mixture fluctuates.

Discrete system optimization chooses the overall species composition based on the optimal defined system or group of defined systems. There is no continuum over which the model may select any possible mixture, as is the case with the continuous system approach.

Employment of the discrete system approach is more commonly found in the agroforestry literature, particularly with case studies based on actual data. Some examples include Hoekstra [4], Raintree [7], Raintree and Turay [8], Verinumbe et al. [9], and Wojtkowski et al. [11].

Combining Continuous and Discrete Systems

It is possible to combine the two methods to obtain a model with both continuous and discrete decision variables. The continuous decision variables would be assigned to crops or crop mixtures with LERs equal to one, while the discrete system decision variables would be assigned to crop mixtures with dynamic LERs as described above. For example, in a model with two decision variables, the first decision variable could be the number of hectares allocated to a corn monoculture (continuous system), and the second could be the number of hectares allocated to a coffee and banana cropping system where the coffee and bananas are grown in fixed proportions, (e.g., a discrete system where there is one banana tree for every ten coffee trees).

The advantage of the combined approach is the increase of cropping system options when planning agroforestry systems with one or more crops with LERs equal to 1.0. In practice, it is unnecessary to exclude cropping combinations of one type, for example dynamic LERs, when considering cropping systems with constant LERs, or vice versa.

Conclusion

By understanding the relative advantages and disadvantages of the continuous and discrete modeling approaches, more realistic mathematical models can be formulated. The continuous system approach is sufficient when crop interactions are negligible. While these circumstances may be infrequent, the myriad of possibilities in crop mixtures offered by this approach is not without interest.

The discrete system approach is more accurate when modeling systems with dynamic LERs. Although the crop mixtures are limited to systems with known yields, more confidence can be placed in the reliability of the model.

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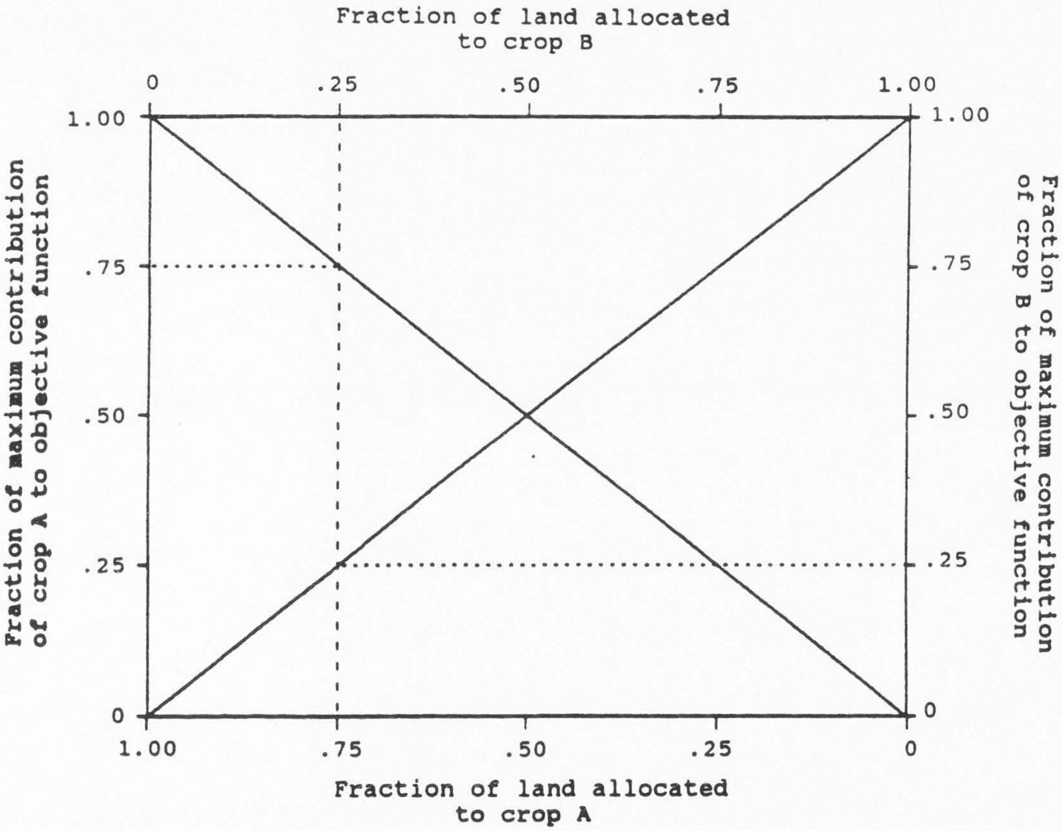

Figure 2. The relationship between the allocation of land to two crops with linear yield functions (LERs = 1) and the resulting affect on the objective function.

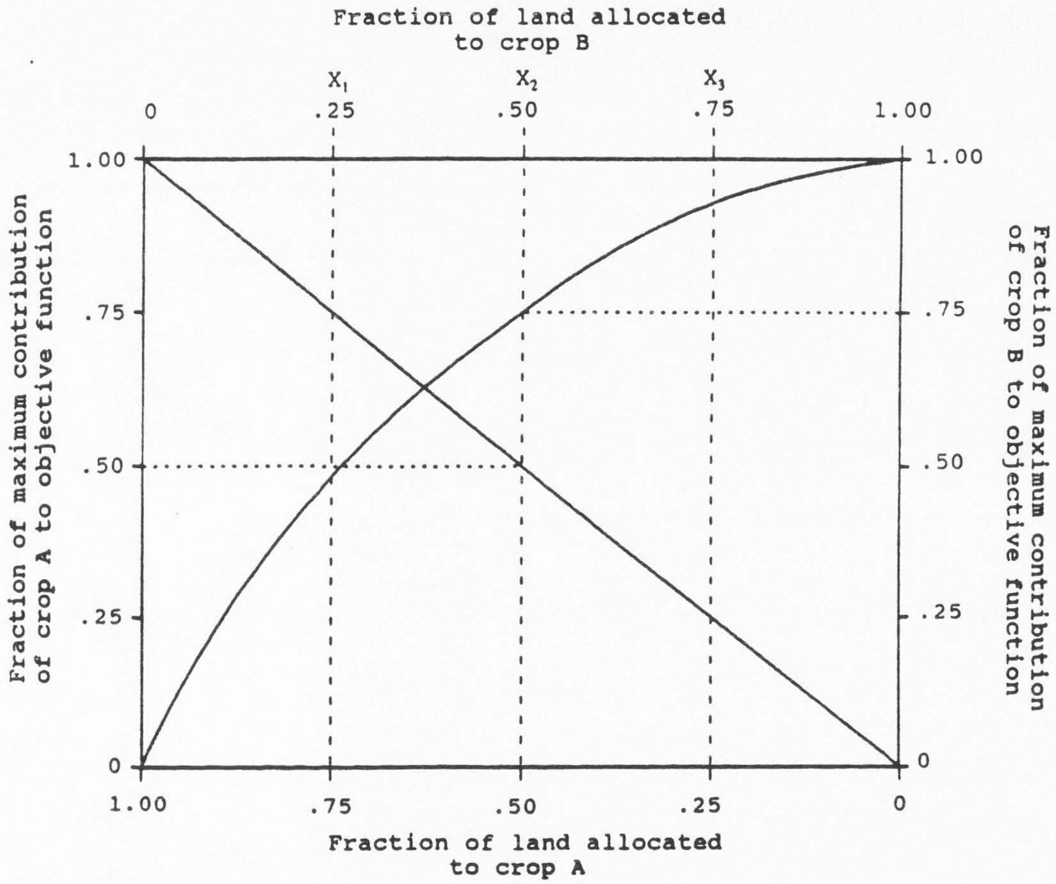

Figure 3. The relationship between the allocation of land to two crops, crop A with LER = 1 and crop B with a dynamic LER, and the resulting affect on the objective function.

CHAPTER III

REDUCING FINANCIAL RISK IN AGROFORESTRY PLANNING: A CASE STUDY IN COSTA RICA

Abstract. The ability to use a knowledge of past net revenue fluctuations from alternative cropping systems to reduce the risk of future returns is explored in the context of planning the allocation of land to various cropping systems. Using a case study in Costa Rica, the set of minimum-risk farm plans is derived for a hypothetical farm. The MOTAD model results indicate risk-averse behavior on the part of most of the farmers interviewed, and any further reduction of risk would require a relatively large reduction in expected net revenue. This type of risk analysis can be useful in farm planning where time series net revenue data is available.

Introduction

Risk-aversion on the part of rural agriculturalists makes the ability to assess and reduce the variability of expected net income an important factor in facilitating the adoption of agroforestry systems [1,2,4]. Modern stock portfolio theory allows for the explicit recognition and reduction of financial risk in agroforestry planning by reducing the variance of expected net returns [3,8,9]. This paper extends earlier work to demonstrate the application of portfolio theory through MOTAD (Minimization of Total Absolute Deviations) programming [5] using a case study site in Costa Rica.

Methods

San Vito de Java, Costa Rica, is a settlement with approximately 7,000 inhabitants lying near the Panamanian border at 890 meters elevation. San Vito was colonized after World War II by Italian immigrants and has maintained an agricultural economy based largely on coffee export. San Vito has a mean annual precipitation of 38.9 cm and an annual mean temperature of 22.4°C [7]. Cost, revenue, and yield data for various cropping systems was collected through interviews with farmers in the San Vito area of Costa Rica. Local farmers were cooperative in providing data for their cropping systems. While only one farmer interviewed had written records, other data was corroborated through information provided by the Consejo Nacional de Producción (CNP), Ministerio de Trabajo, Instituto de Café de Costa Rica, and a local agro-chemical supplier. Consumer price indices were provided by the Ministerio de Economía, Industria y Comercio (MEIC).

A mathematical programming model of a hypothetical 15-hectare farm in the San Vito area was developed. The model included five decision variables, each of which represented the number of hectares planted to a particular cropping system (Table 3). Any combination of the five cropping systems are available for selection, the only constraints being available land and chayote squash production (X_4) limited to a maximum of 3 hectares. This latter constraint was used because it is necessary to elevate the squash with an arbor-like structure which may not be practically constructed on a large-scale. Moreover, the chayote patches observed were all under 3 hectares. While labor can be scarce during the coffee harvest, a sufficient supply is assumed for all cropping systems.

For each cropping system, 5-year time series net revenue data was determined on a per-hectare basis, with all costs except labor included (Table 4). The expected net revenue was estimated by fitting a linear regression through the data, with the exception of X_4 and X_5 which were fitted with quadratic functions. All values are presented in 1990 colones.

Labor is divided into three categories at different wage rates: coffee picking, denoted as W_1 (units are per basket picked), general labor (per hour), W_2, and spraying agro-chemicals (per hour), W_3. The type of labor required for each decision variable is formulated as a series of constraints, while the per-hectare cost of labor is subtracted from the objective function to arrive at total net revenue values.

The following deterministic linear programming model was formulated to maximize net returns:

Maximize:

$$192151.44X_1 + 296048.71X_2 + 156730.98X_3 + 45931.10X_4 + 1993.51X_5 - 63.50W_1 - 68.97W_2 - 96.18W_3$$

Subject to:

$X_1 + X_2 + X_3 + X_4 + X_5 = 15$ (hectares available)

$X_4 \leq 3$ (maximum chayote constraint)

$1240X_2 + 466.67X_3 - W_1 = 0$ remaining equations are labor constraints)

$3024X_1 + 186.98X_2 + 156X_3 + 288X_4 + 70.54X_5 - W_2 = 0$

$186.98X_2 + 130X_3 - W_3 = 0$

Risk, measured by the variance of the expected revenue, is incorporated into the model using a linear approximation technique called MOTAD [5]. In this formulation, the mean absolute deviations (MADs) of the net revenues from the fitted trend line are minimized subject to meeting a parametrically-altered scalar of required net revenue, δ (see Hazell and Norton [6]):

Minimize:

$DP_1 + DN_1 + DP_2 + DN_2 + DP_3 + DN3 + DP_4 + DN_4 + DP_5 + DN_5$ (minimizes sum of net revenue deviations)

Subject to:

$NR \geq \delta$ (required net revenue)

$192151.44X_1 + 296048.71X_2 + 156730.98X3$ (net revenue minus labor)

$+ 45931.10X_4 + 1993.51X_5 - 63.50W_1 - 68.97W_2 - 96.18W_3 - NR = 0$

$X_1 + X_2 + X_3 + X_4 + X_5 = 15$ (hectares available)

$X_4 \leq 3$ (maximum chayote constraint)

$1240X_2 + 466.67X_3 - W_1 = 0$ (labor requirement)

$3024X_1 + 186.98X_2 + 156X_3 + 288X_4$
$+ 70.54X_5 - W_2 = 0$

$186.98X_2 + 130X_3 - W_3 = 0$

$3802.44X_1 - 17838.94X_2 - 9950.07X_3$
$+ 232.84X_4 + 18.27X_5 - DP_1 + DN_1 = 0$

$-1616.46X_1 + 31003.32X_2 + 16358.92X_3$

$- 562.34X_4 - 30.87X_5 - DP_2 + DN_2 = 0$

$-5098.44X_1 - 8576.00X_2 - 3676.81X_3$
$+ 289.86X_4 - 17.10X_5 - DP_3 + DN_3 = 0$

$-163.47X_1 - 4502.17X_2 - 1922.79X_3$
$+ 175.71X_4 + 53.59X_5 - DP_4 + DN_4 = 0$

$3075.97X_1 - 86.17X_2 - 809.21X_3$
$- 136.33X_4 - 24.02X_5 - DP_5 + DN_5 = 0$

(remaining rows calculate deviations of net revenue data from the trend for t years of time series data)

where:

$\sum_{t=1}^{5} (DN_t)$ = the sum of the negative net revenue deviations below the trend, and

$\sum_{t=1}^{5} (DP_t)$ = the sum of the positive net revenue deviations above the trend, where

t = year 1 through 5 of the time series data, starting with 1986.

In this second model, the required net revenue constraint was parametrically-altered between model solutions such that δ equaled 5%, 15%, 30%, and 50% reductions from the maximum net return found by the deterministic model. By solving the MOTAD model for different levels of expected net revenue, a set of minimum-risk farm plans was derived.

Results and discussion

The optimal solution to the deterministic model (plan A in Table 5) allocates all available hectares to X_2, the coffee monoculture. This result is intuitive since this cropping system has the highest expected net return (Table 4). However, since this system also has the highest net revenue variance, it may be avoided by risk-averse farmers since it is economically the riskiest system.

Arc AE in Figure 4 represents the minimum-risk frontier of feasible farms plans in the San Vito area. For a risk-averse farmer, only the minimum-risk plans on arc AE are relevant since any plan below the arc has a lower expected return for a given level of risk. Hence, a risk-averse farmer would only choose plans on arc AE.

Across the minimum-risk frontier, as expected net revenue is reduced to lower financial risk, optimal farm plan shifts to include X_4, the chayote squash system, until X_4 reaches the maximum production level of 3 hectares. As risk is further reduced, the coffee/banana cropping system (X_3), begins to replace the coffee monoculture (plan D). Finally, plan E is dominated by X_3, with 3 hectares allocated to chayote and less than a hectare allocated to the coffee monoculture.

The steep slope of arc AE indicates that reductions in net revenue variance require substantial reductions in expected net revenues. Ultimately, the preferred plan on the frontier is determined by an individual farmer's willingness to trade-off expected net income and risk.

Sugar cane and dairy production were not selected in the model because of low net revenues once labor was included. Both of these cropping systems occurred less frequently relative to coffee in the study area. Moreover, both the dairy and sugar cane producers interviewed grew coffee on other plots of land. This was also true of tomato and vegetable producers; despite diversification into other crops, some amount of coffee was always produced. The coffee/banana cropping system was common on farms of all sizes. As the MOTAD model indicates, risk as well as income was reduced using this cropping system.

The one producer interviewed who had the coffee monoculture also had a relatively large land asset. When asked about diversification, the farmer indicated that as long as coffee net revenues were well above those of other crops it was worthwhile. In addition, the farmer seemed to be innovative, experimenting with both leguminous tree intercropping and alternative coffee planting techniques. This behavior indicates a greater willingness to accept risk, which is consistent with the MOTAD model results. This model deals with the dominant systems and systems with sufficient time-series data. In reality, many other farming systems exist in the San Vito area. Because interest in growing vegetables like tomatoes is fairly recent, time series data regarding these crops was scarce and these systems were excluded from the model. While this may limit the model's application, coffee is a dominant component of most systems in the region, and the representation of the coffee-based systems, X_2 and X_3, is consistent with the author's observations in the area. The extensive use of the coffee/banana cropping system, even with moderate variations in coffee and banana tree densities, indicates a certain degree of risk-aversion on the farmer's part.

As with other types of mathematical models, the utility of the results depend on the accuracy of the data and model. While the results of mathematical models can provide valuable insight into farm planning, there are other factors that are not easily incorporated into mathematical models such as cultural influences that should be considered.

Conclusion

Risk reduction in the San Vito area would be accompanied by significant reductions in expected net revenue. This would likely deter any drastic changes in current land use practices. While the coffee/banana cropping system offers a relatively low-risk system which many farmers feel satisfied with, there are apparently no cropping system alternatives which can significantly reduce expected revenue variance without a comparable reduction in expected net revenue. Further research in developing low-risk agricultural systems could benefit this region. Under conditions where complete data were available to produce a more robust model, this type of risk analysis could be an important tool in the planning of agroforestry systems, development strategies, and regional agricultural policies.

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Table 3. Description of cropping system decision variables.

Decision variable	Description
X_1	Number of hectares allocated to monocultural sugar cane production, processed on-farm.
X_2	Number of hectares allocated to high-density coffee monoculture.
X_3	Number of hectares allocated to the production of coffee with two species of bananas.
X_4	Number of hectares allocated to chayote squash production (limited to a 3-hectare maximum land allocation).
X_5	Number of hectares allocated to dairy cow pasture, at a stocking density of approximately 2.2 cows/hectare. Milking is done by hand, pasture fed.

Table 4. Times series net revenue data and variance-covariance matrix for five cropping systems in Costa Rica (1990 colones).

Year	Sugar cane (X_1)	Coffee monoculture (X_2)	Coffee/ bananas (X_3)	Chayote (squash) (X_4)	Dairy cows (X_5)
		net revenue (1000 colones/ha)			
1986	101.9	191.7	100.1	8.8	.5
1987	119.1	163.5	85.1	11.9	.6
1988	138.2	221.7	115.7	15.9	.8
1989	157.1	256.1	132.6	23.8	1.1
1990	173.3	270.9	144.8	33.6	1.5
Expected	192.2	296.0	156.7	45.9	2.0
		covariance matrix (1000 colones/ha)			
X_1	40987.0	-	-	-	-
X_2	-14750.4	1118676.2	-	-	-
X_3	-9541.4	144987.7	305286.1	-	-
X_4	-26.3	-4970.7	-2561.9	460.6	-
X_5	24.8	-275.1	-141.5	5.9	3.2

Table 5. Set of minimum-risk farm plans.

Plan	Total net return	Percent reduction1/	Net return variance	Percent reduction2/	Land Allocation X_1	X_2	X_3	X_4	X_5
		(colones x 10^5)				hectares			
A	28.0	na	9.3	na	0.0	15.0	0.0	0.0	0.0
B	26.6	5%	8.8	6%	0.0	14.1	0.0	0.9	0.0
C	23.8	15%	7.6	18%	0.0	12.4	0.0	2.6	0.0
D	19.6	30%	6.1	34%	0.0	7.7	4.3	3.0	0.0
E	14.0	50%	4.2	55%	0.0	0.9	11.1	3.0	0.0

1/ Percent reduction from total net return.

2/ Percent reduction from net revenue variance.

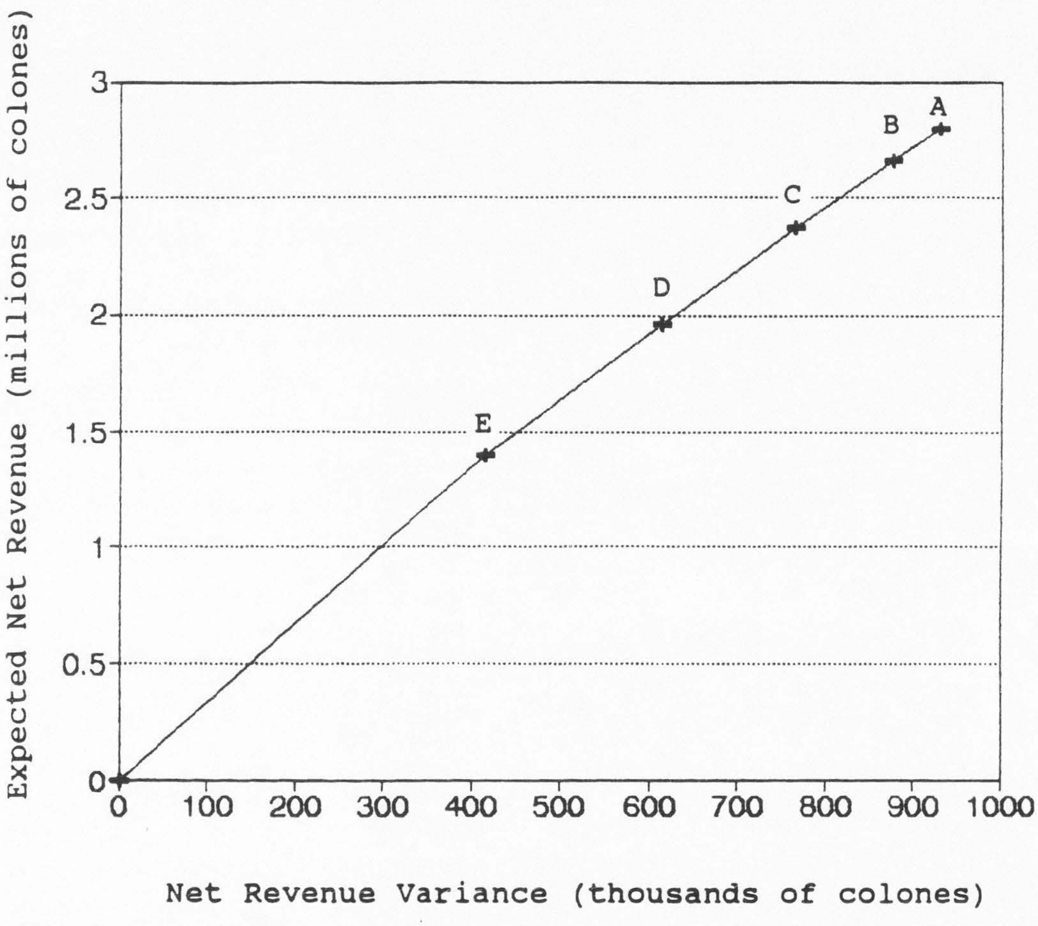

Figure 4. Set of minimum-risk farm plans.

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