Collective modes in strongly coupled complex plasmas and related systems

Sergey Khrapak, Boris Klumov, Lenaic Couedel, and Hubertus Thomas

Aix-Marseille-University, CNRS, Laboratoire PIIM, Marseille, France
Institute of Materials Physics in Space, DLR, Oberpfaffenhofen, Germany
Joint Institute for High Temperatures, RAS, Moscow, Russia

Motivation:

- Theory of collective motion in liquids is active area of research since the second half of XX century (starting from neutron scattering measurements)
- In particular, the theory for monoatomic liquids (liquid metals and rare gases) has been worked out
- To which extent the developed theories are applicable to complex (dusty) plasmas, representing classical systems of strongly interacting particles?
- Alternatively, using complex (dusty) plasmas is it possible to check the accuracy and applicability limits of these early theories?

Quasi-crystalline (quasi-localized charge) approximation (QCA/QLCA)

- Generic expressions for the longitudinal and transverse dispersion relations:

$$\omega_L^2 = \frac{n}{m} \int \frac{\partial^2 V(r)}{\partial z^2} g(r) \left[1 - \cos(kz)\right] d\mathbf{r},$$

$$\omega_T^2 = \frac{n}{m} \int \frac{\partial^2 V(r)}{\partial y^2} g(r) \left[1 - \cos(kz)\right] d\mathbf{r}.$$

are equivalent to the model of collective motion in liquids by Zwanzig (1967), quasi-crystalline approximation (QCA) by Hubbard&Beeby (1969), Takeno&Goda (1971). Similar expressions occur from the analysis of frequency moments of $S(k,\omega)$.

- In the context of plasma physics QCA is also known as QLCA after Kalman and Golden who applied the approximation to one-component-plasma and related charged systems

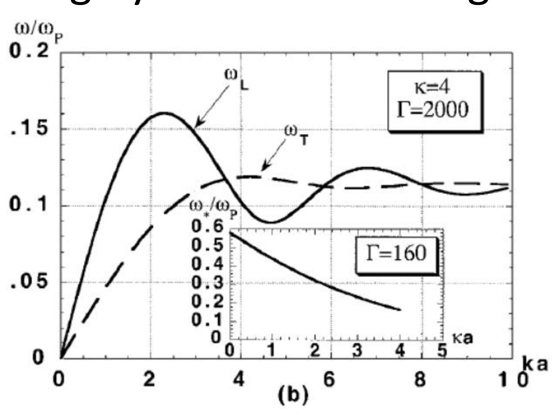
DLR

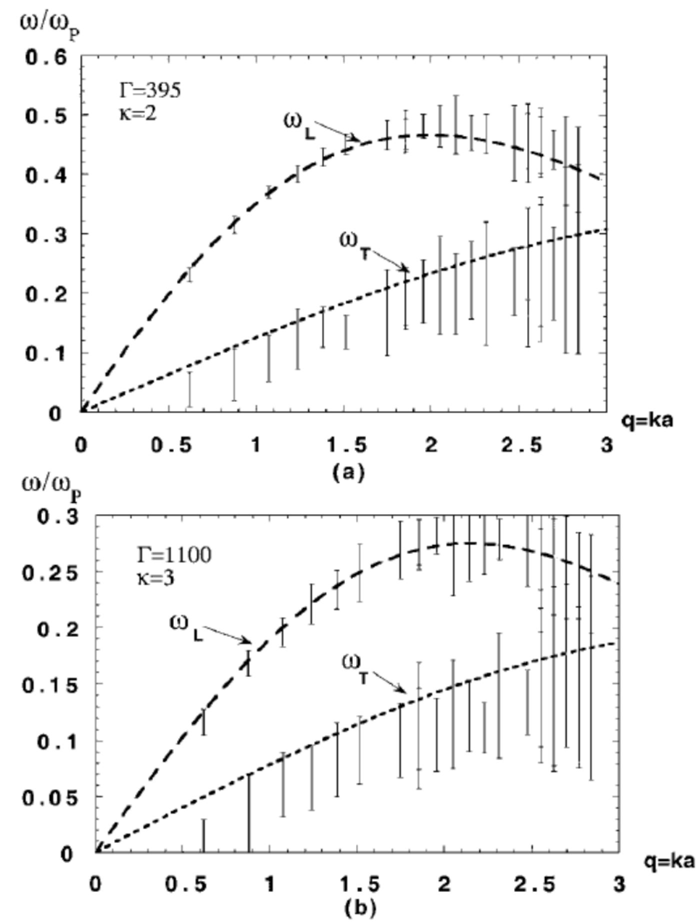
Application to complex (dusty) plasmas

- **Yukawa interaction potential**

- First applied by Rosenberg and Kalman (1997) in the regime of long-wavelengths and weak screening

- Kalman et al. (2000) computed *g*(*r*) using the HNC scheme get results in good agreement with MD modeling by Ohta and Hamaguchi (2000)

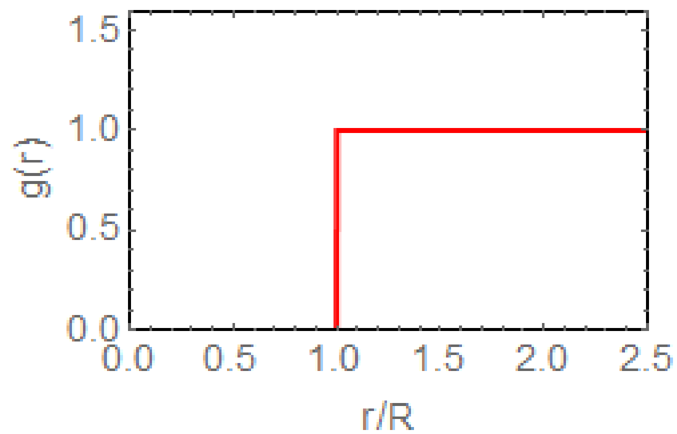




Kalman et al. PRL (2000)

How accurately RDF should be known?

- The simplest model which takes into account excluded volume effects
- The integration can be performed analytically
- The parameter *R* is evaluated requiring consistency for energy or pressure

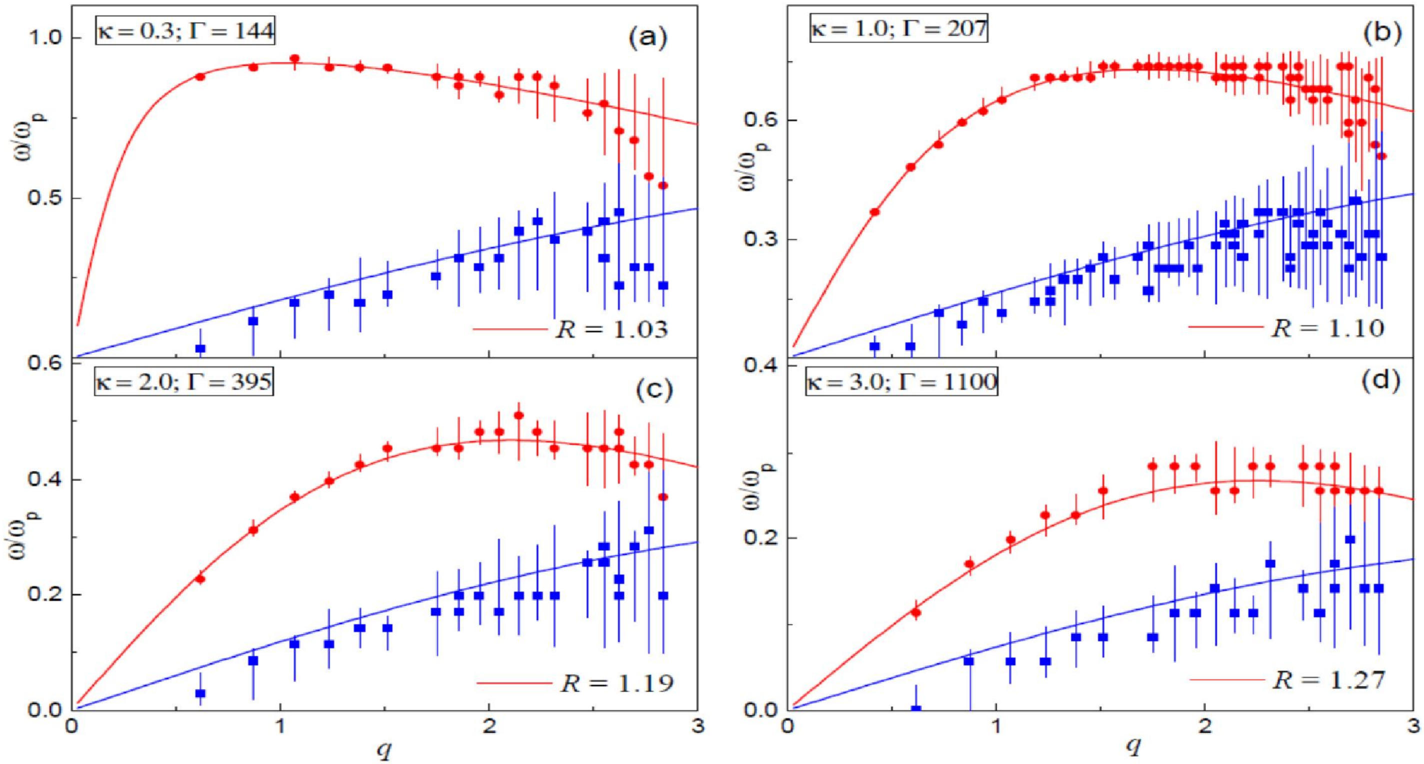


$$\omega_L^2 = \omega_p^2 e^{-R\kappa}\left[(1+R\kappa)\left(\frac{1}{3}-\frac{2\cos Rq}{R^2q^2}+\frac{2\sin Rq}{R^3q^3}\right) - \frac{\kappa^2}{\kappa^2+q^2}\left(\cos Rq + \frac{\kappa}{q}\sin Rq\right)\right].$$

$$\omega_T^2 = \omega_p^2 e^{-R\kappa}(1+R\kappa)\left(\frac{1}{3}+\frac{\cos Rq}{R^2q^2}-\frac{\sin Rq}{R^3q^3}\right).$$

Khrapak et al. (2016)

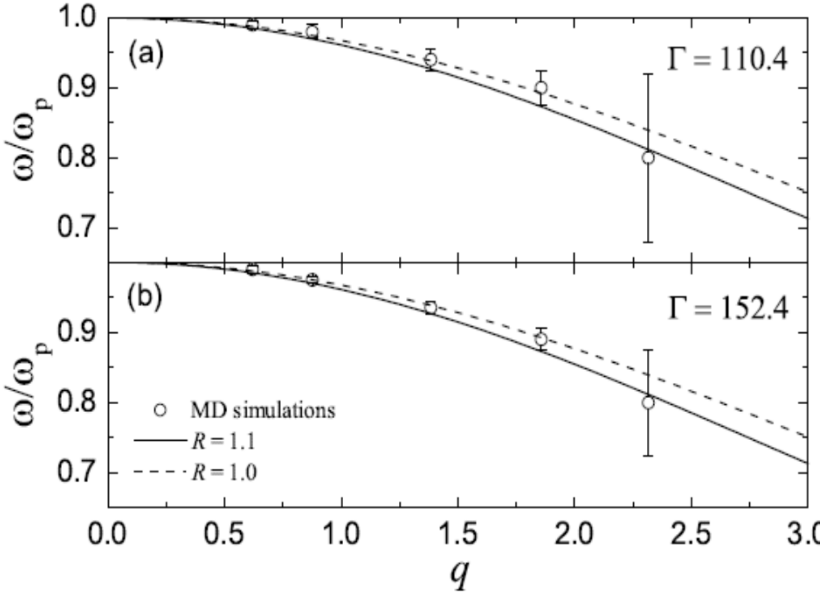
Long-wavelength regime

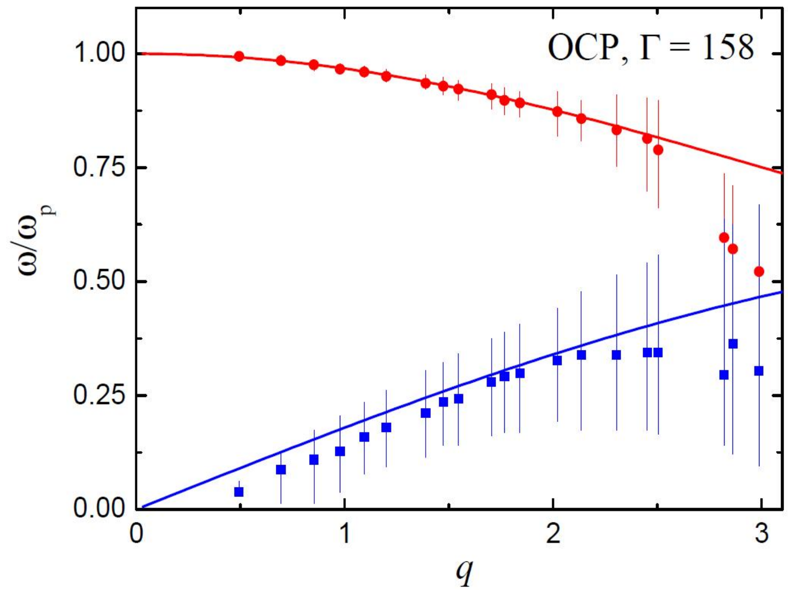


- In the long-wavelength regime such a simple approximation is very useful
- Explicit expression for R: Roughly $R(\kappa) \simeq 1 + \kappa/10$

MD data by Ohta and Hamaguchi (2000)

One-component-plasma (OCP) limit





- Again, simple explicit expressions:

$$\omega_L^2 = \omega_p^2\left(\frac{1}{3} - \frac{2\cos Rq}{R^2q^2} + \frac{2\sin Rq}{R^3q^3}\right), \qquad \omega_T^2 = \omega_p^2\left(\frac{1}{3} + \frac{\cos Rq}{R^2q^2} - \frac{\sin Rq}{R^3q^3}\right).$$

- Good accuracy at strong coupling

Left: Hansen et al. (1974); Right: Schmidt et al. (1997)

Deviations from pure Yukawa interaction in complex (dusty) plasmas

- Electron and ion collection ➔ Power-law long-range asymptotes
- Non-linear ion-particle interaction ➔ Variability of the effective screening length
- Plasma production and loss ➔ Double-Yukawa interaction potential
- Ion flows ➔ Wake-mediated interaction
- **Can QLCA be used to discriminate between different interactions in complex plasmas?**

Representative examples of interaction

- Double-Yukawa potentials

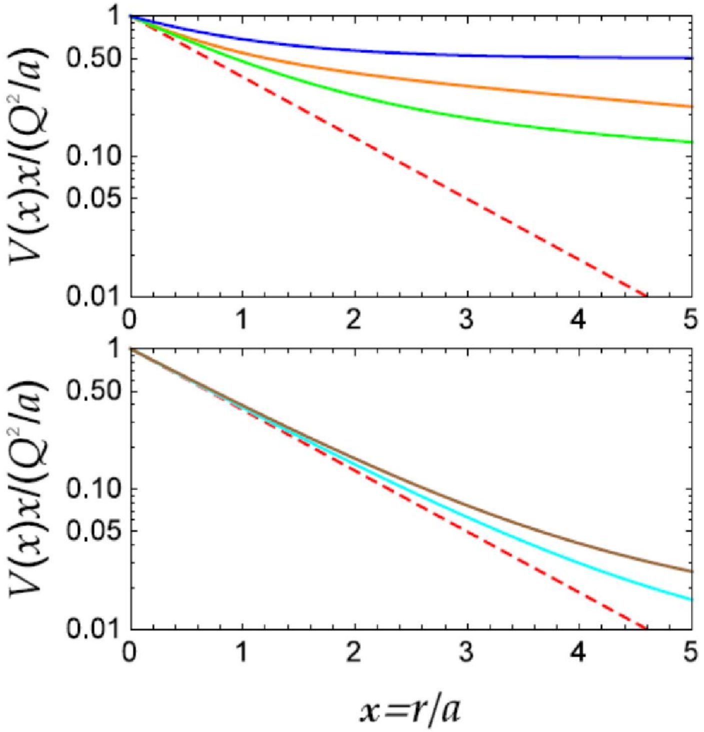
$$V(r) = \frac{Q^2}{r}\left[\epsilon_1 \exp(-r/\lambda_1) + \epsilon_2 \exp(-r/\lambda_2)\right]$$

- Yukawa + inverse square r

$$V(r) = \frac{Q^2}{r}\left[(1-\epsilon)e^{-r/\lambda_D} + (\epsilon\lambda_D/r)\left(1 - e^{-r/\lambda_D}\right)\right]$$

TABLE I. Summary of the model interaction potentials considered in this study (Cases 1 - 5).

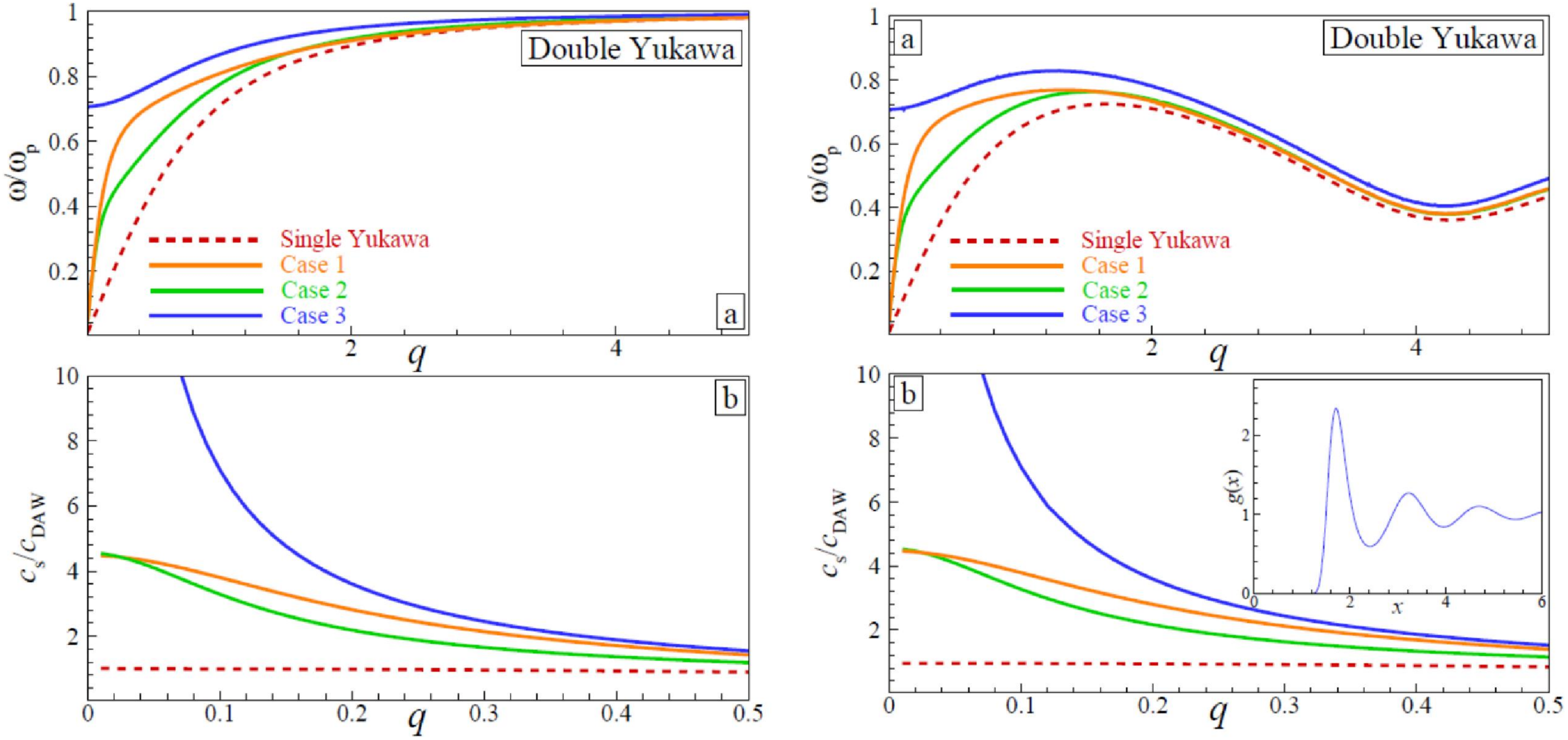
Case	Functional form	Parameters
1	Eq. (2)	$\epsilon_1 = \epsilon_2 = 0.5,\ \lambda_1 = 0.7\lambda_D,\ \lambda_2 = 6.3\lambda_D$
2	Eq. (2)	$\epsilon_1 = 0.8,\ \epsilon_2 = 0.2,\ \lambda_1 = \lambda_D,\ \lambda_2 = 10\lambda_D$
3	Eq. (2)	$\epsilon_1 = \epsilon_2 = 0.5,\ \lambda_1 = \lambda_D,\ \lambda_2 = \infty$
4	Eq. (3)	$\epsilon = 0.05$
5	Eq. (3)	$\epsilon = 0.1$



Khrapak et al. (2017)

Fingerprints of interactions: Double Yukawa class

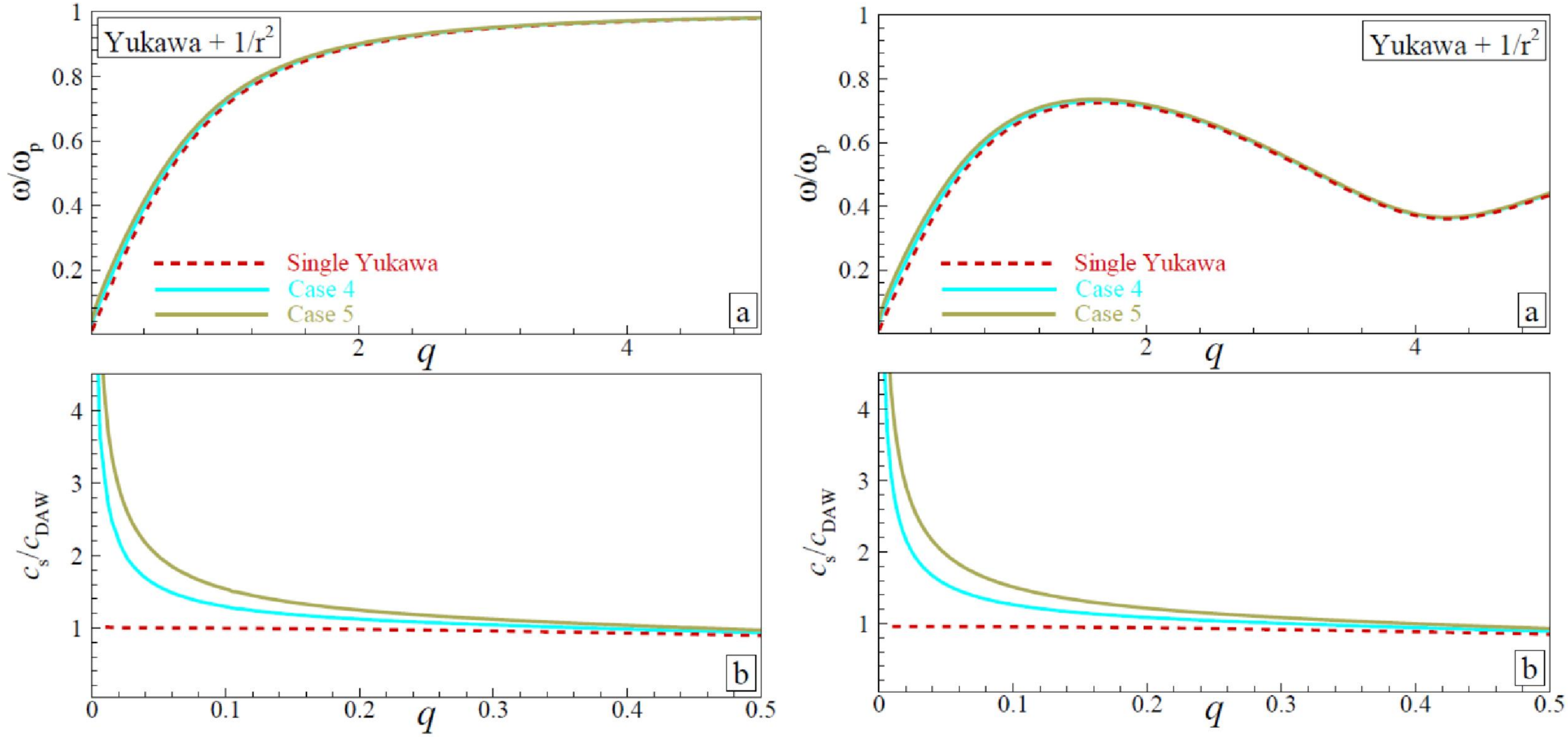
Long-range asymptote of the interaction potential affects the dispersion at long wavelengths, which can be measured experimentally



Khrapak et al. (2017)

Fingerprints of interactions: Yukawa + $1/r^2$

Long-range asymptote of the interaction potential affects the dispersion at long wavelengths, which can be measured experimentally



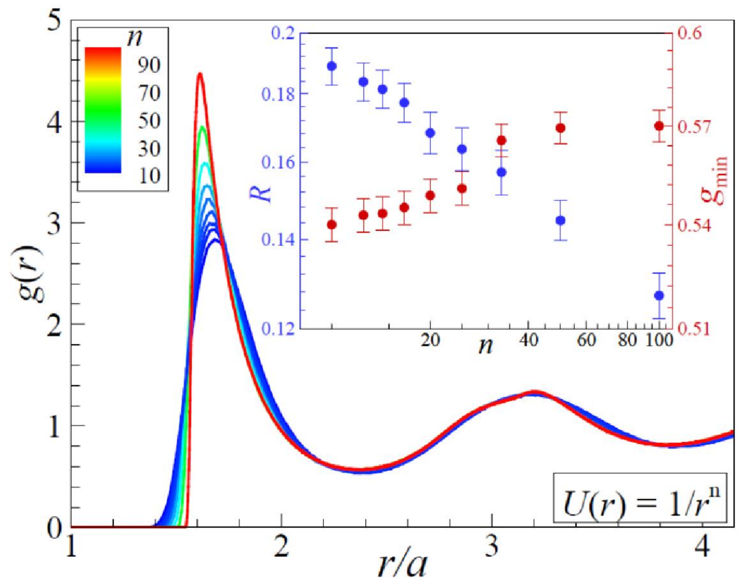
Khrapak et al. (2017)

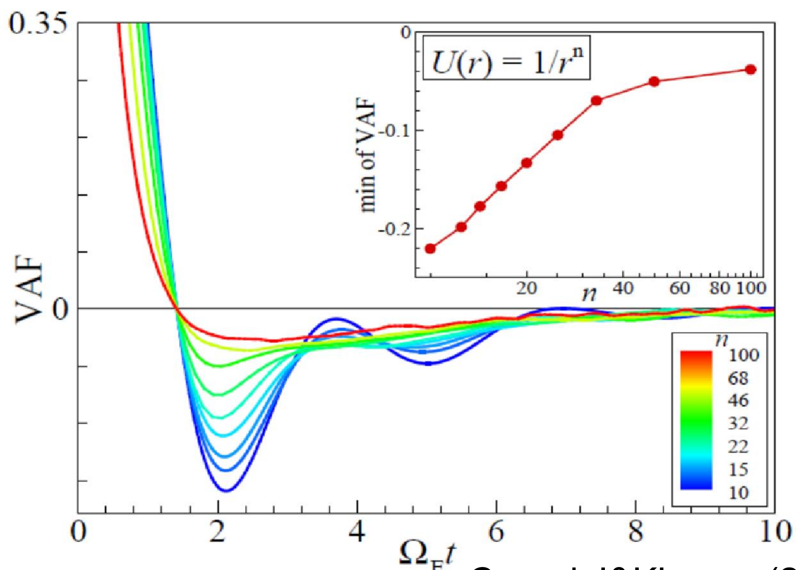
Is QCA/QLCA equally good for hard and soft interactions?

- Systematic study of IPL fluids near the fluid-solid transition
- Agrawal&Kofke (1995) data on coexistence fluid densities of the IPL model
- MD simulations for a number of IPL exponents ($10 \le n \le 100$)
- Analysis: Structure, dynamics, longitudinal mode dispersion

Structure (RDF) and dynamics (VAF)

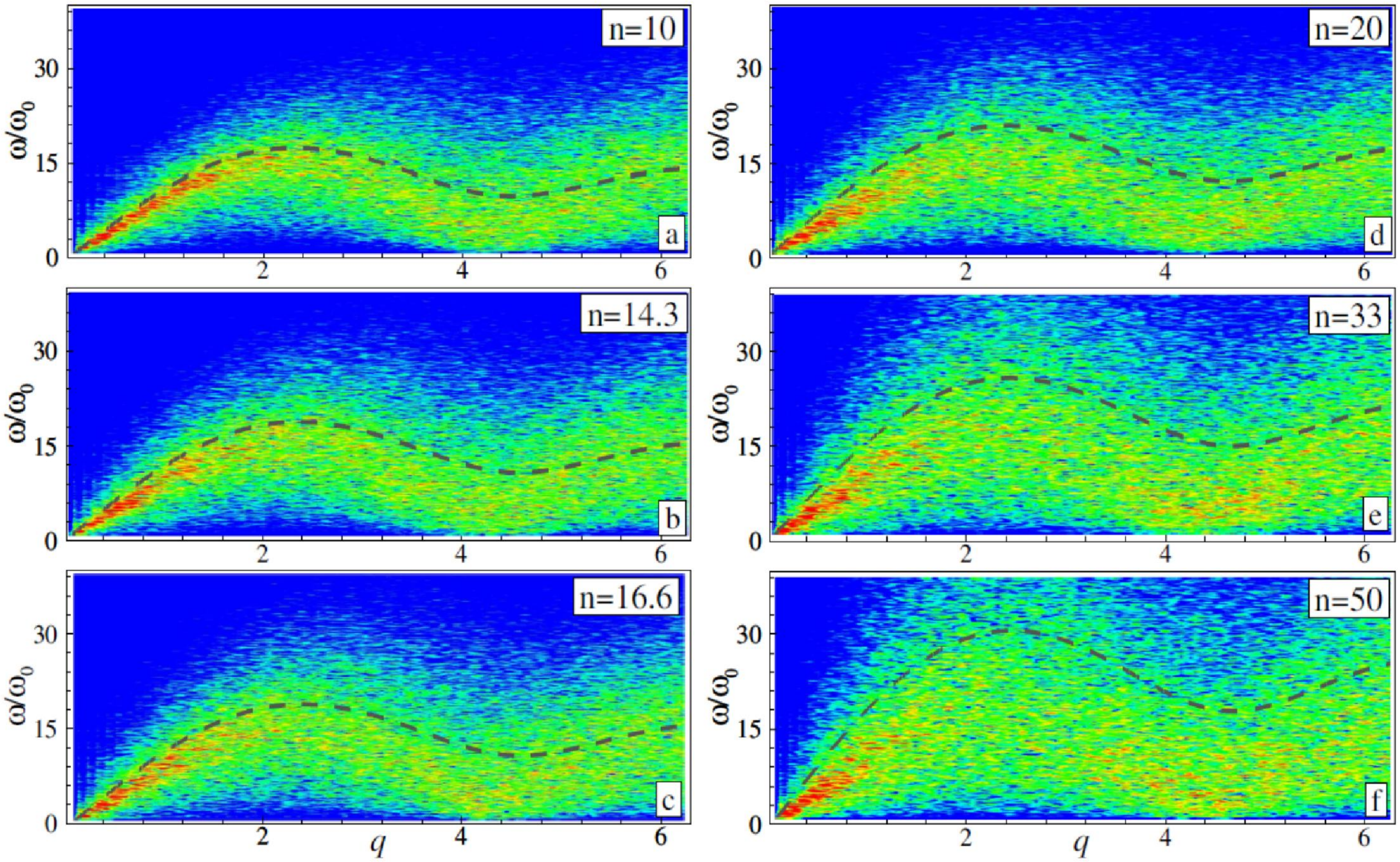
- With increasing the exponent *n* structure and dynamics tend to HS-like
- Raveche-Mountain-Streett criterion of freezing is not very accurate when potential softness varies in a wide range
- More accurate criterion can be based on the height of the minimum of *g*(*r*)





Couedel&Klumov (2016)

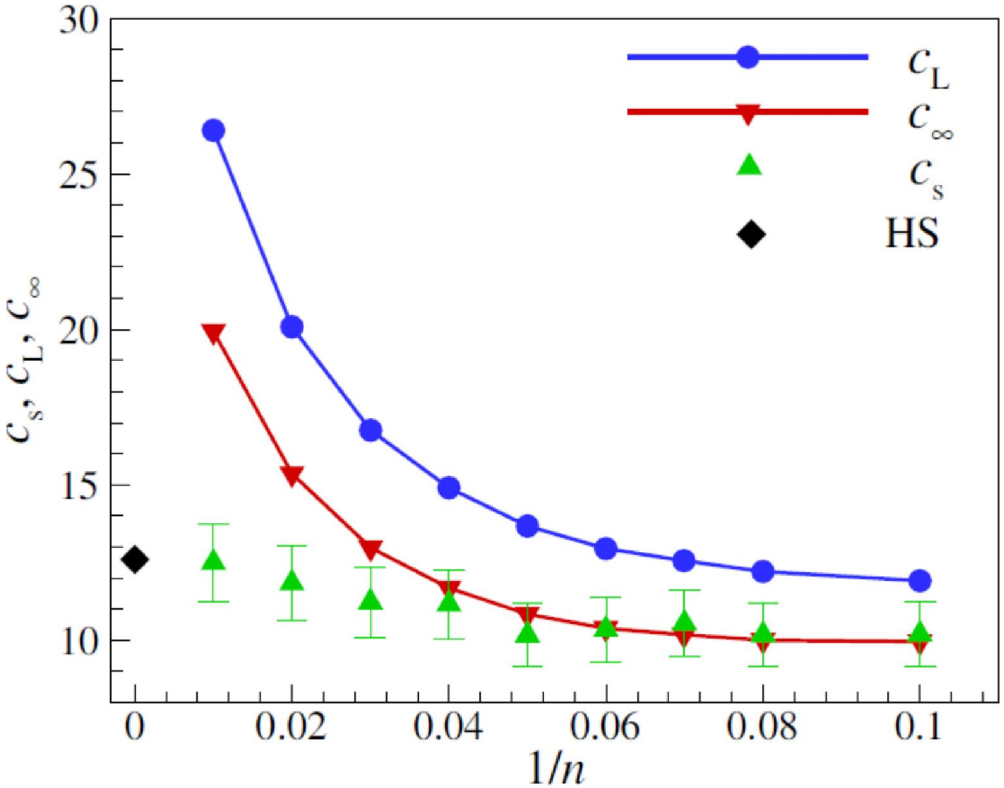
Dispersion of the longitudinal mode



- QCA/QLCA is reasonably accurate only for sufficiently soft potentials with $n < 20$

Khrapak, Klumov, Couedel (2017)

Sound velocities



- Elastic QCA longitudinal sound velocity

$$C_L^2 = (3n+1)v_T^2 p_{ex}/5,$$

overestimates that measured in MD experiment and diverges at large *n*

- Instantaneous sound velocity

$$C_\infty = K_\infty/m\rho = C_L^2 - \tfrac{4}{3}C_T^2$$

Is close to that measured in MD, but also diverges at large *n*

- The HS sound velocity remains finite

Khrapak, Klumov, Couedel (2017)

Conclusion

- We have discussed different aspects of describing theoretically collective modes in simple fluids with applications to complex (dusty) plasmas
- QLCA/QCA approach
  - Simplification based on excluded volume arguments produces useful analytical expressions
  - Possibility to discriminate between different interactions using the long-wavelength dispersion relation
  - Applicability is limited by sufficiently soft interactions

Thank you for your attention!

Acknowledgments

- Studies at Aix-Marseille-University have been supported by by the A*MIDEX grant (Nr. ANR-11-IDEX-0001-02) funded by the French Government "Investissements d'Avenir" program

- The work was also supported by the French-German PHC PROCOPE Program (Project No. 35325NA/57211784).