Designing Precoding and Receive Matrices for Interference Alignment in MIMO Interference Channels

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***Abstract*—Interference is a key bottleneck in wireless communication systems. Interference alignment is a management technique that align interference from other transmitters in the least possibly dimension subspace at each receiver and provides the remaining dimensions for free interference signal. An uncoordinated interference is an example of interference which cannot be aligned coordinately with interference from coordinated part; consequently, the performance of interference alignment approaches are degraded. In this paper, we propose a rank minimization method to enhance the performance of interference alignment in the presence of uncoordinated interference sources. Firstly, to obtain higher multiplexing gain, a new rank minimization based optimization problem is proposed; then, a new class of convex relaxation is introduced which can reduce the optimal value of the problem and obtain lower rank solutions by expanding the feasibility set. Simulation results show that our proposed method can obtain considerably higher multiplexing gain and sum rate than other approaches in the interference alignment framework.**

***Index Terms*—interference alignment, interference MIMO channel, convex relaxation.**

I. Introduction

INTERFERENCE is an important problem in wireless networks and may cause severe limitations in transmitting information; therefore, it is essential to develop communication schemes in order to manage interference. Recently, several methods have been proposed to deal with interference. One of the most effective techniques to manage interference is interference alignment (IA). Aligning the whole interference signals at each receiver, and making interference and desired signal subspaces linearly independent from each other is the key idea of IA. In the high signal-to-noise ratio (SNR) region, degrees of freedom (DOF) also known as multiplexing gain is the first order approximation of the sum capacity. By using linear precoding in a k-user system, IA can provide $k/2$ DOF [1]. The DOF can be interpreted as free signaling dimensions [2]. Practically, to obtain predicted DOF by IA, precoding matrix and receive filter matrix should be designed. Indeed, designing precoding and receive filter matrices is simpler to implement than asymptotic interference alignment schemes such as [1] which needs to decomposition of multi-antenna nodes and infinite symbol extensions.

Rank constrained rank minimization (RCRM) has been proposed in order to design precoding and receive matrices for IA [2] . Authors of [2] show that, minimizing rank of interference matrix can decrease dimension of interference subspaces in order to obtain DOF as large as possible. Due to the fact that the rank minimization problem is NP-hard and non-convex, the nuclear norm heuristic method has been employed [2]. In this method, sum of magnitudes of singular values is minimized as a convex approximation of rank whereas rank is the number of non-zero singular values. Another convex approximation has been proposed in [3] based on log function. Due to the fact that log function is concave, the authors in [3] use reweighted nuclear norm minimization algorithm to solve RCRM problem.

In this paper, we consider one of the most common types of interference in wireless networks which is named as uncoordinated interference. An interference source that is not coordinated by network can cause such interference [4]. For example, in the heterogeneous pico-cell networks, interference caused by femtos and home base stations is considered as uncoordinated interference, and cannot be fully aligned. In the heterogeneous network, the information about the uncoordinated interference can be obtained. The presence of uncoordinated interference degrades the performance of the coordinated part and cannot be ignored [5]. Furthermore, perfect IA is not feasible for such a network [6].

In this paper, we propose a new rank minimization method to enhance the performance of IA, especially when the uncoordinated interference sources exist in the MIMO interference system. In addition, a new class of convex constraint is proposed which expands the feasibility set of the optimization problem. This relaxation considers the possible solutions that are overlooked by existing RCRM-based IA algorithms. Moreover, for analysis we obtain the dual problem of IA. The analysis shows that the proposed constraint can reduce the optimal value of the optimization problem (which leads to higher DOF). Simulation results confirm that our proposed method can achieve higher DOF and sum rate than other approaches.

The organization of the remainder of this paper is as follows. Section II describes the problem formulation. Section III presents our proposed approach. Section IV evaluates numerically the proposed algorithm. Section V concludes this

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paper and represents Appendices.

Notation: Bold uppercase letters such as $\boldsymbol{A}$ denote matrices, bold lowercase letters such as $\boldsymbol{a}$ denote column vectors. The expectation operator is denoted by $E[.]$. Hermitian transpose of matrix $\boldsymbol{X}$ and i-th largest singular value are $\boldsymbol{X}^H$ and $\delta_i(\boldsymbol{X})$, respectively. Trace of matrix $\boldsymbol{X}$ is $tr(\boldsymbol{X})$. $\mathbf{1}_d$ and $\boldsymbol{I}_d$ are all-one and identity d by d matrices, respectively. Nuclear norm of matrix $\boldsymbol{X}$ can be stated as $\|\boldsymbol{X}\|_* = \sum_{i=1}^{rank(\boldsymbol{X})} \delta_i(\boldsymbol{X})$. $\boldsymbol{X}(i,j)$ determines the element in the i-th row and j-th coloumn of $\boldsymbol{X}$.

II. Problem Formulation

We consider a case in which K-user interference wireless system contains K transmitters and K receivers. The number of antennas for each receiver and transmitter is denoted by M_r and M_t, respectively. Transmitters are assumed to be synchronized, and each user transmits the symbol vector $\boldsymbol{x}_k \in \mathbb{C}^{d\times 1}(k=1,...,K)$ to its associated receiver. This K-user MIMO system can be represented by $(M_t \times M_r, d)^K$ [7]. In addition, X sources of uncoordinated interference exist in the system, and $\boldsymbol{y}_k \in \mathbb{C}^{M_r\times 1}$ is considered as the received signal in the k-th receiver. After linear processing, the output can be expressed as follows:

$$\boldsymbol{U}_k^H \boldsymbol{y}_k = \boldsymbol{U}_k^H \boldsymbol{H}_{k,k} \boldsymbol{V}_k \boldsymbol{x}_k + \boldsymbol{U}_k^H \sum_{l=1, l\neq k}^{K} \boldsymbol{H}_{k,l} \boldsymbol{V}_l \boldsymbol{x}_l + \boldsymbol{U}_k^H \sum_{f=1}^{X} \boldsymbol{C}_{k,f} \boldsymbol{F}_f \boldsymbol{q}_f + \boldsymbol{U}_k^H \boldsymbol{w}_k \quad (1)$$

where $\boldsymbol{H}_{k,l} \in \mathbb{C}^{M_r\times M_t}$ represents the flat fading channel between k-th receiver and l-th transmitter. Linear receive filter and precoding matrices are denoted by $\boldsymbol{U}_k \in \mathbb{C}^{M_r\times d}$ and $\boldsymbol{V}_k \in \mathbb{C}^{M_t\times d}$, respectively. $\boldsymbol{C}_{k,f} \in \mathbb{C}^{M_r\times M_f}$ is the channel between the k-th receiver and the f-th uncoordinated source. The number of antennas of the f-th uncoordinated source is denoted by M_f. $\boldsymbol{F}_f \in \mathbb{C}^{M_f\times d_f}$ and $\boldsymbol{q}_f \in \mathbb{C}^{d_f\times 1}$ represent the precoding matrix and symbol vector of the f-th uncoordinated source, respectively. Furthermore, d_f denotes the length of data stream of the f-th uncoordinated source. Finally, $\boldsymbol{w}_k$ is the zero-mean complex additive white Gaussian noise with covariance $\sigma_k^2 \boldsymbol{I}_{M_r}$. We assume that $E[\|\boldsymbol{V}_k \boldsymbol{x}_k\|^2] = P$, $E[\|\boldsymbol{F}_f \boldsymbol{q}_f\|^2] = P_f$ where P is the transmit power of each user and P_f is the transmit power of each uncoordinated source.

The required conditions for perfect IA can be stated as [7]:

$$\boldsymbol{U}_k^H \boldsymbol{H}_{k,l} \boldsymbol{V}_k = 0 \quad (2)$$

$$rank(\boldsymbol{U}_k^H \boldsymbol{H}_{k,k} \boldsymbol{V}_k) = d \quad (3)$$

The system $(M_t \times M_r, d)^K$ is proper if $\frac{M_r+M_t}{K+1} \geq d$. Obviously if this condition is not satisfied, the system is improper [7]. In the improper system, perfect IA cannot be admitted. In improper system the rank of interference is not zero and the goal is to find out linear precoding and linear receive filter matrices that can minimize the rank of interference. Since there is uncoordinated interference in our scenario, the system is improper. In this paper, we try to minimize the rank of interference by RCRM framework.

We define the signal and interference matrices $(\boldsymbol{S}_k, \boldsymbol{J}_k)$ for all $k = 1,...,K$ in the presence of the uncoordinated interference.

$$\boldsymbol{S}_k = \boldsymbol{U}_k^H \boldsymbol{H}_{k,k} \boldsymbol{V}_k \quad (4)$$

$$\boldsymbol{J}_k \triangleq \boldsymbol{U}_k^H \left[\{\boldsymbol{H}_{k,l} \boldsymbol{V}_l\}_{l=1, l\neq k}^{K} \dots \{\boldsymbol{C}_{k,f} \boldsymbol{F}_f\}_{f=1}^{X} \right] \quad (5)$$

where $\boldsymbol{J}_k \in \boldsymbol{C}^{d\times[(K-1)d+D_f]}$. We consider $D_f = \sum_{f=1}^{X} d_f$. In addition, the multiplexing gain of user k can be expressed in terms of rank function: $DOF_k \triangleq rank(\boldsymbol{S}_k) - rank(\boldsymbol{J}_k)$ [2]. We find out precoding and receive filter matrices to minimize the rank of the interference matrix by solving the problem (6):

$$\min_{\boldsymbol{U}_k, \boldsymbol{V}_k, k=1,...,K} \sum_{k=1}^{K} rank(\boldsymbol{J}_k) \quad (6a)$$

$$s.t.: rank(\boldsymbol{S}_k) = d \quad (6b)$$

III. PROPOSED METHODS

In this section, a SDP-based rank minimization method with ability to enhance the performance of IA in the presence of uncoordinated interference is introduced. At the First, we introduce new objective function and study its effects on the performance of IA; then, we propose a new convex relaxation.

One of the most common convex optimization based heuristic approaches to solve rank minimization problem is nuclear norm/trace. In the case of finding sparse vectors, this approach is transformed into l_1 norm minimization method. The advantage of such a method is that can be solved efficiently [8]. Rank of a semidefinite symmetric matrix is equal to the number of its non-zero singular values and nuclear norm of this matrix is sum of the all singular values. In fact, by using the nuclear norm function in (6) it is wished that minimizing the sum of the singular values leads to decrease singular values and consequently, lower rank matrices are obtained. Both small and large positive singular values have equal effect on the rank of positive semidefinite matrices but, the l_1 norm approximation (nuclear norm) sets high emphasis on small singular values. In contrast, it puts the less weight on large singular values [9].

When perfect IA cannot be attainable and therefore rank of the interference matrix is not zero, difference between rank minimization and nuclear norm approach exposes. The amount of DOF is highly affected by weighting of singular values. This motivates us to search for other relaxations to find out better singular values weighting approaches and achieve higher DOF.

In the following, we propose our optimization method (l_2 norm minimization method):

Due to the fact that the rank function is intractable, the rank function should be approximated. In this section, we use l_2 norm approximation to solve RCRM problem (6). In comparison with l_1 norm approximation, large singular values get higher weights in l_2 norm approximation; consequently, l_2 norm approximation yields fewer large singular values than l_1 norm approximation [9]. In the presence of uncoordinated interference, the singular values of interference matrix may increase. This trait encourages us to use l_2 norm approximation in the SDP-based rank minimization in order to obtain higher

DOF. By using l_2 norm approximation, the problem (6) can be rewritten as follows:

$$\min_{\boldsymbol{U}_k,\boldsymbol{V}_k,k=1,\ldots,K} \sum_{k=1}^{K} tr(\boldsymbol{Y}_k) \tag{7a}$$

$$s.t: rank(\boldsymbol{S}_k) = d \tag{7b}$$

$$\boldsymbol{Y}_k = \boldsymbol{J}_k^H \boldsymbol{J}_k \tag{7c}$$

The problem (7) is not a convex optimization problem. To address this, (7c) should be relaxed. Hence, (7c) can be linearized by using the Schur complement, and can be expressed by the following linear matrix inequality (LMI) in (8c):

$$\min_{\boldsymbol{U}_k,\boldsymbol{V}_k,k=1,\ldots,K.} \sum_{k=1}^{K} tr(\boldsymbol{W}_k) \tag{8a}$$

$$s.t: rank(\boldsymbol{S}_k) = d \tag{8b}$$

$$\boldsymbol{W}_k \succeq \boldsymbol{0}_{[Kd+D_f]\times[Kd+D_f]} \tag{8c}$$

$$\boldsymbol{W}_k = \begin{pmatrix} \boldsymbol{I}_d & \boldsymbol{J}_k \\ \boldsymbol{J}_k^H & \boldsymbol{Y}_k \end{pmatrix} \tag{8d}$$

$$\boldsymbol{S}_k = \boldsymbol{U}_k^H \boldsymbol{H}_{k,k} \boldsymbol{V}_k \tag{8e}$$

$$\boldsymbol{J}_k = \boldsymbol{U}_k^H \left[\{\boldsymbol{H}_{k,l}\boldsymbol{V}_l\}_{l=1,l\neq k}^{K} \ldots \{\boldsymbol{C}_{k,f}\boldsymbol{F}_f\}_{f=1}^{X} \right] \tag{8f}$$

Constraint (8b) is not a convex optimization problem constraint. To solve this problem the rank of signal matrices ($\boldsymbol{S}_k$) should be constrained by a convex constraint. It can be easily shown that $\boldsymbol{S}_k$, which satisfies (9) is positive definite and also full rank (Proof: see Appendix (A)):

$$\boldsymbol{S}_k - \gamma \boldsymbol{I}_d \succeq \boldsymbol{0}_{d*d} \tag{9}$$

where $0 < \gamma \ll 1$.

To expand the feasibility set of problem (8), we modify (9) by adding matrix $\boldsymbol{Z}$ as follows:

$$\boldsymbol{S}_k + \boldsymbol{Z} - \gamma \times \boldsymbol{I}_d \succeq \boldsymbol{0}_{d*d} \tag{10}$$

By using (10), new possible solutions are considered which have been disregarded in the previous RCRM-based IA algorithms. To analyze the effect of (10), we obtain dual form of the optimization problem (8) as follows (note that we replace (8b) by (9), and Proof of (11): see Appendix (B)):

$$\max\Big(\sum_{k=1}^{K} tr(\boldsymbol{A}_{1,k} - \boldsymbol{B}_{3,k})\Big) \tag{11a}$$

$$s.t: \boldsymbol{I}_{[Kd+D_f]} - \boldsymbol{A}_{2,k} + \boldsymbol{F}_{1,k} + \boldsymbol{F}_{2,k} = \boldsymbol{0}_{[Kd+D_f]\times[Kd+D_f]} \tag{11b}$$

$$\boldsymbol{Q}\boldsymbol{B}_{1,k} + \boldsymbol{T}\boldsymbol{B}_{2,k} = \boldsymbol{0}_{M_r\times d} \tag{11c}$$

$$\boldsymbol{B}_{1,k} - \boldsymbol{A}_{1,k} = \boldsymbol{0}_{d\times d} \tag{11d}$$

$$\boldsymbol{A}_{1,k} \succeq \boldsymbol{0}_{d\times d} \tag{11e}$$

$$\boldsymbol{A}_{2,k} \succeq \boldsymbol{0}_{[Kd+D_f]\times[Kd+D_f]} \tag{11f}$$

where $\boldsymbol{Q} = \boldsymbol{H}_{k,k}\boldsymbol{V}_k$, $\boldsymbol{T} = \left[\{\boldsymbol{H}_{k,l}\boldsymbol{V}_l\}_{l=1,l\neq k}^{K} \ldots \{\boldsymbol{C}_{k,f}\boldsymbol{F}_f\}_{f=1}^{X}\right]$, $\boldsymbol{F}_{1,k} = \begin{pmatrix} \boldsymbol{0}_{d\times d} & \boldsymbol{0}_{d\times((K-1)d+D_f)} \\ \boldsymbol{B}_{2,k} & \boldsymbol{0}_{((K-1)d+D_f)\times((K-1)d+D_f)} \end{pmatrix}$ and $\boldsymbol{F}_{2,k} = \begin{pmatrix} \boldsymbol{B}_{3,k} & \boldsymbol{0}_{d\times((K-1)d+D_f)} \\ \boldsymbol{0}_{((K-1)d+D_f)\times d} & \boldsymbol{0}_{((K-1)d+D_f)\times((K-1)d+D_f)} \end{pmatrix}$. Lagrange multipliers associated with inequality constraints are $\boldsymbol{A}_{1,k}$ and $\boldsymbol{A}_{2,k}$. $\boldsymbol{B}_{1,k}$, $\boldsymbol{B}_{2,k}$, $\boldsymbol{B}_{3,k}$ are Lagrange multipliers associated with equality constraints.

Proposition 1: Using the relaxation (10) instead of (9) does not change the constraints of the dual problem; however, the objective function of the dual problem of IA is changed to the following statement:

$$\max\left(\sum_{k=1}^{K} tr(\boldsymbol{A}_{1,k} \times (\boldsymbol{I}_d - \boldsymbol{Z}) - \boldsymbol{B}_{3,k})\right) \tag{12}$$

proof: see Appendix (C)

In the convex optimization, the optimal value of the dual problem is the lower bound on the optimal value of the primal problem. Thus, in general case there is a duality gap between optimal values of the primal and dual problems. On the other hand, when strong duality holds, duality gap is zero. This means that the optimal values of the primal and dual problems are equal. We use alternating minimization approach [10] to solve optimization problem (8). In each iteraton of this method we fix one variable and solve optimization problem with respect to another variable. Consequently, in each iteration our optimization problem is convex and SDP (note that we replace (8b) by (9)); therefore, strong duality holds [9]. If $\boldsymbol{Z}$ satisfies that $tr(\boldsymbol{A}_{1,k}\boldsymbol{Z}) \geq 0$, it can be easily shown that the optimal value of the dual problem associated with the constraint (10) is smaller than the case in which $\boldsymbol{Z} = \boldsymbol{0}$. Therefore, adding matrix $\boldsymbol{Z}$ can cause more decrease in the rank of interference matrices than the problem of (8); consequently, matrix $\boldsymbol{Z}$ can enhance performance of IA to achieve higher DOF.

Let the optimal values of problem (8) associated with the constraints (9) and (10) be P^* and $P^*(\boldsymbol{Z})$, respectively. For all $\boldsymbol{Z}$, $P^*(\boldsymbol{Z}) \geq P^* - tr(\boldsymbol{A}_{1,k}^*\boldsymbol{Z})$. In order to be as close as possible to the lower bound of $P^*(\boldsymbol{Z})$, $\boldsymbol{Z}$ should be comparatively small. This means that, to obtain lower rank solution for the interference matrix we choose comparatively small values for the entries of matrix $\boldsymbol{Z}$ in a way that the condition $tr(\boldsymbol{A}_{1,k}\boldsymbol{Z}) \geq 0$ is satisfied.

As long as $\boldsymbol{A}_{1,k}$ is a positive semidefinite, $\boldsymbol{1}_d$ matrix satisfies the constraint $tr(\boldsymbol{A}_{1,k}\boldsymbol{Z}) \geq 0$. Thus the matrix $\boldsymbol{Z}$ is chosen as follows:

$$\boldsymbol{Z} = z \times \boldsymbol{1}_d \tag{13}$$

Although (10) does not guarantee that $\boldsymbol{S}_k$ is a positive definite matrix, our results show that by choosing sufficiently small z our proposed relaxation method provides full rank $\boldsymbol{S}_k$. However, if $\boldsymbol{S}_k$ is not full rank matrix, we decrease z step by step until a full rank matrix is attained. Furthermore, using $\boldsymbol{Z}$ matrix expands the feasibility set of the optimization problem, and this can enable RCRM-based approaches to obtain lower optimal values. The whole procedure of alternating minimization approach is explained in Table I.

IV. SIMULATION RESULTS

In order to evaluate the performance of the proposed methods, numerical results are reported. We run simulations by using MATLAB toolbox CVX [11]. Noise power level is considered as $\sigma^2 = 1$. The algorithms are performed over

TABLE I
ALGORITHM FOR SOLVING THE OPTIMIZATION PROBLEM (8)

1: choose arbitrary matrix $\boldsymbol{U}_k$, k=1,...,K
2: for $n = 1 : n_{max}$ iteration
3: fix $\boldsymbol{U}_k$ and solve optimization problem (8) $\rightarrow \boldsymbol{V}_k$
4: fix $\boldsymbol{V}_k$ and solve optimization problem (8) $\rightarrow \boldsymbol{U}_k$
5: if $rank(\boldsymbol{S}_k) \neq d$, $z \rightarrow z/2$ and jump to step 3
6: end for

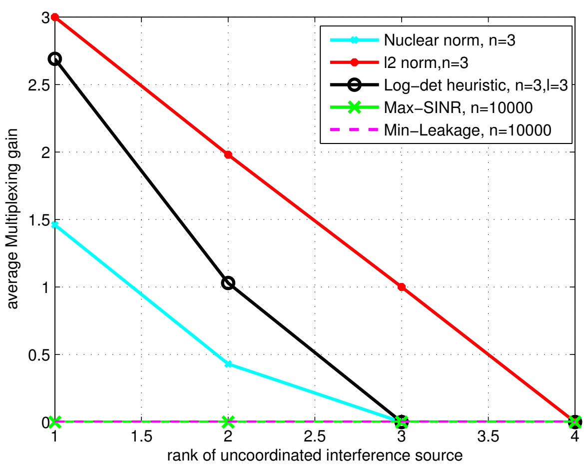

Fig. 1. Average multiplexing gain versus the rank of the uncoordinated interference source ($P/\sigma^2 = 30dB$)

TABLE II
AVERAGE COMPUTATION TIME

Time(S) of Proposed method	3.95
Time(S) of nuclear norm	8.47
Time(S) of Leakage	11.5
Time(S) of Log-det	25.45
Time(S) of SINR	85.89

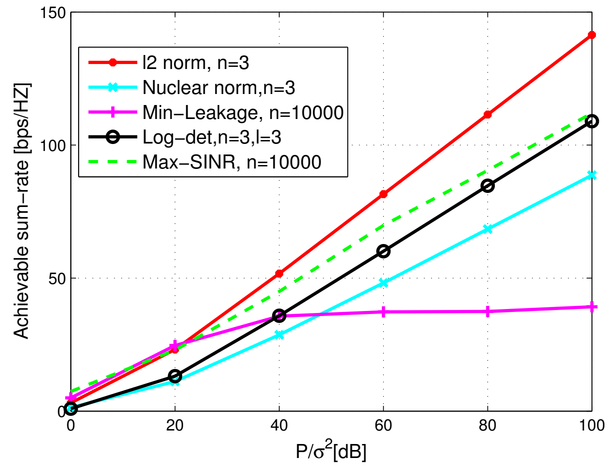

Fig. 2. Average sum rate for $(10 \times 6, 4)^3$ MIMO interference channel

200 channel realizations, where channel elements are drawn i.i.d from a Gaussian distribution with mean zero and variance 1. Matrix $\boldsymbol{Z}$ is set to $\mathbf{1}_d$. The proposed method (l_2 norm minimization) is compared with nuclear norm minimization method [2], The SINR-maximization [4], Log-det heuristic [3] and the interference leakage minimization [12] approaches.

In Fig. 1 and Fig. 2, we consider $(10 \times 6, 4)^3$ MIMO interference system with one uncoordinated source (with fixed power 0dB). The interference free dimensions at each receiver are counted as the number of singular values of $\boldsymbol{S}_k$ greater than 10^{-6} minus the number of singular values of $\boldsymbol{J}_k$ greater than 10^{-6}.

Fig. 1 depicts average multiplexing gain versus rank of uncoordinated interference. The rank of uncoordinated interference varies from 1 to 4. As it can be seen in the Fig. 1, when the rank of uncoordinated interference reaches to 3, other methods cannot provide any average multiplexing gain while our proposed method has considerably better performance.

Fig. 2 presents the average sum rate versus P/σ^2 when the rank of uncoordinated interference is one. From Fig. 2, it is clearly seen that l_2 norm minimization method outperforms the other ones when P/σ^2 is higher than 20dB. Furthermore, the proposed method can achieve higher sum rate than other approaches when P/σ^2 is increased. Expanding the feasibility set of the optimization problem is one of the reasons for such results especially when uncoordinated interferences have strong effect and can considerably degrade the performance of IA.

Table II represents average computation time of the proposed method and the other algorithms for $(10 \times 6, 4)^3$ MIMO interference system. Compared to other algorithms, our proposed method has the least computation time. Indeed, in some cases such as $(10 \times 6, 4)^3$ MIMO interference system at low P/σ^2 (0-20dB) our proposed method and some approaches such as Max-SINR achieve comparatively equal sum-rate; however, our proposed method takes less computation time than the other approaches. In Fig. 3, we use $(10 \times 6, 4)^3$ MIMO interference system with two sources of uncoordinated interference. The rank of the first uncoordinated source varies from 1 to 3, and the rank of the second source is fixed to 1. Fig. 3 displays average multiplexing gain versus rank of uncoordinated interference sources (note that, in Fig. 3, the rank of the first source is just placed on the x-axis). In Fig. 3, it can be observed that the proposed method noticeably outperforms the other approaches. For example, when the rank of the first uncoordinated source is 2 and the second one is 1, other methods cannot obtain any average multiplexing gain, but our proposed method improves the performance of IA significantly.

V. CONCLUSION

In this paper, we propose a rank minimization method to improve the performance of IA in the MIMO interference channel with uncoordinated interference sources. In addition, we expand the feasibility set of rank minimization problem by proposing a new convex relaxation, which can reduce the optimal value of our optimization problem, in order to achieve

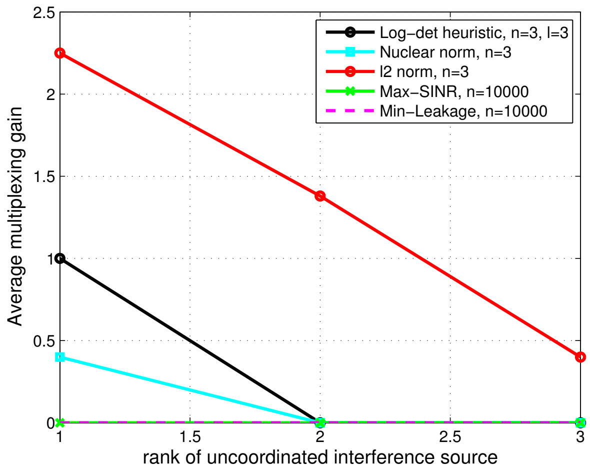

Fig. 3. Average multiplexing gain versus the rank of the uncoordinated interference sources ($P/\sigma^2 = 30dB$)

higher DOF when the rank of interference matrix is large. Simulation results show that our proposed method can achieve noticeably higher number of interference-free dimensions and sum rate compared to recently proposed approaches in IA framework.

Appendix A
Proof of (9)

A symmetric Matrix $\boldsymbol{M}$ is positive definite if the scalar $\boldsymbol{a}^T\boldsymbol{M}\boldsymbol{a}$ is positive for every vector $\boldsymbol{a}$ ($\boldsymbol{a}$ is non-zero column vector). Since (9) is positive semidefinite, we have:

$$\boldsymbol{a}^T(\boldsymbol{S}_k - \boldsymbol{I}_d)\boldsymbol{a} \geq 0 \tag{14}$$

According to (14):

$$\boldsymbol{a}^T\boldsymbol{S}_k\boldsymbol{a} \geq \boldsymbol{a}^T(\boldsymbol{I}_d)\boldsymbol{a} > 0 \tag{15}$$

According to (15), and Due to the fact that $\boldsymbol{I}_d$ is positive definite matrix, $\boldsymbol{S}_k$ is also positive definite and full rank.

Appendix B
Proof of (11)

This section presents the derivation of dual problem of problem (8). Note that, we use alternating minimization approach to solve problem (8). In fact, in each iteration of alternating minimization approach we solve optimization problem with one variable. As it is stated in Table I, firstly we solve the optimization problem which its variables are precoding matrices, and linear receive matrices are fixed; then, in the next step the variables of the optimization are linear receive matrices and precoding matrices are fixed. This procedure is continued until maximum number of iteration is obtained. Indeed, we obtain the dual problem of the optimization problem which its variables are linear receive matrices.

To obtain the dual problem, we use Lagrange dual function. The Lagrangian associated with problem (8) is expressed as follows:

$$\begin{aligned} L(\boldsymbol{U},\boldsymbol{A},\boldsymbol{B}) = &\sum_{k=1}^{K} tr(\boldsymbol{W}_k) + \sum_{k=1}^{K} tr((-\boldsymbol{S}_k + \boldsymbol{I}_d)\boldsymbol{A}_{1,k}) + \\ &\sum_{k=1}^{K} tr((\boldsymbol{S}_k - \boldsymbol{U}_k^H\boldsymbol{Q})\boldsymbol{B}_{1,k}) + \sum_{k=1}^{K} tr((-\boldsymbol{W}_k)\boldsymbol{A}_{2,k}) \\ &+ \sum_{k=1}^{K} tr(\boldsymbol{W}_k(1:d,1:d)\boldsymbol{B}_{3,k}) - \sum_{k=1}^{K} tr(\boldsymbol{B}_{3,k}\boldsymbol{I}_d) \\ &+ \sum_{k=1}^{K} tr((\boldsymbol{J}_k - \boldsymbol{U}_k^H\boldsymbol{T})\boldsymbol{B}_{2,k}) \end{aligned} \tag{16}$$

where $\boldsymbol{T} = \left[\{\boldsymbol{H}_{k,l}\boldsymbol{V}_l\}_{l=1,l\neq k}^{K} \cdots \{\boldsymbol{C}_{k,f}\boldsymbol{F}_f\}_{f=1}^{X}\right]$, $\boldsymbol{Q} = \boldsymbol{H}_{k,k}\boldsymbol{V}_k$. By using (8d), we can represent interference matrix $\boldsymbol{J}_k$ based on $\boldsymbol{W}_k$ as $\boldsymbol{J}_k = \boldsymbol{W}_k(1:d, d+1:Kd+D_f)$. Furthermore, we have:

$$tr(\boldsymbol{W}_k \begin{pmatrix} \boldsymbol{0}_{d\times d} & \boldsymbol{0}_{d\times((K-1)d+D_f)} \\ \boldsymbol{B}_{2,k} & \boldsymbol{0}_{((K-1)d+D_f)\times((K-1)d+D_f)} \end{pmatrix}) = tr(\boldsymbol{J}_k\boldsymbol{B}_{2,k}) \tag{17}$$

In addition, with respect to (8d) $\boldsymbol{W}_k(1:d,1:d) = \boldsymbol{I}_d$, and we should consider this constraint to derivation of the dual problem. Thus, we can conclude following statement:

$$\begin{aligned} tr(\boldsymbol{W}_k \begin{pmatrix} \boldsymbol{B}_{3,k} & \boldsymbol{0}_{d\times((K-1)d+D_f)} \\ \boldsymbol{0}_{((K-1)d+D_f)\times d} & \boldsymbol{0}_{((K-1)d+D_f)\times((K-1)d+D_f)} \end{pmatrix}) = \\ tr(\boldsymbol{W}_k(1:d,1:d)\boldsymbol{B}_{3,k}) \end{aligned} \tag{18}$$

As long as (16) is linear function in order to prevent that it becomes unbounded below the following constraints should be hold. Consequently, the dual problem is given by:

$$\max(\sum_{k=1}^{K} tr(\boldsymbol{A}_{1,k} - \boldsymbol{B}_{3,k})) \tag{19a}$$

$$s.t: \boldsymbol{I}_{[Kd+D_f]} - \boldsymbol{A}_{2,k} + \boldsymbol{F}_{1,k} + \boldsymbol{F}_{2,k} = \boldsymbol{0}_{[Kd+D_f]\times[Kd+D_f]} \tag{19b}$$

$$\boldsymbol{Q}\boldsymbol{B}_{1,k} + \boldsymbol{T}\boldsymbol{B}_{2,k} = \boldsymbol{0}_{M_r\times d} \tag{19c}$$

$$\boldsymbol{B}_{1,k} - \boldsymbol{A}_{1,k} = \boldsymbol{0}_{d\times d} \tag{19d}$$

$$\boldsymbol{A}_{1,k} \succeq \boldsymbol{0}_{d\times d} \tag{19e}$$

$$\boldsymbol{A}_{2,k} \succeq \boldsymbol{0}_{[Kd+D_f]\times[Kd+D_f]} \tag{19f}$$

where $\boldsymbol{F}_{1,k}$ and $\boldsymbol{F}_{2,k}$ are as follows:

$$\boldsymbol{F}_{1,k} = \begin{pmatrix} \boldsymbol{0}_{d\times d} & \boldsymbol{0}_{d\times((K-1)d+D_f)} \\ \boldsymbol{B}_{2,k} & \boldsymbol{0}_{((K-1)d+D_f)\times((K-1)d+D_f)} \end{pmatrix} \tag{20}$$

$$\boldsymbol{F}_{2,k} = \begin{pmatrix} \boldsymbol{B}_{3,k} & \boldsymbol{0}_{d\times((K-1)d+D_f)} \\ \boldsymbol{0}_{((K-1)d+D_f)\times d} & \boldsymbol{0}_{((K-1)d+D_f)\times((K-1)d+D_f)} \end{pmatrix} \tag{21}$$

Lagrange multipliers associated with inequality constraints are $\boldsymbol{A}_{1,k}$ and $\boldsymbol{A}_{2,k}$. $\boldsymbol{B}_{1,k}$, $\boldsymbol{B}_{2,k}$, $\boldsymbol{B}_{3,k}$ are Lagrange multipliers associated with equality constraints.

APPENDIX C
PROOF OF PROPOSITION 1

Adding matrix $\boldsymbol{Z}$ to constraint (9) changes (16) as follows:

$$\begin{aligned}L(\boldsymbol{U},\boldsymbol{A},\boldsymbol{B}) = \sum_{k=1}^{K} tr(\boldsymbol{W}_k) + \sum_{k=1}^{K} tr((-\boldsymbol{S}_k + \boldsymbol{I}_d + \boldsymbol{Z})\boldsymbol{A}_{1,k}) + \\ \sum_{k=1}^{K} tr((\boldsymbol{S}_k - \boldsymbol{U}_k^H \boldsymbol{Q})\boldsymbol{B}_{1,k}) + \sum_{k=1}^{K} tr((-\boldsymbol{W}_k)\boldsymbol{A}_{2,k}) \\ + \sum_{k=1}^{K} tr(\boldsymbol{W}_k(1:d,1:d)\boldsymbol{B}_{3,k}) - \sum_{k=1}^{K} tr(\boldsymbol{B}_{3,k}\boldsymbol{I}_d) \\ + \sum_{k=1}^{K} tr((\boldsymbol{J}_k - \boldsymbol{U}_k^H \boldsymbol{T})\boldsymbol{B}_{2,k}) \qquad (22)\end{aligned}$$

Due to the fact that matrix $\boldsymbol{Z}$ does not depend on any variables of the primal optimization problem, it has not any role in the constraints of dual problem; thus, the cost function of dual problem is changed as follows:

$$\max \left(\sum_{k=1}^{K} tr(\boldsymbol{A}_{1,k} \times (\boldsymbol{I}_d - \boldsymbol{Z}) - \boldsymbol{B}_{3,k}) \right) \qquad (23)$$

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