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Collecting solar power by formation flying systems around a geostationary point

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Abstract Terrestrial solar power is severely limited by the diurnal day-night cycle. To overcome these limitations, a Solar Power Satellite (SPS) system, consisting of a space mirror and a microwave energy generator-transmitter in formation, is presented. The microwave transmitting satellite (MTS) is placed on a planar orbit about a geostationary point (GEO point) in the Earth's equatorial plane, and the space mirror uses the solar pressure to achieve orbits about GEO point, separated from the planar orbit, and reflecting the sunlight to the MTS, which will transmit energy to an Earth-receiving antenna. Previous studies have shown the existence of a family of displaced periodic orbits above or below the Earth's equatorial plane. In these studies, the sun-line direction is assumed to be in the Earth's equatorial plane (equinoxes), and at 23.5° below or above the Earth's equatorial plane (solstices), i.e. depending on the season, the sun-line moves in the Earth's equatorial plane and above or below the Earth's equatorial plane. In this work, the position of the Sun is approximated by a rectangular equatorial coordinates, assuming a mean inclination of Earth's equator with respect to the ecliptic equal to 23.5°. It is shown that a linear approximation of the motion about the GEO point yields bounded orbits for the SPS system in the Earth-satellite two-body problem, taking into account the effects of solar radiation pressure. The space mirror orientation satisfies the law of reflection to redirect the sunlight to the MTS. Additionally, a MTS on a common geostationary orbit (GEO) have been also considered to reduce the relative distance in the formation flying Solar Power Satellite (FF-SPF).

Keywords Solar Power Satellite system · Formation flying · Microwave transmitting satellite · Geostationary point · Two-Body problem · Solar radiation pressure

1 Introduction

Actually about 80% of world energy consumption comes from fossil-fueled sources [1] with significant greenhouse gas emissions into the Earth's atmosphere, resulting in an environmental

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problem [2]. In the last decades, alternative energy sources have been proposed to reduce the effects on our environment, e.g. solar energy, wind energy, geothermal energy and ocean energy. Among these, solar energy is the first energy source in the world. In fact, the global capacity of terrestrial solar-powered devices (e.g. solar photovoltaics plants) has been increasing constantly in the last years [3]. However, terrestrial devices for renewable energy supplying is severely limited by the diurnal day-night cycle. Thus, the collection of solar power in space for global terrestrial power supply has been presented as a option by several authors [4], [5], [6], [7]. A single large light-weight solar reflector (or a constellation of small solar reflectors) could be placed on a low Earth orbit (LEO) or GEO, such that the space mirror area exposed to the Sun would redirect the sunlight (i.e. solar energy), even at night, to an Earth receiving station. Since space mirrors implies the transportation of large amounts of solar energy from orbit thousands of miles above Earth, a new concept of SPS systems configured by formation flying has been presented in the last years [8], [9], [10] [11]. To avoid problems with energy transportation, solar panels in space would reflect the sunlight to an energy generator-transmitter, i.e. collector, such that solar power would be collected from the space and beamed back down to any point on Earth.

Displaced non-Keplerian orbits for solar sails considered by various authors [12], [13], [14], have been also applied to place the SPS system consisting of sunlight reflectors and an energy generator-transmitter. The advantage of a MST in geostationary orbits is that both the satellite and ground station move with the same (Earth) angular velocity, thus greatly reducing the problems with energy transportation. Takeichi et al. [15] present an Earth-pointing MTS on a common GEO and Sun-pointing reflectors that use the solar radiation pressure to achieve orbits parallel (non-identical) to the GEO and have the same radius [15]. However, since the SPS system would suffer perturbations (i.e. periodic drifting) from the large solar pressure, a continuous orbital control by thrusters would be necessary to maintain the longitude of the SPS system [15] [16]. Therefore, although the component of sunlight reflector acceleration perpendicular to the Earth's equatorial plane separates the space mirror from the GEO plane [17], [18], permitting it to levitate few kilometers above or below the Earth's equatorial plane, the component of sunlight reflector acceleration parallel to the Earths equatorial plane does not allow such light levitation [19], [20].

Recently, Baig and McInnes [21] showed that, using the parallel component of sunlight reflector acceleration to the Earths equatorial plane, it is possible to generate a family of parallel periodic orbits for sunlight reflectors, perpendicularly separated from the Earth's equatorial plane, around a GEO point, i.e. a family of displaced periodic orbits with respect to an Earth-fixed rotating frame at a GEO point. Thus, Takeichi et al. [15] and Baig and McInnes [21] show that this kind of orbits are feasible. However, in these studies, it is assumed the sun-line direction to be in the Earth's equatorial plane (equinoxes), and at $23.5°$ below or above the Earth's equatorial plane (solstices). Although this approximation permits to find analytical solutions for the linearized model around GEO point [21], and maintains the sunlight reflector pitch angle constant along the displaced orbit ($45°$ at the autumn/spring equinoxes, and $33.25°$ and $56.75°$ at the winter and summer solstices, respectively), it is only valid for a few days motion. In this paper, the position of Sun is considered on the ecliptic (Earth's orbit plane), assuming a mean inclination of Earth's equator with respect to the ecliptic equal to $23.5°$. As consequence, analytical solutions no longer exist for this more realistic model, and the sunlight reflector pitch angle is no longer constant along the orbit. Thus, the space mirror orientation will have to satisfy the law of reflection to redirect the sunlight to the MTS. Taking into account the effects of Earth's gravitational force and solar radiation pressure, this paper computes a linear approximation of the motion about the GEO point, and an initial guess for finding bounded orbits for the SPS system in the Earth-satellite two-body problem. Finally, a MTS on a GEO are studied as options to reduce the longitude of the SPS system.

This paper is organized as follows: In Sec. 2 the nonlinear equations of motion with respect to the Earth-rotating frame are described for a solar reflector, with the sun-line assumed on the ecliptic. In Sec. 3 the linearized equations of motion around a GEO point are considered. Linear analytical solutions are computed as special cases of displaced orbits above the Earth's equatorial plane at equinoxes and solstices. In Sec. 4 is described a mathematical scheme that uses the analytical linear solutions as initial guesses for finding bounded orbits for the SPS system with the Sun on the ecliptic. Similarly, in Sec. 4 the SPS is simplified, assuming the MST on a GEO, such that, the space mirror reflects practically the sunlight to a GEO, reducing complexity of the system. Again, the linear analytical solutions are used to reduce the relative distance in the formation flying Solar Power Satellite. Finally, in Sec. 5 the conclusions, as well as the discussions, are drawn.

2 Equations of Motion

A GEO is a high circular orbit with a radius $r_{gs} = 42,164.1696$ km measured from the center of the Earth and with zero angle of inclination, i.e. an orbit in the equatorial plane of the Earth [21]. The orbital period τ_e of a satellite in such an orbit is equal to one sidereal day, i.e. $\tau_e = 23$h, 56 min, 4.1 s $= 86,141.1$ s [21]. Therefore, an object on a GEO seems to be fixed in the sky with respect to an observer on the surface of the Earth.

Consider an Earth-centered, Earth fixed rotating reference system $E\left(x_e, y_e, z_e\right)$ that rotates with constant angular velocity $\boldsymbol{\omega}_e = \omega_e \hat{\boldsymbol{z}}_e$ $(\omega_e = \frac{2\pi}{\tau_e})$ with respect to an Earth-centered inertial frame system $I\left(x_I, y_I, z_I\right)$, both with common origin O at the Earth's center of mass, as shown in Fig. 1. The $x_e - y_e$ and $x_I - y_I$ planes coincide with the Earth's equatorial plane, and the z_e and z_I axes are directed along the rotational axis of the Earth. Additionally, the x_I axis points in the vernal equinox direction, the x_e axis points to the GEO point and coincides with the x_I axis at $t = 0$. Assuming the distance r_{gs} between the Earth and GEO point, the gravitational parameter and the magnitude of the angular velocity ω_e equal to unity, the nondimensional vector equation of motion for an ideal (i.e. perfectly reflecting) sunlight reflector can then be written with respect to the rotating frame as [14]

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 2\boldsymbol{\omega}_e \times \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \nabla U = \boldsymbol{a}, \tag{1}$$

where $\mathbf{r} = (x, y, z)^T$ denotes the position vector of the sunlight reflector with respect to the Earth's center of mass, as shown in Fig. 1. The two-body pseudopotential U is the sum of two potentials: the gravitational potential due to a perfectly spherical Earth and the centrifugal potential in the rotating frame, that is,

$$U = -\frac{1}{r} + \frac{\left(x^2 + y^2\right)}{2}. \tag{2}$$

The solar radiation pressure $\boldsymbol{a}$ in Eq. (1) is defined by

$$\boldsymbol{a} = \kappa \left(\hat{\boldsymbol{S}}\left(t\right) \cdot \boldsymbol{u}\right)^2 \boldsymbol{u}, \tag{3}$$

where κ is the reflector characteristic acceleration, $\boldsymbol{u}$ is the reflector normal unit vector, and $\hat{\boldsymbol{S}}\left(t\right)$ is the unit-vector in the direction of the sun-line. Eclipse seasons, i.e. shadow effects, on geostationary satellites have not be included in this investigation.

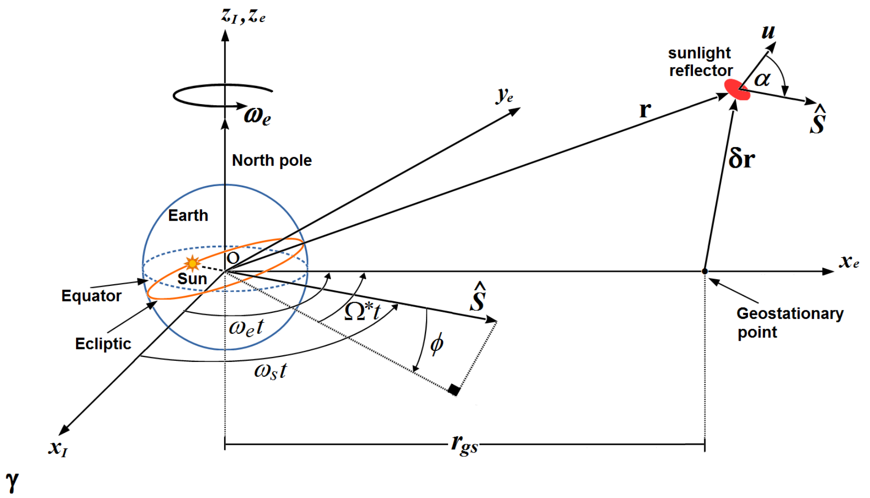

Fig. 1 Rotating reference system $E\,(x_e, y_e, z_e)$ and inertial frame system $I\,(x_I, y_I, z_I)$, both with common origin O at the Earth's center of mass. A solar reflector is also shown around a GEO point, where the sun-line is on the ecliptic.

As a first approximation, the sun-line $\hat{\boldsymbol{S}}\,(t)$ is assumed to be at a fixed angle ϕ above the Earth's equatorial plane (see Figs. 1 and 2). The direction of the sun-line $\hat{\boldsymbol{S}}$ and hence the reflector normal $\boldsymbol{u}$ in the rotating frame are given by [21]

$$\hat{\boldsymbol{S}}\,(t) = \begin{pmatrix} \cos(\mathit{\Omega}^* t)\cos(\phi) \\ -\cos(\mathit{\Omega}^* t)\sin(\phi), \\ \sin(\phi) \end{pmatrix}, \qquad \boldsymbol{u}\,(t) = \begin{pmatrix} \cos(\beta)\cos(\mathit{\Omega}^* t) \\ -\cos(\beta)\sin(\mathit{\Omega}^* t), \\ \sin(\beta) \end{pmatrix}, \tag{4}$$

where α is the reflector pitch angle (i.e. $\hat{\mathbf{S}}\,(\mathbf{t})\cdot\boldsymbol{u} = \cos(\alpha)$), $\beta = \alpha + \phi$ is the angle that the reflector normal makes with the equatorial plane of Earth, as shown in Fig. 2, and $\mathit{\Omega}^* = 1 - \frac{\omega_s}{\omega_e} = 0.9973$ is the nondimensional angular velocity of the rotating frame relative to the sun-line, where $\omega_s = 2\pi/\,(365.25 \times 86,400)$ is the angular velocity of the sun-line with respect to the inertial frame [21]. The angle $\mathit{\Omega}^* t$ is in the Earth's equatorial plane as shown in Fig. 2. Since inclination angle ϕ is considered constant, the angle between the projections of the sun-line in the Earth's equatorial plane is also equal to $\omega_s t$.

3 Linearized Equations

Considering a GEO point at $\mathbf{r}_{gs} = (1, 0, 0)^T$, then the position vector $\mathbf{r}$ of the sunlight reflector can be written as $\mathbf{r} = \mathbf{r}_{gs} + \delta\mathbf{r}$, where $\delta\mathbf{r} = (\xi, \eta, \zeta)^T$ is a small displacement of the reflector from the GEO point (see Fig. 1). Thus, assuming that the solar perturbation $\boldsymbol{a} = (a_\xi, a_\eta, a_\zeta)^T$ does not drive the reflector very far from the GEO point and a uniform solar radiation field, i.e. $\partial\boldsymbol{a}/\partial\mathbf{r} = 0$, a first-order Taylor expansion of each term about $\mathbf{r}_{gs}$ in Eq. (1) yields that the linearized equations of motion can then be written as

$$\delta\dot{\mathbf{X}} = \mathbf{M}\delta\mathbf{X} + \boldsymbol{a}, \tag{5}$$

where $\delta\mathbf{X} = (\delta\mathbf{r}, \delta\dot{\mathbf{r}})^T$, and the matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} -\mathbf{0} & \mathbf{I} \\ \mathbf{K} & \mathbf{\Omega} \end{pmatrix}, \tag{6}$$

where $\mathbf{I}$ is the identity matrix, and the components of the matrices $\mathbf{K}$ and $\mathbf{\Omega}$ are given by

$$\mathbf{K} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{\Omega} = \begin{pmatrix} 0 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

Note that the ζ-component is decoupled from ξ, η in the linearized equations.

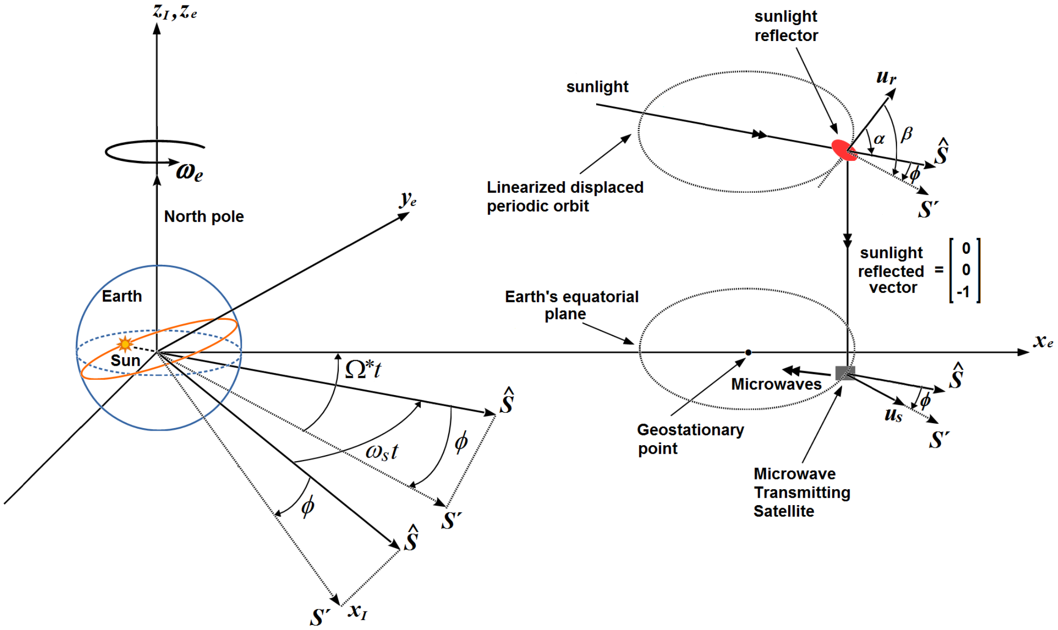

Fig. 2 Sun-line $\hat{\boldsymbol{S}}$ is shown at a constant angle ϕ out of the equatorial plane of the Earth. The sun-line projection $\boldsymbol{S'}$ and the angle $\Omega^* t$ are in the equatorial plane of the Earth. The angle between the projections of the sun-line in the Earth's equatorial plane is equal to $\omega_s t$. A SPS system is also illustrated orbiting in formation around the GEO point, the MTS on a periodic orbit in the Earth's equatorial plane with respect to the rotating frame, and the sunlight reflector, pitched at an angle $\alpha \neq 0$, on a displaced orbit. The sunlight reflector vector is $(-1, 0, 0)^T$.

The periodic solutions for linearized equations of motion, Eq. (5), are given by [21]

$$\xi(t) = -a_p \left(\frac{2\Omega^* + \Omega^{*2}}{\Omega^{*4} - \Omega^{*2}} \right) \cos(\Omega^* t), \tag{8}$$

$$\eta(t) = a_p \left(\frac{\Omega^{*2} + 2\Omega^* + 3}{\Omega^{*2} + 2\Omega^*} \right) \left(\frac{2\Omega^* + \Omega^{*2}}{\Omega^{*4} - \Omega^{*2}} \right) \sin(\Omega^* t), \tag{9}$$

$$\zeta(t) = (\zeta_0 - a_\zeta) \cos(t) + a_\zeta, \tag{10}$$

where $a_p = \sqrt{a_\xi^2 + a_\eta^2} = \kappa \cos^2 \alpha \cos \beta$ is the reflector acceleration component in the equatorial plane. To switch off the out-of-plane periodic oscillation to get an elliptic displaced periodic orbit for $(\xi, \eta, \zeta)^T$, it is set $\zeta_0 = a_\zeta = \kappa \cos^2 \alpha \sin \beta$ in Eq. (11). Therefore, the size of the orbit is determined by the reflector acceleration component parallel to the equatorial plane a_p, and the displacement above the equatorial plane is determined by the reflector acceleration component out of the equatorial plane a_ζ.

Denote κ_r and κ_s as the characteristic acceleration for the sunlight reflector and the MST, respectively. To ensure a SPS system in formation, with a sunlight reflector vector equal to $(-1,0,0)^T$, it is necessary that the elliptic orbits for the system must be the same size (i.e. same in-plane acceleration a_p) as shown in Fig. 2. On the other hand, Eqs. (11) shows that for a MTS with characteristic acceleration κ_s, the MTS pitch angle must be equal to $-\phi$ (i.e. $\beta = 0$) to obtain an elliptic orbit in the equatorial plane. Thus, a SPS system with the same in-plane acceleration a_p implies that for a sunlight reflector, with characteristic acceleration κ_r and pitch angle α, the MST will be always below the pitched reflector if

$$\kappa_s = \left(\frac{\cos^2 \alpha \cos \beta}{\cos^2 \phi} \right) \kappa_r. \tag{11}$$

Figure 3 shows a sunlight reflector on three displaced orbits in formation with a MST in the Earth's equatorial plane for different seasons. The angle ϕ is then equal to zero at the autumn/spring equinoxes, and equal to $+23.5°$ and $-23.5°$ at the winter and summer solstices, respectively. The reflector is pitched at $\alpha = 45°, 33.3°, 56.7°$ on each orbit at the autumn/spring equinoxes and at the winter and summer solstices, respectively [15], [21]. The reflector characteristic acceleration corresponds to $\kappa_r = 0.15$ mms^{-2} and the displacement $\zeta_0 < 18$ km in each simulation run.

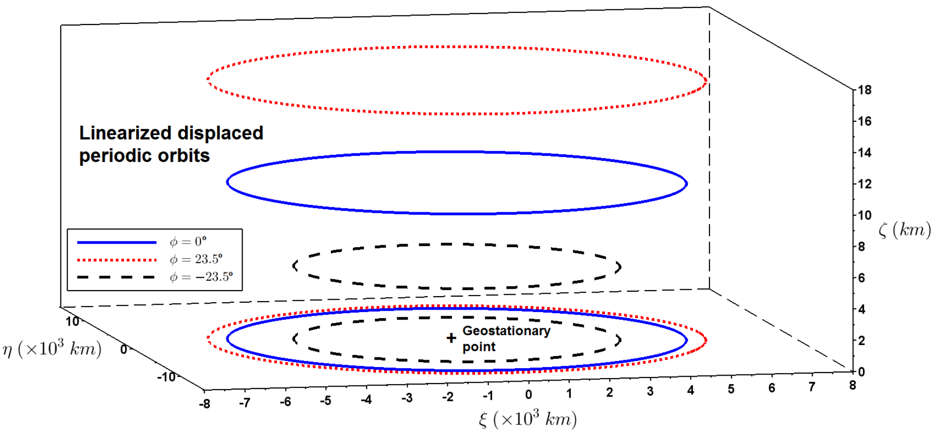

Fig. 3 Sunlight reflector in formation with a MST and pitched at $\alpha = 45°, 23°, 50°$ on each orbit at the autumn/spring equinoxes ($\phi = 0$), at the winter ($\phi = +23.5°$) and summer solstices ($\phi = -23.5°$), respectively [15], [21]. The SPS system is on elliptic periodic orbits around the GEO point and the reflector characteristic acceleration corresponds to $\kappa_r = 0.15$ mms^{-2} in each simulation run.

4 Linear Analysis including the Sun's position on the ecliptic

So far, the sun-line $\hat{\boldsymbol{S}}(t)$ is assumed at a constant angle ϕ over one period. This a reasonable assumption due to the period of the elliptic orbit $T = 2\pi/\Omega^* \approx 1$ day $<<$ 1 year. Although the SPS system is in formation in each season, the size of the periodic orbits and the MTS characteristic acceleration vary with the sun-line angle ϕ, i.e. depend on the season, as shown in Fig. 3. In this section, the path of the Sun is considered on the ecliptic, obtaining a more realistic expression for the solar perturbation $\boldsymbol{a} = (a_\xi, a_\eta, a_\zeta)^T$ in the linearized model Eq. (5). Then the law of reflection is used to redirect the sunlight to the MST for any Sun's location on the ecliptic plane. Finally, the MTS is placed on a common GEO as option to reduce the longitude of the SPS system.

The equatorial components of the sun-line with respect to the inertial frame $I\,(x_I, y_I, z_I)$ are $(-\cos\lambda_\odot, -\cos\varepsilon\cos\lambda_\odot, -\sin\varepsilon\cos\lambda_\odot)^T$, where $\varepsilon = 23.5°$ is the mean Earth's ecliptic and $\lambda_\odot = \lambda_{\odot 0} + \omega_s t$ is the longitude of the Sun and $\lambda_{\odot 0}$ is the initial solar longitude as shown in Fig. 4 [22]. The sun-line $\hat{\boldsymbol{S}}\,(t)$ in the rotating frame $E\,(x_e, y_e, z_e)$ can be obtained as

$$\hat{\boldsymbol{S}}\,(t) = \begin{pmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\cos\lambda_\odot \\ -\cos\varepsilon\cos\lambda_\odot \\ -\sin\varepsilon\cos\lambda_\odot \end{pmatrix} = \begin{pmatrix} \hat{S}_\xi\,(t) \\ \hat{S}_\eta\,(t) \\ \hat{S}_\varsigma\,(t) \end{pmatrix}. \tag{12}$$

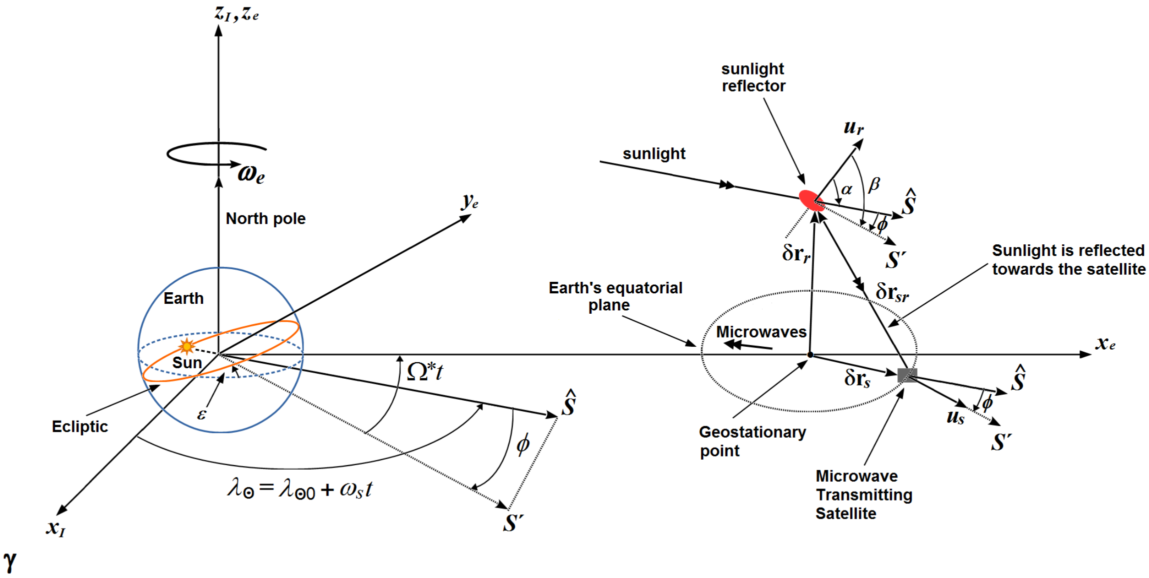

Fig. 4 MTS on an in-plane orbit around the GEO point, the space reflector redirecting the sunlight to the MTS, and the Sun on the ecliptic.

Figure 4 shows the MTS on an in-plane orbit around the GEO point, the space reflector redirecting the sunlight to the MTS, and the Sun on the ecliptic. Denote $\delta\mathbf{r}_i$, $\boldsymbol{u}_i$, $\boldsymbol{a}_i$ as the position vector, the unit vector of the reflector normal, and solar radiation pressure for the MTS $(i = s)$ and the space mirror $(i = r)$, respectively, as shown in Fig. 4. The vector $\delta\mathbf{r}_{sr} = \delta\mathbf{r}_r - \delta\mathbf{r}_s$ then determines the sunlight reflector position relative to the MTS (see Fig. 4). Since $\delta\mathbf{r}_i$ satisfies the linearized equations of motion, Eq. (5), then differentiating the vector $\delta\mathbf{r}_{sr}$ with respect to time t, a linear equation for $\delta\mathbf{r}_{sr}$ is obtained:

$$\delta\dot{\mathbf{Y}} = \mathbf{M}\delta\mathbf{Y} + \delta\boldsymbol{a}_{sr} \tag{13}$$

where $\delta\mathbf{Y} = (\delta\mathbf{r}_{sr}, \delta\dot{\mathbf{r}}_{sr})^T$, $\delta\boldsymbol{a}_{sr} = \boldsymbol{a}_r - \boldsymbol{a}_s$, and $\boldsymbol{a}_i = \kappa_i \left(\hat{\boldsymbol{S}}\,(t) \cdot \boldsymbol{u}_i\right)^2 \boldsymbol{u}_i$. Since the MTS is on an in-plane orbit, it is assumed that the MTS normal unit vector is aligned with the projection of the sun-line $\hat{\boldsymbol{S}}\,(t)$ in the equatorial plane. The expression for $\boldsymbol{u}_s$ in the rotating frame is then $\boldsymbol{u}_s = \left(\hat{S}_\xi\,(t), \hat{S}_\eta\,(t), 0\right)^T$. On the other hand, the sunlight must be redirected towards the MTS, i.e. the reflected sunlight vector must be equal to $-\delta\mathbf{r}_{sr}$, as shown in Fig. 4. Therefore, the law of reflection requires that [?]

$$\boldsymbol{u}_r = \frac{\hat{\boldsymbol{S}}(t) + \hat{\delta}\mathbf{r}_{sr}}{\left|\hat{\boldsymbol{S}}(t) + \hat{\delta}\mathbf{r}_{sr}\right|}. \tag{14}$$

where $\hat{\delta}\mathbf{r}_{sr}$ is the unit vector of the sunlight reflector position relative to the MTS. Substituting the expressions for $\boldsymbol{a}_i$ in Eq. (13), a solution for the linear motion of the SPS system about the GEO point can be computed numerically, i.e. analytical solutions no longer exist for this model.

Figure 5 shows a numerical simulation of the variation of the sunlight reflector relative position coordinates with a characteristic acceleration $\kappa_r = 0.018$ mms^{-2} during a three years span. The periodic solutions, Eqs. (9)-(12), were used as initial conditions in the numerical simulation with $\phi = 0$, $\alpha = 45°$ and $\lambda_{\odot 0} = 180°$, such that the sun-line is aligned with the vernal equinox at $t = 0$. The MTS characteristic acceleration κ_s is determined using Eq. (11). Figures 5(a)-(c) show a bounded behavior of the reflector orbit with respect to the MTS. The out-of-plane motion presents an amplitude $|\zeta_{sr}| < 5$ km. However, the in-plane motion is strongly affected by the solar perturbation. Figure 5(d) shows the variation of the reflector pitch angle α (i.e. control history) when the law of reflection is applied in the space mirror attitude along the time. Although the linear system in Eq. (5) has periodic displaced solutions only for a constant sun-line inclination, this approximation is useful to determine a bounded orbit considering the law of reflection and the effects of sun-line inclination. Finally, Figs. 6(a)-(c) and Fig.6(d) show the bounded orbits for the reflector and MTS in the rotating frame, respectively. Note that when the variation of the sun-line inclination is not neglected, a periodic solution for the MTS becomes an in-plane quasi-periodic orbit.

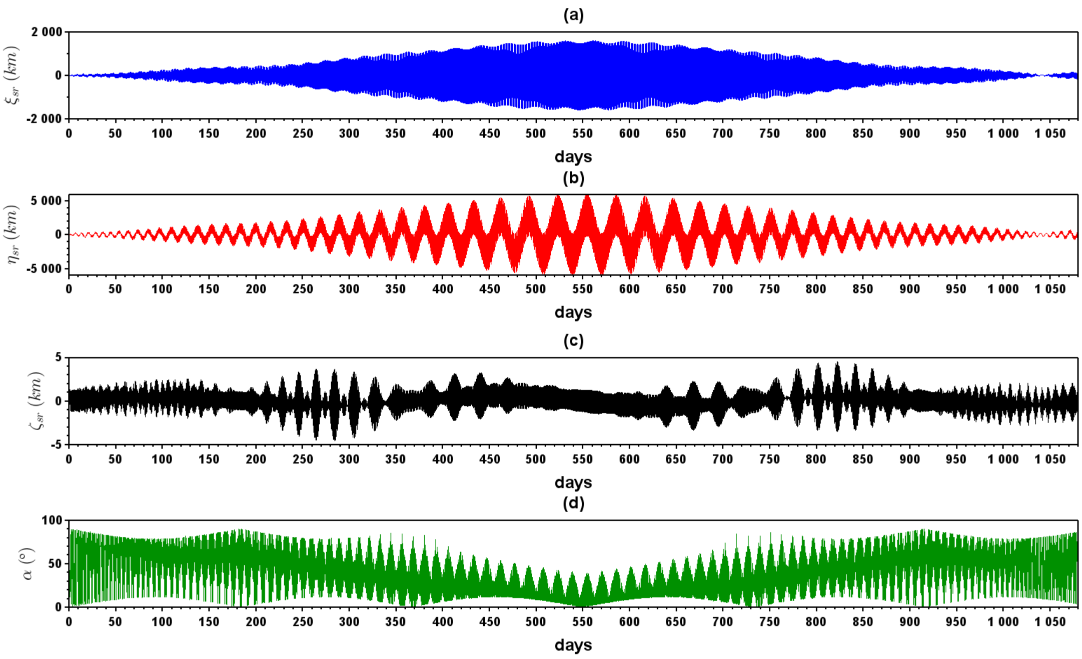

Fig. 5 (a)-(c) Variation of the sunlight reflector relative position coordinates with a characteristic acceleration $\kappa_r = 0.018$ mms^{-2} during a three years span. (d) Variation of the reflector pitch angle α (i.e. control history)

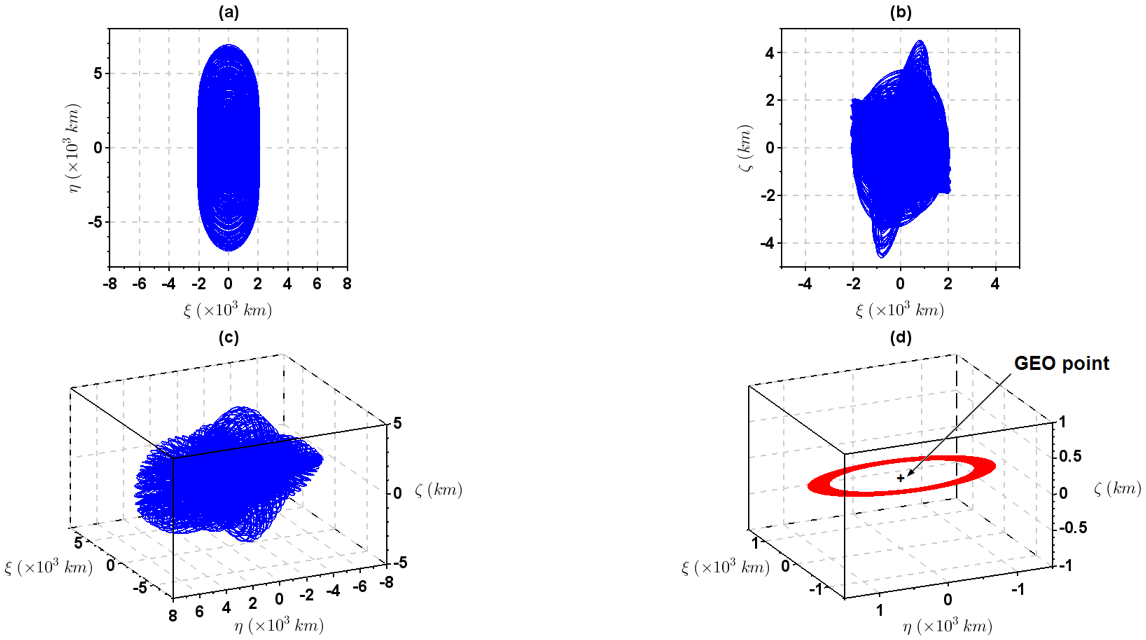

Fig. 6 (a)-(b) Projections in the $\xi - \eta$ and $\xi - \zeta$ planes for the sunlight reflector motion in the rotating frame corresponding to the relative position vectors in Fig. 5. (c)-(d) Bounded and quasi-periodic orbits for the sunlight reflector and MTS in the rotating frame, respectively.

To achieve a quasi-periodic orbit for the sunlight reflector, so that the longitude of the SPS system, as well as the large variations on the reflector pitch angle, can be reduced, the MTS is assumed on a common GEO. Thus, the MTS position vector $\delta\mathbf{r}_s = (1, 0, 0)^T$ in the rotating frame (see Fig. 4), and the reflector relative position vector in the law of reflection, Eq. (14), is substituted by $\delta\mathbf{r}_r$. Figure 7 shows a numerical simulation of the variation of the sunlight reflector position coordinates with a characteristic acceleration $\kappa_r = 0.018$ mms^{-2} during a three years span. Similarly, the initial conditions were determined with $\phi = 0$, $\alpha = 0°$ and $\lambda_{\odot 0} = 180°$. As can be noted in Fig. 7, the sunlight reflector orbit presents a quasi-periodic behaviour around the GEO point. So, the pitch angle variation also presents a quasi-periodic behaviour, with a maximum amplitude of 15°. Figures. 8(a)-(c) show the projections on the $\xi - \eta$, $\xi - \zeta$, $\eta - \zeta$ planes for the reflector in the rotating frame, respectively. Figure 8(d) also shows the quasi-periodic orbit for the reflector in the rotating frame. In the same manner, a displaced periodic solution around the GEO point for the reflector (see Fig. 3) becomes a quasi-periodic orbit when the variation of the sun-line inclination is not neglected.

5 Conlusions

In this study a SPS system have been investigated considering a space mirror and a MTS in formation around a GEO point with respect to an Earth-centered rotating frame that rotates with the same angular velocity of the Earth. As a first approximation, it was assumed the sun-line inclination as constant depending on the season, i.e. equinoxes (0°) and summer and winter solstices (±23.5°). In these seasons, a family of displaced ecliptic orbits of different sizes around the GEO point were determined for the SPS system in the linearized Earth-reflector two-body problem with solar perturbation. Although these results show the existence of periodic orbits

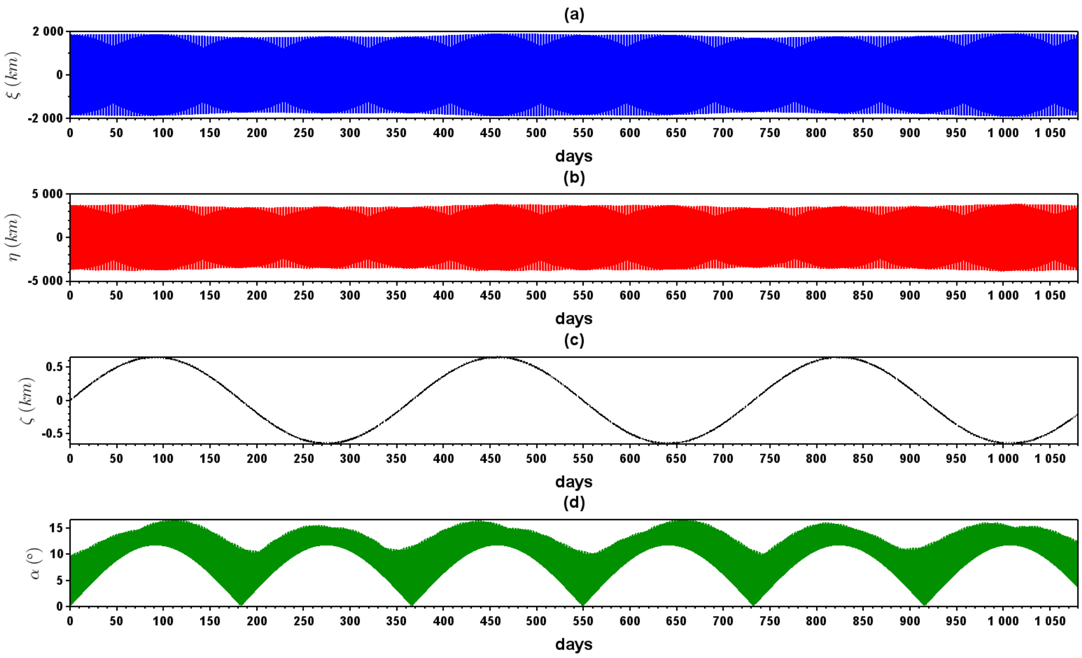

Fig. 7 (a)-(c) Sunlight reflector coordinates with a characteristic acceleration $\kappa_r = 0.018$ mms^{-2} during a three years span. (d) Variation of the reflector pitch angle α (i.e. control history)

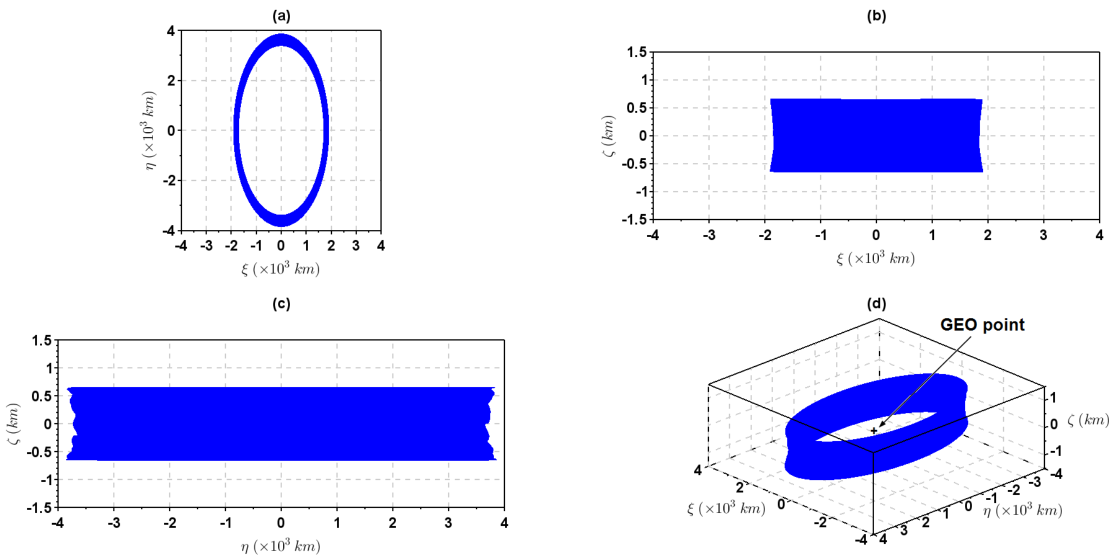

Fig. 8 (a)-(c) Projections in the $\xi - \eta$, $\xi - \zeta$, $\eta - \zeta$ planes for the sunlight reflector motion in the rotating frame corresponding to the coordinates in Fig. 7. (c)-(d) Quasi-periodic orbit for the sunlight reflector in the rotating frame.

around geostationary points, the sun-line changes with time, affecting the periodic behaviour of the displaced orbit. In order to include the direction of the sun-line, the position of the Sun was approximated by a circular ecliptic trajectory, assuming a mean inclination of Earth's equator with respect to the ecliptic equal to 23.5°. To redirect the sunlight to the MTS, the law of reflection was applied in the reflector attitude, so that a bounded orbit for the sunlight reflector and a quasi-periodic orbit for the MTS were obtained using as initial conditions the ecliptic orbits found in the first approximation. As was noted, solar perturbation affected the synchronization of the SPS system, increasing the longitude and producing large variations of the reflector pitch angle.

The possibility of reducing the longitude of the SPS system and the variation of the reflector pitch angle was studied placing the MTS on the GEO point. In this case, the periodic ecliptic orbits become quasi-periodic orbits, not driving the system very far from equilibrium, such that the linearized model is still valid. Thus, this scenario would be more useful for a real mission interested in increasing the solar energy supplying since the sunlight reflector would remain around a geostationary point, requiring a smaller correction of the reflector attitude compared with a MTS around a GEO point. Future studies should also consider nonlinearities and the potential of the Earth's gravity field to obtain a more realistic model.

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