Realistic Modeling of Leakage and Intrusion Flows through Leak Openings in Pipes

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Abstract: The hydraulics of leakage and intrusion flows through leak openings in pipes is complicated by variations in the leak areas owing to changes in pressure. This paper argues that the pressure–area relationship can reasonably be assumed to be a linear function, and a modified orifice equation is proposed for more realistic modeling of leakage and intrusion flows. The properties of the modified orifice equation are explored for different classes of leak openings. The implications for the current practice of using a power equation to model leakage and intrusion flows are then investigated. A mathematical proof is proposed for an equation linking the parameters of the modified orifice and power equations using the concept of a dimensionless leakage number. The leakage exponent of a given leak opening is shown to generally not be constant with variations in pressure and to approach infinity when the leakage number approaches a value of minus one. Significant modeling errors may result if the power equation is extrapolated beyond its calibration pressure range or at high exponent values. It is concluded that the modified orifice equation and leakage number provide a more realistic description of leakage and intrusion flows, and it is recommended that this approach be adopted in modeling studies. **DOI: [10.1061/(ASCE)HY.1943-7900.0001346.](https://doi.org/10.1061/(ASCE)HY.1943-7900.0001346)** *This work is made available under the terms of the Creative Commons Attribution 4.0 International license, http://creativecommons.org/licenses/by/4.0/.*

Introduction

Pressurized pipes are used in various industries to transport fluids, with water supply one of the most important applications in civil engineering. Pipes, joints, and fittings invariably develop leaks such as holes and cracks owing to several internal and external factors (Kleiner and Rajani 2001).

Normally the pressure differential over a pipe wall (defined here as internal minus external pressure) is positive, resulting in leakage flow exiting the pipe through any existing leak openings. However, under certain circumstances the pressure differential may be negative, resulting in intrusion flow entering the pipe.

Since both leakage and intrusions are undesirable in pipe systems, a realistic model of their behavior is important for engineers and researchers in pipeline systems.

The hydraulic behavior of both leaks and intrusions is described by an orifice equation, which is derived from the conservation of energy principle and describes the conversion of potential pressure energy to kinetic energy (e.g., Finnemore and Franzini 2009). Orifice flow has been extensively studied, and much is known about the behavior of orifices under a wide range of conditions (Idelchik 1994).

When applying the orifice equation, the orifice area A is normally assumed to be fixed. However, several laboratory and modeling studies have shown that the areas of real leak openings are not fixed but vary with fluid pressure under both leakage (May 1994; Greyvenstein and van Zyl 2007; van Zyl and Clayton 2007;

Cassa et al. 2010; Ferrante et al. 2011; Massari et al. 2012; De Marchis et al. 2016; Fox et al. 2016a, b) and intrusion (Meniconi et al. 2011; Mora-Rodríguez et al. 2014, 2015) conditions.

Changes in a leak area with pressure means that the conventional orifice equation cannot accurately describe the flow through leak openings in real pipes. To overcome this problem, a power equation is commonly used to model the flow–pressure relationship, both in hydraulic network modeling (Giustolisi et al. 2008; Gupta et al. 2016; Mora-Rodríguez et al. 2014; Ferrante et al. 2014; Schwaller and van Zyl 2014) and water loss management practice (Ogura 1979; Hiki 1981; Lambert 2001; Farley and Trow 2003).

The purpose of this study was to investigate the validity of the power equation approach for modeling leakage and intrusion flows through leak openings in pipes in light of the latest research findings on the behavior of leak areas with pressure.

The study assumed a pipe that is filled with, and submerged in, the same fluid. The effect of factors such as axial flow in the pipe, variations in the discharge coefficient, the flow regime, and soils outside the pipe were ignored.

The next section of the paper describes the behavior of leak areas with pressure and how the orifice equation can be modified to account for these. A dimensionless leakage number is defined. The conventional approach of using a power equation for leakage and intrusion modeling is then evaluated: the properties of the modified orifice equation are explored for different classes of leak openings, and a link between the power and modified orifice equations is established. Finally, the implications for realistic modeling of leakage and intrusion flows are discussed.

Modified Orifice Equation

Orifice Equation

The orifice equation applies to flow through leak openings in pipes, irrespective of whether the flow is exiting the pipe (i.e., leakage) or entering the pipe (i.e., intrusion). Defining the pressure head differential h over the leak opening as $h = h_{internal} - h_{external}$ allows a single equation to describe both leakage and intrusion flows:

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$$Q_o = \text{sgn}(h)C_d A\sqrt{2g|h|} \quad (1)$$

where Q_o = orifice flow rate; sgn() = signum or sign function; C_d = discharge coefficient; A = orifice area; and g = acceleration owing to gravity. A positive Q indicates leakage flow and a negative Q intrusion flow.

Leak Area Variation

It has been shown that leak areas are often not fixed but vary with fluid pressure owing to the resulting changes in internal pipe wall stresses. These changes may result in four types of deformation: elastic, viscoelastic, plastic, and fracture.

Several experimental and modeling studies have shown that under elastic conditions, variation in leak area with pressure is a linear function. This linearity holds true irrespective of the leak opening type and size, pipe material, section properties loading state (Cassa and van Zyl 2013; van Zyl and Cassa 2014; Malde 2015).

Ssozi et al. (2016) used a finite-element model to show that the aforementioned linearity also applies to viscoelastic deformation at any given time after loading for different leak opening types and sizes, pipe materials, and loading states. In practice, viscoelastic deformation is likely to result in hysteresis, with the leak area depending not only on the pressure but also on the loading time history and variation rate [as clearly shown by Ferrante et al. (2011)]. However, Ssozi et al. (2016) argue that linearity can still be assumed, particularly under consistent operational conditions.

Linear variations in a leak area with pressure cannot be assumed for plastic deformation and fracture, and further work is required to better understand these processes in pipes. However, these mechanisms are unlikely to be dominant in a system at any given time for the following reasons:

- They are irreversible processes and can thus can only continue for a limited period of time. For instance, plastic deformation of a crack may result in the wall stresses being redistributed in such a way that the deformation becomes elastic (and thus stable), or the leak opening may become so large that the leak is discovered or the pipe fails catastrophically.
- They will only occur when the pressure in the pipe, and thus the wall stresses, are increased and not when the pressure is decreased.

In this study it was assumed that only elastic viscoelastic deformation occur and thus that the areas of all leak openings are linear functions of pressure. Thus the leak area can be described using the function

$$A = A_0 + mh \quad (2)$$

where A_0 = initial area (area of leak opening when head differential is zero); and m = head-area slope.

Substituting Eq. (2) into Eq. (1) results in the modified orifice equation (known in leakage practice as the *fixed and variable area discharge*, or FAVAD, equation) (May 1994):

$$Q_{mo} = \text{sgn}(h)C_d\sqrt{2g}(A_0|h|^{0.5} + m|h|^{1.5}) \quad (3)$$

Leakage Number

Eq. (3) consists of two terms. The first term describes orifice flow [Eq. (1)] based on the initial leak area and is called the initial area flow $Q_{Ao} = \text{sgn}(h)C_d A_0\sqrt{2g|h|}$. The second term describes the flow through the expanded portion of the leak opening and is called the expanded area flow $Q_{Ae} = \text{sgn}(h)C_d\sqrt{2g}m|h|^{1.5}$.

van Zyl and Cassa (2014) introduced a dimensionless leakage number L_N as the ratio of the expanded area flow to the initial area flow, or

$$L_N = \frac{mh}{A_0} \quad (4)$$

It is possible to write the modified orifice Eq. (3) in terms of L_N, with the term $(1 + L_N)$ a pressure-dependent correction term to the orifice equation to account for changes in the leak area:

$$Q_{mo} = (1 + L_N)Q_{Ao} \quad (5)$$

Power Equation

As mentioned earlier, the behavior of leaks is commonly modeled using a power equation in the form

$$Q_p = sgn(h)C|h|^{\alpha} \quad (6)$$

where C = leakage coefficient; and α = leakage exponent. In modeling practice, such as the widely used *EPANET* software, the power equation is called an emitter and is used to model pressure-dependent consumer demands and leakage.

In leakage management practice, Eq. (6) is called the power leakage equation and α is replaced by the symbol N1 (Lambert 2001). The leakage exponent is popular with practitioners for predicting the likely effect of changing the average system pressure on system leakage. It is commonly expressed as the ratio between the measured leakage at two different pressures in the form

$$\frac{Q_1}{Q_2} = \left(\frac{h_1}{h_2}\right) \quad (7)$$

where Q_1 and Q_2 = leakage flow rates at pressures h_1 and h_2, respectively.

Mathematically the power equation is a generalized form of the orifice equation that allows α to be different from 0.5. However, this generalization severs the equation from fundamental mechanics principles on which the orifice equation is founded. Thus the power equation becomes nothing more than an empirical equation.

Other disadvantages of the power equation are that both C and α are not constant for a given system but vary with pressure and that the equation is dimensionally awkward (van Zyl and Cassa 2014).

Finally, as will be shown in this paper, there are instances of a modified orifice where no power equation solution exists and the leakage exponent approaches infinity.

Properties of Modified Orifice Equation

The finding that the area of leaks in pipe systems will expand linearly with pressure implies that the hydraulic behavior of a leak can be fully characterized by three parameters: the initial area A_0, head-area slope m, and discharge coefficient C_d, as described by Eq. (3). The discharge coefficient C_d may be estimated experimentally or from the large body of empirical knowledge in sources such as Idelchik (1994). It is generally constant for higher Reynolds numbers. The number of characteristic leak parameters may be reduced to two by defining effective area $A' = C_d A$, effective initial area $A_0' = C_d A_0$, and effective head-area slope $m' = C_d m$. Eqs. (1)–(3) now become

$$Q_O = \text{sgn}(h)A'\sqrt{2g|h|} \quad (8)$$

$$A' = A_0' + m'h \quad (9)$$

$$Q_{mo} = \text{sgn}(h)\sqrt{2g}(A_0'|h|^{0.5} + m'|h|^{1.5}) \qquad (10)$$

Derivatives of the Modified Orifice Equation

Eq. (3) has two terms, initial area flow and expanded area flow, with pressure head exponents of 0.5 and 1.5, respectively. For small pressures or head-area slopes, the first term will tend to dominate, while the second term will dominate at large pressures or head-area slopes and an inflection point will occur. A typical case is shown in Fig. 1 for positive pressures.

The slope of a tangent line on the modified orifice equation can be obtained from the first derivative of Eq. (3) with respect to the pressure head. Without loss of generality, this is only done for positive h to obtain

$$\frac{dQ_{mo}}{dh} = C_d\sqrt{2g}[0.5A_0 h^{-0.5} + 1.5mh^{0.5}] \qquad (11)$$

Eq. (8) is also plotted in Fig. 1, reaching a minimum at the inflection point. The location of the inflection point may be determined by first taking the second derivative of the modified orifice equation with respect to h:

$$\frac{d^2Q_{mo}}{dh^2} = C_d\sqrt{2g}[-0.25A_0 h^{-1.5} + 0.75mh^{-0.5}] \qquad (12)$$

then setting it equal to zero and simplifying to obtain

$$\frac{mh}{A_0} = L_N = \frac{1}{3} \qquad (13)$$

Thus the inflection point will occur at a leakage number of $1/3$. It is worth noting that the inflection point may occur at either a positive or negative pressure (but not at both), depending on the values of the leak's initial area and head-area slope.

Leak Openings with Positive Head-Area Slopes

For leak openings with positive head-area slopes, leak areas increase with an increasing head differential and decrease with a decreasing head differential. This is true irrespective of whether the head differential itself is positive or negative. For negative pressure differentials, an absolute limit exists since the pressure inside the pipe cannot be reduced below absolute zero. Thus, for any given pressure on the outside of the pipe, an absolute limit exists for negative head differentials. Under an increasingly negative head
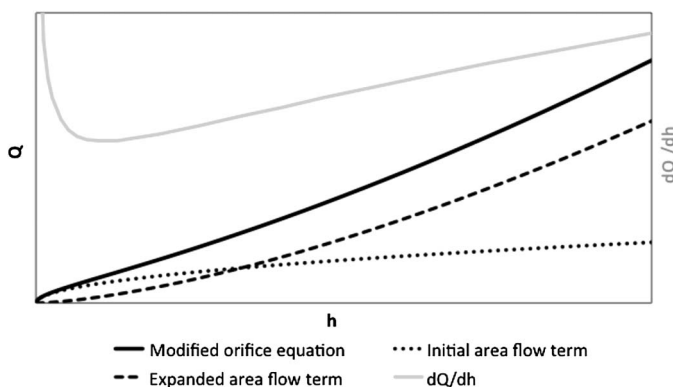
Fig. 1. Modified orifice equation for a typical case under positive pressure; shown are the initial and expanded area components, as well as the first derivative of the equation

differential, leak areas may reduce until the leak opening closes fully (assuming no local effects preclude this), preventing further flow through the leak opening. However, leak areas may not yet have closed when the absolute head differential limit is reached. Leak openings may have a zero initial area, for instance a crack that closes fully when no head differential exists. Finally, it is possible for a leak opening to remain closed up to a certain positive head differential before it starts opening, for instance as a result of external soil pressure acting on the pipe.

Based on the preceding considerations, four categories of behavior may be identified for positive head-area slopes as defined by the following cases and shown in Fig. 2(a). The resulting flow rates are shown in Fig. 2(b):
- Case P0: $A_0 > 0$, with the leak opening not closing fully even at the absolute head differential limit.
- Case P1: $A_0 > 0$, with the leak opening closing fully at or before the absolute head differential is reached.
- Case P2: $A_0 = 0$, that is, the leak opening closes fully at a zero head differential.
- Case P3: $A_0 < 0$: while a negative initial area is not physically possible, it may occur in the mathematical model when a leak opening remains closed at head differentials above zero, for instance as a result of external forces.

Leak Openings with Negative Head-Area Slopes

For leak openings with negative head-area slopes, leak areas decrease with increasing head differential and increase with decreasing head differential. This is true irrespective of whether the head differential itself is positive or negative. Negative head-area slopes are not common but have been shown to occur, for example, in circumferential cracks (Cassa et al. 2010; Malde 2015).

Owing to the absolute negative pressure differential limit, these leak areas have a maximum achievable value. On the other hand, the leak areas will eventually close fully if a high enough positive pressure differential is achieved (assuming that no local effects preclude this), preventing further flow through the leak opening. Finally, mirroring the behavior of positive head-area slopes, leak openings may have a zero initial area or remain closed up to a certain negative head differential before it starts opening.

Based on the preceding considerations, three categories of behavior may be identified for negative head-area slopes as defined by the following cases and shown in Fig. 3(a). The resulting flow rates are shown in Fig. 3(b):
- Case N1: $A_0 > 0$, with the leak opening closing fully at a high enough positive head differential.
- Case N2: $A_0 = 0$, that is, the leak opening closes fully at a zero head differential.
- Case N3: $A_0 < 0$: while a negative initial area is not physically possible, it may occur in the mathematical model when a leak opening remains closed at head differentials below zero, for instance as a result of external forces.

### Linking the Modified Orifice and Power Equations

van Zyl and Cassa (2014) found that the leakage number can be used to link the modified orifice and power equations. They proposed an empirically derived equation to calculate the leakage exponent for any given leakage number, and vice versa. In this section, a mathematical proof is provided for this relationship before discussing its implications for leakage and intrusion modeling and providing an example.
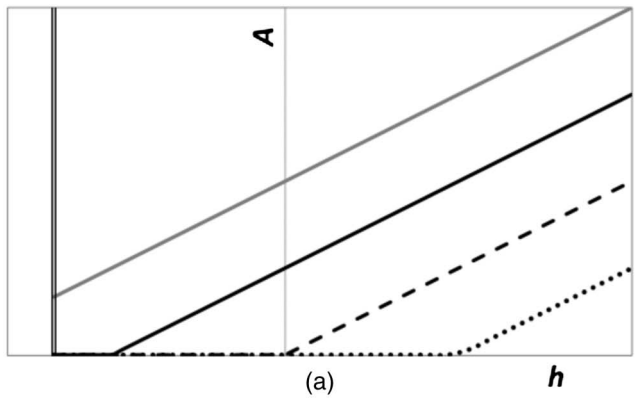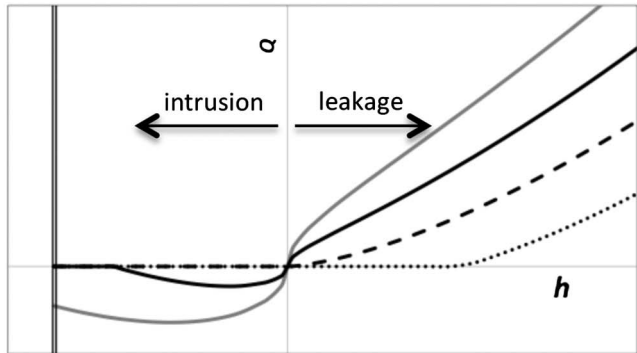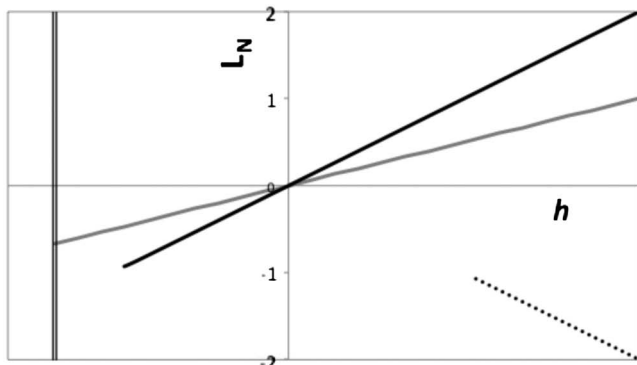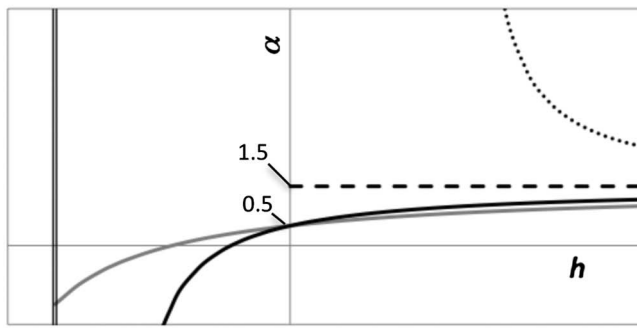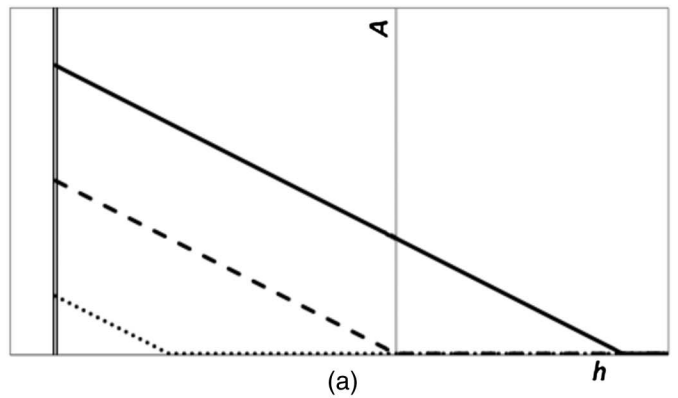
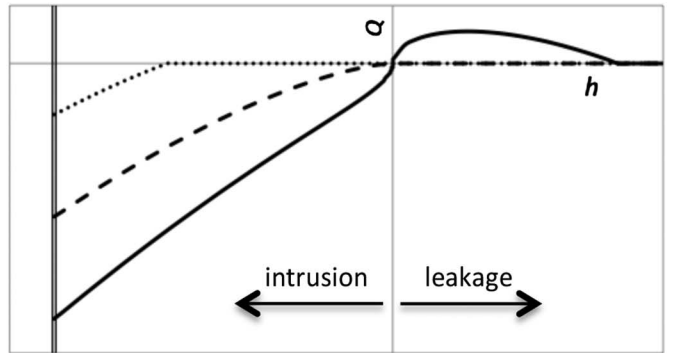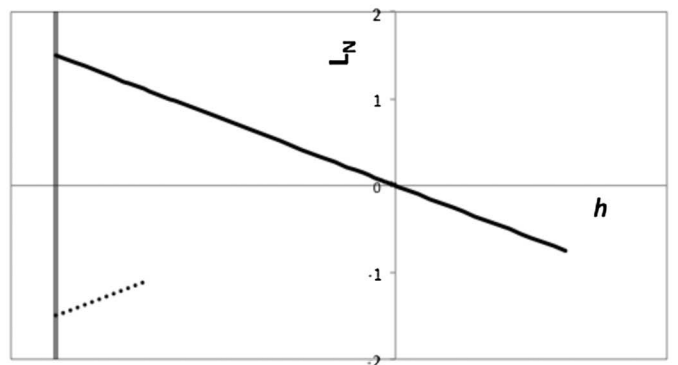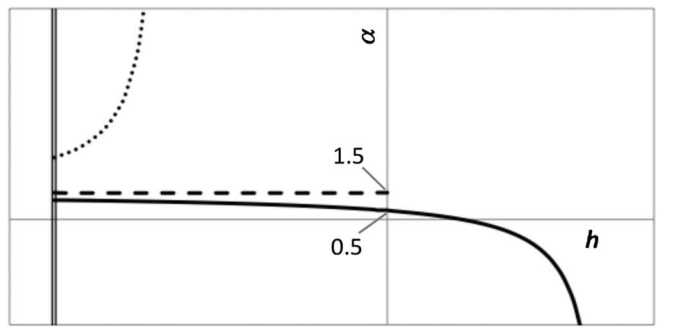**Fig. 2.** Behavior of leak openings with positive head-area slopes as a function of pressure differential: (a) leak area A; (b) flow rate Q; (c) leakage number L_N; (d) leakage exponent α

Fig. 3. Behavior of leak openings with negative head-area slopes as a function of pressure differential: (a) leak area A; (b) flow rate Q; (c) leakage number L_N; (d) leakage exponent α

Proof

First, the flow rates in the modified orifice [Eq. (3)] and power [Eq. (7)] equations are equated. Without loss of generality this may be done for positive h only:

$$C_d\sqrt{2g}(A_0h^{0.5} + mh^{1.5}) = Ch^\alpha \qquad (14)$$

The first derivatives of the two equations are also equated:

$$C_d\sqrt{2g}(0.5A_0h^{-0.5} + 1.5mh^{0.5}) = \alpha Ch^{\alpha-1} \qquad (15)$$

Now Eq. (14) is divided by Eq. (15):

$$\frac{0.5A_0h^{-0.5} + 1.5mh^{0.5}}{A_0h^{0.5} + mh^{1.5}} = \frac{\alpha}{h} \qquad (16)$$

Finally, the left-hand term is divided by A_0 above and below the line and simplified to obtain the equation originally proposed by van Zyl and Cassa (2014):

$$\alpha = \frac{1.5L_N + 0.5}{L_N + 1} \qquad (17)$$

This equation may be rearranged to express L_N as a function of α:

$$L_N = \frac{\alpha - 0.5}{1.5 - \alpha} \qquad (18)$$

Eq. (17) describes a hyperbolic function with asymptotes $L_N = -1$ and $\alpha = 1.5$, as shown graphically in Fig. 4. The function crosses the L_N-axis at $\alpha = 0.5$ and the α-axis at $L_N = -1/3$.

Discussion

To illustrate the implications of this relationship for leak openings with positive and negative head-area slopes, the leakage numbers are shown in Figs. 2(c) and 3(c) and the leakage exponents in Figs. 2(d) and 3(d), respectively. Since both the initial area and head-area slope of any given leak opening are constant, the leakage number is a linear function of the pressure head. The only case not shown in Figs. 2 and 3 is that of a leak opening with a fixed area, that is, with a head-area slope of zero. For this special case, the leakage number and leakage exponent are always zero and 0.5, respectively.
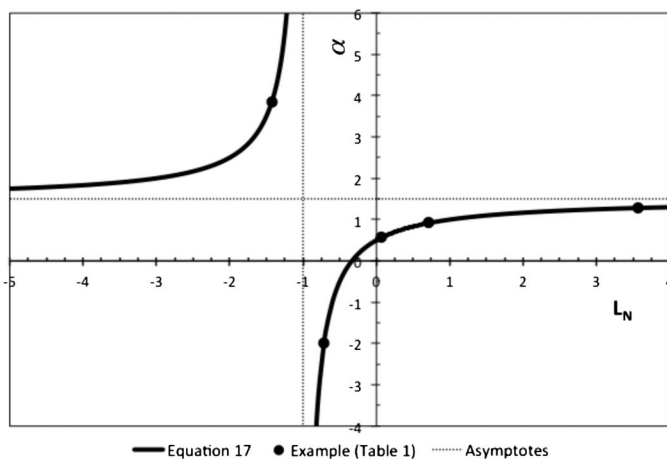
Fig. 4. Relationship between the leakage exponent α and leakage number L_N

Other special cases occur when the initial area is zero ($A_0 = 0$), as shown as Cases P2 and N2. The leakage number and leakage exponent for these two cases are infinity and 1.5, respectively.

For positive leakage numbers (i.e., leaks openings with positive head-area slopes at positive pressures or negative head-area slopes at negative pressures) the leakage exponent varies between 0.5 at $L_N = 0$ and 1.5 as the leakage number approaches infinity. Thus the leakage exponent can be interpreted as the equivalent power exponent for varying flow contributions of the fixed and expanding flow terms in Eq. (3).

As noted earlier, the leakage exponent is asymptotic at $L_N = -1$, which from Eq. (4) will occur at pressure differential $h = -A_0/m$. From Eq. (2) it can be shown that the leakage number will be minus one when the leak area is equal to zero at any non-zero pressure differential. Thus the leakage exponent will approach infinity as it approaches a positive or negative pressure where a leak opening will close. This can be observed in Fig. 2(d) for Cases P0 and P1 and in Fig. 3(d) for Case N1.

An investigation of the behavior of the leakage coefficient C shows that when the leakage exponent approaches infinity, the leakage coefficient approaches zero. Thus the power equation will still accurately model the leak hydraulics at the exact pressure where its parameters were calibrated. However, as Figs. 2(d) and 3(d) show, and as noted by van Zyl and Cassa (2014), the leakage exponent of a given leak opening is generally not fixed but varies with pressure. Thus using the power equation to model the behavior of a leak is likely to result in significant errors when the equation is used at pressures different from the one at which it was calibrated, particularly when working near a leakage number of minus one.

An important conclusion from these results is that the leakage exponent is not an ideal measure to describe the sensitivity of leakage and intrusion flow rates to changes in pressure (as is currently the practice in leakage management) in all circumstances. One reason for this is that, unlike the initial area and head-area slope, the power equation parameters are functions of pressure. A second reason is that a leak with a given head-area slope can adopt a large range of leakage exponents under different conditions.

Example

The range of leakage exponents that a leak can adopt is illustrated using the example of a 100 mm diameter longitudinal crack in a uPVC pipe with an internal diameter of 105.5 mm and a wall thickness of 4.54 mm. The head-area slope of this crack was experimentally determined by Malde (2015) as 4.75 mm²/m. It has been established that the head-area slope is not sensitive to crack width (Cassa and van Zyl 2013). The leakage number and head-area slopes were calculated at a pressure of 15 m for different crack widths using Eqs. (4) and (17), respectively, and are shown in the first four lines of Table 1. The last two lines of Table 1 represent a hypothetical Case P3 with the crack closed at low pressures and

Table 1. Leakage Exponents Calculated for 100 mm Longitudinal Crack in 100 mm Diameter uPVC Pipe under Different Conditions at Pressure of 15 m

Case (from Fig. 2)	Crack width (mm)	Initial area A_0 (mm²)	Leakage number L_N	Leakage exponent α
P0	10	1,000	0.071	0.57
P0	1	100	0.713	0.92
P0	0.2	20	3.563	1.28
P2	0	0	∞	1.50
P3	N/A	−100	−0.713	−1.98
P3	N/A	−50	−1.425	3.85

only starting to open at pressures of 21 and 10.5 m, respectively. Table 1 shows that a crack with the same head-area slope can result in a wide range of leakage exponents; in the example, it is between −1.98 and 3.85, but technically the range is unlimited. The example's leakage numbers and exponents are plotted on the theoretical relationship between the leakage number and exponent in Fig. 4.

While these examples were chosen for illustrative purposes only, the asymptote of the leakage exponent at a leakage number of minus one may be responsible for some of the high leakage exponents reported in field and laboratory studies.

Finally, it should be noted that in field studies it is common practice to determine the leakage exponent from the leakage flows obtained from two or more different pressures. If the example is repeated using such an approach, a smaller range of leakage exponents is likely to be observed.

Implications for Leakage and Intrusion Modeling

It is clear that using a power equation to model leakage and intrusion flows in pipe systems should be done with care to ensure accurate results. As an empirical equation, the power equation should produce acceptable results when applied within its calibration data range, but extrapolating the power model outside this range is likely to result in significant modeling errors. This error is likely to be greater for larger leakage exponents, particularly near a leakage number of minus one.

If the power equation is used for leakage modeling, the change in the leakage exponent with pressure may be estimated by first converting the leakage exponent to a leakage number using Eq. (18), then proportionally adjusting the leakage number for the new pressure in Eq. (4), and finally converting the adjusted leakage number back to a leakage exponent using Eq. (17).

As shown in the example, leak openings with the same head-area slope will result in higher leakage exponents when they have smaller initial areas. This means that experimental studies on slots created by removing pipe material (typically to a width of 1 to 2 mm) will underestimate the leakage exponent of the equivalent crack with a much smaller average width.

Since the areas of leaks can reasonably be assumed to be linear functions of pressure, the total leakage area of a system with many individual leaks will also be a linear function of pressure. The initial area and head-area slope of the total system leakage will be respectively equal to the sum of the initial areas and head-area slopes of all the individual leaks in the system. This technique has been verified by Schwaller et al. (2015) on a spreadsheet model.

This means that if the leakage flows at two or more average pressures are measured for a distribution system or district metered area (DMA), the total effective initial area and head-area slope of all the leaks in the system can be estimated from Eq. (10). The total initial area provides a useful physical condition assessment parameter for the system, while the total head-area slope may be used, in combination with the known behavior of leak head-area slopes from published studies, to estimate the dominant type of leaks in the system. Unrealistic values of the system leakage parameters may be used to diagnose potential problems, such as measurement errors or leaking boundary valves. The leakage number may also be used as a more stable alternative to the leakage exponent for characterizing the sensitivity of network leakage to pressure.

Finally, it is clear that realistic modeling of leakage and intrusion flows through leak openings in pipes requires the implementation of the modified orifice equation in network modeling packages such as *EPANET*. This will be relatively simple to implement by adding a second emitter to junctions and using exponents of 0.5 and 1.5 respectively for the two emitters.

Conclusion

Several published studies have shown that the areas of leak openings are linear functions of pressure under both elastic and viscoelastic conditions. Since the other deformation mechanisms (plastic deformation and fracture) only occur when pressure increases and are irreversible, these are transient events, and it is reasonable to assume that distribution system leak areas will generally be linear functions of pressure.

The power equation, which is currently used to model leaks and intrusions, is an empirical equation and should only be used within its calibration pressure range. The leakage exponent of a given leak opening is generally not constant with variations in pressure and was found to approach infinity as the leakage number approaches a value of minus one. Significant errors are possible if the power equation is extrapolated beyond the calibration pressure range or at high exponent values.

It is recommended that the modified orifice equation is used to describe the pressure-leakage response leakage and intrusion flows through leak openings in pipes, both in steady-state and transient flow studies. The modified orifice equation should also be incorporated into hydraulic modeling software and used in modeling studies that incorporate leakage. Finally, it may be necessary to reassess the findings of earlier power equation–based modeling studies where leakage played a significant role.

Acknowledgments

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