# PRACTICAL APPLICATIONS
# OF RIGID THICK ORIGAMI
# IN KINETIC ARCHITECTURE

A DARCH PROJECT SUBMITTED TO THE GRADUATE DIVISION OF THE UNIVERSITY OF HAWAI‘I AT MĀNOA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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## Abstract

Folding elements have already been used in architecture as either: (a) simple or negligibly thin folds such as tent-like structures; (b) thick panels with single straight hinges; or (c) flat, faceted forms that appear to have been folded. What is seldom seen is folding in more complicated patterns that also use thick panels. The more complicated crease patterns inspired from origami cannot be used interchangeably between thin and thick materials. Further, once a folding feature is designed, it must have a way to attach to the main/super structure and have a means to deploy.

If design parameters and attachments can be better presented and understood, more origami patterns that are rigid and thick may be incorporated into kinetic architecture or rigid-thick origami kinetic architecture.

This research creates a useful primer for understanding and designing rigid-thick origami structures by simplifying and organizing existing knowledge on rigid-thick origami into a more accessible format for designers and architects without the need for deep mathematical background. It also presents a variety of design patterns which can be altered or adapted along provided guidelines, as well as propose some methods in which to attach and operate some of these designs on a superstructure through documentation of a working prototype. The hope is that more rigid-thick origami concepts will be available to allow for more practical and aesthetic design opportunities in the field of kinetic architecture.

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# 1. Introduction

Both architects and origami artists can see the potential relationships between respective disciplines. This is very much so when considering that both heavily involve the building of forms with practical constraints while using well-defined geometry. However, according to Iwamoto, “Critics have argued that the mere physicalization of the fold can in no way approach the complexities embedded in the concept.”[1]

Most forms of architecture are made up of rigid structures using built up layers of materials, while most origami assumes zero-thickness for design purposes. Thus much folding in architecture has mainly occurred with tent-like pliable membranes stretched across collapsible structural frames, or with simple parallel corrugated patterns. But rigid folding can begin to bring these disciplines closer together because it, as stated by Tachi, “can realize a deployment mechanism using stiff panels and hinges, which has advantages for various engineering purposes, especially for designs of kinetic architecture.”[2] On the other hand, the methods to do this have many complications.

Such complications involve analyzing crease patterns (from flat-foldable, to rigid, to rigid thick) and adjusting crease patterns, including some edge and vertex conditions, and simulations. Therefore, this dissertation will be broken into several sections including these issues as well as some historical background, precedents and case studies, and some proposed crease patterns. The aim is to provide a primer of design techniques for various rigid thick situations so the study can be expanded further in the future. There will also be numerous terms native to origami with which the general user may not be familiar. A glossary is included in this document for the most relevant terms.

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[1] Lisa Iwamoto, *Digital fabrications: architectural and material techniques*. (New York: Princeton Architectural Press, 2009) 62. Print.
[2] Tomohiro Tachi, " Rigid-Foldable Thick Origami, " *Origami 5: Fifth International Meeting of Origami Science, Mathematics, and Education* (Boca Raton, FL: CRC Press, 2011) 253. Print

## 2. Literature Review

The research of current literature for this dissertation falls into two areas: kinetic architecture and origami. This can be further analyzed by examining tent-like architecture or rigid panel architecture, and the historical development of origami, starting from art, into math, arriving at thin and thick origami. Precedents of origami in architecture can be seen in section 10, and the math in section 11.2.

**General Origami History**

Origami has developed steadily over the centuries, but most noticeably in the past few decades, as new origami bases and design techniques have advanced greatly. These techniques have not only been used to develop the final shape of more advanced representational models, but recently have been adapted to understand the mathematics of the folding motions of origami during the folding process. It is therefore useful to understand the transition from traditional origami to the contemporary.

Material regarding origami history can be found in the publications of each of the Origami Science, Mathematics, and Education Conferences, the latest publication at the time of this writing is being *Origami 5*. One paper from Origami 5, which contains a wealth of history in particular is David Lister's prolific analyses of origami in both the east and west, and tied in well with Norman Brosterman's book, *Inventing Kindergarten*, which details the exercises of Friedrich Fröbel's child education program, one of them being origami, which explains how origami was introduced into many other cultures.

Another author who wrote on this subject is Peter Engel, an origami artist who also attended architecture school. His book, *Origami from Angelfish to Zen*, is considered one of the classic origami books in the origami community, not only for the quality of his models and hand drawn diagrams, but also for the book's lengthy introduction, which describes ties between origami and many other disciplines, to which a third of the book was devoted.

This review also cites the use of Leland Stowe's article on Akira Yoshizawa in the *Hawaii Beacon*, and how that had an influence on the international folding community. To serve as a bridge between then and now, texts such as Robert J. Lang's

*Origami Design Secrets* and Kazuo Haga's *Origamics* are used to reference the advancement of origami design theory.

### Rigid Origami

The majority of the current literature pertaining to the use of rigid origami came from articles written by Tomohiro Tachi and a few other colleagues such as Jun Mitami, Tom Hull and Robert Lang. These artists have been exploring methods to simulate and predict the rigidity of crease patterns. Much of this literature focuses on the analysis of localized areas of the crease pattern, as analyzing a crease pattern on the whole is very difficult. Thus, only limited methods for determining rigidity have been developed.

### Kinetic Architecture

This is different from simple buildings or shapes that use faceted forms that appear to have been folded. Rather, kinetic architecture is defined as buildings or parts of buildings which move in some way, with the emphasis on continuous folded motion.

Earlier precedents of this type of architecture were not often referred to as origami, but rather, terms such as "mobile", "nomadic", and "deployable" were used to describe the subject, probably since origami hadn't yet been popularized in the west. As such, these areas were heavily explored to find examples, but the majority tended to be mainly forms that slid, rolled, popped-out, rotated or otherwise did not fold. What few examples remained tended to use fabrics and plastics and deployed more like tents, which tended to use negligible thickness. Even then, specific crease patterns were not important since fabric is so flexible.

More contemporary examples acknowledge the influence of origami, and are thus termed as such. These tend to be more sophisticated technologies, using simple folds (which avoid multiple creases meeting at a single vertex), or if higher degree vertices are used, the amount of tiles used are low, thus forming small, localized sets of panels, that do not interact as an entire folding plane.

## 3. A Brief History Leading to Contemporary Origami

Many people may be familiar with origami from childhood. It was first developed in the first or second century A.D. and although popularized in Japan, actually had its first origins in China.[3] One of the first publications of origami is the *Senbazuru Orikata* from 1797.[4] However, not until the 1830's Friedrich Froebel integrate origami crafts as the

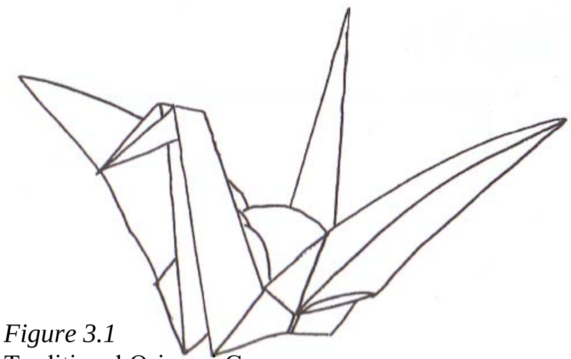

*Figure 3.1*
Traditional Origami Crane

'18th gift' of his kindergarten program. The program, which gained international recognition, still introduces arts and crafts as "gifts" to young children into these varying 'gifts' as arts and crafts.[5] Whereas traditional origami used a limited set of folds and bases (different sets of initial folds in which different origami models can be folded from), the art of origami was later advanced with many new designs and new bases in the 1960s. According to Stowe, "Origami's recent upsurge [as of 1970], both at home and abroad, is chiefly due to the extraordinary inventiveness and dedication of Akira Yoshizawa."[6] In the origami community he is considered the father of modern day origami.

Origami has continued to advance with the discovery of new origami bases, techniques, and sub-genres such as box-pleating (a folding method comprised mainly of a folded

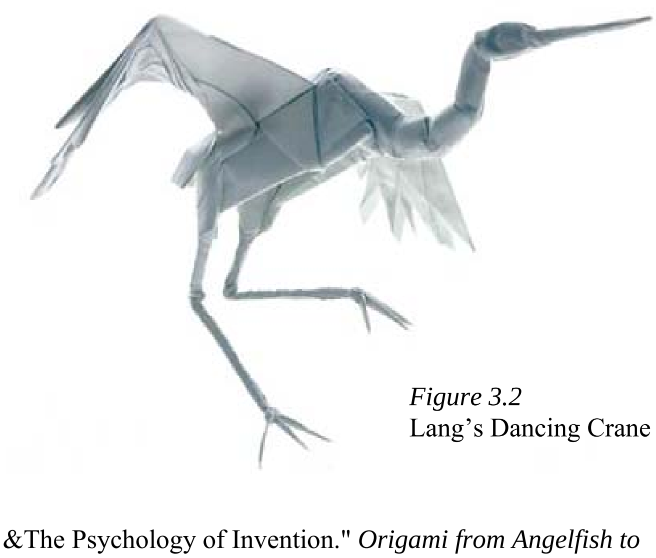

*Figure 3.2*
Lang's Dancing Crane

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[3] Peter Engel, "Folding: A compact History, &The Psychology of Invention." *Origami from Angelfish to Zen*. (New York: Dover Publications, 1989) 23. Print.
[4] David Lister and Koryo Miura, "Some Observations on the History of Paperfolding in Japan and the West." 1994, 157. Print.
[5] Norman Brosterman, *Inventing kindergarten*. New York, 1960: H.N. Abrams, 1997, 78. Print.
[6] Leland Stowe, "The Paper Magic Of Origami." *Hawaii Beacon* July 1970, 36. Print

grid), modular/unit origami (many congruent units that attach together), tessellations, curved folding, wet folding, etc.

The mathematics governing the geometry of folding has also been developed with concepts such as the Maekawa-Kawasaki theorems and the Huzita-Hatori axioms (see section 5, Flat-foldable origami). [7]

These foundations led to more complex origami design theories, such as Robert Lang's Tree-Theory and Circle/River packing techniques.[8] Origami design has been pushed so far to the limits that it begs the question of "what shapes *can't* be made?"[9] However, these new techniques however, are applied specifically to the art and science of paper folding.

The subgenre of flat-foldability (rigid-thin) is typically of most interest to other disciplines, but adaptation of this technique to rigid-thick foldability is of particular interest in this research.

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[7] Kazuo Haga, *Origamics.* (Singapore: World Scientific, 2008) Print.
[8] Robert Lang, *Origami design secrets: mathematical methods for an ancient art.* (Wellesley, Mass.: A.K. Peters, 2003) Print.
[9] Erik Demaine, *Geometric folding algorithms: linkages, origami, polyhedra.* (New York: Cambridge University Press, 2007) 2. Print.

# 4. Rendering Line Types in Typical Origami Crease Patterns

Before exploring different types of origami from flat-foldable to rigid-thin to rigid-thick, a brief explanation should be made about crease patterns (CPs) and their various line types. Although several lines types are used (*Fig 4.1*), the mountain and valley line types are the most important. The modern convention of describing these two orientations of folds are rendered as a dash-dot-dash (or dash-dot-dot-dash in some cases) to indicate mountain folds, and a dashed line to indicate valley folds.[10] Yoshizawa is credited as being the one to develop this system, now used internationally. These two line types are named as such because of how they visually look when folded. A piece of paper with a mountain fold looks like two slopes coming to a 'peak' of a mountain, whereas a 'valley' has the two sides of the paper descending into what looks like a valley.

This understanding is critical for reading the CPs of the different types of folding. The Yoshizawa line types have traditionally been used in the context of step-by-step diagrams (*Fig 4.2 (a)*), which works well against the illustration of the model. If these line types were to be used in an entire CPs, they would become difficult to read. There may be cases where a solid continuous line type like Lang's is clearer than the Yoshizawa's line types (*Fig 4.2(b)*). In these cases the color and line

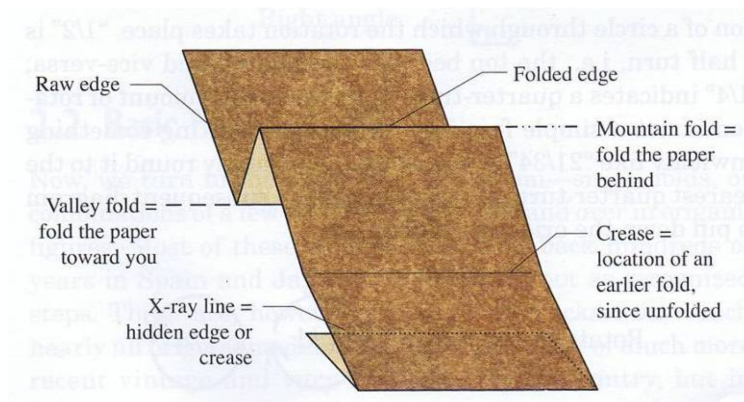


*Figure 4.1*
Lang's illustrations of several line types.

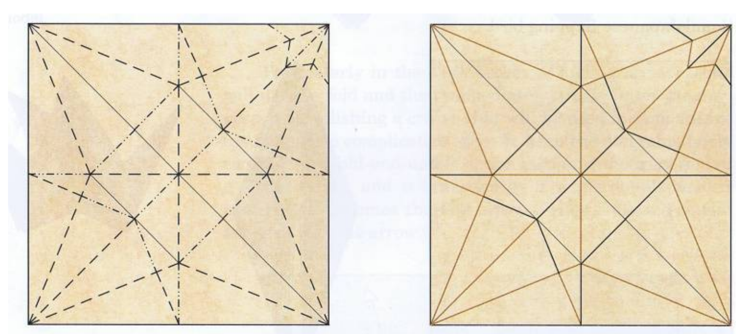

*Figure 4.2*
(a) Yoshizawa's Line Type Crease Pattern
(b) Lang's Line Type Crease Pattern

[10] Lang, 15 & 22

weight is altered. There does not appear to be an established designation for the mountain/valley (M/V) assignments. I will be using **thick red lines for mountains** and *thin blue lines for valley* (although with the rigid-thick patterns, orientation doesn't matter as much as differentiation). The use of color makes CPs very easy to read, and line weights ensure the pattern can still be read even if converted to black and white. Other CPs will have more complicated and various line assignments based on uni- or multi-axis bases, sink folds, detail folds, etc; but these pertain to representational origami, and will not be discussed here.

In later sections relating to rigid-thick origami, various line types will be used to demarcate conditions on the CP. Those will be discussed in Section 8.

## 5. Flat-Foldable Origami

Before rigid or thick origami can be discussed, flat foldable origami needs to be described. This is an origami concept in which all folds on a CP have been folded 180 degrees, the model undergoes no collisions, and rests completely flat.

Kawasaki's Theorem (the sum of every other angle at a vertex must cancel the other set of angles i.e. $\alpha_1 - \alpha_2 + \alpha_3 - \cdots + \alpha_{2n-1} - \alpha_{2n} = 0$) explains that one of the conditions needed is flat-foldability at each individual vertex (*Fig 5.1*). If this is *false* for any vertex, the model **will not fold flat**. If this is *true* for every vertex, then that model **might** fold flat.

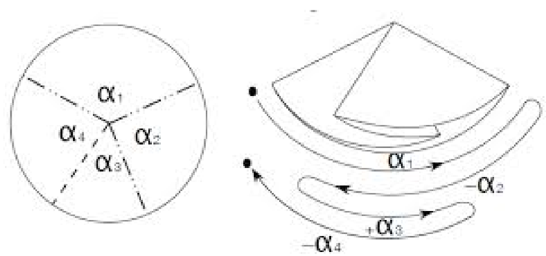


*Figure 5.1*
Illustration of Kawasaki Theorem

His theorem is a great guide for preliminarily deciding how to make a model flat-foldable, but even if a CP is Kawasaki Theorem compliant, that does not mean it will be easy to fold or even possible to fold.[11] Figure 5.2(a) is a CP with a series of folds that are possible, albeit difficult to fold because the paper must be bent oddly in order to tuck into flaps. Figure 5.2(b) is the same pattern, but sections have been elongated. Since these sections tuck into the middle sections, and each middle section is bounded by folds to create a shallow pocket, the elongated tabs will collide into the bottom of the pockets. This is an example of the theorem being satisfied at each vertex, but the larger system is not foldable as a whole. Once it has been determined that a CP is flat-foldable, it must be determined if it is rigidly-flat-foldable, or rather a rigid-thin fold. After a CP is determined rigid-thin, then rigid-thick can be explored.

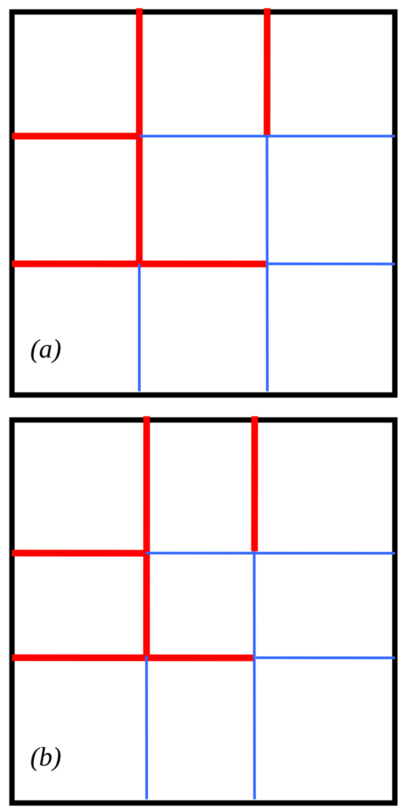


*Figure 5.2 a-b*
Flat foldable vertices.
Each vertex is flat foldable, however, (a) is foldable, whilte (b) is not, due to collisions.

[11] Demaine, 4

## 6. Rigid-Thin Origami

The concept of rigid origami is a topic that many may have stumbled upon without realizing. For example, a typical cardboard box with flaps can be thought of as rigid folding. If one folds an opposing pair of flaps down, then the second pair down, none of the flaps will become distorted and they all fold nicely. However, it is not always this simple. If the first, then the second, and then the third flap are folded, in a clockwise or counterclockwise fashion, the last flap will need to fold under the first, and this will require distortion. Both of these situations use the same crease set, but one is not rigid because of the behavior of the entire system. The CPs (*Fig 5.2*) in the previous section are examples of CPs that are foldable, but not rigidly foldable.

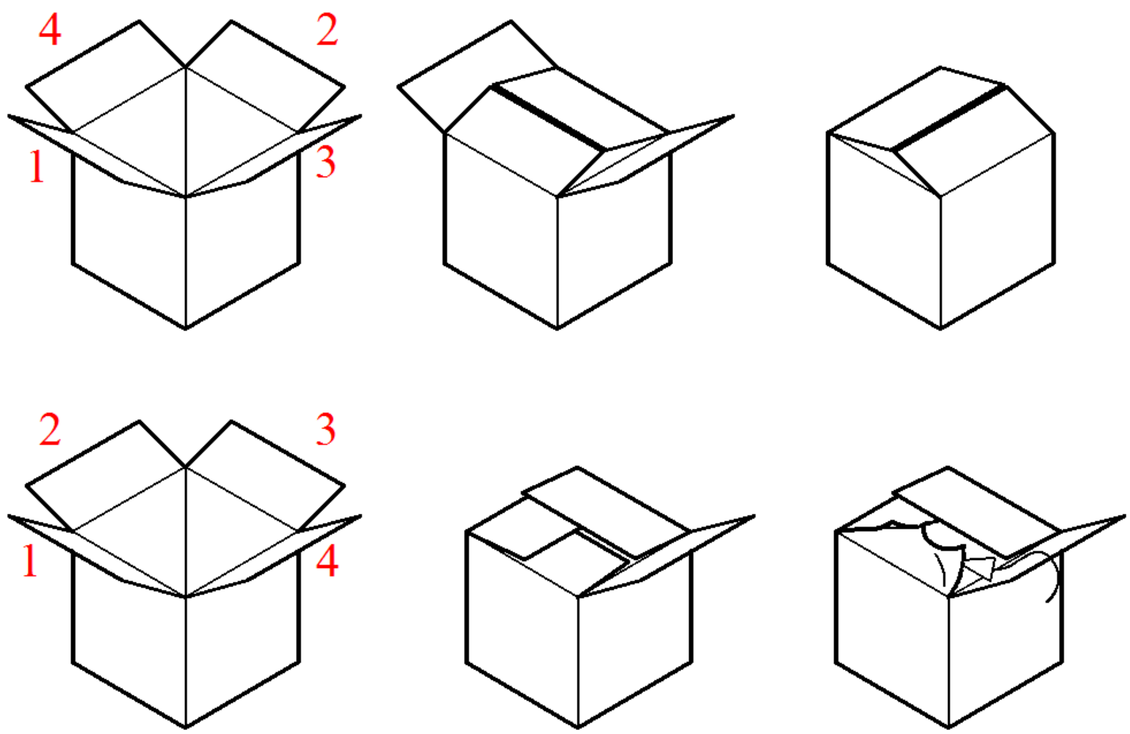


*Figure 6.1*
An example of Rigidness, using the box analogy.

Before proceeding, the CP design rigidity should be determined. Origami artists and researchers such as Watanabe, Kawaguchi, and Tachi state: "*Rigid Origami* is defined as origami in which each surface surrounded with crease lines neither stretches nor bends"[12] or "a piecewise linear origami that is continuously transformable along its folds without deformation by bending or folding of any facet."[13] A common analogy is folding a series of unbendable metal plates only at the hinges, represented by a CP.[14] This can be seen in the example of the box (*Fig 6.1*).

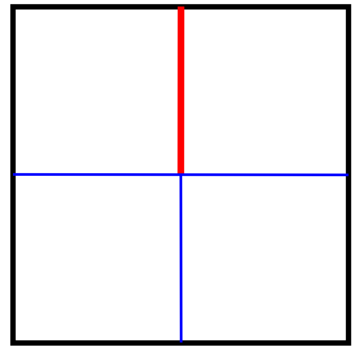

*Figure 6.2*
A fold with no deformation or collision. However all four folds cannot happen at the same time. Since the horizontal valley crease must fold completely first.

Another characteristic of rigid origami requires a unified movement for all creases. Typical step-by-step origami employs a fold or group of folds in sequence, whereas rigid origami moves all at once, (however, it should be noted all movements are not linear; see Section 10.02 regarding crease movement.) A rigid fold should not require a CP in which one fold or set of folds need to be folded before the next fold or set can be folded (*Fig 6.2*). This is especially so since these patterns will be using thick panels, in which a second fold after a complete fold would 'split' on the mountain side of the second fold.

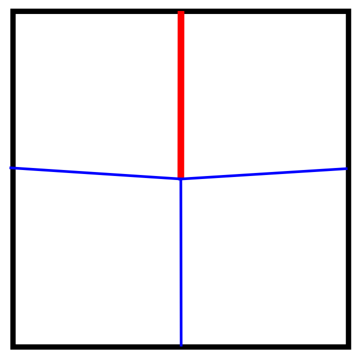

*Figure 6.3*
This model, with folds near 90° can fold. However the rate of change of motion is very fast at the start of the fold.

There are now several constraints incorporated into rigid-thin origami, which will be carried over into rigid-thick origami. If the following four questions are answered in the affirmative, then the pattern might make a good candidate for a rigid-thick model:

**Is it Kawasaki compliant?** **Will each polygon remain rigid?**

**Will any polygons collide?** **Does it fold in one motion?**

If all four questions are in the affirmative, then the model is rigid-thin compliant. Some of these questions are easy to solve, some are harder. Kawasaki's theorem requires

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[12] Naohiko Watanabe & Ken-ichi Kawaguchi "The Method for Judging Rigid Foldability." In *Origami 4* (Wellesley, MA: K Peters, 2009) 165
[13] Tachi, 253
[14] Watanabe & Kawaguchi, 165

the measuring of angles, and with a little practice, these can be recognized easily without computation. Determining uniform motion is easy, too. A non-uniform motion can be spotted easily if any vertex has a continuous mountain or valley assignment through the vertex, such as the horizontal valley fold in *Fig 6.2*. Note that a model can be uniform, but will have rapid rates of change in the motion. If two similar assigned creases are near 90 degrees (*Fig 6.3*), there will be rapid change at the start of the fold.

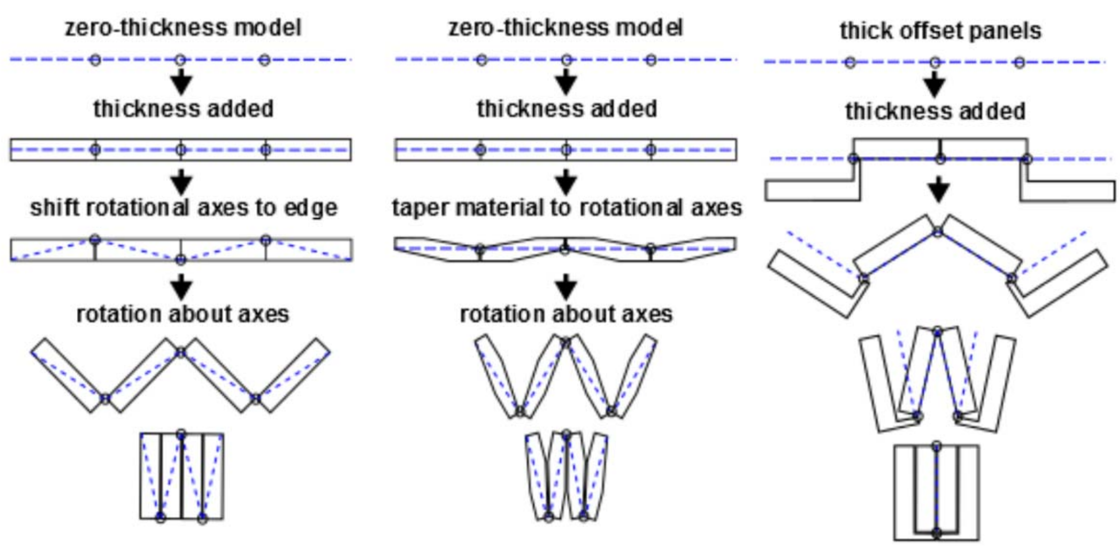
Global rigidity and collisions are much more difficult to efficiently determine from the CP alone. There is work in this field, but the application is rather complex. In fact, thus far determining flat-foldability in these parameters has been shown to be an NP-hard problem, that is, no efficient algorithm to determine flat-foldability can be found.[15]

To go into the specifics of this research would further complicate matters and would undermine the goal of this research to simplify the process; nonetheless, it should be noted that these problems exist. Luckily, the examples given of required deformation and collisions are fairly contrived, and will most likely, should not be encountered. If encountered, they can be observed empirically fairly quickly. Examples of when deformation and collisions may be an issue are discussed in section 9.

[15] Demaine, 214

## 7. Rigid-Thick Origami

Many origami artists have faced folding origami models with paper too thick and discovered how the paper warps, buckles, stretches, and eventually tears. Indeed, most origami models due to complexity cannot accommodate a rigid-thick framework. Artists only circumvent this problem by using very thin and/or very strong fibered papers. Many non-folders have probably discovered the aspects of rigid folding without realizing it. One such example is the exploration of folding a piece of paper in half over and over. The paper quickly starts to build in thickness, and becomes unfoldable because of the thickness. This relationship, in fact, has been solved and an equation dependent on the paper width has been developed.[16] Non-folders probably found that the thickness of the paper plays a large role in the limit of folding. Most origami patterns rely on papers that have fibers that will bend and stretch slightly during the folding process.




(a) (b) (c)

*Figure 7.1*
Three methods to shift the fold axis in a given thickness

For the most part this thickness is negligible and "origami is commonly regarded as an ideal zero-thickness surface."[17] Rigid-thick folding applies to many of the similar principles of the infinitely-thin counterpart, except it assumes a non-negligible non-zero thick foldable plane. Hoberman, Trautz, and Künstler were some of the first to explore

[16] Britney Gallivan. "Folding Paper in Half 12 Times" 2002 http://pomonahistorical.org/12times.htm (accessed Nov 9, 2011) Web.
[17] Tachi, 253

rigid thick origami. [18] They proposed symmetric degree-4 vertices using shifted axis (*Fig 7.1*(a)). Tachi Tomohiro, in his paper, describes his process as “a novel geometric method for implementing a general rigid-foldable origami.”[19] His methods include modifying an idealized plane with tapered details (*Fig 7.1* (b)) or incorporating sliding hinges. His intention was to create a more generalized theory that can be applied to more CPs. While this uses an idealized plane, there must also be thickness, and the model can only fold around 95% of the way due to the thickness of the panels meeting. The methods developed in this research will use less generalized techniques and more case-specific details, which used in combination will allow for a greater variety of design solutions, and thus a greater variety of designs, allowing for more novel applications or greater aesthetic values.

Another novel idea by Bryce Edmondson offsets the thickness further (*Fig 7.1c & 7.2*). This system, however, sometimes needs to shift the axis beyond the limits of the surfaces of either thick panel, which then causes collisions in other panels. [20] Thus, this model of shifted axis will not be pursued.

This research includes more specific techniques of how to work with shifted axis (*Fig 7.1a*), as covered in more detail in the design examples in Section 9, as these details tend to vary with each design. A shifted axis is used so the patterns can be developed in a flat state, and the squared edges can be assumed to be square. This research also goes beyond the theoretical 3D modeling and studies details for fabricating panels and attaching the folding system to a structure (*Sec 12.3*).

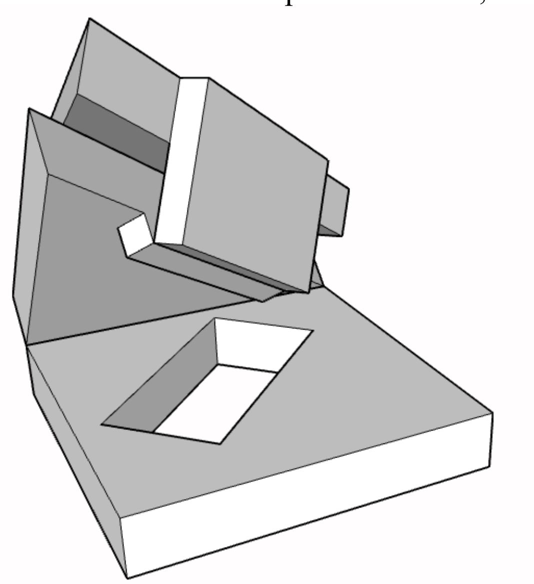


*Figure 7.2*
An example of Edmondson’s offset hinge and the hole it needs to avoid collisions.

[18] Tachi, 253
[19] Tachi, 254
[20] Edmondson 2015, 14

This research also focused mainly on 4 degree folds (a vertex in which 4 creases meet at a single point) that is symmetrical; these are generally referred to as reverse folds (*Sec 9.1*). Additionally, 6 degree folds are explored, as well as even 5 degree and non-Kawasaki compliant CPs (these cannot lie flat). The symmetric 4 degree vertices conform with a Bennett linkage condition, which is a loop of four rods of equal length that rotate at specific angles in relation to each other, such that the rods continue to be connected.[21] Without belaboring the math behind it (which is outside the aims of this dissertation), in a rigid thick asymmetric 4 degree vertex the angles needed for a Bennett linkage are not maintained during the folding process. Thus, in these folds the thickness of the model starts translating geometry away from edges which should remain in unison (*Fig 7.3*). This drifting causes the panels to no longer be one continuous model (*Fig 7.4-5*) and the shifted axis method no longer works, which is most likely why Tachi used a tapered method since the CPs he was working with are asymmetric 4 degree vertices. For this reason that twist folds also rarely work (see *Sec 9.3*).

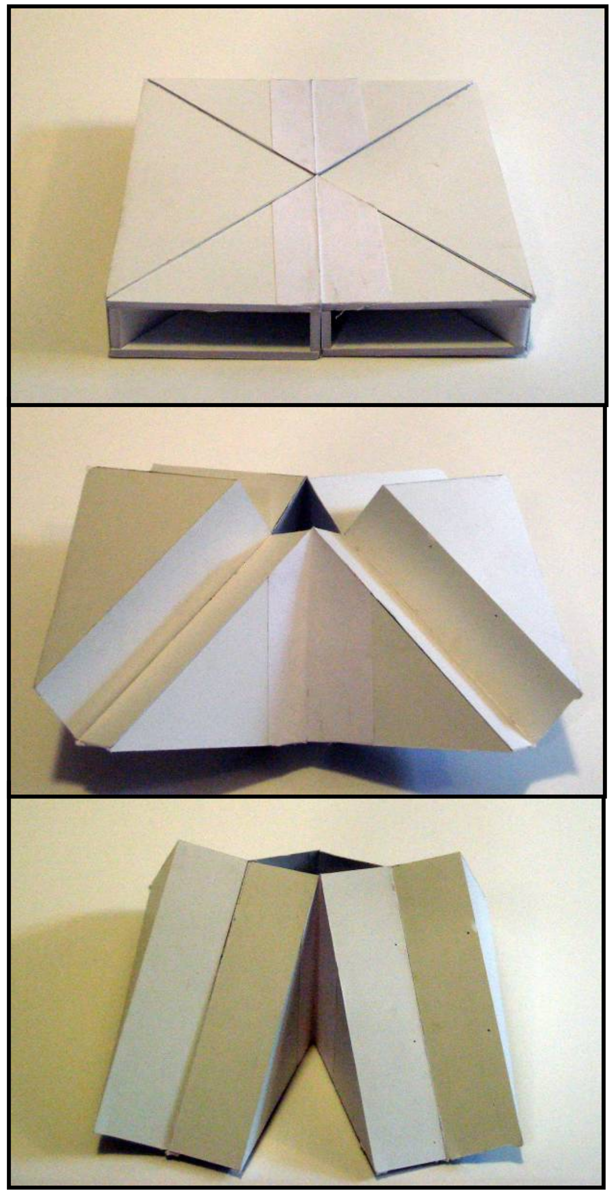


*Figure 7.3*
A Rigid Thick Waterbomb Base folding, with a Bennett linking in the center

[21] Zhong You and, "*Motion Structures*" (London: Spon Press 2011)

However, these asymmetric 4 degree vertex models were explored in the context of rigid thick folding, but with little success. It was hoped that CPs that used such vertices could be employed with other design strategies, such as carving material, or providing cuts, to create a type of pattern that would be a useful design in the kinetic architecture environment. But this linkage generally causes too many problems. With some different strategies perhaps, there may be a novel means to create a rigid thick asymmetric 4 degree fold in the future.

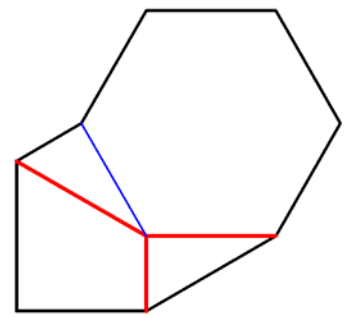


*Figure 7.4*
CP of a single vertex of a Hex Twist Fold

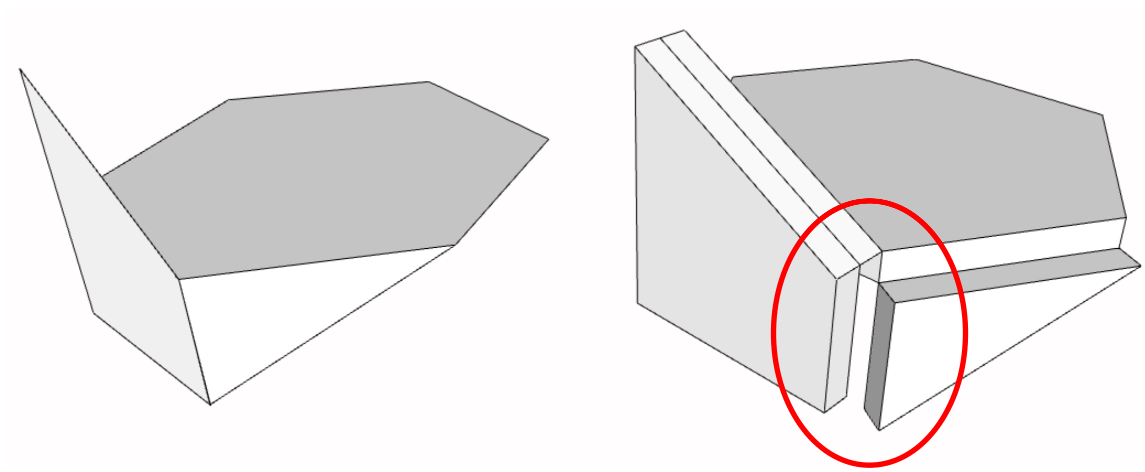


*Figure 7.5*
An example of the Hex Twist Fold in a thin and thick condition. Note the gap.

## 8. Rendering Rigid-Thick Crease Patterns

As discussed with the rigid-thin origami, a CP normally would consist of only lines representing valley and mountain folds (and sometimes a cut line for the more abstract works). These are indicated as either dashed / thin / blue lines and dash-dot-dot / thick / red lines, respectively. It should be noted that artists are increasingly using additional line types and colors to communicate other design structures.[22]

In this rigid-thick context, additional line types are needed to communicate cuts and mid-width/added creases. Thus, there will be seven total line types: mountain, valley, mid-mountain, mid-valley, carve (from top), carve (from bottom), and cut. This is discussed more in depth in the CP studies in Section 9.

Note that (when looking down at the material) there will never be a mountain fold on the top or a valley fold on the bottom (otherwise there would be collisions of material). For models that have carved material, if there is a middle crease, there is probably a carve line nearby, and vice versa.

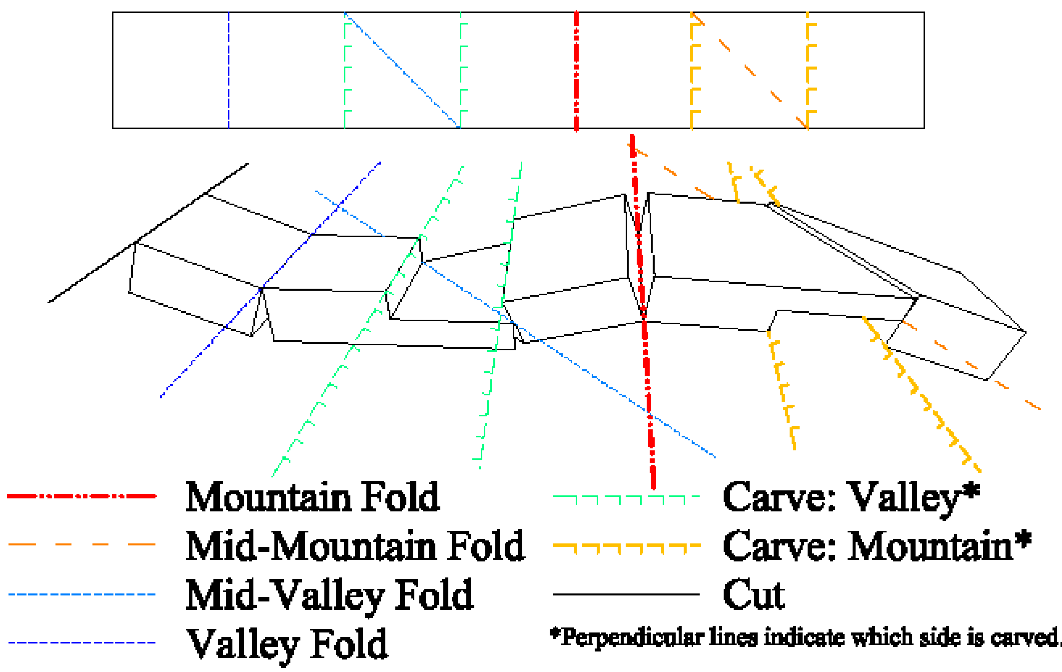



*Figure 8.1*
Proposed Rigid-Tick Line Types

[22] Lang, "Crease Patterns as Art", http://www.langorigami.com/art/creasepatterns/creasepatterns_art.php

## 9. Design Families

When folding and experimenting with these rigid-thick structures, two primary design types arose; reverse-folds and tile tessellations. There are some comparisons to make with these two design types. Based on observations thus far, they both primarily use shifted axes (*Fig 7.1a*), most will need to alter the rigid-thin CP to accommodate the thickness, and both will need through-cuts to allow parts to separate, or allow a stack of layers to fold out of the way. Cutting holes was avoided in these patterns because this tends to allow more parts to operate more independently rather than a single unified system and because it would no longer be a continuous CP. Reverse-folds tend to have more flexibility regarding arbitrary pleats, whereas the tile tessellations tend to be orthogonal. There are also some failed attempts included in this section. These are initial thoughts regarding how to accommodate thickness in certain panels that ultimately did not work.

The table describes some of the attributes that these patterns possess. Some need more explanation, while others, such as in the flawed design types, don't apply at all since these don't really exist. It should also be noted that the names denoted here are of my own creation. Since they are not much more than geometric patterns in origami, there are no formally recognized names for many of these.

| | # | Name | Axis | Carve | Thin CP Alt | Cuts? | Holes? |
|---|---|---|---|---|---|---|---|
| Reverse Fold | 1 | Simple RF | shifted | Yes | No | No | No |
| | 2 | Arbitrary RF | shifted | Yes | No | No | No |
| | 3 | Arbitrary RF w Controls | shifted | Yes | No | No | No |
| | 4 | Split Hinge | shifted | No | Yes | Yes | No* |
| | 5 | Flat Hinge | coplanar | Yes | Yes | No | No* |
| | 6 | Double Tier RF | shifted x2 | Yes x2 | No | Yes | No |
| | 7 | Triangle Array | shifted x2 | Yes x2 | No | No | No |
| Tile | 1 | Rotated Pop-Up Tabs at 90 | shifted | No | Yes | Yes | No |
| | 2 | Rotated Pop-Up Tabs at 60 | shifted | No | Yes | Yes | No |
| | 3 | Alternating Pop-Up Tabs | shifted | No | Yes | Yes | No |
| | 4 | Scale Tessellation | shifted | No* | No | Yes | No |
| | 5 | Pop-Up Square | shifted | No | No | Yes | No |
| Flawed | 1 | Wedge-Miura-Map | shifted | No | Yes | Yes | n/a |
| | 2 | Square-Twist Fold | shifted | n/a | n/a | No | n/a |
| | 3 | Square Waterbomb | shifted | No | n/a | Yes | n/a |
| | 4 | Stacked Waterbomb Pleat | shifted | No | n/a | Yes | n/a |
| | 5 | Tri-Twist Fold | shifted | No | n/a | n/a | n/a |
| | 6 | Hex Twist Fold | shifted | No | n/a | n/a | n/a |

*Table 9.1*
Attributes of rigid thick types

## 9.1 Reverse Fold Family

### 9.1.1 Simple Reverse Fold

This type employs a series of pleats in a M-V-M-V arrangement. A pleat is just a series of single fold conditions, which has been well documented and can be seen in the precedents and case studies of many kinetic architecture works (*See Xile and Cardborigami, Sec 10.19-20*). In a simple reverse fold array, all the pleats start parallel, then the reverse fold is introduced. This crease has three properties: (1) It maintains the same M/V assignment all the way from one side of the array to the other. (2) The crease path itself can be drawn by either (a) as a line traveling across the unfolded CP reflecting over each pleat line it meets (*Fig 9.2.1.1*), or (b), as a single line transcribed onto each pleat when struck across the folded pleat (*Fig 9.2.1.2*). (3) Once incorporated, the RF reverses the M/V assignments of all the pleats on the other side (hence the name). This is an easy fold in origami, but can be harder to simulate and design on a computer or produce in a fabrication process.

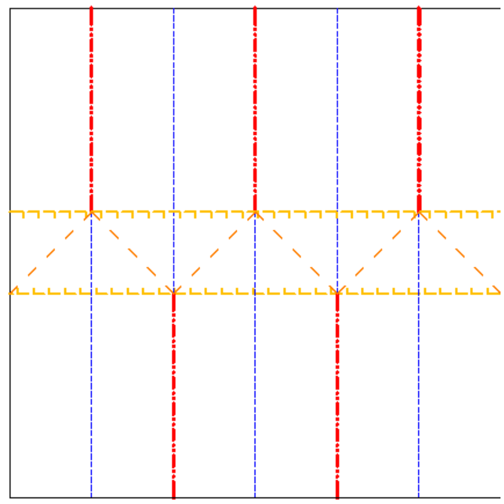


*Figure 9.1.1.1*
An example of a simple RF, with a 45° RF, producing a 90° bend in the final model

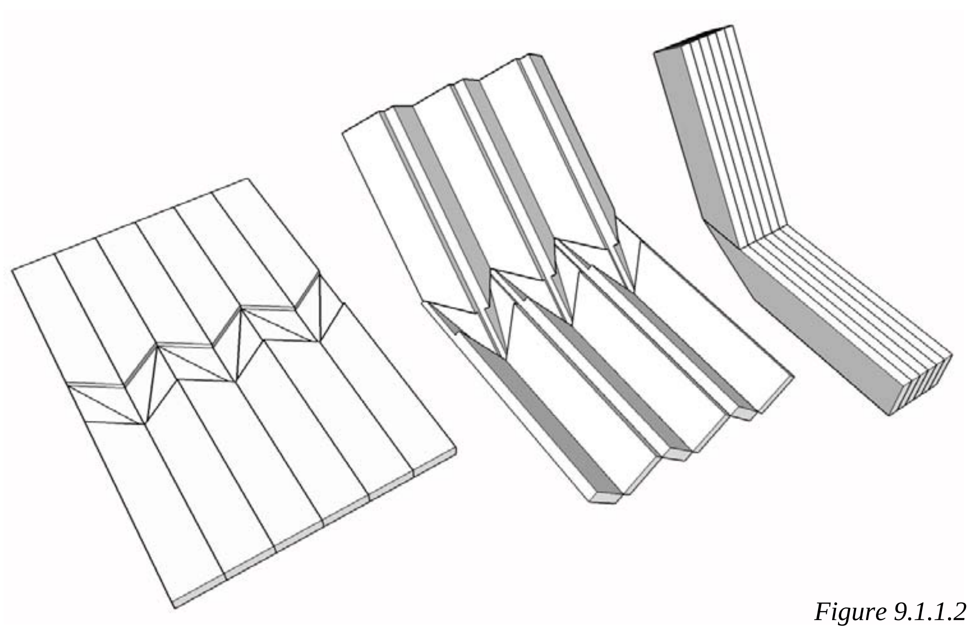


*Figure 9.1.1.2*
An array set at 30°

In section view, these pleats rotate 90 degrees back and forth while translated such that the entire sum of panels can be stacked to one end, much like a series of multiple bi-fold doors. This system has the advantage in that the final width of the system is just the product of the number of panels and the thickness of each panel.

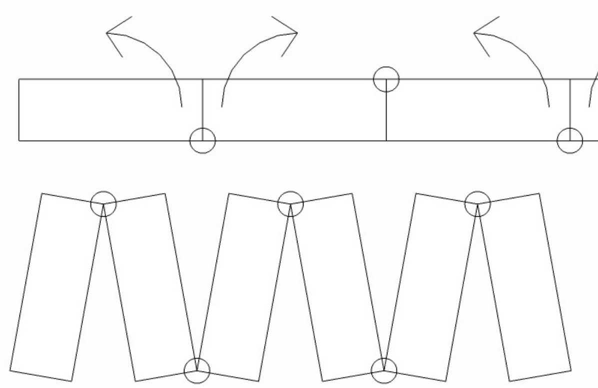


*Figure 9.1.1.3*
Pleat rotation in section

Additionally, material will need to be carved where there are intersections. *Figure 9.1.1.1* shows the CP of a RF from which the material is carved. *Figures 9.1.1.2 & 4* show models of patterns in which areas have been carved out. The second model is based on equilateral triangles, which requires a significant amount of material to be carved out.

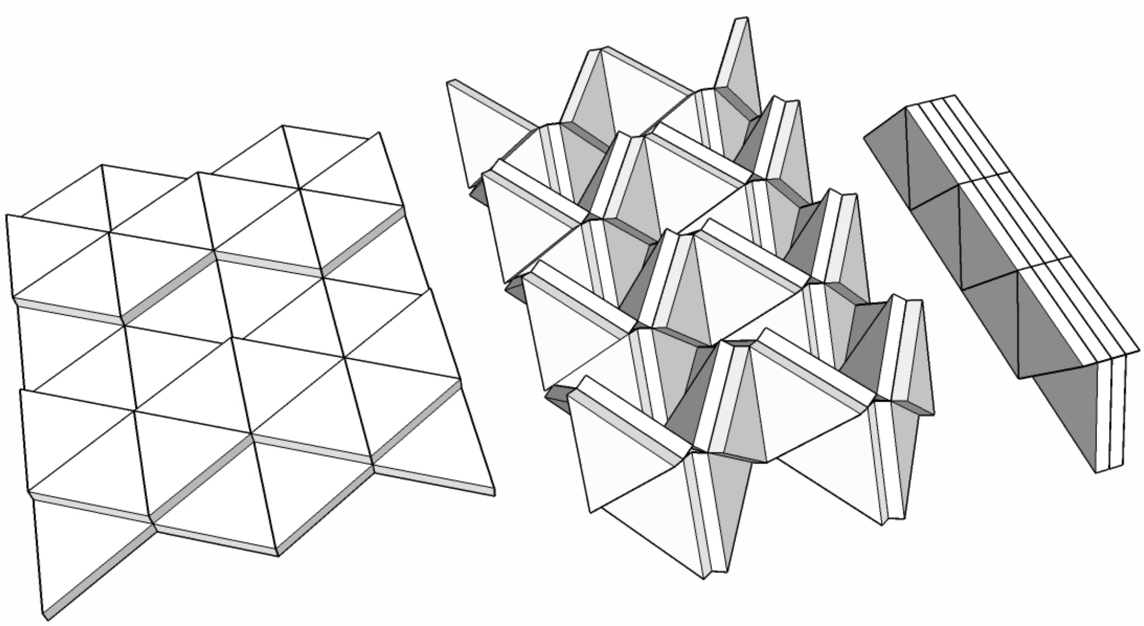


*Figure 9.1.1.4*
A more complicated array set at 60°

### 9.1.2 Arbitrary

The simple reverse fold can be modified quite a bit if the vertical creases are set at arbitrary angles. Just as in the simple reverse fold, creases travel from one edge of the pattern to the other, following Kawasaki's theorem (and in this case, a much simpler law of reflection (*Fig 9.1.2.1)*, where θa = θa'). Interesting variations can be created depending on the combination of reverse folds used.

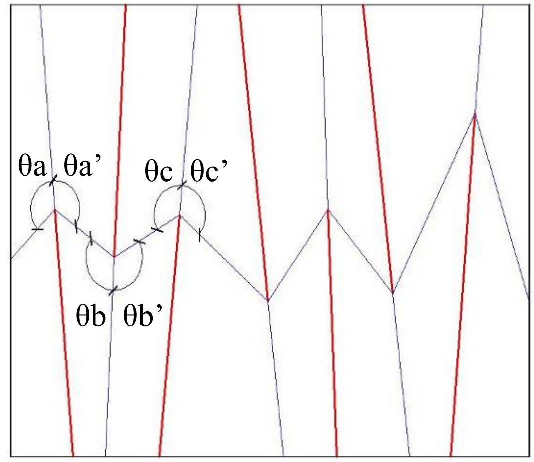



*Figure 9.1.2.1*
Creating a reverse fold via angles

These also have the advantage in that the nature of the pleats and RF can allow more arbitrary folds. The vertical pleats can vary quite a bit, as long as the vertical pleats do not cross and the reverse fold can traverse from one end of the array to the other. For examples and explanation on the reverse fold see the section on rigid simulation (*Sec 11*), a typical example is shown *Fig 9.1.2.2*.

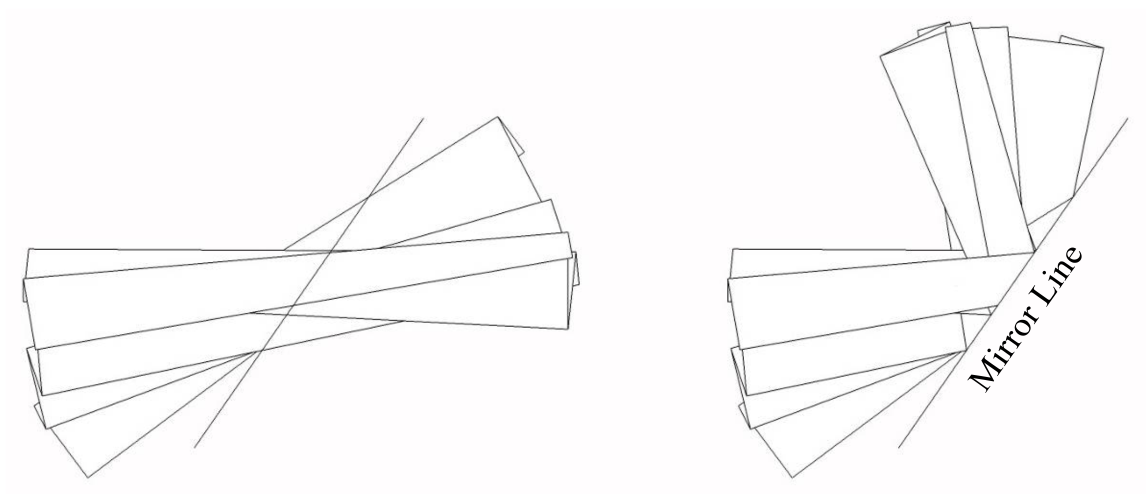



*Figure 9.1.2.2*
Creating a reverse fold via mirror line

With most reverse folds, a carved space will need to be provided since layers will intersect with each other (see section 8 for more detail on carving line types). When there are multiple RFs, that CP can become quite complicated.

The two patterns below (*Fig 9.1.2.4*) are the two layers of material and their respective cuts and hinges of an arbitrary RF pattern, which has three RF joints. Maintaining multiple layers can become cumbersome, so a single crease pattern was made to convey all the necessary information (*Fig 9.1.2.5*).

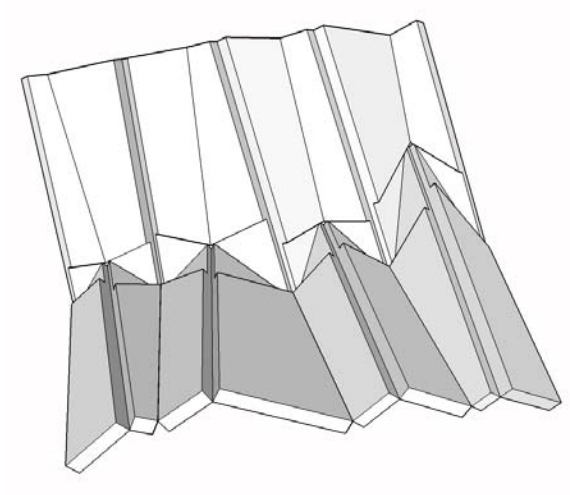


*Figure 9.1.2.3*
An Arbitrary RF with a single RF joint

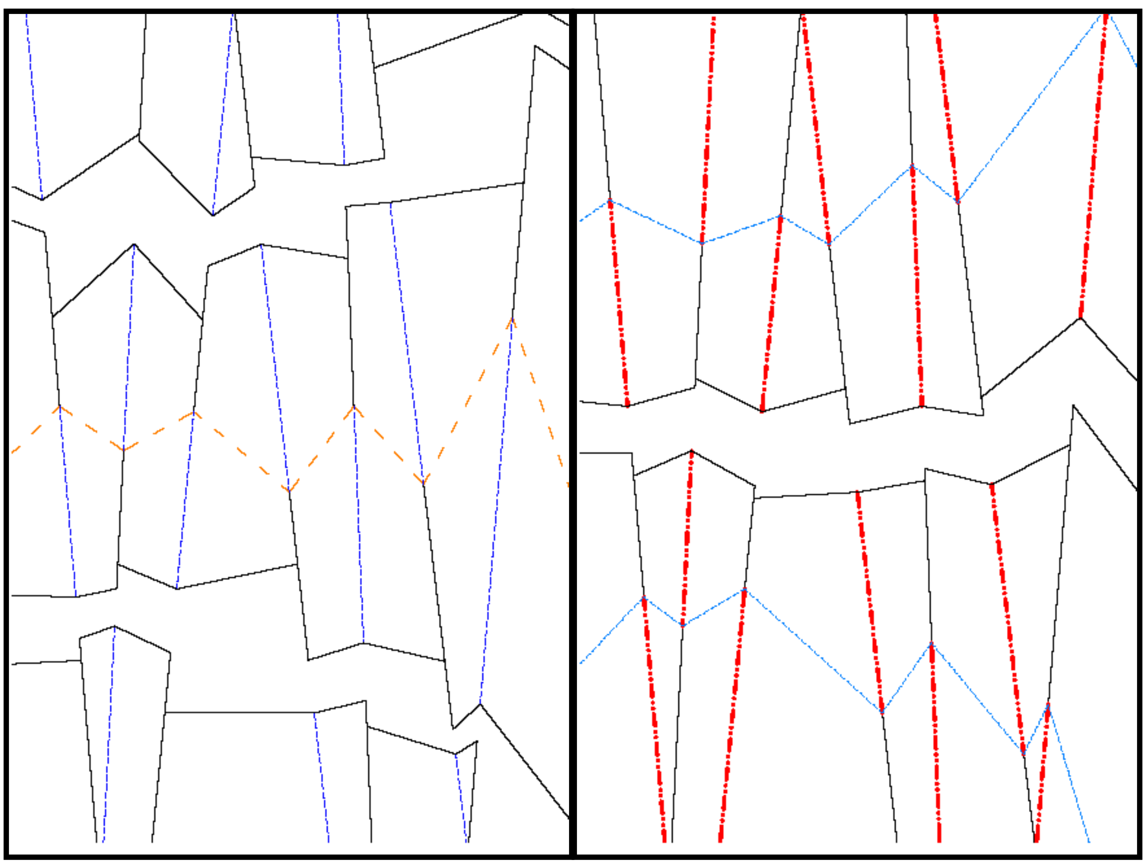


*Figure 9.1.2.4*
Two CPs, each for a different layer

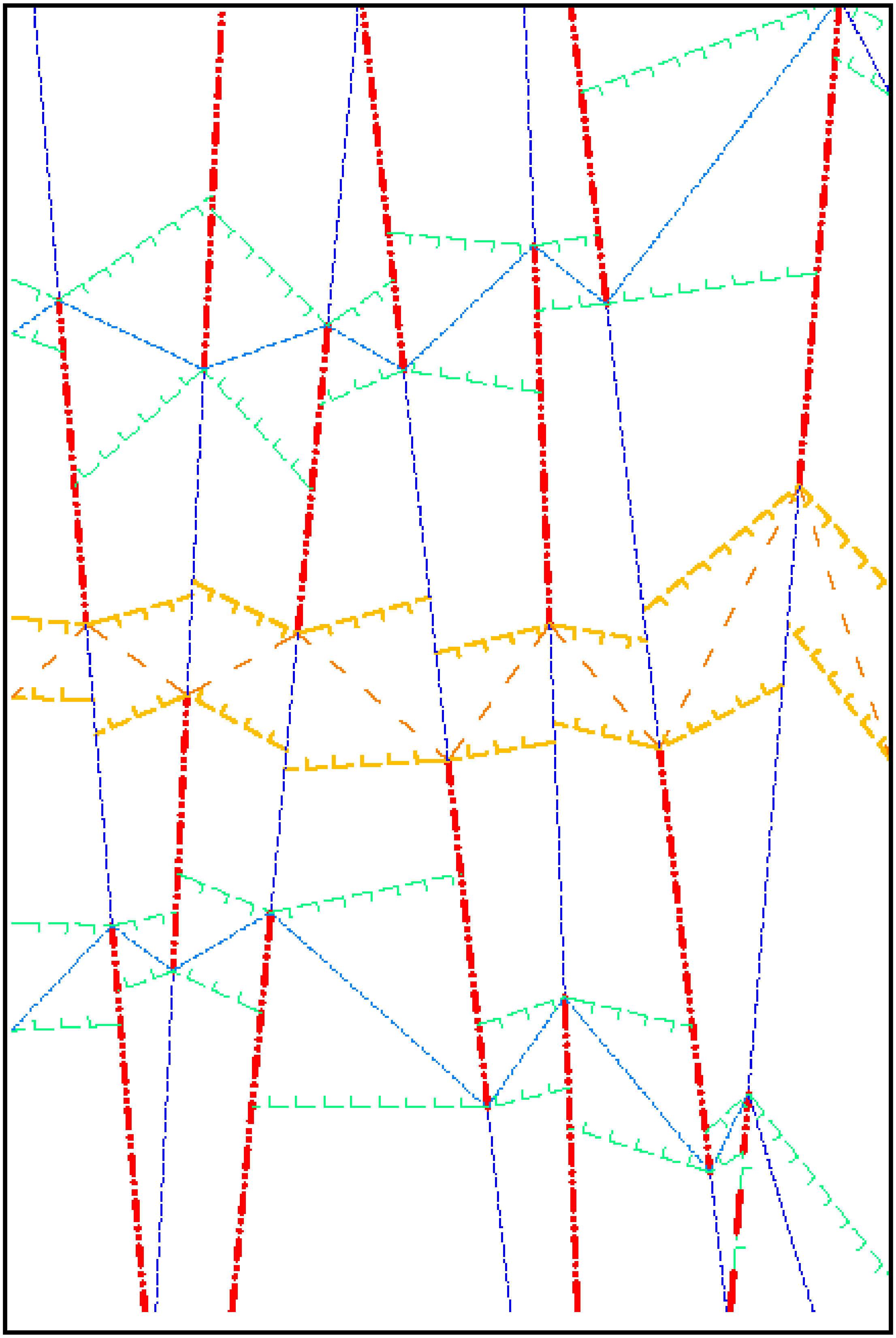


*Figure 9.1.2.5*
A Single CP for 3 Arbitrary Reverse Folds

### 9.1.3 Arbitrary with Controls

The previous design is a great way to design a pattern that is not the typical repeating pattern. But this design causes a lot of irregularities at the top and bottom edges that will create difficulties when mounting it onto a linear track system. This design makes a trade-off, where every 4$^{th}$ crease is a vertical crease set at a regular interval (*Fig 9.1.3.1*). These creases are the control creases, which will all line up, regardless of what the other three creases are doing, and which would at least allow for a consistent bearing point, if only for 25% of the panels. Depending on the size, span, and weight of the panels, this may be fine.

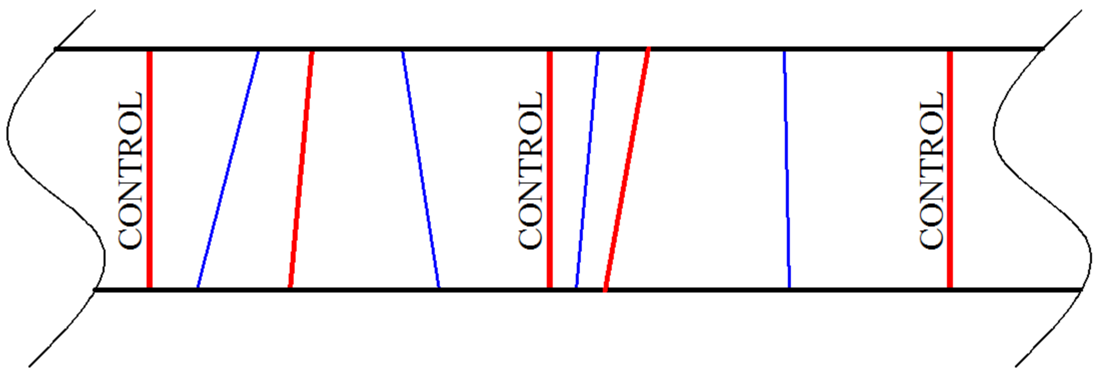



*Figure 9.1.3.1*
Arbitrary Creases with control joints

Folding instructions are as follows (*Fig 9.1.3.2*), and are focused on a single panel between two control creases: (1) Valley fold an arbitrary fold where desired. (2) Valley fold the center point of the right control edge to the center point of the folded left control edge. (3) Mountain fold through the counterpoints, bisecting the two control edges, such that the two control edges will lie on each other.

These steps can be repeated for each section with variations as desired. Once the pleats are determined, the RF and carves can be computed as in the two previous RF examples.

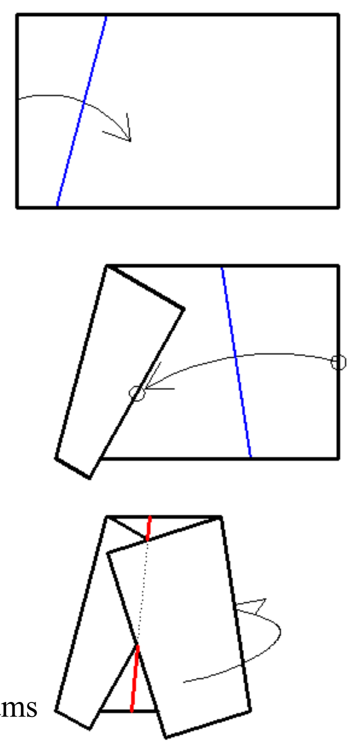


*Figure 9.1.3.2*
Folding diagrams

### 9.1.4 Split Hinge

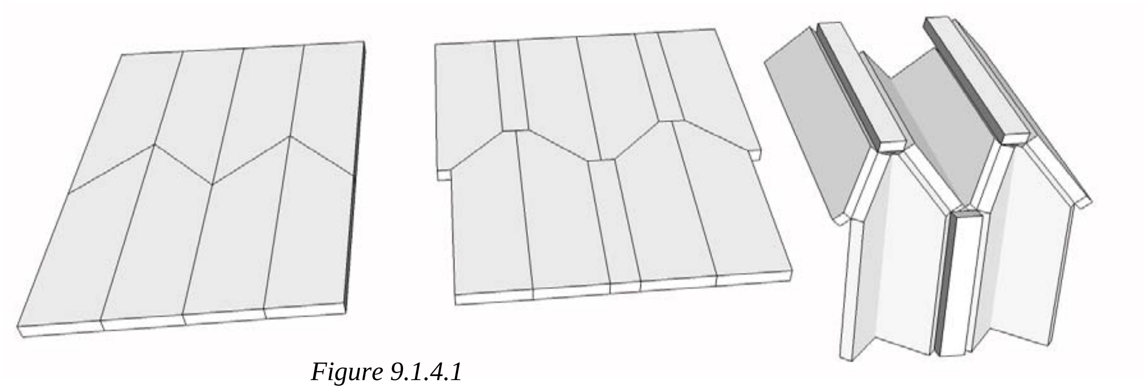


*Figure 9.1.4.1*
The first attempt at a RF-Pleat – CP altered model

In this model (*Fig 9.1.4.1*), the goal was to find out how to modify the original CP so the extra thickness will have a place to go, and carving would not be needed. This would be done by creating two pleats at the mountain folds, at a distance apart of twice the thickness. This would solve some problems, but also create some other obstacles to overcome.

There are some instances where the diagonal across the material is greater than a gap in the material (*Fig 9.1.4.2*). If this were to occur, the excess material would push the planes apart, possibly causing damage to the kinetic work. This is a case of flat foldability, but not rigidly foldable.

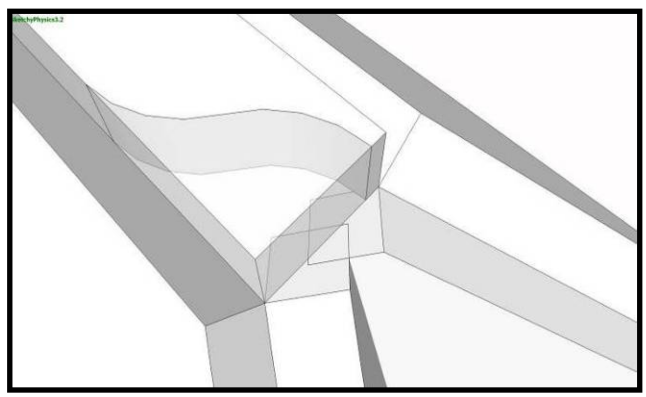


*Figure 9.1.4.2*
Throughout folding, the thickness of the panels collide.

To solve this, the mountain pleat is split (*Fig 9.1.4.3*), to account for the extra expansion. However, this detail will add to the hypotenuse expansion problem.

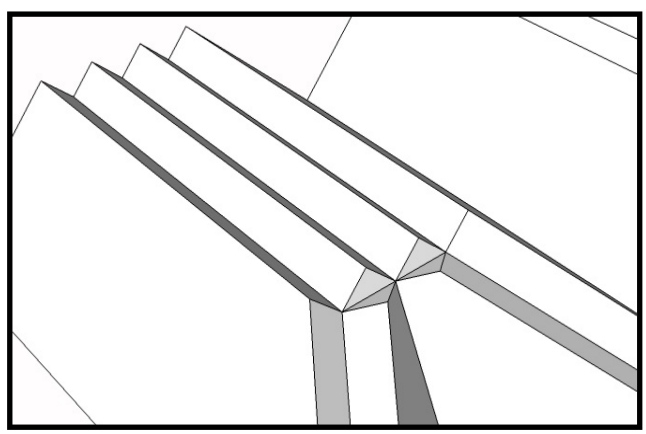


*Figure 9.1.4.3*
An extra hinge would allow for expansion.

This CP still needs a very minor adjustment. The corners localized at the vertex will still collide. This is much better then the entire length of the crease colliding, as it is easier to solve. The solution is to remove some of the material at the corners (*Fig 9.1.4.4)*. This can be done by creating a hole; in conjunction with a split pleat. This is the same solution used in the Grand Central Table case study.[23] (*Sec. 10.16*).

In fact, if really needed, the top surface can remain continuous while some of the corner on the bottom side can be removed. On the front, the surface would appear flat, but on the back there would be an occasional facet cut into it (*Fig 9.1.4.5*).

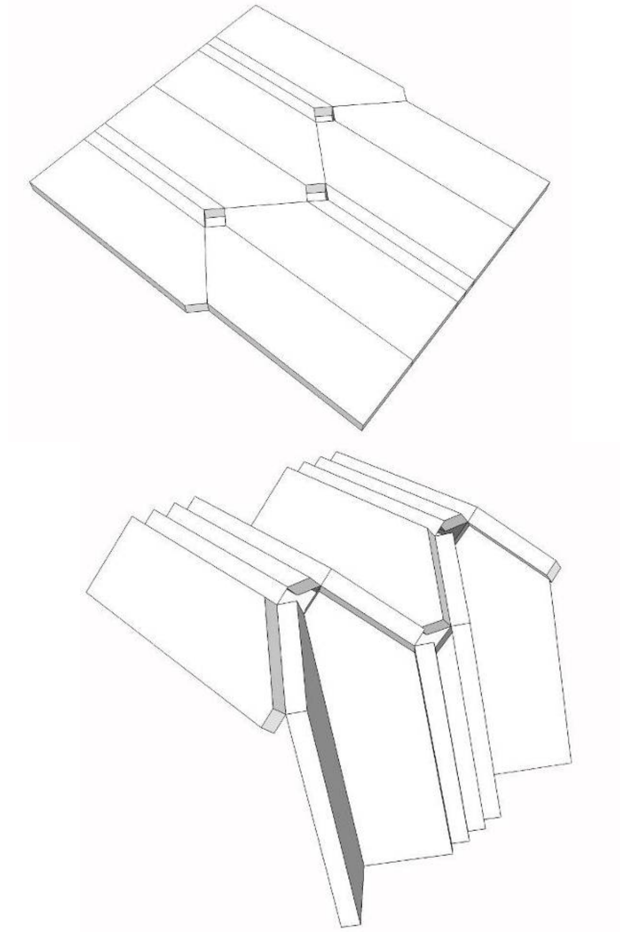


*Figure 9.1.4.4*
Split Hinge with holes

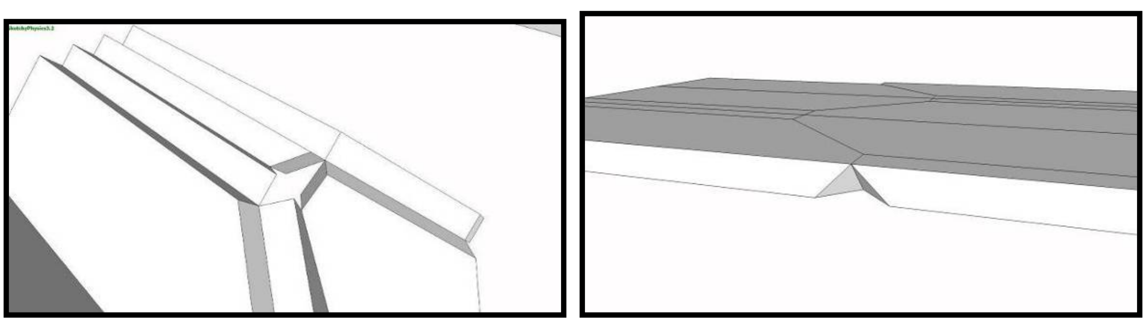


*Figure 9.1.4.5*
Split Hinge with vertex carve

[23] Cliff Kuang, Grand Central Table Folds Like a Subway Map, 2010, http://www.fastcompany.com/1613745/table-unfolds-like-a-subway-map

## 9.1.5 Flat Hinge

As was seen in the Split Hinge (*Fig 9.1.4.2)* the hypotenuse of the thickness of the split hinges causes expansion. A method to circumvent this is to remove the split pleat, and assemble joints with widths to match this hypotenuse length, or rather w = $2\sqrt{t}$.

One way to do this could be to round the edges to a radius (*Fig 9.1.5.1*) to create a valid linkage. This radius needs to be equal to the thickness of the material. This would be a rather interesting mechanical detail, since the material needs to roll, or use a gear, rather than use a conventional pin.

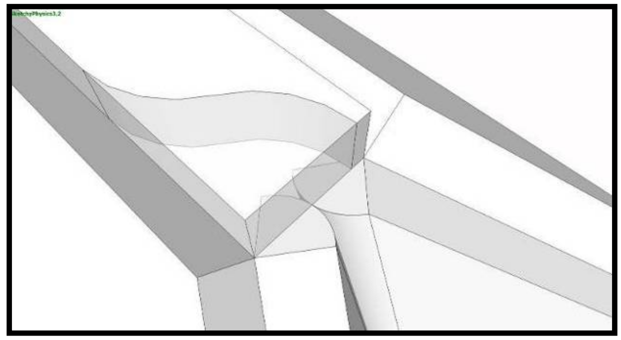


*Figure 9.1.5.1*
Edge collision modified to radius

A different approach would be to omit a portion of the panel and use a double hinge instead (*Fig 9.1.5.2-4)*. This acts more like a carve, in which the entire thickness is carved out, rather than a hole in the CP. A hole is usually created to allow rigidity, whereas this condition is to allow for thickness. Just as with the radius solution these middle sections need to be w = $2\sqrt{t}$ .

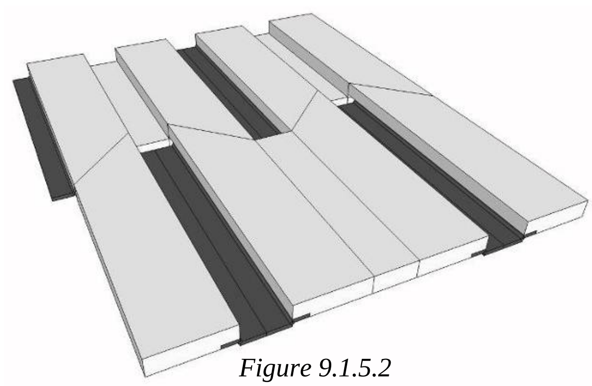


*Figure 9.1.5.2*
The same CP, using a hinge instead

As opposed to splitting the middle portions of the pleats, they are simply ignored. Perhaps this can be considered a hole, as the thick material needs to be removed, but the connection of panels are still continuous through the CP.

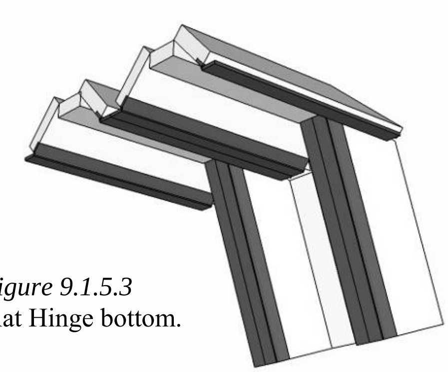


*Figure 9.1.5.3*
Flat Hinge bottom.

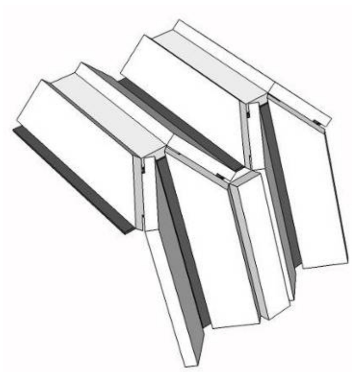


*Figure 9.1.5.4*
Flat Hinge top

### 9.1.6 Double Tier

This is similar to the simple RF and arbitrary RF, but this uses additional layers to allow more of the layers to RF out in more ways. It was an exploration to study what other possibilities remain for unique designs. The example pictured (*Fig 9.1.6.1-2*) is a simple RF, but an arbitrary RF can be used just the same. Additional layers can be added as well. Note that taking two complimentary thick designs and pasting them together is not simple. Each design will need extra space carved, or rather relocated to other sections of the layers. Also, the location of hinges and cuts will need to be altered. This pattern is most likely significantly weaker than other designs due to the amount of cutting needed, and the reduced area for hinges to be attached.

A means to represent this pattern has not yet been attempted. It would probably require multiple CPs to cover the different layers.

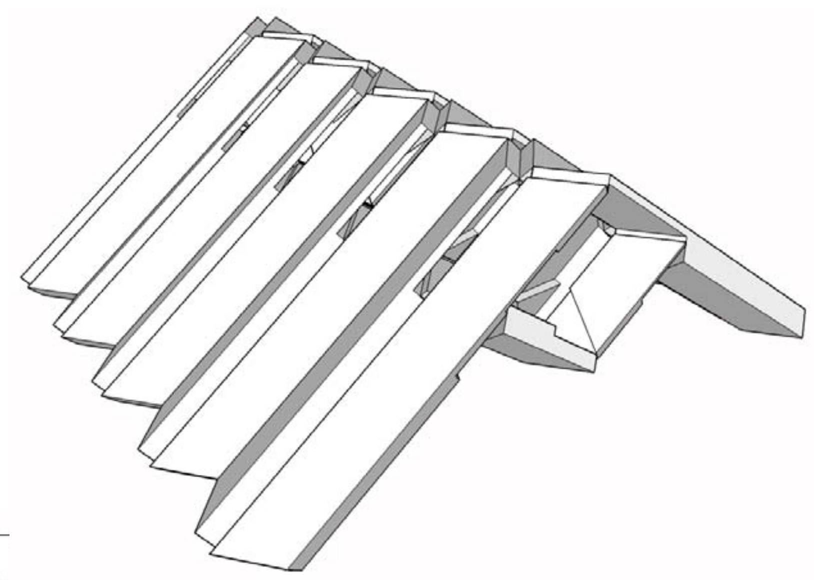


*Figure 9.1.6.1*
Double Tier top

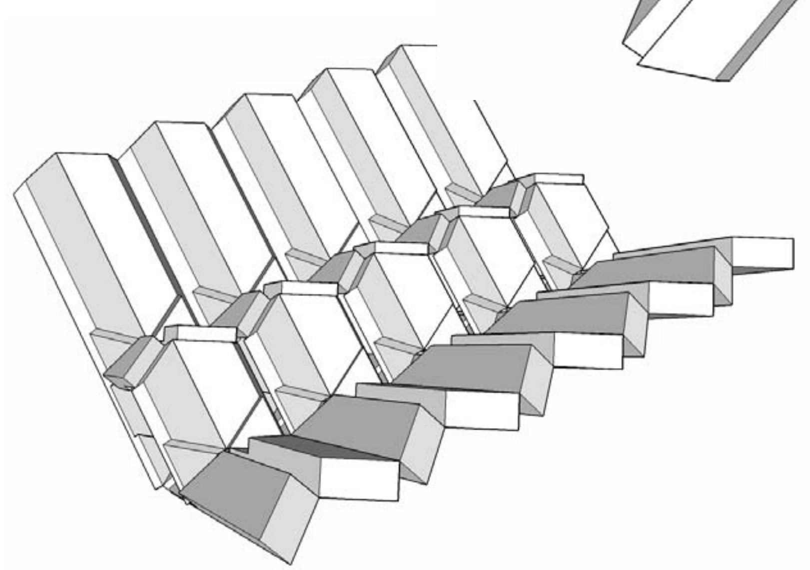


*Figure 9.1.6.2*
Double Tier bottom

### 9.1.7 CP Triangle Array

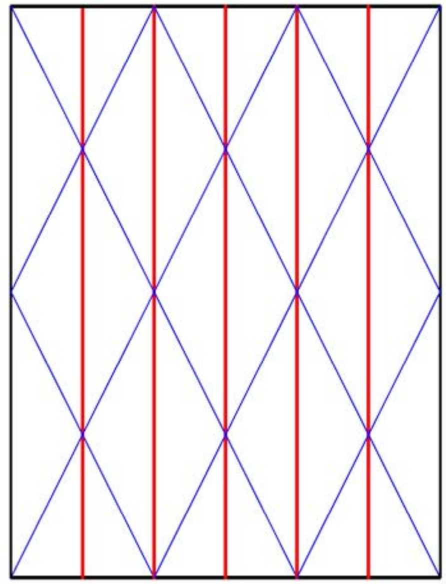


*Figure 9.1.7.1*
The Triangle Array CP

This is one of the most prevalent examples of origami used in projects, as can be seen in case studies such as Mats Karlsson: Xile and Tine Hovsepian: Cardborigami (*Sec 10.14-15*).

This pattern is simple, easy to understand, easy to fabricate, and the structure lends itself nicely to forming a quick tube-like architecture. However, the problem of how to attach it will still remain. (For detail on connections, see Sec 12.2-3 for the working prototype.)

This pattern is simply a series of RFs. The folds are identical, and each new RF occurs directly after the previous. Hence, the 'reverse' in the fold cannot be seen, and all the reverse folds seem to line up. Because of this, no techniques such as carving or cutting are needed. Each vertex has its own waterbomb base, and since the creases line up, the system as a whole remains flat-foldable. Only hinges on the front side for valleys, and on the back for the mountains.

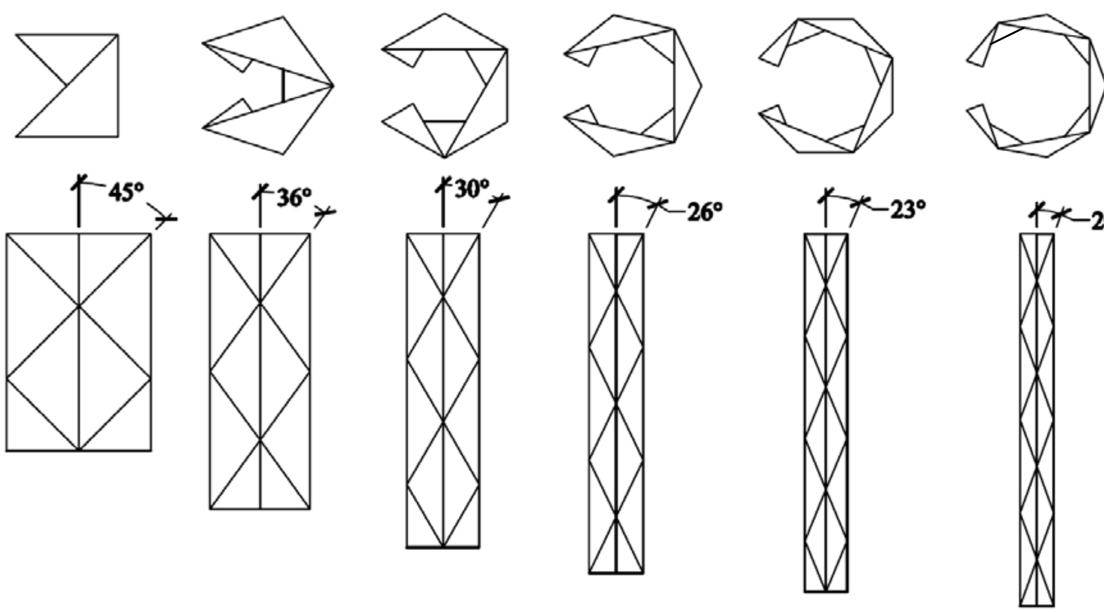



*Figure 9.1.7.2*
Variations of the Triangle Array

Because of this, the pattern is made of a series of identical triangles, except at the edges where the triangles are cut in half. The proportions of the triangle can vary though it changes the overall angle. Angles approaching 45 degrees tend to cause the model to fold back on itself too quickly. *Figure 9.1.7.2* shows some variations based on regular polygons. As these angles become more parallel with the corrugated pleats, the creases become longer, and the bend shallower. These shallower angles can either fit more panels, or have more panels removed and still retain a curve. Arrays with angles more perpendicular to the corrugated pleats become shorter, and not as many panels can fit. A foamcore model set around 30° and only two full panel lengths long, they can form a very nice tube structure (*Fig 9.1.7.3*).

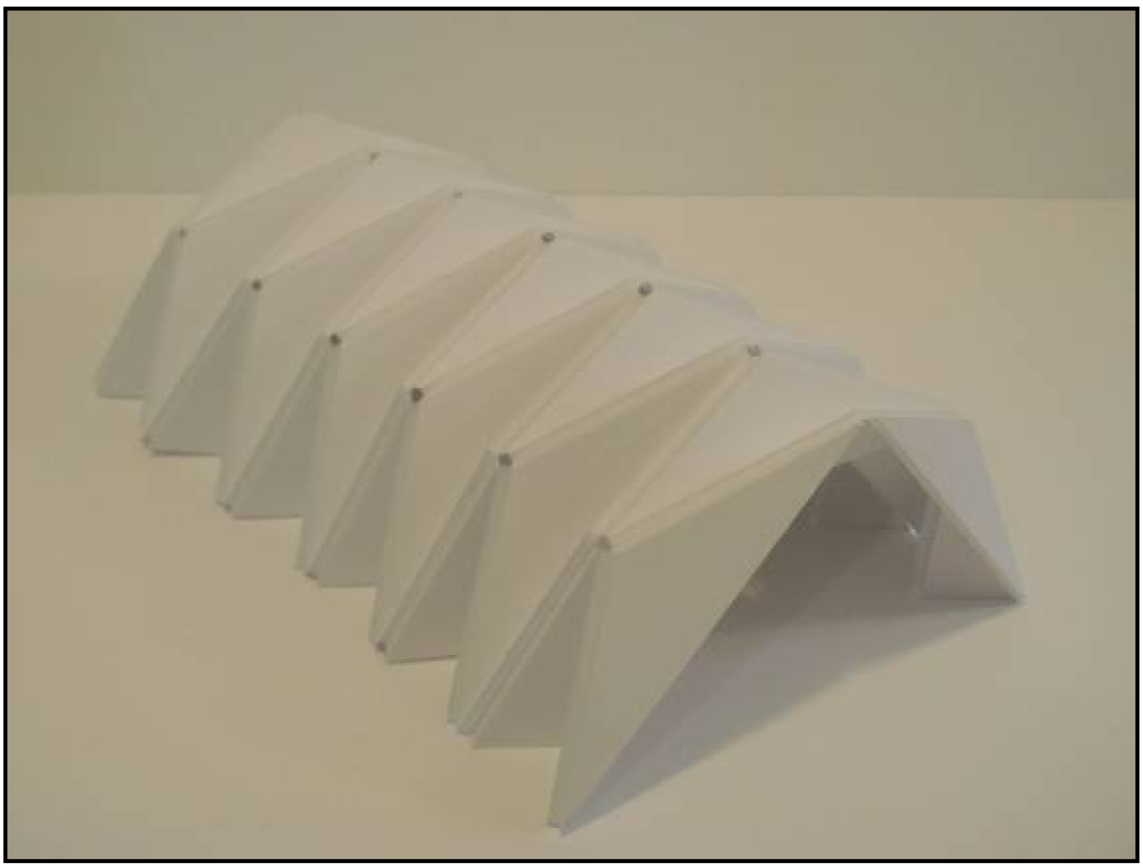


*Figure 9.1.7.3*
Triangle Array foamcore study

## 9.2 Tile Tessellations

This method uses polygons that tessellate, typically rectangles. Origami art uses other polygonal tiles, such as triangular, hexagonal, or even aperiodic tiles. These do not tend to work well, as the tiles do not just translate, but also rotate, and with that, gain rotational symmetry, causing twist folds which can be problematic (as can be seen in *Sec 9.3.2-5*). Hence, the following patterns can be decomposed into squares or rectangles.

When translating these tiles into an array two pairs of M-V pleats that form an alternating M-V-V-M-M-V-V-M… arrangement are used. Unlike the RF-Pleats with each panel rotating 90 degrees, this rotates 180 degrees with every other pleat. This means that every other pleat only translates, without rotating, thus these patterns can only

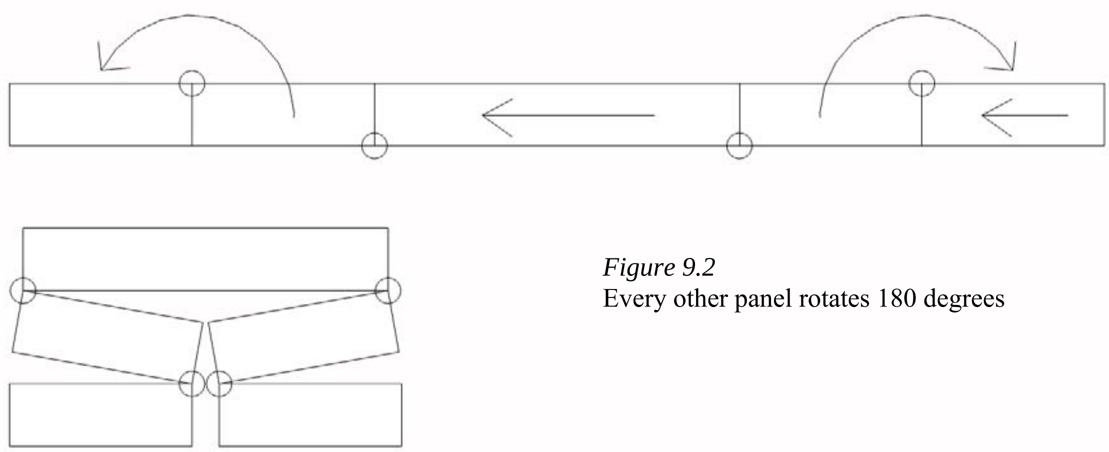


*Figure 9.2*
Every other panel rotates 180 degrees

shrink a max of 1/3 the total width (see *Fig 9.2*). Note that it may be possible to create a pattern that has double stacking pleats, potentially shrinking the final fold to 1/5 the original width. However, there would have to be significantly more consideration for the first set of folds as each additional layer of thickness complicates the pattern more (for example Double Tier *Sec 9.1.6*).

### 9.2.1 Rotated Pop-Up Tabs at 90°

This pattern was originally derived from the water bomb base with pleats radiating out. In a rigid-thin, non-cut surface, these four pleats must be 45 degrees from the horizontal or vertical (*Fig 9.2.1.1*).

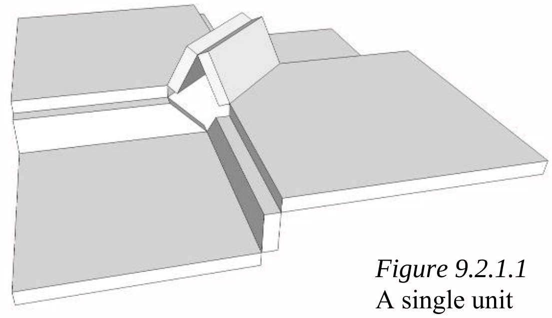


*Figure 9.2.1.1*
A single unit

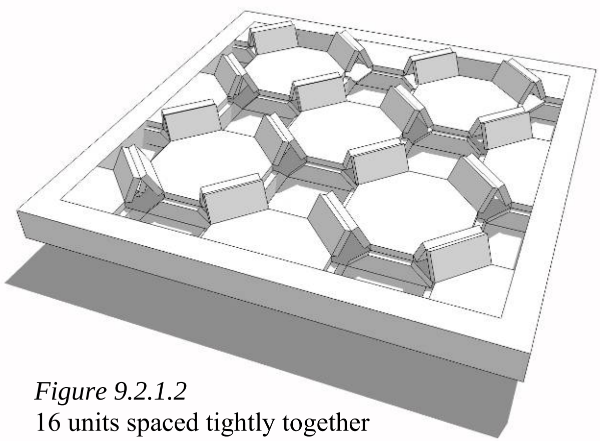


*Figure 9.2.1.2*
16 units spaced tightly together

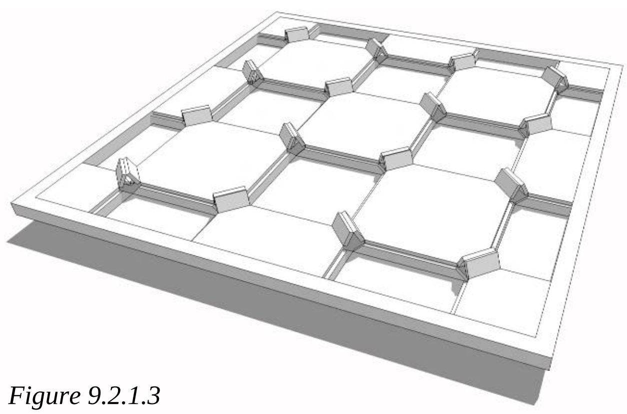


*Figure 9.2.1.3*
The same 16 units, scaled with a larger spacing

It was found when adapting this pattern for rigid-thick use, and when slits were being cut to allow the waterbomb base to rise, that the extra reverse folds that make up the base in the middle were superfluous. Thus, half the thickness required compensation at the middle (*Fig 9.2.1.2*). Furthermore, with these additional changes, elements of the crease pattern could easily be modified, and the resulting repercussions easily predicted or perhaps even sought. The extruding tab could be extended; thus this portion never returns to being completely flat, despite the rest being flat. Since these are set to 45° angles, many kinds of symmetry can take place, in particular, rotational symmetry. Additionally, with two sets of parallel creases, multiple rows and columns can each be translated to larger spacing if desired (*Fig 9.2.1.3*).

### 9.2.2 Rotated Pop-Up Tabs at 60°

After developing the previous CP, it seemed as if the tile had to be rotated at 90 degree intervals in order to create a larger tile pattern made up of 4 units.

Further investigation found that the tile also worked with a triangle grid. Six units formed a larger hexagonal tile, which could be translated very easily.

There are some additional restrictions in the pattern, however; since the folds are set at 60° and 120° from each other, there is a possibility that the pleats can collide on the underside if care is not taken to create the right proportions. This can be avoided by

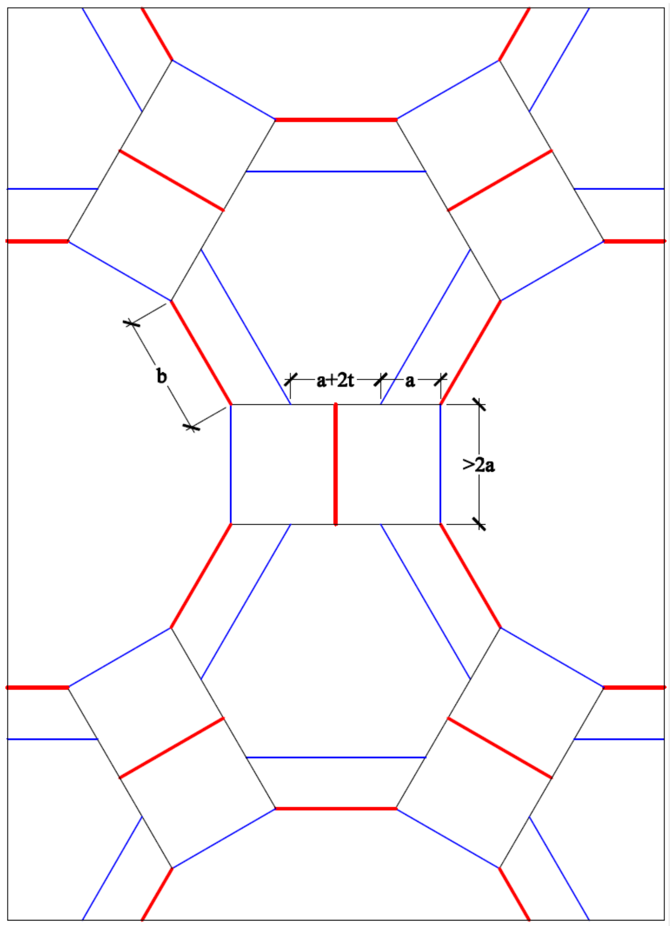



*Figure 9.2.2.1*
Pop-up 60 CP

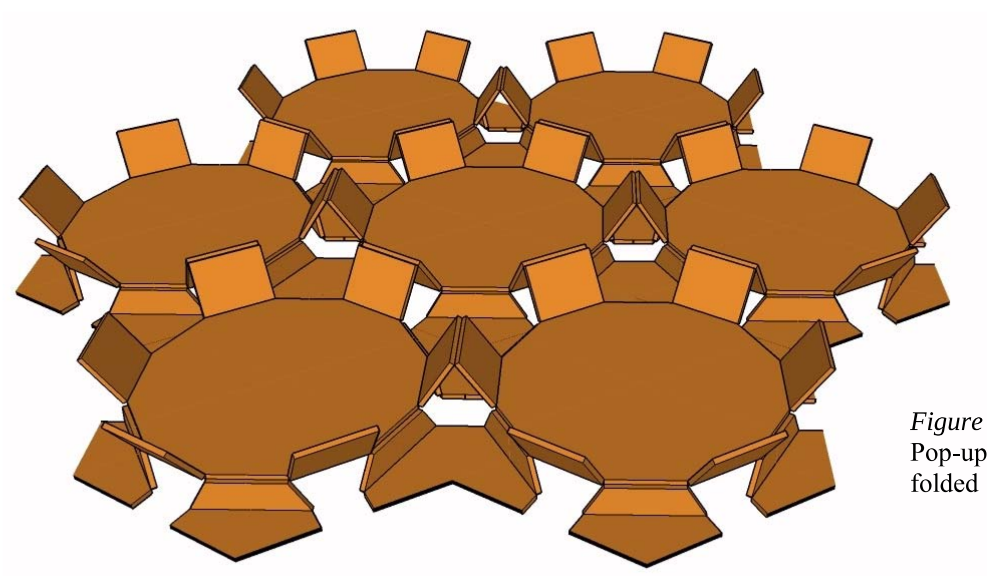


*Figure 9.2.2.2*
Pop-up 60 partially folded

keeping the width of the tabs equal to or greater than twice of ‘a’ and the space between the two pleats at ‘a’ plus twice the thickness. Length ‘b’ has no correlation to length ‘a’. Length b, geometrically, could be reduced to 0, but doing so would disconnect the truncated triangles.

Working with a triangular concept brings in an added amount of difficulty in regards to attachment. But if the end tiles are cut as in *Fig 9.2.2.1* then it should be able to accept an orthogonal mounting system.

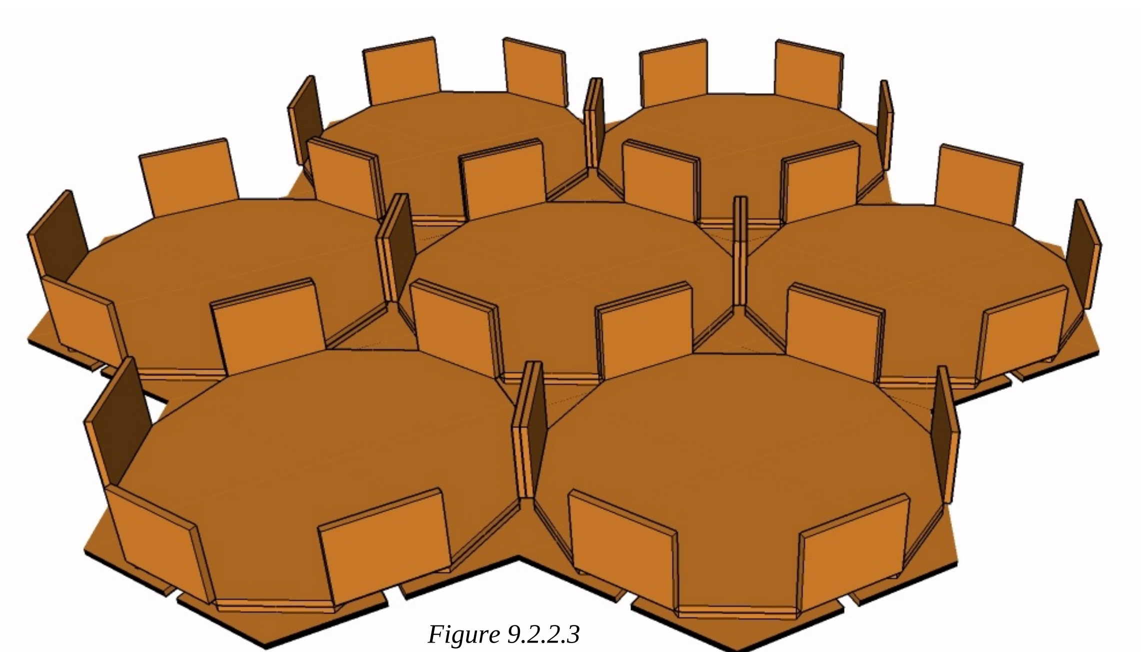


*Figure 9.2.2.3*
Pop-up 60 folded

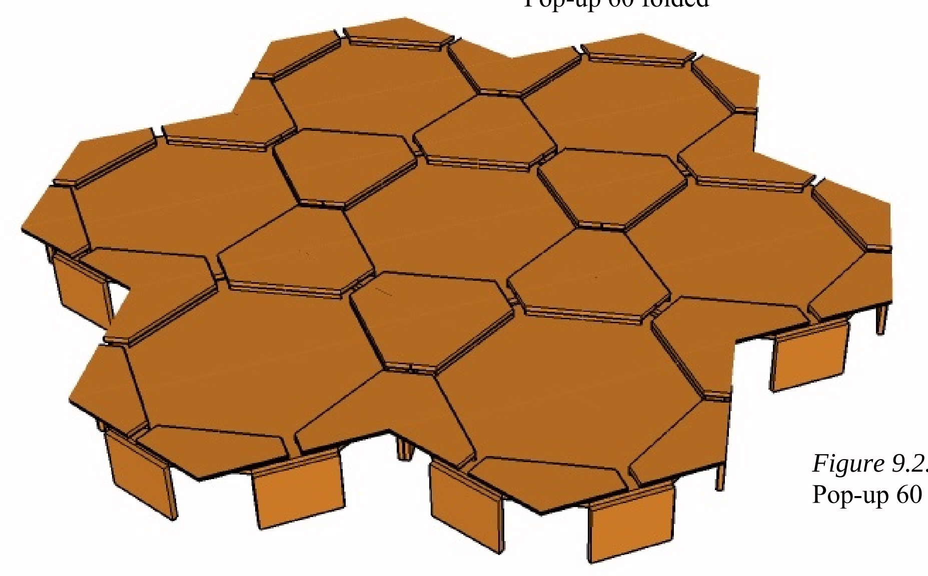


*Figure 9.2.2.4*
Pop-up 60 underside

### 9.2.3 Alternating Pop-Up Tabs

The previous two patterns can also be altered so that arbitrary angles can be used. The angle can be set anywhere between 0 and 90 degrees (although the extremes of these limits will create a very long tile). However, once a 90 or 60 degree pattern is broken, rotation symmetry cannot be used. Instead the tiles can only be translated, except in one case where; every other column has to be reversed along the z axis. Thus, some of the 'tabs' will point 'in' or 'up' while other will point 'out' or 'down'.

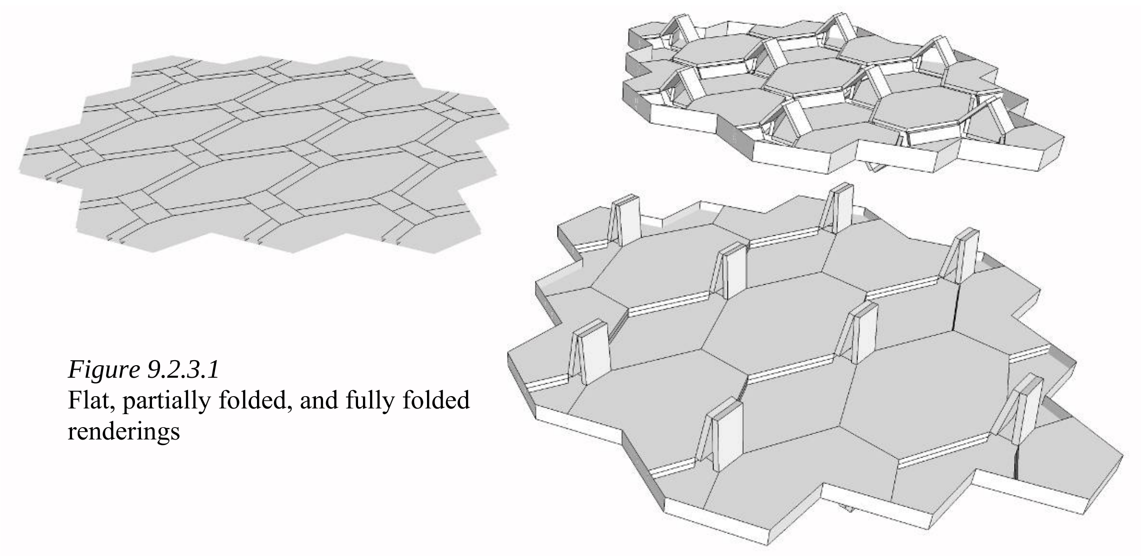


*Figure 9.2.3.1*
Flat, partially folded, and fully folded renderings

### 9.2.4 Scale Tessellation

This tessellation, when folded, is similar to the shape of rounded koi scales on one side and more pointed lizard scales on the other. It has many interesting properties that other models have not exhibited. It is easily constructed, if made as individual units, and tiles nicely. It has many overlapping portions and cavities on the ‘lizard’ scale side. It is best to construct each tile separately, and then join them at their connecting hinges. Finally, small hinges are added in key gaps which add more strength and stiffness.

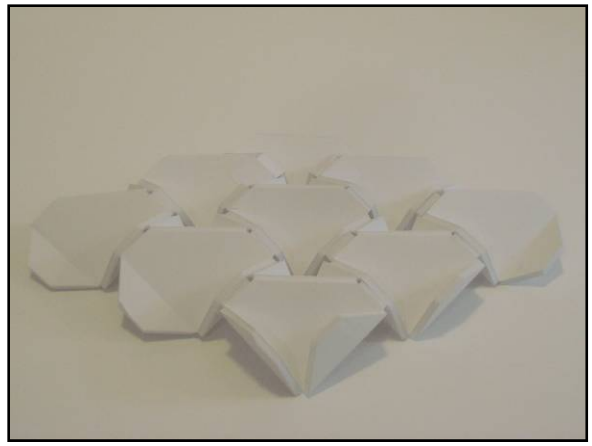
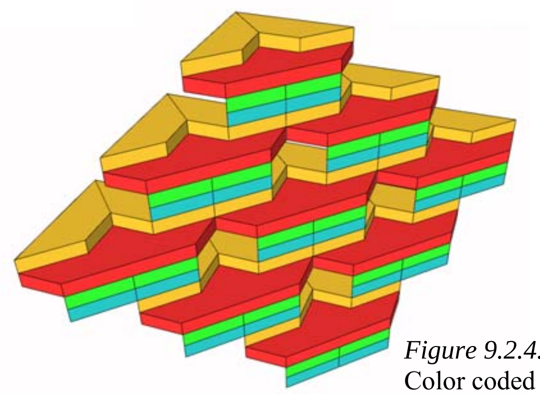
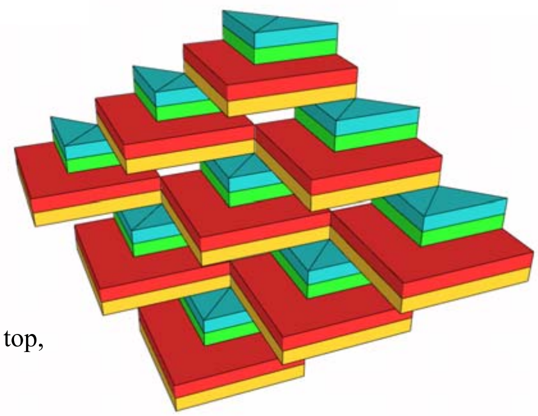


*Figure 9.2.4.1*
Foamcore study





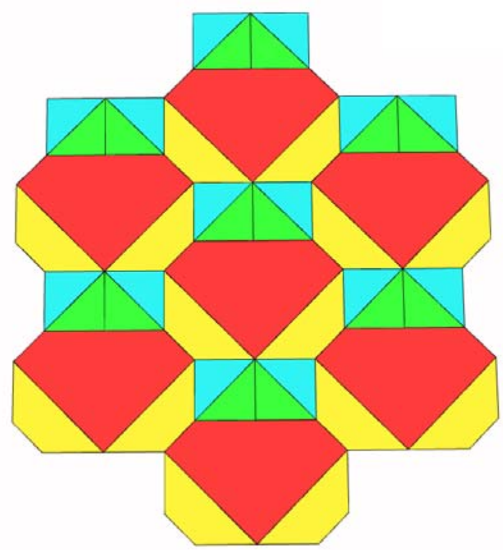


*Figure 9.2.4.2*
Color coded bottom, top, and flat renderings

This model is quite different from the other tile tessellations, in that it does not conform to the 1/3 rule nor the MVVMMV rule. The scales stack up considerably faster. It could be compared to a model that would fold up to form a staircase. Each tread and riser is square to the two floors, but the staircase overall is at an inclined plane. Attaching a model such as this would have to allow for the extra rotation. The scales in the tessellation in the images here may *seem* to be in the same plane, but the scales are layered back.

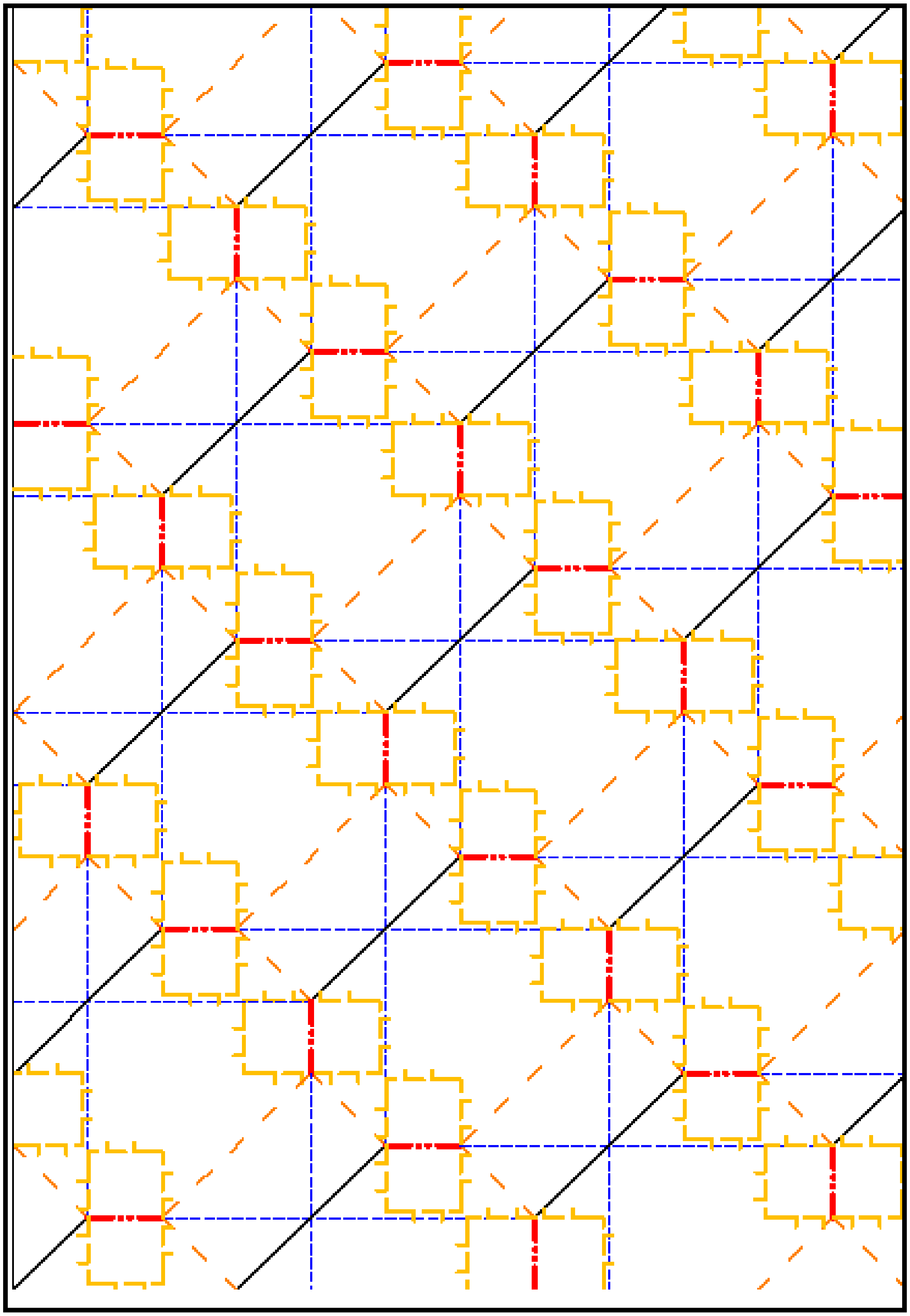


*Figure 9.2.4.3*
Crease Pattern, using proposed line types.

### 9.2.5 Pop-Up Square

This tile is a composite of four preliminary bases at the intersection of two VMMV pleats. The CP does not need alterations to become thick. However, cuts do need to be inserted since planes are being separated. A single tile of this CP contains eight vertices that are Kawasaki theorem compliant with 3DOF, and another eight vertices that are only Kawasaki theorem compliant (thus requiring cuts).

Test folds made from foamcore yield some interesting attributes not easily seen from CP analysis alone. When folded flat, the model becomes quite locked, requiring some pressure at the four outer 3DOF points to induce collapse. This is a result from a hypotenuse plane overexpansion condition occurring locally in the CP. On a side note, it was found the best way to construct the pattern was to first score the continuous horizontal and vertical folds, then cut out the inscribed octagon completely, then score the remaining folds on each element separately. After both elements have been test folded (which is much easier to do in this state), the parts can be put back together by adding the hinge back (in the foam core test glued paper served as a hinge).

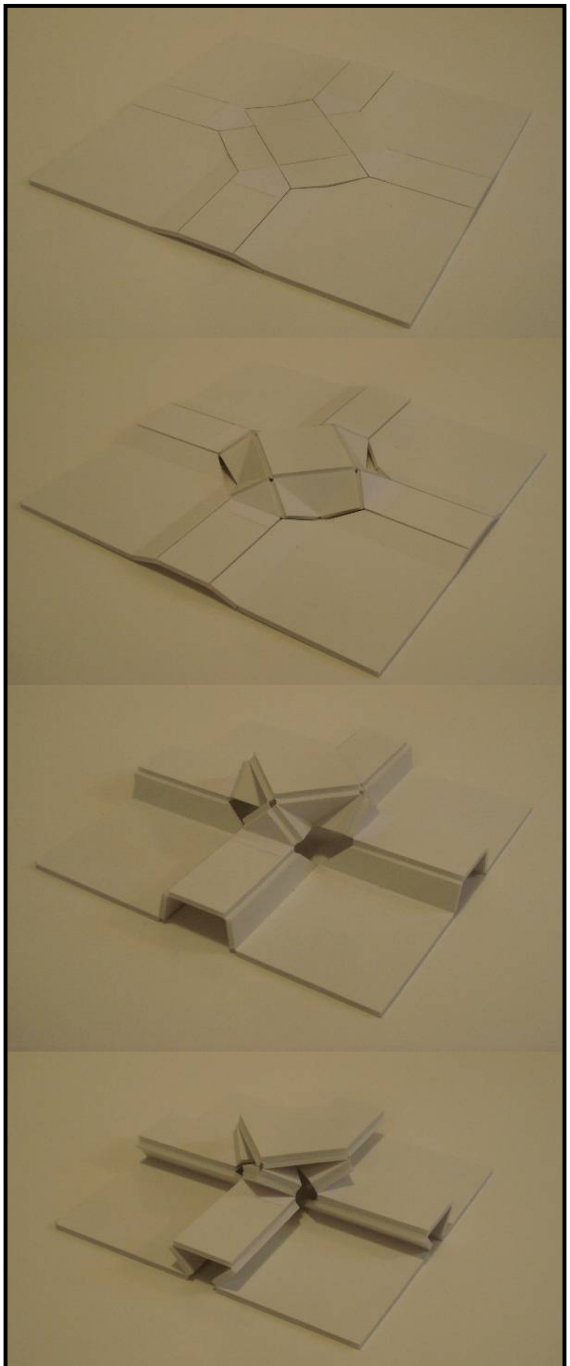


*Figure 9.2.5.1*
Photos of folding process

In theory, the final application would be mechanical hardware that can be assembled / disassembled. Additionally, the folding elements of each of these separated are very interesting in themselves. The pleated backing now has large octagonal holes which can close completely when folded and the entire backing has 2 DOF resulting in an interesting planar joint. While the separated octagon has several combinations of idle linkages, when all eight creases and 16 triangles around the square begin to move, all of them have to move together, creating a 1DOF condition. Thus, when this gadget is added back onto the backing, the entire system is back to 1DOF (except for the vertical and horizontal valley creases). This tile can be folded from a uniform thickness without carving or adding material. The resulting thickness of the final folded form is seven times the thickness of the material. This pattern can have many variations since some of the dimensions are not locked in proportions. Lengths ‘a’, ‘b’, and ‘c’ are all independent of each other. In fact, there can be multiple rows and columns, which can add much variety. However, length ‘a’ cannot be less than 1/3 of length ‘b’ (*Fig 9.2.5.2*). It should also be noted that as length ‘a’ becomes shorter relative to length ‘b’, the cuts become proportionally longer on the pattern as a whole. As ‘a’ is shortened, the overall strength decreases proportionally.

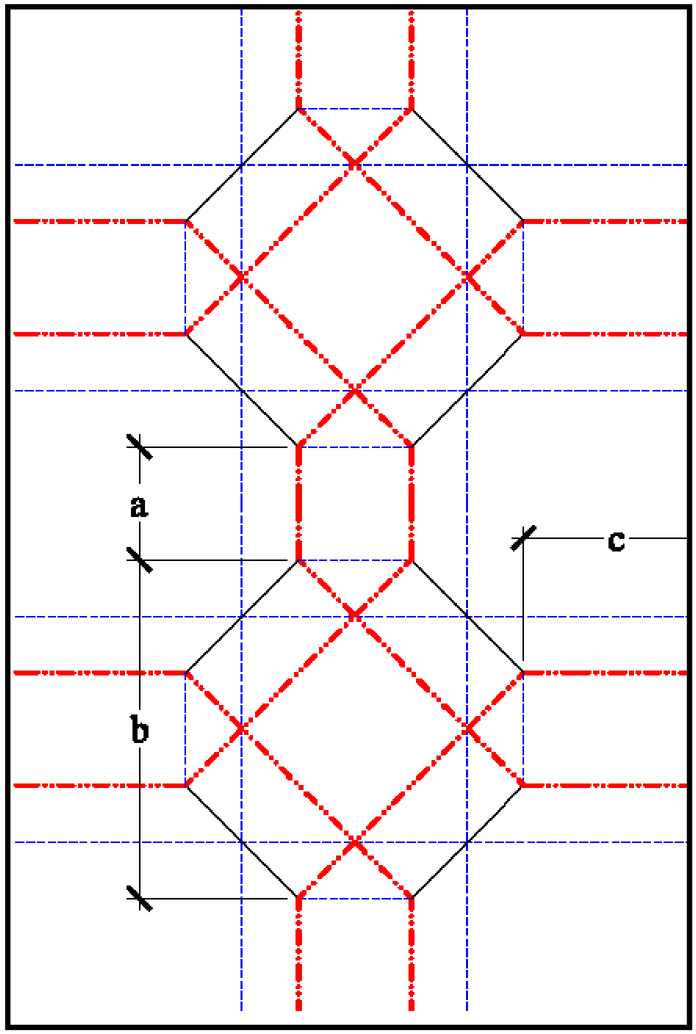



*Figure 9.2.5.2*
The CP and modifiable lengths

Two other foam core mockups were also created. A 2x2 arrangement was created to explore this tile set a little further. Once this tile set was created, a 4x4 tile set, of 16 ‘Pop-Up Squares’ (*Fig 9.2.5.3*) was made in which length ‘a’ was at its minimum.

This last model started to have additional resistance to collapsing it into the final folded form. This is probably due to the foam core materials, and the fibers of the top and bottom ply resisting bending. The fact that length ‘a’ was minimized caused problems in this instance as well. The top and bottoms plys of the foamcore began to peel off of the foam with some parts breaking. However, this is most likely due to the physical properties of the foamcore, and more durable materials and hinges should work well.

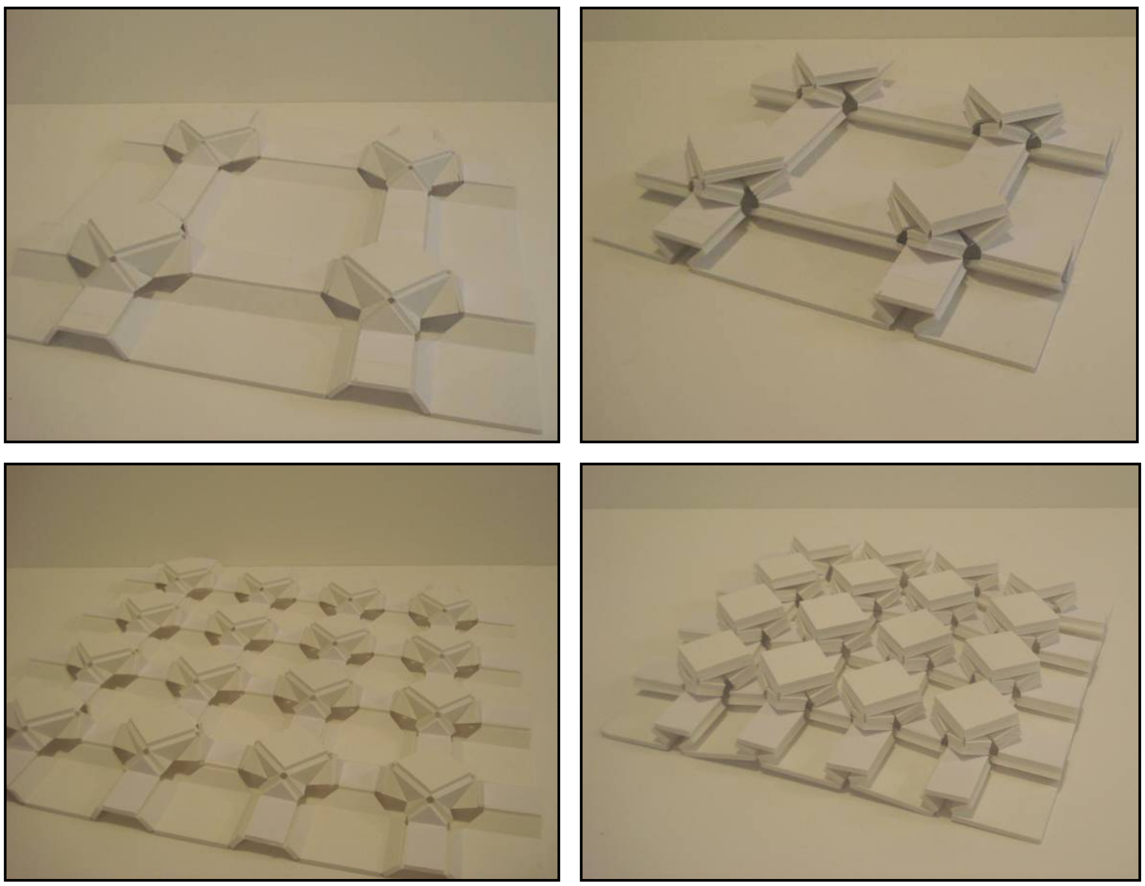


*Figure 9.2.5.3*
Photos of folding process for an array of 4 and 16.

### 9.3 Flawed Designs

The research regarding the following flawed examples may be just as helpful as the successful examples in informing how to design, or rather, how not to design rigid-thick origami designs. The successful examples show that reverse folds and tile tessellations work best. But these flawed examples show that simply moving vertices apart in relation to material thickness was not as straight forward a solution as initially thought. It was also discovered that no type of twist fold can fold in either a rigid-thin nor rigid-thick context. Although it was known that twist folds do not satisfy rigid-thin conditions, there was an attempt to see if there was a way to modify the CP so it could rigidly thick fold.

#### 9.3.1 Wedge Miura Map

The concepts of this pattern were simple enough; adjust the CP so the thickness has a space to fit once folded. In the Miura-map fold, essentially, the pattern is an array of repeating reverse folds which pack tightly once folded. The points are moved apart, creating triangles. However, this has the same problem documented in the split hinge CP (*Sec* 9.1.4). Even though the CP has been altered to match 2x the thickness, once the folding starts, the diagonal through the model becomes too great, and the material collides.

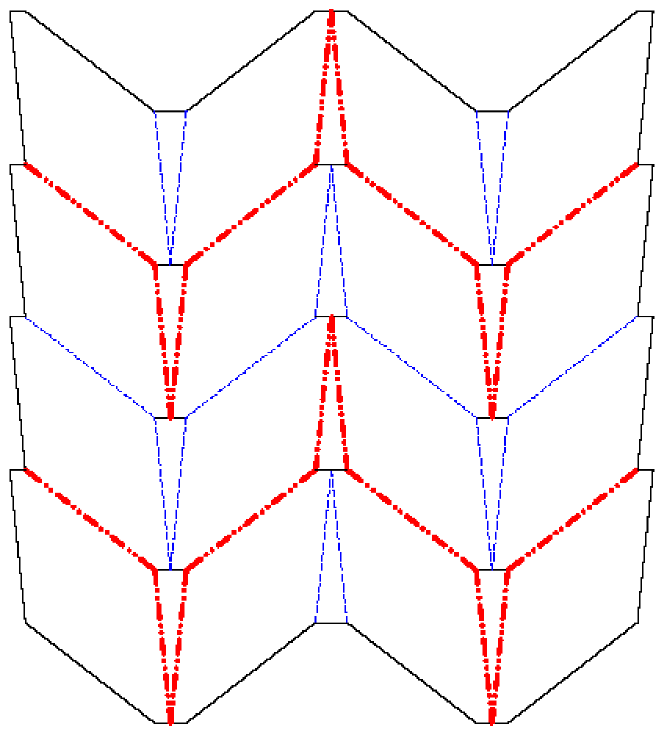


*Figure 9.3.1.1*
CP of Wedge Miura Map.

Although this model failed, it was atually the model that let to the development of the successful counterparts discussed earlier in this section.

### 9.3.2 Square Twist Fold

This model was unlikely at the start, but like so many of the candidates in this paper it was worth exploring to see where it would lead, especially since twist folds make up a large body of the art of origami tessellations. Clearly there would be some problems, as it is not rigid-thin compliant. Furthermore, the alternating M/V folds in rotational symmetry causes a kind of never ending 'Escher staircase' to form (*Fig 9.3.2.1*). Just as this Escher staircase could not work, neither could a rigid-thick model. Attempts to fold just ~170 degrees followed by carving were attempted, but to no avail.

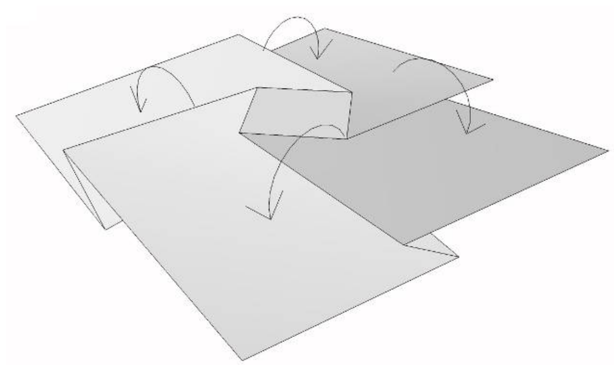


*Figure 9.3.2.1*
A Twist Fold, much like a never ending staircase.

The model was primarily tested on the standard square twist fold. However, investigations into other variations of the square twist fold indicate that they would have the same result. For example, the square tessellation angle can be less than 45 degrees (*Fig 9.3.2.2*), or twist folds can be made based on other polygons. These can vary in shape, with variation usually being regular triangles, hexagons, octagons, dodecagons, or any number of sides, and need not even be regular, just convex. In any of these cases though, the underlying problem of accommodating the thickness remains. Thus, at this time, flat foldable twist folds are abandoned as a possible rigid-thick concept. Since most origami tessellations use flat twist folds, a large portion of origami tessellations has been eliminated as rigid-thick candidates. However, there are many artistic origami tessellations that use non-flat-foldable twist-fold models, and it was thought this *might* be a successful avenue for rigid thick, but this was found not to be the case.

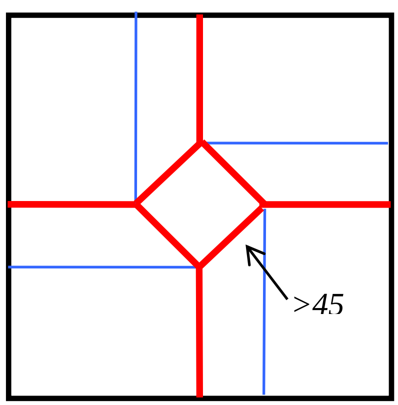



*Figure 9.3.2.2*
The CP of the typical twist fold.

### 9.3.3 Square Waterbomb

This model is a type of twist fold, but since the waterbomb bases stand upright, instead of layering in a twist, it was thought it might not suffer the same problems of a twist fold. An attempt was made to modify tessellations which purposefully finish in a 3D state, i.e. the final model is not flat-foldable. Since it is not flat-foldable, the rigid-thin condition was harder to determine without physical tests. Studies of this pattern found that a single unit of this tessellation can fold rigidly, but when combined, the entire system is not rigid, and deformation must occur.

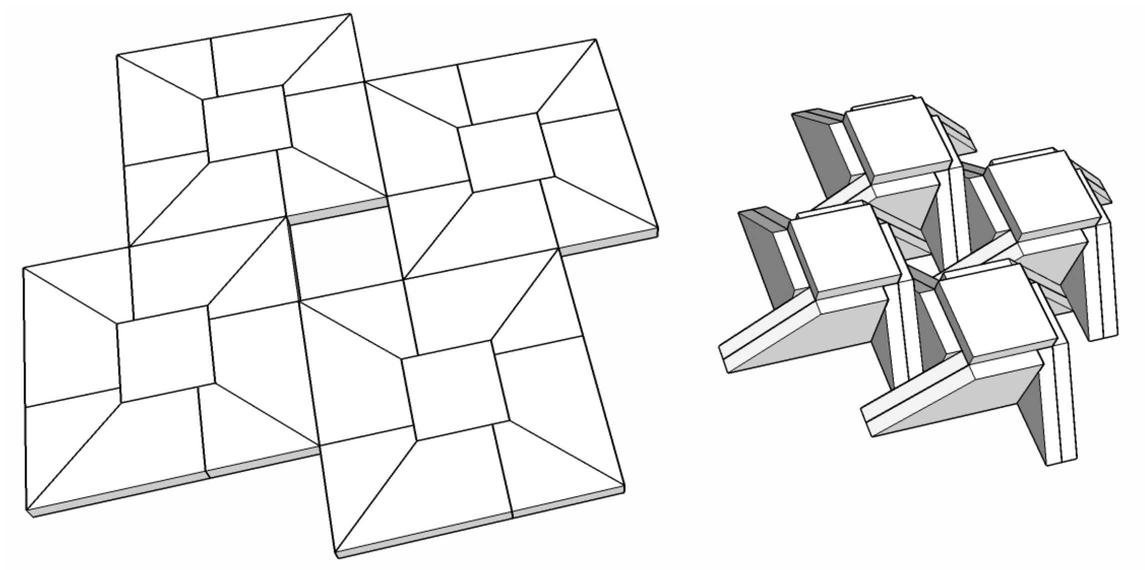


*Figure 9.3.3.1*
The Square Waterbomb flat and folded

### 9.3.4 Stacked Waterbomb Pleat

This was an early attempt to translate a rigid thin CP into a rigid thick pattern while also making alterations to allow for the thickness. Based on a tessellation which uses an array of water bomb bases that interlock with each other, radical changes were needed where these flaps interlocked since there were 4 layers of paper in each of these interlocking systems. Thus, some folds shifted away by four times the thickness of the panels.

This model, like the thin version, is not rigid, and the foamcore study model underwent great stress and deformations while undergoing folding.

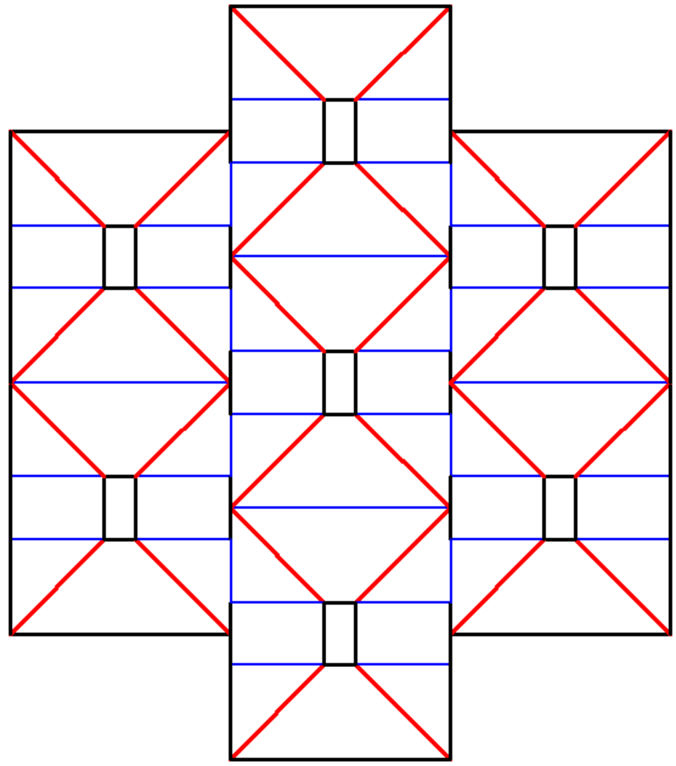


*Figure 9.3.4.1*
Stacked Waterbomb Pleat CP

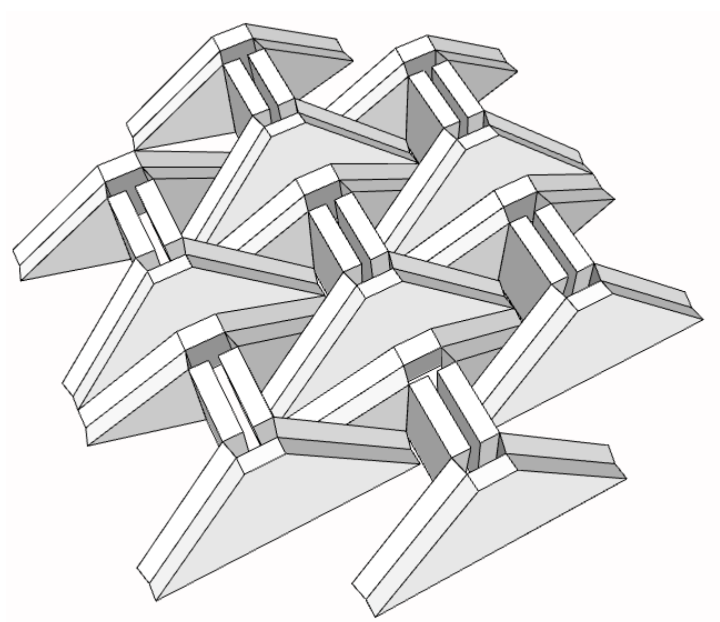


*Figure 9.3.4.2*
The Stacked Waterbomb folded

### 9.3.5 Tri-Twist Fold

This was another attempt at a twist fold. However, this arrangement of a twist fold is different, such that it does not lay flat. Of the tessellation types in origami art, three dimensional (3D) tessellations are becoming more popular, as they are more challenging to design and are more visually interesting and dynamic than flat tessellations. This design is similar to a flat twist, but slightly off-center, causing a triangle to pop up, and beneath it, a 3D void formed akin to an octahedron form. Again, it was thought that since this thin version finishes folding in a 3D form, an exception might be found for rigid-thick folding. In fact, the rotational symmetry still causes the panels to move farther apart, reducing the ability to create a continuously connected and foldable system. Many other arrangements of this geometry were tried, and none could fold rigidly.

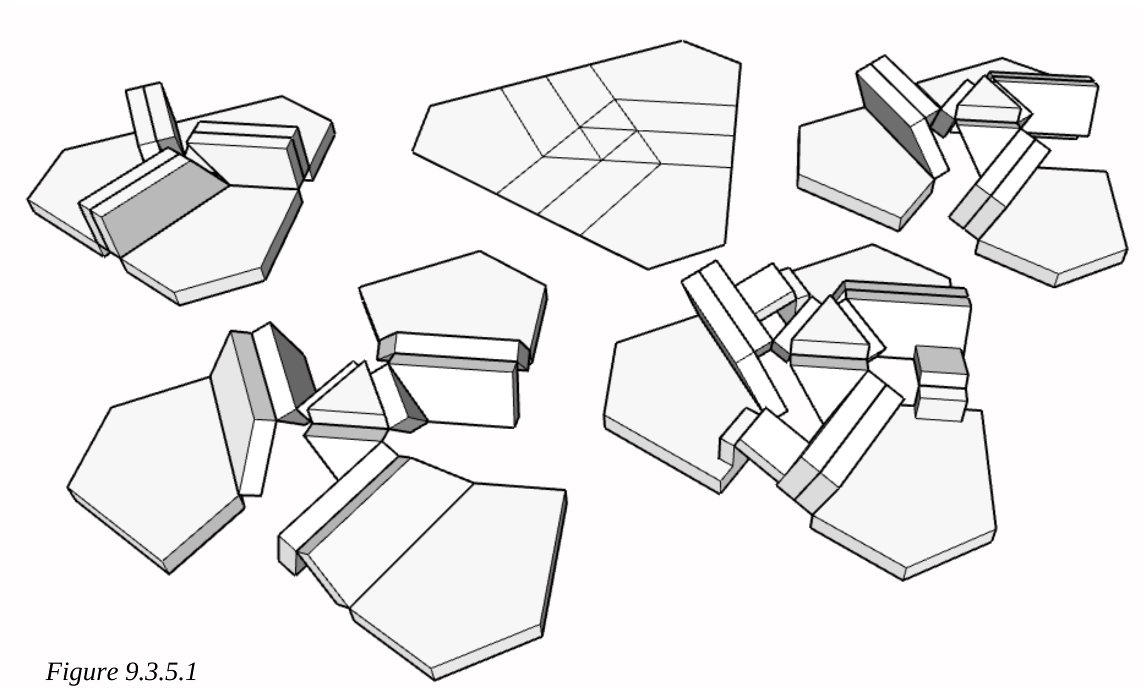


*Figure 9.3.5.1*
The Tri-twist flat and folded

### 9.3.6 Hex Twist Fold

This was the last attempt at developing a twist folding into a thick pattern. This was based on the Solar Power Origami satellite (*Sec 10.07*). First the traditional layout (*Fig 9.3.6.3a*) as in the solar panel prototype was attempted. With five degree vertices forming around the hexagon, many problems quickly arose just as in the other twist folds. The hexagon corners collided, the pleat edges drifted and holes formed (*Fig 9.3.6.1-2*). An alternate pattern was attempted in which three sections were popped up (*Fig 9.3.6.3b*). This removed a crease from each section, and allowed for fewer creases at each vertex. It was thought that this would minimize collisions. However, the asymmetric 4 degree vertices again made the fold impossible in a rigid thick context.

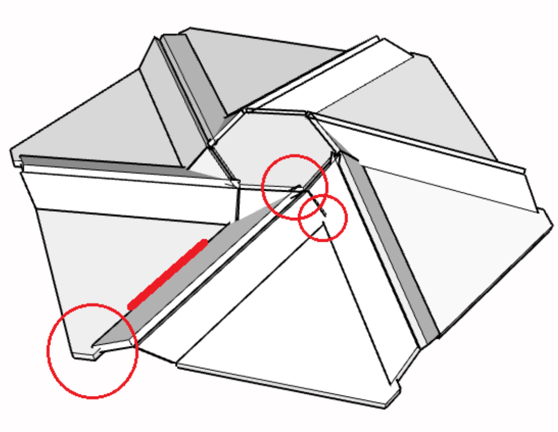


*Figure 9.3.6.1*
Numerous collisions are created in this model

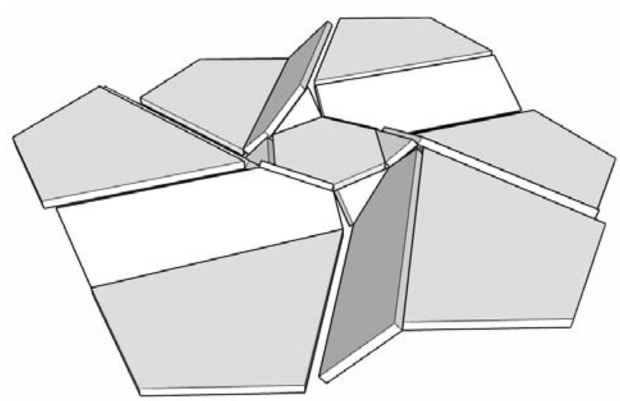


*Figure 9.3.6.2*
Numerous gaps are created in this model

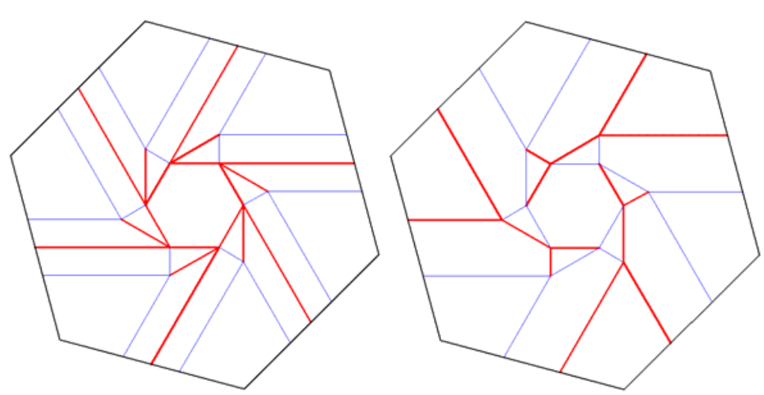


*Figure 9.3.6.3* (a) (b)
The original and altered CPs of the models above

## 10. Historical Precedents and Case Studies

The majority of historical examples are taken from the 19th century. Although there are some historical narratives relating to Native American tepees and Bedouin tents as structures that are portable and transportable, little else is covered between these time periods. [24]Most of this 'non-static' architecture seems to take place after WWII, when housing demands were especially high. Since then, this style has mainly been used in the context of large (sometimes traveling) event spaces. Only recently has this technology become more sophisticated.

As far as folding and bending forms, there are numerous examples of the use of origami as minor details including: "the curved plywood walls of the Office for Metropolitan Architecture's *Educatorium*, the wrapped metal corner panels of Daniel Libeskind's Jewish Museum Berlin, and the structural cladding of Foreign Office Architects' Yokohama Port Terminal."[25] Whereas others use origami as a design concept for the whole form giving a very geometric faceted shape such as with Tadao Ando's hhstyle.com furniture store[26], McBride Charles Ryan's Klien Bottle House[27] and Monaco House[28], and Yasuhiro Yamashita's Reflection of Mineral home.[29] There are still others that go as far as using origami in the rigid-thick sense but usually in a simplified manner without many vertices.

However, for the purposes of this review, since the goal is to also explore the practical application of origami into architecture, there will be many studies of kinetic, portable, movable, and smart/reactive buildings. The goal is to better understand the materiality and mechanizations of these joints. These various examples of works which fold or otherwise move, slide, pivot, collapse, or expand can offer insight in the design process later. Some of these examples are as followss:

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[24] Kronenburg 1995, 18-23
[25] Iwamoto 2009, 62
[26] "MoCo Tokyo: hhstyle.com/casa" 2008 http://mocoloco.com/tokyo/archives/000973.php (accessed Oct 20, 2011)
[27] McBride Charles Ryan. "Klien Bottle House" 2008 http://www.mcbridecharlesryan.com.au/#/projects/klein-bottle-house/ (accessed Oct 13, 2011)
[28] McBride Charles Ryan. "Monaco House" 2007 http://www.mcbridecharlesryan.com.au/#/projects/monaco-house/ (accessed Oct 13, 2011)
[29] "Reflection of Mineral" 2006 http://www.tekuto.com/blog_eng/wp_english/wordpress/?p=29 (accessed Oct 20, 2011)

### 10.01 Acorn House: Carl Koch, Huson Jackson, John Callender

Designed in 1945, this transportable home could be towed by truck with all the building materials already attached.[30] Once at site, the pre-attached panels that were the walls and roof would fold out farther increasing the living space by approximately 500%. This is a great example of having many thick surfaces folded together to form a more compact shape, but all of these examples still uses simple folds.

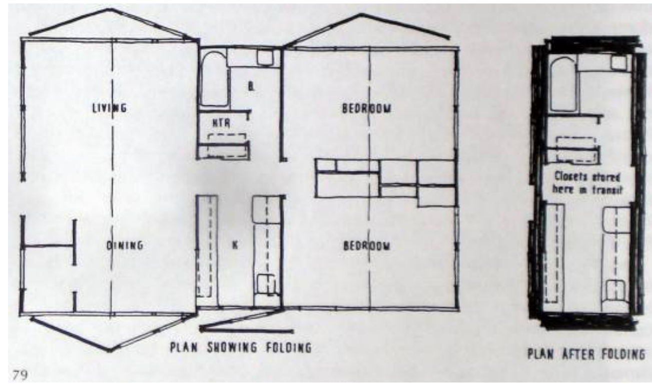


*Figure 10.01*
Acorn House

### 10.02 Motto Markies: Eduard Böhtlingk

Although the geometry in this design is relatively simple and uses a tent like structure, it seems important to note some of the earlier examples of folding structures, if only for a historical context. This prototype was designed as part of a competition for temporary living shelters in the Netherlands in the 1980's. This architect took a different position, where instead of designing a temporary structure that would eventually breakdown, which he felt was 'wasted energy,' he was the only one proposing a portable design. One of the smallest of the 17 selected, it took 10 years to complete the design. Whereas most of the portable prototypes of the time housed the users within the linearity of the transportable pod, this design primarily employed folding windows and membranes which created extra space.[31]

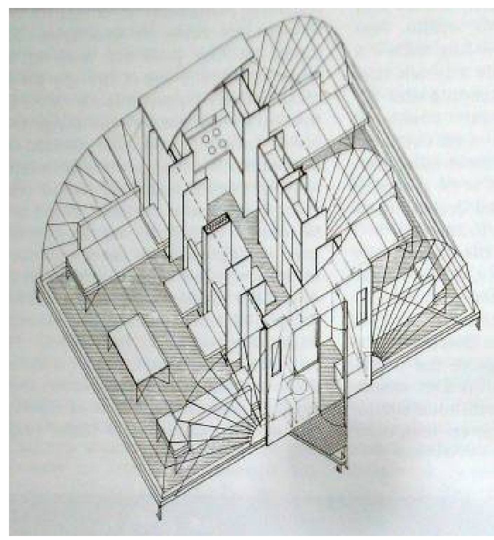

*Figure 10.02*
Motto Markies

[30] Kronenburg 1995, 70-71
[31] Kronenburg 2003, 161-167

### 10.03 Klein Bottle House

The architects of McBride Charles Ryan, designed this Australian house[32] to have an origami feel about it. The origami aspects of the house are in aesthetics only. This structure used no novel folding to construct it nor does anything fold or unfold on the finished structure. It is simply a form inspired by folded origami. Although this building can be critiqued in different architectural terms, in regards of deployable rigid-thick origami described in this dissertation, this example is to be avoided, and it is important to denote the distinction between origami used as forms or as functions.

*Figure 10.03*
Klein Bottle House

### 10.04 Bengt Sjostrom Starlight Theatre: Studio Gang Architects

This theater was designed to maintain an open air experience as much as possible, and even extend the schedule of performances into the rainy season. The sculptural roof panels can open "like petals of a flower in fair weather."[33] This would be a great example to use for analyzing weathering and water proofing conditions. In the pictures one can see some of the flashing, including a main 'cap' that covers all the points of the roof panels. This could imply that the panels must fold up and down in a certain order.

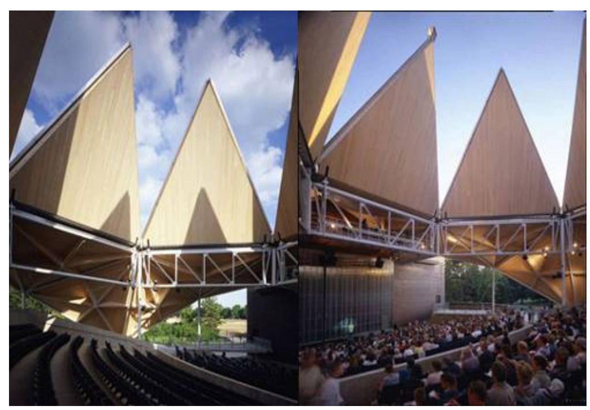

*Figure 10.04*
Starlight Theatre

[32] Amy Frearson 2012, http://www.dezeen.com/2012/01/11/klein-bottle-house-by-mcbride-charles-ryan/
[33] Studio Gang Architects. "Bengt Sjostrom Starlight Theatre" 2003 http://www.studiogang.net/work/1998/starlighttheatre (accessed Sep 3, 2011)

### 10.05 Resonant Chamber

This team calling themselves ‘rvtr’ sought to make an interior envelope system that incorporated rigid origami.[34] These models have pistons on the inside of the triangles that allow them to be opened and closed by remote. Although the crease pattern chosen for this system is rigid, the model needs to curl up. In order for the triangles to swing out, the bottoms need to swing out, which will cause the flat triangles to swing up, as can be seen in the middle unit (*Fig 10.05*). If enough tiles are added, eventually the ends of the curls will collide. Thus, this model cannot transform and stay co-planar, and it is because of this that only a few tiles are used in each array.

*Figure 10.05*
Resonant Chamber

[34] rvtr, “Resonant Chamber” 2011, http://www.rvtr.com/research/resonant-chamber/

**10.06 Appended Space**

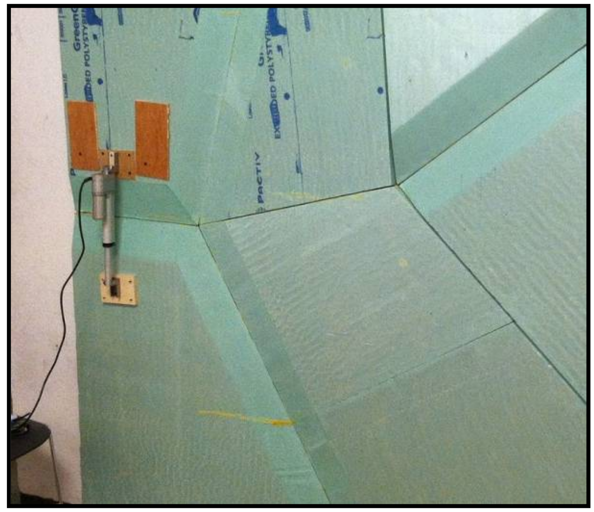

*Figure 10.06.1*
Appended Space

This is an excellent example of a rigid thick origami panel system in use. Sam Rosen investigated scalability and fabrication of asymmetric and irregular rigid thick origami designs using the shifted axis method.[35] He controlled the model through the use of a piston in which one end was attached to a fixed, immobile panel. This panel would keep its place while all the others moved. Again, this system kept to Tachi's 4 degree vertex model and shifted axis technique, but it goes a long way in proposing a means to operate these types of rigid-thick models. It pushes the limits by offering some large doors, most of which is cantilevered off the wall and held through the structure of the creases.

*Figure 10.06.2*
Appended Space

[35] Sam Rosen, "Appended Space" 2015, http://www.curvedfolding.com/profiles/blogs/appended-space

### 10.07 Solar Power, Origami-Style

This concept was developed as an alternate method to pack solar arrays into satellites.[36] The way the panels twist and fold up around the fuselage of the satellite creates some interesting design implications. This model cannot fold in one continuous unified fold, as would be seen in a uniformly rigid model. Instead, this model folds in steps, as each section wraps around one side of the central hexagon.

Additionally, there are a number of perpendicular folds which aid in the array wrapping around the central hexagon. In these types of folds, the fold must complete the 180 degree rotation, before folding over again. This results in an interesting hinge condition in a rigid-thick case as the hinge would have to allow for this offset fold. It seems in this case, the outside hinge is required to be extra wide and flexible.

With closer inspection of this particular case study it is found to be a rigidly thin model. However, like the Resonant Chamber, the model cannot remain flat and coplanar. The planes must curve down as it folds and returns to a coplanar condition. This rigid-thin model *could* tessellate, but again, like Resonant Chamber, it would curl. The model gets much more complicated when applying rigid-thick concepts.

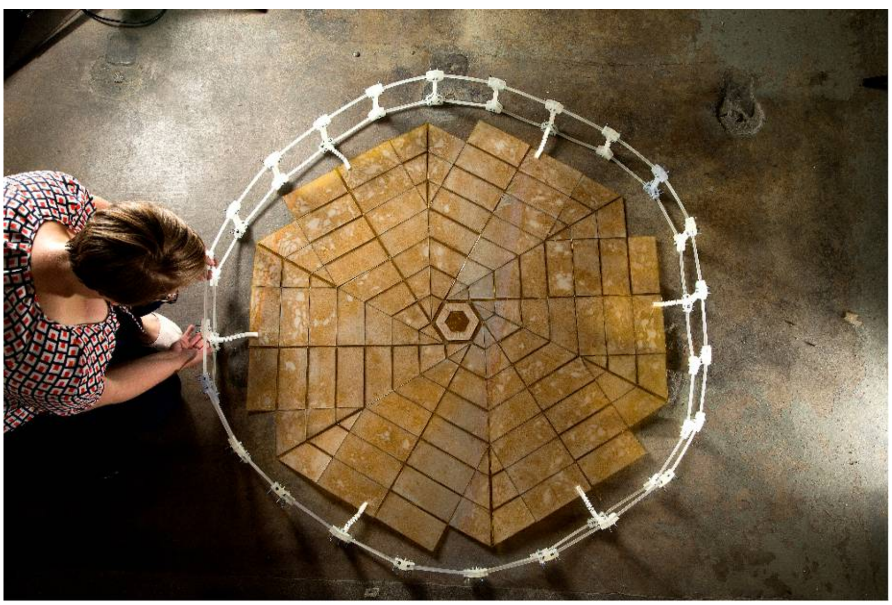

*Figure 10.07*
Solar Power, Origami Style

[36] Robert Lang, Brian Trease, Shannon Zirbel, "Solar Power, Origami-Style" 2014, http://www.jpl.nasa.gov/news/news.php?release=2014-277

### 10.08 Al Bahr Towers

This pair of towers uses origami tiles to control sun penetration throughout the day.[37] It does this by using triangular waterbomb bases controlled by pistons which push the center point of the triangle out, and thus pulls in the edges, allowing more light to enter. Unlike Resonant Chamber, which used a continuous CP (and curls because of it), the towers use individual tiles so they can be controlled independently of other tiles. The system seems to handle the thickness of the panels very well. The hinges may even be oversized and offset to allow room for the piston inside. As a result, the panels are fairly thin when folded up. Overall, this is a very successful use of rigid thick origami in kinetic architecture. However, this is still not a continuous pattern, and does not use more complicated vertices of mountain and valley folds.

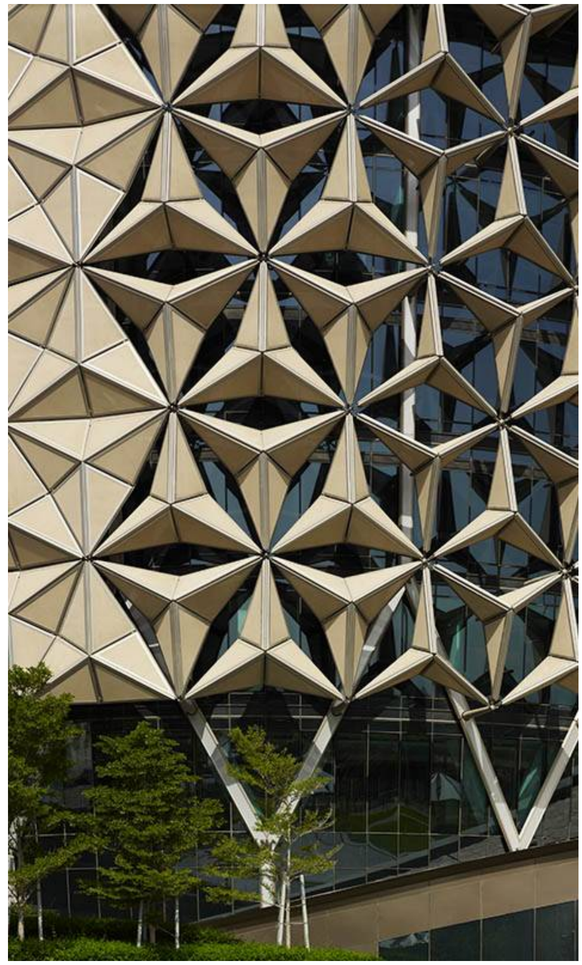

*Figure 10.08*
Al Bahr Towers

[37] AHR Architects "Al Bahr Towers" 2013 http://www.ahr-global.com/Al-Bahr-Towers (accessed Sep 20, 2014)

### 10.09 Kenetura: Kine Tower

This tower starts as a smooth rectilinear shape. It will interact with sunlight and the user activates controls to pull up a "material that is rigid when taut but flexible enough to bend." This then pulls up stripes that allow sunlight to enter but the differing lengths create a very dynamic shape. [38]

*Figure 10.09*
Kine Tower

### 10.10 Hofman Dujardin: Bloomframe

Bloomframe is a window which is flush with the wall and that can be folded down to turn into an extended balcony. There are many ways to configure the window including colors and materials. All the folding and mechanization takes place in a very small area.[39] [40]

*Figure 10.10*
Bloomframe

This is a great design for many interesting reasons. It not only has great hinges, mechanisms and is rigidly foldable, but even creates an extra geometrical dimension, turning a flat surface with two faces, to a three-dimensional box with four sides. The extra sides come from a technique using the thickness of the wall. The sides of the balconies are small individual rectangles approximately four inches wide that stack when stowed, but are situated next to each other when expanded.

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[38] Turner, Troy. "Kinetower by Kinetura" 2011 http://www.yankodesign.com/2011/03/16/fascinating-kinetic-architecture/ (accessed Sep 3, 2011).
[39] Hofman Dujardin Architects. "Bloomframe" 2007 http://www.hofmandujardin.nl/index.php?HofmanDujardin=03&id=19&language=english (accessed Sep 27, 2011).
[40] Poucke, Oliver van. "Bloomframe" 2008 http://blog.kineticarchitecture.net/2008/04/bloomframe-ein-balkon-zum-ausklappen/ (accessed Sep 27, 2011).

### 10.11 Schlaich, Jörg: Folding Bridge over the Förge


Folding bridge near Kiel, Germany. Engineer: Jörg Schlaich. Courtesy of M. Winzenburg.

*Figure 10.11*
Folding Bridge

This is a small foot bridge near Kiel that is able to fold into three panels and the entirety is pulled to one side of the shore. During the entire folding process the bridge is designed to be a statically determined cable-stay bridge. There are no hydraulics or springs used to keep the cables in tensions during the folding.[41] This is a very unique folding sequence in which even the posts and railings fold along with the system. The hinges and cables were designed so that a very simple winch is all that is needed to fold the bridge up and down. Even though the folding is not complicated, there are still a lot of interesting folds and hinges developed.

### 10.12 Aegis Hyposurface

Another example of an interactive surface, the Hyposurface uses what seems to be a right isosceles triangle mesh (waterbomb base tessellation) along with small motors underneath that move each triangle in different directions.[42] [43] Again, another departure from continuous folding patterns, but this pattern of triangles may yield some interesting results.

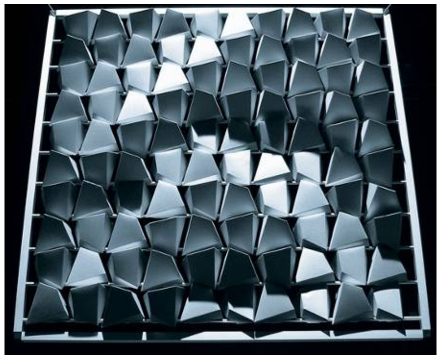

*Figure 10.12*
Hyposurface

[41] Nielsen, Stuart S. "Solar Tower and Jörg Schlaich" http://www.tallbridgeguy.com/2011/08/24/solar-tower-and-jorg-schlaich/ (accessed Oct 20, 2011).
[42] Hyposurfae Corp. "Hyposurface" 2007 http://hyposurface.org/ (accessed Sep 27, 2011)
[43] Oliver van Poucke. "Aegis Hyposurface" 2008 http://blog.kineticarchitecture.net/2008/12/aegis-hyposurface/ (accessed Sep 27, 2011)

### 10.13 WHITEvoid interactive art & design: FLARE kinetic ambient reflection membrane

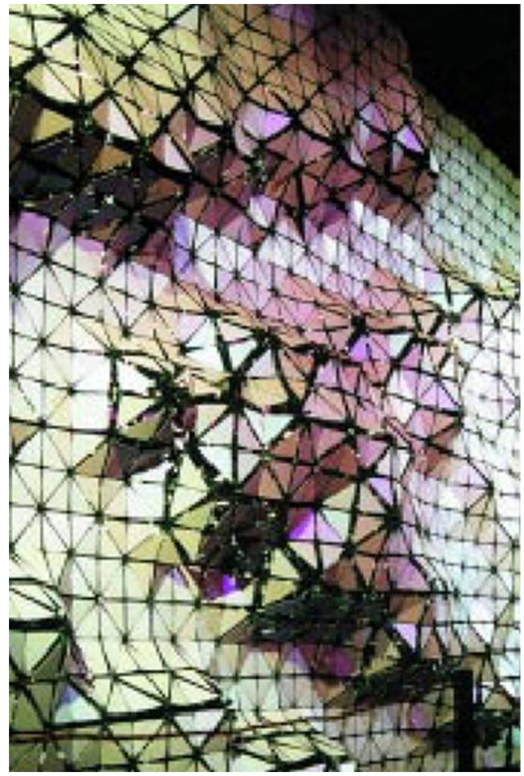

*Figure 10.13*
WHITEvoid

This is a membrane designed to go on the surface of a building, to be controlled by a central computer. In this design the panels are shaped to reflect light up or down via pneumatic cylinders and hinges to change the reflective appearance of the building façade.[44] [45]This is a departure from the more strict definition of an origami principle that is being sought, especially since each of these panels fold individually. It is an exploration into smart buildings and interactive surfaces. Nonetheless, all options are being explored.

### 10.14 Mitsuru's Magnolia Stadium: Yanko Design

*Figure 10.14*
Magnolia Stadium

Inspired by the Magnolia, the flower of Shanghai, this tennis stadium uses eight massive 'petals' that rotate and slide relative to each other to open and close the top. This operation takes about 8 minutes to complete. [46] Although there is a lot of sliding in this example, the way these petals interact with relation to their curves is unique.

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[44] WHITEvoid interactive art & design. "FLARE-façade" 2008 http://www.flare-facade.com/ (accessed Sep 27, 2011)
[45] Poucke, Oliver van. "Flare Kinetic Façade" 2008 http://blog.kineticarchitecture.net/2008/04/flare-dynamische-fassade/ (accessed Sep 27, 2011)
[46] Berrones, Ansley. "Mitsuru's Magnolia Stadium" 2007 http://www.yankodesign.com/2007/11/07/mitsurus-magnolia-stadium/ (accessed Sep 1, 2011)

**10.15 Hoberman Arch: Hoberman Associates**

One of the largest of the Hoberman Associates structures, this arch was used at the stage of the 2002 Winter Olympics in Salt Lake City, Utah. This created a very unique kinetic backdrop for the medal ceremonies, and even presented the Olympic flame as it opened.[47] [48] This structure could provide some insight on how to design the hinges of the foldable structure, even though in this particular case uses many pivoting joints. The mechanization to move all the geometrical movements could also be very useful. Very little of the geometry will probably be incorporated into this research. It uses less folding techniques and more pivoting arms and panels, which also creates many holes through the structure.

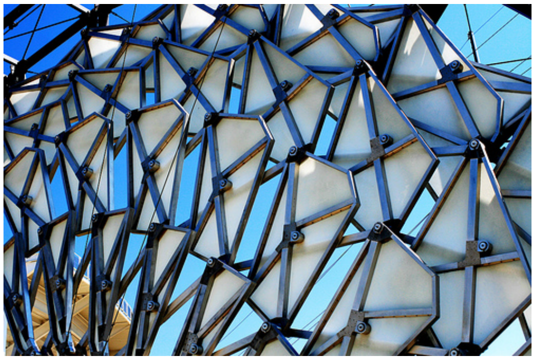

*Figure 10.15*
Hoberman Arch

**10.16 Kiefer Technic Showroom: Ernst Giselbrecht & Partner**

Although only incorporating simple folds, the showroom uses many electrically and centrally controlled panels for the façade of the building. These panels change position as the day progresses because it is a shading device.[49] This precedent would be a great source for finding out the mechanization and durability of hinges that fold often.

*Figure 10.16*
Kiefer Technic Showroom

[47] Hoberman Associates Inc. "Hoberman Transformable Design" 2002 http://www.hoberman.com (accessed Sep 27, 2011)
[48] Oliver van, Poucke. "Hoberman Arch" 2008 http://blog.kineticarchitecture.net/2008/06/hoberman-arch/ (accessed Sep 29, 201)
[49] Irina Vinnitskaya. "Kiefer Technic Showroom / Ernst Giselbrecht + Partner." 2010 http://www.archdaily.com/89270/kiefer-technic-showroom-ernst-giselbrecht-partner/ (accessed Sep 27, 2011)

### 10.17 Heatherwick Studio: Rolling Bridge

This footbridge had to allow access for a small boat to the inlet. “The aim was to make the movement the extraordinary aspect of the bridge.” The mechanism uses hydraulic rams that push the railings up and thus lift the bridge segments.[50] This is an especially interesting bridge in that each hinge can be operated individually, and stopped at any point. Using the hydraulic ram rods at the railings to bring the sides together seems like an unconventional solution.

*Figure 10.17*
Rolling Bridge

### 10.18 Dominique Perrault Architecture: Olympic Tennis Centre

These tennis courts designed for the 2016 Olympics utilize three massive slabs on the roofs that can be propped up with giant hydraulic jacks. The center roof, the largest of the three, is 102m x 70m and is 1,200 tons in weight. In addition to the simple hinging up motion, the roofs and jacks can all slide back to allow the courts to be completely open to the sky.[51] The massiveness of these hinges and mechanisms, when viewed as being an extreme, could help inform how the mechanics of folding should act on a smaller scale.

*Figure 10.18*
Olympic Tennis Centre

[50] Heatherwick Studio. “Rolling Bridge” 2004 / (accessed Sep 27, 2011).
[51] Dominique Perrault Architecture. “Olympic Tennis Centre” 2009 http://www.perraultarchitecte.com/en/projects/2461-olympic_tennis_centre.html (accessed Sep 27, 2011).

### 10.19 Mats Karlsson: Xile

In this example, the designer was able to create a unique and easy way to deploy structure to connect two spaces using some more intricate but already well establisheded geometry.[52] However, to accomplish this, the structure was made out of thin plastic, and thus is not a thick rigid structure; nor do the hinges utilize any kind of hardware, but rely solely on the pliability of the plastic material. This unit as pictured needs a separate sub-flooring structure in order to walk in it.

*Figure 10.19*
Xile

### 10.20 Tine Hovsepian: Cardborigami; Shelters for natural disaster victims

Similar to the Xile structure, it is a scaled down version of the same tube-like structure for deployment as temporary shelters in sites struck by disasters. It uses the same geometry and also relies on the outer layer being durable for weather but still pliable enough for thickness to be negligible. An improvement over Xile is that the floor is integrated into the CP of the structure so the entire unit is one piece. [53]

*Figure 10.20*
Cardborigami

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[52] Etherington, Rose. “Xile by Mats Karlsson” 2008 http://www.dezeen.com/2008/02/07/xile-by-mats-karlsson/ (accessed Sep 29, 2011)
[53] Kaur Sukhmani. “Simply designed cardboard shelters to house victims of natural disasters” 2010 http://www.designbuzz.com/entry/simply-designed-cardboard-shelters-to-house-victims-of-natural-disasters/ (accessed Sep 27, 2011)

### 10.21 Sanna Lindström and Sigrid Strömgren: Grand Central Table

In all the literature review and precedent research, this table, designed by Swedish designers Sanna Lindström and Sigrid Strömgren, is the best example of a prototype of something that is rigid, thick, and has 4 degree vertices. This table was inspired by a New York City folded subway map, which presumably unfolded in a similar manner. Although there is little that can be seen about the actual hinge connections in this table, the geometry does seem to work smoothly, without any deformations. What stands out in this example, and which has served as a great inspiration to this research, is the use of an extra hinge to make the pattern collapsible. This is rather difficult to see, as no diagrams are published, but can be found when carefully reviewing videos of the folding and unfolding motion. When viewing this video closely (which is available publically online), there is a third fold in portions that compensate for added thickness there is a third fold which allows for expansion from the flat length to the hypotenuse length.[54]

*Figure 10.21*
Grand Central Table

[54] Cliff Kuang, "Grand Central Table Folds Like a Subway Map" 2010 http://www.fastcompany.com/1613745/table-unfolds-like-a-subway-map (accessed April 11, 2012)

### 10.22 Rigid Twist Detail: Tom Crain

In this design, Tom Crain, wood worker turned origami artist, built a twist fold out of plywood and a novel hinge of screw eyelets and dowels.[55] His photos demonstrate the same problems discovered in my explorations of ultimately flawed designs (*Sec 9.3*). Additionally, this hinge is neither on a shifted axis, nor has material been carved to accommodate the folds. However, this model can fold about halfway, but the model needs to curl in one direction to do this, Hence the model can only be perhaps 3 or 4 units wide, otherwise the curling ends collide with each other.

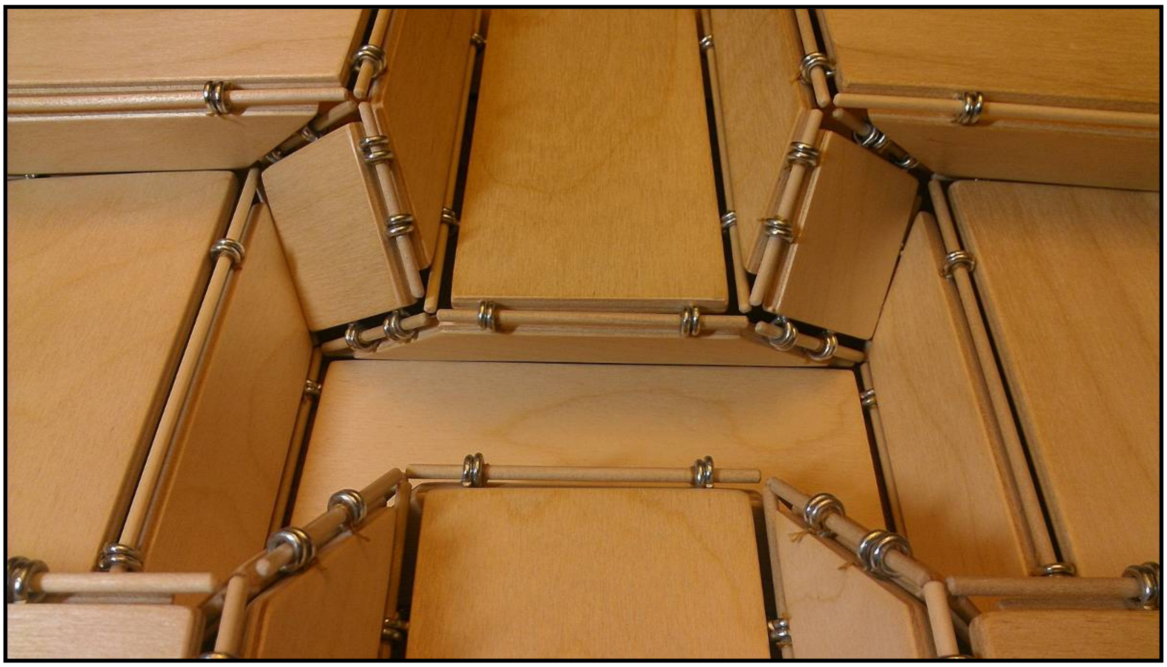


*Figure 10.22*
Rigid Twist Detail

[55] Tom Crain, "Rigid Twist Detail" 2015
https://www.flickr.com/photos/tomcrainorigami/15859844354/in/photostream/

**10.23 Singapore's National Design Centre: SCDA Architects**

SCDA Architects restored a former Convent School for the new Design Centre.[56] In particular, the 46' x 30' chapel was retrofitted with an origami-like structure that enhanced the performance and look of the space.

These panels were cut individually and anchored into place, but do not fold. Nonetheless, they mimic the style of reverse folded pleats. If they are actual representations of such a CP, an advantage would be that mesh is cut with minimal waste, as all the geometry would have been from a single plane. In any case, this example is only origami-like in appearance.

*Figure 10.23*
Singapore's National Design Centre

[56] Shonquis Moreno, "Building-Scale Origami in Metal" 2014, http://www.architectmagazine.com/technology/detail/building-scale-origami-in-metal_o

# 11 Simulating Rigid Thick Origami

When designing rigid-thick origami, it may be desired to first simulate the design in a 3D modeling program to see if the initial pattern, the final folded form, and the transformation process, all work according to the design intent being sought. There can be several programs which can accommodate this exploration. This section will introduce some of the underlying concepts, mathematical formulas needed, and suggest some specific techniques for modeling in Rhino Grasshopper.

## 11.1 Modeling Concepts

It is helpful to understand some general concepts and strategies for developing 3D models of rigid (both thin and thick) origami. Concepts will first be discussed in generalities, in case different programs are to be used. Section 11.3 will apply the concepts in Rhino Grasshopper.

Before creating a folding simulation, one may want to check if the CP conforms with Kawasaki's theorem at each vertex (*Sec 5*). Ultimately, a series of planes will be needed to represent the CP polygons, and a means of selecting edges in which to rotate the design. Rotate commands are used the most since a fold is essentially a plane rotating about the axis (crease line).

Using planar segments is the easiest way to develop and control the manipulation of the model. If a parametric modeling program is used for developing the CP, decide how to divide the CP, which aspects of the CP can be easily modified to aid in the parametric design, and by what controls these aspects will be modified. The pattern can be defined by planes, lines, or points, or a combination thereof. Once all the geometry has been devised, it can be modified by the software and developed into a folded form. If it is just a rigid-thin model it is a matter of selecting edges and rotating faces. If it is thick, then an additional step of extruding the planes is needed.

## 11.2 Formulas

The typical designer may not be familiar with the math behind the folding which involves more complicated terms and expressions. For the purposes of simplifying the subject matter, all that is needed is a means of expressing the motion of all of the polygons in the CP in relation to a given “control crease”, that is, the crease can be varied between 0° and 180°. Since all folding moves in a uniform matter, all creases will be dependent on the control crease.

The formula in this relationship is based on the angle of the reverse fold with respect to the pleat. This angle is called the dihedral angle and is denoted as $\phi$ (*Fig 11.2.2*). Note that this reverse fold must be symmetrical (*Fig 11.2.1*). An asymmetric fold is much harder to formulate, but asymmetric folds are seldom used in these cases.

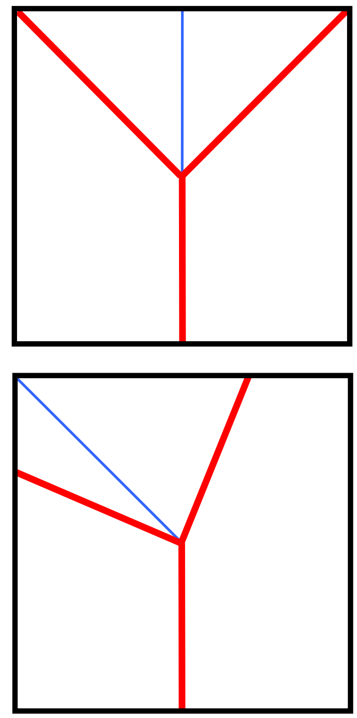
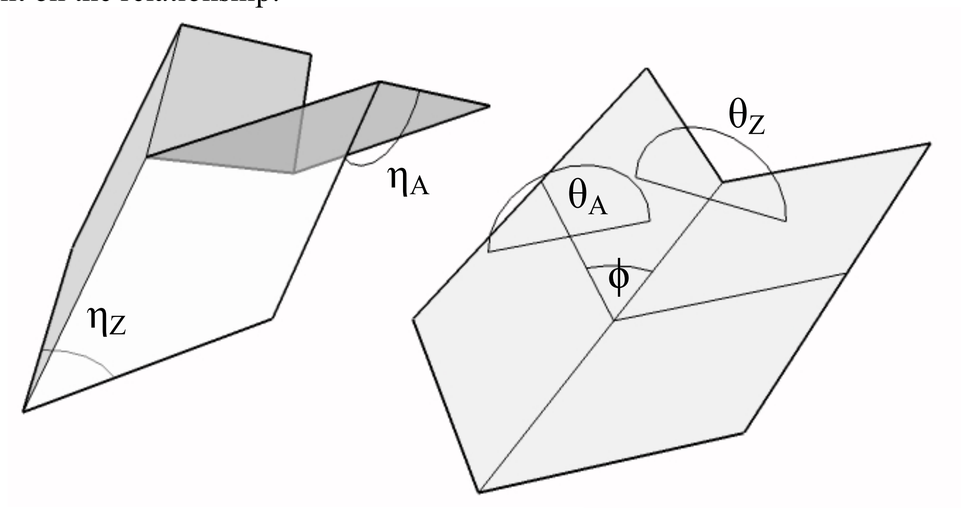


*Figure 11.2.1*
Both of these examples are rigid, but only the first is symmetrical.

Note that these formulas will primarily be used with the reverse fold family, and seldom on the tile tessellations, since the tiles have pleats which stay orthogonal and conform to a “control crease.” The last remaining folds can measure the state of the fold, and a simple deduction can be made for the remaining creases. For example, in the tile tessellations with tabs (*Sec 9.2.1-3*), all the pleats will be controlled, except for the tabs. As the pleats move, the tabs for a very simple triangle, the program can measure where the tabs should be, and draw it based on the geometry, so no complicated formulas are needed.

The difficulty lies in the fact that $\theta_A$ does not have a linear relationship with $\theta_Z$. There have been many mathematical papers discussing this relation across a CP globally. What a designer needs are simple functions in which to input the variables and determine the fold value. The following three formulas,[57] in the context of the following figure, start to shed some light on the relationship:




$$(1 + \cos \eta_Z)(1 - \cos \eta_A) = 4 \cos^2 \phi$$

*Figure 11.2.2*
Dihedral angle formulas

$$\cos \eta_Z = \sin^2 \phi \cos \theta_A + \cos^2 \phi$$

$$\cos \eta_A = 1 + \frac{4(\cos \theta_Z - 1)}{(\cos \theta_Z + 1)(\cos 2\phi - 1) + 4}$$

These formulas are not quite in a form that is helpful to designers. There are too many variables and it is not in the format of a simple function. The variables $\eta_A$ and $\eta_Z$ are additional variables especially helpful for Miura-map foldings (a particular repeating tessellation as can be seen in *Figure 11.2.2*), but these extra variables are not relevant to a broader range of CPs. To eliminate these variables, the second and third formulas can be substituted into the first and simplified in terms of $\theta_A$, such that only $\theta_Z$ and $\phi$ are needed to produce a formula (*Fig 11.2.3*). Incorporating this formula into a modeling program can be an easy way simulate the dihedral folding angles.

[57] Joseph Gattas, Weina Wu, and ZhongYou. "Miura-Base Rigid Origami: Parameterizations of First-Level Derivative and Piecewise Geometries." *Journal of Mechanical Design J. Mech. Des.*, 2013, 2-3.

$$\theta_A = \cos^{-1}\left( \frac{\dfrac{4\cos^2\phi}{1-(\sin^2\phi\cos\theta_Z-\cos^2\phi)} - 1 - \cos^2\phi}{\sin^2\phi} \right)$$

*Figure 11.2.3*
Dihedral angle formula

As can be seen, the relationship between the angles of creases is quite complicated, but for the purposes of designing rigid-thick origami, the understanding of the inner workings of this formula are not necessary. However, understanding how to input this formula into 3D modeling can be tricky, so here is the same formula, as a single line of text:

$\theta_A$ =Acos(((((4*(cos($\phi$)^2))/(1−(((sin($\phi$)^2)*(cos($\theta_Z$)))−
(cos($\phi$)^2)))−1)−cos($\phi$)^2)/(sin($\phi$)^2)))

Note that in using this formula, some scripts or programs may need a value other than the absolute 0 or a full 180 degrees. To avoid difficulties with the program, the numeric range should be kept between 0.001° to 179.99°.

This formula was entered into Rhino Grasshopper for parametric modeling to compute the $\theta_A$ for each pleat (*Fig 11.2.4*). Note that $\theta_A$ is negated every time the pleat is computed so that the CP will have *alternating* mountain and valley folds.

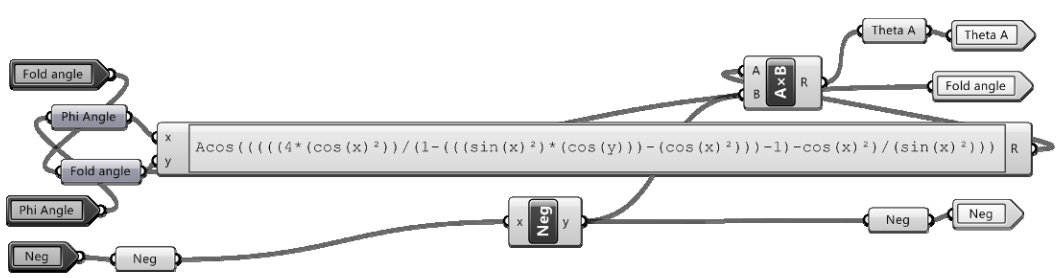



*Figure 11.2.4*
Dihedral angle formula in
Grasshopper

### 11.3 Rhino Grasshopper

Grasshopper is a plug-in script for Rhino which was found to be very useful in designing rigid-thick models, observing the folding transition, and making updates to the CP while in any state of folding. However, as described in the concepts, the script must be set up correctly at the start such that any input values will maintain the integrity of the model.

There are a few things to note when using Grasshopper. The scripts developed for this research grew very large and complicated and are difficult to publish in their entirety, thus specific examples will be given on key points. Also, since Grasshopper is a plug-in, it can work in conjunction with geometry made in stand-alone Rhino, however, the examples here will use solely Grasshopper, relying on no geometry being made in Rhino (the benefit being that no Rhino save files are required).

As discussed in the concepts, the two families are treated differently. The reverse fold family relies on the control of the creases of the pleats and the start of the reverse fold, and tiles seek to model one tile, and then use an array command for all the rest.

The set up of rigid-thick grasshopper scripts consists primarily of three parts. These can be thought of as the start, middle, and end.

In general, the commands that will be used most frequently are:

Loft – to create a surface from two lines
Rotate on Axis – for folding
Extrude – to provide thickness
List Item – to find the lines to fold about and points to intersect with
Merge – to regroup lists of items
Join – to make the bottom loft, top loft, and extruded side into a single panel

These are the general commands used most, and will vary on how the geometry is formed for each different case.

### The Start – User Defined Parametric Inputs

This is the organization of the means to control the initial geometry, the folding of the geometry, and the thickness. It is important that this be organized and clear as to how the modifications are occurring. Thickness can be a simple slider. A dial ranging from 0-179.9 for controlling the fold works well. For reverse folds, there were two sets of sliders for the x and y components on points, which create the initial lines for the reverse folds. There is another set of points forming lines to produce the pleats. All of the lines for the pleats get merged in a single list, which is passed through the clusters that form the middle.

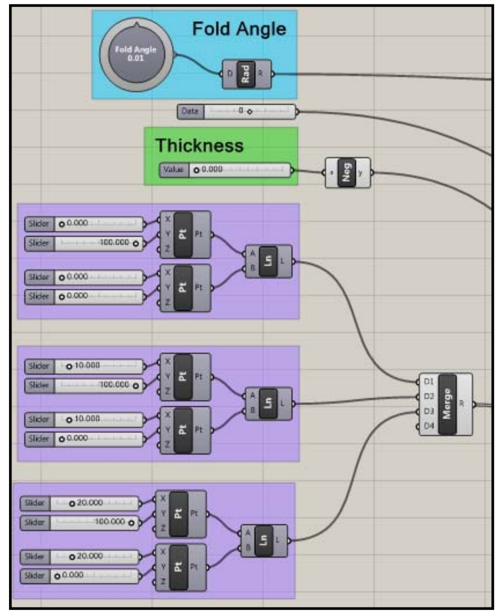



*Figure 11.3.1*
Start Parameters

A tile definition would have the dial and thickness too, but these sliders would work better if they controlled dimensions specific to the tiles, rather than each individual point.

### The Middle – Geometry Construction of Pleats and Reverse Folds

For reverse fold CPs, this middle section can consist of several actions. It can compute the reverse fold, the fold angle ($\theta_Z$), compute carving lines; create thickness, and make a fold. These actions work best if they are organized into columns, one for each pleat, and also rows for different functions; such as creating the reverse fold line, adding thickness, created carved geometry, etc.

It is imperative that each pleat angle is computed in terms of $\theta_A$, $\theta_Z$, and $\phi$. Although it may seem that all the pleats can be folded at an arbitrary angle so a plane can intersect and mirror thus creating a reverse fold, this will not be accurate. The fold angle ($\theta_Z$) of each

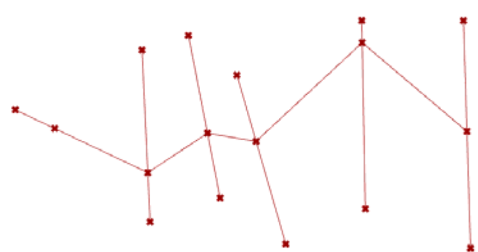


*Figure 11.3.2*
Reverse Fold

pleat is dependent on each reverse fold line (ϕ). Thus, the reverse fold line must be created in whole or concurrently, but not after folding that has begun. However, once the pleats have been folded according to calculations, all reverse folds can use a plane to intersect and mirror.

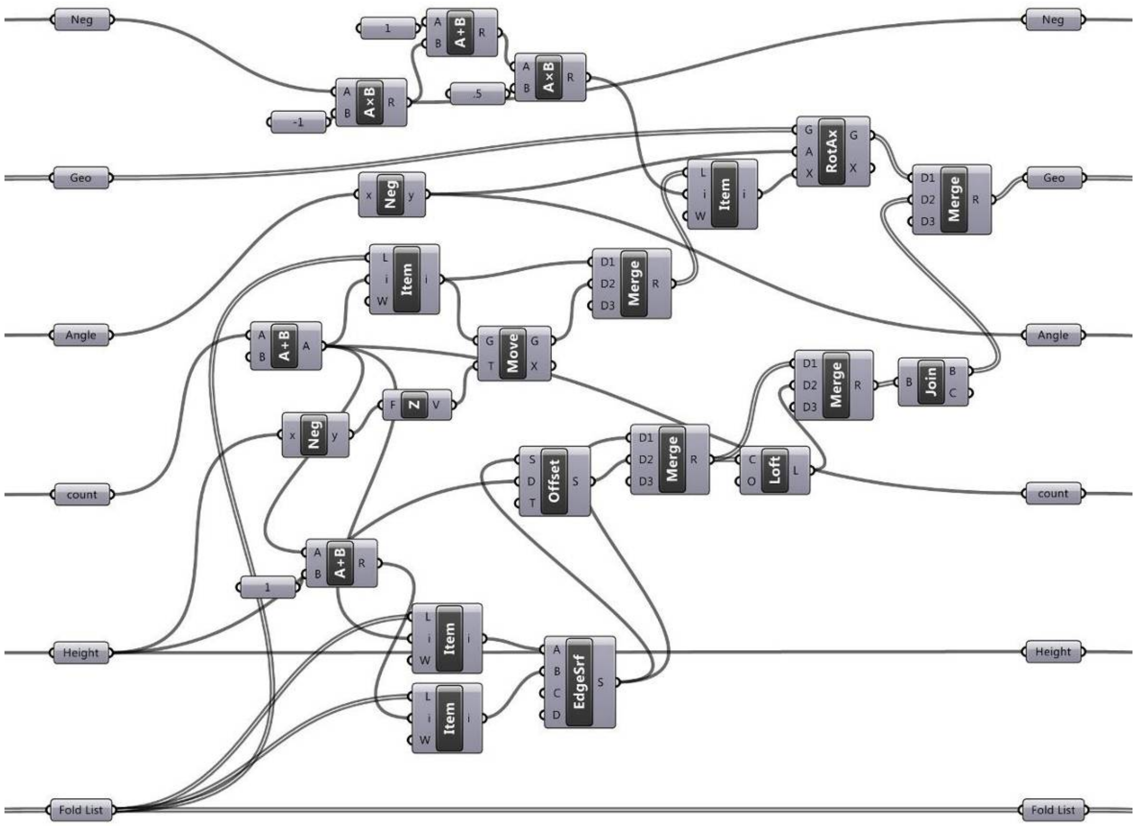



*Figure 11.3.3*
Middle section of code

**In what order to fold?**

This may seem odd, but the program must make a first fold somewhere. As long as the formula for the control angle is used, the pleats can be folded from either end, as long as the geometry stays together. Although one *can* start anywhere in the middle, it is best to start at an end, of which there are primarily two methods of organizing the movement: (a) Fold all geometry that has not yet been folded. (b) Fold all geometry that had been already folded.

Both methods are valid, however, the first example is easier in that the reverse fold crease line does not have to be rotated as well. If the unfolded panel moves, then the reverse fold crease must also travel to serve as the $\phi$ reference when it does come time to fold. Note that none of this will be seen, only the final model to be rendered. However it should be mentioned that large sums of pleats will require more computing time as each pleat doubles the effort for the computer.

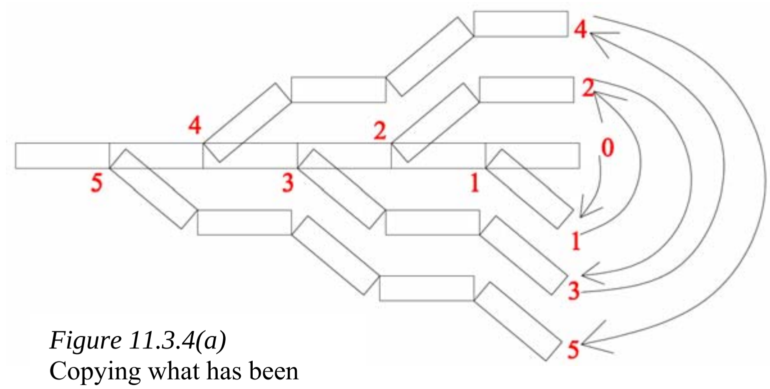



*Figure 11.3.4(a)*
Copying what has been folded

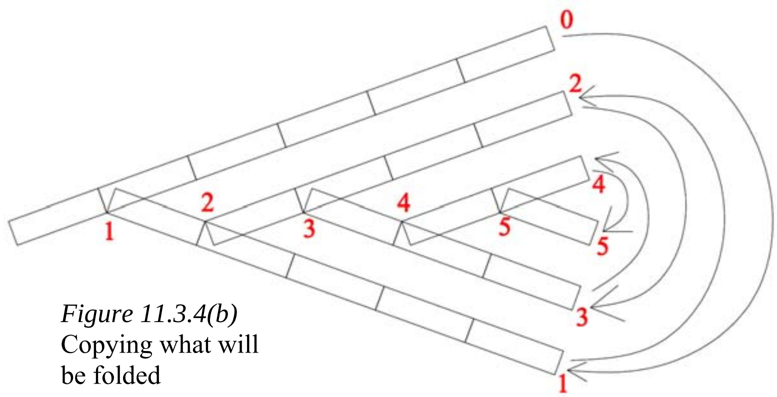



*Figure 11.3.4(b)*
Copying what will be folded

**Folding all at once**

One last note: It is *possible* that each pleat can be folded at once according to $\theta_A$, $\theta_Z$, and $\phi$ with respect only to the pleat in question. But there will then be many disconnected unaligned pleats in the model space. Each pleat could be reunited with the neighbor through a series of rotations and translations. Since the reference edge in each pleat changes with each edge reunification, the next reunification cannot start until the current one is done. Hence, even this method has to be done sequentially. Additionally, this motion requires several more translations, which means more individual computations per fold. However, the amount of transformations remains linear, despite the fact that many pleats exist.

There are no means (no simple means, at least) through which all of the pleats, that is, the $\theta_Z$ creases, can be folded at the same time. For all of them to move at once would mean they can all relate to a common origin. Rather each pleat is linked to its neighbors, much like a chain. The geometry of the numbering pleat determines the rotation.

**The Middle – Geometry Construction of Tile Tessellation Folds**

For Tiles, the process is much simpler since most of the geometry translates along a single plane, and the goal is just to make a single tile, but that does not mean it is less *messy* (*Fig 11.3.5*). This code is generating a simple rigid thick waterbomb base that

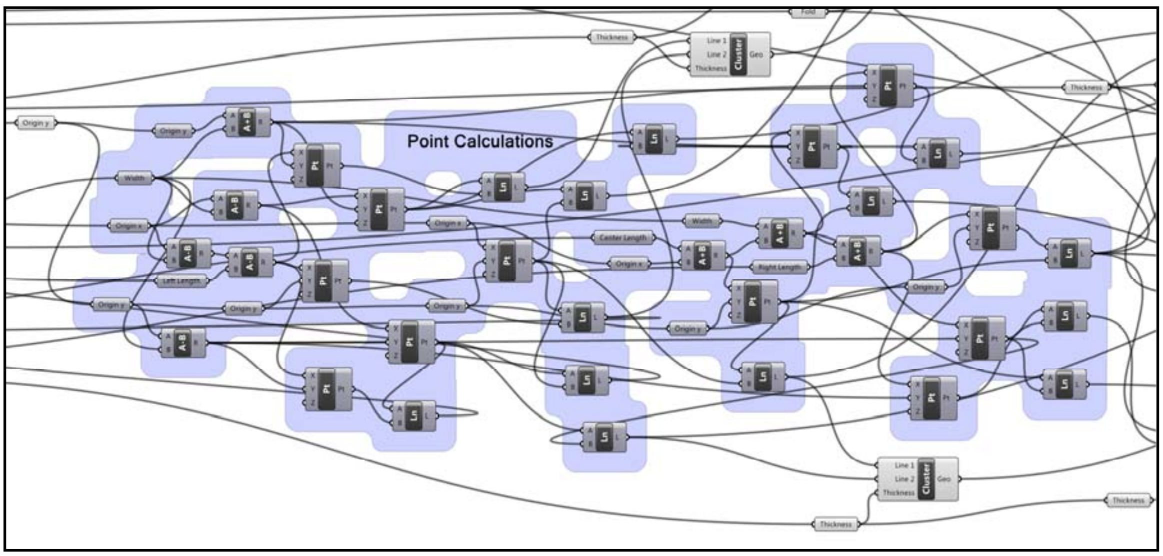
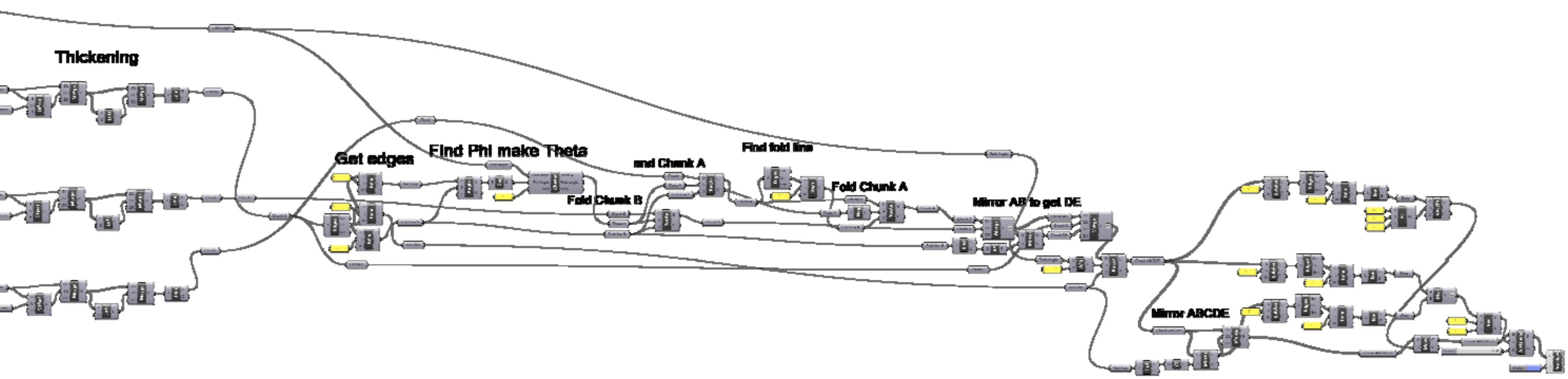



*Figure 11.3.5*
Computing points for a waterbomb array

starts as a simple square, but can be manipulated to different sizes and thickness. This section just determines the movement of the base points as set from the initial parameters. After this, geometry is rotated and mirrored to create the rest of the model. The exact script is different for each tile tessellation model. But in most cases, the initial CP will use as few initial points as possible, and extrapolate the rest based on the tile and what can vary in the tile. Even most of the geometry can be mirrored. The dihedral angle formula will be used very little only when waterbombs or other such reverse folds are used as part of the CP.

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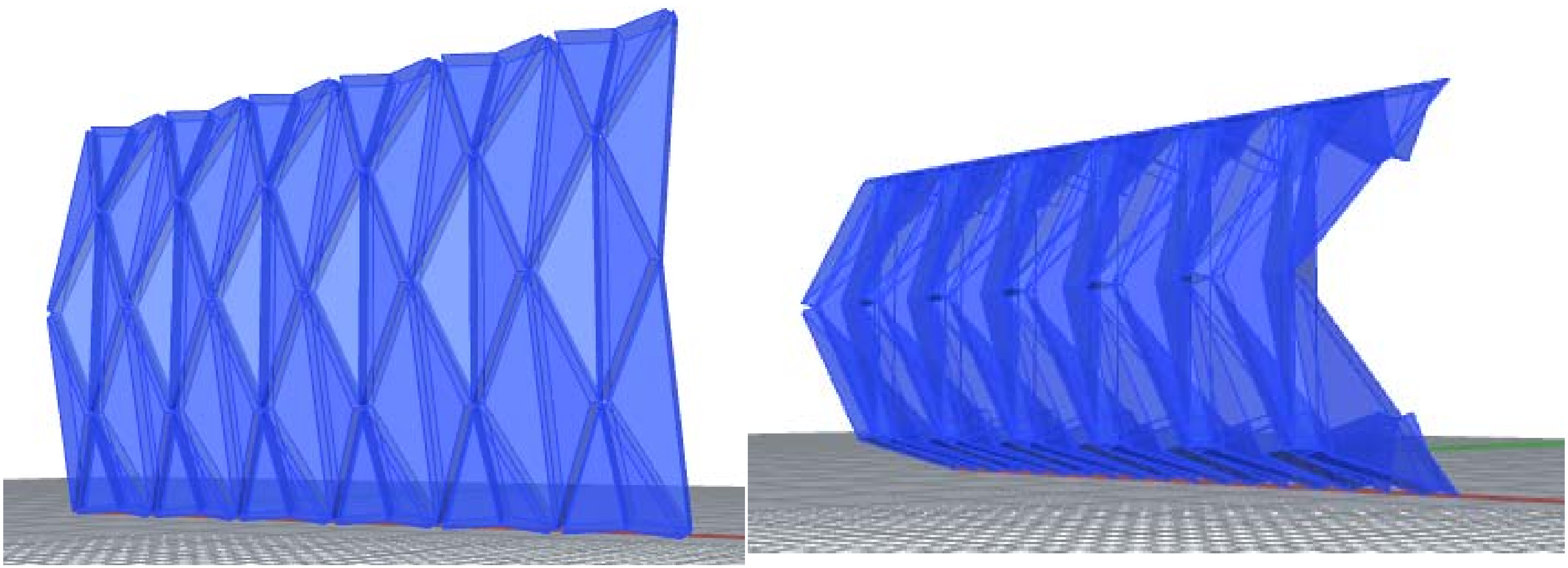
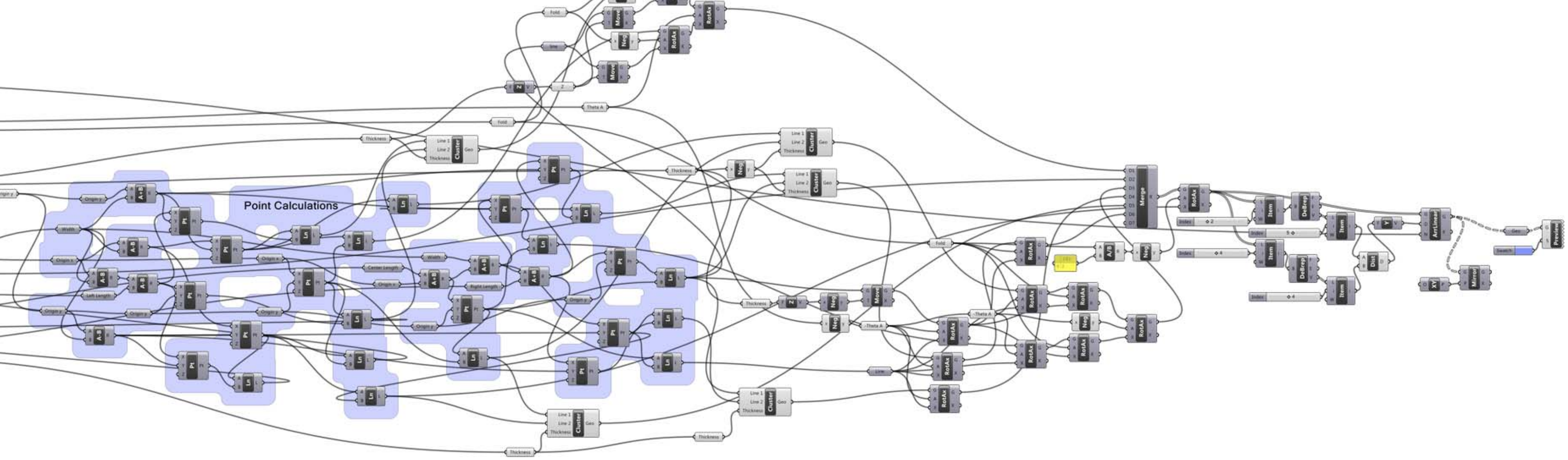


*Figure 11.3.6*
An array scripted as a tessellation.

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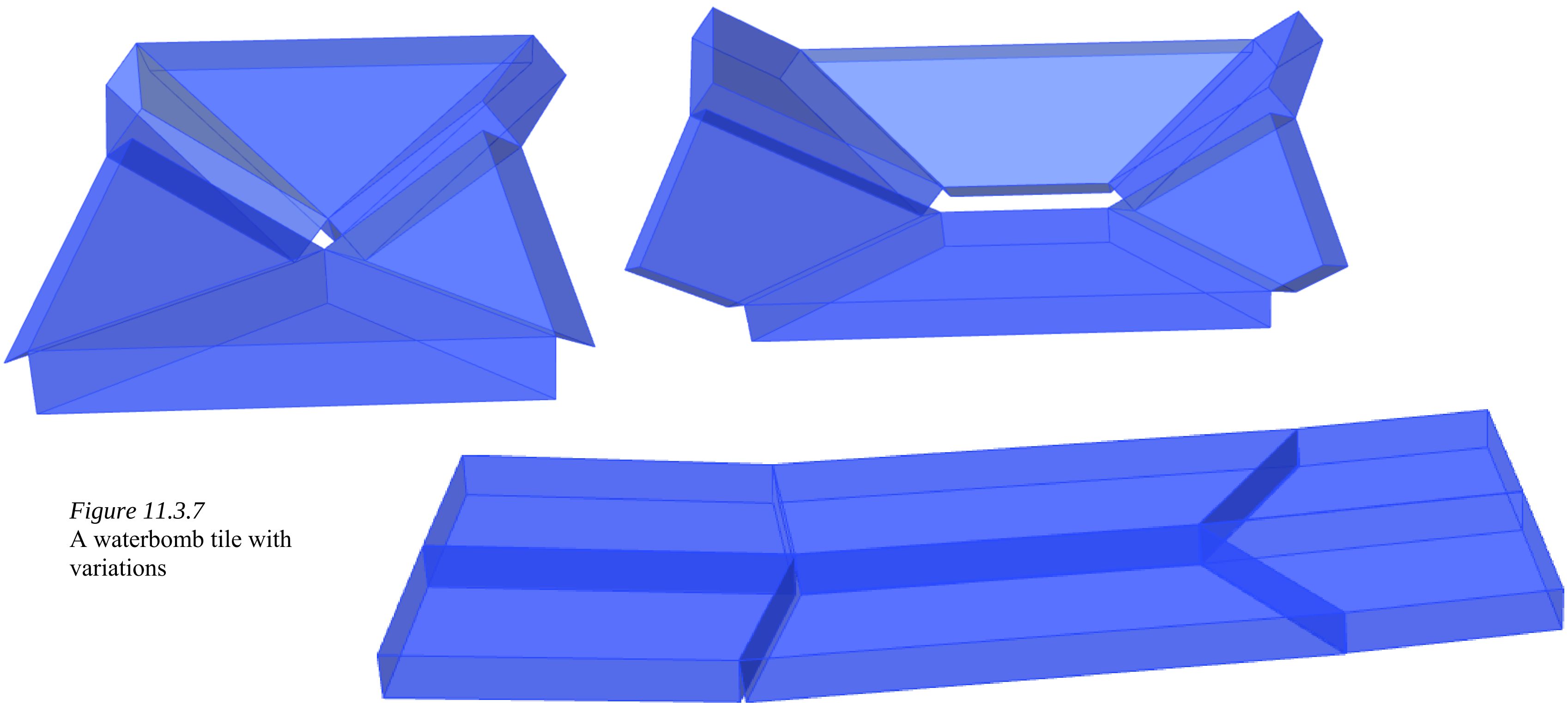


*Figure 11.3.7*
A waterbomb tile with variations

## 12. Construction Prototypes

There were three physical prototypes that were built to test the practicality of rigid-thick origami used in kinetic architecture. The first was a small scale test that investigated the actual workings of the hinges along the edges and at the vertex. After this model, a small mock up of the final prototype was made, followed by the full scale prototype.

### 12.1 Simple Reverse Fold

This simple four tile mock up was constructed with a 60° reverse fold carved tile. For simplicity and expediency, the material was doubled up to simulate the carved away spaces.

*Figure 12.1.1*
Small prototype in stages of folding

Construction started with four standard planks of 1”x8”x4’ pine lumber. From these planks eight copies of 60°/120° trapezoids were cut. Using a sled with a pivoting fence, any arbitrary angle could have been used because the cutting method would force all angles to be identical and complementary, however a 60°/120° ratio was especially nice as it made the carved spaces in the shape of equilateral triangles.

Once these trapezoids were made, a slot for the hinges was made so the panels could close flush, and the pivot point of the hinge would lay precisely where the theoretical hinge should be located. This was done by cutting a dado at the edge of the long side of the trapezoid with a depth of half the thickness of the closed hinge and a width half of the width of the open hinge. The eight planks were then separated into left/right pairs, where, in each pair, one plank had material cut from the left, then the other from the right. Standard 1½” x 30” hinges were cut, 20” for the long, pleated creases, 10” for the shorter, reverse fold creases. These were all installed, and then the left/right pairs were glued together. All hinge insets were cut, so it is possible to add additional tiles to this complete array of four panels.

This mid-size mockup was very strong and folded easily. It worked so well that when the mock up was held vertically by the top two panels without the bottom two panels having any support, and those top panels were folded (with a decent amount of force), the bottom two panels would lift and fold into place.

## 12.2 Small Triangle Array on a Frame

The previous 3D models and foam core studies displayed many strengths and weaknesses of rigid-thick designs, but ultimately these designs mean little as isolated folding patterns. Part of the goal of this research was to study how to adapt these patterns to a structure or building and develop a method for controlling the folding (*Fig 12.2.1-2*). This prototype served as a quick mock up to study any potential problems in the full scale mockup when attaching a rigid-thick pattern to a structure. The selection of the triangle

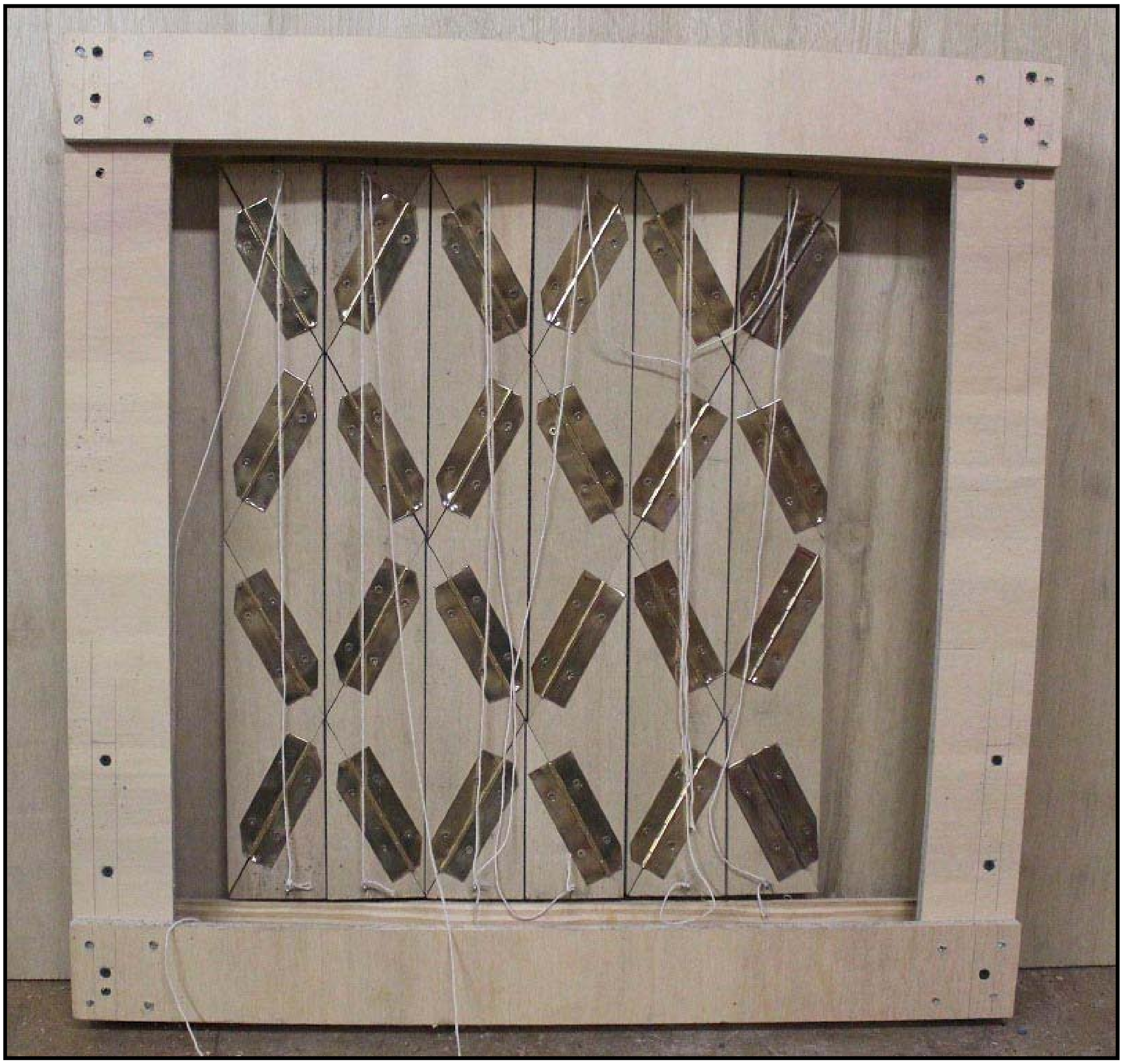


*Figure 12.2.1*
Small prototype in closed position

array was chosen because of its ease of fabrication. This way the emphasis of the study can be placed on mounting and controlling the array. Although this pattern was selected, in practicality, many other patterns can be mounted onto the frame in similar methods.

Several lessons were very quickly learned from building this mockup. 1) The motion of the panels worked best in an orientation where the bottom edge is pulled up to the folded position, and allows gravity to pull the panels back down to an unfolded position. 2) The points at the ends of the panels, which continuously stay collinear throughout folding, must be located. 3) The top panels travel in 3 degrees of freedom, while the bottom travels in 4 degrees of freedom. 4) A vertical pull would only fold the

*Figure 12.2.2*
Small prototype in open position

panel about 95% of the way. A final sideways manipulation would be needed for the final collapse. 5) Overexpansion of the hypotenuse plane may become an issue. The frame width will need to be oversized to allow for this.

For this model, the array was cut from 3/4” birch plywood, and all pieces were connected using small hinges. A connection that could rotate and slide was needed at the center points of the ends of the top and bottom row of panels. A screw head in a keyhole slot worked in this case. These top and bottom beams were also connected to the frame with just screw heads.

This small model functioned very poorly, but this was mainly due to the small scale of the array and relatively high inaccuracy of the alignment between the hinges and panels. As more panels were attached these inaccuracies built, and the entire model started to bind. It was suspected that the same pattern, scaled up, even if it was the same amount of error, the overall size of the model would reduce the error to a relatively small amount, and the model would bind significantly less. This was proven correct.

### 12.3 Large Triangle Array on a Frame

Many of the observations from the small mock-up were incorporated into the large prototype. The construction details, movements needed, and controls to move the panels were all incorporated into this design.

**The Array**

This was made from (2) sheets of 1/8” plywood measuring 4’-0” x 5’4”. These dimensions divide into an array with (12) 8”x16” right-triangle panels and (18) 8”x32” obtuse triangular panels. This pattern required (36) hinges measuring 14” long and (7) hinges 28” long. The panels were cut from a full sheet of plywood via CNC machine. The thicknesses of the panels were then achieved using 1”x2” lumber on edge between two panels of plywood.

Unlike the small version, an inset for the hinges was not cut. Not only would this require extra time and engineering, but since this was a sandwich panel made from 1”x2” lumber and 1/8” plywood, even a 1/16” inset would reduce half of the material holding the panel together. Thus it was decided to attach the hinges directly to the plywood.

**The Track**

Johnson sliding door hardware was chosen for the track because of the casters used. They have wheels which place the load evenly on both sides of the track, and they connect to the panel ends via a pin connection, which allows rotation as well. This 48” aluminum track is at the top and bottom and the panels. Extra casters were purchased so a total of 12 casters attach the array to the frame. Note that these casters are the sole connection points for connecting the array to the frame. After working with the frame adding track to the sides of the frame might be useful to help guide the bottom beam up and down.

**The Frame**

As with the small mock-up, the frame is a representation of an opening in a wall or roof. This was built from 2”x4”s and 2”x6”s with braces to keep the frame up. The design of this frame is completely irrelevant to the mounting hardware other than simply being a canvas to mount onto. In the context of an actual building, braces and many of the other features would not be needed (however, a wall might need additional structure points where the force of the bottom chord of the array may push laterally into the wall).

*Figure 12.3.1*
A simplified diagram of the DOF

**Degrees of Freedom (DOF)**

Analyzing and designing for the movement of the rigid-thick panels is a key component of mounting the array onto a frame. Since the panels move in so many different ways, it can be complicated to determine the points that will remain constantly collinear throughout the folding process.

This pattern, being of the reverse fold variety will have 3 DOF at the top, and 4 DOF at the bottom (*Fig 12.3.1*). The folded panels can slide as a whole to either side of the frame. There is also a rotation of the top and bottom beams as these panels rotate out. The panels also rotate about themselves as they swivel into the folded position. Lastly, the bottom chord has a $4^{th}$ DOF, as it is raised up from the array, curling into the folded position.

Tile tessellations, however, have primarily 2 DOF since the tiles translate into their final positions. Some sections do fold the full 180°, but since all the panels are primarily translating, none of the tracks need to be on hinges. If the casters are attached to the translating pieces, they can be attached via a plate. The casters can also be attached to the panels which rotate 180 degrees. Vertical tracks may be needed for tile tessellations, since there would be a series of panels there as well. However, since there will already be horizontal tracks with either the top or bottom or both moving, the vertical tracks will need to be in a different plane.

There are various designs that can be incorporated into this framework, but it is essential to use methods to either determine or design the key points that will remain collinear through the folding process, whether it be reverse folded or tile tessellation.

**Hypotenuse Overexpansion**

It was thought that the overexpansion of the hypotenuse plane, which occurs after folding starts, but before the panels start to collapse. The designer could either give extra space between posts to accommodate for this, or surface-mount the panels and hardware on the face of the wall, so the opening and panels do not correlate at all. The large frame was designed with the vertical supports placed about 4’6” apart, and that was **not** enough space for the over expansion. However, attaching the panels to a track (that is mounted on the top and bottom beams that fold out and away from the frame) moves the entire panel system outside the confines of the frame, before the panels expand too much. Since the casters are mounted on the centers of 8” wide panels, there is an extra 4” of track for the casters to move out before collapsing back inward.

### Mechanical Manipulation

Studies of rigid-thick design research had a shared commonality in that the manipulation of the designs tended to be done through direct physical grasping and moving with the users' hands. There is little specific research on attachment, let alone locomotion. If this is to be a system for walls or roofs, especially if used at larger scales, there needs to be a means to control it from a console. This could be pistons or motors and cables that are operated by computers. In this model, rope and pulleys hoist and lower the panels. In the prototype, rope from the top beam, looping through the bottom ends, and back up once more at the top provided a method to hoist up the bottom beam at half the speed while pulling down with half the force.

*Figure 12.3.2*
The large scale prototype in the open, unfolded position

*Figure 12.3.3*
The large scale prototype half way between open and closed

*Figure 12.3.4*
The large scale prototype in the closed, folded position

*Figure 12.3.5*
The reverse side of the large scale prototype in the open position


*Figure 12.3.6*
Close up of reverse side of the large scale prototype in the closed position

**Conclusion**

Although origami-like forms have been used in architecture, there are few that have incorporated more advanced forms of folding. Foldable membranes such as tents or simple thick folding panels arranged as pleats have been used. Small modular units with 4 or 6 degree vertices have also been explored, but to a lesser degree, and with fewer panels, or when used as a larger array, each tile is designed to be independent. Overall, there still seems to be a limited amount of origami inspired kinetic architecture.

This dissertation explored several origami design types in Section 9.1-2 which could allow for various types of design functions and intents. These design types vary in form and shape, and offer a great degree of variation and means of alterations. Reverse folds can be heavily modified and utilized in a variety of details, while the tile tessellations movements are very controlled, and the areas of dimensional variability are easy to define.

There are also several areas of origami which seem incompatible with rigid-thick design as seen in Section 9.3. These seem to involve all iterations of twist folds, or otherwise rotationally symmetric CPs. In general, flawed rigid-thick designs seem to usually have non-symmetric 4 degree folds, which do not have compliant Bennett linkage conditions, creating separations in the model during the folding process. These models serve as great examples of conditions to avoid, and provide the means to analyze if a particular CP will create such problems, or at least require further investigation.

The examples covered in Section 9 are by no means exhaustive, but this representative sampling covers a large proportion of typical origami tessellations. There are probably additional variations of rigid-thick origami types that can be employed. Hopefully, this guide will aid in the search and development of these additional designs.

Grasshopper parametric modeling was incorporated into the design process to see how the origami models moved during the folding process and how they could be modified. The real world mockups made it difficult to see if the folding was successful because it was indeed rigid thick compliant, or if the materiality of the model was flexible enough to give that impression. These computer models gave a means to see how the crease patterns moved in a truly rigid sense, since the computer geometry has no flexure.

These parametric models also gave a means to see how changes to the initial CP would impact the model, in whatever folded state it was in. This aided greatly in detecting collisions and measuring the over expansion of the hypotenuse plane.

In addition to the suggested designs of Section 9, a proposed means to attach them into architecture has also been developed. The large scale prototype used a reverse fold example, specifically the triangle array, which has more complicated translations and rotations, and thus, creates extra complications for attachments. The system of tracks and pulleys described can effectively manipulate the geometry mechanically for the 3 DOFs at the top track, and the 4 DOF at the bottom track. This prototype also showed that the over expansion of the hypotenuse plan can be circumvented, which is useful to understand when designing this type of kinetic architecture, especially if it is to be designed in the context of a frame or other surrounding materials.

It is hoped that with these new design techniques and examples, many different kinds of rigid-thick origami can be developed, and incorporated in unforeseen ways, while avoiding *seemingly* plausible designs which are in fact not rigid. These patterns and design techniques were not developed for a specific application, but instead presented so that any of these can be adapted and modified further for any application.

There are still many other areas within this topic that can be explored. The prototype models used plywood and lumber, but rigid foam panels, folded sheet metal, glazing, or a myriad of other materials can be further explored, provided the geometry discussed here is used.

There are still many questions pertaining to drive of the mechanics of the model. Although pulleys can, for the most part, pull up and let down the model, there are still some movement and binding issues that need to be studied further in greater detail.

Incorporating thickness into origami would combine the flexibility and various design intents of mobile or transforming building elements, with the benefits that a static wall has, such as durability, water proofing, insulation, and security. The thickness could even house the mechanics needed to drive the folding. Designing with thickness could accommodate designs as utilitarian as furniture, or as high-tech as solar panels for satellites. Perhaps there are even more applications outside of architecture.

## 14. Glossary

n-degree vertex - A point in which *n* creases arrive at a single point. Usually, these are 4 or 6 creases.

Base – Different sets of initial folds in which different origami models can be folded from. Traditional include waterbomb, fish, bird, and frog base. Contemporary origami has introduced many, many more.

Bennett Link – A constrained system of four linkages connected by four revolute joints.

Crease Pattern – An origami diagram that consists of all or most of the creases of a model mapped onto a square, as if unfolded, usually abbreviated to CP.

Flat foldable – A crease pattern which can be folded such that the final product is flat. Deformation can occur during the folding process, unlike in rigid origami.

Kinetic Architecture – A component of architecture that allows part of a building or structure to move or operate without reducing structural integrity. This tends to have a high degree of aesthesis not directly related to function.

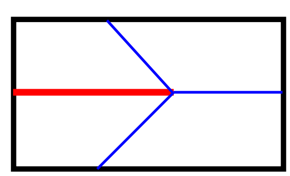
Mountain Fold – A crease in origami in which the paper on either side of the fold slopes down from the crease when partially folded. When turned over, the same crease will then be considered a valley fold.

NP-Hard – A mathematical problem, usually in computer science, in which there is no efficient algorithm to solve, and brute force computations are needed, in which each set must be computed, usually increasing exponentially as set n increases linearly. Determine flat foldability, rigidity, and collisions efficiently in origami tends to be difficult.

Origami - the art, craft, or science of folding paper, usually from a uncut square piece of paper.

Origami Tessellation – A sub-genre of origami, in which a series of folds can be tiled such that all creases align with creases in neighboring tiles, and the entire tessellation is flat-foldable.

Pleat Fold – A series of parallel (or near parallel) alternating mountain / valley folds



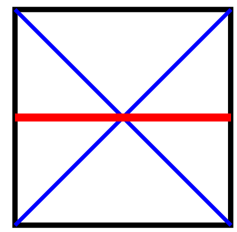
Reverse Fold – A fold that changes M/V assignment of part of a crease by adding two other creases <90° from the changed crease.

Rigid Origami - Origami in which each surface surrounded with crease lines neither stretches nor bends.

Tessellation – The tiling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps.

Valley Fold – A crease in origami in which the paper on either side of the fold slopes up from the crease when partially folded. When turned over, the same crease will then be considered a mountain fold.

Waterbomb Base – Named after the origami waterbomb, this base is used in many other origami models. It is simple two diagonal valley folds, with a single horizontal mountain fold through the middle.



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