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Models and Algorithms for the Integrated Planning of Bin Allocation and Vehicle Routing in Solid Waste Management

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Abstract

The efficient organization of waste collection systems based on bins located along the streets involves the solution of several tactical optimization problems. In particular, the bin configuration and sizing at each collection site as well as the service frequency over a given planning horizon have to be decided. In this context, a higher service frequency leads to higher routing costs, but at the same time less or smaller bins are required which lead to lower bin allocation investment costs. The bins used have different types and different costs and there is a limit on the space at each collection site as well as a limit on the total number of bins of each type that can be used. In this paper we consider the problem of designing a collection system consisting in the combination of a vehicle routing and a bin allocation problem in which the trade-off between the associated costs has to be considered. The solution approach combines an effective VNS metaheuristic for the routing part with a MILP-based exact method for the solution of the bin allocation part. We propose hierarchical solution procedures in which the two decision problems are solved in sequence, as well as an integrated approach where the two problems are considered simultaneously. Extensive computational testing on synthetic and real-world instances with hundreds of collection sites show the benefit of the integrated approaches with respect to the hierarchical ones.

Keywords: Waste collection, Logistics, Matheuristics

1 Introduction

The collection and treatment of waste is one of the most difficult and expensive operational problems faced by local authorities. It involves several types of waste to be collected, such as the Municipal Solid Waste (MSW) produced by houses and commercial activities including separated waste (paper, glass, organic, ...) and Industrial Waste (IW) produced by factories and construction activities, some of which are hazardous or toxic. Each type of waste requires specific collection procedures and technologies, vehicles, as well as treatment and disposal facilities. For an overview of most operational problems related to waste management planning we refer to Golden et al. (2001), and Ghiani et al. (2011), whereas reviews of practical applications are given in Vigo et al. (2007).

The volume of MSW to be managed for each town or province is really large and constantly increasing over time. Each year in the European Union more than 3 billion tons of waste are produced, among which 15-25% are MSW and the remaining are IW (see OECD (2007)). Moreover, the estimated increase in total waste production for the European Union in the period 1995-2020 is about 45% and similar figures can be found for the United States and other countries. Such volumes require very high levels of services from the collection systems. In urban areas collection sites must be visited several times during the week, and in some extreme cases even two or three times during each day. In addition, a large network of plants for the treatment and disposal is required. As a consequence the costs associated with waste management are huge and a large part of them is related to the logistics activities. For example, in Italy the treatment of the more than 30 million tons of MSW produced during 2004 cost more than 4.5 billion Euro and about half of that was due to collection and transport (see APAT (2006)).

In recent years, due to a number of cost, health and environmental concerns, many municipalities, particularly in industrialized nations, have been forced to assess their solid waste management and examine the cost-effectiveness and environmental impacts. The result is that both private and municipal haulers are giving serious consideration to new technologies such as computerized vehicle routing software that, compared with the traditional manual design of the collection routes, allows reduction of the operational time, taking into account a large set of variables and constraints and carrying out scenario what-if analyses.

The design and management of an effective collection service involves several steps, which in turn can be grouped into three main phases. The first one is the "Demand Analysis" where through dedicated forecast models and specific data collection, the quantification of the waste to be collected over a territory is performed. The second phase, which we call "Supply Planning" involves the choice of the specific collection model, namely a door-to-door service or a collection based on large street bins, that are used for each type of waste. In addition, the appropriate service frequency (i.e., number of bin emptyings or visits to individual houses in door-to-door systems) and capacity allocation to collection sites (i.e., the size and number of bins present) must be determined. Note that these two decisions are strongly interrelated. Given the demand of a collection site, by increasing (resp. reducing) the number of bins located there, one may reduce (resp. increase) the required service frequency of the site. Therefore, varying the capacity allocation criteria on the one hand impacts the fixed investment costs associated with service setup. On the other hand, it may influence the variable costs associated with the collection as a consequence of the corresponding service frequency. Finally, the last phase involves the design of the vehicle routes for the collection vehicles on each day of a planning horizon.

Almost all literature on optimization methods applied to waste collection, described in detail in the next section, is concentrated on the last phase of the planning process, i.e., on routing optimization, where several sophisticated approaches were developed. However, it is clear that a large opportunity for improvement is represented by the definition of integrated planning approaches that allow various steps of the process to be incorporated. For example, preliminary experiences conducted in early 2000 in Northern Italy show that simultaneously considering capacity allocation and vehicle routing may lead to much larger savings compared to simply optimizing the routing (see Vigo et al. (2007), ch. 6.5). Starting from these assumptions, in this paper we define an approach that integrates the routing of the waste collection vehicles and the allocation of waste bins to the sites. For a given planning period we develop a schedule for the waste collection vehicles that can be applied repeatedly and we also determine the number and type of bins to put at each waste-collection site. As previously mentioned, this problem that we denote as the Waste Bin Allocation and Routing Problem (WBARP), has not been tackled in the literature so far.

More precisely, we consider a collection system based on street bins where the location of the collection sites is fixed and known a priori. We also know the waste generation rate (e.g., daily) of each type of waste for each collection site. For each waste-collection site we want to determine the service frequency as well as the particular days of visit associated with this service frequency. We also want to decide the capacity allocation, namely type and number of bins to be put at each site. Bins differ in size, space requirement and purchase cost. Finally, we have to plan the daily routing for a fleet of vehicles based at a depot and using Intermediate Facilities (IFs) such as landfills, incinerators or recycling facilities for dumping. In this paper we study different versions of this problem, e.g., by considering one or more types of waste and by considering an existing initial bin distribution as starting point.

We developed solution methods based on a combination of Variable Neighborhood Search (VNS) and Mixed Integer Linear Programming (MILP). The VNS is used to solve the routing part whereas the MILP is used to solve the bin allocation part. Such approaches, where mathematical optimization techniques, such as MILP, are combined with metaheuristics, such as VNS, are often called Matheuristics. To better understand the role of the various optimization problems included in the integrated approach (i.e., capacity allocation and routing) we will both consider hierarchical approaches in which the problems are solved in sequence as well as an integrated one.

The remainder of this paper is organized as follows. In Section 2 we review the most relevant approaches for waste collection optimization presented in the recent literature. In Section 3 we describe the problem and in Section 4 the solution approaches. The results of our computational testing on both generated and real-world data are discussed in Section 5. Finally, Section 6 draws some conclusions.

2 Related Work

As previously mentioned most of the work presented in the literature dealing with optimization of collection systems refer to the development of algorithms for the routing of vehicles used for collection. The model most often used is the so-called Periodic Vehicle Routing Problem (PVRP) that was first proposed within the waste collection context by Beltrami and Bodin (1974). The main reference for the routing literature up to the end of 20th century is represented by Golden et al. (2001) where the distinction between commercial collection, residential collection and roll-on-roll-off problems is examined. While the first one involves the collection of bins/containers at commercial locations and is therefore a node routing problem, residential collection involves collecting household refuse along a street network and these are therefore generally modeled as an arc routing problem. We note however, that also several residential collection cases are appropriately modeled as node routing problems. This is when (as happens in several countries) waste is not collected along the street network at individual houses, but from large street bins located at waste collection points. Finally, Roll-on-roll-off problems occur when large containers or trailers have to be picked up at construction sites or industries and then transported to dumping facilities and unloaded. This problem was studied by Bodin et al. (2000) and recently generalized by Archetti and Speranza (2004).

The most recent literature on waste collection routing optimization attempted to incorporate into the PVRP model several new features of the real world application. In the following we briefly review only the recent papers that considered issues related to the specific problem we are studying here.

For example, Angelelli and Speranza (2002b) considered that collection vehicles use Intermediate Facilities (IF) for unloading along their routes. The corresponding problem was called PVRP with Intermediate Facilities (PVRP-IF) for which a tabu search algorithm was developed. In Angelelli and Speranza (2002a) such an algorithm was used to measure the operating cost of three different waste-collection systems. Recently, Hemmelmayr et al. (2011) studied a real world application in waste collection. In this paper, they proposed a successful VNS algorithm for the PVRP-IF and the multi-depot vehicle routing problem with inter-depot routes.

One of the first cases in which service frequency is combined with routing is given by Baptista et al. (2002) who examined the collection of recycling papers in the Almada municipality in Portugal. Their problem is modeled as a PVRP. However, unlike in the classical PVRP the visit frequency is not fixed, but is a decision of the model. The problem was solved through a heuristic based on the method proposed by Christofides and Beasley (1984): First initial frequencies and visit day-combinations are assigned to the points, followed by an interchange procedure that tries to find better visit day-combinations. These concepts are further developed by Francis et al. (2006) who proposed an extension of the PVRP, called PVRP with Service Choice (PVRP-SC) motivated by an applications arising in interlibrary loan delivery services. In the PVRP-SC the visit frequency is not given, but chosen during the search. Moreover, a cost is defined to measure the benefit of a service frequency. They developed an exact solution method for the PVRP-SC and a heuristic that can be used for larger problem instances. In Francis et al. (2007) they explore the trade-offs between operational flexibility and complexity in periodic distribution problems by incorporating different flexibility options in a tabu search heuristic.

3 Problem Description

In this section we describe in detail the problem occurring in the waste collection industry where the integration between supply planning and vehicle routing is explicitly considered. The problem is called the Waste Bin Allocation and Routing Problem (WBARP) and requires to balance the trade-off between the service frequency of a given waste-collection site over a planning period, and the number of bins that can be placed there. More precisely, if a site is associated with a higher service frequency, the routing cost will increase, since this site has to be visited more often, but at the same time the allocation cost is reduced, because a smaller number of bins is required at the site. Bins used may be of different types, each characterized by different capacity and cost. Moreover, due to possible space limitations at each collection site there may be a limit on the total number of bins that can be used.

Therefore, WBARP consists of two aspects. On the one hand there is the routing with the appropriate service frequencies, which turns out to be a PVRP-SC. Moreover, since intermediate facilities for dumping are almost always used, the actual routing problem is a PVRP-IF-SC. On the other hand, there is the problem of deciding how many bins to place at each site given the restrictions on space and on the total number of bins to be used.

We now describe the WBARP more precisely and introduce the necessary notation. For the sake of simplicity we initially consider the case where only a single type of waste is present. The case with multiple types of waste is introduced in Section 3.2.4.

The problem considers the service along a *time horizon* of T consecutive days. For MSW typically $T = 7$, i.e. one week, but it may be larger for separated and industrial wastes. We are given a set V of n *collection sites*, each characterized by the volume of waste produced per day, q_i, and by the total space, U_i available for the bins. The space is measured in a one dimensional fashion, i.e., how much space is available for bins placed side by side. Service at collection sites may be performed according to different possible *service profiles* which specify the relative days of the time horizon when the bins are served (i.e., emptied). For our purposes, we are given H service profiles, each associated with the maximum number of days between two consecutive visits, a_h, $h = 1, \dots, H$. For example, given a service profile with a frequency of three visits per week and where emptyings are performed on Monday, Wednesday and Friday, the associated value of a_h is 3 (i.e. the days between Friday and the next Monday). We only consider periodic visit day-combinations, where all the combinations belonging to the same service frequency have the same a_h and we, therefore, use the terms service profile and service frequency interchangeably. Note indeed that, under the reasonable assumption of a constant production rate at the sites, to appropriately define the total capacity required at a given collection site the only important information about the service profile is the maximum number of days between two consecutive visits.

We are also given a set of m *bin types*, each characterized by a volume capacity Q_j and space u_j it requires. Each bin type has a *purchase cost*, C_j^P that may also include the maintenance costs, and that is the actual quota of total purchase and maintenance costs relative to a single time horizon T. Moreover, specific costs exist for the *transfer* of a bin from the depot to a site, C_j^T, and for the *removal* from the site to the depot, C_j^R. Note, that the transfer of a bin from one site to another consists of a removal from one site and a transfer to the other one. In practice transfers are indeed performed by first collecting all containers, then bringing them to the depot for cleaning and servicing and finally distributing all containers to their destination. At most M_j bins of type j can be purchased.

For the situations in which an initial distribution of bins exist, let p_{ij} denote the initial number of bins of type j present at site i, with $i = 1, \dots, n$ and $j = 1, \dots, m$.

Finally, when multiple types of waste are considered we have K types of waste and some input parameters, such as the volume of generated waste, require an additional index k to identify the waste type. For each bin type $j = 1, \ldots, m$ a binary coefficient W_{jk} takes value 1 if the bin can be used for waste type $k = 1, \ldots, K$.

Table 1 summarizes all the parameters used hereafter for the bin allocation. Where appropriate, the specific index for the waste type is also given.

Table 1: The parameters of the Bin Allocation Problems

Parameters	
n	number of sites
m	number of bin types
K	number of waste types
H	number of possible frequencies or service profiles
a_h	maximum number of days between two consecutive visits for service frequency h
Site data	
q_i, q_{ik}	volume of waste (of type k) produced per day at site i
U_i	maximum total space for bins at site i
p_{ij}	initial number of type j bins at site i
f_{ih}	flag taking value 1 if service frequency h is used for site i.
	(In some scenarios it can be a binary decision variable)
Bin data	
u_j	space needed by a type j bin
Q_j	volume capacity of a type j bin
M_j	maximum total number type j bins that can be purchased
C_j^P	purchase cost of a type j bin
C_j^T	transfer cost of a type j bin
C_j^R	removal cost of a type j bin
W_{jk}	flag taking value 1 if type j bin can be used for waste type k
p_{ij}	initial number of bins of type j present at site i

In the following we give a detailed description of the problem by separately considering the two main components it is divided into: namely the routing and the bin allocation parts. We keep the description of the two parts separate since also our solution methods generally solve them separately either in a hierarchical fashion or by introducing the required communication between the two problems through the service frequencies decisions.

3.1 Routing Part

The routing part of our problem is a PVRP-IF-SC. As in the classical VRP we have a limited fleet of homogeneous vehicles based at a common depot, each having the same capacity. Each vehicle may perform a route which uses IFs for the unloading and there is an overall limit to the route duration. Each portion of a route between either the depot and one of the IFs, or between two IFs, is called a *trip*.

The PVRP extends the Vehicle Routing Problem (VRP) to a planning horizon of T days. A set of customers requires regular visits during the planning horizon. The timing of the visits is not given, but every customer has a certain visit frequency, e_i, that must be respected. Moreover, for every customer a set C_i of allowable *visit combinations* (i.e., a set of days in which the service must occur) is given. For example, if $e_i = 2$ and we have a given set of periodic combinations $C_i = \{\{1, 4\}, \{2, 5\}, \{3, 6\}\}$, then customer i must be visited twice during the planning period and these visits can take place either on days 1 and 4, or on days 2 and 5, or on days 3 and 6. Note that, in all such cases the maximum number of days between two consecutive visits is always equal to $a_h = 4$ days. Hence, the PVRP consists of choosing for each customer a visit combination and defining a set of routes for each day of the planning horizon with overall minimum routing cost associated with the arc traversal.

In the PVRP-SC the visit frequency is not given, but chosen during the search. Moreover, a cost is defined to measure the impact of a service frequency: in our problem such an impact is based on the bin allocation cost associated with the frequency. Moreover, as previously mentioned in PVRPs arising in solid waste collection there is a set of IFs where vehicles can unload and continue their trip afterwards. In our problem those can be either landfills or incinerators where the waste is dropped. Note that in the routing problem included in WBARP, customers are actually the collection sites, and we will use the two terms interchangeably in the following sections.

3.2 Bin Allocation

In order to better understand the impact on the overall solutions of the various components included in the bin allocation part of the problem we study four different versions, or scenarios, of increasing complexity. Each scenario is an extension of the previous one, i.e., Scenario 4 includes all aspects of Scenarios 1-3 plus a new one. The four scenarios are the following

- Scenario 1 - Basic Case: Only site capacity requirements and no starting bin configuration
- Scenario 2 - Space Restricted Case: Space restriction at each site
- Scenario 3 - Consistent Case: Consider a starting bin configuration
- Scenario 4 - Multiple Waste Types Case: Multiple types of waste

The first two scenarios can be seen as so called *greenfield* scenarios where we plan the allocation from scratch. Scenarios 3 and 4 are instead *brownfield* scenarios that also consider the previous site configurations and use them as a starting point for a re-planning activity. Moreover, since in the first two scenarios we do not consider explicitly a limit on the total number of bins that can be used, such problems are separable by collection site, i.e., they can be solved separately for each bin site.

3.2.1 Scenario 1 - Basic Case

The first scenario is the so-called basic scenario. A set of sites has to be served with a service profile h and a set of bin types is available to equip the sites. The bin types differ in capacity and purchase cost. The objective is to minimize purchase cost so that the set of bins chosen provide sufficient capacity for the waste produced. As previously mentioned, we do not consider a limit on the total number of bins to be used. The problem can be formulated by using a set of integer variables x_{ij}, that give the number of bins of type $j = 1, \dots, m$ to be placed at collection site $i = 1, \dots, n$.

$$\text{(P1)} \quad \min \quad \sum_{i=1}^{n} \sum_{j=1}^{m} C_j^P x_{ij}$$

subject to

$$\sum_{j=1}^{m} Q_j x_{ij} - q_i a_h \geq 0 \qquad \forall\, i = 1, \dots, n, \tag{1}$$

$$x_{ij} \geq 0 \text{ and integer}, \qquad \forall\, i = 1, \dots, n; j = 1, \dots, m. \tag{2}$$

The objective function minimizes the total purchase cost. Constraints (1) state that the maximum volume of waste accumulated at the site when served with frequency h does not exceed the capacity of the bins put there. It is clear that such a problem is separable for each collection site $i = 1, \dots, n$ and the resulting problem for a given service frequency is a bounded knapsack problem in minimization form which can be solved very efficiently (see, e.g., Pisinger (2000)). It is therefore possible to determine the cost for every separate collection site i and every possible service frequency h, independently.

3.2.2 Scenario 2 - Space Restricted Case

The second scenario is the extension of the first scenario where a further constraint on the total space available at each site $i = 1, \dots, n$ is given. This constraint is necessary in urban areas, where space along the streets is limited and few large bins are preferable with respect to several smaller ones. The model, called (P2) is the same as that of the previous section with the addition of the space constraints:

$$\sum_{j=1}^{m} x_{ij} u_j \leq U_i \qquad \forall\ i = 1, \dots, n. \tag{3}$$

Again, it is possible to separate the problem and determine the cost for every collection site i and every possible service frequency h, independently.

3.2.3 Scenario 3 - Consistent Case

In the third scenario we also consider the existing configuration of bins at the collection sites. This may happen in brownfield cases where a given allocation of bins is present and a re-planning is needed. The existing bin allocation is provided by the input parameter p_{ij} that represents the bins of type $j = 1, \dots, m$ initially present at site $i = 1, \dots, n$. Note that, by setting p_{ij} to zero one has the greenfield situation. Another frequent issue that happens in re-planning brownfield applications is that there is a limit on the total number of bins that can be purchased. To handle such a situation we introduce a maximum total number M_j of type j bins that may be purchased, $j = 1, \dots, m$. Note also, that within these settings it may be convenient to move bins from a site to another one or to permanently remove them from a site. As previously mentioned the movement of a bin from a site to another is always decomposed in separate removal and add movements, as this is what in practice is performed in these situations.

In addition to the x variables, the model makes use of the following decision variables. Since we need to distinguish between the bins that are actually purchased and those that are simply moved between sites, we introduce a new set of continuous variables. In particular, for each pair (i, j) of site and bin type we let z^+_{ij} and z^-_{ij} denote the number of bins of type j that are added or removed from site i, respectively. Moreover, for each bin type j ($j = 1, \dots, m$) let w_j denote the total number of bins of that type to be purchased. Finally, the binary variable f_{ih} indicates that frequency h is chosen for site i. Note that f_{ih} may be either considered as a binary variable or it may be an input parameter provided by the routing part of the problem.

The model is then:

$$\text{(P3)} \quad \min \quad \sum_{j=1}^{m} C^P_j w_j + \sum_{i=1}^{n}\sum_{j=1}^{m} C^R_j z^-_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} C^T_j z^+_{ij}$$

subject to

$$\sum_{j=1}^{m} Q_j x_{ij} - q_i \sum_{h=1}^{H} a_h f_{ih} \geq 0, \qquad \forall\ i = 1, \dots, n, \tag{4}$$

$$\sum_{j=1}^{m} x_{ij} u_j \leq U_i \qquad \forall\ i = 1, \dots, n, \tag{5}$$

$$w_j \leq M_j \qquad \forall\ j = 1, \dots, m, \tag{6}$$

$$z^+_{ij} \geq x_{ij} - p_{ij} \qquad \forall\ i = 1, \dots, n; j = 1, \dots, m, \tag{7}$$

$$z^-_{ij} \geq p_{ij} - x_{ij} \qquad \forall\ i = 1, \dots, n; j = 1, \dots, m, \tag{8}$$

$$x_{ij} - z^+_{ij} + z^-_{ij} = p_{ij} \qquad \forall\ i = 1, \dots, n; j = 1, \dots, m, \tag{9}$$

$$w_j \geq \sum_{i=1}^{n}(z_{ij}^{+} - z_{ij}^{-}) \qquad \forall\, j = 1, \ldots, m, \qquad (10)$$

$$\sum_{h=1}^{H} f_{ih} = 1 \qquad \forall\, i = 1, \ldots, n, \qquad (11)$$

$$x_{ij} \geq 0 \text{ and integer}, \qquad \forall\, i = 1, \ldots, n; j = 1, \ldots, m, \qquad (12)$$

$$z_{ij}^{+}, z_{ij}^{-} \geq 0, \qquad \forall\, i = 1, \ldots, n; j = 1, \ldots, m, \qquad (13)$$

$$w_j \geq 0, \qquad \forall\, j = 1, \ldots, m, \qquad (14)$$

$$f_{ih} \in \{0, 1\}, \qquad \forall\, i = 1, \ldots, n; h = 1, \ldots, H. \qquad (15)$$

The objective function minimizes purchase, removal and transfer cost. Constraints (4) impose that the total capacity of the bins at a given site is not smaller than the maximum amounts of waste produced at the site between two consecutive visits of the selected service frequency h. Moreover, constraints (5) limit the total number of bins that may be associated with a site, and constraints (6) limit the total number of bins that may be used for each bin type. Constraints (7), (8) and (9) control the number of bins that are added and the number of bins that are removed. Constraints (10) state that the number of bins to be purchased for every bin type is the difference between the number of bins that are added and those that are removed and constraints (11) state that one frequency must be chosen for every customer. Note that this model can be rather easily modified to include further issues arising in real-world applications. For example, if the presence of bins of a certain type should be reduced or totally eliminated it is sufficient to add a value M'_j, $j = 1, \ldots, m$, as a limit on the number of bins of that type that can be used and adding a constraint set similar to (6) on variables x.

As will be further illustrated in the rest of this paper, the resulting Mixed Integer Linear Programming problem (P3) appears to be relatively easy to solve by commercial solvers such as CPLEX, even for the typical sizes associated with practical applications, involving hundreds of sites and several types of bins.

3.2.4 Scenario 4 - Multiple Waste Types Case

When we consider multiple types of waste the model changes slightly. More precisely, x_{ij} changes to x_{ijk} that corresponds to the number of bins of type j for waste type k at site i. The demand is now given for every customer and for every type of waste q_{ik}. Moreover, there is an indicator W_{jk} that is 1 if bin type j can be used for waste type k, and 0 otherwise. Furthermore, f_{ih} changes to f_{ikh}, indicating the frequency chosen per customer and per waste type.

Moreover, constraints (4), (5), (7), (8), (9), (11), (12) and (15) change to:

$$\sum_{j=1}^{m} Q_j x_{ijk} W_{jk} - q_{ik} \sum_{h=1}^{H} a_h f_{ikh} \geq 0 \qquad \forall\, i = 1, \ldots, n, \forall\, k = 1, \ldots, K \qquad (16)$$

$$\sum_{j=1}^{m} \sum_{k=1}^{K} x_{ijk} u_j \leq U_i \qquad \forall\, i = 1, \ldots, n, \qquad (17)$$

$$z_{ij}^{+} \geq \sum_{k=1}^{K} x_{ijk} - p_{ij} \qquad \forall\, i = 1, \ldots, n; j = 1, \ldots, m, \qquad (18)$$

$$z_{ij}^{-} \geq p_{ij} - \sum_{k=1}^{K} x_{ijk} \qquad \forall\, i = 1, \ldots, n; j = 1, \ldots, m, \qquad (19)$$

$$\sum_{k=1}^{K} x_{ijk} - z_{ij}^{+} + z_{ij}^{-} = p_{ij} \qquad \forall\, i = 1, \ldots, n; j = 1, \ldots, m, \qquad (20)$$

$$\sum_{h=1}^{H} f_{ihk} = 1 \qquad \forall\ i = 1, \ldots, n; k = 1, \ldots, K, \tag{21}$$

$$x_{ijk} \geq 0 \text{ and integer}, \qquad \forall\ i = 1, \ldots, n; j = 1, \ldots, m; k = 1, \ldots, K, \tag{22}$$

$$f_{ihk} \in \{0, 1\}, \qquad \forall\ i = 1, \ldots, n; h = 1, \ldots, H; k = 1, \ldots, K. \tag{23}$$

Also in this case the resulting MILP model can be solved within relatively short computing time by commercial solvers such as CPLEX.

4 Solution Approaches

The overall solution approaches we implemented for WBARP are of two types. The first type is represented by hierarchical approaches in which the two main problems, namely routing and bin allocation, are solved in sequence. In particular, we considered both Bin Allocation-First, Route-Second (BAFRS) and Route-First, Bin Allocation-Second (RFBAS) approaches. These sequential approaches are those typically used in practice and will be mainly used here as a comparison to assess the efficacy of integrated methods. We next describe the various solution approaches and start by describing the specific algorithms that we used for solving the routing and the bin allocation parts of WBARP.

4.1 VNS Algorithm for the Routing Part

As described in Section 3.1 above, the routing problem is a PVRP-IF-SC, therefore we propose for its solution a VNS framework that extends to the PVRP-IF-SC the successful algorithm proposed by Hemmelmayr et al. (2011) to efficiently solve the PVRP-IF. The modifications we introduced are aimed at incorporating the service choice into the previous framework without altering too much its overall structure, thus possibly preserving its effectiveness.

Starting from an initial solution, the VNS consists of the following phases: shaking, local search and acceptance decision. A set of neighborhoods $N_\kappa(\kappa = 1, \ldots, \kappa_{max})$ needs to be defined for the shaking phase, where a solution is generated at random from the current neighborhood. The goal of the local search phase is to reach a local optimum. Finally, in the acceptance phase it is decided whether or not to accept the current solution. If it is accepted, the search restarts from the first neighborhood, otherwise the neighborhood index is increased. The various components of the VNS algorithm we used for PVRP-IF-SC are shown in Figure 1 and are described in detail hereafter.

Initial Solution

The initial solution is based on an extension of the Savings Algorithm by Clarke and Wright (1964). First, the highest frequency is assigned to each customer and visit day-combinations are chosen randomly. Then, for every day of the time horizon the corresponding VRP is solved with the savings algorithm, where only the overall tour length constraint is respected. Afterwards, the IFs are inserted with the dynamic programming (DP) procedure explained below. In our extensive preliminary testing we have also tested the use of different frequency choices, as the lowest or a random one, for the initialization step. However, the impact of the starting frequency on the final result is negligible.

Shaking

In the shaking phase, a random move is performed in the current neighborhood κ. In our case the neighborhoods are *change combination*, *move*, *cross*, and *change frequency*, which handles the service choice. The first operator assigns a new visit day-combination to one or more customers. The customers are moved to the days of the new visit day-combination, while the frequency of visit remains the same. The next two operators only change the routing of a given day. In the *move* operator a sequence of customers is moved from one route to another and in *cross*, sequences of customers are swapped between routes. Both *move* and *cross* are performed in an *inter-trip* and *inter-tour* version. Inter-trip means that

Algorithm 1 Basic VNS

```
s, s′ ← InitialSolution, choose N_κ(κ = 1, ..., κ_max)
repeat
  κ ← 1
  repeat
    s′ ← Shaking(N_κ(s))
    s″ ← LocalSearch(s′)
    if AcceptanceDecision(s, s″) then
      s ← s″
      κ ← 1
    else
      κ ← κ + 1
    end if
  until κ > κ_max
until stopping condition is met
```

Figure 1: Basic steps of the VNS

sequences of customers between different trips, i.e., parts of a route between two IFs or the depot, are swapped, while inter-tour refers to moving or swapping sequences of customers between different vehicles. Note also that the sequence that is moved, the length of the sequence and the insertion position are chosen randomly. Finally, in *change frequency* a new visit frequency is assigned to one or more customers. *Change frequency* represents the largest neighborhood, since it increases or decreases the total number of visits and also the demand that has to be picked up.

Neighborhoods are ordered in increasing size. More precisely, we have 18 neighborhoods. The first three are *change combination*, where the number of customers to which a new visit day-combination is assigned is chosen randomly between one and the index of the neighborhood. For example in the third neighborhood the maximum number of customers that can be assigned a new visit day-combination is three. The next three neighborhoods are inter-trip *move* and inter-trip *cross*. Here again, the maximum length of the sequence depends on the index of the neighborhood. Afterwards three times the inter-tour *move* operator and three times the inter-tour *cross* operator can be performed. The last three neighborhoods are *change frequency*. Table 2 lists the neighborhoods and their size. Preliminary tests showed that this order of neighborhoods is better than other combinations.

Local Search

Local search consists of a dynamic programming (DP) procedure to insert the IFs and the well known 2-opt operator. First the IF are removed and then reinserted with DP. The method is based on that described by Beasley (1983) for a route-first, cluster-second method for the VRP that was also used in a genetic algorithm by Prins (2004). In our case the IF which is the closest one between two customers is considered for insertion. Then 2-opt is used to improve the routes, where only solutions that are feasible with respect to capacity can be accepted. See Hemmelmayr et al. (2011) for more details.

Acceptance Decision

In the move or not phase, the decision on whether to accept a solution or not is taken. In the basic VNS only improving solutions with a better objective function value are accepted. For solution acceptance we use instead a condition inspired by Simulated Annealing (SA) (Kirkpatrick et al. 1983). More precisely, solutions that yield a better objective function value are always accepted and inferior solutions are accepted with a probability $e^{\frac{-(f(x'')-f(x))}{\tau}}$, where

κ	operator	min. customers	max. customers
1	*change-combination*	1	1
2	*change-combination*	1	2
3	*change-combination*	1	3
		min. segment length	max. segment length
4	*inter-trip move*	1	min(1, n)
5	*inter-trip move*	1	min(2, n)
6	*inter-trip move*	1	min(3, n)
7	*inter-trip cross-exchange*	1	min(1, n)
8	*inter-trip cross-exchange*	1	min(2, n)
9	*inter-trip cross-exchange*	1	min(3, n)
10	*inter-tour move*	1	min(1, n)
11	*inter-tour move*	1	min(2, n)
12	*inter-tour move*	1	min(3, n)
13	*inter-tour cross-exchange*	1	min(1, n)
14	*inter-tour cross-exchange*	1	min(2, n)
15	*inter-tour cross-exchange*	1	min(3, n)
16	*change-freq*	1	1
17	*change-freq*	1	2
18	*change-freq*	1	3

Table 2: Set of neighborhood structures with $\kappa_{max} = 18$

$f(\cdot)$ is the objective function value corresponding to a solution, x is the incumbent solution and x'' is the new solution obtained after shaking and local search. Therefore, the acceptance of inferior solutions depends on the difference between the costs of the new solution and that of the incumbent solution, and on the current temperature value τ. We decrease τ in η/k stages during the search process, where η represents the total number of iterations executed (τ is increased by a small positive quantity in the beginning to avoid that it becomes zero). Thus, every k iterations τ is decreased by an amount $\frac{\tau \cdot k}{\eta}$. Infeasibility in the tour length or capacity constraint is penalized in the objective function with a constant value.

4.2 The Bin Allocation Part

We have analyzed and tested the bin allocation models of Section 3.2 and we found that CPLEX can solve the problems within seconds, when applied to instances with practically relevant size, i.e., having several hundreds of collection sites.

Since the bin allocation model is solved very fast, we can directly incorporate it in the solution procedure for the routing part whenever new solutions are found that change the service profile of at least one customer. Note that for the bin allocation problem a different service profile means a service profile that has a different maximum number of days in between two consecutive visits, i.e., a different a_h. Changes between visit frequencies with the same a_h do not affect a solution to the bin allocation model. In our computational testing we adopted for the solution of the bin allocation problem the different models described in Section 3.2 for the various scenarios. As previously mentioned, note that f_{ih} can be either a decision variable or a parameter when the frequencies are given as an input.

4.3 Solution Approaches for WBARP

We propose and analyze four different algorithms: two hierarchical approaches that optimize in sequence the two parts of the problem, one integrated method that solves the overall problem by dynamically varying service frequencies and bin allocations and one method that uses a heuristic estimation for the bin allocation part.

The first hierarchical approach is called Bin Allocation-First, Route-Second (BAFRS). First we solve the bin allocation part of the problem. Then the frequencies yielded are used for the routing part of the problem. In the solution of the bin allocation problem, we adopted a lexicographic strategy. First, according to the current scenario, we determine the bin allocation which minimizes the bin cost. Then, with the given bin cost we determine the highest compatible service frequencies for the customers. Note that, since in this case the frequencies are fixed, the routing part simply consists of a PVRP-IF.

The second hierarchical algorithm starts by solving the routing problem, i.e., a PVRP-IF-SC, by using the VNS algorithm described in Section 4.1. Then, it passes the frequencies from the routing solution as an input to the bin allocation problem. We refer to this algorithm as Route-First, Bin Allocation-Second (RFBAS). Observe that, for Scenario 1 and 2 it is possible to determine the frequencies that lead to feasible bin allocations for each site in a preprocessing phase so that we can avoid considering infeasible combinations during the execution of the algorithm.

Note also that it can happen for RFBAS and BAFRS that the solution to the second problem is infeasible. For example, in a good solution to the routing part the service frequencies tend to be rather low, since an additional visit increases the travel cost. Therefore, when we minimize the routing cost in RFBAS the frequencies that are given to the bin allocation problem may yield an infeasible solution requiring an excessive number of bins either at specific collection sites with space limitation or with respect to their total number. On the other hand, in a good solution to the bin allocation the service frequencies tend to be rather high, thus requiring only a small number of bins. Hence, in BAFRS it can happen that the tour length constraint cannot be respected because the service frequencies are too high.

The third algorithm, called Integrated Method (IM), solves the routing and the bin allocation in an integrated fashion. More precisely, the VNS to solve the routing part is executed. Whenever the frequency of at least one customer is changed, i.e., when the change-frequency step in shaking is performed, we have to determine the new bin allocation cost. Recall that for Scenarios 1 and 2 the bin allocation problem is solved separately for each site and frequency. Therefore, in a preprocessing step we can compute the bin allocation cost for every pair of site and frequency, and simply use this information along the execution of the IM algorithm. For Scenarios 3 and 4 this is not possible. Hence, we have to solve the model again whenever the frequency of at least one customer is changed.

We have also developed a fourth solution method for Scenarios 3 and 4, the Heuristic Integrated Method (HIM), where instead of solving the entire model for bin allocation when it is required we use a heuristic estimation corresponding to the solution of a Scenario 2 model for that site and frequency. Note that such a solution need only to be computed once in the preprocessing step so the computation for the bin allocation in HIM is much shorter.

Finally, in Scenario 4 the methods are a bit different since we consider multiple waste types. We do not consider vehicles with multiple compartments, so the routing problem has to be solved separately for each type of waste. In addition, in the shaking step of the VNS the type of waste which will be considered is chosen randomly.

5 Computational Study

The impact of the integration of the bin allocation and routing optimization in WBARP is analyzed through extensive computational testing on both randomly generated and real world instances. The algorithms are implemented in C++ and the experiments are performed using an Intel Core 2.40 GHz PC with 4GB of memory. Bin allocation problems are solved by using CPLEX 12.1 solver directly called within the VNS. The VNS is run 5 times for 10^7 iterations and the average, the best and the worst solutions over such 5 runs are reported.

We first describe the results on the set of random instances by analyzing all the scenarios described in Section 3.2. Then we present results for a sensitivity analysis regarding the purchase cost, the number of customers and regarding service profile and bin capacity options. We also report the results on small instances that we compare to the solution by CPLEX. Finally, we discuss the result achieved on a real world instance.

Table 3: Service frequency, in emptyings over T, and service day-combinations

T	Frequency	Day-combinations
4	1	$\{1\}\{2\}\{3\}\{4\}$
	2	$\{1,3\}\{2,4\}$
	4	$\{1,2,3,4\}$
6	1	$\{1\}\{2\}\{3\}\{4\}\{5\}\{6\}$
	2	$\{1,4\}\{2,5\}\{3,6\}$
	3	$\{1,3,5\}\{2,4,6\}$
	6	$\{1,2,3,4,5,6\}$

5.1 Synthetic Instances

The first set of instances is obtained by extending a set of PVRP-IF instances from the literature (see Hemmelmayr et al. (2011)). In the dataset there are ten different instances, ranging from 48 to 288 collection points, with a time horizon T of 4 and 6 days, respectively. In Table 3, we list the visit frequencies and the corresponding visit day-combinations for each value of T.

The number of sites, n, as well as the number of frequencies, H, and the days in between two consecutive visits for each frequency, a_h, was taken from the PVRP-IF instances. Also the daily waste production for each site, q_i, is derived from the PVRP-IF instances, by dividing the original demand by T so as to transform it into a daily demand. Inspired by the values we observed in real world instances as described in Section 5.4, we added the data for the bin allocation as follows. We consider $m = 3$ different bin types, having volume capacities, Q_j, equal to 10, 15 and 25, $j = 1, 2, 3$. In addition the space requirements, u_j are 1, 2 and 4, whereas the purchase costs, C_j^P, are 10, 11 and 13, respectively. Finally, transfer costs C_j^T and removal costs C_j^R, are set to 1, $j = 1, 2, 3$. A sensitivity analysis of the problem solution with respect to these costs is discussed in Section 5.2.1.

For Scenario 4, we considered $K = 3$ different waste types and we assume that the waste volume generated at each site for type 1, 2 and 3 is equal to 50%, 17% and 33% of the total waste volume generated at that site.

The space U_i available at each site $i = 1, \ldots, n$, is an integer value randomly generated in the interval $[\underline{\nu_i}, \overline{\nu_i}]$, where

$$\underline{\nu_i} = \min_{h=1,\ldots,H;k=1,\ldots,K;j=1,\ldots m:W_{jk}=1} \left\lceil \frac{a_h q_{ik}}{Q_j} \right\rceil u_j,$$

and

$$\overline{\nu_i} = \max_{h=1,\ldots,H;k=1,\ldots,K;j=1,\ldots m:W_{jk}=1} \left\lceil 1.1 \cdot \frac{a_h q_{ik}}{Q_j} \right\rceil u_j.$$

Similarly, the maximum number M_j of bins for $j = 1, \ldots, m$ is an integer value randomly generated in the interval $[\underline{\mu_j}, \overline{\mu_j}]$, where

$$\underline{\mu_j} = \min_{h=1,\ldots,H;k=1,\ldots,K:W_{jk}=1} \sum_{i=1}^{n} \left\lceil \frac{a_h q_{ik}}{Q_j} \right\rceil,$$

and

$$\overline{\mu_j} = \max_{h=1,\ldots,H;k=1,\ldots,K:W_{jk}=1} \sum_{i=1}^{n} \left\lceil 1.15 \cdot \frac{a_h q_{ik}}{Q_j} \right\rceil.$$

Finally, as to the initial configuration of bins at each site used in Scenarios 3 and 4, for each site (resp. site and type of waste) we select randomly a bin type j and we set the corresponding value of p_{ij} (resp. p_{ijk}) to one.

The computational experiments confirmed some basic properties of the problem. As to the routing problem, it is generally beneficial to adopt a small service frequency. There are, however, a few exceptions to this general rule. One of these arises when there exist isolated clusters of customers making it profitable to visit all of them with the same frequency, which for some of the sites may not be the smallest possible one. Another example may be represented by a "split delivery" effect, i.e., when it is required to visit a customer more frequently due to vehicle capacity restriction, thus increasing its service frequency. On the other hand, for the bin allocation problem the best option is to adopt high service frequencies, because when customers are visited more often, less waste accumulates and therefore less bins are needed.

5.1.1 Results for Scenario 1 - Basic Case

In the basic case, only the capacity constraint at each site is taken into account. Therefore, the solution to the bin allocation problem is always feasible irrespective of the frequency imposed for the service at that site. Table 4 summarizes the results of the algorithms BAFRS, RFBAS, and IM for the first scenario and shows the average, minimum and maximum deviation with respect to IM for each instance in the test set.

As to RFBAS we considered two different ways of defining the frequency at the sites to analyze the impact of fixed frequencies versus a free choice of frequencies. In particular, in the first case, called RFBAS *fixed freq*, the service frequency is fixed to the value defined in the corresponding PVRP-IF instance. In the second case, called RFBAS *free freq*, all the frequencies reported in Table 3 can be used. The table reports the average values over the ten instances of the dataset for all costs in the solutions, as well as the average running time in hours on a 2.40 GHz PC. The detailed results achieved by algorithm IM for this and the other scenarios are given in the Appendix.

Table 4: Average results for Scenario 1 - Basic Case. Running times are in hours on a 2.40 GHz PC.

	cost			run	% dev wrt IM		
	routing	bin allocation	total	time (h)	avg	min	max
BAFRS	6175.53	1972.90	8148.43	0.51	49.08	33.43	75.43
RFBAS *fixed freq*	5265.42	2146.30	7411.72	0.42	36.59	18.76	76.32
RFBAS *free freq*	2255.03	3888.82	6143.85	1.27	9.22	-0.04	19.54
IM	2902.21	2630.50	5532.71	1.25			

The results of Table 4 clearly show the benefit of an integrated approach with respect to classic hierarchical solution methods. Indeed, IM obtains solutions which are much less expensive and more balanced with respect to those obtained by both BAFRS and RFBAS while the overall computing time is not exceedingly different for all algorithms. As expected, BAFRS yields the smaller bin allocation cost whereas RFBAS with free frequency yields the best routing cost, but in both cases the other component of the problem is heavily affected and provides a large overall cost of the solution. By comparing IM with RFBAS with free frequencies we immediately appreciate the positive impact of an integrated solution where the two components are jointly optimized in contrast to a sequential one. In the IM solution the bin allocation cost is drastically reduced with only a limited increase of the routing cost and an overall average cost reduction larger than 9%. This shows the ability of the IM approach to determine the sites for which the increase of the frequency that reduces the bin allocation cost will have a limited impact on the routing, e.g. because the sites are close to others visited with the same frequency. This is clearly not possible within a hierarchical approach in which the two problems are treated separately. In addition, by comparing IM with RFBAS with the initial frequencies of the PVRP-IF instances we can observe that an appropriate investment on the capacity spread on the territory may have a tremendous impact in terms of variable costs which are almost halved. Among the three approaches, BAFRS appears to be the worst performing one. Moreover, we should also note that it can happen that the routing solutions for this approach are infeasible, since with the minimum possible frequencies the total volume to be collected may be too large for the available fleet.

5.1.2 Results for Scenario 2 - Space Restricted Case

As already mentioned, in Scenario 2, where we consider the space restriction at the sites, we can again decompose the problem and solve it separately for each site. Therefore, we can identify the feasible frequencies as well as their costs for each customer in a preprocessing step by solving the small MILP from Section 3.2.2. The set of feasible frequencies can be used for RFBAS and the cost for every customer and every frequency is used for IM.

Table 5: Average results for Scenario 2 - Space Restricted Case. Running times are in hours on a 2.40 GHz PC.

	cost			run	% dev wrt IM		
	routing	bin allocation	total	time (h)	avg	min	max
BAFRS	6174.62	1986.20	8160.82	0.51	24.33	16.85	36.57
RFBAS *free freq*	3662.41	3831.12	7493.53	1.09	12.71	5.21	24.49
IM	4129.40	2414.36	6543.76	0.90			

Table 5 shows the routing cost, the bin allocation cost and the total cost of BAFRS, RFBAS with free frequencies and IM for Scenario 2 and the average, minimum and maximum deviation with respect to IM for each instance in the test set. We note that RFBAS *fixed freq* cannot be used anymore since the solution to the bin allocation problem with the space capacity constraints is generally infeasible with the original PVRP-IF frequencies.

The interplay between the algorithms is similar to what we already observed for Scenario 1. In terms of bin allocation cost, BAFRS has the lowest value, whereas RFBAS obtains the best routing costs. However, the IM solution is more balanced and gets a considerable improvement with respect to the hierarchical approaches.

Comparing the results from Table 4 and Table 5 shows that a capacity restriction at the sites clearly raises the total cost. In fact, a space restriction at the sites can make the low frequencies infeasible because the demand accumulated between two consecutive visits is too high to be accommodated at a site. Therefore, the frequency has to be increased, leading to a worse routing cost. In addition, even if an increase in frequency is not necessary it can happen that the best cost combination of bin allocation cannot be used anymore because it consumes too much space, thus leading to an increase in bin allocation cost. However, for RFBAS and IM the increase in bin allocation cost is out-weighed by the increase in routing cost. In these approaches the routing frequencies were rather low in Scenario 1, so the increase in frequencies leads to a decrease of the bin allocation cost. Further comments on the frequencies used within the various scenarios can be found later in Section 5.2.1.

5.1.3 Results for Scenario 3 - Consistent Case

In this scenario we assume that the constraint on the total number of bins is added to the previous model. As a consequence, the problem is no longer separable per collection site and finding a feasible solution is no longer guaranteed for our approaches, and in particular for RFBAS. To this end we included in the bin allocation model two sets of slack variables that report the violations of Constraints (5) and (6). We note here that with the instances used in our computational testing we never detected infeasibilities with respect to the second set of constraints. In addition we remind that in this scenario we assume that an initial distribution of bins on the sites is given.

The results are shown in Table 6 where we first of all note that the bin allocation costs are just a fraction of those reported for Scenarios 1 and 2. In fact here part of the bins are already present on the territory and the costs are mostly associated with removal and transfer of bins. Nevertheless, also in this case the IM approach has the best performance in terms of total cost, but the running times are considerably larger than for the other approaches, due to the fact that the bin allocation problem has to be solved every time the service frequency of a site is changed. Therefore, it is

Table 6: Average results for Scenario 3 which consider previous configuration for bins. Running times are in hours on a 2.40 GHz PC. Column Avg Inf. U_i reports average infeasibilities that occurred with respect to total number of bins, i.e., the average additional number of bins required to make the solution feasible.

	cost			run	% dev wrt IM			Avg Inf.
	routing	bin allocation	total	time (h)	avg	min	max	U_i
BAFRS	5233.28	201.60	5434.88	0.46	17.07	9.67	28.29	–
RFBAS	2258.88	3451.68	5710.56	1.62	17.92	-14.91	45.89	120.76
HIM	4123.51	515.54	4639.05	0.97	0.44	-0.24	1.56	–
IM	4100.06	517.38	4617.44	4.83				

worth using the heuristic integrated approach HIM that has computing times similar to those of the hierarchical approaches but yields much better results.

For this more constrained scenario the BAFRS approach, which puts more emphasis on the bin allocation part, is not only feasible but also on overall better than the RFBAS approach, which turns out to violate the space restriction constraints, as denoted by the non-zero value for the Avg Inf. U_i column.

5.1.4 Scenario 4 - Multiple Waste Types Case

We also studied the case where different types of waste are present. Since the three waste types must be collected separately, the routing cost and the bin allocation cost are much higher when we treat multiple waste types. Consider, for example, a site with a very low demand of undifferentiated waste such that a single bin suffices and which only needs one visit in the planning period. When we consider multiple waste types, the site has to be visited for each type of waste separately and we also need at least one bin for each type of waste. Therefore, both bin allocation cost and the marginal routing cost to serve the site are approximately three times as large as those for the single waste type case.

In Table 7 we show the results of BAFRS, RFBAS, HIM and IM for the case with multiple waste types. We note here that HIM and RFBAS are able to improve the total cost slightly with respect to IM. However, since the results for these algorithms are infeasible, as can be seen by the value of the Avg Inf. U_i column, a direct comparison is difficult to make. In addition, the computing times for algorithm IM are now much larger than the hierarchical and heuristic methods. All other features of the algorithms observed in the previous scenarios are also confirmed in the case with multiple waste types.

Table 7: Average results for Scenario 4 - Multiple waste types case. Running times are in hours on a 2.40 GHz PC. Column slack U_i reports average infeasibilities that occurred with respect to total number of bins.

	cost			run	% dev. wrt IM			Avg Inf.
	routing	bin allocation	total	time (h)	avg	min	max	U_i
BAFRS	9816.70	3838.20	13654.90	1.38	21.68	11.19	31.55	–
RFBAS	5592.55	5406.14	10998.69	1.39	-3.04	-21.76	5.19	137.00
HIM	6589.00	4342.26	10931.26	1.36	-3.14	-17.15	1.41	15.64
IM	7028.18	4159.28	11187.46	16.12				

5.2 Sensitivity Analysis

5.2.1 Analysis of visit frequencies

Some interesting information about the behavior of the different approaches we propose can be derived by observing the average visit frequency at the sites that is achieved by each algorithm on the different scenarios. Table 8 shows the average, over all instances of the data set, of the visit frequency at the collection sites. Note that, for Scenario 4 the frequency reported is the sum of the frequencies for each waste type. As expected, RFBAS that optimizes the routing first, has the lowest average frequencies for all scenarios, whereas BAFRS has the highest. It is, however, interesting to see that the integrated approach, by accurately planning the capacity distribution over the territory is able to establish service frequencies, which lie between the two extremes represented by the hierarchic approaches. The frequencies for the heuristic HIM are slightly larger than those of IM but recall that it requires a smaller computing time.

Table 8: Average values, over all instances, of the visit frequencies at the collection sites determined by the different approaches.

Scenario	RFBAS *fixed freq*	RFBAS *free freq*	BAFRS	IM	HIM
1 - Basic	2.40	1.02	3.08	1.56	=IM
2 - Space Restricted	–	1.55	3.08	2.09	=IM
3 - Consistent	–	1.02	2.40	2.05	2.08
4 - Multi Waste	–	3.01	4.42	3.58	3.62

When we compare Scenario 1 to Scenario 2, the frequencies increase for each algorithm except for BAFRS. When we put a capacity constraint at each site, less bins can be used and therefore an increase in the visit frequency is necessary to cope with the demand accumulated at each site. BAFRS has already a high visit frequency in Scenario 1 and therefore this effect cannot be observed.

Instead, from Scenario 2 to Scenario 3 the frequencies decrease slightly since we have an initial configuration and we can move bins at a lower cost between the different sites compared to purchasing new ones. Having larger potential capacities available at the sites helps reducing the visit frequencies.

5.2.2 Sensitivity Analysis Regarding Purchase Cost

We performed a sensitivity analysis of the solution with respect to the bins purchase cost, which is clearly one of the most important cost parameters of the problem. To this end, we considered Scenario 3 solved by our integrated approach IM and we constructed four additional sets of instances where the purchase cost of the bins is multiplied by 0.1, 0.5, 5 and 10 with respect to the standard instances analyzed in Section 5.1.3, that corresponds to a multiplier equal to 1 (also called the *default case*, hereafter).

Table 9 shows for each of these cases the routing cost, the bin allocation cost, the total cost, the run time, expressed in hours of a 2.40 GHz PC, the percentage deviation of the total cost with respect to the default case, the average visit frequency per customer and the average number of bins purchased for each type of bins. The table clearly shows that the overall cost increase is directly connected with the purchase cost. When it is low, several tens of bins are purchased and added to the existing ones, so as to reduce the visit frequencies and impact on the routing cost. On the other hand, we see that when the purchase cost increases with respect to the standard scenario, only a very limited number of bins is purchased. Nevertheless, the IM algorithm is still able to keep a considerably good solution quality with relatively low service frequencies, given its ability to move bins from one site to another to get an homogenous service along the territory. We

Table 9: Analysis of results for algorithm IM applied to Scenario 3 when the purchase cost of the bins is multiplied by a factor. The avg. visit frequency per customer as well as the average number of bins of type 1 - 3 is shown

C^P	cost			run	%	avg.	average n. of bins		
multiplier	routing	bin all.	total	time (h)	dev.	freq.	#1	#2	#3
0.1	3755.23	328.19	4083.42	45.50	-11.57	1.72	89.41	17.24	1.50
0.5	3922.62	472.47	4395.09	8.80	-4.82	1.90	42.24	4.84	6.04
1(default)	4100.06	517.38	4617.44	4.83	0.00	2.05	29.49	3.16	0.33
5	4354.45	468.42	4822.88	5.19	4.45	2.32	6.53	0.00	0.00
10	4558.87	493.82	5052.69	5.24	9.43	2.35	3.71	0.00	0.00

also observe that when the purchase cost is small, the running time of IM increases considerably due to a larger number of potentially improving decisions related to purchasing or moving bins.

5.2.3 Sensitivity Analysis Regarding the Number of Customers

By considering the single instances of our test set, which range from 48 to 288 customers we may perform some analysis on the behavior of the proposed approaches with respect to the instance size. Table 10 and Figure 2 show the deviation with respect to IM for each instance of Scenario 3. The column marked n, shows the number of customers for each instance, which were sorted by nondecreasing values of n. For the RFBAS algorithm, the deviation is approximately increasing with the number of customers, which means that the performance of the algorithm gets worse for larger instances. However, it is important to keep in mind that it produces infeasible solutions with respect to U_i, which can be seen in the Avg Inf. U_i column. The deviation of the HIM algorithm does not change too much by increasing the size of the instance.

Table 10: Deviation from IM for Scenario 3

Instance	n	BAFRS	RFBAS	Avg Inf. U_i	HIM
pr01	48	9.67	-14.91	19.00	0.13
pr07	72	28.29	1.09	66.00	-0.24
pr02	96	11.64	-3.92	54.60	0.30
pr03	144	19.36	7.91	59.40	0.81
pr08	144	22.50	21.79	151.20	0.36
pr04	192	10.92	16.60	97.80	1.56
pr09	216	19.16	37.37	173.60	0.26
pr05	240	14.27	37.95	131.80	0.24
pr06	288	11.52	29.40	149.60	0.39
pr10	288	23.38	45.89	304.60	0.55
avg		17.07	17.92	120.76	0.44

5.2.4 Sensitivity Analysis Regarding Service Profile Options

We studied two cases with reduced service profile options. For the first case (denoted as Freq Case 1), all day-combinations with frequency of one visit were deleted, while for the second case (denoted as Freq Case 2) all day-combinations with frequency of two visits were deleted. Table 11 shows the average percentage deviation from IM and to the corresponding default case (i.e., the basic instance with all day-combinations) over all instances (see Table 6 for the default case of Scenario 3).

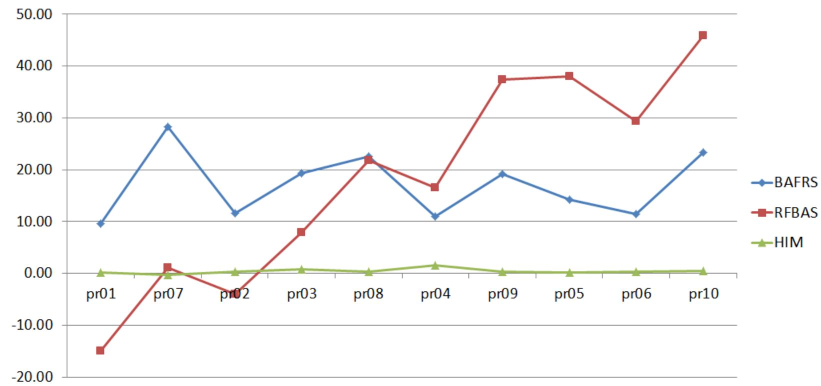

Figure 2: Deviation from IM for Scenario 3

Table 11: Comparison of the algorithms to IM for different frequency cases for Scenario 3

	Freq Case 1			Freq Case 2		
	%dev IM	%dev default	Avg Inf. U_i	%dev IM	%dev default	Avg Inf. U_i
BAFRS*	12.64	5.17	–	15.79	11.70	–
RFBAS	-9.64	-13.82	22.58	4.23	0.67	124.16
HIM	0.48	9.40	–	0.38	15.22	–
IM	–	9.36	–	–	15.30	–

* The tour length constraint was violated in instance pr01

For the RFBAS and BAFRS algorithms, the deviation from the IM algorithm gets smaller in Freq Case 1 and 2 compared to the default setting. This shows that the hierarchical algorithms benefit from less degrees of freedom. RFBAS is even improving IM. However, it is important to keep in mind that RFBAS produces results that violate the site restriction constraints and are therefore difficult to assess.

When comparing with the default case, an increase in cost can be observed for all algorithms except for the RFBAS algorithm. For the BAFRS, HIM and IM algorithms, the cost increase compared to the default case is larger for Freq Case 2. This is due to the fact that in the default case the average visit frequency was around 2 visits, which is not available in this scenario.

For Freq Case 1, the RFBAS algorithm improves its performance compared to the default case, because the average visit frequencies in the default case were too low (1.02, see Table 8) and frequencies of one visit are not available in Freq Case 1. However, in Freq Case 2, the behavior is similar to the default case, since in this case the frequencies of one visit can be taken again.

5.2.5 Sensitivity Analysis Regarding Bin Capacity Options

To analyze the impact of different bin capacity options, we changed the number of bin types m. In the first scenario (denoted as Bin Case 1) two bin types are considered and in the second scenario (denoted as Bin Case 2) four bin types are available. For the first scenario we deleted the medium-sized bin, i.e. the bin with capacity 15 and cost 11 from the default case. The maximum number of bins that can be purchased, M_j, of the deleted bin was split and added to the remaining two bin types. For the second scenario we added another medium-sized bin with capacity 20, purchase cost 12 and space requirement 3 to the default case. In this case, the M_j of the last bin was split and added to the new bin type. The p_{ij} were adapted accordingly.

Table 12 shows the two bin assortment scenarios and the deviation of these cases from the default case (see Table 6 for the default case) as well as the deviation from the IM algorithm.

Table 12: Different bin cases for Scenario 3

	Bin Case 1			Bin Case 2		
	%dev IM	%dev default	Avg Inf. U_i	%dev IM	%dev default	Avg Inf. U_i
BAFRS	18.49	8.39	–	17.62	-0.72	–
RFBAS	26.23	14.69	110.67	17.49	-1.55	121.92
HIM	1.71	8.47	–	0.25	-1.33	–
IM –	–	7.10	–	–	-1.15	

From the table it can be seen that when less bin options are available (Bin Case 1), the total cost is increasing compared to the default case, while it is decreasing when more bin options are available.

The algorithms have in general a similar deviation from IM as in the default case. For Bin Case 1, the deviation from IM is slightly larger. This shows that IM can better cope with the situations where fewer bin options are available.

For the RFBAS algorithm a larger deviation from the default case, as well as from IM, can be observed for Bin Case 1. In this algorithm the bin allocation is performed after the routing phase with the fixed frequencies taken from the routing, which results in a poor performance when less bin options are available.

5.3 Results on small-sized instances

To validate the effectiveness of our solution procedures we run them on small-sized instances and compare them to the results generated by CPLEX. The formulation we used is given in the appendix. In order to make it tractable we limited the number of IFs to one and the number of waste types to one. We generated 8 instances based on the synthetic instances pr01,pr02,pr07 and pr08 by deleting random customers. The maximum number of bins that can be purchased M_j was adjusted. The instances range from 5 to 20 customers and 4 to 6 days. The size of the instances is given in the instance name where n refers to the number of customers and t to the number of days. A time limit of two hours was set for CPLEX.

Table 13: Results on the small-sized instances

	Gap UB-LB	Gap UB-Avg.				Avg Inf.	Run time in seconds				
Instance	CPLEX	IM	HIM	BAFRS	RFBAS	U_i	CPLEX	IM	HIM	BAFRS	RFBAS
n5t4	0.0	0.00	0.00	2.33	436.74	3.0	2.6	0.8	0.6	0.2	0.6
n5t6	0.0	0.00	0.00	164.69	0.00	–	9.0	8.3	3.6	0.0	2.6
n10t4	0.0	0.00	0.00	89.67	1.78	–	139.0	27.5	24.6	0.0	9.0
n10t6	5.9	0.00	0.00	68.96	1104.37	18.0	7200.0	27.2	10.6	172.6	17.8
n15t4	12.6	0.00	0.00	8.69	573.24	8.0	7200.0	518.0	675.6	138.2	25.6
n15t6	40.4	-1.62	-1.25	69.03	174.13	2.0	7200.0	1946.8	885.0	0.0	844.6
n20t4	8.6	0.00	0.00	39.36	400.30	3.0	7200.0	2767.4	3022.2	180.6	1554.6
n20t6	14.8	-4.94	-4.94	25.47	830.23	19.8	7200.0	4138.0	1505.0	201.6	3936.8

For CPLEX we report the gap between the upper and lower bound (Gap UB-LB) and for the remaining methods we report the gap between the upper bound by CPLEX and between the solutions found by the corresponding algorithms averaged over five runs (Gap UB-Avg).

The results show that only tiny instances up to 10 customers and 4 days can be solved to optimality by CPLEX. The algorithms HIM and IM find the optimal solutions for the instances where optimality is proven. For the remaining instances they either find the same solution as the upper bound computed by CPLEX or they improve it. The hierarchical solution procedures RFBAS and BAFRS yield worse results and in the RFBAS approach the maximum space restriction at the sites is violated, which is shown in column Avg Inf. U_i.

5.4 Results on a real world instance

We have also tested the different approaches on a real world instance relative to a district of a town in Northern Italy. The instance includes 201 collection sites where three types of waste are accumulated, namely Organic, Multi-material (i.e., Plastic and Paper) and Undifferentiated waste. Each type of waste has its own dedicated unloading intermediate facility. The available fleet is made up by 20 vehicles based at a common depot and that can perform trips at most 6.5 hours long. As to the bin allocation, there are 12 different bin types available, and 657 bins are in total initially present in the territory. For some bin categories it is possible to purchase additional bins. The purchase cost is 140.5 units per bin and no upper limit on the quantity to be purchased was considered. The planning is relative to a time horizon of six out of seven days (i.e., on Sunday no service can take place), and the possible service frequencies range from 1 to 3 visits per week.

Table 14: Results for the real world data

	cost			run	% dev.	Avg Inf.
	routing	bin allocation	total	time (h)	wrt IM	U_i
BAFRS	3081.09	171.2	3252.29	1.28	15.49	–
RFBAS	1906.45	1106.93	3013.39	0.87	7.01	11.6
HIM	1872.39	1116.48	2988.87	0.93	6.14	11.0
IM	2417.71	398.37	2816.08	19.17	0.00	–

As can be observed BAFRS performs relatively well on that instance, whereas both RFBAS and HIM, which get better solution values, were not able to find feasible ones. The integrated approach IM again shows its ability of not only finding feasible solutions but also very good ones. We note that being a strategic problem whose solution is implemented and run for relatively long periods, even a moderate saving of some percent points with respect to the traditional hierarchical approaches may add up to significant amounts of money quite quickly. In addition, such a saving fully justifies spending relatively long computing times as those required by IM.

6 Conclusions

In this paper we have considered a new routing problem arising in a real-world context of solid waste collection aimed at integrating capacity allocation issues within routing decisions. Traditionally, such problems are solved by adopting sequential hierarchical approaches where one first determines the capacity to be allocated at each collection site and then determines the corresponding visit frequencies and solves the routing. We examined in detail the bin allocation problems associated with the various scenarios that may be encountered in practical applications, from the very simple ones to those considering global constraints and multiple types of waste. All such problems are formulated as mixed integer linear programs (MILP) that turn out to be relatively easy to solve even for medium-sized instances. Therefore, we proposed an overall matheuristic approach where the bin allocation solved through the MILP is integrated within a well established VNS algorithm for the solution of the routing phase.

The proposed approach is extensively tested by using both a real world instance and a test set obtained by adapting instances from the literature to incorporate the bin allocation issues. Our results clearly show that adopting integrated approaches where capacity allocation and routing decisions are simultaneously evaluated has a great potential impact with respect to traditional sequential planning allowing for savings that are by far larger than those that can be achieved by simply redesigning the collection routes. In general the integrated approaches also proved able to tackle instances with hundreds of collection sites, corresponding to the service performed by several vehicles during a week, within computing times that are overall compatible with the strategic nature of the problem.

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Appendix

6.1 Additional results

We give here the detailed results for each instance of the random data set found by Algorithm IM in the four Scenarios.

Table 15: Detailed Results for the IM Algorithm in Scenario 1

Instance	n	routing cost	bin allocation cost	total cost	run time (h)
pr01	48	1303.21	771.20	2074.41	0.40
pr02	96	1950.57	1461.60	3412.17	0.40
pr03	144	2545.77	2133.40	4679.17	0.62
pr04	192	2907.88	2714.80	5622.68	1.33
pr05	240	3258.90	3425.60	6684.50	1.65
pr06	288	3372.58	4025.40	7397.98	2.01
pr07	72	1906.39	1330.60	3236.99	1.12
pr08	144	3082.44	2424.20	5506.64	1.26
pr09	216	3925.90	3488.00	7413.90	1.61
pr10	288	4768.43	4530.20	9298.63	2.13
avg		2902.21	2630.50	5532.71	1.25

Table 16: Detailed Results for the IM Algorithm in Scenario 2

Instance	n	routing cost	bin allocation cost	total cost	run time (h)
pr01	48	1959.01	696.80	2655.81	0.04
pr02	96	3006.05	1323.60	4329.65	0.14
pr03	144	3455.72	2054.80	5510.52	0.30
pr04	192	4176.81	2560.20	6737.01	0.61
pr05	240	4187.44	3230.20	7417.64	1.01
pr06	288	4882.60	3635.00	8517.60	1.39
pr07	72	3036.41	1130.80	4167.21	0.10
pr08	144	4455.71	2182.40	6638.11	0.91
pr09	216	5415.96	3196.40	8612.36	2.49
pr10	288	6718.34	4133.40	10851.74	2.03
avg		4129.40	2414.36	6543.76	0.90

Table 17: Detailed Results for the IM Algorithm in Scenario 3

Instance	n	routing cost	bin allocation cost	total cost	run time (h)
pr01	48	1943.41	181.60	2125.01	2.70
pr02	96	2965.59	305.20	3270.79	2.15
pr03	144	3459.64	432.00	3891.64	3.66
pr04	192	4090.53	545.40	4635.93	3.70
pr05	240	4163.48	583.20	4746.68	4.51
pr06	288	4828.87	586.40	5415.27	5.76
pr07	72	3050.90	312.00	3362.90	2.62
pr08	144	4427.50	557.20	4984.70	6.48
pr09	216	5374.37	767.00	6141.37	8.15
pr10	288	6696.30	903.80	7600.10	8.55
avg		4100.06	517.38	4617.44	4.83

Table 18: Detailed Results for the IM Algorithm in Scenario 4

Instance	n	routing cost	bin allocation cost	total cost	run time (h)
pr01	48	3182.64	1147.00	4329.64	7.28
pr02	96	5147.19	2291.40	7438.59	10.00
pr03	144	6325.70	3462.00	9787.70	14.28
pr04	192	6996.11	4472.40	11468.52	15.81
pr05	240	7062.28	5667.20	12729.48	14.84
pr06	288	8579.48	6686.60	15266.08	22.24
pr07	72	6083.96	1845.00	7928.96	12.94
pr08	144	7244.01	3608.80	10852.82	17.53
pr09	216	9074.67	5317.80	14392.46	21.62
pr10	288	10585.78	7094.60	17680.38	24.67
avg		7028.18	4159.28	11187.46	16.12

6.2 Model

In this section we present the formulation that was used for the computational experiments in section 5.3. In order to make it more tractable we limited the number of waste types and of intermediate facilities to one. In addition to the variables f_{ih}, w_j, x_{ij}, z_{ij}^+ and z_{ij}^- that were introduced in Section 3, we make use of the following variables. Variables χ_{ijlt} are binary variables indicating whether vehicle l visits node j immediately after node i on day t. Binary variables y_{ir} indicate whether visit combination $r \in C_i$ is assigned to customer i or not. Variables v_{ijlt} represent the amount of waste transported from node i to node j on day t by vehicle l. Table 19 gives an overview of the variables and parameters used.

$$\min \sum_{i=0}^{n+s}\sum_{j=0}^{n+s}\sum_{l=1}^{o}\sum_{t=1}^{T} c_{ij}\chi_{ijlt} + \sum_{j=1}^{m} C_j^P w_j + \sum_{i=1}^{n}\sum_{j=1}^{m} C_j^R z_{ij}^- + \sum_{i=1}^{n}\sum_{j=1}^{m} C_j^T z_{ij}^+$$

subject to

$$\sum_{r \in C_i} y_{ir} = 1 \quad \forall i = 1, ..., n, \tag{24}$$

$$\sum_{r \in C_i} y_{ir}\lambda_{rh} = f_{ih} \quad \forall i = 1, ..., n, \forall h = 1, \dots, H, \tag{25}$$

Table 19: Variables and parameters used

Variables	
f_{ih}	a binary variable indicating whether service frequency h is used for site i
v_{ijlt}	a real variable indicating the load of vehicle l on arc ij on day t
w_j	total number of bins of type j to be purchased
x_{ij}	is the number of bins of type j allocated to site i
χ_{ijlt}	a binary variable indicating whether vehicle l visits node j immediately after node i on day t
y_{ir}	a binary variable indicating whether visit combination $r \in C_i$ is assigned to customer i
z_{ij}^{+}	number of bins of type j added to site i
z_{ij}^{-}	number of bins of type j removed from site i
Parameters	
a_h	maximum number of days between two consecutive visits for service frequency h
a_{rt}	indicator whether day t is contained in visit combination r
C_j^P C_j^R,C_j^T	purchase, removal and transfer cost of bin
C_i	set of possible visit day combinations for customer i
c_{ij}	travel time from node i to node j
d_i	amount of waste collected from customer i
D	maximum tour length
H	number of possible frequencies or service profiles
M_j	maximum total number of a type j bins that can be purchased
m	number of bin types
n	number of customers/sites
o	number of vehicles
p_{ij}	initial number of type j bins at site i
Q_j	volume capacity of a type j bin
Q	vehicle capacity
s	number of intermediate facilities
s_i	service duration of customer i
T	number of days in the planning horizon
u_j	space needed by a bin of type j
U_i	maximum total space for bins at site i
λ_{rh}	an indicator stating whether combination r belongs to frequency h

$$\sum_{j=0}^{n+s}\sum_{l=1}^{o}\chi_{ijlt} - \sum_{r\in C_i} a_{rt}y_{ir} = 0 \quad \forall i = 1, ..., n,\ t = 1, ..., T, \tag{26}$$

$$\sum_{i=0}^{n+s}\chi_{ihlt} - \sum_{j=0}^{n+s}\chi_{hjlt} = 0 \quad \forall h = 0, ..., n+s,\ l = 1, ..., o,\ t = 1, ..., T \tag{27}$$

$$\sum_{j=1}^{n}\chi_{0jlt} \le 1 \quad \forall l = 1, ..., o,\ t = 1, ..., T \tag{28}$$

$$\sum_{i=0}^{n+s}\sum_{j=0}^{n+s}(c_{ij} + s_i)\chi_{ijlt} \le D \quad \forall l = 1, ..., o,\ t = 1, ..., T, \tag{29}$$

$$v_{ijlt} \le Q\chi_{ijlt} \quad \forall i = 0, .., n+s,\ j = 0, .., n+s,\ l = 1, ..., o,\ t = 1, ..., T \tag{30}$$

$$\sum_{i=0}^{n+s} v_{ijlt} + q_j\sum_{h=1}^{H} a_h f_{jh} \le \sum_{i=0}^{n+s} v_{jilt} + Q(1 - \sum_{i=0}^{n+s}\chi_{ijlt}) \quad \forall j = 1, .., n,\ l = 1, ..., o,\ t = 1, ..., T, \tag{31}$$

$$v_{pjlt} = 0 \quad \forall p = n+1, .., n+s,\ j = 0, .., n+s,\ l = 1, ..., o,\ t = 1, ..., T, \tag{32}$$

$$v_{0jlt} = 0 \quad \forall j = 0, .., n+s,\ l = 1, ..., o,\ t = 1, ..., T, \tag{33}$$

$$\chi_{i0lt} = 0 \quad \forall i = 1, .., n,\ l = 1, ..., o,\ t = 1, ..., T, \tag{34}$$

$$x_{ijlt} \le \sum_{h=1}^{n+s} x_{0hlt} \forall i = 1, .., n+s, \forall j, .., n+s, l = 1, .., o,\ t = 1, .., T, \tag{35}$$

$$\sum_{j=1}^{m} Q_j x_{ij} W_j - q_i\sum_{h=1}^{H} a_h f_{ih} \ge 0 \quad \forall i = 1, \ldots, n \tag{36}$$

$$\sum_{j=1}^{m} x_{ij}u_j \le U_i \quad \forall i = 1, \ldots, n, \tag{37}$$

$$w_j \le M_j \quad \forall j = 1, \ldots, m, \tag{38}$$

$$z_{ij}^{+} \ge \sum_{k=1}^{K} x_{ijk} - p_{ij} \quad \forall i = 1, \ldots, n; j = 1, \ldots, m, \tag{39}$$

$$z_{ij}^{-} \ge p_{ij} - \sum_{k=1}^{K} x_{ijk} \quad \forall i = 1, \ldots, n; j = 1, \ldots, m, \tag{40}$$

$$\sum_{k=1}^{K} x_{ijk} - z_{ij}^{+} + z_{ij}^{-} = p_{ij} \quad \forall i = 1, \ldots, n; j = 1, \ldots, m, \tag{41}$$

$$w_j \ge \sum_{i=1}^{n}(z_{ij}^{+} - z_{ij}^{-}) \quad \forall j = 1, \ldots, m, \tag{42}$$

$$f_{ih} \in \{0, 1\}, \quad \forall i = 1, \ldots, n; h = 1, \ldots, H; \tag{43}$$

$$v_{ijlt} \geq 0 \quad \forall i = 0,..,n+s,\ j = 0,..,n+s,\ l = 1,...,o,\ t = 1,...,T, \tag{44}$$

$$w_j \geq 0, \quad \forall j = 1,\dots,m, \tag{45}$$

$$x_{ij} \geq 0 \text{ and integer}, \quad \forall i = 1,\dots,n; j = 1,\dots,m \tag{46}$$

$$\chi_{ijlt} \in \{0,1\} \quad \forall i = 0,..,n+s,\ j = 0,..,n+s,\ l = 1,...,o,\ t = 1,...,T, \tag{47}$$

$$z^+_{ij}, z^-_{ij} \geq 0, \quad \forall i = 1,\dots,n; j = 1,\dots,m, \tag{48}$$

$$y_{ir} \in \{0,1\} \quad \forall i = 1,..,n,\ r \in C_i \tag{49}$$

The objective is to minimize the total cost, which is the sum of total travel cost over all days plus the total bin allocation cost, which is in turn the sum of purchase, movement and transfer costs.

Constraints (24) state that every customer must be assigned to one feasible visit combination. Constraints (25) ensure that the visit frequency corresponds to the chosen combination. Constraints (26) ensure that a customer is visited exactly on the days specified by the assigned visit combination. Constraints (27) guarantee that if a vehicle visits a customer on one day, it also leaves that customer on that day. Constraints (28) say that every vehicle can be used at most once every day and tour length restrictions are ensured by constraints (29). Constraints (30) ensure that the capacity constraint of the vehicle is respected and they also link v_{ijltk} with χ_{ijltk}. Constraints (31) guarantee that the outbound flow of customer j equals the inbound flow plus the waste collected from customer j and therefore forbid subtours. Constraints (32) guarantee that the flow out of every IF is zero, while constraints (33) state that the initial flow out of the depot is zero. Constraints (34) ensure that before we go back to the depot, we unload at an IF. Constraints (35) ensure that we leave the depot on the day t by vehicle l if one or several customer visits are scheduled.

Constraints (36) impose that the total capacity of the bins at a given site is not smaller than the maximum amounts of waste produced at the site between two consecutive visits of the selected service frequency h. Constraints (37) impose a capacity constraint for each site, and constraints (38) limit the total number of bins that may be used for each bin type. Constraints (39), (40) and (41) control the number of bins that are added and the number of bins that are removed. Constraints (42) state that the number of bins to be purchased for every bin type is the difference between the number of bins that are added and those that are removed. Finally, constraints (43) to (49) define the type of the decision variables.