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# Orderly Production and Marketing in the Beef-Pork Sector

By John E. Trierweiler[1]
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## INTRODUCTION

Meat is basic in the modern diet and meat animals are a mainstay of modern agriculture. Expenditures for beef and pork were about six percent of each income dollar (after taxes) in 1968. The sale of meat animals provides a third of all dollars earned by the United States farmer.

Livestock is produced in virtually every part of the United States. On January 1 of 1969 there were approximately 110 million cattle and calves, 57 million hogs, and 21 million sheep and lambs on United States farms. Except for minor increases and decreases, the general trend of cattle and calf inventories for the last 40 years has been increasing. The number of hogs for the same period increased to a peak of 84 million head in 1944, but has generally decreased with minor fluctuations since then. Similarly, sheep and lamb numbers increased to a peak inventory of 56 million head in 1942, but have decreased since then.[3]

The livestock sector has characteristically experienced large variations in the slaughter price of beef and pork. During the period from 1957 to 1969, for example, the price of 900 to 1,100-pound Choice slaughter steers at Omaha averaged $25.38 a hundredweight. During the same period, however, steer prices ranged from a high of $29.98 a hundredweight in May, 1959, to a low of $20.12 a hundredweight in February 1957.[4]

The price of 220 to 240-pound No. 1–3 barrows and gilts during the same period varied between $11.95 and $28.46 with an average

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[3] "Livestock and Meat Statistics," Agricultural Marketing Service Statistical Reporting Service, Economic Research Service, U.S.D.A., Washington, D.C., Statistical Bul. No. 333 and Supplements.

[4] *Ibid.*, Table 166.

price of $18.30 a hundredweight.[5] Variations in price have not reflected changes in the cost of production.

Uncertainty about the level of future product prices in the beef-pork industry has resulted in alternate periods of over production and under production. Price uncertainty has existed because of a number of biological and economic factors. It requires 18 months to two years from the time a calf is born until eventual slaughter as beef. Gestation adds an additional nine months to the production period. The production period for hogs is from five to nine months, plus four months for gestation.

A second factor contributing to price uncertainty is seasonal variation in consumption, of which a large part results from seasonal and annual variations in production levels. Tradition, climate and geographic differences shift the level of the supply costs seasonally for beef and pork. Variations also result from shifts in demand as consumers substitute between beef, pork and poultry products.

Exaggerated price variation over time suggests disorderly production and marketing. If production and marketing were orderly the additional costs of carrying cattle or hogs to heavier weights would equal the extra returns, hereafter referred to as equilibrium. In this definition carrying costs include all variable and fixed costs of production.

Within the last decade there have been times when it would have been profitable for producers to carry cattle or hogs to heavier weights, and several times the opposite situation existed. Generally, with rising product prices it is profitable to carry animals to heavier weights since the added value exceeds the carrying costs. Conversely, during periods of falling product prices producers would find it more profitable to market at lighter weights because carrying costs exceed the added return of carrying to heavier weights.

## Objectives of the Study

This study will present economic models for analyzing temporal equilibrium positions for the beef-pork sector. The issues of spatial distribution, storage, and meat forms will not be directly analyzed. Grade and yield changes for various carcass classes will, however, be implied in the carrying costs of the animal. The models will be designed such that a number of different factors that affect the production and marketing of beef and pork can be analyzed simultaneously.

Consistent with overall objectives of this study the specific tasks are:

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[5] *Ibid.*, Table 173.

1. To evaluate the price structure relationships of the beef-pork sector and to delineate areas of inefficient performance.

2. To develop and test temporal equilibrium flow models consistent with estimated relevant supply cost and price relationships necessary for the orderly production and marketing of beef cattle and hogs.

3. To indicate the usefulness of research procedures and results and suggest data and informational needs to further improve industry performance in production and marketing for the beef-pork sector.

# VALUE STRUCTURE FOR BEEF AND PORK

Value relationships between prices and consumption rates for the beef-pork sector will be developed in this section. Special attention is given to time, form and spatial relationships from primary production at the farm or ranch level, to final retail consumption as beef or pork. Lengthy or elaborate discussion of theoretical estimation techniques is omitted. Primary concern is presentation and evaluation of statistical and economic results to ascertain critical inefficient performance conditions in the industry.

## Beef and Pork at Retail

The analysis assumes that domestic civilian quantities consumed during the period were equivalent to production less changes in storage inventories, import-export balances, and military consumption. Price levels were assumed to be flexible and dependent on the predetermined supply volumes that had to clear the market. Separate price functions for beef and pork were fitted using the simple least squares multiple regression technique. The usual tests of goodness of fit were applied.

The beef and pork equations estimated quarterly for the United States from 1957 to 1966 in linear form are:

$$(1)\quad \hat{P}_1 = \underset{}{78.33983} - \underset{(.34372)}{2.72784\,X_1} - \underset{(.75145)}{1.45827\,S_1} - \underset{(.74229)}{0.93549\,S_2} + \underset{(.78934)}{2.08688\,S_3} + \underset{(.00423)}{0.03673\,X_3} \qquad R^2 = .693$$

$$(2)\quad \hat{P}_2 = 132.28963 - \underset{(.59060)}{1.54889\,X_1} - \underset{(.53225)}{5.02065\,X_2} - \underset{(1.43948)}{6.96960\,S_1} - \underset{(1.66744)}{11.05847\,S_2} - \underset{(1.83342)}{8.43848\,S_3} + \underset{(.00723)}{0.02559\,X_3} \qquad R^2 = .806$$

Where:

$\hat{P}_1$ = retail beef price (cents per pound).

$P_2$ = retail pork price (cents per pound).

**$X_1$ = quarterly per capita consumption of beef and veal (carcass equivalent).**

**$X_2$ = quarterly per capita consumption of pork (carcass equivalent).**

**$X_3$ = per capita disposable income at annual rates (dollars), deflated by the index of consumer prices (1957–1959 = 100).**

**$S_1$ = dummy variable for seasonal variation in winter quarter.**

**$S_2$ = dummy variable for seasonal variation in spring quarter.**

**$S_3$ = dummy variable for seasonal variation in summer quarter.**

Signs of all coefficients in the estimating equations agree with the economic theory. The standard errors of the coefficients are in parentheses. Results of the t-test in the retail beef price equation indicate that per capita consumption of beef and disposable income are significant at the .01 level, while seasonal variation in the summer is significant at the .05 level. Although seasonal variations in winter or spring are not significant in this equation, they were thought sufficiently important to include on the grounds that they removed some of the interaction through time within the estimating system.

Results of the t-test in the retail pork equation indicate that all variables except per capita consumption of beef and veal are significant at the .01 level, while consumption of beef and veal is significant at the .05 level. The independent variables in the beef equation explain 69 percent of the variation in retail beef price, while the independent variables in the pork equation explain 81 percent of the variation in retail pork prices. The values of the coefficients of determination are significant in both equations at the .01 level. The standard errors of estimate indicate 1.65 and 2.82 cents per pound error above and below the means for the beef and pork equations respectively.

Both beef and pork equations were estimated in linear form, therefore, the property of nonconstant elasticity, a feature of linear demand functions, presents a problem for the selection of meaningful points for quantification. The following quarterly consumption rates and annual income rates were selected for evaluating the elasticities of the beef pork estimates: $X_1 = 27.5$, $X_2 = 16.25$ and $X_3 = 2350$. These values appear consistent for the present and near future. In the quantity dependent form, the beef and pork demand functions are:

$$(3)\quad X_1 = 28.71863 - .36659\,P_1 - .53459\,S_1 - .34294\,S_2 + .76503\,S_3 + .01346\,X_3$$

$$(4)\quad X_2 = 26.34910 - .30850\,X_1 - .19918\,P_2 - 1.38819\,S_1 - 2.20260\,S_2 - 1.68076\,S_3 + .00510\,X_3$$

The most influential factor in the retail prices of beef and pork is the per capita consumption of beef.[6] The price elasticities of demand for beef and pork are –1.19 and –.84 respectively. Although not a precise measure of consumer demand, the values obtained suggest an elastic response in the quantity of beef consumed for changes in the retail price of beef, while pork consumption is inelastic in response to changes in the price of pork.

The income elasticity of demand, a measure of the change in the consumption rate to a corresponding change in personal disposable income, was 1.15 for beef and .74 for pork. This would indicate, other things being equal, that per capita consumption of beef would increase at a faster rate than pork with increases in personal disposable income. Past evidence has tended to bear out that beef is preferred to pork in the consumer's diet. If trends continue into the future with rising personal income, beef producers will benefit more than pork producers. However, the benefit would not be expected to occur at the expense of pork.

In the estimated function for retail beef prices it was found that per capita consumption of pork was not statistically significant. However, per capita pork as well as beef consumption was found significant in the retail price of pork. This appears contrary to past analyses of these two products.

There are two plausible arguments that may help to explain these results. First, because of the longer cycle for beef production, variations in beef supply were not as great as variations in pork supply. Thus, small fluctuations in beef price as a result of variations in pork supplies were not statistically measurable during the period of analysis.

Second, due to the rapid rise in personal income during the last decade, variation in beef and pork prices were offset by the income effect. When the price of beef increased relative to pork, consumers preferred to maintain their consumption of beef and accept a temporary decrease in real income rather than substitute pork for beef. Conversely, during periods of increased pork prices relative to beef prices consumers shifted from pork to beef consumption. The argument maintains that once a level of beef consumption has been attained, the consumer will not retreat to a less preferred good for small changes in retail price. In any case, some caution should be taken in accepting this relationship too readily. Further work should be devoted to these relationships.

The cross elasticity for pork with respect to beef is .31. The rela-

[6] The reader is reminded that per capita consumption is defined as the equivalent of per capita production with minor adjustments on an annual or quarterly basis.

tionship indicates that for a one percent increase in the retail price of beef, per capita consumption of pork would increase .31 percent.

The coefficients for seasonal variation shift the demand curve to the right or left, to reflect the different quantities consumed (above and below the average level in the fall quarter) during the seasons of the year, at the same price level. The coefficients for seasonal variation for beef indicate that the consumption rates decrease .53 pounds per capita in the winter quarter, .34 pounds in the spring quarter, and increase .77 pounds in the summer quarter, relative to fall quarter levels. The fall quarter of consumption is contained in the constant term of the estimate. In the pork estimates, per capita consumption decreases below the fall quarter rate by 1.39 pounds in the winter, 2.20 pounds in the spring, and 1.68 pounds in the summer quarter.

## Retail to Wholesale Structure

An evaluation of the value structure for the beef and pork sectors requires an analysis of the relationships between the retail and wholesale levels. The price system at the wholesale level serves as a mechanism for distributing beef carcasses and semi-dressed pork cuts from the meat packers to retail outlets. Generally, beef is quoted on a whole carcass basis at car-lot loads. Pork, on the other hand, is quoted as wholesale cuts, at less than car-lot loads, in the form of semi-dressed loins, boston butts, hams, bacon slabs, and other minor cuts.

Because of the aggregation problem and the complexity of the wholesale pork market, no attempt at the quantification of the performance between the retail and wholesale markets was attempted in this study. The study did, however, measure the industry performance from retail to the slaughter level, which includes the wholesale level.

The price relationship functions between levels were estimated for the period from 1957 to 1967 by months as follows:

$$(5)\quad \hat{P}_{w_1} = 9.96677 + \underset{(.03842)}{.74830\,P_1} - \underset{(.82630)}{12.95590\,W_r} - \underset{(.15065)}{.60680\,X_1}$$

$$R^2 = .804$$

Where:

$\hat{P}_{w_1}$ = price of Choice beef carcasses 600 to 700 pounds at Chicago (dollars per hundredweight).

$P_1$ = retail price of beef and veal (cents per pound).

$W_r$ = wage rate for food and kindred product workers (dollars per hour).

$X_1$ = first difference in per capita consumption between quarters ($X_t - X_{t-1}$).

The signs on the first two coefficients agree with economic theory. The standard errors of the coefficients are in parentheses. Results of the t-test indicate that all of the variables are significant at the .01 level. The three independent variables explain 80 percent of the variation in the dependent variable. The standard error of estimate indicates an error of plus or minus $1.49 per hundredweight at the mean.

The coefficient for the retail beef price is .75, reflecting the approximate carcass yield between the wholesale and the retail level for Choice beef carcasses. The coefficient tends to indicate good price performance between the two levels. The variable for wage rates represented the cost of preparing carcass beef for retail sales. The coefficient implies a cost of $13 per hundredweight, reflecting the retail cost for cutting and trimming, advertising, overhead, transportation, labor, and locker shrinkage.

The constant term of $10 per hundredweight represents the value of the by-products salvaged in the cutting and trimming process. The coefficient for the difference in per capita consumption represented the delay in adjustment to consistency between wholesale and retail prices. This results in alternate periods of gain or loss to the wholesale or retail level depending on increasing or decreasing supplies of beef which create short period "buyer" or "seller" marketing conditions. In general, the above estimating equation indicates good price performance between the wholesale and retail levels.

## Between Carcass Grades and Weights

The following relationships estimated the horizontal price performance at the wholesale level between various weights and grades of beef carcasses. Beef carcasses in the range of 600 to 700 pounds Choice grade were chosen as the base. Theoretically, if the price mechanisms were performing efficiently, the regression coefficient should be near one (reflecting relative yield rates), and the constant term equal to zero, for the various weights of a given grade of beef carcasses. Relationships between grades for a given weight should have a constant term reflecting grade differentials for quality and a slope coefficient of one. The equations were estimated as follows:

$$(6)\quad \hat{P}_{w_2} = .99811 + \underset{(.02068)}{.95982}\, P_{w_1} \qquad r^2 = .948$$

$$(7)\quad \hat{P}_{w_3} = .08362 + \underset{(.03456)}{.96180}\, P_{w_1} \qquad r^2 = .868$$

$$(8)\quad \hat{P}_{w_4} = .49322 + \underset{(.02231)}{.93471}\, P_{w_1} \qquad r^2 = .937$$

$$(9)\quad \hat{P}_{w_5} = 4.54895 + \underset{(.02629)}{.84482}\, P_{w_1} \qquad r^2 = .897$$

Where:

$\hat{P}_{w_1}$ = wholesale price of 600 to 700 pound Choice beef carcasses at Chicago (dollars per hundredweight).

$\hat{P}_{w_2}$ = wholesale price of 700 to 800 pound Choice beef carcasses at Chicago (dollars per hundredweight).

$\hat{P}_{w_3}$ = wholesale price of 800 to 900 pound Choice beef carcasses at Chicago (dollars per hundredweight).

$\hat{P}_{w_4}$ = wholesale price of 600 to 700 pound Good beef carcasses at Chicago (dollars per hundredweight).

$\hat{P}_{w_5}$ = wholesale price of 500 to 600 pound Good beef carcasses at Chicago (dollars per hundredweight).

The signs of all of the slope coefficients and most of the constant terms agree with economic theory, and the t-test shows them to be significant at the .01 level. The coefficients of determination ranged from .87 to .95. The standard errors above and below the mean were .75, 1.25, .81, and .95 per hundredweight, respectively.

Generally, the relationships were sufficiently strong and suggest efficient price performance between the various weights and grades at the wholesale level. Price variations in the 600 to 700 Choice carcasses explained from 87 to 95 percent of the variations in price of the selected grades and weights of carcasses. The regression coefficients were slightly less than one for the two Choice and high Good grade carcasses, while the constant terms were slightly greater than zero.

The coefficient for the 500 to 600 pound Good grade carcasses was considerably lower than expected at .84, while the constant term was higher than expected at 4.54. Some distortion in the estimated functions could have resulted from a narrow range in the data during the time period tested and the usually flattening fit from simple least squares. These two factors combined to tip the regression plane, yielding regression coefficients which were less than their expected values, while at the same time forcing the constant terms to intercept the axis at higher than their expected values. The above results however are quite adequate for forecasting purposes, and would probably yield estimates with no more than one or two percent error at the means.

## Wholesale to Slaughter Structure

An evaluation of the farm to retail transfer for beef requires an examination of the relationship between carcass and slaughter beef prices. In addition to examining the slaughter to wholesale level, this section will link the farm to retail price structure. The price of 900 to 1100 pound Choice steers was used as the base price. The relationships were estimated using monthly prices from 1957 to 1966 as follows:

$$(10)\ \hat{P}_{c_1} = -4.35784 + .68975\ P_{w_1} + .19009\ t \qquad R^2 = .951$$
$$(.01482) \qquad (.01702)$$

$$(11)\ \hat{P}_{c_1} = -3.06539 + .52421\ P_1 - 6.27350\ W_r \qquad R^2 = .760$$
$$(.02768) \qquad (.59443)$$

**Where:**

**$\hat{P}_{c_1}$ = price of Choice slaughter steers 900 to 1100 pounds at Omaha (dollars per hundredweight).**

**$P_{w_1}$ = price of Choice beef carcasses 600 to 700 pounds at Omaha (dollars per hundredweight).**

**$P_1$ = retail price of beef and veal (cents per pound).**

**t = annual trend (1957 = 1).**

**$W_r$ = wage rates for food and kindred product workers (dollars per hour).**

Signs of all coefficients agree with economic theory. Results of the t-test indicate that all of the coefficients are significant at the .01 level. The two independent variables in the farm to wholesale relationship explain 95 percent of the variation in slaughter beef prices, while in the farm to retail estimate, 76 percent of the variation is explained. The standard errors of the estimate above and below the mean are $0.49 and $1.07 a hundredweight, respectively.

The coefficient for carcass beef price in farm to wholesale equation indicated a liveweight to carcass yield of 69 percent. The value is high for the yield rate. On the other hand, the constant term which should measure the net cost of processing at the slaughter level is low at $4.36 per hundredweight. It is reasonable to assume that in beef carcass pricing, by-products are subtracted from the cost of processing, and the constant term should approximate the cost (negative) of processing plus the value of the salvaged by-products. Although somewhat more complicated, the result is similar to the relationship estimated in equation 5, between the wholesale and retail level. The coefficient for annual trend indicated an increase in the slaughter prices of 19 cents per hundredweight per year. Secular trends in plant efficiency and more direct selling could support this result.

In the second equation, the coefficient for retail price indicates a liveweight to retail yield of 52 percent. This yield rate appears to be consistent with yield rates found in the earlier analyses, from wholesale to retail and slaughter to wholesale. The value of the salvaged by-products are contained in the constant term, along with the processing, storage, and retail merchandising costs uncorrelated with wage rates. Thus, the value of the constant term at –$3.07 per hundredweight is a net cost, along with the correlated costs contained in the coefficient for wage rates.

In an earlier analysis no attempt was made to estimate the price relationship between the wholesale and retail levels for the pork industry. Therefore, in this section, the relationship between the retail and slaughter levels is examined directly. Theoretically, the price margin between the two levels should reflect the retail pork yield, minus the cost of processing and distribution, plus the value of any by-products salvaged in the slaughter to retail movement. The relationship was estimated as follows:

$$(12)\ \hat{P}_{h_1} = -12.77266 + \underset{(.02180)}{.55540}\ P_2 - \underset{(.72409)}{1.34337}\ W_r \qquad R^2 = .855$$

**Where:**

**$\hat{P}_{h_1}$ = price of No. 1–3 butcher hogs at Omaha, 220 to 240 pounds (dollars per hundredweight).**

**$P_2$ = retail price of pork excluding lard (cents per pound).**

**$W_r$ = wage rates for food and kindred product workers (dollars per hour).**

The signs of the coefficients agree with economic theory. Results of the t-test indicate that the retail price of pork is significant at the .01 level. The coefficient for wage rates is not significant, but was included on the grounds that its presence gave stability over time to the estimating system. The two independent variables explain 86 percent of the variation in the slaughter price of hogs. The standard error of estimate indicates an error of plus and minus $1.35 per hundredweight at the mean.

The uncorrelated marketing costs of processing, storage, cutting and trimming, plus the value of the salvaged by-products and lard are contained in the constant term. Costs correlated with wage rates are represented in the coefficient for wage rates. Their combined amounts indicate a net cost of approximately $15.50 per hundredweight of live hog when wage rates were at the mean. This value seems reasonably close to present industry standards. The coefficient on retail price accurately indicates a 55 percent yield rate of live to retail conversion. Overall, the estimating equation suggests efficient marketing performance for the pork industry from the slaughter to retail level.

## Between Slaughter Cattle Weights, Grades and Classes

The interrelationships of the price structure at the slaughter level for beef cattle were examined. Theoretically, the relationships between prices for the various weights, grades and class categories should reflect the relative yields, and the value of premiums or discounts for weight, grade or class. In the analysis, Choice steers at Omaha in the 900 to 1100 pound weight class were chosen as the base. The relationships were estimated as follows:

$$(13)\ \hat{P}_{c_2} = -.56483 + \underset{(.01143)}{1.02122}\ P_{c_1} \qquad r^2 = .985$$

$$(14)\ \hat{P}_{c_3} = .35661 + \underset{(.02002)}{.89833}\ P_{c_1} \qquad r^2 = .945$$

$$(15)\ \hat{P}_{c_4} = -2.26548 + \underset{(.02712)}{1.14684}\ P_{c_1} \qquad r^2 = .938$$

$$(16)\ \hat{P}_{c_5} = .81024 + \underset{(.01787)}{.93793}\ P_{c_1} \qquad r^2 = .959$$

$$(17)\ \hat{P}_{c_6} = .90758 + \underset{(.02561)}{.85018}\ P_{c_1} \qquad r^2 = .903$$

**Where:**

$P_{c_1}$ = price of Choice slaughter steers at Omaha, 900 to 1100 pounds (dollars per hundredweight).

$\hat{P}_{c_2}$ = price of Choice slaughter steers at Omaha, 1100 to 1300 pounds (dollars per hundredweight).

$\hat{P}_{c_3}$ = price of Good slaughter steers at Omaha, 900 to 1100 pounds (dollars per hundredweight).

$\hat{P}_{c_4}$ = price of Prime slaughter steers at Omaha, 1100 to 1300 pounds (dollars per hundredweight).

$\hat{P}_{c_5}$ = price of Choice slaughter heifers at Omaha, 800 to 1000 pounds (dollars per hundredweight).

$\hat{P}_{c_6}$ = price of Good slaughter heifers at Omaha, 700 to 900 pounds (dollars per hundredweight).

Results of the t-test indicate that all coefficients are significant at the .01 level. The coefficients of determination range from .90 to .98. The standard errors of estimate above and below the mean are 27, 47, 64, 42 and 60 cents per hundredweight, respectively.

The coefficients of determination suggested strong associations between the various weights, grades, and classes, indicating efficient performance in the price mechanism at the slaughter level. Examination of the constant terms and regression coefficients generally indicated price premiums paid for weight and quality differences in the heavy Choice and Prime grades. Discounts were evident in the Good grade and heifer classes. Improvements in the estimating technique, by using an error-in-variable model, could improve the consistency of the regression slopes and intercept points. For purpose of forecasting, the above estimating equations accurately measure the price differentials for weight, grade, and class.

## Between Slaughter Hog Weights and Locations

The following horizontal analysis estimates the price relationships between various weights and locations for slaughter hogs. The price for

220 to 240 pound slaughter hogs on the Omaha market served as the basis for the analysis. In an efficient market the price relationships should reflect the relative yields, and the resulting premiums or discounts for each weight class. Differences in price between locations, for the same weight class, should be equal to the differences in transportation cost between the locations and a common outlet. The equations estimated monthly from 1957 to 1966 are as follows:

$$(18)\quad \hat{P}_{h_2} = -.09246 + \underset{(.00154)}{1.00708}\, P_{h_1} \qquad r^2 = .999$$

$$(19)\quad \hat{P}_{h_3} = -.19895 + \underset{(.00662)}{.99157}\, P_{h_1} \qquad r^2 = .995$$

$$(20)\quad \hat{P}_{h_4} = -.38904 + \underset{(.01950)}{.89066}\, P_{h_1} \qquad r^2 = .946$$

$$(21)\quad \hat{P}_{h_5} = -.28552 + \underset{(.00580)}{1.00124}\, P_{h_1} \qquad r^2 = .996$$

$$(22)\quad \hat{P}_{h_6} = .36762 + \underset{(.00593)}{.97938}\, P_{h_1} \qquad r^2 = .996$$

$$(23)\quad \hat{P}_{h_7} = .53809 + \underset{(.00701)}{.97951}\, P_{h_1} \qquad r^2 = .994$$

**Where:**

- $\hat{P}_{h_1}$ = price of No. 1–3 slaughter hogs at Omaha, 220 to 240 pounds (dollars per hundredweight).
- $\hat{P}_{h_2}$ = price of No. 1–3 slaughter hogs at Omaha, 200 to 220 pounds (dollars per hundredweight).
- $\hat{P}_{h_3}$ = price of No. 1–3 slaughter hogs at Omaha, 240 to 270 pounds (dollars per hundredweight).
- $\hat{P}_{h_4}$ = price of No. 1–3 slaughter sows at Omaha, 330 to 400 pounds (dollars per hundredweight).
- $\hat{P}_{h_5}$ = price of No. 1–3 slaughter hogs at St. Paul, 220 to 240 pounds (dollars per hundredweight).
- $\hat{P}_{h_6}$ = price of No. 1–3 slaughter hogs at St. Louis, 220 to 240 pounds (dollars per hundredweight).
- $\hat{P}_{h_7}$ = price of No. 1–3 slaughter hogs at Indianapolis, 220 to 240 pounds (dollars per hundredweight).

All regression coefficients are significant at the .01 level. The coefficients of determination range between .99 and .94. The standard errors of estimate above and below the mean are 6, 25, 74, 22, 23 and 27 cents per hundredweight, respectively.

The values of $r^2$ for the above equations indicate strong competitive forces exist in the bidding for slaughter hogs. The evidence supports an efficient performance by the price mechanisms at the slaughter level. The prices paid for 220 to 240 pound hogs explain from 95 to 99 percent of the variation in the prices of hogs for various weights, classes and location. The regression coefficients and constant terms accurately reflect yield and price differentials for grades and weights of slaughter hogs. In the location estimates, transportation cost differences per hundredweight are represented by the values of the constant terms. The constant terms tend to reflect the proximity to major pork consumption centers.

## Slaughter to Feeder Structure

An evaluation of the slaughter to feeder relationship requires an analysis of the prices paid for feeder animals and their eventual slaughter price. The analysis requires lagging the price of feeder animals equivalent to a period of time necessary to fatten the animal from a feeder to slaughter weight. The price of 900 to 1100 pound Choice slaughter steers at Omaha was chosen as the base price.

Theoretically, if equilibrium producing decisions were being made and were reflected in the price structure, the regression coefficients and the constant terms should reflect cost of gain and feeding efficiency margins between the feeder and slaughter price level. Feeding costs were generally constant over the period so no feed price was used in the functions. The relationships were estimated using average monthly prices from 1957 to 1966 as follows:

$$(24)\quad \hat{P}_{f_{2t-3}} = 9.08457 + \underset{(.08627)}{.53576}\, P_{c_{1t}} \qquad r^2 = .246$$

$$(25)\quad \hat{P}_{f_{3t-4}} = 11.59478 + \underset{(.10504)}{.47352}\, P_{c_{1t}} \qquad r^2 = .147$$

$$(26)\quad \hat{P}_{4t-6} = 17.10627 + \underset{(.12893)}{.32310}\, P_{c_{1t}} \qquad r^2 = .050$$

**Where:**

**$\hat{P}_{c_{1t}}$ = price of Choice slaughter steers at Omaha, 900 to 1100 pounds (dollars per hundredweight).**

**$\hat{P}_{f_{2t-3}}$ = price of feeder steers at Omaha, 800 to 900 pounds lagged three months (dollars per hundredweight).**

**$\hat{P}_{f_{3t-4}}$ = price of feeder steers at Omaha, 700 to 800 pounds lagged four months (dollars per hundredweight).**

**$\hat{P}_{f_{4t-6}}$ = price of feeder steers at Omaha, 500 to 700 pounds lagged six months (dollars per hundredweight).**

Results of the t-test indicate that the coefficients on the independent variables in the first two equations are significant at the .01 level, while in the third equation the coefficient is significant at the .05 level. The standard errors of estimate are $1.94, $2.34 and $2.84 per hundredweight, respectively, above and below the means.

The $r^2$ values are extremely low in all three equations, which indicate very poor performance in the consistent pricing of feeder steers. The $r^2$ values indicate that variations in the price of the base slaughter steer class explain only 25, 15 and 5 percent, respectively, of the variations in the price of feeder steers.

If the marketing system were performing efficiently, the base slaughter prices for 900 to 1100 pound steers would be expected to explain at least 80 percent of the variations in feeder steer prices. The $r^2$ on each of the estimating equations indicates that the lighter the feeder steer the less efficient is price performance. The more distant in time the feeder animal was from slaughter, the greater the inability of feedlot operators to anticipate future slaughter prices.

If the marketing system for feeder animals is performing efficiently, the regression coefficient would be approximately the ratio of slaughter to feeder weight, and the constant term would reflect the average supply cost per hundredweight of moving the animal from a feeder class to a slaughter class. However, in the above three equations, none of the regression coefficients or constant terms consistently reflect these relationships—the lighter the feeder animal, the greater the distortion from the expected value. The estimates indicate that prices paid for feeder animals are not based on expected future slaughter price.

If the previous three equations were estimated without lags, the results would indicate the influence of existing slaughter prices on the bidding for feeder animals. The relationships were estimated in a manner similar to the previous equations, with the exception of the lags, and are as follows:

$$(27)\quad \hat{P}_{f_{2t}} = -.51070 + \underset{(.04920)}{.93168}\, P_{c_{1t}} \qquad r^2 = .752$$

$$(28)\quad \hat{P}_{f_{3t}} = -1.16446 + \underset{(.06496)}{.99561}\, P_{c_{1t}} \qquad r^2 = .666$$

$$(29)\quad \hat{P}_{f_{4t}} = -1.39168 + \underset{(.08290)}{1.07438}\, P_{c_{1t}} \qquad r^2 = .587$$

The prices paid for feeder animals seem to be influenced more by current slaughter prices than by expected slaughter prices (or expected prices are strongly based on current prices), since the $r^2$ values are considerably higher in these latter estimates. The relationships are not sufficiently strong to conclude that feeder prices are based solely on existing slaughter prices. The evidence is strong enough to conclude, however, that existing slaughter prices strongly influence the prices paid for feeder animals. The values of the constant terms and the regression coefficients tend to support these conclusions.

## Implications

The relationships in the preceding analyses reflect strong competitive forces which produce efficient performance from the slaughter to the retail level. Vertically, between the various market levels, and horizontally, in space and form for the various weights, grades and classes, the performance of the transfer mechanism was found to be good. This would indicate that the effects of improved order in the production and marketing of cattle and hogs could yield substantial industry-wide payoff.

Relationships between the price of slaughter and feeder cattle were found to be weak and distorted. Results showed the value bid for feeder animals did not efficiently reflect their future value in demand as slaughter animals. This appeared to be due to the inability of feedlot operators to correctly anticipate future slaughter prices. Consequently, ranchers and feeders alternately suffered windfall gains or losses, depending on the directional forces of supply and demand.

Besides the price system's role as a mechanism of trade, it must also serve as a signal for producers to organize the level of future production. If producers make the mistake of basing future production levels on existing product prices, they may run the risk of over or under production. When existing prices are above the equilibrium level, producers respond by increasing production, until product prices are driven down to the equilibrium level, due to increased product supplies.

If, however, producers overrespond to price, the increased supplies may drive price below the equilibrium level. Conversely, during periods of low product price, producers respond by decreasing production, driving the price up to the equilibrium level. Again, if producers overrespond, price may be driven above the equilibrium level. Overresponse to the price signals results in alternate periods of overproduction and underproduction accompanied by the corresponding inverse cyclical patterns in product price.

The inefficiency in the price mechanism between the primary production and feeding levels is both a production and marketing problem. In the long run, producers must try to anticipate correctly future demand levels, and then organize parallel production patterns. In the short run, producers must adjust marketing weights to maintain orderly flows of product into the market channels if they want to maintain equilibrium level prices.

An inherent biological problem in the beef-pork sector is the length of the production process. In beef production it may take from two and one-half to four years from the time a production decision is made until eventual marketing as a slaughter animal. In pork production, the time required is from nine months to a year. Short-run adjustments in marketings can be implemented much faster due to the ease of slaughter or carrying of animals to heavier weights. The problem therefore becomes one of establishing long-run production levels to meet expected future demands, and then allowing for short-run adjustments in marketing as the animal reaches the slaughter point, to correct for errors in the production level. Hopefully, both performances can be made consistently optimal.

## THEORETICAL MODEL

The economic logic and programming procedures employed in the equilibrium solution for orderly production and marketing of beef and pork are presented in this section. The discussion will be limited to a presentation of the models and assumptions. No lengthy or elaborate mathematical proofs will be attempted.

The theoretical model used in this study is a time and form equilibrium allocation model. The model is designed to allocate competitively the flow of reproduction, production and marketing over time, such that supply costs are minimized. Simplifying assumptions treat the beef and pork sectors as being completely competitive, with price patterns over time being dependent on flows of supplies into the market against the demands. It is also assumed that demands through time can be represented by known linear demand functions, and for any point in time, animal inventory numbers by categories can be determined.

Given the initial animal inventories and the projected set of linear demand functions for meat and final inventories, the problem becomes one of determining the equilibrium minimum cost allocation over time of animals to reproduction, feeding, and final slaughter for consumption.

Remember that the final meat demands are represented by a series of linear demand functions, but for solution purposes only single points are selected on each demand function. The duality theorem of linear programming offers a mechanism for finding the optimum equilibrium solution.

In equilibrium, the linear programming shadow prices on the demand constraints should equal the demand prices used and the shadow prices on the final inventories should equal the supply costs.[7] Consequently, to obtain an equilibrium solution, the procedure necessarily becomes iterative. First, an initial solution is computed. Second, the prices on the demand functions are adjusted to agree with the dual solution. Third, the model is recomputed for a new solution.

[7] For a more theoretical treatment see: G. G. Judge and T. D. Wallace, "Estimation of Spatial Price Equilibrium Models," *Journal of Farm Economics,* Vol. XLI, No. 4, Nov., 1959, pp. 801–820.

The iterative process is continued until a consistent optimal equilibrium solution is obtained, which by definition is also the competitive equilibrium solution.

## SOLUTION PROCEDURES

Two iterative linear programming models were developed for the orderly production and marketing of beef and pork. The models were designed to determine optimum inventory flows for beef and pork to aid the industries in making economic adjustments from situations of disequilibrium to equilibrium and maintenance of that position. The models not only have the capability of simultaneously allocating existing inventories to future market demands, but also the determination of the proper consistent levels of replacement and culling of the reproduction base for production in the future, while minimizing total supply costs associated with production and marketing. In addition, the derived value of future demand for any class of animal in the various stages of growth can be determined from the shadow prices. The models have the capability of generating an equilibrium inventory flow starting from any initial animal inventory position above, below, or at equilibrium.

### Beef Model

The inventory flow model for beef, hereafter referred to as the beef model, contains 253 restrictions and 912 activities.[8] The model was designed for a six-year production and marketing period. The first 25 restrictions are initial inventory categories. The categories, subdivided by weight classes, include: five classes of feedlot steers, five classes of feedlot heifers, five classes of non-feedlot steers, and five classes of non-feedlot heifers. The reproduction base was divided into four categories of cows according to season of calf drop. One category in the initial inventory was reserved to represent cull cows from the reproduction base moving into the meat demand sector.

Following the initial inventory are 73 restrictions allowing for reproduction, culling, and heifer replacement for the six-year production period. The primary function of these restrictions is to generate calf production and the maintenance of the cow herd. Calves saved percentages are based on past industry performance subdivided seasonally. Calves saved are arbitrarily divided equally between steers and heifers. A 14 percent culling rate is assumed to represent the physical deterioration of the cow base, plus some nonbreeders. Herd replacements are made dependent on the model's evaluation of the

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[8] A schematic diagram outlining the various functions of the beef model is contained in Appendix A.

heifer's opportunity value as slaughter beef opposed to her value in future reproduction. The result is that although culling is strictly arbitrary, replacement is based on economic choice.

The next 144 restrictions in the beef model represent monthly quantities of beef demanded during the six-year marketing period. The first 72 restrictions represent the monthly quantities of beef demanded.[9] The second 72 restrictions are necessary to limit the quantity of Prime grade beef during any month to less than 20 percent of the month's total beef demand.[10] No restrictions are placed on the quantity of Good and Choice beef demanded during any month.

The last 11 restrictions in the beef model are final animal inventory classes. The classes are similar but not identical to the initial inventory classes. The purpose of the final inventory is to terminate the model with a carryover of animals necessary for the production and marketing in the seventh and ensuing years.

The 914 columns or activities in the beef model proved for marketings of steers and heifers from the initial animal inventory to the meat demand sector, cow culling and heifer replacement, the production and marketing of program generated supplies, and the termination of the program into final animal inventory. Corresponding to each activity is the estimated supply cost of producing the animal to that stage of growth or marketing weight.

In the programming model, the supply costs were defined as the $C_j$ values, or the total costs associated with the unit level of the $j^{th}$ activity. For animals in the initial inventory, all prior costs of production are disregarded. The $C_j$ values are only the costs necessary to carry the animals forward to one of the alternative marketing weights or utilization states. This is tantamount to asserting that in the short-run prior costs fixed into the animal do not affect the future allocation decision. The decision process should only be affected by the additional costs and returns of carrying the animal forward.

The $C_j$ values on the reproduction activities are set at zero. All costs incurred during the one-year reproduction period, including the carrying cost of the cow, are carried forward on the calf. Thus, at marketing, the $C_j$ values represent all the supply costs associated with the animal, and are equal to the total added value of producing the animal.

The beef model provides for a number of production and marketing alternatives. The production alternatives include: high energy feedlot finishing, medium energy feedlot finishing, and low energy non-feedlot finishing. Steers and heifers less than 800 pounds are not

[9] The nature and form of the demand restrictions are presented in more detail in a later section.

[10] This was a judgment to avoid the addition of many more column vectors having varying $C_j$ values reflecting changing prime price differentials.

considered for slaughter. In the model, steers greater than 800 pounds have 18 marketing alternatives, while heifers have 16. Figures 1 and 2 trace the production and market alternatives for steers and heifers, respectively.

## Pork Model

The pork model has 229 restrictions and 520 activities, and was designed for a three-year production and marketing period.[11] The first 29 restrictions are initial inventory categories. Represented in the initial inventory are eight classes of barrows and gilts, subdivided by weight. These classes account for all live hogs in the inventory, ranging from newborn pigs to fat butcher hogs, not yet committed to slaughter or reproduction.

Also in the initial inventory are two categories of the reproduction base. The first category contains 12 classes of bred sows, subdivided by the number of previous farrowings and the expected month of next farrowing. The second category is made up of nonbred and lactating sows, classified by number of farrowings and month of last farrowing.

Following the initial inventory are 144 reproduction restrictions. The first 108 allow for reproduction, culling, and gilt replacements. In the programming model, multi-farrowing is limited to three litters per sow. It was thought that no appreciable reduction in the cost of reproduction could be achieved after three farrowings, and would

[11] A schematic diagram outlining the various functions of the pork model is contained in Appendix A.

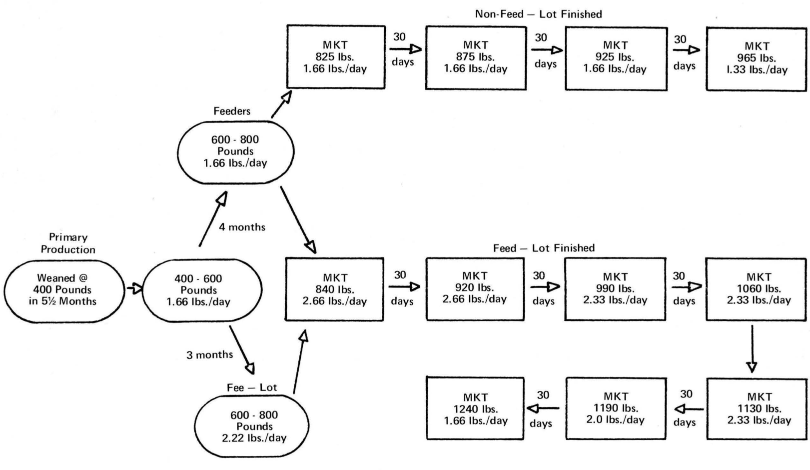


**Fig. 1. Flow diagram for steers from primary production to slaughter.**

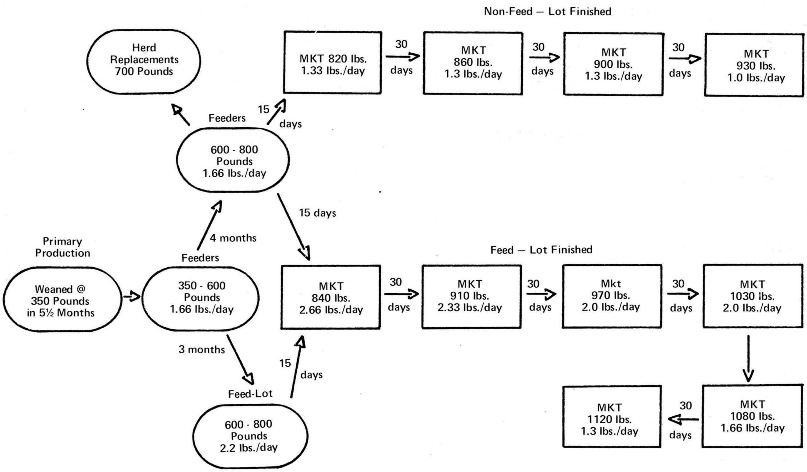


**Fig. 2. Flow diagram for heifers from primary production to slaughter.**

only complicate computation and interpretation. Arbitrarily, 25 percent of the gilts are culled after first farrowing. These represent an estimate of the percentage of poor quantity and quality breeders. All other culling and gilt replacement is dependent on the model evaluation of the gilt's or sow's opportunity value in slaughter versus future production. The remaining 36 restrictions in the reproduction category pool the number of pigs saved by months. These restrictions are necessary because of the greater number of pigs saved by second and third litter sows.

The last 56 restrictions are the meat demands and final animal inventory. The demands are 36 monthly demand functions. No quality restrictions are used in the pork model. The last 20 rows are used to terminate the model into final animal inventory classes, similar in specification to the initial animal inventory categories. The purpose of the final inventory is identical to that in the beef model.

The functions of the 520 activities in the pork model are similar to those in the beef model. The activities provide for production and marketing of barrows and gilts, plus the maintenance and adjustments necessary in the reproduction base. The $C_j$ values associated with each activity are the estimated carrying costs or added value of the animal. The $C_j$ values on initial inventories are treated the same as in the beef model. All costs incurred during the reproduction period are carried forward on the pigs.

In the pork model only a single feeding alternative is considered on butcher hogs. The butcher hog could be marketed during a four-month period from five months to nine months of age. Multi-farrowing allows for single, double and triple farrowings. Movements from disequilibrium to equilibrium through adjustments in births and weights can be achieved much faster in pork production.

## Estimation of Supply Cost

In conventional transportation-storage models, the transfer costs are those associated with the movement and storage of a commodity from the $i^{th}$ supply sector to the $j^{th}$ demand sector. The commodity moves into storage and is released as essentially the same product with only negligible changes in quality or quantity. In the inventory flow model for beef and pork, cattle and hogs held for later marketing grow and mature changing in both weight and grade. Adjustments were necessary to account for differences in quality and quantity.

Partial budgets were developed for each production and marketing activity. The budgets represent all costs of production including land, labor, capital and management. The supply cost of an animal at any marketing weight is the sum of the transfer costs of all the individual production activities necessary to move the animal from birth. Stated differently, the $C_j$ value of any activity represents the total cost of production for the animal up to that stage of growth. As mentioned earlier, however, prior production costs on initial inventory animals are disregarded. Total supply costs per animal, along with the average and marginal costs per hundredweight for the various production and marketing activities, are contained in Tables 1, 2 and 3.

Changes in quality or grade during growth are accounted for by additions or subtractions from Choice beef supply costs. Premiums paid for Prime grades are subtracted from supply costs, while discounts for Good grade animals are added to supply costs. No adjustments are necessary on Choice grade animals. Changes in grade are not directly accounted for in hog production. The quality of pork as meat is not thought to change significantly with added weight. The changes in carcass yield rates with increases in liveweight adequately accounted for changes in quality and quantity.

## Estimation of the Demand Functions

This section presents demand functions used in the temporal equilibrium analyses. Demand for agricultural products as well as other goods is normally measured at the retail level. However, our interests are most effectively evaluated by standardizing at the slaughter level. In the previous section, per capita retail price relationships were estimated for beef and pork on a quarterly basis. Converting the retail quarterly estimates of beef and pork demand (Equations 3 and 4) to the quantity dependent form on a monthly basis we obtain:

Table 1. **Estimated transfer cost and total cost for various weights of steers, unadjusted for seasonal variation.**

| Description | Transfer cost per animal | Total cost per animal | Average cost per cwt. | Marginal cost per cwt. |
|---|---|---|---|---|
| Feedlot at 600 pounds | | | | |
| 600– 800 pounds | $38.60 | $219.70 | $27.47 | $19.30 |
| 800– 840 pounds | 7.15 | 226.85 | 27.00 | 17.87 |
| 840– 920 pounds | 14.76 | 241.61 | 26.26 | 18.45 |
| 920– 990 pounds | 14.63 | 256.24 | 25.88 | 20.90 |
| 990–1060 pounds | 15.50 | 271.74 | 25.63 | 22.14 |
| 1060–1130 pounds | 16.28 | 288.02 | 25.48 | 23.22 |
| 1130–1190 pounds | 17.28 | 305.30 | 25.66 | 28.80 |
| 1190–1240 pounds | 19.95 | 325.25 | 26.23 | 39.90 |
| Feedlot at 800 pounds | | | | |
| 600– 800 pounds | 46.54 | 227.54 | 28.44 | 23.27 |
| 800– 840 pounds | 7.15 | 234.69 | 27.94 | 17.87 |
| 840– 920 pounds | 14.76 | 249.45 | 27.11 | 18.45 |
| 920– 990 pounds | 14.63 | 264.08 | 26.67 | 20.90 |
| 990–1060 pounds | 15.50 | 279.58 | 26.37 | 22.14 |
| 1060–1130 pounds | 16.28 | 295.86 | 26.18 | 23.22 |
| 1130–1190 pounds | 17.28 | 313.14 | 26.31 | 28.80 |
| 1190–1240 pounds | 19.95 | 333.09 | 26.86 | 39.90 |
| Non Feedlot | | | | |
| 600– 800 pounds | 46.54 | 227.54 | 28.44 | 23.27 |
| 800– 825 pounds | 5.94 | 233.48 | 28.30 | 23.76 |
| 825– 875 pounds | 12.25 | 245.73 | 28.08 | 24.50 |
| 875– 925 pounds | 12.86 | 258.59 | 27.96 | 25.72 |
| 925– 965 pounds | 13.05 | 271.64 | 28.15 | 32.62 |

$$(30)\quad Q_B = 9.57287 - 0.12220\ P_{BR} - 0.17820\ S_1 - 0.11431\ S_2 + 0.25501\ S_3 + 0.01346\ Y_D$$

$$(31)\quad Q_P = 8.78303 - 0.30850\ Q_B - 0.06639\ P_{PR} - 0.46273\ S_1 - .73420\ S_2 - 0.56025\ S_3 + 0.00170\ Y_D$$

Also in a previous section retail to slaughter price relationships (Equations 11 and 12) were developed using monthly data. Transforming these relationships into the retail price dependent form and substituting them for beef and pork prices in the retail demand estimates, the derived demands at the slaughter level are:

$$(32)\quad Q_B = 8.85829 + 0.2331\ P_{C_1} + 1.46243\ W_R - 0.17820\ S_1 - 0.11431 + 0.25501\ S_3 + 0.00429\ Y_D$$

$$(33)\quad Q_P = 7.25624 - 0.308050\ Q_B - 0.11954\ P_{H_1} - 0.16058\ W_R - 0.46273\ S_1 - 0.73420\ S_2 - 0.56025\ S_3 - 0.00170\ Y_D$$

The above derived demand relationships have the effect of making the per capita quantity of beef demanded dependent on the slaughter price of 900 to 1,100 pound Choice steers at Omaha. Per capita quan-

**Table 2. Estimated transfer cost and total cost for various weights of heifers, unadjusted for seasonal variation.**

| Description | Transfer cost per animal | Total cost per animal | Average cost per cwt. | Marginal cost per cwt. |
|---|---|---|---|---|
| Feedlot at 600 pounds | | | | |
| 600– 800 pounds | $40.32 | $226.38 | $28.30 | $20.16 |
| 800– 840 pounds | 7.23 | 233.61 | 27.81 | 18.08 |
| 840– 910 pounds | 13.94 | 247.55 | 27.20 | 19.91 |
| 910– 970 pounds | 13.58 | 261.32 | 26.94 | 22.63 |
| 970–1030 pounds | 14.46 | 275.59 | 26.75 | 24.10 |
| 1030–1080 pounds | 15.44 | 291.03 | 26.92 | 30.88 |
| 1080–1120 pounds | 15.25 | 306.28 | 27.34 | 38.13 |
| Feedlot at 800 pounds | | | | |
| 600– 800 pounds | 46.68 | 232.57 | 29.07 | 23.34 |
| 800– 840 pounds | 7.23 | 239.80 | 28.54 | 18.08 |
| 840– 910 pounds | 13.94 | 253.74 | 27.88 | 19.91 |
| 910– 970 pounds | 13.58 | 267.32 | 27.55 | 22.63 |
| 970–1030 pounds | 14.46 | 281.78 | 27.36 | 24.10 |
| 1030–1080 pounds | 15.44 | 297.22 | 27.52 | 30.88 |
| 1080–1120 pounds | 15.25 | 312.47 | 27.90 | 38.13 |
| Non Feedlot | | | | |
| 600– 800 pounds | 46.68 | 232.57 | 29.07 | 23.34 |
| 800– 820 pounds | 5.98 | 238.55 | 29.09 | 29.90 |
| 820– 860 pounds | 11.91 | 250.46 | 29.13 | 29.77 |
| 860– 900 pounds | 12.64 | 263.10 | 29.23 | 31.61 |
| 900– 930 pounds | 12.42 | 275.52 | 29.63 | 41.40 |

tity of pork demanded was made dependent on the price of 220 to 240 pound No. 1–3 hogs at Omaha and the per capita consumption of beef.

The estimated demand functions for beef and pork contained a number of exogenous variables, all of which, except for product price, were considered predetermined in the programming analysis. This required fitting relationships for each exogenous variable. In addition, per capita consumption had to be converted to total con-

**Table 3. Estimated transfer cost and total cost for various weights of barrows and gilts, unadjusted for seasonal variation.**

| Description | Transfer cost per animal | Total cost per animal[a] | Average cost per cwt. | Marginal cost per cwt. |
|---|---|---|---|---|
| Barrows and gilts | | | | |
| 50–180 pounds | $19.82 | $30.82 | $17.12 | $15.25 |
| 180–230 pounds | 7.73 | 38.55 | 16.76 | 15.46 |
| 230–270 pounds | 8.17 | 46.72 | 17.30 | 20.42 |
| 270–300 pounds | 8.37 | 55.09 | 18.36 | 27.90 |

[a] Assumes an $11 cost of production for a 50 pound weaned pig. In the programming model the farrowing costs through weaning are expressed as total cost per litter.

sumption for the programming analysis. The conversion of per capita to total consumption required multiplying per capita consumption by population. The estimates of the exogenous variables and population using time as the independent variable are:

$$(34)\quad \hat{I}_{CPij} = 97.34538 + \underset{(.00975)}{0.12528}\ t_{ij} \qquad r^2 = .975$$

$$(35)\quad \hat{W}_{Rij} = 1.92290 + \underset{(.00008)}{0.00507}\ t_{ij} \qquad r^2 = .969$$

$$(36)\quad \hat{T}_{YDik} = 275.466961 + 5.33843\ t_{ik} \qquad r^2 = .977$$

**Converted to $P_i = (181.6)\,(1.0069)^i$**

**Where:**

$\hat{I}_{CPij}$ = Consumer price index (1957 to 1959 = 100).

$\hat{W}_{Rij}$ = Wage rates for food and kindred product workers.

$\hat{T}_{YDik}$ = Total personal disposable income on an annual basis.

$\hat{P}_i$ = United States civilian population, annual estimates.

t = Time.

i = Years (1950 = 1).

j = Months (January = 1).

k = Quarters (Winter = 1).

The coefficients on all variables are significant at the .01 level. The values of $r^2$ indicate a high degree of correlation between the dependent and independent variable, suggesting good forecasting relationships. The values for the exogenous variables in slaughter level demand functions in the programming analysis are estimated from the equations.

## Estimation of Net Demand

The estimated net demand functions presented above considered the total demand for beef and pork. Included in the total consumption of beef are cull cows and calves from the dairy herd, plus the import-export balance. The total consumption of pork includes the import-export balance. However, in the programming analysis, interest is in domestically produced and marketed cattle and hogs. This required adjustment for the quantity of beef supplied by the dairy sector. Import-export balances are assumed to be positive at a constant percentage over the projected demand periods at six and three percent of total demand for beef and pork, respectively. The estimates of the by-products of the dairy sector are:

$$(37)\quad \hat{D}_{C_i} = 11{,}027.12 - 300.22\, t_i \qquad r^2 = .873$$
$$(50.92)$$

$$(38)\quad \hat{D}_{V_i} = 1{,}322.54 - 46.18\, t_i \qquad r^2 = .675$$
$$(20.71)$$

**Where:**

$\hat{D}_{C_i}$ = **Total carcass beef equivalent supplied by dairy cows per year in millions of pounds.**

$\hat{D}_{V_i}$ = **Total carcass veal equivalent supplied by dairy calves per year in millions of pounds.**

$t_i$ = **Years (1950 = 1).**

The net beef demands to be used in the programming analysis are obtained by subtracting the cull beef and veal supplied by the dairy sector and the import-export balances from the total demands. Net pork demands are equal to total pork demand minus the import-export balance.

# SOLUTION RESULTS AND GENERAL EVALUATION (BEEF MODEL, 1968-1973)

At this point attention will be given to presentation and evaluation of solution results for the beef model—with special emphasis directed towards the estimated equilibrium price pattern, the form and time allocation of steers and heifers to slaughter, the size and seasonal distribution of the program-generated reproduction base, and the value of the objective function. The specific setting which was chosen begins with inventory conditions on January 1, 1968.

## Initial Inventory

The beginning animal inventory used in the beef model is shown in Table 4. The basic inventory numbers were taken from published reports by the United States Department of Agriculture. Determination of specific inventory numbers by weight and class were made by a balance sheet method using seasonal reproduction levels in prior years and normal growth weights. This information was supplemented by slaughter levels in previous years, by weight and by class. The size of the reproduction herd was assumed equal to the number of beef cows two years old and older. Seasonal breakdown by quarters of calf crop was determined from reported distributions of steers and heifers by weight and number in previous year inventories of feedlot and slaughter activities.

Although the described accounting method may not yield correct weight and disposition numbers for the beef cattle inventory, it was the best method available at the time. *The intent of this study was*

**Table 4. Initial inventory of steers and heifers on feed, feeder steers and heifers, and reproduction base, January 1, 1968.**

| Description | Number (1000 head) |
|---|---|
| Feedlot steers | |
| > 1100 pounds | 380 |
| 900–1099 pounds | 2011 |
| 700– 899 pounds | 2494 |
| 500– 699 pounds | 1904 |
| < 500 pounds | 1061 |
| Feedlot heifers | |
| > 1100 pounds | 0 |
| 900–1099 pounds | 382 |
| 700– 899 pounds | 1102 |
| 500– 699 pounds | 1108 |
| < 500 pounds | 802 |
| Feeder steers and calves | |
| > 750 pounds | 2739 |
| 550– 749 pounds | 4795 |
| 400– 549 pounds | 2640 |
| 299– 399 pounds | 4714 |
| < 200 pounds | 2595 |
| Feeder heifers and calves | |
| > 700 pounds | 573 |
| 500– 699 pounds | 4638 |
| 350– 499 pounds | 4973 |
| 200– 349 pounds | 2640 |
| < 200 pounds | 2595 |
| Reproduction base | |
| Cows to calf in winter quarter | 9990 |
| Cows to calf in spring quarter | 13061 |
| Cows to calf in summer quarter | 6107 |
| Cows to calf in fall quarter | 6142 |
| Culls from previous year cow base | 4856 |

*not to provide a precise or accurate adjustment method for incomplete inventory data, but rather to demonstrate the need for and the accuracy requirements of such data if used in normative economic models for industry order.*

## Equilibrium Price Path

The duality theorem of linear programming gives the necessary conditions for an optimum equilibrium solution. The shadow prices on the demand constraints can be interpreted as the marginal value product of the commodity in demand. The necessary conditions of temporal and form equilibrium are met when each of the marginal value products are equal to the corresponding prices used to specify the demand constraints, and when price levels between months differ by marginal supply cost differences.

In the programming model, units of supply are expressed in terms of animal units, while units of demand are expressed in pounds of carcass beef. To compare the demand constraint shadow prices with the Choice grade live prices used in the demand functions, the latter must be converted to carcass equivalent. The most common marginal supply category is the Choice grade. A 60 percent conversion rate is used to simplify the identification and evaluation of the equilibrium solution. Therefore, to equate live prices used and shadow prices obtained in the programming solution, the live prices are multiplied by the reciprocal of the 60 percent.

The estimated equilibrium Choice steer prices, the corresponding net beef demands, and the program shadow prices by months are presented in Table 5. The results were obtained from the linear programming solution based on minimizing the total supply cost of producing and marketing beef during a six-year period beginning January 1, 1968. The estimated prices are based on the Omaha price for 900 to 1100 pound Choice grade steers. To obtain Prime or Good grade price equivalents or the price equivalents between classes, the values must be adjusted up or down according to the competitive price differentials between grades, weights and classes.

## Evaluation

The equilibrium Choice steer price path decreased from a high of $28.45 a hundredweight in January 1968 to a low of $22.12 in July of the same year. When marketings were completed from the initial animal inventory, the price path moved into regular cyclical patterns of increasing slaughter beef prices, starting in October and peaking in May, followed by decreasing paths from June to September. The pattern can be seen more clearly in Figure 3.

The range between the peaks and the valleys of the cycles becomes smaller as production and marketing are adjusted. It is obvious that the equilibrium price path in the optimum beef solution does not parallel actual market prices during the 1968 and 1969 demand period. The different shape of the price path is largely due to an inability to correctly identify and estimate the composition of the beginning inventory of cattle and calves from currently reported statistics.

In equilibrium, a cyclical pattern in beef price is expected as opposed to a flat or constant price pattern. The expected cyclical price patterns result from seasonal differences in production costs resulting from biological, climatic, and traditional factors affecting the timing of the calf drop.

Historically, the winter and spring calf crops have accounted for about two-thirds of the total calves produced. Cow-calf operators in the traditional calf producing areas of the Western Plains and Moun-

**Table 5. Choice steer prices, program shadow prices, and differences for the six-year production and marketing period (Omaha slaughter level basis).**

| Year | Month | Net beef demand (mil. lb.) | Choice steer prices (dollars/cwt.) | Program shadow prices (dollars/cwt.) | Converted live prices (dollars/cwt.) | Differences (dollars/cwt.) |
|---|---|---|---|---|---|---|
| 1968 | January | 1460 | 28.45 | 47.23 | 48.70 | 1.47 |
| | February | 1500 | 27.62 | 45.85 | 47.41 | 1.56 |
| | March | 1560 | 26.34 | 43.72 | 45.05 | 1.32 |
| | April | 1620 | 25.05 | 41.59 | 41.45 | – .14 |
| | May | 1670 | 24.00 | 39.84 | 40.69 | .85 |
| | June | 1710 | 23.17 | 38.47 | 37.71 | – .76 |
| | July | 1760 | 22.12 | 36.73 | 37.22 | .49 |
| | August | 1730 | 22.88 | 37.98 | 38.10 | .12 |
| | September | 1730 | 22.96 | 38.11 | 38.48 | .37 |
| | October | 1740 | 22.81 | 37.86 | 37.93 | .08 |
| | November | 1700 | 23.79 | 39.49 | 38.90 | – .59 |
| | December | 1670 | 24.54 | 40.74 | 39.95 | – .79 |
| 1969 | January | 1640 | 25.29 | 41.99 | 40.97 | –1.02 |
| | February | 1630 | 25.59 | 42.48 | 41.88 | – .60 |
| | March | 1630 | 25.66 | 42.60 | 42.11 | – .49 |
| | April | 1630 | 25.74 | 42.73 | 41.54 | –1.19 |
| | May | 1620 | 26.04 | 43.23 | 42.62 | – .61 |
| | June | 1680 | 24.76 | 41.11 | 39.59 | –1.52 |
| | July | 1740 | 23.49 | 39.00 | 39.08 | .08 |
| | August | 1720 | 24.01 | 39.87 | 40.13 | .26 |
| | September | 1770 | 22.97 | 38.13 | 38.28 | .15 |
| | October | 1770 | 23.05 | 38.26 | 38.30 | .04 |
| | November | 1740 | 23.79 | 39.49 | 39.26 | – .23 |
| | December | 1700 | 24.75 | 41.10 | 40.30 | – .80 |
| 1970 | January | 1660 | 25.72 | 42.70 | 42.89 | .19 |
| | February | 1620 | 26.68 | 44.30 | 43.80 | – .50 |
| | March | 1610 | 26.98 | 44.79 | 44.66 | – .13 |
| | April | 1610 | 27.05 | 44.90 | 43.90 | –1.00 |
| | May | 1610 | 27.12 | 45.02 | 44.75 | – .27 |
| | June | 1690 | 25.40 | 42.17 | 41.54 | – .63 |
| | July | 1750 | 24.14 | 40.07 | 40.89 | .82 |
| | August | 1710 | 25.10 | 41.67 | 41.88 | .21 |
| | September | 1770 | 23.83 | 39.57 | 39.80 | .23 |
| | October | 1770 | 23.90 | 39.68 | 39.81 | .13 |
| | November | 1750 | 24.42 | 40.54 | 40.71 | .17 |
| | December | 1720 | 25.16 | 41.77 | 41.75 | – .02 |
| 1971 | January | 1710 | 25.44 | 42.24 | 42.49 | .25 |
| | February | 1670 | 26.40 | 43.82 | 43.42 | – .40 |
| | March | 1640 | 27.13 | 45.04 | 44.51 | – .53 |
| | April | 1650 | 26.97 | 44.78 | 44.19 | – .59 |
| | May | 1640 | 27.26 | 45.26 | 45.18 | – .08 |
| | June | 1740 | 25.12 | 41.70 | 41.94 | .24 |
| | July | 1760 | 24.74 | 41.07 | 41.26 | .19 |
| | August | 1740 | 25.25 | 41.92 | 42.23 | .31 |
| | September | 1780 | 24.45 | 40.59 | 40.49 | – .10 |
| | October | 1810 | 23.84 | 39.57 | 39.79 | .22 |
| | November | 1790 | 24.35 | 40.42 | 40.70 | .38 |
| | December | 1760 | 25.07 | 41.62 | 41.74 | .12 |
| 1972 | January | 1750 | 25.36 | 42.10 | 42.33 | .23 |
| | February | 1720 | 26.08 | 43.30 | 43.30 | .00 |
| | March | 1700 | 26.59 | 44.14 | 44.40 | .26 |
| | April | 1700 | 26.65 | 44.24 | 44.02 | – .22 |
| | May | 1680 | 27.15 | 45.08 | 45.02 | – .06 |
| | June | 1760 | 25.46 | 42.27 | 41.79 | – .48 |

**Table 5. (continued)**

| Year | Month | Net beef demand (mil. lb.) | Choice steer prices (dollars/cwt.) | Program shadow prices (dollars/cwt.) | Converted live prices (dollars/cwt.) | Differences (dollars/cwt.) |
|---|---|---|---|---|---|---|
| | July | 1800 | 24.65 | 40.91 | 41.12 | .21 |
| | August | 1770 | 25.37 | 42.11 | 42.09 | - .02 |
| | September | 1860 | 23.46 | 38.94 | 38.99 | .05 |
| | October | 1860 | 23.55 | 39.09 | 39.00 | - .09 |
| | November | 1840 | 24.02 | 39.88 | 39.93 | .05 |
| | December | 1810 | 24.74 | 41.08 | 40.98 | - .10 |
| 1973 | January | 1790 | 25.24 | 41.91 | 42.10 | .19 |
| | February | 1770 | 25.74 | 42.73 | 43.08 | .35 |
| | March | 1740 | 26.46 | 43.92 | 44.18 | .26 |
| | April | 1740 | 26.52 | 44.02 | 44.17 | .15 |
| | May | 1720 | 27.01 | 44.85 | 45.16 | .31 |
| | June | 1800 | 25.33 | 42.05 | 41.92 | - .13 |
| | July | 1810 | 25.17 | 41.79 | 41.89 | .10 |
| | August | 1790 | 25.67 | 42.61 | 42.84 | .23 |
| | September | 1840 | 24.64 | 40.91 | 41.30 | .39 |
| | October | 1860 | 24.27 | 40.29 | 40.54 | .25 |
| | November | 1840 | 24.76 | 41.11 | 41.42 | .31 |
| | December | 1810 | 25.47 | 42.29 | 42.46 | .17 |

tain States, historically have timed their calf crop to coincide with the more favorable spring and summer grazing and climatic conditions. However, with technological advances in range management and herd improvements, plus the rapid expansion of cow-calf operations into the southeastern states, production levels between seasons at the primary production level could be expected to even out.

The principal reason for variations in production numbers between seasons is that costs of production vary seasonally at both the range and feedlot levels. Factors which seasonally affect the costs of production include: caloric intake required per pound of gain, quantity and opportunity cost of labor for production activities, fixed and variable cost of capital equipment, and transportation and storage costs for feed grain and roughage inputs.

In general, relative costs of production are higher on both range and feedlot activities in the Northern Corn Belt, Northern Plains, and Mountain States from mid-fall until early spring, while often

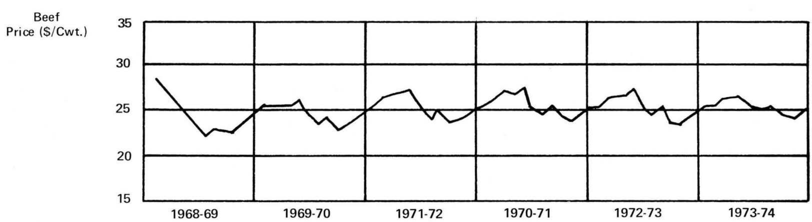


**Fig. 3. Equilibrium price path for choice beef steers for the six-year production and marketing period 1968–73 (Omaha basis).**

times, the southeastern, southwestern and portions of the Pacific slope states may enjoy relatively lower production costs during the same seasons.

On the other hand, cow-calf and feedlot operators in the north experience relatively lower costs of production in the spring, summer, and early fall months, while these same seasons may be the high cost months for the southern operator. Thus, seasonal comparative advantages affect the cost of production resulting in cyclical equilibrium price patterns.

On the demand side, cyclical forces occur which cause seasonal variations in the quantity of beef demanded. Recalling the discussion of demand (Equations 3 and 4), seasonal shifts indicated lower consumer demand levels in the winter and spring than the summer and fall.

Differences between the demand shadow prices and the converted live prices are shown in Figure 4. Note that the larger deviations occur in the first three years of the production and marketing period, while in the last three years the deviations become relatively minor as the program moves to a stable production and marketing pattern.

However, even the largest deviations in the first three years account for less than one dollar per hundredweight on slaughter cattle, while in the last three years of the period the deviation represents less than 25 cents per hundredweight. The closeness of fit suggests that the solution is sufficiently near to optimum to be used in developing short- and long-run normative marketing and production goals for the beef sector.

## Market Allocation

The previous discussion centered around an evaluation of the equilibrium price path and the consistent marginal value products or shadow prices on the demand constraints. This section will present the marketing allocations of steers, heifers, and cull cows.

The allocation of steers (Table 6), heifers and culls (Table 7) to net monthly demands, are presented by weight classes on a live animal basis and in units of thousands of head. Net beef demands are ex-

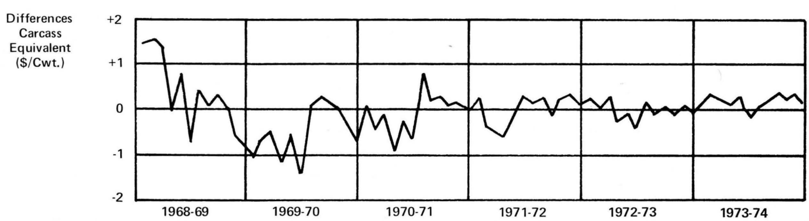


**Fig. 4. Differences between converted live price and the program shadow price.**

**Table 6. Optimum steer marketing, by month and weight, in thousands of head.**

| Year | Month | Net beef demand (mil. lb.) | 840 pounds | 920 pounds | 990 pounds | 1060 pounds | 1130 pounds | 1190 pounds | 1240 pounds |
|---|---|---|---|---|---|---|---|---|---|
| 1968 | January | 1460 | .... | .... | .... | 1121 | .... | 380 | .... |
| | February | 1500 | .... | 1323 | .... | .... | 890 | .... | .... |
| | March | 1560 | 778 | .... | .... | .... | .... | .... | .... |
| | April | 1620 | .... | .... | .... | 2251 | .... | .... | .... |
| | May | 1670 | .... | .... | .... | .... | 1659 | .... | .... |
| | June | 1710 | .... | .... | .... | 1290 | .... | .... | .... |
| | July | 1760 | .... | .... | .... | .... | 2248 | .... | .... |
| | August | 1730 | .... | .... | .... | .... | .... | 2375 | .... |
| | September | 1730 | .... | .... | .... | 284 | .... | .... | .... |
| | October | 1740 | .... | .... | .... | .... | 2128 | .... | .... |
| | November | 1700 | .... | .... | .... | .... | .... | 1535 | .... |
| | December | 1670 | .... | .... | .... | .... | 630 | 928 | 454 |
| 1969 | January | 1640 | .... | .... | .... | .... | .... | 1510 | 446 |
| | February | 1630 | .... | .... | .... | .... | .... | .... | 444 |
| | March | 1630 | .... | .... | .... | 112 | .... | .... | 56 |
| | April | 1630 | .... | .... | .... | .... | 2097 | .... | .... |
| | May | 1620 | .... | .... | .... | .... | .... | 386 | .... |
| | June | 1680 | .... | .... | .... | 396 | .... | .... | .... |
| | July | 1740 | .... | .... | .... | .... | 2214 | .... | .... |
| | August | 1720 | .... | .... | .... | .... | .... | 1536 | .... |
| | September | 1770 | .... | .... | .... | .... | .... | .... | .... |
| | October | 1770 | .... | .... | .... | .... | 2162 | .... | .... |
| | November | 1740 | .... | .... | .... | .... | .... | 1236 | .... |
| | December | 1700 | .... | .... | .... | .... | .... | 1563 | 463 |
| 1970 | January | 1660 | .... | .... | .... | .... | 724 | .... | 452 |
| | February | 1620 | .... | .... | .... | .... | .... | 1525 | .... |
| | March | 1610 | .... | .... | .... | 598 | .... | .... | 438 |
| | April | 1610 | .... | .... | .... | .... | 2043 | .... | .... |
| | May | 1610 | .... | .... | .... | .... | .... | .... | .... |
| | June | 1690 | .... | .... | .... | 1436 | .... | .... | .... |
| | July | 1750 | .... | .... | .... | .... | 2199 | .... | .... |
| | August | 1710 | .... | .... | .... | .... | .... | 1542 | .... |
| | September | 1770 | .... | .... | .... | .... | .... | .... | .... |
| | October | 1770 | .... | .... | .... | .... | 1757 | .... | .... |
| | November | 1750 | .... | .... | .... | .... | .... | 1549 | .... |
| | December | 1720 | .... | .... | .... | .... | .... | 1554 | 468 |
| 1971 | January | 1710 | .... | .... | .... | .... | 22 | .... | 465 |
| | February | 1670 | .... | .... | .... | .... | .... | 1583 | .... |
| | March | 1640 | .... | .... | .... | .... | .... | 1542 | 446 |
| | April | 1650 | .... | .... | .... | .... | 2103 | .... | .... |
| | May | 1640 | .... | .... | .... | .... | .... | 168 | .... |
| | June | 1740 | .... | .... | .... | 659 | .... | .... | .... |
| | July | 1760 | .... | .... | .... | .... | 2216 | .... | .... |
| | August | 1740 | .... | .... | .... | .... | .... | 1578 | .... |
| | September | 1780 | .... | .... | .... | 221 | .... | .... | .... |
| | October | 1810 | .... | .... | .... | .... | 2187 | .... | .... |
| | November | 1790 | .... | .... | .... | .... | .... | 1597 | .... |
| | December | 1760 | .... | .... | .... | .... | .... | 1589 | .... |
| 1972 | January | 1750 | .... | .... | .... | .... | .... | .... | 476 |
| | February | 1720 | .... | .... | .... | .... | .... | 1593 | .... |
| | March | 1700 | .... | .... | .... | .... | .... | 1598 | 463 |
| | April | 1700 | .... | .... | .... | .... | 2166 | .... | .... |
| | May | 1680 | .... | .... | .... | .... | .... | 426 | .... |
| | June | 1760 | .... | .... | .... | 2071 | .... | .... | .... |

Table 6. (continued)

| Year | Month | Net beef demand (mil. lb.) | 840 pounds | 920 pounds | 990 pounds | 1060 pounds | 1130 pounds | 1190 pounds | 1240 pounds |
|---|---|---|---|---|---|---|---|---|---|
| | July | 1800 | .... | .... | .... | .... | 2262 | .... | .... |
| | August | 1770 | .... | .... | .... | .... | .... | .... | .... |
| | September | 1860 | .... | .... | .... | .... | .... | .... | .... |
| | October | 1860 | .... | .... | .... | .... | 2245 | .... | .... |
| | November | 1840 | .... | .... | .... | .... | .... | 1407 | .... |
| | December | 1810 | .... | .... | .... | .... | .... | 1644 | 493 |
| 1973 | January | 1790 | .... | .... | .... | .... | .... | .... | 487 |
| | February | 1770 | .... | .... | .... | .... | .... | 1581 | .... |
| | March | 1740 | .... | .... | .... | .... | .... | 1637 | 473 |
| | April | 1740 | .... | .... | .... | .... | 1758 | .... | 44 |
| | May | 1720 | .... | .... | .... | .... | .... | 1230 | .... |
| | June | 1800 | .... | .... | .... | .... | .... | .... | .... |
| | July | 1810 | .... | .... | .... | .... | 2245 | .... | .... |
| | August | 1790 | .... | .... | .... | .... | .... | 1613 | .... |
| | September | 1840 | .... | .... | .... | 2043 | .... | .... | .... |
| | October | 1860 | .... | .... | .... | .... | 2234 | .... | .... |
| | November | 1840 | .... | .... | 181 | .... | .... | 1629 | .... |
| | December | 1810 | .... | .... | .... | .... | .... | .... | 492 |

pressed in terms of millions of pounds of carcass beef. For approximate purposes, a 60 percent yield rate can be used to compute the meat equivalent from the liveweight numbers of steers and heifers. Cull cows were assumed to be marketed at 1,000 pounds liveweight and yielding 54 percent carcass meat equivalent.

## Evaluation

Marketings of steers in the first three months of 1968 were extremely lightweight fed cattle. The equilibrium solution indicated that about 2.1 million head of steers weighing between 840 and 920 pounds should be marketed in February and March. However, once out of the immediate short-run period, the marketing allocation of steers to slaughter developed a definite cyclical pattern. Steers were generally marketed at heavier weights in the months from November through February, and at lighter weights from March through October.

The marketing allocation of heifers, although not as well defined, paralleled that of the steers. Generally, heavier weight heifers were marketed from November through February, and lightweight heifers from March through October. Unlike steers, heifers can be marketed as slaughter beef, or returned to the cow herd for future reproduction. The particular alternative selected is dependent on the opportunity value in immediate slaughter versus the opportunity value in future production.

**Table 7. Optimum heifer and cull cow marketing, by month and weight, in thousands of head.**

| Year | Month | Net beef demand (mil. lb.) | 840 pounds | 910 pounds | 970 pounds | 1030 pounds | 1080 pounds | 1120 pounds | Cull Cow 1000 pounds |
|---|---|---|---|---|---|---|---|---|---|
| 1968 | January | 1460 | .... | .... | .... | 382 | .... | .... | 421 |
| | February | 1500 | .... | .... | .... | .... | .... | .... | 340 |
| | March | 1560 | .... | .... | 1675 | 675 | .... | .... | 350 |
| | April | 1620 | .... | .... | .... | .... | .... | .... | 332 |
| | May | 1670 | .... | 611 | .... | .... | .... | .... | 350 |
| | June | 1710 | .... | .... | 1125 | .... | .... | .... | 376 |
| | July | 1760 | .... | .... | .... | .... | .... | .... | 385 |
| | August | 1730 | .... | .... | .... | .... | 274 | .... | 439 |
| | September | 1730 | .... | .... | 1300 | 291 | .... | .... | 439 |
| | October | 1740 | .... | .... | .... | .... | .... | .... | 502 |
| | November | 1700 | .... | .... | .... | .... | .... | 511 | 484 |
| | December | 1670 | .... | .... | .... | .... | .... | .... | 439 |
| 1969 | January | 1640 | .... | .... | .... | .... | .... | .... | 428 |
| | February | 1630 | .... | .... | .... | .... | 1758 | .... | 346 |
| | March | 1630 | .... | .... | 2207 | .... | .... | .... | 356 |
| | April | 1630 | .... | .... | .... | .... | .... | .... | 338 |
| | May | 1620 | .... | .... | .... | .... | 388 | .... | 356 |
| | June | 1680 | .... | .... | 1635 | .... | .... | .... | 383 |
| | July | 1740 | .... | .... | .... | 2031 | .... | .... | 392 |
| | August | 1720 | .... | .... | .... | .... | .... | .... | 447 |
| | September | 1770 | .... | .... | 2546 | .... | .... | 529 | 447 |
| | October | 1770 | .... | .... | .... | .... | .... | .... | 510 |
| | November | 1740 | .... | .... | .... | 381 | 535 | .... | 492 |
| | December | 1700 | .... | .... | .... | .... | .... | .... | 447 |
| 1970 | January | 1660 | .... | .... | .... | 904 | .... | .... | 469 |
| | February | 1620 | .... | .... | .... | .... | 498 | .... | 379 |
| | March | 1610 | .... | .... | 1157 | .... | .... | .... | 390 |
| | April | 1610 | .... | .... | .... | .... | .... | .... | 370 |
| | May | 1610 | .... | 2544 | .... | .... | .... | .... | 390 |
| | June | 1690 | .... | .... | 906 | .... | .... | .... | 419 |
| | July | 1750 | .... | .... | .... | .... | .... | .... | 429 |
| | August | 1710 | .... | .... | .... | .... | 526 | .... | 489 |
| | September | 1770 | .... | .... | 2508 | .... | .... | .... | 489 |
| | October | 1770 | .... | .... | .... | 401 | .... | .... | 559 |
| | November | 1750 | .... | .... | .... | .... | 538 | .... | 539 |
| | December | 1720 | .... | .... | .... | .... | .... | .... | 489 |
| 1971 | January | 1710 | .... | .... | .... | 1732 | .... | .... | 466 |
| | February | 1670 | .... | .... | .... | .... | 514 | .... | 377 |
| | March | 1640 | .... | .... | .... | .... | .... | .... | 387 |
| | April | 1650 | .... | .... | .... | .... | .... | .... | 367 |
| | May | 1640 | .... | 1785 | .... | .... | 505 | .... | 387 |
| | June | 1740 | .... | .... | 1821 | .... | .... | .... | 417 |
| | July | 1760 | .... | .... | .... | .... | .... | .... | 427 |
| | August | 1740 | .... | .... | .... | .... | 535 | .... | 486 |
| | September | 1780 | .... | .... | 2292 | .... | .... | .... | 486 |
| | October | 1810 | .... | .... | .... | .... | .... | .... | 556 |
| | November | 1790 | .... | .... | .... | .... | 551 | .... | 536 |
| | December | 1760 | .... | .... | .... | .... | 542 | .... | 486 |
| 1972 | January | 1750 | .... | .... | .... | 1793 | .... | .... | 482 |
| | February | 1720 | .... | .... | .... | .... | 529 | .... | 390 |
| | March | 1700 | .... | .... | .... | .... | 40 | .... | 400 |
| | April | 1700 | .... | .... | .... | .... | .... | .... | 380 |
| | May | 1680 | .... | 1601 | .... | 428 | .... | .... | 400 |
| | June | 1760 | .... | .... | 335 | .... | .... | .... | 431 |

Table 7. (continued)

| Year | Month | Net beef demand (mil. lb.) | 840 pounds | 910 pounds | 970 pounds | 1030 pounds | 1080 pounds | 1120 pounds | Cull Cow 1000 pounds |
|---|---|---|---|---|---|---|---|---|---|
| | July | 1800 | .... | .... | .... | .... | .... | .... | 441 |
| | August | 1770 | .... | 2371 | .... | .... | 298 | .... | 502 |
| | September | 1860 | .... | .... | 2646 | .... | .... | .... | 502 |
| | October | 1860 | .... | .... | .... | .... | .... | .... | 574 |
| | November | 1840 | .... | .... | .... | 261 | 566 | .... | 554 |
| | December | 1810 | .... | .... | .... | .... | .... | .... | 502 |
| 1973 | January | 1790 | .... | .... | .... | 1834 | .... | .... | 493 |
| | February | 1770 | .... | .... | .... | 110 | 545 | .... | 398 |
| | March | 1740 | .... | .... | .... | .... | .... | .... | 409 |
| | April | 1740 | .... | .... | .... | 447 | .... | .... | 388 |
| | May | 1720 | .... | 500 | .... | .... | 529 | .... | 409 |
| | June | 1800 | .... | .... | 2602 | .... | .... | .... | 440 |
| | July | 1810 | .... | .... | .... | 27 | .... | .... | 451 |
| | August | 1790 | .... | .... | .... | .... | 551 | .... | 514 |
| | September | 1840 | .... | .... | 423 | .... | .... | .... | 514 |
| | October | 1860 | .... | .... | .... | .... | .... | .... | 587 |
| | November | 1840 | .... | .... | .... | .... | 566 | .... | 566 |
| | December | 1810 | .... | .... | 1511 | .... | .... | .... | 514 |

If the marketing allocation of steers and heifers is considered simultaneously, a definite marketing pattern emerges. The marketings of heavy steers and heifers came during the higher price periods of the season price cycles, while the marketings of lighter animals came during periods of lower seasonal prices. In equilibrium, the above would be the expected marketing and price behavior. Market prices would be positively correlated with the marginal cost of production.

During periods of short supply, competitive market prices are forced upwards and, in turn, beef producers should respond by carrying some animals to heavier weights. Producers should continue to add weight up to the point where the marginal cost per pound of gain is equal to the value of marginal product.

Conversely, during periods of larger supply numbers, some marketings should occur at lighter weights. Thus, in normal short-run situations, to maintain an equilibrium price or to move to equilibrium producers should market at light weights. To maintain or move to equilibrium prices when product prices are above equilibrium, producers should market at heavier weights.

However, the marketing allocations in Tables 6 and 7 indicate a marketing behavior that seems to deviate from the normal strategy described above. The solution indicates that in the short run when product prices are higher than equilibrium price level (due to a short supply of finished animals, but an excess inventory of forthcoming feeder animals, which would have the effect of forcing future product prices to drop below the normal equilibrium level), the best marketing strategy would be to immediately slaughter a relatively large number of light animals. This should have the effect of depressing present product price as the over supply of feeder animals reach conventional slaughter weights.

Given an initial inventory by number, weight, class, and disposition, plus a series of future demand functions, the programming model can determine the proper future production and marketing strategy which minimizes the carrying cost for each producing activity, for each animal form, in terms of alternative temporal marketing or demand periods.

The particular marketing allocation selected is the one that meets the dual criteria of a least cost and an equilibrium solution. The specific producing and marketing activities selected are least cost efficiency points and may or may not be the least cost point on a specific total cost growth curve. This is possible because the level of production costs through time are assumed constant, while carrying cost between different temporal and form marketings reflect individual efficiency points on the production growth curve. The linear programming model simultaneously determines the consistent combination of points on the growth curve to secure a least cost solution. Thus in equilibrium, the marketing allocations developed by the linear programming solution are the combined least cost efficiency points, and by definition the competitive market solution.

## Production Allocation

The previous marketing analysis focused attention on short-run adjustments necessary to maintain or move toward general price equilibrium. The prime adjustment mechanism was slaughter weight. In this part of the analysis emphasis will shift from the short run to the long run, where adjustments are achieved through increases or decreases in the aggregate size of the beef cattle inventory. Primary concern will be directed toward changes in the size and seasonal composition of the reproduction herd. The major adjustment mechanism in the long run is birth numbers.

Culling of cows from the reproduction base is not treated directly as an economic decision within the model. Cows are culled from the reproduction herd at a rate of 14 percent a year. The cull rate used allows for slightly more than seven years per cow of reproduction capacity. This cull rate is not meant to imply that the reproduction capacity of any particular cow is terminated upon seven years of reproduction, but represents average physical deterioration of the cow herd, plus some nonbreeders and producers of poor performance calves.

Heifer replacement into the herd is treated as an economic decision. Heifers can be slaughtered or returned as *replacements to the* reproduction herd. The particular activity selected by the programming model depends on the immediate value as slaughter beef in the demand sector or the value in reproduction for future meat demands. Treating replacement as an economic decision rather than determining purely arbitrary replacement levels, has the effect of generating a long-run reproduction herd based on a criteria of equilibrium least cost production and marketing.

## Evaluation

Changes in the size and seasonal composition of the reproduction herd can be stated in terms of heifer replacement, cow culling or the net difference between replacement and culling. Replacement, cull, and net replacement for the six-year production period by year and season are presented in Table 8.

Net replacement in 1968 out of the initial inventory was about 3.3 million heifers, while in 1969, net replacement was a negative 228,000 head. This was due to the large initial inventory of inter-

**Table 8. Cow cull and heifer replacement in the cow herd for the six-year production period, by year and season.**

| Year | Description | Cow herd (1000 head) | Cow cull (1000 head) | Heifer replacement (1000 head) | Net replacement (1000 head) | Net percent change in cow herd |
|---|---|---|---|---|---|---|
| 1968 | Total[a] | 35300 | 4942 | 8291 | 3349 | .95 |
| | 1st quarter | 9990 | 1399 | 3736 | 2337 | 23.4 |
| | 2nd quarter | 13061 | 1828 | 1643 | − 185 | − 1.4 |
| | 3rd quarter | 6107 | 855 | 2913 | 2058 | 33.7 |
| | 4th quarter | 6142 | 860 | 0 | − 860 | −14.0 |
| 1969 | Total | 38649 | 5411 | 5183 | − 228 | − 0.6 |
| | 1st quarter | 12327 | 1726 | 0 | −1726 | −14.0 |
| | 2nd quarter | 12875 | 1802 | 2415 | 613 | 4.8 |
| | 3rd quarter | 8165 | 1143 | 1284 | 141 | 1.7 |
| | 4th quarter | 5282 | 740 | 1484 | 744 | 14.1 |
| 1970 | Total | 38421 | 5379 | 6661 | 1282 | 3.3 |
| | 1st quarter | 10601 | 1484 | 1201 | − 283 | − 2.7 |
| | 2nd quarter | 13488 | 1888 | 2347 | 459 | 3.4 |
| | 3rd quarter | 8306 | 1163 | 1346 | 183 | 2.2 |
| | 4th quarter | 6026 | 844 | 1767 | 923 | 15.3 |
| 1971 | Total | 39703 | 5559 | 6452 | 893 | 2.5 |
| | 1st quarter | 10318 | 1445 | 311 | −1134 | −11.0 |
| | 2nd quarter | 13946 | 1952 | 2685 | 733 | 5.3 |
| | 3rd quarter | 8490 | 1188 | 1292 | 104 | 1.2 |
| | 4th quarter | 6949 | 973 | 2163 | 1190 | 17.1 |
| 1972 | Total | 40596 | 5684 | 5791 | 107 | 0.3 |
| | 1st quarter | 9185 | 1286 | 2101 | 815 | 8.9 |
| | 2nd quarter | 14679 | 2055 | 432 | −1623 | −11.1 |
| | 3rd quarter | 8593 | 1203 | 1247 | 44 | 0.5 |
| | 4th quarter | 8139 | 1140 | 2012 | 872 | 10.7 |
| 1973 | Total | 40703 | 5698 | 6895 | 1197 | 2.9 |
| | Final inventory | 41800 | | | | |

[a] Parts might not exactly add to totals because of rounding.

mediate stocker and feeder heifers and steers. Comparatively, the program found the opportunity cost greater to replace heifers in the first production year, when prices on slaughter animals were low, than in the second year when slaughter prices were at a higher level (Figure 3).

Over the six-year production period, proportionately greater replacements took place in the summer and fall quarters than the winter and spring quarters. This was partially due to an adjustment of the seasonal distribution of the cow herd in the final animal inventory. However, a certain amount of the proportional change in the seasonality of the cow herd was probably due to needed adjustment in production levels to meet future demands and reflect differences in costs of production between seasons.

## Economic Contribution

The contribution to the general economy by the beef sector, *from primary production to slaughter* as finished beef, can be determined by the added value concept. The concept implies that in equilibrium, where the price of a good or service is equal to the unit cost of producing the good or service, the general economy receives a benefit equal to the total cost of producing the good or service. Because the linear programming model is both a cost minimization and an equilibrium solution, the value of the objective function can be directly interpreted as the added value of the beef sector. The value of the objective function indicated that the beef sector contributed 53.3 billion dollars during the six-year production and marketing period, or slightly less than nine billion dollars per year.

In the next section attention will be directed towards the pork sector. The combined output of the beef and pork sector constitutes the largest single producing segment of agriculture. If orderly production and marketing that reflected equilibrium prices could be established in, and between, these two sectors, considerable stability could be expected in agricultural incomes and prices.

# SOLUTION RESULTS AND GENERAL EVALUATION (PORK MODEL, 1968-1970)

The format and content of this section parallels the previous discussion. Presentation and evaluation of the solution results from the pork model will be given. Emphasis will be placed on the discussion of the equilibrium pork price pattern, the form and time allocation of butcher hogs to slaughter, the size and composition of the program-generated reproduction base, and the value of the objective function. As in the beef model, the initial inventory conditions are those on

January 1, 1968. However, the short gestation period, multiple farrowing, and multiple births for swine make it possible to use a three-year production and market model to compute the equilibrium solution.

## Initial Inventory

The beginning inventory used in the pork model is given in Table 9. The basic initial inventory numbers were the total numbers of hogs and pigs on farms on January 1, 1968, by weight and disposition. No class distinctions are made between barrows and gilts. Data were taken from published reports of the U.S. Department of Agriculture. The composition of the reproduction herd in the initial inventory was arbitrarily assumed to be 45 percent gilts, 30 percent one-litter sows, and 25 percent two-or-more-litter sows. *The objective of this study*

**Table 9. Initial inventory of barrows and gilts, and sows, on January 1, 1968.**

| Description | Number (1000 head) |
|---|---|
| Barrows and gilts | |
| < 25 pounds | 1374 |
| 25– 50 pounds | 4037 |
| 51– 85 pounds | 6575 |
| 86–130 pounds | 10568 |
| 131–180 pounds | 7963 |
| 181–230 pounds | 5300 |
| 231–270 pounds | 1425 |
| 271–300 pounds | 390 |
| Bred sows to farrow | |
| First litter in January | 326 |
| First litter in February | 427 |
| First litter in March | 666 |
| First litter in April | 675 |
| Second litter in January | 217 |
| Second litter in February | 285 |
| Second litter in March | 444 |
| Second litter in April | 450 |
| Third litter in January | 181 |
| Third litter in February | 237 |
| Third litter in March | 370 |
| Third litter in April | 375 |
| Lactating sows | |
| First litter, first month | 236 |
| First litter, second month | 109 |
| Second litter, first month | 157 |
| Second litter, second month | 246 |
| Third litter, first month | 131 |
| Third litter, second month | 191 |
| Non-bred sows (being conditioned for sale) | |
| One litter sows | 90 |
| Two litter sows | 200 |
| Three litter sows | 124 |

*was to demonstrate the need for and the accuracy requirements of basic inventory data, and not to provide a method for estimating inventories from incomplete data.*

## Equilibrium Price Path

In the analysis of the retail price relationships for beef and pork (Equations 3 and 4), it was pointed out that beef appeared to be a substitute for pork, but pork was not a significant substitute for beef. This made it possible to compute the equilibrium temporal and form price path consistent for beef, and then substitute the corresponding per capita consumption rates into the estimated demand function for pork. From this point, the programming procedures for handling the demand functions in the pork model are similar to those described for the beef model.

The necessary conditions of an optimum equilibrium solution are again provided for by the duality theorem of linear programming. Conditions of temporal and form equilibrium are achieved in the pork model when the sum of the shadow price equivalents on the demand constraints and the pigs saved categories are equal to the pork prices used to specify the net demand levels, and when the price levels between months differ by the marginal supply cost differentials.

In the pork model, the units of supply are expressed in animal units, while units of demand are expressed in pounds of carcass pork (excluding lard). In the programming model for pork, reproduction activities are treated as transfers from the reproduction constraints to pigs-saved constraints. Barrows and gilts can then be carried forward from the pigs-saved constraints through the finishing activities to either the demand sector as pork, or be returned to the reproduction herd as gilt replacements. The treatment of reproduction in this manner (transfer through the pigs-saved constraints to finishing activities as in the beef model) provides some difficulty in the identification and evaluation of an equilibrium solution.

The shadow prices on the pigs-saved constraints are interpreted as the marginal value product per pig up to weaning. Multiplying the shadow prices on the pigs-saved constraints by 45 percent (approximately the hundredweight equivalent for a 220 pound butcher hog) converts the marginal value product from a per-pig to a hundredweight basis. The shadow prices on the demand constraints when converted to live price equivalents represent the marginal value products of live pork on a hundredweight basis from weaning to slaughter. Conversion of the shadow prices on the demand constraints from a meat equivalent to a liveweight basis is accomplished by multiplying the shadow prices by a 56 percent liveweight to carcass pork conversion rate.

When the converted shadow prices on the pigs-saved constraints are summed with the appropriate (approximately seven months forward) shadow prices on the demand constraints, the results are total marginal value products or shadow prices for producing and marketing hogs. Since the total shadow prices are expressed in terms of live prices on a per hundredweight basis, they may be compared directly with the appropriate specified live prices used in the demand functions. Temporal and form equilibrium is achieved when the converted total shadow prices equal the specified live prices.

The estimated equilibrium slaughter hog prices, the corresponding net pork demands, and the converted live shadow price equivalents by months are contained in Table 10. Results in Table 10 are obtained from the linear programming solution based on minimizing the total supply cost over a three-year period beginning January 1, 1968, for producing and marketing pork. The computed slaughter hog prices are based on the Omaha price for No. 1–3, 220 to 240 pound barrows and gilts. Other grade and weight price equivalents can be determined according to the competitive price relationships. Net pork demands by month are equal to total pork demands minus the import-export balance. The converted live shadow prices by months are the sum of the per hundredweight shadow price equivalents for the pigs-saved constraints and the shadow prices on the demand constraints coverted to live price equivalents. The differences are the converted live shadow prices minus the specified live prices in the demand function.

## Evaluation

The equilibrium slaughter hog price path generally increased from a low in the $14 range during the first three months of 1968, to the equilibrium level of $17 to $17.50 per hundredweight in the latter months of 1968. The computed equilibrium price path for pork does not parallel actual market prices during 1968. This is due to an over estimation of the beginning inventories of hogs and pigs. With the exception of January 1969, the equilibirum price path moves to a relatively flat price level with minor fluctuations reflecting variations in seasonal costs of production and demand levels. Generally the low points in the price path come during the winter and spring months from February to May, while slightly higher than average prices occur in the fall months from October to January. The pattern can be seen more clearly in Figure 5.

Differences between the converted live shadow prices and the butcher hog prices specified from the demand function are shown in Figure 6. The largest deviations from zero occur in the first seven months of the production and marketing period, while deviations in

**Table 10. Specified live prices, converted shadow prices and differences for the three-year production and marketing period, 1968–70 (Omaha slaughter level basis).**

| Year | Month | Net pork demand (mil. lb.) | Specified live prices[a] | Converted shadow prices | Differences |
|---|---|---|---|---|---|
| | | | Dollars per hundredweight | | |
| 1968 | January | 1150 | 14.56 | 15.20 | .64 |
| | February | 1130 | 14.98 | 16.23 | 1.25 |
| | March | 1130 | 14.26 | 15.81 | 1.55 |
| | April | 1090 | 15.26 | 16.76 | .50 |
| | May | 1050 | 16.39 | 17.57 | 1.18 |
| | June | 1050 | 15.95 | 16.84 | .89 |
| | July | 990 | 17.94 | 19.39 | 1.45 |
| | August | 1010 | 17.58 | 17.40 | - .18 |
| | September | 1020 | 17.25 | 17.05 | - .20 |
| | October | 1020 | 17.21 | 17.07 | - .14 |
| | November | 1060 | 17.29 | 17.08 | - .21 |
| | December | 1060 | 17.49 | 17.78 | .29 |
| 1969 | January | 1030 | 18.42 | 18.35 | - .07 |
| | February | 1050 | 17.81 | 17.99 | .19 |
| | March | 1060 | 17.61 | 17.80 | .19 |
| | April | 1070 | 17.14 | 17.38 | .24 |
| | May | 1080 | 16.95 | 16.94 | - .01 |
| | June | 1060 | 17.08 | 16.92 | - .16 |
| | July | 1040 | 17.22 | 17.13 | - .08 |
| | August | 1040 | 17.58 | 17.33 | - .25 |
| | September | 1030 | 17.42 | 17.27 | - .15 |
| | October | 1030 | 17.38 | 17.32 | - .16 |
| | November | 1050 | 17.17 | 17.27 | .10 |
| | December | 1060 | 17.37 | 17.35 | - .02 |
| 1970 | January | 1070 | 17.58 | 17.41 | - .17 |
| | February | 1100 | 16.94 | 17.01 | .07 |
| | March | 1110 | 16.75 | 16.59 | - .16 |
| | April | 1110 | 16.84 | 16.60 | - .24 |
| | May | 1110 | 16.93 | 16.80 | - .13 |
| | June | 1080 | 17.22 | 17.09 | - .13 |
| | July | 1060 | 17.35 | 17.51 | .16 |
| | August | 1070 | 17.56 | 17.33 | - .23 |
| | September | 1060 | 17.27 | 17.29 | .02 |
| | October | 1060 | 17.36 | 17.25 | - .11 |
| | November | 1070 | 17.29 | 17.21 | - .08 |
| | December | 1070 | 17.65 | 17.48 | - .17 |

[a] Equivalent to the Omaha price for No. 1–3, 220–240 pound barrows and gilts.

the last two and one-half years become relatively minor, as the program moves towards a stable production and marketing pattern. The closeness of fit indicated in Figure 6 suggests that the solution is sufficiently near to optimum to be used for developing short- and long-run production and marketing goals for the pork sector.

The pork sector has the ability to adjust production numbers and marketing weights much more rapidly than the beef sector to move towards and maintain equilibrium price levels. The major reasons on the supply side are the faster rate of inventory turnover, the growth and reproductive characteristics of swine, and the linear shape of the growth cost curve.

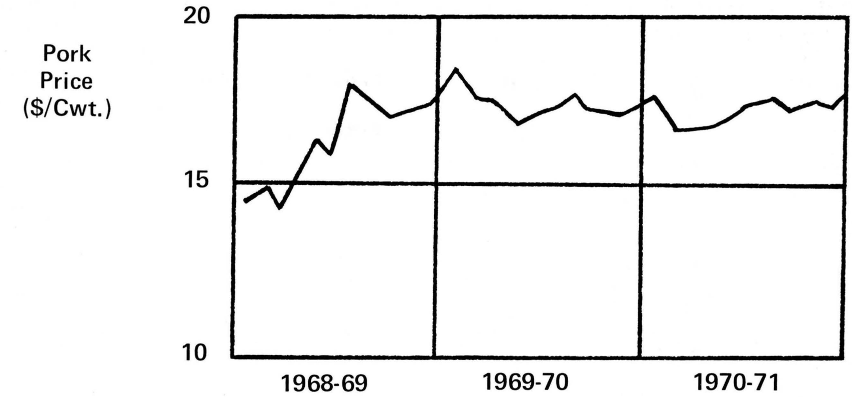


**Fig. 5. Equilibrium price path for barrows and gilts for the three-year production and marketing period, 1968–70 (Omaha basis).**

Parts of each of the three reasons are interrelated, so they will be discussed simultaneously. The gestation period for hogs is about 114 days, plus from 5 to 9 months from farrowing to finishing as butcher hogs, for a total production period of 9 to 13 months.

Given an initial inventory of pigs and hogs, the very youngest will clear the market in nine months. Given an inventory that includes bred sows, the normal maximum inventory turnover would still be no more than 13 months. Compared with cattle feeding, the marginal cost per pound of gain to finish butcher hogs is relatively constant at usual slaughterable weights. The effect is that minor marketing adjustments across close temporal demand periods can be made with relative ease.

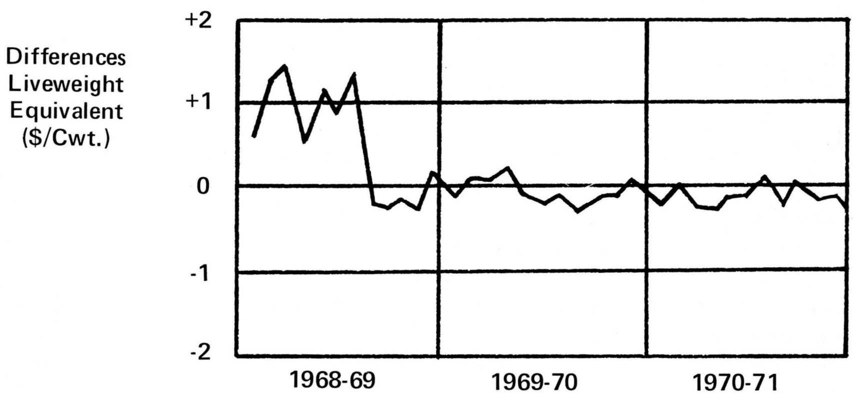


**Fig. 6. Differences between specified live prices and converted shadow prices.**

If product prices are below equilibrium levels, producers should market some animals at lighter weights, forcing product prices back toward equilibrium levels as excess supplies of pork are diminished. Conversely, if product prices are above equilibrium levels, producers should hold some animals to heavier weights forcing product prices down to equilibrium levels.

On the demand side, adjustments towards equilibrium price levels can be made relatively fast because of the inelastic shape of the demand curve for pork. A one percent increase or decrease in the supply of pork offered results in a greater than one percent increase or decrease in product price. Thus minor adjustments in production and marketing patterns may have magnified effects in movements towards price equilibrium.

The very characteristics of the pork sector that make adjustment from disequilibrium to equilibrium price levels relatively easy, make the industry vulnerable in attempting to maintain a long-term equilibrium. Because the industry can rapidly adjust production numbers and marketing weights with only minor increases or decreases in the direct marginal cost of production, and because of the inelastic nature of pork demand, short- and long-run equilibrium production and marketing goals need constant monitoring.

## Market Allocation

In this section the marketing allocation to monthly demands by weight and disposition will be discussed. Major concern will be focused on the form and timing of slaughter. The marketing allocation of barrows and gilts, and cull sows from the reproduction herd to meet net pork demands are presented in Table 11. The allocations by weight class are on a live animal basis and expressed in units of thousands of head. Net pork demands by month are expressed in terms of millions of pounds of carcass pork (excluding lard). For approximative purposes, a 56 percent yield rate can be used to compute the meat equivalent from liveweight numbers of barrows and gilts. Cull sows are marketed at 310, 400, and 460 pounds, yielding 52, 50, and 48 percent carcass pork equivalent (excluding lard), respectively.

## Evaluation

The greatest numbers of marketings of barrows and gilts during the first five months are from the 230 pound weight category. About two-thirds of the residual marketings are from the 270- and 300-pound weight categories, with the remainder from the 180 pound category. Throughout the rest of the three-year production and marketing period no barrows and gilts are marketed at greater than 230 pounds. After the first five months, the marketings out of the 180- and 230-

**Table 11. Programmed marketings of barrows and gilts, and sows, by month and weight, in thousands of head.**

| Year | Month | Net pork demand (mil. lbs.) | Barrows and gilts | | | | Sows | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 180 lbs. | 230 lbs. | 270 lbs. | 300 lbs. | 310 lbs. | 400 lbs. | 460 lbs. |
| 1968 | January | 1150 | .... | 5099 | 1425 | 390 | 117 | 446 | 315 |
| | February | 1130 | 1148 | 6650 | 201 | ... | 59 | 157 | 131 |
| | March | 1130 | .... | 7410 | .... | ... | 81 | 217 | 181 |
| | April | 1090 | .... | 5559 | 992 | ... | 107 | 285 | 237 |
| | May | 1050 | 1001 | 4037 | 562 | ... | 166 | 444 | 370 |
| | June | 1050 | 5180 | 2117 | .... | ... | 169 | ... | 375 |
| | July | 990 | 5482 | .... | .... | ... | 328 | 532 | ... |
| | August | 1010 | 7190 | .... | .... | ... | 255 | 177 | ... |
| | September | 1020 | 4562 | 3080 | .... | ... | 113 | 245 | ... |
| | October | 1020 | 2504 | 5415 | .... | ... | ... | 320 | ... |
| | November | 1060 | .... | 5911 | .... | ... | 189 | 500 | ... |
| | December | 1060 | .... | 7356 | .... | ... | ... | 506 | ... |
| 1969 | January | 1030 | 2562 | 4162 | .... | ... | 31 | 985 | ... |
| | February | 1050 | 5260 | .... | .... | ... | 367 | 764 | ... |
| | March | 1060 | 4050 | 3405 | .... | ... | 143 | 340 | ... |
| | April | 1070 | 8172 | .... | .... | ... | 253 | ... | ... |
| | May | 1080 | 7701 | .... | .... | ... | 194 | 567 | ... |
| | June | 1060 | .... | 6844 | .... | ... | 170 | ... | ... |
| | July | 1040 | .... | 6429 | .... | ... | 185 | 92 | ... |
| | August | 1040 | 1585 | 5043 | .... | ... | ... | 1102 | ... |
| | September | 1030 | .... | 6769 | .... | ... | 62 | 428 | ... |
| | October | 1030 | 3947 | 3691 | .... | ... | .... | 758 | ... |
| | November | 1050 | 6794 | 957 | .... | ... | 127 | 581 | ... |
| | December | 1060 | 4960 | 1381 | .... | ... | 280 | 511 | ... |
| 1970 | January | 1070 | 5734 | .... | .... | ... | 383 | 556 | ... |
| | February | 1100 | 7954 | .... | .... | ... | 305 | ... | ... |
| | March | 1110 | 8873 | .... | .... | ... | 184 | 187 | ... |
| | April | 1110 | 6972 | 1919 | .... | ... | 163 | ... | ... |
| | May | 1110 | .... | 6703 | .... | ... | 160 | 381 | ... |
| | June | 1080 | .... | 6824 | .... | ... | 21 | 839 | ... |
| | July | 1060 | 538 | 5289 | .... | ... | 83 | 1148 | ... |
| | August | 1070 | 4525 | 3308 | .... | ... | ... | 914 | ... |
| | September | 1060 | 5061 | 2299 | .... | ... | 142 | 553 | ... |
| | October | 1060 | 5368 | 1867 | .... | ... | 180 | 490 | ... |
| | November | 1070 | .... | 5356 | .... | ... | 275 | 481 | ... |
| | December | 1070 | .... | 5165 | .... | ... | 250 | 63 | ... |

pound categories are approximately equal, with slightly greater total numbers coming from the 180-pound category, but slightly greater total weight coming from the 230-pound category. No consistent pattern developed that would indicate a general strategy for seasonal marketing by weight for barrows and gilts.

The marketings allocation of cull sows showed heavy marketings of 400- and 460-pound sows in the first six months of the production and marketing period. After the first six months, there were no marketings of 460-pound or third-litter sows. It is noted that all of the 460-pound sows marketed during the first six months are contained in the initial inventory as bred or lactating third-litter sows. During the remainder of the three-year production and marketing period, approximately 75 percent of the marketings of sows are 400-pound or second-litter sows, while the remaining 25 percent are 310-pound or first-litter sows. It should be pointed out that 25 percent of the first-litter are arbitrarily culled as sub-standard with reference to reproductive performance. Any additional culling is based on the economic value of the sow in immediate demand as opposed to her future value in reproduction and eventual slaughter.

Considering both the slaughter hogs and cull sows, either simultaneously or independently, no identifiable and consistent seasonal patterns of marketings are evident. Both the equilibrium market allocation and pork price path specified from the demand function appear to be relatively stable when compared with the cyclical movements experienced in the previous analysis of the beef sector. First, the small increase in the marginal cost per pound of gain between 180 and 230 pounds has the effect of evenly dividing the marketing numbers across the two weight categories in response to seasonal variations in demand, without substantially introducing cyclical marketing or price patterns.

A second factor which influences the lack of a marketing pattern is the comparatively short production period from the beginning of reproduction through slaughter. Adjustment in production levels can be made with relative ease when compared to the beef sector. Thus, in the pork sector, the strain of weight adjustments at marketing can be reduced considerably by needed adjustments in production numbers.

Finally, the lack of substantial seasonal variations in the total cost of production diminishes the need for seasonal adjustments in marketing weights. While seasonal differences in farrowing and finishing costs vary considerably, the effect of the two costs of production tend to average out for any particular animal. The length of the production period is the prime reason. Pigs produced during seasons of high farrowing cost generally reach the finishing stages during low seasonal cost points. Conversely, pigs produced during low farrowing cost periods are finished during high cost periods. Although the two periods may not arithmetically balance, the effect of seasonal variation in the total cost of production is reduced.

## Production Allocation

In the analysis of the production allocation, attention shifts from short-run to long-run adjustments necessary to move to and maintain price equilibrium. The mechanism of aggregate inventory adjustment is birth numbers. Therefore increases or decreases in the size of the reproduction herd are of primary concern in long-run equilibrium adjustments.

Culling and replacement in the pork model are treated as economic decisions, with the exception of the partial cull of first-litter sows, representative of normal culling operations to eliminate producers of poor performance pigs, small litters, and nonbreeders. In the programming model, third-litter sows represent sows carried for three or more litters. Gilts can be slaughtered as pork or returned to the reproduction herd as replacements. Treating culling and replacement as an economic decision has the effect of generating a long-run reproduction herd, based on a criteria of equilibrium least-cost production and marketing.

## Evaluation

Changes in the size of the reproduction herd for the three-year production period, started in terms of sow cull, gilt replacement, or the net difference between replacement and culling are presented in Table 12. No definite production pattern exists that would indicate seasonal comparative advantages for culling or replacement. An alternating pattern of negative and positive net replacement intervals is evident, which generally lasts from four to five months in duration. The significance of the positive and negative intervals in the net replacement numbers can be understood more clearly from Table 13.

Table 13 contains the computed composition of the reproduction herd broken down in terms of the month of farrow and the number of farrowings. During the first six months of the first production period the proportion of first-litter sows is substantially greater than either second- or third-litter sows. When the first-litter sows are carried forward as second-litter sows into the second half of the 1968 production year, a greater proportion of the reproduction herd becomes second-litters sows. When the composition of the reproduction herd is predominantly first-litter sows, replacement is less than the cull rate, but when the composition is predominantly second-litter sows, the replacement is greater than the cull rate. Thus, a series of alternating positive and negative net replacements is established.

The composition of the reproduction herd computed by the programming model and contained in Table 13 indicates the least-cost advantage of limiting the number of farrowings per sow to two or less. Although production costs per litter increase only nominally when carrying sows to three or more litters, the salvage value of the cull sow in slaughter demand greatly diminishes. After 400 pounds body weight, both the sow's growth potential and yield rate of live to meat equivalent decreases.

Table 12. Sow cull and gilt replacement, by months for the three-year production and marketing period, 1968–70, in thousands of head.

| Year | Month | Sow cull | Gilt replacement | Net replacement |
|---|---|---|---|---|
| 1968 | January | 878 | 1313 | 435 |
| | February | 347 | 1019 | 672 |
| | March | 479 | 453 | - 26 |
| | April | 629 | .... | - 629 |
| | May | 980 | 757 | - 223 |
| | June | 544 | .... | - 544 |
| | July | 860 | 123 | - 737 |
| | August | 432 | 1470 | 1038 |
| | September | 358 | 571 | 213 |
| | October | 320 | 1011 | 691 |
| | November | 689 | 774 | 85 |
| | December | 506 | 681 | 175 |
| 1969 | January | 1016 | 741 | - 275 |
| | February | 1131 | .... | -1131 |
| | March | 483 | 249 | - 234 |
| | April | 253 | .... | - 253 |
| | May | 761 | 509 | - 252 |
| | June | 170 | 1119 | 949 |
| | July | 277 | 1530 | 1253 |
| | August | 1102 | 1219 | 117 |
| | September | 490 | 737 | 247 |
| | October | 758 | 653 | - 105 |
| | November | 708 | 642 | - 66 |
| | December | 791 | 83 | - 708 |
| 1970 | January | 939 | 333 | - 606 |
| | February | 305 | .... | - 305 |
| | March | 371 | 567 | 296 |
| | April | 163 | 720 | 557 |
| | May | 541 | 1100 | 559 |
| | June | 860 | 1000 | 140 |
| | July | 1231 | 400 | - 831 |
| | August | 914 | 400 | - 514 |
| | September | 695 | 325 | - 370 |
| | October | 670 | 425 | - 245 |
| | November | 756 | 650 | - 106 |
| | December | 419 | 750 | 331 |

## Economic Contribution

The added value to the economy by the pork sector, *from the beginning of production to eventual slaughter* as finished pork and lard, can be approximated from the objective function of the linear program model. The objective function indicates a contribution of nearly 10 billion dollars for the three-year period, or approximately 3.3 billion dollars per year. This compares to the added value of the beef sector of nine billion dollars per year. Thus the total program value of the objective functions for the beef and pork sectors combined account for 12.3 billion dollars per year. *In 1968, the actual*

Table 13. Number of sows farrowed by litter number, and pigs saved, by months for the three-year production and marketing period, 1968–70, in thousands of head.

| Year | Month | First-litter sows | Second-litter sows | Third-litter sows | Pigs saved |
|---|---|---|---|---|---|
| 1968 | January | 326 | 217 | 181 | 5303 |
| | February | 427 | 285 | 237 | 6952 |
| | March | 666 | 444 | 370 | 10841 |
| | April | 675 | 450 | 375 | 10988 |
| | May | 1313 | 82 | .... | 9189 |
| | June | 1019 | 177 | .... | 8037 |
| | July | 453 | 245 | .... | 4903 |
| | August | .... | 320 | .... | 2562 |
| | September | 757 | 499 | .... | 8914 |
| | October | .... | 506 | .... | 4050 |
| | November | 123 | 985 | .... | 8681 |
| | December | 1470 | 764 | .... | 15668 |
| 1969 | January | 571 | 340 | .... | 6429 |
| | February | 1011 | .... | .... | 6573 |
| | March | 774 | 568 | .... | 9573 |
| | April | 681 | .... | .... | 4428 |
| | May | 741 | 92 | .... | 5553 |
| | June | .... | 1102 | .... | 8817 |
| | July | 249 | 428 | .... | 5043 |
| | August | .... | 758 | .... | 6067 |
| | September | 509 | 581 | .... | 7954 |
| | October | 1119 | 511 | .... | 11365 |
| | November | 1530 | 556 | .... | 14395 |
| | December | 1219 | .... | .... | 7924 |
| 1970 | January | 737 | 187 | .... | 6289 |
| | February | 653 | .... | .... | 4246 |
| | March | 642 | 381 | .... | 7224 |
| | April | 83 | 839 | .... | 7253 |
| | May | 333 | 1148 | .... | 11349 |
| | June | .... | 914 | .... | 7315 |
| | July | 567 | 553 | .... | 8108 |
| | August | 720 | 490 | .... | 8600 |
| | September | 1100 | 481 | .... | 11000 |
| | October | 1000 | 62 | .... | 7000 |
| | November | 400 | 250 | .... | 4600 |
| | December | 400 | .... | 250 | 4600 |

gross national product of the total farm sector in current dollars was 25 billion dollars. Although not a precise accounting, the contribution of the beef and pork sectors is about 50 percent of the gross national product of total farm sector.

The beef-pork sector, contributing 50 percent of the total added to the farm sector, but suffering price variation which results from disorderly production and marketing levels, should demand an information service which would provide relief in the form of production and marketing strategies that would have the effect of stabilizing product prices at competitive equlibrium levels.

# GENERAL EVALUATION AND PROGRAM USAGE

The remainder of the discussion will be devoted to issues that would arise in the use of the beef and pork models. Specifically the issues of smoothing and spreading of the production and marketing allocations, input data needs and current availability, sensitivity and stability of solution results, and a general criteria for the use of the models in an information service will be discussed.

## Improvement of Interpretation

The beef and pork models can be used to estimate future production and marketing levels which minimize total supply cost given initial inventory numbers by weight, class, disposition and a series of future demand functions. The particular production and marketing allocations selected by the programming models are those that meet the dual criteria of least cost and an equilibrium solution. The specific activities selected are least-cost efficiency points which, when combined, minimize the value of the objective function and are, by definition, the competitive market solution. However, because of the discrete nature of the beef and pork models, a number of nearby production and marketing activities are not selected, which may not substantially change the value of the objective function, but which would be more compatible with accepted production and marketing behavior.

To illustrate, the marketing allocation of steers and heifers to demand periods presented in Tables 6 and 7 indicated that during a number of months the entire quantity of beef demanded could be met from a single weight class of steers or heifers. In the production allocations in Tables 8 and 13 for the cattle and swine reproduction herd respectively, it was indicated that herd replacement numbers should be large during a particular period, followed immediately by a period of zero replacement. *These elements of discreteness do not diminish the use value of the models.* The model solutions should not be used as tools of precise prediction, but rather to develop strategies for the equilibrium flow of output from the supply to the demand sector.

The presence of discreteness is inherent in all simplifying models and in particular linear programming models. Mathematically, the maximum number of production and marketing activities selected in the optimum solution cannot be greater than the number of rows or constraints in the model. Thus, in the beef model, an average of slightly more than three activities could be used to meet each of the 72 demand constraints, while in the pork model slightly less than three activities could be used to meet each of the 36 pig-saved constraints and the 36 demand constraints. Other factors leading to discreteness in the models are: the linearity assumption of linear programming, the length of the production and demand time intervals, and the lumpiness of form categories (weight, grade, sex, and age).

Practical usage of the programming results for the production and marketing allocations of beef and pork will require interpretation such that the adjustment strategies for the industry will fall within the bounds of acceptable and feasible industry practices and standards. In effect this would require rounding and smoothing of the specific program-generated production and marketing allocation. Instead of one or two program-specified marketing forms for a particular demand period, consistent marketings across a larger number of weight, grade, and classes could be recommended. Adjustments in the reproduction herd could be interpolated across a number of production periods to smooth the discreteness and variations in seasonal patterns. Before producers could be expected to comply with recommended strategies, they must be made reasonably achievable. Rounding and spreading of the production and marketing strategies could be done effectively, without significantly changing the value of the objective function, or increasing the supply cost of the beef and pork.

## Data Needs and Availability

In the course of construction of the beef and pork models, a large proportion of the data inputs had to be synthesized from a number of "best available sources."

The objective of this study was not to provide complete and accurate input data for the beef and pork models, but rather to demonstrate the need for and the accuracy requirements of such data if used in normative economic models for orderly production and marketing at equilibrium price levels. Considerable time and effort was spent in developing reasonably reliable and consistent estimates of input data. Data needs for use in the models include: first, physical performance and time requirements for animals on alternative growth curves, and between animals on identical growth curves; second, demand estimation and prediction; and third, inventory numbers of cattle and hogs classified by weight, disposition and age.

## Growth Curves

Current information concerning physical performance and time requirements necessary to carry cattle and hogs through the various stages of growth is incomplete. Information about performance and time are needed at all levels of growth with respect to alternative feeding intensities, climatic differentials, and environmental conditions. Additional information is needed concerning between animal performance on identical growth curves.

Although considerable data are available on the above two categories, most of the data available are related to specific experiments on specific animal forms, where performance and time requirements were only secondary issues. In addition, the specific nature and objectives of these research efforts make it difficult, if not impossible, to interpolate physical and biological research results across various stages of animal growth.

## Demand Estimation

Estimates of the retail demand for beef and pork need further investigation. Of particular importance are more precise measurements of the substitution and income effects. In the light of the rapid changes in the demand shifters during the period from 1967–1969, further research about the relationships between the substitution and income effects, and their own individual relationship on per capita consumption and product price, are critical to any normative use of the models for developing production and marketing strategies. Additional data are also needed on the quantity of per capita consumption of beef and pork consisting of processed and unprocessed meat.

## Inventory Numbers

Difficulties in developing an initial inventory have been discussed in the previous sections. Much of the needed inventory input data for the programming models are available only in aggregate forms, and not delineated into class, disposition, and age categories.

First, data availability for cattle and calf numbers on feed by weight and classes are reasonably adequate. However, for the programming model it would be desirable to have further knowledge about the level of intensity of feeding programs. This delineation would not have to be a detailed categorization by protein and energy input levels. For example, two categories on feeding described as full-feed and liberal-roughage would be helpful in determining the potential growth performances and time dimensions of various animal forms in the feed-lot inventory.

Second, data availability for determining numbers in the beginning inventory at the stocker-feeder level is incomplete, while no data are available on animals by weight, age, and class. The single biggest improvement at the stocker-feeder level would be a categorization by numbers and age. *A priori* knowledge on growth performance plus an accounting procedure could be used to fill in the missing blanks on the various form and class categories.

Third, data availability on the seasonal distribution of reproduction at the primary level is of extreme importance in solving for production and marketing allocations. Because of the length of the production process, seasonal adjustments at the reproduction level take time, and a substantially longer time is necessary before the effect of the adjustment is evident at the market place. Although some data are available on the spatial distribution of calf production, no data are available on seasonal distribution of reproduction. Quarterly delineation of calf production numbers would be adequate for use in the programming model. Additional information concerning seasonal variation in calf-saved percentages would also be of value for programming purposes.

A number of similar data problems were incurred in developing a beginning inventory for the pork model. Adequate data are available on barrows and gilts between various weight classes; however, the data are reported as of December 1, causing some difficulty in adjusting the numbers and weights to January 1 equivalents. For purposes of model usage, it would be advantageous to have the quarterly inventories of cattle and calves, and hogs and pigs released simultaneously instead of separately on alternate months.

Inventories of gilts and sows bred to farrow are reported as quarterly totals, without reference to a distribution by the number of previous farrowings. For the program, it is necessary that gilts and sows bred to farrow be reported by the month of expected farrowing. The inventory turnover in the pork sector is so rapid that quarterly reporting on the reproduction herd would distort normative production and marketing allocations if used in outlook projection and followed by producers.

In summary, the most critical needs in the beef and pork sector as far as data improvement are concerned, are improved knowledge about physical growth performance and time dimensions of the various animal forms; accurate and reliable estimates of retail demand, with particular reference to the income and substitution effects; and the need for new and more detailed data on cattle and swine inventories, especially at the stocker-feeder and primary production levels.

## Data Accuracy Requirements

Some of the implications of errors in data and the effects of data errors on the optimum solution will be discussed in this section. Primary concern will be directed to the issue of random errors in data. Attention will also be given to data errors resulting from uniform over-or-under estimation of an entire data series. The question of data accuracy requirements depends on the economic cost of data error times the probability of data error, if the data were used in models of industry order and results of the models were followed by cattle and swine producers.

## Beginning Inventory Constraints

The effect of errors in data can be analyzed by observing the range over which a constraint at limit level can be varied without changing the activity structure within the solution. If the magnitude of the range on a row or constraint at limit level is small, the activity structure in the solution is sensitive to small changes in the value of that constraint. If the range on a constraint is large, the activity structure is stable for adjustments in the value of the constraint.

In the beginning inventory, if the range of a constraint is small, the optimum solution is sensitive to changes in the value of the constraint, thus, small errors in measurement of input data in the beginning inventory could effect the optimum solution.

Conversely, if the range of a constraint is large, large errors in measurement could be tolerated before the activity structure in the optimum solution would be affected. The upper and lower values of the constraints at limit level for the beginning inventories in the optimum beef and pork solutions are presented in Tables 14 and 15. The ranges are the differences between the upper and lower values of the constraints at limit level.

In the beef model, the magnitude on the ranges for the beginning inventory are small, varying in size from a low of two thousand head to a high of seven thousand head. As would be expected, animal categories in the beginning inventory closest in time to immediate slaughter are the most sensitive. Animals expected to go into demand period about one year away are slightly less sensitive.

The magnitude of the range indicates that extremely small changes in the value of individual categories in the beginning inventory would affect the activity structure of the optimum beef solution. The changes in composition and level of the activities are unknown, without parametrically programming each of the individual inventory categories outside of the constraint range of the optimum solution. It can be observed, however, that small random errors of measurement in the beginning inventory could potentially affect the optimum solution.

The magnitude of the ranges in the pork model are considerably larger than those observed in the beef model. This suggests that random errors in measurement in the beginning inventory would have less effect on the optimum solution of the pork model than the beef model. As in the beef model, the ranges are smaller on those animals closest in time to immediate slaughter and the ranges grow larger

**Table 14. Upper and lower values of the beginning inventory constraints at limit level for the optimum beef solution, in thousands of head.**

| Description | Lower constraint level | Upper constraint level | Range |
|---|---|---|---|
| Feedlot steers | | | |
| > 1100 pounds | 378 | 382 | 4 |
| 900–1099 pounds | 2009 | 2113 | 4 |
| 700– 899 pounds | 2492 | 2496 | 4 |
| 500– 699 pounds | 1901 | 1907 | 6 |
| < 500 pounds | 1058 | 1064 | 6 |
| Feedlot heifers | | | |
| > 1100 pounds | 0 | 2 | 2 |
| 900–1099 pounds | 380 | 384 | 4 |
| 700– 899 pounds | 1100 | 1104 | 4 |
| 500– 699 pounds | 1105 | 1111 | 6 |
| < 500 pounds | 799 | 805 | 6 |
| Feeder steers and calves | | | |
| > 750 pounds | 2737 | 2741 | 4 |
| 550– 749 pounds | 4792 | 4798 | 6 |
| 400– 549 pounds | 2637 | 2643 | 6 |
| 299– 399 pounds | 4711 | 4717 | 6 |
| < 200 pounds | 2592 | 2598 | 6 |
| Feeder heifers and calves | | | |
| > 700 pounds | 571 | 575 | 4 |
| 500– 699 pounds | 4635 | 4641 | 6 |
| 350– 499 pounds | 4970 | 4976 | 6 |
| < 200 pounds | 2591 | 2598 | 7 |
| Reproduction base | | | |
| Cows to calf in winter quarter | 9988 | 9992 | 4 |
| Cows to calf in spring quarter | 13059 | 13063 | 4 |
| Cows to calf in summer quarter | 6105 | 6109 | 4 |
| Cows to calf in fall quarter | 6140 | 6144 | 4 |
| Culls from previous years cow base | 4853 | 4859 | 6 |

the further the animal is away from immediate slaughter. Therefore, it can be concluded that the *need for individual data accuracy is greatest on animals nearest to slaughter weights. The further the animal is away in time from slaughter, the less the degree of accuracy required.*

## Demand Constraints

The upper and lower constraint values at limit levels for the beef and pork demand functions are presented in Tables 16 and 17, respectively. Interpretation of the upper and lower constraint values and the ranges are similar to that described in the beginning inventory.

The magnitude of the ranges increases from a low of two million pounds in the immediate demand periods to a high of 29 million

Table 15. Upper and lower values of the beginning inventory constraints at limit level for the optimum beef solution, in thousands of head.

| Description | Lower constraint level | Upper constraint level | Range |
|---|---|---|---|
| Barrows and gilts | | | |
| < 25 pounds | 3840 | 3895 | 55 |
| 25– 50 pounds | 4011 | 4053 | 42 |
| 51– 85 pounds | 6552 | 6589 | 37 |
| 86–130 pounds | 10548 | 10580 | 32 |
| 131–180 pounds | 7948 | 7972 | 24 |
| 181–230 pounds | 5287 | 5308 | 21 |
| 231–270 pounds | 1414 | 1432 | 18 |
| 271–300 pounds | 380 | 396 | 16 |
| Bred sows to farrow | | | |
| First litter in January | 322 | 328 | 6 |
| First litter in February | 421 | 431 | 10 |
| First litter in March | 659 | 670 | 11 |
| First litter in April | 666 | 680 | 14 |
| Second litter in January | 213 | 219 | 6 |
| Second litter in February | 277 | 290 | 13 |
| Second litter in March | 436 | 449 | 13 |
| Second litter in April | 435 | 460 | 25 |
| Third litter in January | 177 | 183 | 6 |
| Third litter in February | 230 | 241 | 11 |
| Third litter in March | 362 | 375 | 13 |
| Third litter in April | 365 | 381 | 16 |
| Lactating sows | | | |
| First litter, first month | 230 | 240 | 10 |
| First litter, second month | 104 | 112 | 8 |
| Second litter, first month | 147 | 163 | 16 |
| Second litter, second month | 238 | 251 | 13 |
| Third litter, first month | 122 | 136 | 14 |
| Third litter, second month | 183 | 196 | 13 |
| Non-bred sows | | | |
| One-litter sows | 81 | 96 | 15 |
| Two-litter sows | 192 | 205 | 13 |
| Three-litter sows | 117 | 128 | 11 |

pounds in the last months of the sixth demand year. The value of the ranges in the beef demand functions indicate that very small changes in individual demand quantities affect the optimum solution in the beef model. This suggests that small individual random errors in the estimation of beef demand could potentially affect the production and marketing solution levels for beef. The small magnitude of the constraint range further suggests the need for extremely sophisticated demand estimation and projection techniques. A portion of the extremely narrow constraint range on beef demand functions results from the length of the biological production process and the time needed for adjustments in reproduction levels.

Table 16. Upper and lower values on the demand constraints at limit level for the optimum beef solution.

| Year | Month | Lower constraint level | Upper constraint level | Range |
|---|---|---|---|---|
| | | (millions of pounds) | | |
| 1968 | January | 1459 | 1461 | 2 |
| | February | 1499 | 1501 | 2 |
| | March | 1559 | 1561 | 2 |
| | April | 1619 | 1622 | 3 |
| | May | 1668 | 1672 | 4 |
| | June | 1708 | 1712 | 4 |
| | July | 1758 | 1762 | 4 |
| | August | 1728 | 1732 | 4 |
| | September | 1728 | 1732 | 4 |
| | October | 1738 | 1742 | 4 |
| | November | 1698 | 1702 | 4 |
| | December | 1668 | 1672 | 4 |
| 1969 | January | 1638 | 1642 | 4 |
| | February | 1628 | 1632 | 4 |
| | March | 1628 | 1632 | 4 |
| | April | 1628 | 1632 | 4 |
| | May | 1618 | 1622 | 4 |
| | June | 1678 | 1683 | 5 |
| | July | 1737 | 1743 | 6 |
| | August | 1717 | 1723 | 6 |
| | September | 1767 | 1773 | 6 |
| | October | 1767 | 1773 | 6 |
| | November | 1737 | 1743 | 6 |
| | December | 1697 | 1703 | 6 |
| 1970 | January | 1657 | 1663 | 6 |
| | February | 1617 | 1623 | 6 |
| | March | 1607 | 1613 | 6 |
| | April | 1607 | 1614 | 7 |
| | May | 1606 | 1614 | 8 |
| | June | 1686 | 1694 | 8 |
| | July | 1746 | 1755 | 9 |
| | August | 1706 | 1715 | 9 |
| | September | 1766 | 1775 | 9 |
| | October | 1766 | 1775 | 9 |
| | November | 1745 | 1755 | 10 |
| | December | 1715 | 1725 | 10 |
| 1971 | January | 1706 | 1715 | 9 |
| | February | 1666 | 1675 | 9 |
| | March | 1636 | 1645 | 9 |
| | April | 1645 | 1656 | 11 |
| | May | 1635 | 1646 | 11 |
| | June | 1734 | 1746 | 12 |
| | July | 1754 | 1767 | 13 |
| | August | 1734 | 1747 | 13 |
| | September | 1774 | 1787 | 13 |
| | October | 1803 | 1817 | 14 |
| | November | 1783 | 1798 | 15 |
| | December | 1753 | 1768 | 15 |
| 1972 | January | 1743 | 1757 | 14 |
| | February | 1713 | 1727 | 14 |
| | March | 1693 | 1707 | 14 |
| | April | 1692 | 1709 | 17 |
| | May | 1672 | 1689 | 17 |

Table 16. (continued)

| Year | Month | Lower constraint level | Upper constraint level | Range |
|---|---|---|---|---|
| | | (millions of pounds) | | |
| | June | 1751 | 1770 | 19 |
| | July | 1790 | 1811 | 21 |
| | August | 1760 | 1781 | 21 |
| | September | 1849 | 1872 | 23 |
| | October | 1849 | 1872 | 23 |
| | November | 1828 | 1853 | 25 |
| | December | 1798 | 1823 | 25 |
| 1973 | January | 1779 | 1802 | 23 |
| | February | 1759 | 1782 | 23 |
| | March | 1729 | 1752 | 23 |
| | April | 1729 | 1752 | 23 |
| | May | 1709 | 1733 | 24 |
| | June | 1788 | 1814 | 26 |
| | July | 1797 | 1825 | 28 |
| | August | 1776 | 1805 | 29 |
| | September | 1828 | 1854 | 26 |
| | October | 1847 | 1875 | 28 |
| | November | 1826 | 1855 | 29 |
| | December | 1796 | 1825 | 29 |

In the pork model, the ranges on the demand constraint begin at three in January and February of 1968 and gradually widen in magnitude through April of 1969. In May of 1969, the constraint range immediately widens in magnitude and remains extremely wide for the remainder of the three-year production and marketing period.

The ranges in the pork model on the demand constraints indicate that the seriousness of random errors in demand measurement is not as great as in the beef model. As would be expected, immediate demands are the most sensitive. After about one year, adjustments to the quantity demanded could be flexibly met by the production and marketing activities. This results from the relatively fast rate of inventory turnover in the pork sector compared with the beef sector.

## Nonrandom Errors in Data

The effect of an aggregate over-or-under estimation of a data series can be a serious potential problem in computing orderly production and marketing allocations. A number of the estimators used in developing the beginning inventories and the demand functions are directionally bound together by interrelationships. A random error in the measurment of a single important input variable may result in over-or-under estimation of a whole data series. The activity level of an optimum solution based on a non-random error in measurement may be considerably different than the true optimum solution.

**Table 17. Upper and lower values of the demand constraints at limit level for the optimum pork solution.**

| Year | Month | Lower constraint level | Upper constraint level | Range |
|---|---|---|---|---|
| | | (millions of pounds) | | |
| 1968 | January | 1149 | 1252 | 3 |
| | February | 1129 | 1232 | 3 |
| | March | 1128 | 1133 | 5 |
| | April | 1088 | 1093 | 5 |
| | May | 1048 | 1053 | 5 |
| | June | 1047 | 1054 | 7 |
| | July | 982 | 1003 | 11 |
| | August | 1002 | 1022 | 20 |
| | September | 1010 | 1036 | 26 |
| | October | 1006 | 1040 | 36 |
| | November | 1042 | 1086 | 44 |
| | December | 1041 | 1091 | 50 |
| 1969 | January | 1009 | 1064 | 55 |
| | February | 1031 | 1088 | 57 |
| | March | 1035 | 1110 | 75 |
| | April | 1040 | 1124 | 84 |
| | May | 625 | 1242 | 617 |
| | June | 469 | 1270 | 801 |
| | July | 733 | 1227 | 494 |
| | August | 829 | 1336 | 507 |
| | September | 755 | 1255 | 500 |
| | October | 832 | 1213 | 381 |
| | November | 871 | 1288 | 461 |
| | December | 827 | 1370 | 542 |
| 1970 | January | 820 | 1235 | 415 |
| | February | 1027 | 1357 | 330 |
| | March | 948 | 1427 | 479 |
| | April | 899 | 1523 | 624 |
| | May | 835 | 1646 | 811 |
| | June | 881 | 1236 | 355 |
| | July | 801 | 1251 | 450 |
| | August | 734 | 1156 | 422 |
| | September | 727 | 1172 | 445 |
| | October | 781 | 1206 | 425 |
| | November | 707 | 1260 | 553 |
| | December | 926 | 1279 | 353 |

The ranges on individual constraints measure the sensitivity of the activity structure in the optimum solution to changes in the value of the constraint. The change in the activity structure and level is not indicated; only that they will change if the value of the constraint is adjusted outside of the constraint range. It is believed, however, that the sensitivity of the optimum solution to general nonrandom errors of measurement is approximately equal to the magnitude of the individual ranges divided by the number of constraints.

In other words, the effects of nonrandom errors on the optimum solution is more serious than random errors. The basis for this judgment is the strong substitution equivalence of the constraints by groups. This proposition is not directly measurable from the sensitivity analysis of the rows at limit level, but could be measured by adjusting all the constraints by a percentage and solving for a new optimum solution.

## USE OF NORMATIVE MODELS

The price structure of the beef-pork industry was found to be performing efficiently between the slaughter and retail levels. Inefficient allocation and value performance existed between the primary production level and the feedlot level. The inefficiency resulted from the inability of producers at the primary and feedlot levels to accurately anticipate future product prices and to plan production and marketing levels accordingly.

Temporal and form normative models for equilibrium allocations of production and marketing were developed for the beef and pork sectors. The models were then tested for their ability to compute equilibrium production levels and marketings of beef and pork to satisfy market demands. Data input needs and accuracy requirements for the models were also discussed in previous sections. This section will briefly describe the possible outlook use of the models. No arguments for, or a design of, specific policy programs will be discussed.

The models are primarily designed to estimate temporal and form production and marketing allocations for the beef and pork industry which if the industry would achieve should result in continuous equilibrium. The models are designed such that given a beginning inventory and a series of demand functions, the proper equilibrium inventory flow can be determined. From the optimum solution it is possible to determine temporal and form allocation of marketings. Further, the correct seasonal distribution and size of the reproduction herd, the intermediate inventory composition by weight and number, and the cull and replacement level can be determined.

The beef model is designed so that an optimum solution can be computed every three months. While it is technically possible to solve for a new pork solution monthly, it would more practically be solved simultaneously with the beef model. The production and marketing recommendations based on the models could be disseminated to producers and the public by an information service agency of the U.S.D.A. Instead of the traditional price forecasting presently being done by the Economic Research Service, the time and form production and marketing allocations could be released as normative quantity forecasts.

The use of normative quantity projections instead of positive price forecasting would have the effect of placing the burden of responsibility on the producers. If producers complied with the production and marketing "targets," they could expect a movement toward or the maintenance of equilibrium prices. If the producers did not comply with the production and marketing "targets," disequilibrium performance would continue. On the other hand, if producers complied with the quantity "targets," but data errors had the effect of directionally leading the industry away from rather than toward equilibrium, then the responsibility for disequilibrium results would rest with the agency.

An active information program would require close cooperation between the Economic Research Service and the Statistical Reporting Service. If the models were solved at three month intervals for the production and marketing allocations, it would require collection and estimation of quarterly cattle and hog inventories, plus considerable data not currently collected on physical performance, seasonality of production, etc. Further, it would require a feedback from the industry to determine the degree of industry compliance at each of the production levels, and the various adjustment problems being encountered by producers. Much of the basic information for projection or normative forecasting must come from producers. They have a vital responsibility to support and respond to requests for information and to submit correct data. This is of special significance when high levels of estimation accuracy are sought.

In summary, the use potential of the beef and pork models for actively guiding the livestock industry from positions of disequilibrium to equilibrium and the continued maintenance of the equilibrium is of considerable economic value. The continuation of disequilibrium performance in an industry which has an imputed program value of 12.3 billion dollars of added value per year, or approximately one-half the total added value of the entire agriculture industry, is of serious consequence.

# SUMMARY AND CONCLUSIONS

The livestock sector has characteristically experienced large variations in the slaughter prices of beef and pork. Generally, variations in slaughter price have not reflected changes in the cost of production. Price variations have occurred because of a number of biological and economic factors which have led to alternate periods of over- and under-production.

The objectives of this study were: first, to evaluate the price structure of the beef-pork sector and delineate areas of inefficient performance, and second, to develop and test orderly production and marketing models consistent with competitive temporal and form equilibrium, and third, to indicate the usefulness of the models and suggest data input needs and accuracy requirements that would improve the performance of the industry.

In the evaluation of the value structure of the beef-pork sector, special attention was given to the space, time, and form relationships, from the primary production level at the farm or ranch, to the final consumption level as beef or pork. The value relationships reflected strong competitive forces producing efficient price performance from slaughter to the retail level. Vertically between market levels, and horizontally in space, time and form, the performance of the transfer mechanism was found to be good. This suggested that the effects of improved order in the production and marketing of cattle and hogs could yield substantial industry-wide payoff.

Relationships between the primary production and feedlot levels indicated a weak and distorted price structure. Prices bid for feeder animals did not reflect future value in demand. Evidence suggested that existing slaughter prices strongly influence the prices paid for feeder animals. A primary element appeared to be the inability of producers to correctly anticipate future slaughter prices.

The inefficiency of the price mechanism between the ranch and feedlot levels is both a production and marketing problem, resulting from the difficulty in organizing correct production levels in the long run, and then finally adjusting marketing weights of the animals for slaughter in the short run.

A time and form equilibrium model was designed that would simulate competitive allocations of the flow of reproduction, production, and marketings of cattle and hogs. The model assumes that demands through time can be represented by known linear demand functions and that, for any point in time, animal inventory numbers by categories can be determined and allocated to demand such that supply costs are minimized.

For solution purposes, an iterative linear programming model was used to determine the optimum production and marketing allocation of beef and pork. Single points from a series of linear demand functions that represent the optimum equilibrium solution were selected using the duality theorem of linear programming. The economic model was designed for the purpose of aiding the industry in making adjustments from situations of disequilibrium to equilibrium and the maintenance of that position, starting from any animal inventory position, above, below, or at equilibrium.

The inventory flow model for beef contains 253 restrictions and 912 activities, and was designed for a six-year production and marketing period. The beef model provides for marketing from the initial animal inventory, cow cull and heifer replacement, and the production and marketing of program-generated supplies. The $C_j$ values of the activities are to total supply costs of producing the animal to that stage of growth. All prior costs on animals in the initial inventory were disregarded. The objective function was designed to minimize total supply cost associated with production and marketing during the six-year adjustment period.

Initial inventory conditions in the beef model tested were those on January 1, 1968. Inventory numbers were taken from published reports of the U.S.D.A. Results of the model indicated that after the first year, the computed equilibrium price path moved into a regular seasonal pattern of increasing slaughter beef prices, starting in October and peaking in May, followed by a decreasing price path from June to September. The range between the peaks and troughs of the price cycles became smaller as production and marketing adjusted to equilibrium conditions. The price pattern resulted because of seasonal variation in production cost, production numbers, and quantity demanded.

The marketing allocation of steers and heifers indicated that in equilibrium, heavier animals should be marketed during seasons of high beef prices, while during seasons of low beef prices light animals should be marketed. Long-run adjustment in production should be made by adjusting the size of the reproduction base. Although culling was arbitrarily assumed to be 14 percent per year, replacement of heifers was treated as an economic decision. Heifers had the alternative of being slaughtered or used as replacements in the reproduction herd depending on the immediate value of slaughter beef in the demand sector or the value of reproduction for future meat demands. This has the effect of basing long-run reproduction on a criterion of equilibrium least cost production and marketing.

The temporal and form equilibrium model for hogs has 229 restrictions and 520 activities, and was designed for a three-year production and marketing period. The functions of the pork model are similar to the beef model.

Initial inventory conditions were those of January 1, 1968, and were taken from published reports of the U.S.D.A. Once the initial inventory animals were marketed, the equilibrium price path became relatively flat with only minor variations reflecting seasonal production costs and demand levels. No consistent seasonal pattern of marketing by weight class for barrows and gilts developed. This was due to the relatively flat marginal cost curve between 180 and 230 pounds, the relatively short production period, and the offsetting nature of the seasonal costs of production at the reproduction and finishing levels.

The production strategies computed in the optimum program indicated a least cost advantage of limiting the number of farrowings per sow to two or less. Although the production costs per litter increase only nominally after two farrowings, the salvage value of the sow as pork generally diminishes.

The values of the objective functions in the beef and pork models indicate that the imputed annual added value from primary production to the slaughter level is about 9 billion dollars in the beef sector and 3.3 billion dollars in the pork sector. The combined program "added values" for beef and pork is about 50 percent of the total national income contribution of agriculture in 1968.

The objectives of this study were not to provide complete and accurate input data for the programming models, but rather to demonstrate the need for, and the accuracy requirements of, such data if used in normative economic models for industry order. Data needs found to be most urgent included physical performance and time differentials between animals on alternative growth curves, and between animals on identical growth curves, accurate and reliable demand estimates and projections; and inventory numbers of cattle and hogs classified by weight, disposition, and age.

The issue of data accuracy required evaluating the sensitivity of the optimum solution to changes in the size of the beginning inventory and the quantity demanded. Categories in the beginning inventories and months in the demand functions closest in time were found to be the most sensitive to errors in data while inventories and demands later in time were found to be less sensitive.

The use values of the beef and pork models are their potential for actively guiding the industry. In practice this would require smoothing and spreading the production and marketing allocations computed in the optimum solution. This could be done without substantially changing the value of the objective function, and would provide credible and acceptable industry production and marketing strategies. The prescriptive use of the models, however, would further require the close cooperation of an active information service in the Economic Research Service and an expanded Statistical Reporting Service.

## Some Further Needs

Several areas closely related to this study are in need of further research efforts. At several places in this study the difficulties encountered in developing an initial inventory were pointed out. Special attention in the future should be given to an investigation of the aggregate inventory data needs by categories for the livestock industry. This study should primarily be concerned with quantitatively establishing the magnitude of tolerable random error in data collection by category, plus the effect of nonrandom errors in data estimation, assuming the data were used and followed in an industry order model.

A related study should be made that would evaluate the informational needs of producers at the firm level. Currently, quantities of raw data in the form of numbers, prices and indexes are available to producers. The study should determine the decision problems at the firm level, develop economic decision models with the appropriate specification of data needs and accuracy requirements, and indicate a proper format for disseminating interpreted rather than raw information to the decision-makers.

Future research efforts should be made in outlining a detailed action program, which would combine the talents of the Economic Research Service and the Statistical Reporting Service with the responsibility of providing normative production and marketing strategies for the livestock-feed sector. This would require consideration of the time dimensions necessary in gathering, interpreting, and disseminating the information, methods and forms of interpretation and distribution of information, feedback from the industry on the progress of the adjustment, the initiation and maintenance cost of the program and the expected economic returns.

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# APPENDIX A

# SCHEMATIC DIAGRAMS

**Appendix Figure 1. Schematic of beef programming model.**

Initial Inventory:

Feedlot Steers (5-classes, 2-feeding intensities, 14 slaughter alternatives)
Feedlot Heifers (5-classes, 2-feed intensities, 12 slaughter alternatives)
Feeder Steers and Calves (5-classes, 3-feeding intensities, 18-slaughter alternatives)
Feeder Heifers and Calves (5-classes, 3-feeding intensities, 16-slaughter alternatives)
Reproduction Base (Seasonally distributed, quarterly)

Intermediate Production
and Transition Inventory:

Beef Demands:

$D_{1,1}$
$D_{1,2}$
.
.
.
$D_{1,12}$
.
.
.
$D_{6,12}$

Terminal Inventory:

Feedlot Steers (3-classes)
Feedlot Heifers (2-classes)
Feeder Steers and Calves (3-classes)
Feeder Heifers and Calves (3-classes)
Reproduction Base

**Appendix Figure 2. Schematic of pork programming model.**