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Time Truncated Efficient Testing Strategy for Pareto Distribution of the 2nd Kind Using Weighted Poisson and Poisson Distribution

(Strategi Ujian Cekap Masa Terpangkas untuk Taburan Pareto Jenis ke-2 Menggunakan Taburan Poisson Berpemberat dan Poisson)

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ABSTRACT

In this study, group acceptance sampling plan (GASP) proposed by Aslam et al. (2011) is redesigned where the lifetime of test items are following Pareto distribution of 2nd kind. The optimal plan parameters are found by considering various pre-determined designed parameters. The plan parameters were obtained using the optimization solution and it also concludes that the proposed plan is more efficient than the existing plan as it requires minimum sample size.

Keywords: Consumer's risk; group acceptance sampling; operating characteristic values; Poisson & Weighted Poisson distribution; producer's risk

ABSTRAK

Dalam kajian ini, pelan pensampelan penerimaan kumpulan (GASP) yang dicadangkan oleh Aslam et al. (2011) direka semula dengan hayat item ujian mengikuti taburan Pareto jenis ke-2. Parameter optimum pelan boleh didapati dengan mengambil kira pelbagai parameter yang telah ditetapkan terlebih dahulu. Parameter pelan diperoleh menggunakan penyelesaian pengoptimuman dan ia membuat kesimpulan bahawa rancangan yang dicadangkan adalah lebih cekap daripada rancangan sedia ada kerana ia memerlukan saiz sampel yang minimum.

Kata kunci: Nilai ciri operasi; pensampelan penerimaan kumpulan; risiko pengeluar; risiko pengguna; taburan Poisson berpemberat & Poisson

INTRODUCTION

Acceptance sampling plan is one of the most important techniques to improve the quality of a submitted item. The main objective of acceptance sampling is to reduce the time and cost of the truncated life test experiment and also very helpful that the producer's increase the quality of the item. For the final item, it may not feasible to examine each and every item at the time of the inspection. Then, a random sample is chosen with the support of acceptance sampling method for the final confirmation of the lot. The selected items are put on the test and lot is acceptable if the number of failures are less than the pre-specified number of failures. In these plans, as the opinion is made on the use of sample observation, therefore two risks are constantly affixed with sampling plans. The probability that a good lot is not accepted is known as producer's risk and the probability of accepting a bad lot is called the consumer's risk denoting by α and β, respectively.

Many researchers proposed acceptance sampling plans based on truncated life test for several distributions, for example, Balakrishnan et al. (2007), Baklizi (2003), Epstein (1954), Goode and Kao (1961), Kantam and Rosaiah (1998), Kantam et al. (2006), Mughal et al. (2011), Radhakrishnan and Mohana Priya (2008) and Tsai and Wu (2006). In ordinary acceptance sampling plan, a single item is inspected but in practice, testers are available that can accommodate more than one item. Therefore, the experimenters can use the group acceptance sampling plan (GASP) to observe the multiple items at the same time. The more details about group acceptance sampling plans (GASP) can be seen in Aslam et al. (2010a, 2010b), Mughal and Aslam (2011) and recently Mughal and Ismail (2013) designed an economic reliability efficient group acceptance sampling plans for family Pareto distributions.

Aslam et al. (2011) designed a comparison of GASP for Pareto distribution of the 2nd kind using Weighted Poisson and Poisson distributions. In practice, the Pareto distribution of the 2nd kind is used for failure life time data. To the best of our knowledge, still no researcher has produced the efficient GASP for Pareto distribution of the 2nd kind using Weighted Poisson and Poisson distribution. Therefore, in our proposed research, the inspection of submitted items from total sample will be conducted rather than inspecting items in each group as discussed by the Aslam et al. (2011). Due to minimum sample size, the proposed plan will be more efficient and economical when it comes to save time, cost, energy and man-hours involved. If sample size is large and p (defective items in a lot) is very small then Poisson distribution is a best approximation of the binomial distribution. In life, testing weighted distribution methods and application are widely used because the reported data is biased. The biased data do not express the parent distribution behavior unless every item is interpreted by assigning the equal chance of selection.

The objective of this study was to obtain the optimal value of group size, operating characteristic value and minimum ratio of true average life by considering the various pre-determined plan parameters.

Methods

Consider μ and μ_0 denote the true and specified average life of an item respectively. An item is acceptable for consumer use if the average life μ is greater than a specified average life μ_0. According to Mughal and Ismail (2013), procedure of proposed acceptance sampling plan under GASP will be: Obtain the number of g groups and allocate r items to each group so the required sample size in the life test is $n = r \times g$; Selecting the acceptance number c and carry out the experiment for the g groups and accept the lot if at most c failures record in all of the groups; and If more than c failures occur at time t_0, truncate the experiment and reject the submitted lot.

Pareto (1897) discussed the importance of Pareto distribution as a model for income. The probability density function (PDF), the cumulative distribution function (CDF) and the mean μ of a Pareto distribution of the 2nd kind can be written, respectively,

$$f(t;\sigma,\lambda)=\frac{\lambda}{\sigma}\left(1+\frac{t}{\sigma}\right)^{-(\lambda+1)} \quad t>0,\sigma>0,\lambda>0. \tag{1}$$

$$F(t;\sigma,\lambda)=1-\left(1+\frac{t}{\sigma}\right)^{-\lambda} \quad t>0,\sigma>0,\lambda>0. \tag{2}$$

$$\mu=\frac{\sigma}{\lambda-1}, \quad \lambda>1. \tag{3}$$

The two parameters (σ, λ) are scale and shape parameters, respectively and it is important to note that the value of shape parameter must be greater than 1. Under the proposed plan the lot acceptance probability function for Weighted Poisson and Poisson distribution can be written in this form,

$$L(p)=\left[\sum_{(i=1)}^{c}\frac{e^{-(rg)p}((rg)p)^{(i-1)}}{(i-1)!}\right], \tag{4}$$

$$L(p)=\left[\sum_{(i=0)}^{c}\frac{e^{-(rg)p}((rg)p)^{i}}{(i)!}\right], \tag{5}$$

where p denotes the probability of failure of an item during the test termination time t_0. The termination time t_0 is a multiple of the specified mean life μ_0 and pre-assumed constant 'a'. For example, $a = 0.5$ means that the test termination time is half of the specified average life. So $t_0 = a\mu_0$ and p is estimated as follow,

$$p=F(t;\sigma,\lambda)=1-\left[1+\frac{a}{(\lambda-1)(\mu/\mu_0)}\right]^{-\lambda} \tag{6}$$

The minimum group size was found when the following two inequalities fulfilled the conditions for both weighted Poisson and Poisson distributions (7), (8), respectively and placed in Tables 1-4.

$$L(p)=\left[\sum_{i=1}^{c}\frac{e^{-(rg)p}((rg)p)^{i-1}}{(i-1)!}\right]\le\beta \tag{7}$$

$$L(p)=\left[\sum_{i=0}^{c}\frac{e^{-(rg)p}((rg)p)^{i}}{(i)!}\right]\le\beta \tag{8}$$

The minimum mean ratio (μ/μ_0) determined in (9) and (10) for given producer's risk which is very helpful tool for the producers to choose the appropriate life testing plan and discussed in Tables 8-12.

$$L(p)=\left[\sum_{i=1}^{c}\frac{e^{-(rg)p}((rg)p)^{i-1}}{(i-1)!}\right]\ge 1-\alpha \tag{9}$$

$$L(p)=\left[\sum_{i=0}^{c}\frac{e^{-(rg)p}((rg)p)^{i}}{(i)!}\right]\ge 1-\alpha \tag{10}$$

The minimum group size, operating characteristics values and minimum mean ratio (μ/μ_0) were obtained for Weighted Poisson and Poisson distribution and allocated in Tables 1-12. In these tables, we considered the various designed parameters such as, number of tester $r = 7(1)12$, acceptance number $c = 5(1)10$, test termination ratio $a = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0$, consumer's risk $\beta = 0.25, 0.10, 0.05, 0.01$ and producer's risk $\alpha = 0.05$. Tables 1-4 illustrate that test termination ratio monotonically decreases as the group size increases. In Tables 5-8, the operating characteristics (OC) values are established using (4) and (5) for $c = 7$, $r = 9$. For various value of designed parameter the OC values can be accessed by using the same technique. From Tables 5-8, it is obvious to see that when mean ratio increases from 2 to 12, the probability of lot acceptance is also increasing. Conversely, as test termination ratio (μ/μ_0) increases from 0.7 to 2.0, we have found the decreasing tendency in probability of lot acceptance. The efficiency and advantages of the proposed plan compared with the existing plan in term of sample size also presented in Tables 13-16. The identical values of designed parameters have been used for the both acceptance sampling plans.

Application in the Industry

The practical use of the proposed plan in the industry for the testing of the items whose lifetime based on Pareto distribution of the 2nd kind (using Poisson distribution) will interpret in the following theoretical example.

TABLE 1. Number of groups required for the proposed plan for the Pareto distribution of the 2nd kind with λ =2, using Weighted Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	2	2	2	2	2	2
	8	6	2	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.10	7	5	2	2	2	2	2	2
	8	6	2	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.05	7	5	2	2	2	2	2	2
	8	6	2	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.01	7	5	3	3	3	3	2	2
	8	6	3	3	3	3	2	2
	9	7	3	3	3	3	2	2
	10	8	3	3	3	3	2	2
	11	9	3	3	3	2	2	2
	12	10	3	3	3	2	2	2

TABLE 2. Number of groups required for the proposed plan for the Pareto distribution of the 2nd kind with λ = 3, using Weighted Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	2	2	2	2	2	2
	8	6	2	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.10	7	5	2	2	2	2	2	2
	8	6	2	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.05	7	5	3	2	2	2	2	2
	8	6	3	3	2	2	2	2
	9	7	3	3	2	2	2	2
	10	8	3	3	2	2	2	2
	11	9	3	3	2	2	2	2
	12	10	3	3	2	2	2	2
0.01	7	5	3	3	3	3	3	2
	8	6	3	3	3	3	2	2
	9	7	3	3	3	3	2	2
	10	8	3	3	3	3	2	2
	11	9	3	3	3	3	2	2
	12	10	3	3	3	3	2	2

TABLE 3. Number of groups required for the proposed plan for the Pareto distribution of the 2nd kind with $\lambda = 2$, using Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	2	2	2	2	2	2
	8	6	2	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.10	7	5	3	2	2	2	2	2
	8	6	3	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.05	7	5	3	3	2	2	2	2
	8	6	3	3	2	2	2	2
	9	7	3	3	2	2	2	2
	10	8	3	3	2	2	2	2
	11	9	3	3	2	2	2	2
	12	10	3	3	2	2	2	2
0.01	7	5	3	3	3	3	3	3
	8	6	3	3	3	3	3	3
	9	7	3	3	3	3	3	3
	10	8	3	3	3	3	3	2
	11	9	3	3	3	3	3	2
	12	10	3	3	3	3	2	2

TABLE 4. Number of groups required for the proposed plan for the Pareto distribution of the 2nd kind with λ = 3, using Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	2	2	2	2	2	2
	8	6	2	2	2	2	2	2
	9	7	2	2	2	2	2	2
	10	8	2	2	2	2	2	2
	11	9	2	2	2	2	2	2
	12	10	2	2	2	2	2	2
0.10	7	5	3	3	2	2	2	2
	8	6	3	3	2	2	2	2
	9	7	3	3	2	2	2	2
	10	8	3	3	2	2	2	2
	11	9	3	3	2	2	2	2
	12	10	3	3	2	2	2	2
0.05	7	5	3	3	3	2	2	2
	8	6	3	3	3	2	2	2
	9	7	3	3	3	2	2	2
	10	8	3	3	3	2	2	2
	11	9	3	3	3	2	2	2
	12	10	3	3	3	2	2	2
0.01	7	5	4	3	3	3	3	3
	8	6	4	3	3	3	3	3
	9	7	3	3	3	3	3	3
	10	8	3	3	3	3	3	2
	11	9	3	3	3	3	3	2
	12	10	3	3	3	3	3	2

TABLE 5. Operating characteristics values of the group sampling plan with c =7, r = 9 for Pareto distribution of the 2nd kind with λ= 2, using Weighted Poisson distribution

β	g	a	2	4	6	8	10	12
0.25	2	0.7	0.2985	0.7677	0.9296	0.9764	0.9911	0.9963
	2	0.8	0.2240	0.6860	0.8908	0.9598	0.9838	0.9929
	2	1.0	0.1301	0.5297	0.7942	0.9112	0.9598	0.9807
	2	1.2	0.0799	0.3991	0.6860	0.8450	0.9225	0.9598
	2	1.5	0.0428	0.2584	0.5297	0.7270	0.8450	0.9112
	2	2.0	0.0193	0.1301	0.3289	0.5297	0.6860	0.7942
0.10	2	0.7	0.2985	0.7677	0.9296	0.9764	0.9911	0.9963
	2	0.8	0.2240	0.6860	0.8908	0.9598	0.9838	0.9929
	2	1.0	0.1301	0.5297	0.7942	0.9112	0.9598	0.9807
	2	1.2	0.0799	0.3991	0.6860	0.8450	0.9225	0.9598
	2	1.5	0.0428	0.2584	0.5297	0.7270	0.8450	0.9112
	2	2.0	0.0193	0.1301	0.3289	0.5297	0.6860	0.7942
0.05	2	0.7	0.2985	0.7677	0.9296	0.9764	0.9911	0.9963
	2	0.8	0.2240	0.6860	0.8908	0.9598	0.9838	0.9929
	2	1.0	0.1301	0.5297	0.7942	0.9112	0.9598	0.9807
	2	1.2	0.0799	0.3991	0.6860	0.8450	0.9225	0.9598
	2	1.5	0.0428	0.2584	0.5297	0.7270	0.8450	0.9112
	2	2.0	0.0193	0.1301	0.3289	0.5297	0.6860	0.7942
0.01	3	0.7	0.0413	0.3859	0.7099	0.8709	0.9409	0.9715
	3	0.8	0.0227	0.2838	0.6096	0.8065	0.9042	0.9510
	3	1.0	0.0076	0.1488	0.4257	0.6597	0.8065	0.8899
	3	1.2	0.0030	0.0776	0.2838	0.5133	0.6898	0.8065
	2	1.5	0.0428	0.2584	0.5297	0.7270	0.8450	0.9112
	2	2.0	0.0193	0.1301	0.3289	0.5297	0.6860	0.7942

TABLE 6. Operating characteristics values of the group sampling plan with c = 7, r = 9 for Pareto distribution of the 2nd kind with λ = 3, using Weighted Poisson distribution

β	g	a	2	4	6	8	10	12
0.25	2	0.7	0.4641	0.8888	0.9750	0.9930	0.9970	0.9991
	2	0.8	0.3669	0.8341	0.9573	0.9871	0.9955	0.9982
	2	1.0	0.2272	0.7082	0.9051	0.9668	0.9871	0.9945
	2	1.2	0.1428	0.5799	0.8341	0.9339	0.9719	0.9871
	2	1.5	0.0755	0.4130	0.7082	0.8625	0.9339	0.9668
	2	2.0	0.0314	0.2272	0.5008	0.7082	0.8341	0.9051
0.10	2	0.7	0.4641	0.8888	0.9750	0.9930	0.9970	0.9991
	2	0.8	0.3669	0.8341	0.9573	0.9871	0.9955	0.9982
	2	1.0	0.2272	0.7082	0.9051	0.9668	0.9871	0.9945
	2	1.2	0.1428	0.5799	0.8341	0.9339	0.9719	0.9871
	2	1.5	0.0755	0.4130	0.7082	0.8625	0.9339	0.9668
	2	2.0	0.0314	0.2272	0.5008	0.7082	0.8341	0.9051
0.05	3	0.7	0.1092	0.6052	0.8648	0.9520	0.9813	0.9920
	3	0.8	0.0645	0.4931	0.7974	0.9203	0.9666	0.9850
	2	1.0	0.2272	0.7082	0.9051	0.9668	0.9871	0.9945
	2	1.2	0.1428	0.5799	0.8341	0.9339	0.9719	0.9871
	2	1.5	0.0755	0.4130	0.7082	0.8625	0.9339	0.9668
	2	2.0	0.0314	0.2272	0.5008	0.7082	0.8341	0.9051
0.01	3	0.7	0.1092	0.6052	0.8648	0.9520	0.9813	0.9920
	3	0.8	0.0645	0.4931	0.7974	0.9203	0.9666	0.9850
	3	1.0	0.0233	0.3089	0.6442	0.8323	0.9203	0.9605
	3	1.2	0.0092	0.1852	0.4931	0.7223	0.8522	0.9203
	2	1.5	0.0755	0.4130	0.7082	0.8625	0.9339	0.9668
	2	2.0	0.0314	0.2272	0.5008	0.7082	0.8341	0.9051

TABLE 7. Operating characteristics values of the group sampling plan with $c = 7$, $r = 9$ for Pareto distribution of the 2nd kind with $\lambda = 2$, using Poisson distribution

β	g	a	2	4	6	8	10	12
0.25	2	0.7	0.4359	0.8705	0.9707	0.9922	0.9976	0.9991
	2	0.8	0.3458	0.8095	0.9497	0.9852	0.9950	0.9981
	2	1.0	0.2202	0.6757	0.8890	0.9610	0.9852	0.9939
	2	1.2	0.1452	0.5468	0.8095	0.9222	0.9670	0.9852
	2	1.5	0.0843	0.3883	0.6757	0.8409	0.9222	0.9610
	2	2.0	0.0415	0.2202	0.4705	0.6757	0.8095	0.8890
0.10	2	0.7	0.4359	0.8705	0.9707	0.9922	0.9976	0.9991
	2	0.8	0.3458	0.8095	0.9497	0.9852	0.9950	0.9981
	2	1.0	0.2202	0.6757	0.8890	0.9610	0.9852	0.9939
	2	1.2	0.1452	0.5468	0.8095	0.9222	0.9670	0.9852
	2	1.5	0.0843	0.3883	0.6757	0.8409	0.9222	0.9610
	2	2.0	0.0415	0.2202	0.4705	0.6757	0.8095	0.8890
0.05	3	0.7	0.0817	0.5329	0.8280	0.9381	0.9763	0.9902
	3	0.8	0.0480	0.4186	0.7468	0.8973	0.9572	0.9812
	2	1.0	0.2202	0.6757	0.8890	0.9610	0.9852	0.9939
	2	1.2	0.1452	0.5468	0.8095	0.9222	0.9670	0.9852
	2	1.5	0.0843	0.3883	0.6757	0.8409	0.9222	0.9610
	2	2.0	0.0415	0.2202	0.4705	0.6757	0.8095	0.8890
0.01	3	0.7	0.0817	0.5329	0.8280	0.9381	0.9763	0.9902
	3	0.8	0.0480	0.4186	0.7468	0.8973	0.9572	0.9812
	3	1.0	0.0180	0.2465	0.5744	0.7885	0.8973	0.9492
	3	1.2	0.0076	0.1417	0.4186	0.6604	0.8125	0.8973
	3	1.5	0.0026	0.0625	0.2465	0.4736	0.6604	0.7885
	3	2.0	0.0007	0.0180	0.0980	0.2465	0.4186	0.5744

TABLE 8. Operating characteristics values of the group sampling plan with c =7, $r = 9$ for Pareto distribution of the 2nd kind with $\lambda = 3$, using Poisson distribution

β	g	a	2	4	6	8	10	12
0.25	2	0.7	0.6130	0.9486	0.9916	0.9982	0.9995	0.9998
	2	0.8	0.5125	0.9153	0.9840	0.9962	0.9989	0.9996
	2	1.0	0.3498	0.8267	0.9577	0.9882	0.9962	0.9986
	2	1.2	0.2381	0.7210	0.9153	0.9728	0.9904	0.9962
	2	1.5	0.1384	0.5614	0.8267	0.9331	0.9728	0.9882
	2	2.0	0.0642	0.3498	0.6485	0.8267	0.9153	0.9577
0.10	3	0.7	0.1898	0.7430	0.9345	0.9816	0.9941	0.9978
	3	0.8	0.1206	0.6412	0.8912	0.9658	0.9882	0.9955
	2	1.0	0.3498	0.8267	0.9577	0.9882	0.9962	0.9986
	2	1.2	0.2381	0.7210	0.9153	0.9728	0.9904	0.9962
	2	1.5	0.1384	0.5614	0.8267	0.9331	0.9728	0.9882
	2	2.0	0.0642	0.3498	0.6485	0.8267	0.9153	0.9577
0.05	3	0.7	0.1898	0.7430	0.9345	0.9816	0.9941	0.9978
	3	0.8	0.1206	0.6412	0.8912	0.9658	0.9882	0.9955
	3	1.0	0.0493	0.4478	0.7758	0.9142	0.9658	0.9855
	2	1.2	0.2381	0.7210	0.9153	0.9728	0.9904	0.9962
	2	1.5	0.1384	0.5614	0.8267	0.9331	0.9728	0.9882
	2	2.0	0.0642	0.3498	0.6485	0.8267	0.9153	0.9577
0.01	3	0.7	0.1898	0.7430	0.9345	0.9816	0.9941	0.9978
	3	0.8	0.1206	0.6412	0.8912	0.9658	0.9882	0.9955
	3	1.0	0.0493	0.4478	0.7758	0.9142	0.9658	0.9855
	3	1.2	0.0212	0.2958	0.6412	0.8374	0.9267	0.9658
	3	1.5	0.0069	0.1514	0.4478	0.6924	0.8374	0.9142
	3	2.0	0.0015	0.0493	0.2205	0.4478	0.6412	0.7758

TABLE 9. Minimum ratio of true average life to specified life for the producer's risk of 0.05, for the Pareto distribution of the 2nd kind with λ = 2, using Weighted Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	8.9	10.1	12.7	15.2	18.9	25.4
	8	6	7.5	8.6	10.7	12.9	16.1	21.4
	9	7	6.6	7.5	9.4	11.3	14.1	18.9
	10	8	5.9	6.8	8.5	10.2	12.8	17.0
	11	9	5.5	6.2	7.8	9.4	11.7	15.7
	12	10	5.1	5.8	7.3	8.7	10.9	14.6
0.10	7	5	8.9	10.1	12.7	15.2	18.9	25.4
	8	6	7.5	8.6	10.7	12.9	16.1	21.4
	9	7	6.6	7.5	9.4	11.3	14.1	18.9
	10	8	5.9	6.8	8.5	10.2	12.8	17.0
	11	9	5.5	6.2	7.8	9.4	11.7	15.7
	12	10	5.1	5.8	7.3	8.7	10.9	14.6
0.05	7	5	8.9	10.1	12.7	15.2	18.9	25.4
	8	6	7.5	8.6	10.7	12.9	16.1	21.4
	9	7	6.6	7.5	9.4	11.3	14.1	18.9
	10	8	5.9	6.8	8.5	10.2	12.8	17.0
	11	9	5.5	6.2	7.8	9.4	11.7	15.7
	12	10	5.1	5.8	7.3	8.7	10.9	14.6
0.01	7	5	13.9	15.8	19.8	23.7	18.9	25.4
	8	6	11.8	13.5	20.2	25.3	16.1	21.4
	9	7	10.4	11.9	14.9	17.9	14.1	18.9
	10	8	9.5	10.8	13.5	16.3	12.8	17.0
	11	9	8.7	10.0	12.5	9.4	11.7	15.7
	12	10	8.2	9.4	11.7	8.7	10.9	14.6

TABLE 10. Minimum ratio of true average life to specified life for the producer's risk of 0.05, for the Pareto distribution of the 2nd kind with λ = 3, using Weighted Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	6.7	7.7	9.6	11.6	14.5	19.3
	8	6	5.7	6.5	8.1	9.8	12.2	16.3
	9	7	5.0	5.7	7.2	8.6	10.8	14.4
	10	8	4.5	5.2	6.5	7.8	9.7	13.0
	11	9	4.2	4.8	6.0	7.2	9.0	12.0
	12	10	3.9	4.4	5.6	6.7	8.4	11.2
0.10	7	5	6.7	7.7	9.6	11.6	14.5	19.3
	8	6	5.7	6.5	8.1	9.8	12.2	16.3
	9	7	5.0	5.7	7.2	8.6	10.8	14.4
	10	8	4.5	5.2	6.5	7.8	9.7	13.0
	11	9	4.2	4.8	6.0	7.2	9.0	12.0
	12	10	3.9	4.4	5.6	6.7	8.4	11.2
0.05	7	5	10.5	7.7	9.6	11.6	14.5	19.3
	8	6	8.9	10.2	8.1	9.8	12.2	16.3
	9	7	7.9	9.0	7.2	8.6	10.8	14.4
	10	8	7.2	8.2	6.5	7.8	9.7	13.0
	11	9	6.6	7.6	6.0	7.2	9.0	12.0
	12	10	6.2	7.1	5.6	6.7	8.4	11.2
0.01	7	5	10.5	12.0	15.0	18.0	22.4	19.3
	8	6	8.9	10.2	12.8	15.3	12.2	16.3
	9	7	7.9	9.0	11.3	13.6	10.8	14.4
	10	8	7.2	8.2	10.3	12.3	9.7	13.0
	11	9	6.6	7.6	9.5	11.4	9.0	12.0
	12	10	6.2	7.1	10.7	13.4	8.4	11.2

TABLE 11. Minimum ratio of true average life to specified life for the producer's risk of 0.05, for the Pareto distribution of the 2nd kind with λ = 2, using Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	6.4	7.3	9.2	11.0	13.8	18.4
	8	6	5.7	6.5	8.2	9.8	12.3	16.4
	9	7	5.2	6.0	7.5	9.0	11.2	15.0
	10	8	4.9	5.6	7.0	8.4	10.5	14.0
	11	9	4.6	5.2	6.5	7.9	9.8	13.1
	12	10	4.3	5.0	6.2	7.5	9.3	12.5
0.10	7	5	10.2	7.3	9.2	11.0	13.8	18.4
	8	6	9.1	6.5	8.2	9.8	12.3	16.4
	9	7	5.2	6.0	7.5	9.0	11.2	15.0
	10	8	4.9	5.6	7.0	8.4	10.5	14.0
	11	9	4.6	5.2	6.5	7.9	9.8	13.1
	12	10	4.3	5.0	6.2	7.5	9.3	12.5
0.05	7	5	10.2	11.6	9.2	11.0	13.8	18.4
	8	6	9.1	10.5	8.2	9.8	12.3	16.4
	9	7	8.4	9.6	7.5	9.0	11.2	15.0
	10	8	7.8	9.0	7.0	8.4	10.5	14.0
	11	9	7.4	8.5	6.5	7.9	9.8	13.1
	12	10	7.1	8.1	6.2	7.5	9.3	12.5
0.01	7	5	10.2	11.6	14.5	17.5	21.8	29.1
	8	6	9.1	10.5	13.1	15.7	19.6	26.2
	9	7	8.4	9.6	12.0	14.4	18.1	24.1
	10	8	7.8	9.0	11.2	13.5	16.9	14.0
	11	9	7.4	8.5	10.6	12.7	15.9	13.1
	12	10	7.1	8.1	10.1	12.2	9.3	12.5

TABLE 12. Minimum ratio of true average life to specified life for the producer's risk of 0.05, for the Pareto distribution of the 2nd kind with λ = 3, using Poisson distribution

			a					
β	*r*	*c*	0.7	0.8	1.0	1.2	1.5	2.0
0.25	7	5	4.9	5.6	7.0	8.4	10.5	14.0
	8	6	4.4	5.0	6.2	7.5	9.4	12.5
	9	7	4.0	4.6	5.7	6.9	8.6	11.5
	10	8	3.7	4.2	5.3	6.4	8.0	10.7
	11	9	3.5	4.0	5.0	6.0	7.5	10.1
	12	10	3.3	3.8	4.8	5.7	7.2	9.6
0.10	7	5	7.7	8.8	7.0	8.4	10.5	14.0
	8	6	6.9	7.9	6.2	7.5	9.4	12.5
	9	7	6.4	7.3	5.7	6.9	8.6	11.5
	10	8	6.0	6.8	5.3	6.4	8.0	10.7
	11	9	5.6	6.4	5.0	6.0	7.5	10.1
	12	10	5.4	6.1	4.8	5.7	7.2	9.6
0.05	7	5	7.7	8.8	11.0	8.4	10.5	14.0
	8	6	6.9	7.9	9.9	7.5	9.4	12.5
	9	7	6.4	7.3	9.1	6.9	8.6	11.5
	10	8	6.0	6.8	8.5	6.4	8.0	10.7
	11	9	5.6	6.4	8.1	6.0	7.5	10.1
	12	10	5.4	6.1	7.7	5.7	7.2	9.6
0.01	7	5	10.5	8.8	11.0	13.2	16.6	22.1
	8	6	9.5	7.9	9.9	11.9	14.9	19.9
	9	7	6.4	7.3	9.1	11.0	13.7	18.3
	10	8	6.0	6.8	8.5	10.3	12.8	10.7
	11	9	5.6	6.4	8.1	9.7	12.1	10.1
	12	10	5.4	6.1	7.7	9.2	11.6	9.6

TABLE 13. Comparisons of sample size when shape parameter λ = 2, *c* =8 and using Weighted Poisson distribution

β	*a*	Existing plan (Aslam et al. 2011)	Proposed plan
0.01	0.7	120	30
	0.8	100	30
	1.0	80	30
	1.2	60	30
	1.5	60	20
	2.0	50	20

TABLE 14. Comparisons of Sample Size '*n*' when shape parameter λ = 3, *c* =8 and using Weighted Poisson distribution

β	*a*	Existing plan (Aslam et al. 2011)	Proposed plan
0.01	0.7	170	30
	0.8	130	30
	1.0	90	30
	1.2	70	30
	1.5	60	20
	2.0	50	20

TABLE 15. Comparisons of Sample Size '*n*' when shape parameter λ = 2, ***c*** = 8 and using Poisson distribution

β	*a*	Existing plan (Aslam et al. 2011)	Proposed plan
0.01	0.7	200	30
	0.8	160	30
	1.0	120	30
	1.2	100	30
	1.5	80	30
	2.0	70	20

TABLE 16. Comparisons of sample size '*n*' when shape parameter λ = 3, *c* =8 and using Poisson distribution

β	*a*	Existing plan (Aslam et al. 2011)	Proposed plan
0.01	0.7	300	30
	0.8	220	30
	1.0	150	30
	1.2	110	30
	1.5	90	30
	2.0	70	20

Consider that the experimenter wants to test the quality of electrical cart for 10000 h. Furthermore, in the laboratory, one has to have the facility to install more than one electrical cart on a tester. If the experimenter would like to choose the acceptance number $c = 8$, number of tester $r = 10$, test termination time $a = 0.70$, consumer's risk $\beta = 0.10$ and $\lambda = 3$, then the required minimum group size $g = 3$ from Table 4. The designed parameters of the proposed GASP are $(g, r, c, a) = (3, 10, 8, 0.70)$. So, the practitioner needs to choose a random sample of size 30 items from the lot and put ten items to three groups on the life testing experiment. The submitted lot is not accepted if more than eight failures occurred in 10000 h, otherwise accepted.

COMPARISON

In this section, we explained the comparative study regarding sample sizes from the proposed plan with the existing plan. As discussed the designed parameters of the proposed and existing GASP are $(g, r, c, a) = (3,10,8,0.70)$ and $(g, r, c, a) = (30,10,8,0.70)$, respectively. From Table 16, the proposed plan need 30 ($n = r \times g$) items and existing plan require 300 ($n = r \times g$) items, respectively, to achieve a similar inference concerning the submitted lot. Therefore, the number of comparison is also shown in Tables 13 to 16. The same problem can also be considered for various lifetime distributions on pre-specified designed parameters to inspect the average life of a submitted item.

CONCLUSION

In this research analysis, tables are disposed for time truncated efficient testing strategy for Pareto distribution of the 2nd kind using Weighted Poisson and Poisson distribution. The Weighted Poisson and Poisson distributions are considered to locate the optimal values group size, mean ratio and probability of lot acceptance. A comparison between the proposed plan and existing plan is displayed for true analysis. The consequence is the proposed technique provides the very smaller sample size as compared to the established plan in literature. Hence, this present research analysis is more beneficial to save cost, time, energy, labor and quick inspection of the submitted items by the vendor. The proposed acceptance sampling plan can be used to test the lifetime of many electronic components such as the mobile phones, transportation electronics system, wireless devices and global positioning system.

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