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The Option Value of Patent Litigation: Theory and Evidence

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December 1, 2003

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Abstract

In this paper I present a real options model of patent litigation when patents are not perfectly enforceable. I consider both finite horizon and infinite horizon models. The theoretical results demonstrate that patent value depends not only on the underlying technology, but also on the degree of uncertainty over the property right. Additionally, uncertain property rights create an effective patent term that is less than the statutory term. Using simulation methods and patent data, I estimate the hazard rate of patent litigation. I find that, contrary to previous results, the most valuable patents are not the primary candidates for litigation.

JEL: K41, O32.

Keywords: patents, litigation, real options, hazard rate

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1 Introduction

Patents are classic examples of real options: a patent holder has the option to develop certain types of products, or to license the technology, or to use it as an input for further research (Dixit and Pindyck 1994). In an early renewal study Pakes (1986) models the patent as an option to renew in patent regimes where renewal fees are required. However, less work has been done modeling the enforceability decision in patents.

Several theoretical papers examine patenting decisions using a real options approach. Reiss (1998) models the decisions of firms to develop and/or patent an innovation when competition arrives stochastically. The paper explicitly examines the trade-off between patents and trade-secrets as modes of protection, and introduces some uncertainty in the patent right, in that competition may develop a substitute technology that does not infringe the patent. Takalo and Kanninen (2000) and Weeds (2002) develop models investigating the patenting decision. Their models find that costly patenting may delay the development of an innovation, even in the face of competition. Neither paper explicitly models the patent enforcement process; however, Takalo and Kanninen (2000) acknowledge that there are enforcement costs (because of infringers), and they incorporate them into the general cost of patenting.

Bloom and VanReenen (2002) adapt empirical patent valuation models (Pakes 1985, Pakes 1986) to explain the timing of the development of patented innovations, and the value of innovating firms. Since the technology is subject to uncertainty, and since development expenditure is irreversible, patents are real options. Bloom and VanReenen (2002) model the value of a firm as the sum of the value of its developed patents and its options to develop, where the profit from patents (whether developed or not) evolves stochastically. They find that higher market uncertainty raises the value of patent options, but decreases the impact of patents on productivity. The model of Bloom and VanReenen (2002), as well as most

models in the theoretical literature, treat the patent holder as having exclusive rights to develop a technology.

However, any option on the patented technology presupposes an enforceable property right. The fundamental value of a patent right is the right to exclude others from using the technology. Since enforcement is imperfect and costly, the right to exclude becomes the right to sue with some probability of success. In patents, both the scope of the technology and the validity of the patent can come into question (Schweizer 1989, Meurer 1989, Llobet 2000, Marco 2000). Empirical work in patent litigation shows that more valuable patents are more likely to be enforced through lawsuits (Lanjouw 1998, Lanjouw and Lerner 1998, Harhoff, Scherer and Vopel 1999, Lanjouw and Schankerman 2001), and the imperfect enforceability of patents affects patenting behavior (Lerner 1995, Hall and Ziedonis 2001) and industry structure (Marco and Rausser 2002).

Some research has been done exploring the option value of litigation in breach of contract. Cornell (1990) investigates the use of option pricing in estimating damages. However, his model does not explain the incidence of litigation.

In this paper, I treat the patent as an option to bring a lawsuit against an alleged infringer. Just like financial options, the option to sue need not be exercised in order for it to have value. Thus, the value of a patent is a function of the enforceability of the property right.

In the analysis below, I present a simple real options model where there is uncertainty over the validity of a patent. Section (2) presents the basic framework of patents as options. Section (3) discusses the results for infinitely-lived patents and Section (4) examines finitely-lived patents. I characterize the value functions and exercise boundaries for both types of patents. While infinitely-lived patents are unrealistic,¹ they are useful from a theoretical perspective because the infinite time horizon enables an analytic solution, which lends itself better to comparative statics. Additionally, if the time horizon is finite but long, then the difference between the infinite and finite cases will be small at the beginning of the patent's life. To understand how patent value and the litigation decision change over time,

it is important to develop a solution for the finite horizon case, which involves numerical methods.

In Section (5) I investigate the incidence of patent litigation, and Section (6) uses numerical methods and simulations to develop some testable implications about the timing and incidence of patent litigation. In Section (7), I compare simulated data to patent litigation data, and I find that while litigated patents tend to be valuable according to standard measures, they are not the *most* valuable patents. Section (8) concludes with some policy implications.

2 Model

In the US, patents are not born with certain validity when issued by the Patent and Trademark Office (PTO). In enforcing a patent right against alleged infringers, it is quite common to encounter a “validity defense” or a validity countersuit, the success of which means invalidity for the patent right. That is, patent enforcement is risky since the patent holder may lose its entire patent. Thus, the opportunity cost can be steep. When these opportunity costs are high, higher levels of infringement will be accommodated before the option to sue is exercised. In order to formally model this decision, I begin with a very simple framework where beliefs about patent validity evolve stochastically.

Suppose that x is the current income stream resulting from a patent right. This income can be explicit in the form of royalty revenue, or implicit in the form of increased revenue from the ability to exclude others from the market. Let p be the patent holder’s belief about the probability that a court will uphold the patent as valid. If there is widespread belief that the patent holder will not enforce its property right, x will be small or non-existent (either because bids for licenses will be small, or because infringement is widespread).

Under these assumptions, the patent can be thought of as a portfolio consisting of two assets: (1) an asset that pays a stochastic profit flow x , and (2) an option to go to court. The option to go to court is a put option: the patent holder has the right to sell the current profit flow in return for the court imposed outcome. Since the patent holder currently owns

the asset on which it owns the put, it is in a covered position. One can think of the court outcome as being a money payoff (damages, or a “reasonable royalty” determined by the court), or as another portfolio consisting of (1) a new profit flow, and (2) a new option to sue, both of which will reflect updated beliefs about the patent.

The strike price of a put option is a specified price that the firm obtains from electing to sell a share of the underlying asset. Here the underlying asset is the current profit flow, and the price that the firm obtains is whatever it obtains from the court ruling. Hence, the strike price is expected damages less litigation costs. The option to sue has finite life, as does the underlying asset, and the option can be exercised anytime during its life. Thus, the option to sue is akin to an American put option on a bond. However, in Section (3), I discuss the extreme case of infinitely-lived patents.

An alternative but equivalent interpretation of the enforcement problem is one of dynamic programming. In particular, we can view the litigation decision as an optimal stopping problem (Dixit and Pindyck 1994). The state variable x is the royalty income from the patent, or the patent holder’s own benefit from using the patent. If x becomes very low due to of infringement, then the patent holder will invest in litigation in order to stop the infringement. Since the firm can either attempt to stop the infringement now, or defer the decision, it can be viewed as a dynamic programming problem.

In the formal model, I aggregate the infringement and validity issues into a single matter for the court. The outcome at trial is winner-take-all, and the patent holder is risk neutral. In addition, I assume:

1. There is a profit flow z that represents the profit that would accrue to the firm were the patent known by all parties to be valid with certainty. z is common knowledge and can be thought of as the profit flow under the most optimistic circumstances. Since z is constant, there is no commercial uncertainty. Though I allow for no uncertainty over scope, z will be dependent upon the scope defined by the patent office. The larger the scope, the larger is z (given the same underlying technological value).² In this way z becomes a policy variable.

2. Issued patents are of two types: valid and not valid. The patent holder believes that its patent is valid with probability p .
3. By paying litigation costs c , the patent holder can go to court to obtain an irreversible and perfectly enforceable decision on validity.
4. The actual profit flow, x , follows geometric Brownian motion with no drift, such that

$$dx = \sigma x dw \tag{1}$$

where dw is the increment of a Wiener process. The profit flow follows a stochastic process (from the perspective of the patent holder) as the beliefs of users vary. The beliefs vary because of the entry and exit of users, or because there is uncertainty as to whether the patent applies to a new product group.

5. The patent is infinitely-lived, or has a life of T years.

3 Infinite horizon

In the case of infinitely-lived patents, one can obtain an explicit analytical solution. To do so, examine the outcome of going to court: at any time t , the patent holder may go to court to obtain a decision about validity. By doing so, it receives a terminal payoff of

$$\Omega = \int_t^\infty pze^{-rs} ds - c = \frac{pz}{r} - c \tag{2}$$

in expectation. The terminal payoff from the perspective of time t is the present value (net of litigation cost, and discounted at rate r) of receiving the profit flow z with probability p , and losing the patent with probability $(1 - p)$. Since the court's decision is perfectly enforceable, the patent holder knows that a validity ruling will lead to a profit flow of z and an invalidity ruling will lead to a profit flow of 0.

Note that there are no damages for past infringement in the model. This is because

the trial is instantaneous. In reality, damages can be assessed retroactively so long as the patent holder was not aware of the infringement. The doctrine of *laches* prevents a patent holder from obtaining damages for a period of time during which it knew of the defendant's actions yet did nothing to stop them. If a patent holder seeks payment from a technology user, it typically sends notice making the user aware of the alleged infringement and asking for compensation. Frequently the date of this notice is used to define the period after which damages can be assessed if the dispute goes to court. In the model, the date of notice and the date of the court's decision are the same, so there are no damages; the value of the terminal payoff is in the injunctive right which allows it to prevent further infringement.³

To determine the value of the patent we must solve simultaneously for the value function and for the patent holder's decision rule. The value function can be expressed generally as

$$V(x) = \max \left\{ \Omega, xdt + e^{-r dt} E[V(x + dx)] \right\}. \quad (3)$$

Equation (3) is the Bellman equation for the dynamic programming problem and states that the patent value will be equal to the maximum of the termination value (if the option is exercised) and the continuation value. The function V assumes that the patent holder will make the current decision optimally, and will also make future decisions optimally.

The value function can be interpreted as follows: over a short period of time dt , the patent holder has the choice to litigate or not litigate. If it litigates it pays c and receives the expected terminal payoff Ω . Thus the left-hand term in the maximand is the termination value of the optimal stopping problem, or, alternatively, the strike price of an American put option. If the patent holder does not litigate, it receives the current income, xdt , plus the discounted expected value of the patent, where x will change by dx over time period dt .

Importantly, the right-hand term in the maximand is greater than current income by an amount equal to the option value. The option puts a floor on the level to which x can fall before the patent holder sues. Owning the option gives the patent holder some insurance and protects it against a small x .

Note that time is not explicitly included in the value function: the infinite horizon assumption means that the optimal decision at time t given profit flow \tilde{x} is the same as the optimal decision at time s given profit flow \tilde{x} . Since the optimal decision rule does not change over time, and all flows are discounted to the current time period, time does not explicitly enter the value function.

The solution involves solving for the value function, $V(x)$, and for the optimal decision rule, \bar{x} , below which the patent holder will sue. \bar{x} reflects the critical value, or exercise boundary, between the continuation region of x (where the patent holder does not sue) and the stopping region.

Define the patent value in the continuation region as

$$G(x) = xdt + e^{-rdt} E[G(x + dx)] \quad (4)$$

with the usual boundary conditions given by value matching and smooth-pasting:

$$G(\bar{x}) = \Omega = \frac{pz}{r} - c \quad (5)$$

$$\frac{\partial}{\partial x} G(\bar{x}) = \frac{\partial \Omega}{\partial x} = 0. \quad (6)$$

Since Ω is not a function of x , the derivatives of the continuation and stopping regions must be zero at \bar{x} .

Using that fact that for small dt , $e^{-rdt} \simeq 1 - rdt$, and $E[G(x + dx)] = G(x) + dG(x)$,

$$G = xdt + (1 - rdt)G + E[dG], \quad (7)$$

where the x has been dropped for simplicity.

Ito's Lemma then yields

$$dG = G'dx + \frac{1}{2}G''(dx)^2. \quad (8)$$

Using the facts that $(dx)^2 = \sigma^2 x^2 (dw)^2$ and $(dw)^2 = dt$ and $E(dx) = 0$, we can write $E[dG]$

as

$$E[dG] = \frac{1}{2}\sigma^2 x^2 G'' dt. \quad (9)$$

From Equations (7) and (9) we obtain a homogeneous differential equation in x :⁴

$$rG = x + \frac{1}{2}\sigma^2 x^2 G''. \quad (10)$$

The solution to Equation (10) is of the form

$$G(x) = \frac{1}{r}x + K_1 x^{\frac{1}{2} \frac{\sigma^2 - \sqrt{(\sigma^2(\sigma^2 + 8r))}}{\sigma^2}} + K_2 x^{\frac{1}{2} \frac{\sigma^2 + \sqrt{(\sigma^2(\sigma^2 + 8r))}}{\sigma^2}}. \quad (11)$$

From an economic standpoint, we know that $K_2 = 0$ because as x gets large, the option value of the patent ($G(x) - \frac{x}{r}$) should go to zero. Therefore the exponent on x should be negative. K_1 is determined by the boundary conditions, Equations (5) and (6).

Solving for \bar{x} and K from the two conditions yields:

$$\bar{x} = (pz - cr) \frac{\gamma - \sigma}{\gamma + \sigma} \quad (12)$$

and

$$K = \frac{\frac{2\sigma}{r}(pz - cr)}{\sigma + \gamma} \left((pz - cr) \frac{\gamma - \sigma}{\gamma + \sigma} \right)^{-\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} \quad (13)$$

where $\gamma = \sqrt{\sigma^2 + 8r} > \sigma$. We can now express the value of the patent in the continuation region as

$$G(x) = \frac{x}{r} + \frac{\frac{2\sigma}{r}(pz - cr)}{\sigma + \gamma} \left((pz - cr) \frac{\gamma - \sigma}{\gamma + \sigma} \right)^{-\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} x^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}}$$

or alternatively as

$$G(x) = \frac{x}{r} + \frac{2\sigma}{r} \frac{(pz - cr)}{(\sigma + \gamma)} \left(\frac{x}{\bar{x}} \right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}}. \quad (14)$$

The entire value function can then be written as

$$V(x) = \begin{cases} \frac{pz}{r} - c & \text{if } x < \bar{x} \\ \frac{x}{r} + \frac{2\sigma}{r} \frac{(pz - cr)}{\sigma + \gamma} \left(\frac{x}{\bar{x}} \right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} & \text{if } \bar{x} \leq x \end{cases} \quad (15)$$

Table (1) shows the effects of the model parameters on the value function and the critical value. A more detailed analysis of these effects is given in Appendices (1) and (2). One important implication of the model, as far as policy implications are concerned, is that explicitly recognition should be given to the role of enforceability in patent rights. Much of the policy debate turns on issues of length and breadth. However, as Table (1) shows, the probability of validity and variance of beliefs both affect patent value. Rational public policy should take these factors into account. p and z enter the model symmetrically, and have the same impact on the value function and critical value. Litigation costs have the opposite effect: higher values serve to diminish value to patent holders.

In exploring the value function, it is instructive to view the results graphically. Figures (1) and (2) show the solution for $V(x)$ for the particular numerical values given in Table (2). V exhibits the usual convex shape in x . Additionally, both the probability of validity (p) and the level of uncertainty (σ) increase the patent value. The probability of validity tends to have a larger impact when litigation is likely (low values of x), and σ has a larger impact when x is high, because for a given x , the likelihood of needing to exercise the option is more likely when σ is high. The first and second derivatives of \bar{x} in Appendix (1) show that the exercise boundary is decreasing and concave in σ .

However, the dynamics of the exercise boundary are much more interesting in the finite case. Additionally, we will need to solve the finite horizon model in order to examine the litigation rate.

4 Finite horizon

To account for finite patent lengths, time will enter the value function explicitly. With finite patent length T , the value of the property right decreases over time, so that $V(x, T) = 0$. This assumption does not imply that the value of the patented technology is zero, only that the ability to appropriate that technology using a patent right is zero. Other forms of appropriation, like first-mover advantage, may still be effective.

The expected terminal payoff from litigating at date t can now be written as

$$\Omega(t) = \int_0^{T-t} pze^{-rs} ds - c = pz \frac{1 - e^{r(t-T)}}{r} - c. \quad (16)$$

The terminal payoff in Equation (16) is very similar to that in Equation (2) except that it depends on the age of the patent, t . When t is near zero, $\Omega(t)$ will be almost the same as Ω (from the infinite time horizon model), since the expiration date is far away. However as $t \rightarrow T$, $\Omega(t) \rightarrow 0$. In contrast to the infinite horizon case, the critical value \bar{x} will also now be a function of t because the payoff to litigating is changing with t .

The finite horizon version of the Bellman equation is

$$V(x, t) = \max \left\{ \Omega(t), xdt + e^{-rdt} E [V(x + dx, t + dt)] \right\}. \quad (17)$$

I will again define G as the value of the patent in the continuation region:

$$G(x, t) = xdt + (1 - rdt) G + E [dG] \quad (18)$$

which is the same as Equation (7), except that G is now explicitly dependent upon time.

Ito's Lemma yields

$$E [dG] = G_t dt + \frac{1}{2} \sigma^2 x^2 G_{xx} dt. \quad (19)$$

And, from Equations (18) and (19):

$$rG = x + G_t + \frac{1}{2} \sigma^2 x^2 G_{xx} \quad (20)$$

which is a partial differential equation in the two state variables x and t . The solution is subject to the finite time version of the boundary conditions, plus a third condition that reflects the fact that the asset value must be zero at the end of its life. It is the third condition that will allow us to use a recursive technique to obtain particular numeric

solutions.

$$G(\bar{x}, t) = \Omega(t) \tag{21}$$

$$\frac{\partial}{\partial x} G(\bar{x}, t) = \frac{\partial}{\partial x} \Omega(t) = 0 \tag{22}$$

$$G(x, T) = 0. \tag{23}$$

The solution strategy involves solving the dynamic program using particular parameter values and the terminal condition that $V(x, T) = 0$. The terminal condition enables the use of an iterative algorithm to solve backward for the patent holder’s optimal choices, starting from the patent’s expiration. I implement the numerical solution by using the standard approach of discretizing time, and using a binomial approximation to geometric Brownian motion (John C. Cox 1979).

4.1 Numerical results

I again use the parameters of Table (2) as a base case for the numerical solution, and in addition I assume a patent term of 20 years. Litigation cost, c , is normalized to one, so that the value for z can be thought of as a multiple of this value. While litigation cost is to some degree fixed institutionally, it is likely to vary to some degree with the size of the stakes. The variance parameter for the equation of motion is $\sigma = 0.4$ and the discount rate is $r = 0.1$. The effect of the parameters on value are not qualitatively different from the infinite horizon solution, so I focus here on the exercise boundary, $\bar{x}(t)$.

Figures (3) and (4) show the exercise boundaries over time for different levels of p and σ . In all cases, the exercise boundary hits zero just prior to expiration. For any patent, there is some \tilde{t} such that $\Omega(\tilde{t}) = 0$ provided $c > 0$. After this time, litigation is never profitable. Because of costly enforcement, the patent’s expiration is *effectively shortened* to the point where $\bar{x}(t) = 0$.

The graphs show that the exercise boundary is increasing when the patent is young, so that in all the contours, $\bar{x}(t)$ reaches its peak in mid-life. Near the beginning of the patent’s

life, there is an incentive for the patent holder to delay litigation. The reason is that there is some probability that the state variable, $x(t)$, will rise tomorrow and make litigation less tempting. This effect is large near the beginning of the patent’s life since there is ample time to determine whether $x(t)$ will rise or fall. As time passes this effect diminishes and it becomes more urgent to exercise the option for a given value of x . Near the expiration date of the patent, litigation becomes too costly and \bar{x} falls.

Figure (3) shows that higher values of p make the curve more “humped.” Additionally, patents with low p ’s force the exercise boundary to zero more quickly. Those patents will only be enforced at early ages, while patents with high p ’s will be litigated almost until expiration, conditional on z . The policy implication of this is that the effective term of patent protection is limited by the probability that the patent is valid. After \bar{x} reaches zero, the patent lapses into the public domain. The reduction in the enforceable life of the patent is an important policy issue that has not been addressed in the literature.

Figure (4) shows that σ one similar effect to that of p : higher values of σ create a more humped shaped exercise boundary. However, higher variance also tends to lower the exercise boundary; This is to be expected since—conditional on a low $x(t)$ — there is a greater likelihood that $x(t + s)$ can reach a relatively high value if σ is high. So, the value of waiting is higher. The value of σ does not alter the enforceable life of the patent like p .

The general shape of the exercise boundary is robust to a number of different parameter values and stochastic processes. However, with near worthless patents, the exercise boundary is flat (at zero), or decreasing and concave over x . With financial options, the exercise boundary approaches the asset price as the expiration nears, and on the day of expiration, they will be equal. This is true in the present case with one caveat; in the context of patents, the underlying asset expires at the same date as the option expires. So, both the asset value and the exercise boundary must approach zero as the patent nears the end of its life.⁵

Unfortunately, investigating \bar{x} in isolation does not inform us about the likelihood of litigation.

5 Litigation

The expected probability of litigation (the litigation rate) is a function of the exercise boundary, the initial conditions, the variance of dx , and time. Determining the impact of changes in parameter values on the litigation rate requires understanding the interaction of these factors. Consider the probability of validity. If p increases, the exercise boundary increases. All other things equal, this would increase the litigation rate. However, the initial value of x is important in this context. If x_0 responds to p (which is reasonable—suppose $Ex_0 = pz$), then the effect on the litigation rate is less clear: the exercise boundary shifts up, but so does the path of $x(t)$.

For σ , the effect is more opaque. Larger values of σ decrease the exercise boundary. However, the reason for this is because a high σ means that $x(t)$ can rise more quickly over a short period of time, making litigation relatively less attractive. Therefore it is not clear *a priori* that a higher σ will lead to a higher or lower litigation rate.

Last, quantitative impacts on the litigation rate depend—in part—on the definition of the litigation rate. One can consider the likelihood of *ever* being involved in litigation over the life of the patent; or, the probability of being involved in litigation at some time t ; or, the hazard rate of litigation at time t (conditional on having survived to time t).

To be more explicit, given the equation of motion for dx in Equation (1), $x(t)$ is lognormally distributed with mean x_0 and variance $x_0^2(e^{\sigma^2 t} - 1)$. Thus, the natural log of $x(t)$ is normally distributed with mean $\ln x_0 - \frac{\sigma^2 t}{2}$ and variance $\sigma^2 t$. We can then transform $\ln x(t)$ into a standard normal random variable,

$$\frac{\ln x(t) - \ln x_0 + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}.$$

Conditional on not having been litigated, the probability at time t that the patent is litigated in the next dt interval of time is the probability that $x(t + dt) < \bar{x}(t + dt)$, which is given

by the cumulative distribution function (cdf) evaluated at \bar{x} :

$$\Pr(x(t + dt) < \bar{x}(t + dt)) = \Phi\left(\frac{\ln \bar{x}(t + dt) - \ln x(t) + \frac{1}{2}\sigma^2 dt}{\sigma\sqrt{dt}}\right) \quad (24)$$

where Φ is the cdf for the standard normal. The hazard rate could be determined from this distribution, if a closed form solution for $\bar{x}(t)$ existed. Without such a solution in the finite horizon case, I turn next to simulation results, which will be compared predictions from patent litigation data.

6 Simulation Results

For the simulations, parameters were drawn from uniform distributions with the ranges given in Table (2). For each iteration i , a patent “regime” is determined by the parameters drawn. The exercise boundary $\bar{x}_i(t)$ is calculated for that regime as described in Section (4.1). 100 “patents” are then created within each regime with different initial conditions. For the n th patent in regime i , $x_{in0} = kpz$, where $k \sim U(.5, 1.5)$. x_{int} is calculated at discrete intervals of time, and tested against \bar{x}_{it} to determine whether litigation takes place at time t . Each x_{int} is recorded, along with the parameters and the litigation date (if any).⁶

The simulated data are used to estimate the hazard rate, $h(t)$, based on the model parameters. In the finite horizon model, and also in the actual litigation data below, not every patent will be litigated by the end of its life. Because of this truncation, I use a duration approach to estimate the relationship between the parameters of the model and the hazard. I estimate the hazard function using a Weibull specification, because of the flexibility of the functional form, as well as the ease of interpreting parameters. The hazard rate for the Weibull specification is

$$h(t, X) = \lambda\rho(\lambda t)^{\rho-1} \quad (25)$$

where X is the simulated data, and

$$\lambda = \exp X\beta_1 \tag{26}$$

$$\rho = \exp X\beta_2. \tag{27}$$

In the Weibull specification, $\rho > 1$ indicates *positive duration dependence* and $\rho < 1$ indicates *negative duration dependence*. If $\rho = 1$, the Weibull reduces to an exponential distribution where the hazard is constant over time. The specification allows me to estimate the effects of the model parameters on the hazard rate, as well as on duration dependence. For instance, it may be the case that higher p increases λ but decreases ρ . Then, for young patents, higher p may lead to a higher hazard, but for old patents, the hazard actually decreases below the previous value; i.e., $h(t)$ becomes more skew.

Table (3) shows the results of the estimation. The first column gives $\widehat{\beta}_1$, and the second column $\widehat{\beta}_2$. I use quadratic forms to allow for flexibility in the estimation of pz , σ , and *forward* citations. Since I am not able to fit the parameters of the model using a structural model, the quadratic form does not limit the analysis. The variables used are those described in Section (2), with the exception of *forward*. The *forward* variable is meant to proxy for forward citations, which are important in Section (7). The simulation data includes data on x_{int} . The actual value of x is known for each patent at each date prior to litigation. For real patent data, profit flow is obviously unavailable. However, economists frequently use the number of citations received by a patent (“forward” citations) as a proxy for value (Hall, Jaffe and Trajtenberg 2000, Lanjouw and Lerner 1998). Thus, I slightly obfuscate the simulation data by transforming x_t into a forward citation-like variable.

I assume that forward citations are received when there is “good news” about the profit flow of the patent. Therefore, I count each “up-tick” in x_t as a citation. Letting u_{int} be the cumulative number of up-ticks at time t for patent n in regime i , I define *forward* as

$$forward_{int} = u_{int}p_i z_i.$$

Also, note that since p and z enter the model in very similar ways, I estimate the hazard model using pz as an independent variable. Again, this intentionally blurs the data in a way that is consistent with what one would expect to find in real patent data. Since p and z are complementary, explanatory variables may tend to be correlated with the product rather than the components.

Using the coefficients from Table (3), I predict the values of $\hat{\rho}_{int}$ and \hat{h}_{int} for each patent. Figures (6) to (9) show boxplots of $\hat{\rho}$ and \hat{h} graphed against *forward*, p , z , and σ .⁷ The predictions show a median hazard rate of 0.007 per year and median duration dependence of 7.8. Since $\hat{\rho} > 1$, the longer a patent has gone without litigation, the more likely it is that it will be litigated in the next period of time. However, the value of 7.8 is quite high. Even the smallest predicted values of ρ are greater than one. Obviously, one should not infer too much from the quantitative results of the simulations.

The results for *forward* are most obvious and dramatic. The hump shape for both ρ and h show that it is *not* the most valuable patents (those receiving more up-ticks) that are most likely to be litigated. A common claim in the literature is that *litigated patents are among the most valuable* (Allison and Lemley 1998, Lanjouw and Lerner 1998, Lanjouw and Schankerman 2001). Instead, my results support a different hypothesis: that patents with very few up-ticks and patents with many up-ticks are *less* likely to be litigated. Those patents in the middle are the most likely to be litigated and also the have the most positive duration dependence. So, the longer they remain without being litigated, the more likely that they will be litigated. This reflects what we know from the model: if $x(t)$ is very high, the patent holder is not likely to litigate. If $x(t)$ is very low, it may be that the underlying technology is not worth much, or that the probability of validity is small; these are patents that may never be litigated, which is why the hazard approaches zero on the left tail of the distribution. So, middle values of *forward* reflect a greater proportion of patents with a balance of high pz and relatively *low* $x(t)$.

Turning to Figures (7) and (8), the duration dependence parameter ρ is increasing in p and z . The hazard rate, on the other hand, is quadratic in p and z , first increasing and

then decreasing. Notably, the highest predicted hazard rates occur when $p = .5$, which is consistent with the hypothesis of Priest and Klein (1984) that cases will self-select until there is a win rate of 50% for the plaintiff. These results support the view that the most valuable patents are *not* the primary candidates for litigation.

Interestingly, σ tends to increase the duration dependence, and *decrease* the hazard rate. That is, more uncertainty leads to *less* litigation. This means that for my parameter values, σ decreases \bar{x} by more than it increases the likelihood of reaching \bar{x} .

While these results are provocative, they do not carry much weight without confirmation from testable implications. The results of the simulation are most interesting when contrasted to the results from the real litigation data.

7 Empirical Results

The data for the empirical analysis come from two sources. Litigation data come from Derwent's LitAlert database. The database contains the patent number and date of filing for most patent suits filed between 1975 and 1995 (approximately 7500). I restrict this set to patents issued between 1975 and 1995, which are matched to patent data from NBER (Hall, Jaffe and Trajtenberg 2001).

Using data from NBER I create a one-to-one matched sample by randomly selecting an equal number of non-litigated patents. Obviously, there is significant censoring in the data since patents granted in 1995 will, other things equal, appear to be litigated less frequently than older patents. Because of the censoring, duration analysis is appropriate.

The NBER data contains detailed information about forward citations for each patent. I use this data to calculate a running total of forward citations by date, up to either the expiration of the patent, or the date of first filed lawsuit. By examining the changes in forward citations at different dates, the censoring effects can be controlled. The alternative is to use the total number of citations that a patent receives over its lifetime, regardless of when (or whether) a patent was litigated. This presents a problem. Citations received prior to litigation are treated the same as those received after litigation. One is not able

to distinguish between the two. Lanjouw and Schankerman (2001) term the latter the “publicity effect,” which may arise *because* of litigation. By explicitly controlling for the timing of forward citations and litigation, I can eliminate the possibility of a confounding publicity effect.

The other patent variables used are patent specific and do not vary over time. Table (4) shows the results of the estimation. The specification is identical to that described in Section (6), except that the independent variables are different. Besides *forward*, I use *cmade* (the number of citations made by the patent to previous patents), *claims* (the number of claims in the patent), *gyear* (the grant year), and *foreign* (whether the patent is owned by a foreign entity). Again, I use a quadratic specification in order to allow for flexibility. Since I am not interested in the precise parameter estimates, this is not a limitation.

The estimate from Table (4) is used to predict $\hat{\rho}$ and \hat{h} for each observation. The median hazard rate is 0.09 (per year) and the median duration dependence is 1.7. Just like the simulation data, the data exhibit positive duration dependence, on average. However, the estimate for $\hat{\rho}$ is less than one for about 10% of the predicted values. Figures (10) to (13) show the results of graphing $\hat{\rho}$ and \hat{h} against each of the explanatory variables, except *foreign*.

Again, the results for *forward* are striking. The curves show almost exactly the same pattern as the simulated forward citations. This result provides some support that: (1) forward citations are an estimate profit flow (or *up-ticks*) in profit flow, and (2) that the most valuable patents are not the most likely candidates for litigation.

Citations made by a patent (backward citations) appear to have some impact on the hazard rate, but not on duration dependence. Since failure to cite prior art is one of the common reasons patents are found invalid, it may be that more backward citations decrease the uncertainty about validity. The effect on the hazard rate is consistent with the view that more backwards citations indicate less uncertainty about the validity of the patent. However, the effect on duration dependence does not match the results of σ above. The effect of claims is equally ambiguous, since it does not have a strong effect in the regression

results, relative to other parameters. It has been proposed that claims are a measure of the technological value of the patent (Lanjouw and Schankerman 2001). If so, claims should be correlated with the parameter z . Because of the weak results, we cannot be sure.

The year in which the patent was granted has a strong effect. There is a consensus in the literature that there was a significant strengthening of patent rights in the early to mid-1980s (Kortum and Lerner 1999a). In the context of my model, this can be interpreted as either an increase in p , or a decrease in σ , or both. The shape of the hazard rate over the grant year shows an increasing litigation rate. According to the simulation results, this is consistent with a decrease in σ . However, again the slightly hump-shaped curve (even after 1982) in duration dependence is somewhat inconsistent with this interpretation.

8 Conclusion

In this paper, I investigated the effects of uncertainty about validity and costly enforcement on patent value using a theoretical real options model. I developed both finite and infinite time horizon models that examined the effects the parameters of the model on patent value and the litigation decision. The models make several predictions about patent value in a world where enforceability is costly and legal outcomes uncertain, and they show that patent enforcement is a clear application of option valuation.

The first model presented was the infinite horizon model. Table ?? summarizes the results of the comparative statics for patent value and the exercise boundary in the infinite horizon model. The table makes clear the policy complementarity of (especially) p , z , and c . This result must be qualified proportionally to the degree of abstraction of the model. However, I believe the most fundamental result to be independent of the specification of the model: to the extent that both the scope of the patent, legal uncertainty regarding that reward, and enforcement costs influence patent value, policy-makers need to take into account the contribution of each to total patent value in order to set good policy.

The second model imposed a finite life on patents. I showed that the effective life of a patent is increasing in the probability of validity, p . Again, the results must be qualified

because of the necessity to impose a particular stochastic process and solve for particular numeric solutions. However, the result that the effective patent life is shorter when enforcement is uncertain and costly to be robust to the particular specification. Again, policy-makers need to be aware of all of their policy instruments.

The work of Gilbert and Shapiro (1990) and Klemperer (1990) show that there may be benefits to long and narrow patents. However, in determining the appropriate level of reward in a dynamic setting, there may be benefits to shorter patent terms. For instance, in new patenting areas, like software patents or business method patents it may be that it is desirable to experiment with intellectual property protection by giving more limited patent rights in these areas. Policy makers (both the patenting authorities and the courts) can do so by issuing intentionally uncertain patents. Over time, as the courts and the patenting authorities define the validity in these areas, beliefs for certain classes of patents may be updated (revised upward or downward based on information from related patent cases). In this way, patents for which the courts and the patenting authorities deem more protection necessary for optimal appropriation will be just those patents that experience an increase in confidence about validity. It is more feasible for the patenting authorities to issue vague patents than to idiosyncratically and retroactively adjust the patent term.⁸

In the final sections of the paper, I compare patent litigation data to simulated litigation data. I find that the forward citation results provide strong support for the model. One important result that is contrary to the literature is that the most valuable patents are *not* the most likely to be litigated. Rather, I find that there exists a middle value of patents that are most prone to litigation. One must remember that the value in this context is a function of all the parameters of the model, as shown in Section (3).

Notes

¹But note that under certain conditions they may be socially preferable to finitely-lived patents if the scope is more narrowly defined (Gilbert and Shapiro 1990, Klemperer 1990).

²See Gilbert and Shapiro (1990) and Klemperer (1990) for economic interpretations of scope with regard to patent value.

³The injunctive right is a right of property owners. Liability rights alone allow for the collection of damages, but not for an injunction. Thus intellectual property falls under the property regime, since the law does not put limits on the owner's right to exclude others. An exception to this rule is compulsory licensing which equates to a liability rule: the property owner is not allowed to exclude, but instead must set some price by which others may use the property.

⁴We will see below that in the finite horizon context, we obtain a partial differential equation in x and t .

⁵Again, this is similar to a put on a bond.

⁶The simulated data are created this way to match—as closely as possible—the structure of the litigation data described in Section (7).

⁷The *initial* conditions are not plotted, as they have the unsurprising effect of decreasing litigation.

⁸This is an application of the evolution of the common law towards efficiency (Posner 1992).

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Table 1: Marginal effects in the infinite horizon model

Parameter		Continuation region $G(x)$	Stopping region (Ω)	Critical value $(x\text{-bar})$
		(sign of derivative)		
Probability of validity	p	+	+	+
Maximum profit flow	z	+	+	+
Litigation cost	c	-	-	-
Variance parameter	σ	+	0	-
Interest rate	r	-	-	?

Table 2: Numerical Parameter Values

Parameter		Base Case	Simulation
Probability of validity	p	0.7	[0.1, 0.9]
Maximum profit flow	z	1	[0.25, 5]
Litigation cost	c	1	1
Variance parameter	σ	0.4	[0.05 4]
Interest rate	r	0.1	[0.01 0.20]
Term	T	20	20

Table 3: Weibull Hazard Estimation: Simulated Data

Variable	Primary: ln(lambda)		Ancillary: ln(rho)	
	Coef	SE	Coef	SE
forward	-1.02	0.022 **	0.028	0.0008 **
forward ²	0.01	0.000 **	0.000	0.0000 **
pz	8.86	0.274 **	-0.077	0.0114 **
(pz) ²	-1.61	0.071 **	0.031	0.0029 **
sigma	-2.41	0.173 **	0.153	0.0092 **
sigma ²	-0.29	0.068 **	-0.008	0.0020 **
initial	-0.80	0.059 **		
rate	4.21	0.343 **		
constant	-13.0	0.3 **	1.57	0.02 **
Observations	186736			
Likelihood Ratio (chi2(8))	8063			
Log Likelihood	-555			

Notes:

Coefficients are in log-relative hazard form

** indicates significance at the .001 level

* indicates significance at the .01 level

Table 4: Weibull Hazard Estimation: Litigation Data

Variable	Primary: ln(lambda)		Ancillary: ln(rho)	
	Coef	SE	Coef	SE
forward	-0.4283	0.0123 **	0.09635	0.00237 **
forward ²	0.0037	0.0001 **	-0.00140	0.00006 **
gyear	-2.0179	0.0960 **	0.30984	0.03298 **
gyear ²	0.0123	0.0006 **	-0.00174	0.00020 **
cmade	0.0285	0.0030 **	-0.00333	0.00107 *
cmade ²	-0.0001	0.0000 **	-0.00001	0.00002
claims	0.0134	0.0021 **	0.00007	0.00056
claims ²	-0.0001	0.0000 **	-0.000001	0.00001
foreign	-0.9712	0.0356 **		
constant	80.3	4.0 **	-13.7	1.4 **
Observations	58311			
Likelihood Ratio (chi2(9))	3746			
Log Likelihood	-16312			

Notes:

Coefficients are in log-relative hazard form

** indicates significance at the .001 level

* indicates significance at the .01 level

Figure 1: Patent Value, infinite horizon (p)

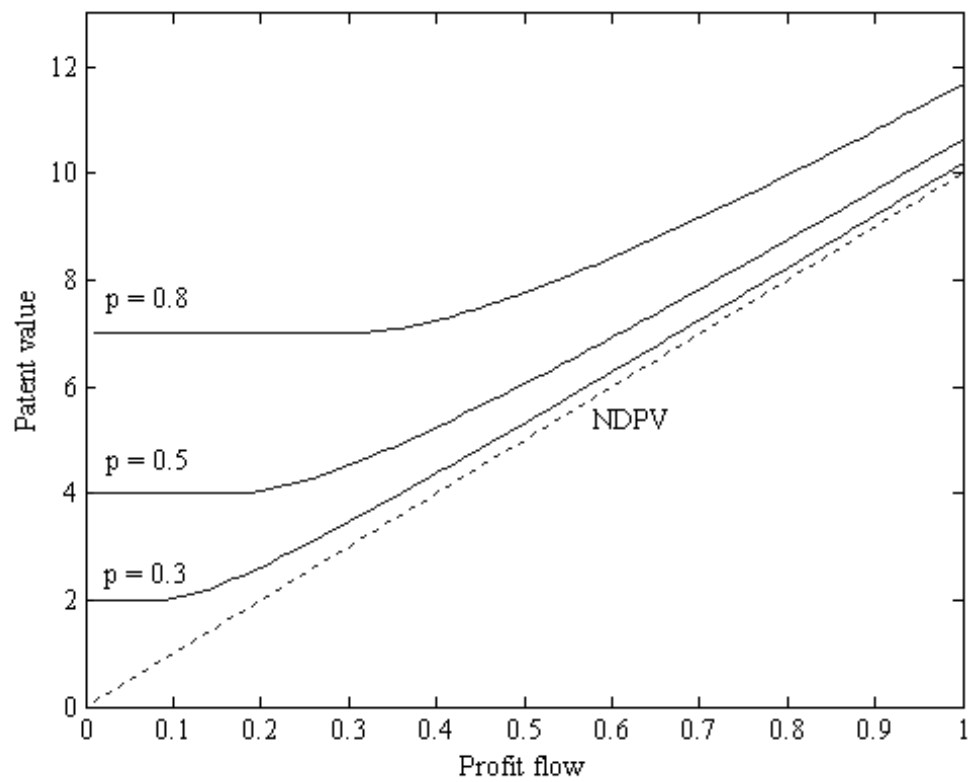


Figure 2: Patent Value, infinite horizon (σ)

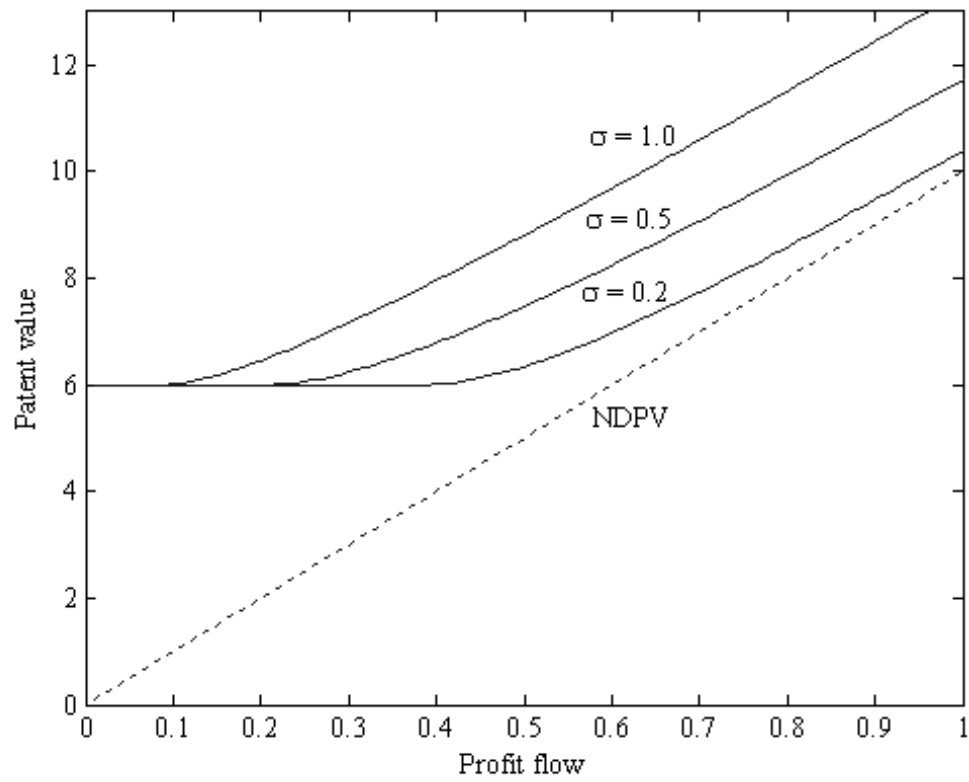


Figure 3: Exercise Boundary, values of p

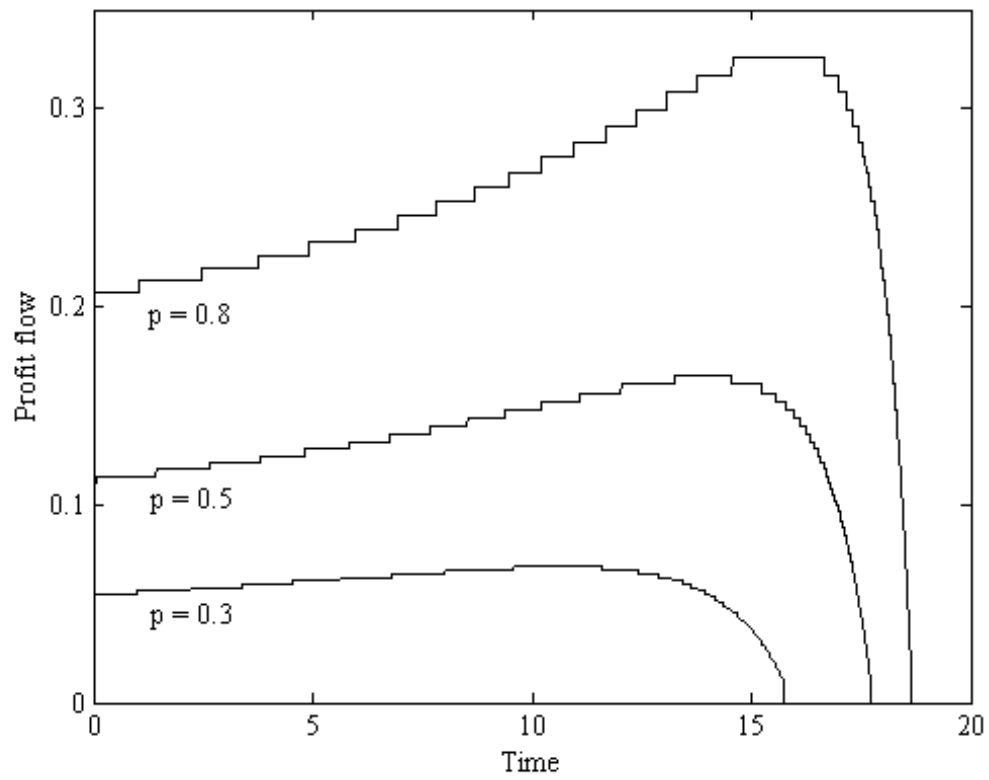


Figure 4: Exercise Boundary, values of σ

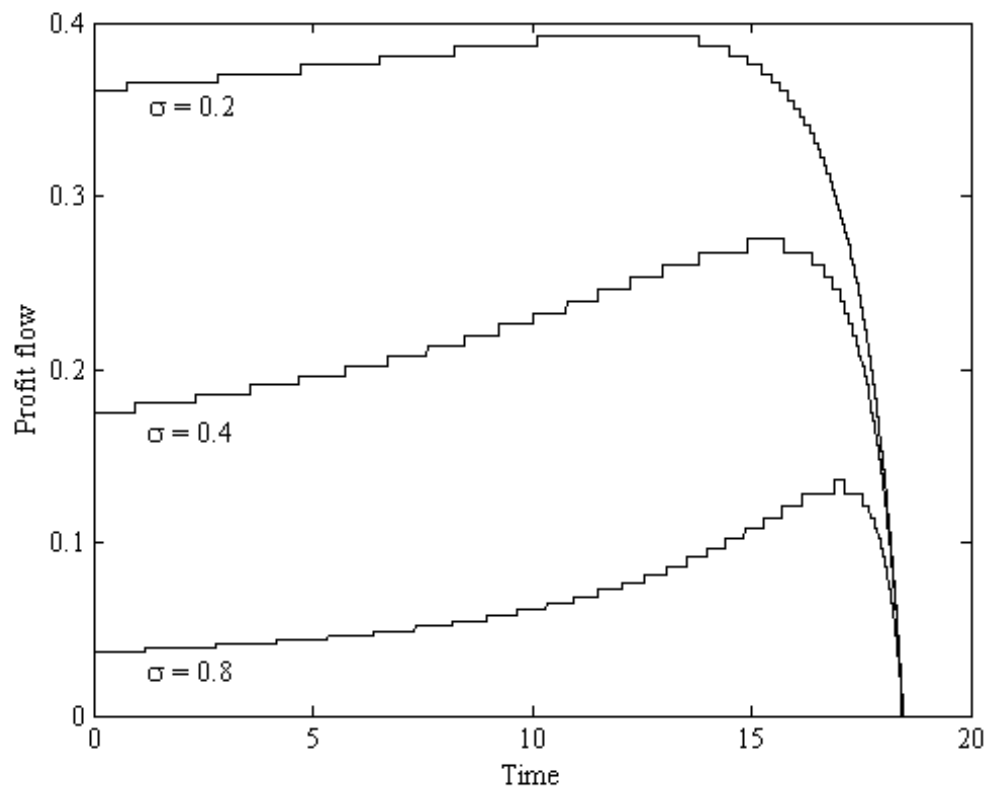


Figure 5: Weibull distribution as a function of *forward*

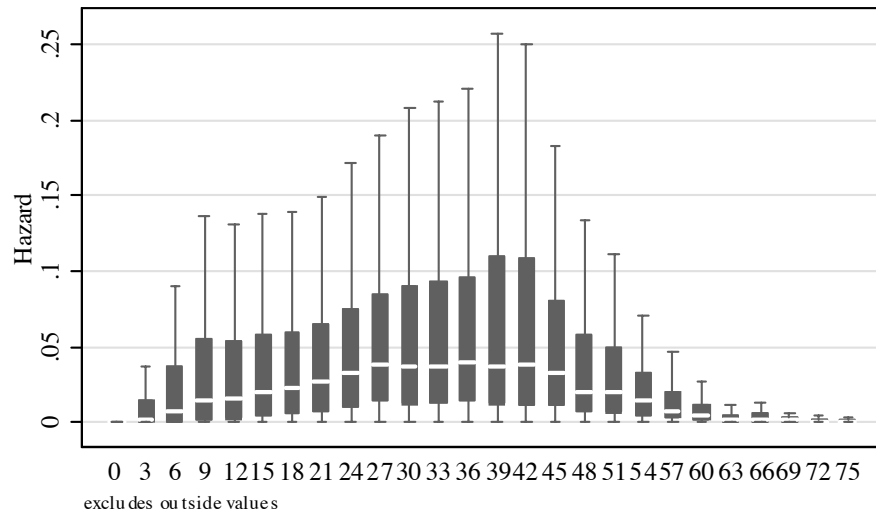
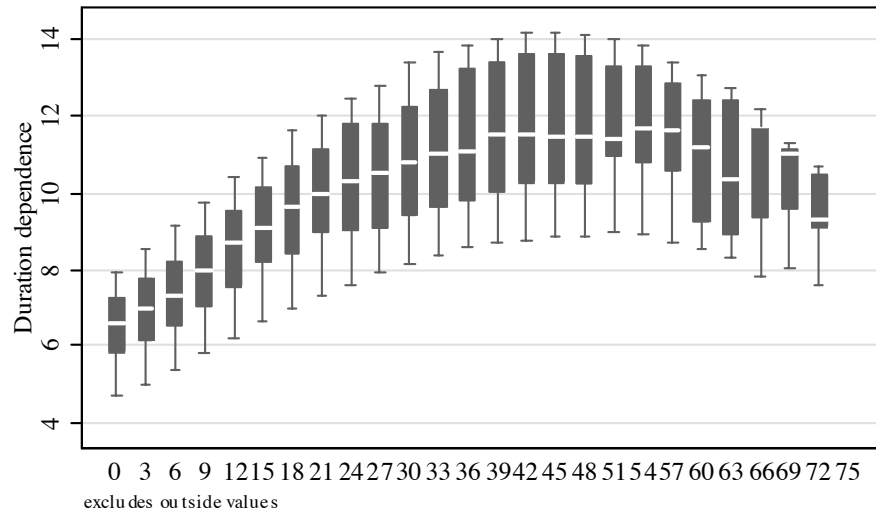


Figure 6:

Figure 7: Weibull distribution as a function of p

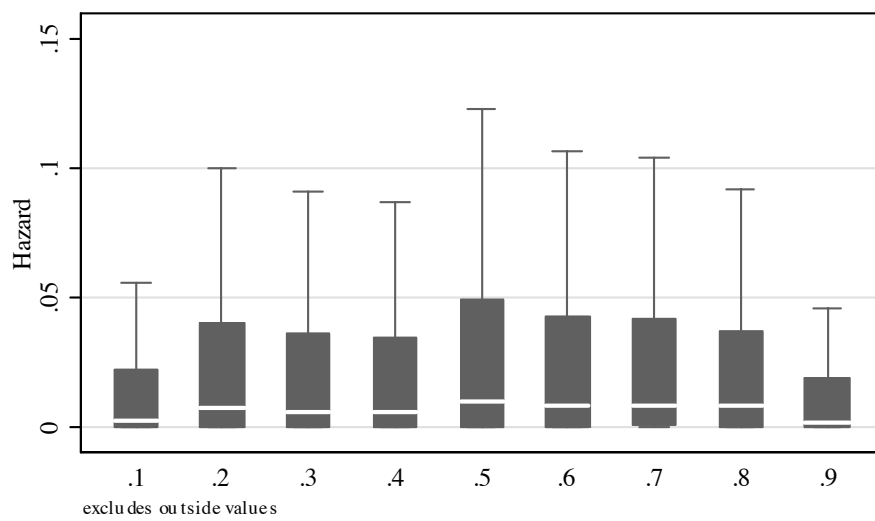
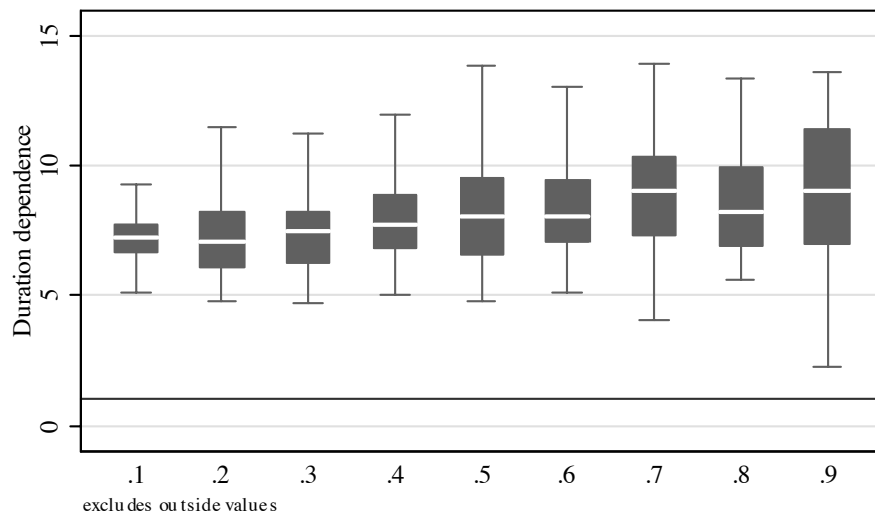


Figure 8: Weibull distribution as a function of z

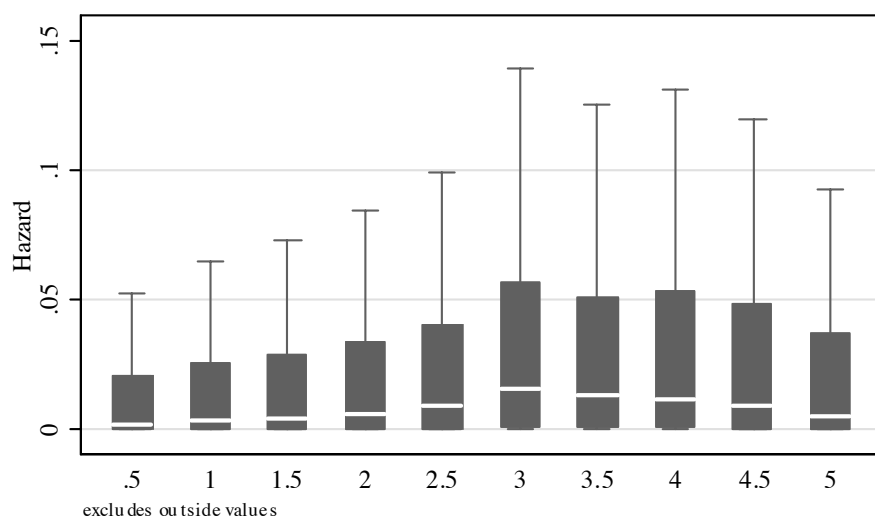
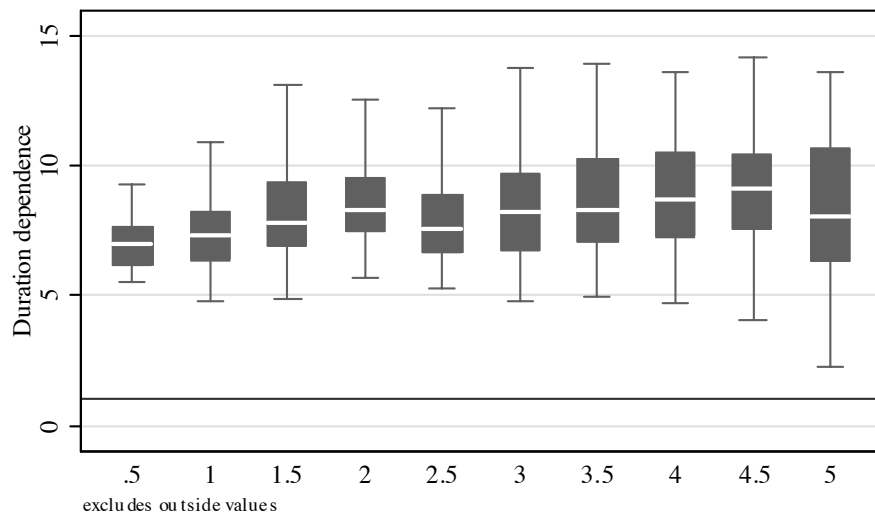


Figure 9: Weibull distribution as a function of σ

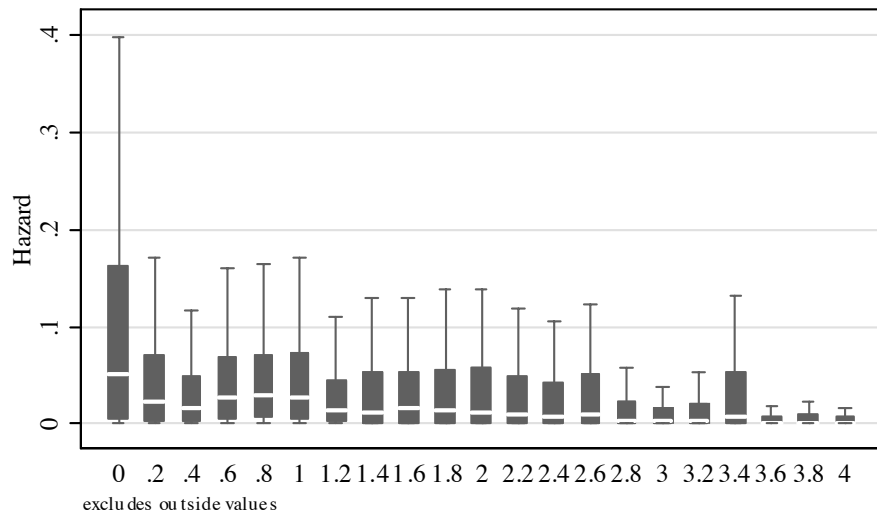
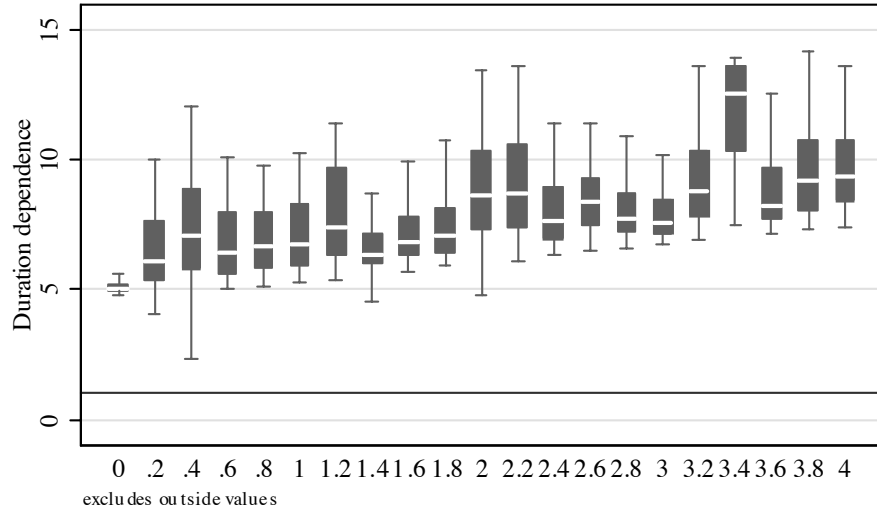


Figure 10: Weibull distribution as a function of *forward*

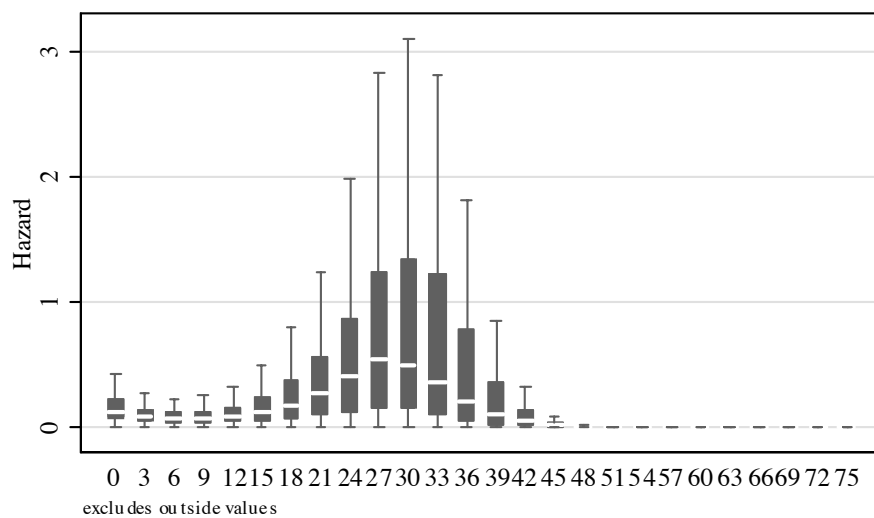
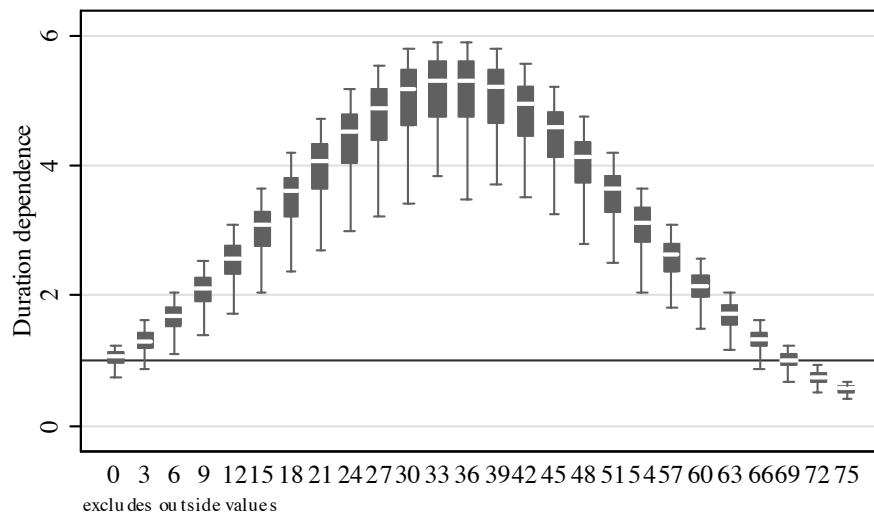


Figure 11: Weibull distribution as a function of *gyear*

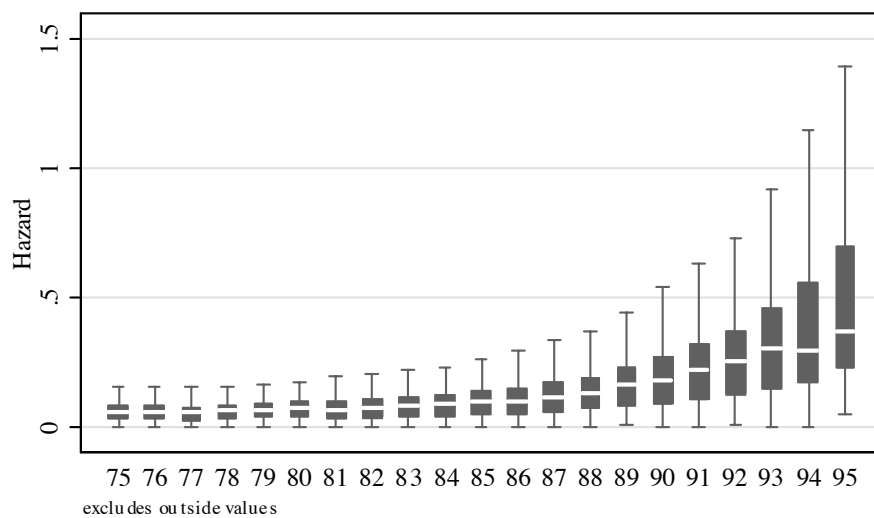
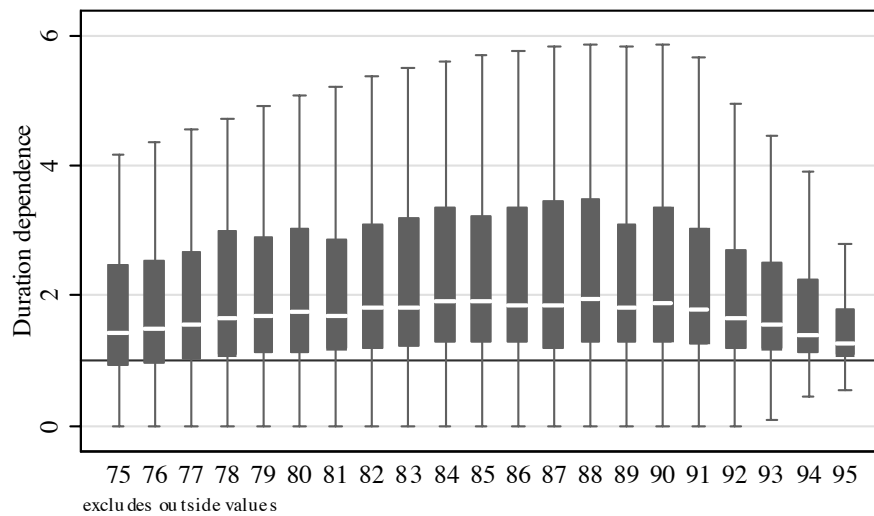


Figure 12: Weibull distribution as a function of *cmade*

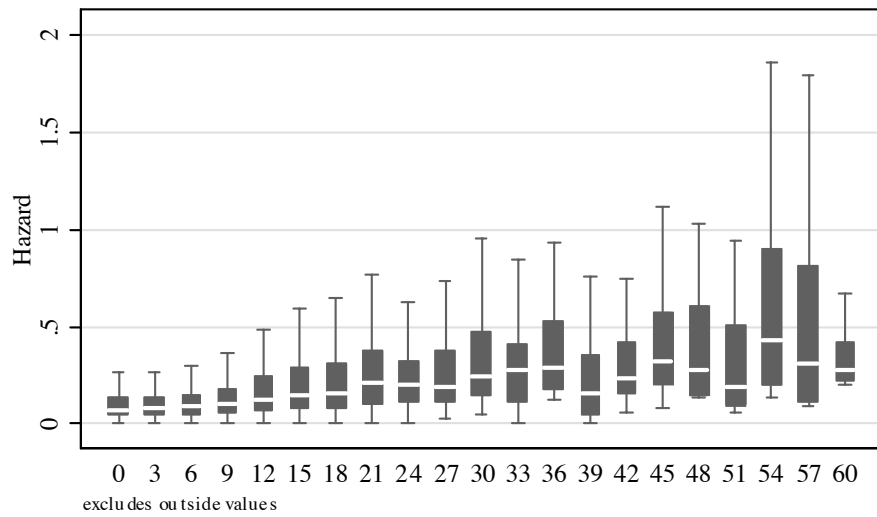
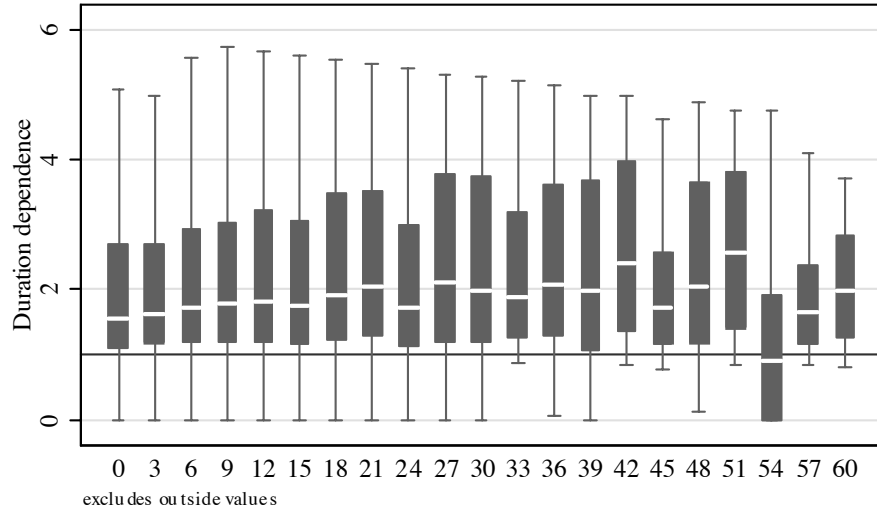
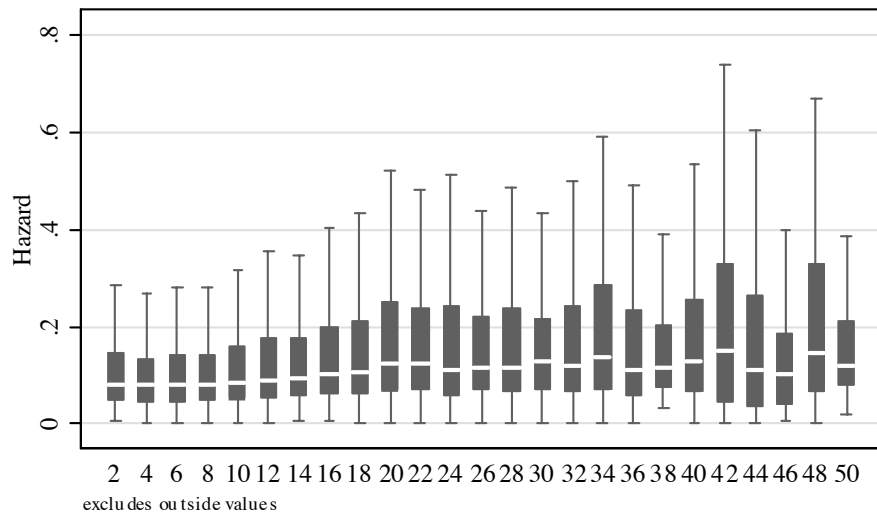
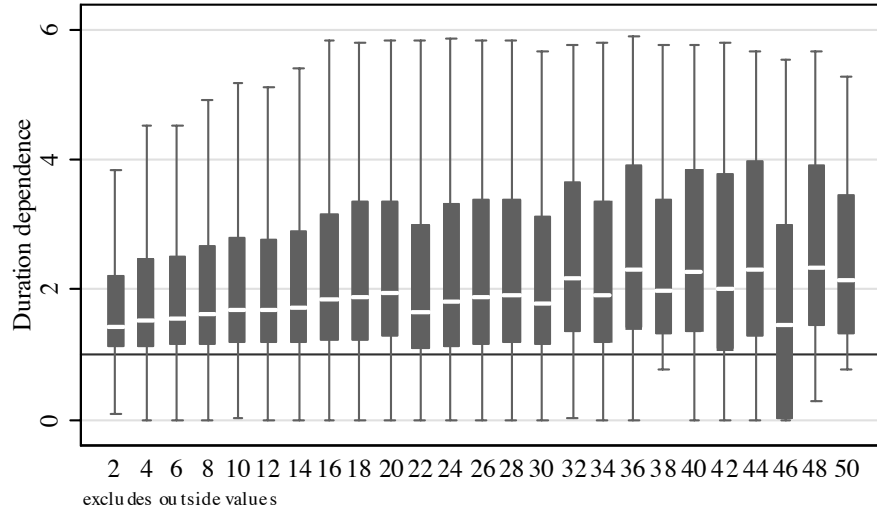


Figure 13: Weibull distribution as a function of *claims*



A Properties of the Exercise Boundary

Recall that the exercise boundary in the infinite horizon model is given by

$$\bar{x} = (pz - cr) \frac{\gamma - \sigma}{\gamma + \sigma} \quad (28)$$

The patent holder will decide to litigate whenever the current flow of profit drops below the critical value \bar{x} . This reflects the patent holder's belief that it can make more (in expectation) by going to court than it can by accepting the amount that other technology users are willing to pay for it. The value of the option is the ability to wait until tomorrow to see if the state of the world improves. When the option is "in the money," x has fallen sufficiently below pz for the patent holder to be willing to expend the litigation cost and risk losing the income stream entirely.

From a policy standpoint it is instructive to examine the ways in which the underlying parameters affect the patent holder's litigation decision. I do this by way of comparative statics on \bar{x} . First, note that for there to be a possibility of enforcement it must be that $pz > cr$.⁹ This condition simply means that the termination value is greater than zero ($\Omega > 0$). I will assume this throughout; those patents for which $pz < cr$ are unenforceable.

Proposition 1 *The critical value \bar{x} is decreasing in the variance parameter σ , but at a decreasing rate (\bar{x} is decreasing and convex in σ).*

$$\begin{aligned} D_{\sigma}\bar{x} &= 16r \frac{cr - pz}{\gamma(\sigma + \gamma)^2} < 0, \text{ when } pz > cr \\ D_{\sigma\sigma}\bar{x} &= 16r \frac{(pz - cr)(3\gamma + 3\sigma^2 + 16r)}{\gamma^3(\sigma + \gamma)^3} > 0, \text{ when } pz > cr. \end{aligned}$$

A high level of σ implies that today's level of x will be a bad predictor of tomorrow's level, since the percent change in x can be very large. Intuitively, as σ increases, the incentive to sue is diminished for any given x since the likelihood of x returning to an "acceptable" level is high when σ is high. In other words, the current level of x is less meaningful when σ is high: the option needs to be *deep* in the money before it is exercised. From Equation

(12), as σ increases without bound, \bar{x} approaches 0.

The equation of motion for x (Equation 1), depends on σ so the effect of σ on the litigation rate is not readily apparent. An increase in σ decreases \bar{x} but it also increases the likelihood that x will drop to a low value, so the effect on the litigation rate is ambiguous.

Proposition 2 *The critical value \bar{x} is increasing over a relevant range of the discount rate r , and then decreasing, and is concave in r .*

$$\begin{aligned} D_r \bar{x} &= 8 \frac{\sigma p z - (\sigma + \gamma) c r}{\gamma (\sigma + \gamma)^2} > 0, \text{ when } p z > c r \left(1 + \frac{\gamma}{\sigma}\right). \\ D_r \bar{x} &< 0, \text{ when } p z < c r \left(1 + \frac{\gamma}{\sigma}\right). \\ D_{rr} \bar{x} &= -16 \frac{(6 r c \sigma + 2 c r \gamma + 6 p z \gamma + c \gamma \sigma^2 + 2 p z \sigma + c \sigma^3) \sigma}{\gamma^3 (\sigma + \gamma)^3} < 0, \text{ for all } r. \end{aligned}$$

The discount rate enters in two ways. First, a higher r increases the incentive to sue because it makes the cost of waiting high. So, if x dips below the expected court outcome (pz), the one time (expected) gain becomes large relative to the benefit of waiting. However, a higher r also reduces the incentive to sue because by paying for litigation tomorrow instead of today, the patent holder can save by waiting since real litigation costs are decreasing. For low values of r , the first effect is larger; for high values of r , the second effect dominates. For very high values of r , \bar{x} will equal 0 and can technically become negative. Recall that I exclude that possibility from the analysis because the patent becomes unenforceable (these cases are uninteresting but not necessarily pathological: from a policy standpoint they may be very important).

Since the discount rate, r , is not in the equation of motion for x (Equation (1)), changes in \bar{x} that are the result of changes in r will be reflected in changes in the litigation rate. In particular, when $r < \frac{pz\sigma}{c(\sigma+\gamma)}$, increases in r will increase \bar{x} . The path of x does not depend on r , so the likelihood that x will drop below \bar{x} increases as does the litigation rate. In contrast, when $r > \frac{pz\sigma}{c(\sigma+\gamma)}$, increases in r will decrease \bar{x} and the litigation rate.

Proposition 3 *The critical value \bar{x} is linear and increasing in p and z , and linear and*

decreasing in c.

It can be seen directly from Equation (12) that \bar{x} is linear in p , z , and c with first derivatives given by

$$\begin{aligned} D_z \bar{x} &= \frac{\gamma - \sigma}{\sigma + \gamma} p > 0 \text{ for all } z, \\ D_p \bar{x} &= \frac{\gamma - \sigma}{\sigma + \gamma} z > 0 \text{ for all } p, \\ D_c \bar{x} &= \frac{\sigma - \gamma}{\sigma + \gamma} r < 0 \text{ for all } c. \end{aligned}$$

Note that the marginal effects on the critical value of suing, \bar{x} , of changes in the probability of validity (p), the underlying patent value (z), and litigation cost (c) are all functions of the variance parameter (σ) and the discount rate (r). I analyze p , z , and c together since they can all be seen as policy tools.

The probability of validity can be affected by the amount of time the patenting authority spends examining patents. The more time spent examining patents (and the more the patent office hires qualified examiners), the more likely a patent will be found valid. This is especially true when validity is attacked on the basis of prior art, the most common legal strategy (Allison and Lemley 1998). The establishment of pro- or anti-patent courts will also affect beliefs that a court will uphold validity (Lerner 1994, Lanjouw 1994, Lanjouw and Shankerman 1997, Kortum and Lerner 1999b).

Since the underlying patent value is affected not only by the technology itself, but also by the scope of the patent, the patent office has some influence over z . Should the patent office wish to increase rewards to patent-holders, one option is to increase the scope of individual patents. Lastly, litigation cost can be affected by policy-makers, at least in discrete ways. First, the establishment of administrative courts may reduce expenditures for the initial stage of disputes. Also, the establishment of specialized patent courts should reduce litigation cost. On the other hand, making appeals easier may tend to raise the expected litigation cost, since more cases will go through a second adjudication.

We can see from the first derivatives of \bar{x} that the marginal effect of p is linear in z and

vice versa. So, p will have a larger marginal effect when z is high, and z will have a larger marginal effect when p is high. This matters from a policy perspective since both p and z are policy tools in principle.

The effects of litigation costs (c) on \bar{x} are similar to the effects of p and z except that it is negatively related to \bar{x} and the marginal effect is dependent only upon the discount rate and σ .

B Properties of the value function

Proposition 4 *Patent value is increasing and convex in x in the continuation region.*

Recall that the value function in the continuation region is given by Equation (14):

$$G(x) = \frac{x}{r} + \frac{2\sigma}{r} \frac{(pz - cr)}{(\sigma + \gamma)} \left(\frac{x}{\bar{x}}\right)^{\frac{\sigma - \gamma}{2\sigma}}.$$

$G(x)$ is composed of a linear increasing function of x , plus a convex decreasing function of x , the sum of which is an increasing convex function. The first term, $\frac{x}{r}$, represents the discounted present value of a stream of profit x . The value of the patent is everywhere greater than $\frac{x}{r}$ because it is composed of $\frac{x}{r}$ plus a positive amount. Since the exponent on x in the second term is negative ($\frac{1}{2} \frac{\sigma - \gamma}{\sigma} < 0$), the option value will be lower for higher values of x , hence we see $V(x)$ approach $\frac{x}{r}$.

Proposition 5 *Patent value is increasing and convex in the probability of validity.*

From the first and second derivatives of G with respect to p , we can see that the patent value is increasing and convex in p .

$$D_p G = \frac{z}{r} \left(\frac{x}{\bar{x}}\right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} > 0$$

$$D_{pp} G = \frac{1}{2} \frac{\gamma - \sigma}{\sigma} \frac{z^2}{(pz - cr)} \frac{z^2}{r} \left(\frac{x}{\bar{x}}\right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} > 0$$

Note the substitution of \bar{x} in the first derivative, and recall that \bar{x} is a function of the other parameters, so $\frac{\partial \bar{x}}{\partial p}$ must be taken into account in the derivatives. Also, one should note the role of z in the derivatives of G with respect to p . The underlying value z serves to linearly change the marginal effect of p .

Proposition 6 *Patent value is increasing and convex in the underlying value.*

Value is also increasing and convex in z , as we again see parallel effects in p and z , as

demonstrated by the first and second derivatives.

$$D_z G = \frac{p}{r} \left(\frac{x}{\bar{x}} \right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} > 0$$

$$D_{zz} G = \frac{1}{2} \frac{\gamma - \sigma}{\sigma (pz - cr)} \frac{p^2}{r} \left(\frac{x}{\bar{x}} \right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} > 0$$

Because of these parallel effects, policy makers should be sensitive to the current levels of validity (p) and underlying value (z) if they are to efficiently set patent value so as to appropriately reward innovation. If policy makers desire to increase V most efficiently, they need to be aware of the trade-offs involved. On one hand, a high z increases the marginal effect of an increase in p and vice versa. So, there is an incentive to increase the smaller of the two policy variables. However, V is convex in z and p , so there is an incentive to raise the higher of the two variables. Additionally, higher litigation rates must be balanced against this policy goal.

Also, higher current values of profit (x) will tend to make the marginal effects of z and p small. So, if p were raised for all patents, a patent holder who happens to be experiencing a low draw on x would be made proportionately better off than a patent holder experiencing a high draw.

Proposition 7 *Patent Value is decreasing and convex in litigation cost.*

From the first derivative of the value function in the continuation region, the marginal effect of an increase in litigation cost is negative for any value of c . Value is also convex in c for any r .

$$D_c G = - \left(\frac{x}{\bar{x}} \right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} < 0$$

$$D_{cc} G = \frac{1}{2} r \frac{\gamma - \sigma}{\sigma (pz - cr)} \left(\frac{x}{\bar{x}} \right)^{\frac{1}{2} \frac{\sigma - \gamma}{\sigma}} > 0$$

From a policy standpoint, it may be worthwhile to attempt to initiate reforms that lower litigation costs if policy-makers desire to increase patent value. This will be the case if the current discount rate is high. A high r increases the marginal impact of c , but it decreases

the marginal impact of raising p or z . Again, policy makers need to keep magnitudes in mind when setting patent policy.