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Repeated Hotelling Duopoly

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Abstract

This paper explores the effect product differentiation has on the ability of firms to collude in setting prices. It is often thought that product differentiation can be an impediment to collusion. However, the results in this paper suggest that product differentiation can enhance the ability of firms to collude. Specifically, in an infinitely repeated Hotelling duopoly, trigger strategies which result in the collusive outcome are subgame perfect equilibria for a larger range of discount factors the more differentiated the products are.

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This paper explores the effect product differentiation has on the ability of firms producing differentiated produces to collude on price. It addresses the question of whether collusion is more or less likely in an infinitely repeated Hotelling model of product differentiation when firms produce differentiated products than when products are homogeneous.

The conventional argument is that collusion is more difficult when products are differentiated because firms may have difficulty agreeing on the correct price or prices.[1] In the context of a Hotelling model, coming to an agreement (whether explicit or implicit) on the joint-profit maximizing price requires that firms know the consumers' reservation value, transportation (or disutility) costs and location of all products within the product space. However, it should be noted that the informational requirements for firms to charge the noncollusive profit maximizing prices are the same. This paper ignores these informational concerns by assuming that firms have perfect information about the demand for the products.

Oftentimes firms may recognize that it is in their best interests to behave cooperatively in setting prices and thus maximize their combined profits. However, the firm may find itself caught in a prisoners' dilemma. By cooperating the firms are able to achieve higher profits than if they behave competitively. However, when firms are cooperating there is often the incentive for the individual firm to defect on the cooperative agreement to individually increase its own profits. In infinitely repeated games, firms can use "trigger strategies" to enforce an implicit collusive agreement. An example of a "trigger strategy" is the "grim trigger strategy" in which a firm's strategy is

[1] See for example, Carlton and Perloff (1994), p. 188.

to charge the collusive price but if a rival ever deviates and undercuts the collusive price the firm reverts to the single-repetition Nash (or competitive) price for the rest of time. Trigger strategies involve punishing firms that deviate from the collusive agreement. A firm considering whether or not to cheat on the collusive agreement weighs the benefit from deviating from the collusive agreement and unilaterally increasing its profits in the short term against the losses incurred when collusion breaks down. Depending on how firms discount the future, the collusive outcome can be supported as an equilibrium outcome.

When products are horizontally differentiated, there is less of an incentive to deviate from the collusive agreement than when products are homogeneous. When products are differentiated, by deviating from the collusive agreement a firm gains less since there are some consumers who remain loyal to the other firm and thus the increase in sales when defecting is not as large as when the products are homogeneous. However, the punishment is less severe when products are since product differentiation serves to relax price competition. Since these effects work in opposite directions, it is not immediately clear whether collusion is more or less likely when products are differentiated.

As shown in this paper, within the context of a Hotelling model of product differentiation, as products become more differentiated collusion becomes more likely; the collusive outcome is an equilibrium outcome for a larger range of discount factors the more differentiated the products are.

The remainder of the paper is organized as follows: Section I describes the spatial model of product differentiation and equilibrium in repeated interaction; Section II solves the model of product differentiation for outcomes under collusion, defection and punishment; Section III illustrates the result of the paper; and Section IV concludes with some suggestions for future research.

I. Model

To analyze the effect that product differentiation has on the ability of firms to collude, the model in this paper incorporates two basic elements, product differentiation and a dynamic model of pricing rivalry.

A. Product Differentiation

The model of product differentiation adopted in this paper is a variant of Hotelling's (1929) spatial model of horizontal product differentiation. In spatial or address models of product differentiation, a product is defined by its location or address within a space of product characteristics. The space of product characteristics is assumed to be a line of unit length. Products are defined by their location along the unit line; $x_i \in [0,1]$.

There are two firms each producing a single product. The firms have zero marginal cost of production. The product locations are assumed to be symmetric; $x_1 = 1 - x_2$. Define d to be the measure of the distance between the two products; $d = x_2 - x_1 = 1 - 2x_1$. d can be thought of as a measure of the degree of product differentiation. When $d = 0$, the products are homogeneous both being located at the center of the line segment, $x_1 = x_2 = ½$. When $d = 1$, the products are located at the opposite ends of the line segment, $x_1=0$, $x_2=1$. Greater values of d reflect a greater degree of differentiation between the products. Since the purpose of this paper is to explore the effect that differentiation has on the ability of firms to collude, it is assumed that locations and the degree of differentiation are exogenously given.[2]

Consumers vary in their preferences for different product characteristics. Consumers can be thought of as being located within the product space, where their location represents the consumer's ideal product, the product (or mix of characteristics) the consumer prefers to all others. Consumers are assumed to be uniformly distributed along the line. The number of consumers is normalized to one.

The utility a consumer located at x^* purchasing a product located at x_i at price p_i receives is $U(x_i,p_i) = V-p_i - t(x^*-x_i)^2$, where V is the consumer's reservation value for its ideal product, and $t(x^*-x_i)^2$ are the transportation costs or disutility that the consumer incurs from not purchasing its ideal product.[3] V and t are assumed to be the same for all consumers known to the firms.

A consumer purchases, at most, one unit of the product that gives them the highest utility, provided that the utility is nonnegative. A consumer located at x^* will purchase the product located at x_i if

$$V- p_i - t(x^*-x_i)^2 \geq V- p_j - t(x^*-x_j)^2 \text{ and}$$

$$V- p_i - t(x^*-x_i)^2 \geq 0.$$

From the consumer's perspective the second inequality might be thought of as a rationality constraint, ie., the consumer only purchases the product if they receive non-negative utility. From the firm's perspective the inequality might be thought of as a

[2] An interesting question for future research is to examine what effect collusion on price has on the locations or degree of differentiation that firms choose.

[3] Quadratic transportation costs are chosen to insure that there exists a pure strategy equilibrium in price for all locations. With linear transportation costs ($t|x^*-x_i|$) a pure strategy Nash equilibrium does not exist if the products are too close together. See D'Aspremont et. al. (1979).

surplus extraction constraint; the firm can adjust p_i so that the constraint binds for a specific consumer to extract as much surplus as possible from the consumer.

For a specific consumer along the line segment the first constraint will hold with equality. At the prices p_i and p_j this marginal consumer, x_m is exactly indifferent between purchasing the product located at x_i and the one located at x_j. This marginal consumer determines the demand for the respective products.

$$V - p_i - t(x_m - x_i)^2 = V - p_j - t(x_m - x_j)^2$$

Recall the product locations are assumed to be symmetric, $x_j = 1 - x_i$ and $d = 1 - 2x_i$. Substituting this relationship in and solving for x_m yields:

$$x_m = ½ + (p_j - p_i)/2td.$$

x_m must be on the unit line (between 0 and 1). Thus, the demand for product i is given by:

$$D_i(p_i,p_j) = \begin{cases} 0 & \text{if } p_i \geq p_j + td \\ ½ + (p_j - p_i)/2td & \text{if } p_j + td \geq p_i \geq p_j - td \\ 1 & \text{if } p_i \leq p_j - td \end{cases}$$

$$\text{as long as } V - p_i - t(D_i - x_i)^2 \geq 0 \quad (1)$$
$$\text{and } V - p_i - t(x_i)^2 \geq 0 \quad (2)$$

(1) requires that the middle or marginal consumer receives non-negative surplus. (2) requires that the consumer located at the endpoint receives non-negative surplus.

B. Equilibrium in Repeated Games

The two firms compete for sales repeatedly facing the demand given above in each period. Each firm seeks to maximize the present discounted value of its profits ($\Sigma_t\delta^t\pi_t$, $\delta \leq 1$) recognizing that what it does in a given period can affect how its rival responds in future periods. In each period t, both firms observe the prices charged by each and recall the prices charged by both in all previous periods. When the game is repeated infinitely, a cooperative or collusive outcome can be supported as a subgame perfect equilibrium outcome by the use of trigger strategies provided that the discount factor is sufficiently large.

Strategies that support the collusive outcome as an equilibrium can take different forms from the unforgiving "grim trigger strategy" to the more forgiving "tit-for-tat". Consider the following grim trigger strategy: charge p^c (the collusive price) in period 0; charge p^c in period t if both firms have charged p^c in all previous periods, otherwise charge p^n (the single-shot Nash equilibrium price) forever. Here a defection from the collusive price triggers a reversion to the Nash equilibrium of the single-shot game forever.

If both firms adopt this strategy, the observed outcome will be the collusive outcome in every period. In considering whether to adopt such a strategy a firm weighs the increased profits from defecting or "cheating" on the collusive "agreement" against the lower profits that result from the reversion to the single-shot Nash equilibirum. It is an equilibrium for both firms to adopt the grim trigger strategy if

$$\sum_{t=o}^{\infty}\delta^t \pi_i^c(p_i^c, p_j^c) \geq \pi_i^d(p_i^*(p_j^c), p_j^c) + \sum_{t=1}^{\infty}\delta^t \pi_i^n(p_i^n, p_j^n)$$

where δ is the rate at which the firm discounts the future, p_i^c is the collusive or joint profit-maximizing price for firm i, p_i^n is the Nash equilibrium price for a single play of the game, and $p_i^*(p_j)$ is firm i's optimal response to p_j, and where π_i^c, π_i^n and π_i^d are the corresponding profits under collusion, single-shot Nash and defection.

Manipulating the equation shows the range of discount factors for which the collusive outcome may be observed as an equilibrium:

$$\delta(d) \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^n}.$$

Define $\delta^*(d)$ to be the δ for which the above holds with equality. For values of $\delta < \delta^*$, the joint-profit maximizing outcome is not a subgame perfect equilibrium outcome; collusion at the prices that maximize the firms joint profits is not sustainable as an equilibrium outcome. For values of $\delta \geq \delta^*$, the collusive outcome can be supported as a subgame perfect equilibrium outcome; collusion is possible. As δ^* falls, the collusive outcome becomes more likely; collusion can be sustained as an equilibrium outcome for a larger range of discount factors the lower δ^* is.

For this problem to be interesting it must be that $\pi_i^d > \pi_i^c > \pi_i^n$. This requires that $V - p_i^n - t(D_i - x_i)^2 > 0$. If $V - p_i^n - t(D_i - x_i)^2 \leq 0$, then the Nash equilibrium of a single play of the game results in the firms having a local monopoly and not facing any competition so that $\pi_i^d = \pi_i^c = \pi_i^n$. As shown below, when $d = 1$, $x_i = 0$, $x_j = 1$, $p_i^n = p_j^n = t$ and $D_i = D_j = ½$. Therefore, since $p_i^n + t(D_i - x_i)^2$ is greatest when $d=1$, $\pi_i^c > \pi_i^n$ requires that $V - 5t/4 > 0$.

II. Prices and Profits

It remains to find the prices and profits under collusion, defection and punishment in order to calculate δ^*.

In each period the firms simultaneously choose prices facing demand:

$$D_i(p_i,p_j) = \begin{cases} 0 & \text{if } p_i \geq p_j + td \\ \tfrac{1}{2} + (p_j - p_i)/2td & \text{if } p_j + td \geq p_i \geq p_j - td \\ 1 & \text{if } p_i \leq p_j - td \end{cases}$$

where $V - p_i - t(D_i - x_i)^2 \geq 0$
and $V - p_i - t(x_i)^2 \geq 0$.

A. Collusion: Joint-Profit Maximization

When the firms seek to maximize their combined profits and locations are symmetric, the profit maximizing prices are equal. To see this, consider prices $p_i>p_j$. The marginal consumer is indifferent between paying the lower price for product j and incurring greater transportation costs and paying the higher price for product i and incurring smaller transportation costs; $p_i + t(x_i - x_m)^2 = p_j + t(x_j - x_m)^2$. The firms can increase their collective profits by raising p_j slightly, so that the (former) marginal consumer now prefers to pay the higher price for product i thereby increasing the collective profits. The firms want to minimize the transportation costs paid by consumers in order to get the consumers to pay higher prices to the firms. When locations are symmetric, minimizing transportation cost requires that the firms set equal prices and split the market evenly.

When $V - 5t/4 > 0$ it is always more profitable to serve the entire market than to leave some of it unserved. Given that the entire market is served, the firms will want to adjust their prices as high as possible so that one of the rationality or surplus extraction constraints bind: either $V - p_i - t(D_i - x_i)^2 = 0$ or $V - p_i - t(x_i)^2 = 0$.

For $d \in [0, ½]$, $x_i \in [1/4, ½]$, the consumers located at the endpoints will be paying the greatest total cost (price plus disutility cost). To maximize profits, the firms will charge prices such that the rationality constraint binds for those consumers located at the endpoints.

$$V - p_i - tx_i^2 = 0$$

$$p_i^c = p_j^c = V - tx_i^2 = V - t((1-d)/2)^2$$

$$\pi_i^c(p_i^c, p_j^c) = p_i^c(D_i) = ½[V - t((1-d)/2)^2]$$

For $d \in [½,1]$, $x_i \in [0,1/4]$, the marginal consumer indifferent between the two products will be paying the greatest price. With symmetric locations and equal prices the marginal consumer is located at ½ (D_i= ½). To maximize profits, the firms will charge prices such that the rationality constraint binds for the consumer located at ½.

$$V - p_i - t(½ - x_i)^2 = 0$$

$$p_i^c = p_j^c = V - t(½ - x_i)^2 = V - t(d/2)^2$$

$$\pi_i^c(p_i^c, p_j^c) = ½[V - t(d/2)^2]$$

Note that collusion is most profitable when d= ½, x_i=1/4, x_j=3/4, the welfare maximizing locations.[4]

B. Best Response Function: Defection and Punishment

To calculate the profits when there is defection from the collusive outcome and when there is punishment, it is necessary to find the firm's best response function, $p_i^*(p_j)$. If firm i were to defect from the collusive outcome, it would choose p_i to maximize $\pi_i(p_i, p_j^c)$ subject to the constraint that $D_i(p_i,p_j) \le 1$; the player optimally defects by choosing the price that is the best response to p_j^c. Similarly, the single-shot Nash equilibrium or punishment prices are best responses to one another.

Firm i's best response function, found by choosing p_i to maximize $\pi_i(p_i, p_j)$ subject to the constraint that $D_i(p_i,p_j) \le 1$, is:

$$p_i^*(p_j) = \begin{cases} (p_j+td)/2 & \text{if } p_j \le 3td \\ p_j - td & \text{if } p_j \ge 3td \end{cases}$$

$$\pi_i(p_i^*(p_j),p_j) = \begin{cases} (p_j+td)^2/8td & \text{if } p_j \le 3td \\ p_j - td & \text{if } p_j \ge 3td \end{cases}$$

[4] It is interesting to note that in a Hotelling duopoly with quadratic transportation costs when firms first choose location and then prices, competition results in lower welfare than monopoly. Competitive choice of locations results in firms locating at opposite ends of the line segment, x_i=0, x_j=1. Of course, the drawback of monopoly is that the firm is able to extract most of the surplus from consumers.

Note that when $p_j \leq 3td$ firm i will share the market with firm j (D_i, $D_j>0$) but when $p_j \geq 3td$, it is optimal for firm i to serve the entire market ($D_i=1, D_j=0$).

In defecting, firm i will choose $p_i^*(p_j^c)$ where p_j^c is the joint-profit maximizing price found in section II. A. There is a technical issue when the products are homogeneous. When $d=0$, $p_i^*(p_j) = p_j^c - \varepsilon$, the firm slightly undercuts the collusive price in order to capture the entire market.[5]

The resulting defection profits are:

$$\pi^d = \begin{cases} V-t(d/2)^2 - td, & d \in [\tfrac{1}{2},1],\ V-t(d/2)^2 \geq 3td \\ V-t((1-d)/2)^2 - td, & d \in [0,\tfrac{1}{2}],\ V-t((1-d)/2)^2 \geq 3td \\ (V-t(d/2)^2 + td)^2/8td, & d \in [\tfrac{1}{2},1],\ V-t(d/2)^2 \leq 3td \\ (V-t((1-d)/2)^2 + td)^2/8td, & d \in [0,\tfrac{1}{2}],\ V-t((1-d)/2)^2 \leq 3td. \end{cases}$$

It can be shown that $\partial\pi^d/\partial d < 0$. The profits when defecting from the collusive prices are smaller the more differentiated the products are.

The Nash equilibrium prices and corresponding profits are:

$$p_i^n = td$$

$$\pi_i^n = td/2.$$

Note that when the products are homogeneous ($d=0$), the equilibrium prices and profits are zero. Also, $\partial\pi^n/\partial d = t/2 > 0$. Punishment is less severe the more differentiated the products are.

III. Effect of Differentiation on Collusion: δ^*

The critical values for the discount factor, $\delta^*(d)$, can now be calculated by plugging the corresponding profits into the equation $\delta^*(d) = (\pi^d - \pi^c)/(\pi^d - \pi^n)$.

Proposition 1: $\partial\delta^*/\partial d < 0$.

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[5] For $d>0$, this is not of concern since the marginal consumer is of size ε, whereas when $d=0$, all consumers are marginal.

Proof: See Appendix.

As the products become more differentiated (d increases), the critical value for the discount factor below which the collusive outcome is not an equilibrium outcome falls. This suggests that in a spatial model of product differentiation firms find it easier to collude the more differentiated their products are.

IV. Concluding Comments

This paper has demonstrated the counter-intuitive result that product differentiation may make collusion easier. When firms interact repeatedly they may be able to implicitly collude in setting prices. In an infinitely repeated Hotelling duopoly, the collusive outcome is an equilibrium outcome for a larger set of discount factors the more differentiated the products are.

The relatively simple Hotelling model can be used to explore other questions involving and gain insight into collusion and product differentiation. In this paper the location or characteristics of products were exogenously determined. However, one might ask how the ability of firms to collude on prices affects the firms' choice of product location.

Consider a game in which in the first stage the firms simultaneously choose product location and in the second stage engage in a repetition of the resulting price game. If firms choose prices competitively and consumers have quadratic disutility costs, D'Aspremont et al. (1979) show that the equilibrium is for the firms to locate at opposite ends of the product space. However, if firms are able to collude on price, they face "fixed prices" in the second stage. Thus it may be that the original Hotelling result in which firms locate at the center might be restored as the equilibrium provided that $\delta > ½$.

In addition in the model in this paper, each firm produces only a single product. Defection from the collusive price is met with punishment by reversion to the single shot Nash equilibrium prices. However, in a model of product differentiation, punishment might take other forms than reversion to price competition. For example, firms might produce a new product (a fighting brand) or relocate their existing product to make punishment more severe. It might be interesting to explore what effect the use of alternative punishments might have on the ability of firms to collude.

Appendix

Proof of Proposition 1: To prove proposition 1, it is necessary to go through some algebraic manipulations to sign $\partial\delta^*/\partial d$, where

$$\delta^*(d) = \frac{\pi^d - \pi^c}{\pi^d - \pi^n}.$$

There are four cases to be examined:

Case 1a: $d\in[1/2,1]$, $V\text{-}t(d/2)^2 \geq 3td$

Case 1b: $d\in[0,1/2]$, $V\text{-}t((1\text{-}d)/2)^2 \geq 3td$

Case 2a: $d\in[1/2,1]$, $V\text{-}t(d/2)^2 \leq 3td$

Case 2b: $d\in[0,1/2]$, $V\text{-}t((1\text{-}d)/2)^2 \leq 3td$.

For all cases π^n=td/2.

Note that for Cases 1a and 1b, $\pi^d = p^c - td = 2\pi^c - td$. Therefore, $\delta^*(d)$ can be written as:

$$\delta^*(d) = \frac{2\pi^c - td - \pi^c}{2\pi^c - td - td/2} = \frac{1}{2} - \frac{td}{8\pi^c - 6td}$$

$$\frac{\partial\delta^*}{\partial d} = -\frac{t(8\pi^c - 6td) - td(8\frac{\partial\pi^c}{\partial d} - 6t)}{(8\pi^c - 6td)^2} = -\frac{8t(\pi^c - d\frac{\partial\pi^c}{\partial d})}{(8\pi^c - 6td)^2}$$

For Cases 1a and 1b sign $\partial\delta^*/\partial d$ = -sign $[\pi^c\text{-}d(\partial\pi^c/\partial d)]$.

Case 1a: π^c=1/2(V-t(d/2)^2), $\partial\pi^c/\partial d = -td/4$. $\pi^c\text{-}d(\partial\pi^c/\partial d)$= ½(V-td^2/4)+td^2/4=V/2+td^2/8>0. Therefore, $\partial\delta^*/\partial d < 0$.

Case 1b: π^c=1/2(V-t((1-d)/2)^2), $\partial\pi^c/\partial d = t(1\text{-}d)/4$. $\pi^c\text{-}d(\partial\pi^c/\partial d)$= ½(V-t(1-d)^2/4)-td(1-d)/4 =V/2-t(1-d^2)/4=1/2(V-t(1-d^2)/2)>0, since V-5t/4>0 and (1-d^2) $\leq$ 1 implies that V-t(1-d^2)/4 >0. Therefore, $\partial\delta^*/\partial d < 0$.

For Cases 2a and 2b note that $\pi^d = (2\pi^c+td)^2/8td$. Therefore $\delta^*(d)$ can be written as:

$$\delta^* = \frac{(2\pi^c + td)^2/8td - \pi^c}{(2\pi^c + td)^2/8td - td/2} = \frac{2\pi^c - td}{2\pi^c + 3td}$$

$$\frac{\partial \delta^*}{\partial d} = \frac{(2\frac{\partial \pi^c}{\partial d} - t)(2\pi^c + 3td) - (2\frac{\partial \pi^c}{\partial d} + 3t)(2\pi^c - td)}{(2\pi^c + 3td)^2}$$

Case 2a: $\pi^c = 1/2(V - t(d/2)^2)$, $\partial\pi^c/\partial d = -td/4$

$$\frac{\partial \delta^*}{\partial d} = \frac{(-\frac{td}{2} - t)(2\pi^c + 3td) - (-\frac{td}{2} + 3t)(2\pi^c - td)}{(2\pi^c + 3td)^2} < 0$$

since $(-td/2-t)<0$, $(2\pi^c+3td)>0$, $(3t-td/2) >0$ $(d\leq 1)$ and, $(2\pi^c-td)>0$. To see this last inequality, note that $\pi^c>\pi^n=td/2$, $2\pi^c>td$.

Case 2b: $\pi^c=1/2(V - t((1-d)/2)^2)$, $\partial\pi^c/\partial d = t(1-d)/4$.

$$\frac{\partial \delta^*}{\partial d} = \frac{(\frac{t}{2}(1-d) - t)(2\pi^c + 3td) - (\frac{t}{2}(1-d) + 3t)(2\pi^c - td)}{(2\pi^c + 3td)^2} < 0$$

for similar reasons as Case 2a.

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