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THE ANATOMY OF VIRTUAL MANIPULATIVE APPS: USING GROUNDED
THEORY TO CONCEPTUALIZE AND EVALUATE
EDUCATIONAL MATHEMATICS APPS

by

Jennifer M. Boyer-Thurgood

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF PHILOSOPHY

in

Education

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Logan, Utah

2017

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ABSTRACT

The Anatomy of Virtual Manipulative Apps: Using Grounded Theory to Conceptualize
and Evaluate Educational Apps that Contain Virtual Manipulatives

by

Jennifer M. Boyer-Thurgood, Doctor of Philosophy

Utah State University, 2017

Major Professor: Patricia S. Moyer-Packenham, Ph.D.
Department: School of Teacher Education and Leadership

This exploratory qualitative study used grounded theory to investigate the anatomy of educational apps that contain virtual manipulatives. For this study 100 virtual manipulatives within educational apps designed for the iPad were observed by the researcher in order to expand the explanations of and build theory about virtual manipulatives within apps. Affordance theory was used to frame all six phases of the study in which the researcher identified virtual manipulatives situated within educational apps, conducted observer-as-participant structured and unstructured observations, analyzed component data including field notes and memos using open and axial coding, created a conceptual framework, developed an evaluation tool prototype to evaluate virtual manipulatives within educational apps, and used the evaluation tool prototype to evaluate additional virtual manipulatives within educational apps.

The constant comparative method of open and axial coding was used to analyze

the observation data that included field notes, memos, and video recordings. This in-depth qualitative analysis led to the emergence of six study results concerning the components and relationships within educational apps that contain virtual manipulatives. The results revealed that: (1) virtual manipulatives within apps are comprised of two components: dynamic mathematical objects and features; (2) there are three distinct types of dynamic mathematical objects; (3) there are eight categories of features; (4) within one virtual manipulative there can be one or multiple objects; (5) varying relationships can exist among the dynamic object and the features within a virtual manipulative; and (6) varying relationships can exist among the virtual manipulatives within an education app in terms of the number, type, and ways the user proceeds.

A conceptual framework was also developed during the study to illustrate the components and relationships that emerged from the analysis and to serve as the basis for the development of an evaluation tool prototype to evaluate educational apps that contain virtual manipulatives. The components, relationships, framework, and evaluation tool prototype developed during this study advance the literature on virtual manipulatives and provide researchers with a common language to evaluate these apps.

(185 pages)

PUBLIC ABSTRACT

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Jennifer M. Boyer-Thurgood

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DEDICATION

To my parents, Keith and Kathy Marsh,
who taught me to treasure learning.

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To my chair, mentor, and friend, Dr. Moyer-Packenham, how can I begin to thank you for your belief in me, your trust in me, and the time you have invested in my development over the past 5 years? Sitting in your office during our first meeting I had no idea how much growth I would experience during my time in the doctoral program, but I felt your encouragement that first day and have every day since. Thank you for being a sustaining positive influence in my life both academically and personally. To my dissertation committee members, thank you for so generously sharing your time, talents, and insights with me. Thank you also to my doctoral cohort from all across the state of Utah and my fellow graduate assistants at USU. You have encouraged me every step of the way and have been true examples of mathematics leaders.

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Jennifer M. Boyer-Thurgood

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CHAPTER 1

INTRODUCTION

The invention and rapid adoption of touchscreen mathematics apps as tools for learning has dramatically changed the potential avenues and routines of learning mathematics. Both adults and children are quick to recognize the unique uses and benefits of apps and the potential for learning inherent in them (Cohen, Hadley, & Frank, 2011). While mathematics apps are a relatively new learning tool, the use of classroom technologies for learning has been highly encouraged for over 15 years by the National Council for Teachers of Mathematics (NCTM). The NCTM technology principle (2000) states that, “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (p. 24). The Common Core State Standards for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) also place importance on the use of technology for learning mathematics. Mathematical Practice 4, Use Appropriate Tools Strategically, describes mathematically proficient students as those who can consider the uses of available tangible and technological tools. Educational apps delivered via touchscreen devices are quickly becoming common tools that support the development of students’ mathematical understanding when used strategically.

Background and Problem Statement

While it may seem simple, the question, “What are ‘good’ mathematics apps?” is currently difficult to answer. Few research- or theory-based studies have been conducted

that provide a sufficient response to this question. Unfortunately, when selecting apps, teachers, educational leaders, and researchers are left to rely on the recommendations of bloggers, review websites, distribution store ratings, and distribution store descriptions.

The purpose of this exploratory qualitative study was to expand the explanations of and build theory about educational applications containing virtual manipulatives. A virtual manipulative is “an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge” (Moyer-Packenham & Bolyard, 2016, p. 16). Educational apps that contain virtual manipulatives and are designed for touchscreen devices are the focus of this exploratory qualitative study based in grounded theory. To date, no known research has been done on this specific group of apps designed for touchscreen devices.

Educational apps delivered via touchscreen devices are just beginning to be explored by researchers. The current research mathematics educational consists mainly of broad app-in-education overviews (Murray & Olcese, 2011) and narrow examinations of specific mathematics apps (Cohen et al., 2011). App-in-education overviews inform teachers and researchers about how the use of apps delivered via touchscreen devices may be affecting children in general and how children are using apps. App-specific studies link learning with a specific app, but cannot address which components of the apps leading to learning because no language or framework currently exists for doing so. No known research provides a way for teachers or researchers to discuss, compare, and evaluate apps based on their components so that they may select the mathematics apps

best suited for supporting students learning.

There is a continually growing body of research on computer-based mouse-driven educational apps that contain virtual manipulatives. A recent meta-analysis identified 32 studies comparing virtual manipulatives with other instructional treatments. When virtual manipulatives were compared with other instructional treatments, this produced an overall moderate effect (0.34; Moyer-Packenham & Westenskow, 2013). Research continues to point to the positive impact virtual manipulatives have on student learning. However, specific virtual manipulative components within apps have yet to be defined or categorized. These definitions, categorization, and an evaluation tool are needed to allow researchers to investigate and communicate about specific virtual manipulative components that may be contributing to learning.

Research also points to the idea that touchscreen applications may allow educators and students to do things in educational settings that they might not otherwise do. To take advantage of these affordances developers must make use of effective app components (Byers & Hadley, 2013; Murray & Olcese, 2011). In order to know what these components are, app components must be described and categorized. This work could then lead to future studies about specific app components, their affordances, and their effectiveness.

Educational apps that contain virtual manipulatives vary greatly. Currently, there is no common language for mathematics education researchers to use to communicate about these app components. Definitions of educational app components along with an evaluation tool are needed to unify the research base centered on virtual manipulatives

and on apps. This is needed in order to make future research on how and what children learn from specific app components possible. Teachers believe they can recognize a “good” app when they see it, but there is no language or framework for discussing what makes them good. Good for whom, good for what mathematical topics, and good when during instruction are unanswered questions. There is an urgent need to establish language, a framework, and an evaluation tool that can be used to describe and evaluate virtual manipulative educational apps.

Significance of the Study

With the overwhelming popularity of touchscreen devices and the existence of thousands of educational apps, definitions and descriptions detailing what educational apps are comprised of and how they are organized is needed. This exploratory qualitative study was designed to directly answer the recent call within the field to identify and better understand the anatomy of apps for children (Cohen et al., 2011), the need for new forms of app evaluation (Byers & Hadley, 2013), and to blend the educational research on virtual manipulatives with technical design research through affordance theory (Gibson, 1986).

During this study the researcher developed a conceptual framework to describe the anatomy of virtual manipulatives educational apps and the relationships among components of the apps. This framework is significant because there was no theory- or research-based framework available for teachers and researchers when considering which apps to select or describing app components. During the study, the researcher also

developed an evaluation tool prototype. This evaluation tool prototype is significant because, although many evaluation tools and criteria attempt to aid users in examining educational apps in general, no known research-based evaluation tool exists for evaluating the components of virtual manipulative educational apps.

Research Questions

The purpose of this exploratory qualitative study was to expand the explanations of and build theory about educational apps that contain virtual manipulatives. Because this study employed grounded theory methodology, the research questions were broad (Cresswell, 2012), but were focused by affordance theory, which served as a lens through which data were collected. The research questions guiding this study were as follows.

1. What components comprise the anatomy of virtual manipulative apps and how can these components be described and categorized?
2. How can the relationships among these components be described?
3. How can the anatomy of virtual manipulative apps be conceptualized?
4. How can this conceptualization aid in evaluating virtual manipulative apps?

Summary of Research Study Design

This study utilized an exploratory qualitative design, based in grounded theory. Grounded theory methodologies and affordance theory were used to frame all six phases of the study in order to: (1) identify 75 virtual manipulatives situated within educational applications (apps) designed for touchscreen devices; (2) conduct observer-as-participant structured and unstructured observations (Dunn, 1988; Pretzlik, 1994) of the apps; (3) categorize component data including field notes and memos using open and axial coding

(Charmaz, 2006; Glaser, 1978); (4) create a conceptual framework including an integrative diagram and written narrative based on theory generated through the observation, coding and analysis phases (Phases 2 and 3) to explain the relationships among categories; (5) develop an evaluation tool prototype based on the conceptual framework to evaluate educational apps that contain virtual manipulatives; and (6) use the evaluation tool prototype to evaluate 25 additional virtual manipulatives within educational apps. This systematic approach allowed the researcher to develop a theory based on the anatomy of the virtual manipulative apps, a conceptual framework, and an evaluation tool prototype for further research (Cresswell, 2012). Four data sources were used during the study including field notes and memos taken during observations, analytic memos created during axial coding (Phase 2), and panel member memos created during Phases 3 and 6. All of these data sources were appropriate when implemented as part of the constant comparative method (Glaser, 1992; Glaser & Strauss, 1967; Strauss, 1987). To increase the rigor of the qualitative analysis, the researcher followed the steps for carrying out the constant comparative method suggested by Boeije (2002). Affordance theory was used as the lens through which to view the data.

Scope of the Study

The purpose of this study was to expand the explanations of and build theory about educational apps that contain virtual manipulatives, not examine how students interact with apps. How students interact with educational applications and what they learn from these apps are very important research topics, but were not within the scope of

this study. Future research studies linking specific components with student learning are a logical next step. The terms defined, theory generated, and tools created within this study provide vocabulary and an understanding of educational apps that allow for future research on how these components affect student interaction and learning.

Only mathematics apps that contain virtual manipulatives were included in this study. The reason for this was two-fold. First, apps that are simply question/answer or text-based quizzes with little pictorial support and no digital conceptual experiences simply transfer old, nonconceptual, ways of experiencing mathematics to a new environment (Byers & Hadley, 2013). Manipulating a virtual manipulative on a touchscreen device is a completely different and conceptual experience when compared to tapping the touchscreen to select the right answer in a multiple-choice quiz. Apps that take advantage of the unique experiences touchscreen devices afford, were the focus of this study. Second, virtual manipulatives have been shown to be an effective means by which to teach mathematical concepts using mouse-driven input. Initial research shows that interactions with virtual manipulatives on touchscreen devices may be even more powerful and engaging than using mouse-driven input, especially for novice users (Cohen et al., 2011). For this reason, this study was limited in scope to apps that contained virtual manipulatives.

Last, this study was limited to apps available for use on iPads. The main reason for this limitation was access. The researcher had access to multiple iPads necessary for conducting all six phases of the study. Although only iPads were used, 20 of the apps included in the study were available in GooglePlay, the app store for iPad's biggest

mobile device competitor Android, in order to represent a small portion of the apps available there. Future studies could compare iPad and Android app components.

Definition of Terms

The following terms were defined for this study.

Affordance: a combination of features that acts as a cue for potential virtual manipulative use. This definition is based on the definition by Burlamaqui and Dong (2014).

Educational app: an application designed to teach educational content on a touchscreen device. This study specifically refers to educational apps that teach a mathematical concept.

Virtual manipulative: As defined by Moyer-Packenham and Bolyard (2016) a virtual manipulative is, “an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge” (p. 16).

Virtual manipulative app: An educational app that contains a virtual manipulative.

Virtual manipulative component: the parts of an app including the dynamic mathematical object and the features. The term “components” does not denote that these app elements are directly transferrable to other apps with no additional coding, but that these elements are parts of an app and that similar parts could be found in multiple apps. Identifying and describing additional app components is a major focus of this study.

CHAPTER II

LITERATURE REVIEW

The purpose of a literature review within a grounded theory study is to illustrate the gaps in the existing knowledge, thus providing a rationale for a theory-generating study (Cresswell, 2012). The first section of this review presents a more detailed overview of the role of the literature review in a study using grounded theory methodology. The remaining sections highlight the gaps present in the current knowledge concerning educational apps and support the expansion of explanations and theory concerning the nature of the anatomy of educational apps that contain virtual manipulatives. These sections include research related to educational apps including virtual manipulatives and affordances. The chapter concludes with a review of literature concerning the current state of learning app evaluation.

The Role of the Literature Review in a Study Using Grounded Theory Methodology

Several researchers (Cohen et al., 2011; Murray & Olcese, 2011) have appealed to the educational community for research designed to describe the anatomy of learning apps. Grounded theory methodology was adopted specifically as one appropriate response to these appeals. The research questions for this study were developed based on this research methodology and affordance theory. Because this study employed grounded theory methodology, it is generally considered most appropriate to synthesize related theories and frameworks in the Discussion section (Creswell, Clark, Gutmann, &

Hanson, 2003). This is because as data are collected, grounded theory methodology allows for theory to be generated as a result of a “zigzag” between data collection, analysis, and the related research literature (Creswell, 2012). During this inductive process a literature review evolves simultaneously with the data collection. While the Discussion section of this study includes the literature related to the analysis and theory building, a review of research is presented here for another purpose.

Charmaz (2004) pointed out that even when conducting a grounded theory study a review of literature is needed to produce initial information adequate to identify the broad areas of inquiry that ought to be considered by the qualitative researcher. While a researcher conducting a ground theory study is expected to approach the data on its own basis, without making conjectures on where it could lead before collecting it (Charmaz, 2004; Glaser, 1992; Strauss, 1987; Strauss & Corbin, 1994) the researcher is not assumed to be a blank slate.

Grounded theorists attempt to use their background assumptions... to sensitize them to look for certain issues and processes in their data... [They] often begin their studies with certain research interests and a set of general concepts...[or] *points of departure* (Charmaz' emphasis) to look at data...and to think analytically about the data... developing, rather than limiting, their ideas (Charmaz, 2004, p. 501).

In an attempt to conform to the prevailing wisdom of these qualitative scholars, this review presents the research that sensitized the researcher to this particular topic and the general concepts or *points of departure* that were considered prior to the study, including information that served to (a) guide the selection of a particular type of learning app, educational apps that contain virtual manipulatives, to (b) use affordance theory as a lens through which to consider the relationships among app components and what uses

they may afford the learner, as well as (c) inform the researcher concerning the current state of app evaluation to ensure that this study was relevant to the appeals of the educational community for a description of the anatomy of learning apps and a way to evaluate them. The following sections present these concepts.

Selection of App Type

The first point of departure that helped to focus the proposed study was the decision to focus on educational apps that contain virtual manipulatives. A review of literature concerning educational apps and virtual manipulatives is presented below to provide background information for readers concerning this particular point of departure.

Virtual Manipulatives

The first requirement when selecting apps for inclusion in the study was that they include at least one virtual manipulative. Moyer, Bolyard, and Spikell (2002) defined a virtual manipulative as “an interactive, Web-based visual representation of a dynamic mathematical object that presents opportunities for constructing mathematical knowledge” (p. 373). Dorward (2002) also described virtual manipulatives as “computer based renditions of common mathematics manipulatives and tools” (p. 329). In 2002, virtual manipulatives were largely Java-based applets designed for use on personal computers using a mouse as in the input device. The National Library of Virtual Manipulatives (NLVM; <http://nlvm.usu.edu>) is a large collection of these types of Java-based virtual manipulative applets. The Interactives available through the National Council of Teachers of Mathematics (NCTM) Illuminations site

(<http://illuminations.nctm.org>) are an example of similar computer-based virtual manipulatives that utilize Adobe Player.

In recent years, virtual manipulatives have also become available on a new platform, touchscreen devices, as part of educational apps. These apps utilize the touch and multi-touch technologies that touchscreen devices of all sizes afford (Chiong & Shuler, 2010). Virtual manipulatives designed for use on the touchscreen platform have been programmed in many different languages for both Android and Apple devices. In short, virtual manipulatives are now readily available in many different formats as a part of apps on a variety of different touchscreen devices and other media.

Educational Apps that Contain Virtual Manipulatives

Educational apps are different from computer-based virtual manipulative applets designed for mouse-driven interaction. These applets are small applications designed to perform one very simple specific task within the scope of a larger program. The term “applet” most commonly refers to a small Java program designed to be embedded in a web page and to function within a program separate from the web browser itself. This design was originally used to allow virtual manipulatives to be mouse-driven and run very quickly without having to call on the server. Apps designed for use on touchscreen devices are more robust than virtual manipulative applets, often containing elaborate app environments, are designed specifically to function as individual applications on handheld devices, and require a completely different input modality. However, the continually growing body of research on virtual manipulative applets may help inform

research on virtual manipulative apps.

Educational Apps

The term app, short for application program, describes a small self-contained computer program designed to perform a specific function. Applications have a long history in computing, but the shortened term “app” is most commonly used to describe applications specifically designed for mobile devices. An *educational app* is program intended for use on a handheld device specifically designed to teach or support learning for one or more mathematics concepts.

Since 1998 when the first highly recognized handheld device app Snake, for the Nokia mobile phone, appeared demand has driven a rapid expansion in the number and variety of apps (Bates, 2014). Increase in apps has also been fueled by the availability of app developer tools and the introduction of the app-based iPhone in 2007. Rapid app adoptions and app creation created a need for organization of these apps within distribution stores such as Apple’s App Store and Google Play. As a result, a wide-range of app categories have been assigned to apps by app developers, reviewers, and curators (Bates, 2014). Apps relevant to learning, like educational apps, appear most often in either in the Education category or the Games category of distribution stores (Shuler, Levine, & Ree, 2012).

Although apps delivered via touchscreen devices are increasingly suggested as tools for educational (McKenna, 2012), research pertaining to educational apps is currently limited to broad educational app overviews (Choing & Shuler, 2010; Murray & Olcese, 2011) and narrow examinations of specific mathematics apps (Cohen et al., 2011;

Paek, Hoffman, Saravanos, Black, & Kinzer, 2011; Risconscente, 2011). Only one known study addresses the middle ground of educational app design (Cayton-Hodges, Feng, & Pan, 2015). Practitioner articles that examine pedagogical and administrative implications for using apps in the classroom (Aronin & Floyd, 2013; Attard, 2013; Henderson & Yeow, 2012; McKenna, 2012) do not apply to this study and have not been included because they focus on practices and suggestions for using touchscreen devices in the classroom, not on describing and evaluating the apps themselves.

Educational App Overviews

Several educational app overviews describe the state of educational apps in general. In a 2012 study, Shuler described apps labeled as educational. In this study Shuler found that 80% of the top selling paid apps in the Educational category of Apple's App Store target children. The study also revealed that apps for toddlers and preschoolers (ages 2, 3, and 4) are particularly prominent. Apps for this age group make up 58% of the Educational category and mathematics is the second most popular category of Educational app (13%) behind general early learning (47%). This shows that a very large number of educational apps, especially those for young children, are available to users and that there is a demand for such apps. Because of this there is an urgent need for teachers and researchers to be able to adequately describe and be able evaluate educational apps.

Choing and Shuler (2010) examined how children use and learn from apps. In this study, parents reported that when children are given time to use mobile devices, 80% of the time they are using apps. In the same study, 64% of the children said that touchscreen

devices were “easy” or “very easy” to use and 53% of the children in the study did not need an adult to help them begin or continue to work with an app during the observation. Children also showed learning gains on a post-test after using a specific app. This shows that touchscreen apps are frequently used by children and provide opportunities for accessible learning experiences that are linked to learning gains. Because of this, educational apps are likely to continue to be considered important learning tools and must be able to be described and evaluated.

Murray and Olcese (2011) reviewed educational apps of many types in order to identify apps that provide novel learning experiences for children. In this study, 315 apps from Apple’s App Store were reviewed and categorized using Means’ (1994) four categories of technology type: tutor, exploration, tool, and communicate. Interestingly, only 56 of the apps identified as educational by the researchers were found in the Educational category within Apple’s App Store. Because of this, Murray and Olcese noted that the categories offered by distribution stores are of little use to educators in helping them make decisions or identify apps. The study found that the majority of the apps listed in the Educational category of the App Store were tutorial in nature, pointing to a misconception about what types of apps may be categorized as educational by app developers and distributors. The significance of Murray and Olcese’s study to this study is the arbitrary categorization of apps and the limitations that this categorization imposes on users trying to locate and select apps. In this study educational apps that contain virtual manipulatives were selected from the Apple App Store regardless of their category. This study also investigated different VL environments, including but not

limited to tutorial environments. This was done to add clarity to the types of environments that can be categorized as educational.

Cohen et al. (2011) conducted a study concerning the iPad's potential educational use for children ages 2- to 8-years old. In this study, 60 children were observed using iPads in one-on-one interviews and small group settings while their caregivers were interviewed separately about their family's technology ownership and usage. The findings of the study indicate that touchscreen technology offers accessible and meaningful experiences for children in this age group and that app design is critical in moving users from novice to mastery levels and in sustaining engagement. Simply delivering educational content on an iPad does not necessarily improve student achievement (Carr, 2012). Findings from the Cohen et al. study also indicated that it is potentially possible to identify and understand the components and anatomy of apps and that the adjustment of these components may help optimize learning. The findings of Cohen et al. supported their initial hypothesis that research is needed to identify the components that comprise educational apps, specifically those components that may promote learning. This study directly addresses this finding by expanding the explanations of and building theory concerning educational app components.

Educational App Research

While the studies presented in the previous section focus on the broad view of educational apps in general, very specific research has also been carried out on individual educational apps. Risconscente (2011) conducted a study in which 122 children used one app Motion Math Fractions, a fractions game app designed for the iPad, 20 minutes per

day for 5 consecutive days. The researcher reported that children's fractions test scores improved 15% on an assessment given at the end of the five days, student's self-efficacy increased, and students reported liking fractions more after using the app. The research hypothesizes that these positive results were due to the features of the app including instant feedback, scaffolding, time-limits, entertainment, physically tilting the iPad, and the high number of practices each child engaged in. However, without a clearly defined framework of components including features, researchers are limited in being able to link features and learning, which is why the current study was an important contribution to the research.

In another study (Paek et al., 2011), 59 second-grade students learned multiplication concepts via touchscreen applications and mouse-driven applications. These applications involved various instances of visual and auditory feedback. This study revealed that while there were no significant differences in input modality, touchscreen versus mouse-driven apps, there were significant differences in apps containing different feedback features. Students in groups where auditory feedback features and visual feedback features were provided showed significantly higher learning outcomes than students in groups where only visual feedback was provided. As a result, a combination of features was directly linked to learning outcomes. This study suggests that virtual manipulatives within apps on touchscreen devices can be effective learning environments for young children and also points to the importance of virtual manipulative components, understanding them, and leveraging them to improve students' learning experience.

Educational App Design Research

Cayton-Hodges et al. (2015) conducted a survey of educational apps as part of a larger research project aimed at developing a tablet-based assessment prototype for the Education Testing Service (ETS). In this survey 16 mathematics apps from Apple's App Store were each surveyed for 10 to 25 minutes. The survey focused on four dimensions: (1) the quality of the mathematical content, (2) feedback and scaffolding, (3) richness of interactions, and (4) adaptability of the applications. While these apps were surveyed with the purpose of identifying elements pertinent to assessment, several findings are relevant to this study. Researchers reported that the majority of educational apps available in the Apple App Store target preschool to elementary-age students. They also reported that existing educational apps include a variety of learning environments such as tutor environments, demonstration environments, game environments, test-prep environments, and e-textbook environments, with game environments being the most common. Additionally, feedback and scaffolding features are reported to be available in the majority of educational apps and take on a variety of different forms.

Affordances

The second point of departure was the selection of affordance theory as a lens through which to consider the anatomy of educational apps and how affordance theory informs the known categories of virtual manipulative affordances.

Affordance Theory

Affordance theory states that tools are perceived not only in terms of their

features, such as shape, color, and spatial relationships, but also in terms of possibilities for action called affordances (Gibson, 1977). Gibson suggested that affordances are likely noticed before individual features of an object and that affordances help to distinguish the entity from other entities. Affordances indicate possibilities for action and are perceived in a direct, immediate way with no sensory processing, usually before individual features are seen. Examples of physical objects perceived in terms of their affordances include buttons for pushing, knobs for turning, and handles for pulling. While these objects have features such as smooth rounded surfaces, levers, and springs that make these affordances possible, the user is most likely to perceive the affordances first and sometimes exclusively.

Within the context of educational apps, as with physical objects, affordances are the result of specific features, combinations of features, and the relationships among features. Building theory concerning app features, components, and the uses they afford users was a main focus of this study.

Virtual Manipulative Affordance Categories

Virtual manipulative affordances are defined for this study as, "...cues of the potential uses of an artifact by an agent in a given environment" (Burlamaqui & Dong, 2014). In this context, virtual manipulatives are the artifacts. Moyer-Packenham and Westenskow conducted a meta-analysis of studies comparing virtual manipulatives with other instructional treatments that resulted in the identification of five categories of virtual manipulative affordances shown to promote mathematical learning. These affordance categories helped to focus the application of affordance theory to the data

gathered concerning educational apps in this grounded theory study. The affordance categories identified by Moyer-Packenham and Westenskow are simultaneous linking, efficient precision, focused constraint, motivation, and creative variation. Four of these affordance categories are relevant to this study and will be discussed in detail. Identifying the presence or absence of these affordance categories among the data collected concerning educational apps and the programmable features that make them possible was a major focus of this study.

Focused constraint. According to Moyer-Packenham and Westenskow (2013), there are currently 17 studies that include empirical evidence showing that focused constraint contributes to student learning. Evans and Wilkins' (2011) study was an example in which focused constraint is evident. In this study, Evans and Wilkins observed that children using virtual manipulatives were more focused on the underlying geometric concepts they were studying than on simply solving the problem they were given because of the focusing and constraining features of the virtual manipulative. Students spent more time considering how to move, rotate, grasp, and flip geometric shapes when using the virtual tool as compared to those who used the physical tool. This extra attention in turn caused those using the virtual tool to attend more to the underlying mathematical concepts.

Creative variation. Creative variation is the result of features that “allow students to generate their own representations, [encourage] creativity and novelty, and [prompt] experimentation” (Moyer-Packenham & Westenskow, 2013). When students experience creative variation when using a virtual manipulative, they experiment more

(Clements et al., 2001) and exhibit significant learning gains (Trespacios, 2010).

Simultaneous linking. The affordance category of simultaneous linking refers to programmable features that allow a virtual manipulative to present multiple representations of a mathematical concept at the same time. Simultaneous linking is an affordance of a virtual manipulative if two representations change simultaneously in response to learner interaction. This allows the learner to experience multiple embodiments of a concept (i.e. dynamic, symbolic, numerical; Botzer & Yerushalmy, 2008) and affords learners the opportunity to link concrete representations with the symbolic (Sarama & Clements, 2009).

Haisting's (2009) research compared two different versions of one specific virtual manipulative and the versions' effect on student learning. An internet base-10 block virtual manipulative that included symbolic representation (numerals) along with interactive base-10 blocks was used with the first treatment group. The second treatment group used a virtual manipulative specifically designed for the study that did not include symbolic representation, only block representations. The researcher taught an instructional unit over a 4-week time period in which 71 first-grade students and their teachers participated. Findings suggested students who worked with virtual manipulatives that afforded simultaneously linking via symbolic representation features and a visual representation features developed a strong connection between the two.

Efficient precision. The efficient precision affordance category refers to the efficient manipulation and accurate presentations made available by virtual manipulatives (Moyer-Packenham & Westenskow, 2013). The combination of efficiency and precision

allow learners to learn at a faster rate than with physical manipulatives (Beck & Huse, 2007).

Understanding the affordances of a virtual manipulative was important in this study because in order for explanations of and theory about virtual manipulatives within educational apps to expand, the features that make affordances available to be perceived by the user must be considered. Virtual manipulative affordances are the result of less noticeable features (Gibson, 1977).

Current State of App Evaluation

The third point of departure prior to the study was the confirmation that learning app evaluations had no theory by which they may be designed. The research review that led to this confirmation was important to the study because it ensured that this study was relevant to the appeals of the educational and research communities for a description of the anatomy of learning apps and a way to evaluate them. There are currently many websites and individuals dedicated to reviewing educational apps. These sources vary greatly in their focus and priorities and offer several different tools to help educators identify ‘good’ apps. Specific tools were selected that represent large segments of evaluation tool types currently available to teachers and researchers. These representative tools, presented below, are of varying types and represent varying priorities

Evaluation Tools

One of the most well-known tools for evaluating educational apps is the Evaluation Rubric for iPod apps developed by Walker (2010). This rubric was originally

designed to help educators evaluate iPod apps and includes six evaluation categories: curriculum connection, authenticity, feedback, differentiation, user friendliness, and student motivation. Walker added a seventh category, student performance, in 2012.

While Walker's rubric suggests several criteria that may be important to educators, it has several key deficiencies. First, the criteria descriptions are not specific enough to provide a framework for thorough evaluation of educational apps. For example, optimal user friendliness is described by the rubric statement, "Students can launch and navigate within the app independently." However, User Friendliness could be attributed to many features in addition to navigation features. The category of User Friendliness needs to be described in much more detail in order for it to be more descriptive and useful to a teacher or researcher.

Second, many descriptions within the rubric are limiting. An example of this is the criteria of Student Motivation. Ideal student motivation according the Walker rubric is described as, "Students [who] are highly motivated to use the app and select it as their first choice from a selection of related choices of apps." This description does not describe the motivational aspects of the app, but the choice of apps at any given moment by a student. Motivation has a much broader implication and is the most frequently cited virtual manipulative affordance attributed to impacting student learning (Moyer-Packenham & Westenskow, 2016). In order for a tool to be effective, the evaluation criteria must be well defined.

There are also several other well-known tools that have been developed based on Walker's Rubric. The Critical Evaluation Instrument for Mobile Content-Based Apps

(Schrock, 2011) is one of these. This checklist includes 12 “content and components” criteria for educators to consider. These include Walker’s six original areas with the addition of reporting, sound, instructions, support page, navigation, and modalities. This checklist also has a place for reviewers to record general information such as app name, cost, size, creator, content area, grade level, iTunes application ranking, and an overall summary of the app and its potential classroom uses. Schrock’s tool is more comprehensive than Walker’s tool, but also has several deficiencies.

The first deficiency is that many of the criteria are subjective and not descriptive enough to allow for thorough evaluation. For example, Schrock’s Navigation criteria requires evaluators to check yes, no, or n/a after reading the following question, “Does the app use the touchscreen effectively throughout its use?” There could be many interpretations of this question and many elements that may contribute to effective touchscreen use. It is also dependent on the evaluator’s independent knowledge of touchscreen abilities. More objective, descriptive, feature based questions may be, “Does the app take advantage of the iPad gyroscope?” or “Does the app take advantage of multi-touch capabilities?”

A second deficiency of Schrock’s tool is its failure to provide a final recommendation. The checklist is just that, a checklist of criteria based on user perception and belief. It helps users to see what their overall opinions of the app are and to consider several criteria while using the app, but does not provide a way to summarize these results. It acts more as a log of information about the app than a tool that leads to app selection and evaluation. A final deficiency of both Schrock’s and Walker’s

evaluation tools, and a deficiency in all known educational app evaluation tools, is the omission of a comprehensive examination of app features at the component level.

Online Review Criteria

Nearly every website that reviews apps has its own set of review criteria. The majority of the review criteria are based on the priorities of the reviewer. One of these sites is <http://bestappsforkids.com>. The criteria for these reviews are based upon what the site authors believe educators need to know about the app. The evaluation criteria include quality, entertainment, value, and child friendliness. While these can be important criteria, they are not comprehensive, reflect subjective views, and lead users to think that apps that meet the review criteria lead to better educational.

In contrast, there is one known website <http://www.balefirelabs.com/apps/> that conducts reviews of educational apps using criteria based on researched-based literature on learning. The apps reviewed on this website are reviewed for instructional design (feedback for correct answers, error feedback, adapting difficulty, error remediation, mastery-based instruction, meaningful interaction, clearly stated learning objectives), usability design (relevant screen and sound use, learner support, easy-to-use interface, reports with actionable data), and additional data (in-app purchases, links to social media, push notifications, in-app advertisements). This website does attempt to evaluate apps based on research-based principals and is the most comprehensive criteria that was located during this review. However, it is unlike this study because it does not evaluate apps at the component level and does not consider features as discussed in the previous section.

Summary

Grounded theorists generally emphasize that researchers who propose grounded theory methodology should not allow previous knowledge of related research to excessively influence their analysis of the data. Rather they should use research findings to create points of departure from which to structure the study. Then as the data collection and analysis progress, they should look for literature that may help the researchers to synthesize and interpret the themes and categories emerging from the data. The points of departure that guided this study evolved from a study of concepts in the literature concerning educational apps and virtual manipulatives. These concepts served to guide the selection of a particular type of learning app, educational apps that contain virtual manipulatives, to the use of affordance theory as a lens through which to consider the relationships among app components and what uses they may afford the learner, as well as to inform the researcher concerning the current state of app evaluation.

This review illustrates that without a clear conceptual framework to describe mathematics apps and no research-based way to evaluate app components, educational apps are often reviewed and selected haphazardly. Apps are chosen for inclusion in studies or for classroom use based on convenience or based upon similarity to current classroom practices (Henderson & Yeow, 2012) and are reviewed based on noncomprehensive and often subjective criteria. There was an urgent need for research concerning the components of educational apps so that they may be effectively reviewed and described.

CHAPTER III

METHOD

The purpose of this exploratory qualitative study was to expand the explanations of and build theory about educational applications containing virtual manipulatives. This study was conducted in six phases: (1) 100 virtual manipulatives situated within educational apps designed for touchscreen devices were identified; (2) observer-as-participant structured and unstructured observations (Dunn, 1988; Pretzlik, 1994) of the virtual manipulatives were conducted; (3) component data including field notes and memos was categorized using open and axial coding (Charmaz, 2006; Glaser, 1978); (4) a conceptual framework including an integrative diagram and written narrative based on theory generated through the observation, coding and analysis phases (Phases 2 and 3) to explain the relationships among categories was created; (5) an evaluation tool prototype based on the conceptual framework to evaluate educational apps that contain virtual manipulatives was developed; and, (6) the evaluation tool prototype was used to evaluate 25 additional virtual manipulatives within educational apps.

Research Design

This study was an exploratory qualitative research design. *Exploratory qualitative research* is research that specifically “emphasizes developing theory from data” (Stebbins, 2001, p. 5). Exploratory qualitative research is an appropriate research type for this theory-generating study because it is “an act of gradual, structured, and theory-led heuristic expansion from an original set of models, explanations, and questions” (Reiter,

2013, p. 11).

Grounded theory methodology was employed as a means to carry out this exploratory qualitative research. *Grounded theory methodology* is a “general methodology for developing theory that is grounded in data systematically gathered and analyzed” (Strauss & Corbin, 1994, p. 273). Grounded theory methodology dictates that, “theory is developed during the actual research through continuous interplay between analysis and data collection” (Strauss & Corbin, 1994, p. 273). A methodology that facilitates this continuous interplay is the constant comparative method (Glaser, 1967, 1992; Strauss, 1987). The constant comparative method was used throughout the six phases of this study and included the constant comparison procedure developed by Boeije (2002). The details concerning the use of these methods are described in detail in the Procedures and Analysis section of this chapter. In addition, affordance theory (Gibson, 1986) was used as a lens through which to examine and evaluate the data.

Data Sources

There were four data sources used in this study. The first two sources were researcher field notes and memos taken during structured and unstructured observations of virtual manipulatives situated within educational apps. The third data source was analytic memos created by the researcher during axial coding that took place during Phase 3 of the study. The fourth data source was panel expert memos created by panel members during Phases 3 and 6

Field Notes

Information collected during both the structured and unstructured observations was considered field notes and was taken through inscription and transcription methods (Clifford, 1990). Inscription refers to the moment of observation in which the flow of observation is interrupted in order to jot down a word, phrase, or drawing so that details may be recalled later. Transcription is more interruptive and includes moments when the observer takes time to write down detailed information (Clifford, 1990). During structured observation, field notes were inscriptive with predetermined codes while during unstructured observations, field notes were more transcriptive allowing for open-coding to occur and new and novel data to be recorded. Fieldnotes for the initial 10 observations were recorded in Microsoft Word documents. The fieldnotes for the remainder of the observations were recorded in the observation protocol.

Memos

Three types of memos were utilized during the study. Memos as defined by Clifford (1990) are overall impressions and provide “thick descriptions” (Clifford, 1990, p. 52; Geertz, 1973), reflection, analysis, and interpretation of an observation. They are intended to be more coherent representations than field notes and to be used to generate a finished account of a specific happening or a summary. Researcher memos were made following each observation and will allowed the researcher to make meaning directly from the observation and field notes. Analytic memos were made during Phase 2 as the researcher engages in axial coding. Additionally, panel members made expert memos similar to researcher memos, during Phase 3 and Phase 6.

Instruments

One instrument, a researcher-developed observation protocol (Mulhall, 2003), was used during this study. The observation protocol was created at the onset of coding and analysis phases as an Excel Workbook. The protocol included an area to record detailed field notes about each observation and specific information about the app during the structured observation. During coding and analysis phases of the study, areas for open codes, axial codes, and axial code categories were added to each observation worksheet.

Validity and Reliability

Establishing validity and reliability in an exploratory qualitative research study is achieved through implementing several strategies put forth by Moschkovich and Brenner (2000). The strategies of prolonged engagement, persistent observation, and expert panel review assure that the data collected is adequate for capturing the major features to be studied. Prolonged engagement means that enough time has been spent gathering and analyzing data so that recurrent patterns become clear and very few new types of data are being collected. By observing 100 apps during the study, the strategy of prolonged engagement was met. Details about this can be found in Phase 2 of the Procedures and Analysis section. Persistent observation (Eisenhart & Howe, 1992) entails taking an analytical view of the data, looking for important patterns, and then seeking out further information that confirms or contradicts the emerging understanding of the researcher. As patterns emerged throughout the data analysis the researcher continually returned to the literature for this purpose. Expert panel member review is a strategy in which the

researcher's constructed understanding is presented to others for comment or revision.

This occurred twice during the study. Each time a panel of experts conducted app observations, reviewed codes, and reviewed descriptions created by the researcher.

Details about these two expert panel reviews can be found in Phase 3 and Phase 6 of the Procedures and Analysis section.

The strategies of thick description and purposeful sampling ensure that the results of the study are relevant in a broader context (Moschkovich & Brenner, 2000). Thick descriptions are detailed specifications about the dynamics and context of the study. By creating an observation protocol, these thick descriptions were easily captured as organized field notes and memos during each observation and throughout the coding and analysis process. Purposeful sampling (Palinkas et al., 2015), detailed in Phase 1 of the Procedures and Analysis section, ensured that apps included in the study were chosen to optimally inform the researcher.

The strategies of using audit trails and recording devices also ensured consistency within the study so that under the same circumstances, the same results may be found (Moschkovich & Brenner, 2000). All observation memos and field notes were kept within the protocol along with memos and field notes concerning coding decisions, and expert panel memos. This was done so that others may examine the protocol easily outside of the study. Recording devices were used during Phase 2 of the study to achieve and prove the dependability of the study.

Finally, the strategy of thoroughly defining the researcher's role at the onset of the study helped to ensure the neutrality of the research (Moschkovich & Brenner, 2000). For

this study the researcher took on the observer-as-participant role and completed structured and unstructured observations of apps. The researcher also explicitly acknowledged her own subjectivity (Peshkin, 1988) and used this subjectivity to produce a distinctively qualitative product by enlisting background knowledge to begin the study and to inform the analysis of the data (Howe & Eisenhart, 1990). Details about this role can be found in Phase 2 of the Procedures and Analysis section.

Procedures and Analysis

The nature of an exploratory qualitative study is that exploration leads to the development of theory. This theory can then support the development of a conceptual framework based on phenomena evident in emergent patterns in the data (Marshall & Rossman, 2010). In addition, grounded theory methodology explicitly involves “generating theory and doing [research as] two parts of the same process” (Glaser, 1978, p. 2). Because of this, the study procedures and analysis are intertwined.

This study was conducted in six phases (see Table 1). Phase 1 included app identification and the selection of 100 virtual manipulatives. Phase 2 included app use, component observation, and data cataloguing. Phase 3 included qualitative data coding and qualitative analysis of the data collected during. During Phase 4 a conceptual framework was developed based on the coding and analysis in the previous phase. Phase 5 was the development of an evaluation tool prototype for educational apps and Phase 6 included use of the evaluation tool prototype to evaluate 25 additional virtual manipulatives within educational apps.

Table 1

Study Phases

Phase	Activities
1	App identification and virtual manipulative selection
2	App use, component observation, and data cataloguing
3	Qualitative data coding and qualitative analysis
4	Conceptual framework development
5	Evaluation tool prototype development
6	Evaluation tool prototype use

Phase 1: App Identification and Virtual Manipulative Selection

Phase 1 of the study included identifying virtual manipulatives within educational apps appropriate for children ages 3 to 10 and selecting 100 virtual manipulatives from these apps. Educational apps that contain virtual manipulatives were selected for this study because they potentially offer the novel educational situations for users (Byers & Hadley, 2013). Each of the apps selected also aligns to at least one Common Core State Standard for Mathematics (CCSSM). A list of the selected apps and one associated CCSSM can be found in Appendix D.

App identification. In exploratory research, the choice of inclusion in the study is not random. Inclusion is “predicated by the logic of analyzing the richest, most telling, [instances] and to unveil the thickest and most telling connection” (Reiter, 2013, p. 8). This mandated that the selection of apps was intentional (Palinkas et al., 2015). To do this several steps were carried out. First, educational apps available in the Apple iTunes store that contained virtual manipulatives were identified and procured. From these apps 100

apps were intentionally selected. This process required the researcher to download 137 apps because it took this number until the researcher was able to locate 100 apps that contained virtual manipulatives. An excerpt from an observation in which the app was determined to be a nonvirtual manipulative app is included in Appendix B. The apps that did contain virtual manipulatives and were selected ranged in cost from free to \$10 (Byers & Hadley, 2013). The researcher received a graduate student enhancement award to fund the purchase of these resources. The selected apps came only from the Apple iTunes store. iTunes apps were used because of the devices available to the researcher. However, 20 of the selected apps were also available for purchase or free download in other app stores such as Google Play and the Amazon App store. This was done in order to ensure that the study included resources available for a variety of touchscreen devices.

Virtual manipulative selection. The virtual manipulatives used in this study were limited to those that address mathematical topics appropriate for children ages 3 to 10 as determined by app developers. This age group was selected because the majority of the researcher's teaching experience has been with this age group and because there are a wide variety of educational apps currently available for these ages. Over 80% of the top selling paid apps in the Education section of the iTunes store target children (Shuler et al., 2012). Early learning is the most popular app subject (47% of all apps) and mathematics the second most popular subject (13% of all apps). A growing body of research also points to the effectiveness of computer-based virtual manipulatives for children in early childhood and elementary age groups (Moyer-Packenham & Westenskow, 2013). For these reasons, mathematics apps for children ages 3 to 10 that

included virtual manipulatives were an important area of research.

Educational apps were chosen from the Apple iTunes store based on several criteria to ensure purposeful sampling of virtual manipulatives and features. Apps were selected based on: (1) popularity as suggested by Byers and Hadley (2013), (2) online reviewer recommendations, and (3) awards received. Selecting apps based on this range of criteria ensured that a wide variety of virtual manipulatives were represented and that the breadth of the industry was considered. Because of the exploratory nature of this study and the grounded theory methodology, the researcher was open to other selection criteria emerging, but none did.

Recommendations by private app review organizations. A Google search on September 1, 2015, revealed that over 45 websites advertise mathematical learning iPad app reviews and rankings. Unfortunately, these reviews and rankings are based upon various criteria, most of which are not research-based and evaluate only subjective app qualities and not educational app components. However, apps ranked highly by these websites are often among the most downloaded and most used apps. Because of that, it was important to include apps that were ranked as excellent using other metrics.

Awards and recognition received. Apps receive awards and recognition from the app stores in which they are available and from private technical, educational, and parental groups. Well known technical award groups include Best Mobile App Awards and the Apple Design Awards. Educational and parental entities often review and recommend educational apps based on a wide variety of subjective criteria. Prominent recommenders include EducationWorld.com, BalefireLabs.com, Graphite.com, and

TeachersWithApps.com. These organizations and individuals endorse and award apps based on subjective impressions and checklists. It was important to consider the apps recommended by these reviewers because recommenders who are familiar with the field see them as valuable or well-designed resources. No known iPad research includes apps for this reason. Table 2 shows the assignment of virtual manipulatives within apps to the development and testing groups.

Phase 2: Observations and Data Cataloguing

In Phase 2 the researcher conducted an observation of each educational app on iPads owned by the researcher and linked to one iTunes account. Each iPad was running the latest version of the Apple iPad software and was connected to a high speed Wi-Fi connection. The volume, gyroscope, accelerometer, multi-touch capabilities, and front and rear cameras were enabled. The cameras were used for features within the apps and not used to record the researcher. Each iPad was housed in a protective case with no screen cover to allow for full movement of the iPad and full function of all possible features.

Table 2

Assignment of Virtual Manipulatives within Apps to Development and Testing Groups

Selection criteria	Total #	Assigned to the development group	Assigned to the testing group
Popularity	60	45	15
Online review recommendation	20	15	5
Awards received	20	15	5
Total	100	75	25

Observer-as-participant role. Due to the nature of both structured and unstructured observations the researcher assumed the observer-as-participant role (Atkinson & Hammersly, 1994; Gold, 1958; Junker, 1952) throughout the study. Thoroughly defining this role adds validity to the study by detailing the neutrality of the researcher (Moschkovich & Brenner, 2000). Observers-as-participants complete unstructured and structured observations in tandem with participation and strive to strike a balance between observation and natural participation. They have brief interactions with numerous informants (e.g., in this case, Bugs and Numbers and Motion Math Fractions, apps in this study) and are more likely to recognize overall patterns and trends than other (Gold, 1958) researcher types. In this study, the researcher alone interacted with one app at a time through unstructured and structured observations and recorded thick descriptions in the observation protocol. These observations were done in much the same way and employed the same techniques as human participant observations (Dunn, 1988; Pretzlik, 1994). For example, in a human-to-human observation the participant might be prompted to complete some task while the researcher observes. In a similar way, each app was prompted to act or react while the researcher observed and reacted to the app's response.

To bring additional validity to this role, seven app observations were videotaped using a camera built into the researcher's laptop so that she could act as a full observer of these apps. The researcher reviewed these video recordings, made field notes and memos based on the videos, and coded the seven observations during the review of the video to ensure that the observations made while the researcher was taking on the observer-as-

participant role aligned with the observations made while the researcher was just an observer. The use of recording devices adds validity to the study (Moschkovich & Brenner, 2000).

Unstructured and structured observations. The two different observational approaches, unstructured and structured observations, stem from two different paradigms and are both used by the observer-as-participant researcher. Researchers who use unstructured observations contend that it is impossible to separate the researcher from that being researched no matter how objective one attempts to be (Mulhall, 2003). On the other hand, researchers who rely upon structured observations attempt to eliminate their own objectivity by relying on predetermined observation protocols. They attempt to remain objective and not “contaminate” the data (Mulhall, 2003). Pretzlik (1994) claimed that both observation types may be used in the same study. In this study the researcher used both structured and unstructured observations. The purpose of the unstructured observations was for the researcher to examine educational apps from a user point-of-view, to have an authentic exploratory interaction, and to take extensive and detailed field notes on each app. The purpose of the structured observation was simply to gather basic information about each app (e.g., the name of the apps, the publisher, and the cost).

An unstructured observation was carried out first with each educational app. During this observation the researcher approached the app with no predetermined goals, categories, or coding scheme. The aim was to investigate the app as a user might for the first time and develop ideas about the components and their relationships. Over the course of the study, as data were gathered and experience gained (Mulhall, 2003, p. 307),

the observations also included a comparison to other apps and the presence or lack of features that were initially expected.

The focus of the unstructured observations was what the virtual manipulative “said,” what it “did,” and how it “reacted.” Virtual manipulatives within apps often include audio features that provide users with information. This audio information, what the app “said,” was recorded using field notes during the observation in much the same way that human participants’ verbal utterances might be recorded (Mulhall, 1998). Visual images or displays present in the virtual manipulative were recorded in the field notes similarly to the physical behavior and body language of a human participant (Mulhall, 1998). In addition, screenshots were taken at times to capture events that were difficult to describe within the field notes. What the virtual manipulative “did” and how the virtual manipulative “responded” to touch or user movement, was also be recorded and treated similarly to human participant actions and behaviors (Dunn 1988; Mulhall, 2003). Throughout the unstructured observation the researcher recorded extensive narrative field notes and wrote memos to accompany those field notes. The aim was to record thick descriptions of all observations within the observation protocol to add validity to the study (Moschkovich & Brenner, 2000).

During the structured observations the researcher collected the following information about each app: (1) app name, (2) publisher name, (3) intended age group, (4) related Common Core State Standards for Mathematics (CCSSM) as determined by the app publisher, (5) mode(s) of interaction, (6) selection criteria, (7) awards received, (8) iTunes ranking, and (9) recommendations from private reviewers. Interestingly, the

researcher initially planned to conduct the structured observations first. However, it became evident that the structured portion of the observation needed to be conducted last. Conducting the structured observation prior to the unstructured observations gave the researcher detailed information about the app that a typical user would most likely not know and this affected the way the app was observed. For example, if in the structured observation, the researcher had recorded that the app had received a particular award or that reviewers really enjoyed a particular aspect of the app, the researcher's interaction with that app was influenced. By conducting the unstructured observation first, the researcher was better able to maintain an open perspective consistent with grounded theory methodology and to write open and extensive field notes based solely on experiences with the app.

Phase 3: Qualitative Data Coding and Analysis

The constant comparative method (Glaser, 1992; Glaser & Strauss, 1967; Strauss, 1987) was employed throughout the coding and analysis processes. Through this method the researcher was able to develop theory inductively by “categorizing, coding, delineating categories and connecting them” (Boeije, 2002, p. 393). The constant comparative method calls for comparison during all stages of coding and analysis, effectively blurring the lines between what may be clearly considered coding and what may clearly be seen as analysis. In order to ensure rigorous qualitative coding and analysis and to engage in using the validity strategy of persistent observation (Moschkovich & Brenner, 2000) three of the five steps for carrying out the constant

comparative method suggested by Boeije were followed. The two steps that were omitted apply only to observations of pairs of participants.

Coding and analysis step 1: Comparison within a single observation. During this step, open coding occurred (Charmaz, 2006) as every field note section for the first observation was studied and coded with a descriptive open code. This same process was conducted individually for each of the remaining 74 individual app observations that followed. In some studies, a rigid coding scheme is created prior to the study. However, grounded theory mandates that the codes should emerge from the data itself. Each open code was assigned to the portion of the field notes it described by being placed in the cell to the right of it. By coding in this way an audit trail was created detailing the origin of the initial codes within the field notes. This helped the researcher to know exactly where the code had emerged.

The aim of the initial observations and initial open coding was to begin to understand the overall nature of the apps. The results of this coding and analysis step were the development two initial open codes- object being manipulated and elements outside of the object.

Coding and analysis step 2: comparison of apps with similar components. After each of the 75 apps were observed and given an open code, apps with similar components were compared. Axial coding was used to sift through data in an effort to focus and synthesize the open codes, to explain larger segments of the data, and to connect the most significant and/or frequent open codes (Charmaz, 2006). Decisions were made during this step concerning which codes made the most analytic sense when

considered in the context of both apps being compared. If codes were combined the researcher returned to all previous observations and reevaluated the instance to which the code was assigned to assure it adequately described the instance and was applied to all observations (Boeiji, 2002). Apps were also analyzed during this step for positive and negative instances codes.

During this step categories of codes also emerged. These codes were then grouped within the Excel spreadsheet and given a category code. During this step the researcher also returned to the literature to gain clarity concerning emerging codes and categories. Each major coding change within the Excel file was saved as a new file so that an audit trail of changes was created.

The aim of this step was to further conceptualize educational apps and to refine the study codes and categories. This process continued until all apps had been compared to all other apps with similar components. The results of this coding and analysis step included an extensive number of axial codes and categories necessary to adequately describe the various components of apps that contain virtual manipulatives.

Coding and analysis step 3: Comparison of observation. In this third step, a panel of three experts member-checked three virtual manipulative educational apps and provided feedback about the codes and categories created to describe the app. The panel members were each mathematics education faculty members at three different universities. Each panel member had extensive experience with educational apps, had published virtual manipulative research, and is currently involved in app research. The aim of this step was to further improve the conceptualization of the educational apps,

ensure the rigor of the constant comparative method, and to present the researcher's observations, codes, and categories to others for comment and revision (Moschkovich & Brenner, 2000). The results of this coding and analysis step included a more refined list of codes and categories and expert panel memos.

Phase 4: Conceptual Framework Development

In Phase 4 the goal was to create a conceptual framework, including an integrative diagram and a written narrative based on theory generated through the observation, coding and analysis phases (Phases 2 and 3), to illustrate and describe the composition of educational apps and the relationships among the components within them. The first step in framework creation was to conceptually define the categories created during Phase 3. To do this the literature was consulted to help clarify and validate terms and concepts that have been used and developed during coding. Apps and data were also revisited, as dictated by the constant comparative method, in order to create and refine detailed definitions and descriptions of each category.

Once the categories were conceptually defined, axial coding techniques (Strauss, 1987; Strauss & Corbin, 1990, 1998) were used to describe the interconnected relationships that exist among the categories. One purpose of axial coding is to sort, synthesize, and continue to organize data (Creswell, 1998). Also during this stage an integrative diagram was created to illustrate the interactive nature of the app components and to illustrate a theory of description and evaluation for future analysis (Charmaz, 2006). This framework, including the integrative diagram and a written narrative, were used during Phase 5 and also stand alone as a product of the study. The framework will

serve to eliminate ambiguity during future educational app descriptions and evaluation as well as provide terms by which to explain future educational app observations.

Phase 5: Evaluation Tool Prototype Development

In Phase 5, the conceptual framework developed during Phase 4 was used to develop an evaluation tool prototype to evaluate educational apps that contain virtual manipulatives. Historically, Likert-type scales have been most commonly used and deemed effective for computer-based software evaluation tools (Bangert-Drowns & Kozma, 1989; Chang & Osguthorpe, 1987; Rowley, 1993). However, these tools have come under severe scrutiny (Squires & Preece, 1999) as computer-programs have become more sophisticated, the number of computer device types has increased, and technical knowledge for teaching has become increasingly important (Mishra & Koehler, 2006).

The evaluation tool prototype contains detailed questions about app components organized by category based-on the theory and conceptual framework developed earlier. The questions focus on components that are factually present in the app, not how the user would rate these components. Based on the theory developed during this study and the conceptual framework, it is hypothesized that known that certain features and combinations of features make certain affordances and environments available to users. Once users answer all of the questions presented by the tool, the evaluation tool prototype generates information about what possible affordances the app affords, the possible environment type afforded by the app.

Phase 6: Evaluation Tool Prototype Use

During Phase 6, the researcher evaluated the 25 apps reserved for testing using the evaluation tool prototype developed during Phase 5. This phase allowed the researcher to apply the results of the analysis to new and novel apps in order to identify how the analysis results were confirmed by new data and how they may be lacking. Returning to the data once again to improve the analysis is in line with the constant comparative method and grounded theory.

The educational apps evaluated during this phase were selected at the onset of the study and reserved for this phase. None of them were used during Phases 1 through 5, but proportionally represented all of the app selection criteria. Each app was reviewed in much the same way that the previous 75 development apps were reviewed. The researcher evaluated each app using the evaluation tool prototype and also repeated the coding and analysis steps described in Phase 3. Field notes were taken when evaluation tool prototype elements were lacking.

Following the researcher's analysis of the 25 evaluation apps, one member of the expert panel was trained to use the evaluation tool prototype and then asked to review three of the 25 evaluation apps. This double-coding provided data about the usefulness of the tool and whether or not the tool was transferable to educational apps beyond those used to develop it. At the conclusion of Phase 6, the data gathered through researcher evaluations, the panel member evaluations, and the researcher's final reflections concerning the study were used to inform improvements in the evaluation tool prototype.

CHAPTER IV

RESULTS

The purpose of this exploratory qualitative study was to expand the explanations of and build theory about educational apps that contain virtual manipulatives. The research questions guiding this study were as follows.

1. What components comprise the anatomy of virtual manipulative apps and how can these components be described and categorized?
2. How can the relationships among these components be described?
3. How can the anatomy of virtual manipulative apps be conceptualized?
4. How can this conceptualization aid in evaluating virtual manipulative apps?

In order to answer these questions this exploratory qualitative study, based in grounded theory, was conducted. The results of this study are presented based on the six procedure and analysis phases that led to the results for each study question. The conceptual framework will be presented first, even though it was not developed until Phase 4, so that the remainder of the results can refer back to and support the framework. Following the conceptual framework, the procedure and analysis phases will be presented in the order in which they took place.

Conceptual Framework

The conceptual framework was developed during Procedure and Analysis Phase 4 in order to answer research question 4. This research question focused on how the anatomy of educational apps that contain virtual manipulatives can be conceptualized. The conceptual framework is based on the six results of the study (see Figure 1).

Study result 1 was that there are two components of virtual manipulatives within

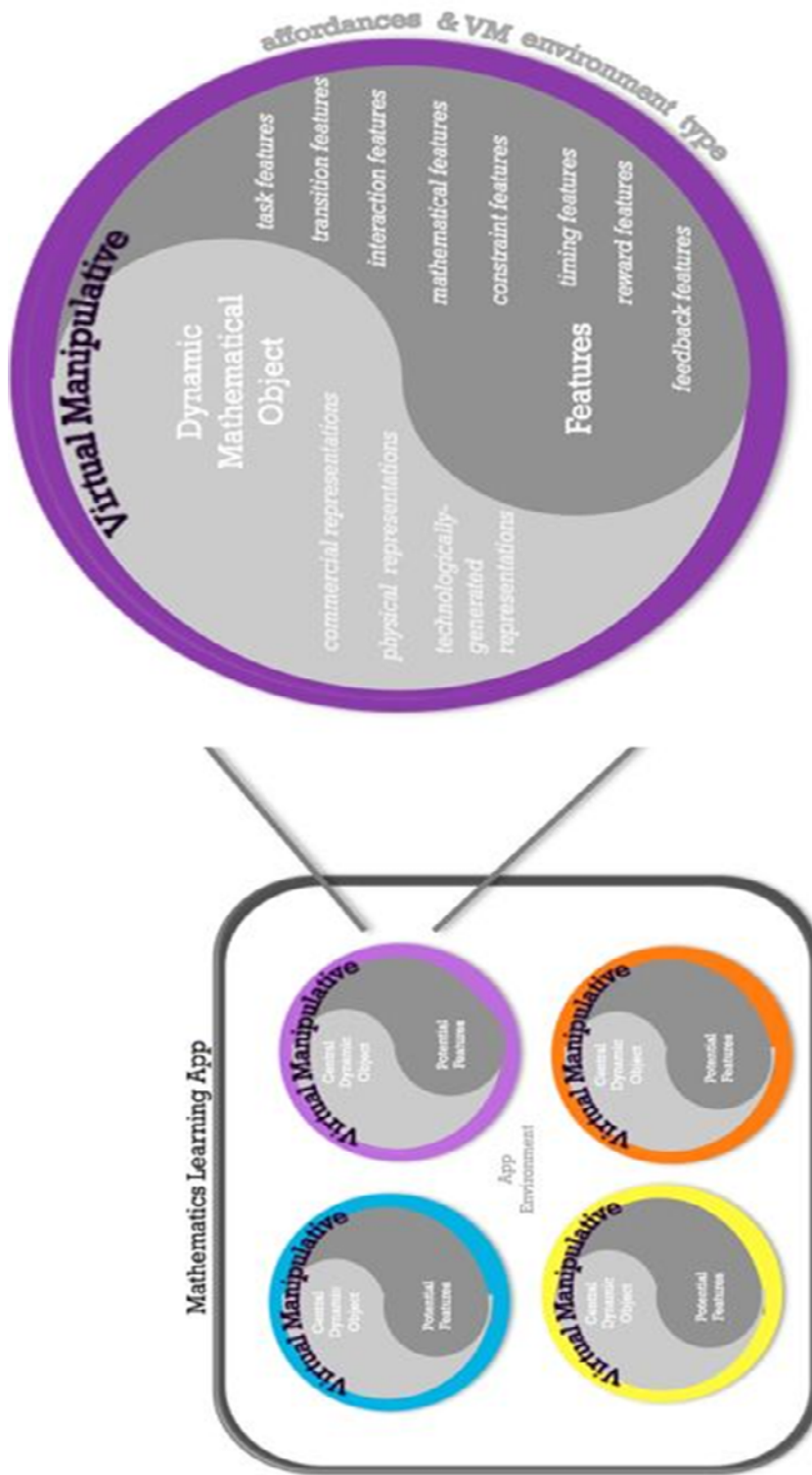


Figure 1. Conceptual framework: Anatomy of educational apps that contain virtual manipulatives.

apps: a dynamic mathematical object and features. This result emerged during Procedure and Analysis Phase 3 and is discussed in detail within that section of this chapter. A dynamic mathematical object is the central component within a virtual manipulative that the user manipulates, meant to be a representation for a mathematical concept that can be manipulated. Features are the programmable elements of a virtual manipulative that determine how the dynamic mathematical object behaves when operated on by a user. To represent this result within the conceptual framework, the structure of a virtual manipulative, including these two components, is shown enlarged on the right side.

Study result 2 was that there are three different types of dynamic mathematical objects. To represent this result within the conceptual framework the three possible types of dynamic mathematical objects: (1) commercial manipulative representations, (2) physical object representations, and (3) technologically generated representations were listed in the left of the large circle in the framework. Study result 3 was that there are eight categories of features. These eight categories are listed to the right of the large circle in the framework and include: task, transition, interaction, mathematical, constraint, timing, reward, and feedback. Study result 4 was that within one virtual manipulative there can be one or multiple dynamic mathematical objects. Study result 5 was that varying relationships can exist among the dynamic mathematical object and features within one virtual manipulative. This is represented by the swirling image in the framework. Study result 6 was that virtual manipulatives can appear in different ways and have varying relationships within one educational app. This is represented in the framework a small conceptualization of an app.

The conceptual framework represents a summary of the study results. However, the many details of the results that led to the creation of the framework and the results have yet to be explained in detail. The subsequent sections of this chapter are presented in order of the Procedure and Analysis Phases that yielded the results that answer the researcher questions and were used to construct the conceptual framework. Within each section the results are discussed and tied back to the conceptual framework. The following table illustrates the organization of this chapter (Table 3).

Table 3

Chapter Organization

Section heading	Research question addressed	Result(s)
Conceptual framework	Question 3	A conceptual framework was developed
Phase 1: app identification and selection	Precursor to answering question 1 and question 2	N/A
Phase 2: app observation and data cataloguing	Precursor to answering question 1 and question 2	N/A
Phase 3: data coding and qualitative analysis	Question 1	Result 1 Result 2 Result 3 Result 4
	Question 2	Result 5 Result 6
Phase 5: evaluation tool prototype development	Question 4	An evaluation tool prototype was created
Phase 6: evaluation tool prototype use	Question 4	The evaluation tool prototype was tested and refined

Phase 1: App Identification and Selection

Procedure and Analysis Phase 1 included the initial setup for the data collection within the study. No results emerged during this phase, but this phase was the precursor for the results that emerged in later phases to answer research questions 1 and 2. Phase 1 included identifying and selecting 100 virtual manipulative educational apps. These 100 apps were selected and assigned to either the development group or the testing group as shown in Table 4. A table including the names of the apps assigned to each group can be found in Appendix A.

Popular apps were identified first. To identify these apps, the researcher searched the education category of the iTunes app store. The most popular paid educational apps in the education category were considered first. The name and publisher of apps that looked like they may include a virtual manipulative were recorded in the App Identification Sheet within Excel. Additionally, awards or recognitions the app received that were listed in the app description within iTunes were also recorded. The most popular free

Table 4

App Selection and Assignment to the Development or Testing Group

Selection criteria	Total # of VMs	Assigned to the development group	Assigned to the testing group
Popularity	60	45	15
Online review recommendation	20	15	5
Awards received	20	15	5
Total	100	75	25

educational apps in the educational category were then considered. The information for apps in this category that looked like they may include a virtual manipulative were then recorded in the App Identification Sheet. App publishers often list awards their app has won and groups who recommend their app in the app description on iTunes. This information was also recorded.

Following the search for the most popular paid and most popular free apps in the education category a similar search was conducted in the games category, because many educational apps are often found there (Shuler et al., 2012). The most popular apps within the subcategories of the education category were then considered. This included apps in the Preschool and Kindergarten, Elementary School, and Math subcategories. Table 5 shows an excerpt from the identification sheet used during this search.

Once over 60 popular apps that appeared to contain virtual manipulatives were identified through the search of the iTunes store for popular apps, the researcher began searching for apps that had won awards. This search began by doing a Google search for the awards listed for apps on the App Identification Sheet. The researcher reviewed the recipients of 15 different awards including the Parents Choice Award, The Best Mobile App awards, the Kapi Awards, the Tabby Awards, Common Sense Media award winners, Educational App Grand Challenge Award winners, Best App Ever Awards, and the Webby Awards. Virtual manipulative educational apps that had received awards were then added to the identification sheet. In many cases apps that received awards were also popular apps in iTunes and were already on the app identification sheet. The search for award winning apps not listed as top apps in iTunes continued until 25 apps were

Table 5

App Identification Sheet

App	Publisher	Popularity	Award(s) listed in iTunes	Recommendation listed in iTunes
Todo telling time	Enuma, Inc.	Education category> preschool and kindergarten	Best design-launch educational conference	N/A
Motion math: Hungry guppy	Motion Math	Education> elementary	Parent's choice silver award winner	Moms with Apps
Math planet	Playpower Labs	Education> elementary	National stem game competition Educational app grand challenge NY "gap app" competition	N/A
Busy shapes	Edoki Academy	Games category	Parents' choice award 2014 Children's technology review awards	The Good App Guide Common Sense Media <i>USA Today</i>

identified. Only 20 apps were needed for the study, but the researcher had not yet downloaded the apps to verify that the apps contained a virtual manipulative, so five extra apps were identified in case some did not meet the criteria for inclusion.

Next, apps recommended by online reviews were identified. This search began in a very similar manner to the search for award winning apps. The online recommendation sources listed for apps on the app identification sheet were used to begin the search. Online review sites were searched until 25 apps emerged that had not already been selected based on popularity or awards. Similar to the process for the award winning apps, five extra apps were included in case some of the apps did not include virtual

manipulatives and meet the criteria for inclusion. Last, the apps were downloaded to the researcher's iPad from iTunes.

Phase 2: App Observations and Data Cataloguing

Once the apps were downloaded, the researcher commenced Phase 2. Phase 2 included 75 app observations and data cataloguing. This Phase was a precursor to answering research questions one and two. In Phase 2 the researcher took on the observer-as-participant role as described in detail in Methods Chapter III. In this role the researcher participated with each app and also recorded field notes and memos as an observer. To add validity to the study, seven app observations were video recorded and then reviewed by the researcher to ensure that the observations made while the researcher was taking on the observer-as-participant role aligned with the observations made while the researcher was just an observer. When the videos were reviewed there were very few differences between the field notes taken while watching the video and the original field notes, validating the researcher's observer-as-participant role.

Each of the 75 app observations included both an unstructured and a structured observation. During the unstructured observations the researcher took extensive field notes and memos as she interacted with each of the 75 development apps. The field notes were a detailed description of what the researcher did, what the app "said," what it "did," how it "reacted," and memos detailing the researcher's thoughts. Each field note was entered into a new cell within an Excel spreadsheet. Columns were then created for future coding and the worksheet format allowed sections of the field notes to potentially

be moved, sorted, and color-coded efficiently. An excerpt from the field notes taken during one of the 75 observations is presented in Figure 2.

During the structured observation the researcher collected the following information about each app: (1) app name, (2) publisher name, (3) intended age group, (4) version number, (5) cost, (6) “freemium” purchase option (7) related Common Core State Standards for Mathematics (CCSSM) as determined by the app publisher, (8) selection criteria, (9) awards received, (10) iTunes ranking, and (11) recommendations from private reviewers.

Field Notes and Memos	
Researcher:	Opened the app and triangle grid paper appears Dragged double hexagon out
App:	Hexagon snaps to grid
Researcher:	Tried to double tap the double hexagon
App:	Hexagon didn't move onto the grid or respond in anyway
Researcher:	Overlaid a triangle onto the hexagon Drug another triangle out Rotated the second triangle using arrow on overlay
Memo:	Would be really cool if a gesture could rotate the shape
App:	Clicked on drop icon- can change color and set text. Color and text are duplicated for every similar shape.
Researcher:	I typed in 1/3.
Memo:	This label might work for one example, but not another... the labels can't be customized for different examples unless you save this sheet and open another one
Researcher:	Plus sign icon copies the shape
Memo:	Shape is set so that it will not resize. This is so it will fit the grid- focused constraint!
Researcher:	Changed shapes to solids instead of transparent using toggle switch

Figure 2. Field notes excerpt from the unstructured observation of the Pattern Blocks App by Braining Camp observation.

Phase 3: Qualitative Data Coding and Analysis

Phase 3 involved the qualitative data coding and analysis for the data collected during Phase 2. Within Phase 3 results emerged to answer research questions one and two. During Phase 3, three coding and analysis steps for carrying out the constant comparative method suggested by Boeije (2002) were implemented to ensure rigorous qualitative coding and analysis and to engage in using the validity strategy of persistent observation (Moschkovich & Brenner, 2000). These coding and analysis phases are explained in detail in Method (Chapter III). During coding and analysis Step 1, comparisons occurred within single observations. During coding and analysis Step 2, comparisons occurred among apps with similar open codes. During coding and analysis Step 3, comparisons occurred between observations.

Coding and Analysis Step 1: Comparisons Within Single Observations

Coding and Analysis Step 1 began as soon as the first observation had been completed. The researcher stopped after this observation, reread field notes, considered what could emerge, and then completed the next observation. After the second observation the same analysis process occurred and the researcher began to record descriptive open codes for sections of the field notes. These open codes were later used during Coding and Analysis Step 2 to create axial codes that would describe groups of open codes. This process of taking field notes and creating open codes was then repeated for all 75 apps in the development group. Excerpts from the field notes of three observations and the open codes for these excerpts are presented in Figure 3.

Coding and Analysis Step 2: Comparisons Among Apps with Similar Open Codes

During coding and analysis Step 2 axial coding occurred as observations with similar open codes were group and assigned axial codes. Groups of axial codes were then grouped and became axial code categories. From this process four results answering

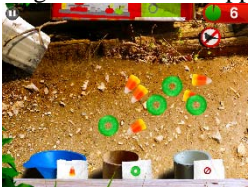
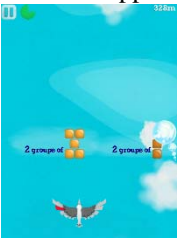

Virtual Manipulative	Field Notes	Open Codes
<p>Train Station virtual manipulative from the Bugs and Numbers app.</p> 	<p>Researcher: set the hands on the analog clock to match the time shown on the digital clock.</p> <p>App: When the hands are set correctly a train moves across the background on a track and whistles.</p>	<p>change analog clock telling time drag hands Helps user know the response was correct</p>
<p>Store virtual manipulative from the Bugs and Numbers app.</p> 	<p>Candies lying on a table. Pipes are labeled with a picture of one type of candy. The user drags or flicks the candy into the right pipe.</p> <p>If you choose the wrong pipe a red X appears and you hear “Opps!”</p>	<p>candies to sort sorting drag or flick candies Helps the user know the response was incorrect</p>
<p>Level 2 virtual manipulative from the Motion Math: Wings app</p> 	<p>Bird flies and comes to two different representations. The user tilts the iPad to fly the bird over the representation of the largest quantity.</p> <p>If the bird flies over the lesser quantity the bird is pushed backwards in the air, you hear, “Uh!,” the quantities change position and the user has to try the task again.</p>	<p>Arrays to quantify Comparing quantities Fly the bird to make the array do something. Not actually touching the array. Helps the user know the response was incorrect. Makes the user try again. Changes the task</p>

Figure 3. Excerpt from field notes illustrating the open descriptive codes developed during Coding and Analysis Step 1.

research question 1 and two results answering research question 2 emerged. Although only 100 observations were planned, it took 137 observations of apps in order to identify 100 educational apps that contained virtual manipulatives and met the criteria for inclusion in the study. This was because the distinction emerged during Coding and Analysis Step 2 between apps that teach a mathematical concept and require user interaction and apps that are virtual manipulative apps. This distinction is discussed at the end of this section.

Result 1: Components of virtual manipulatives within apps. The first result of the study was that there are two components of virtual manipulatives within apps: a dynamic mathematical object and features. These two components emerged as the two major axial code categories. Figure 4 displays several examples of apps and how field notes on each app led to open codes which led to axial codes which led to the axial code categories. This figure provides a glimpse of the process the researcher used during coding an analysis when going from the detailed narrative field notes to establishing the two major component categories during observations of the first 75 apps.

This result answered the first part of research question 1 that focused on components that comprise anatomy of virtual manipulatives apps. Although it is possible through coding and description to separate the dynamic mathematical object and features of a virtual manipulative app, the two are really dependent upon each other and a virtual manipulative app cannot exist unless both components are present. This is why the two components are represented as if they are swirling together in the conceptual framework.

Dynamic mathematical object. A dynamic mathematical object is the central




Virtual Manipulative	Field Notes	Open Codes	Axial Codes	Axial Code Category
Train Station virtual manipulative from the Bugs and Numbers app	The user sets the hands on the analog clock to match the time shown on the digital clock.	change analog clock drag hands telling time	Interactive object Mathematical concept	Dynamic mathematical object
	When the hands are set correctly a train moves across the background on a track and whistles.	Helps user know the response was correct	Communication to the user about response	Feature
Store virtual manipulative from the Bugs and Numbers app	Candies lying on a table. Pipes are labeled with a picture of one type of candy. The user drags or flicks the candy into the right pipe.	candies to sort drag or flick candies sorting	Interactive object Mathematical concept	Dynamic mathematical object
	If you choose the wrong pipe a red X appears and you hear "Opps!"	Helps the user know the response was incorrect	Communication to the user about the response	Feature
Level 2 virtual manipulative from the Motion Math: Wings app	Bird flies and comes to two different representations. The user tilts the iPad to fly the bird over the representation of the largest quantity.	Arrays to quantify Fly the bird to make the array do something. Not actually touching the array.	Interactive object	Dynamic mathematical object
	If the bird flies over the lesser quantity the bird is pushed backwards in the air, you hear, "Uh!," the quantities change position and the user has to try the task again.	Comparing Quantities Helps the user know the response was incorrect.	Mathematical concept Communication to user about response	Feature

Figure 4. Excerpt from field notes and memos illustrating how the axial codes and axial codes categories emerged from the field notes.

component within a virtual manipulative that the user manipulates, meant to be a representation for a mathematical concept. In order for the tool to be considered a virtual manipulative app, the user must be able to manipulate one or more dynamic mathematical objects. This is because the definition of a virtual manipulative stipulates that the visual representation be dynamic not static.

In addition to being able to be manipulated, the dynamic mathematical object within a virtual manipulative must also represent a mathematical concept. This means that the manipulations made with the dynamic mathematical object, lead to the potential understanding of a mathematical concept. An example of a dynamic mathematical object that is both able to be manipulated and represents a mathematical concept is Fraction Ninja by Interactive Elementary. In this app the dynamic mathematical object is a falling wooden bar. The user manipulates the object by swiping up to slice the bar into the indicated fraction pieces. The mathematical concept represented by the bar is a fraction region model. A coding excerpt illustrating selected open and axial codes that led to the bar being coded as a dynamic mathematical object is shown in Figure 5. Additional open


Virtual Manipulative	Field Notes	Open Code	Axial Codes
Level 2 of Fraction Ninja App 	Researcher: Open app to see ninja standing on a platform. Memo: Unsure what to do. App: Bar is wiggling at the top of the hut Researcher: Noticed the fraction shown near the roof of the hut. Tried to swipe down to cut the bar. App: No response Memo: I should have read the directions, but the whole page of text was overwhelming. Memo: The ninja is holding a star. I think the star is supposed to cut the bar. Researcher: Tried swiping down multiple times App: No response Researcher: Swiped up out of frustration. App: The bar split and smoke appeared Memo: What does the smoke mean?	Throw the ninja star by swiping up to cut the fraction bar into the indicated fraction.	Interactive object Teaches a mathematical concept

Figure 5. Excerpt from the field notes of the Fraction Ninja App (Interactive Elementary©) adventure mode level 2 virtual manipulative, illustrating the emergence of the dynamic mathematical object axial code category.

and axial codes were created for this excerpt concerning the difficulty of interaction, the instructions within the app, and the feedback features of the virtual manipulative; however, these codes are not shown because the purpose of this figure is to illustrate the emergence of the axial code category Dynamic Mathematical Object.

Features

The programmable features of a virtual manipulative determine how the dynamic mathematical object behaves, functions or is supported. Features include all of the virtual manipulative elements besides the dynamic mathematical object itself and dramatically affect the nature of the virtual manipulative app. During axial coding, it became evident that there were features that were programmed in the app for different purposes. For example, in some cases features determined the behavior of, the function of, and supported the dynamic mathematical object. A coding excerpt from the Fraction Ninja app (see Figure 6) illustrates how the axial code category features emerged from the field notes, open codes, and axial codes.

The purpose and nature of a virtual manipulative within an app may change dramatically based on its features. For example, within the Base Ten Blocks Math app by Tap Fun Inc. the dynamic mathematical object is a set of base ten blocks. How these dynamic mathematical objects are programmed to behave and function, and how they are supported are the features of the virtual manipulative within an app. Figure 7 shows the field notes and open codes that led to the axial codes for features and dynamic mathematical object.


Virtual Manipulative	Field Notes	Open Code	Axial Codes
Level 2 of Fraction Ninja App 	<p>Researcher: I can see how many correct and wrong answers I have gotten on the top banners to the side of the hut.</p> <p>I can see how many correct and wrong answers I have gotten in a row on the bottom banners to the side of the hut.</p> <p>I can see how many problems I have left until the end of this level on the top right corner.</p> <p>App: the bar is falling towards the ninja at a very slow but constant speed</p> <p>Memo: I wonder if the bar would squish the ninja? I bet students would wait to see.</p> <p>Researcher: smoke appears when I cut the bar in the correct place.</p> <p>Memo: However, it seems that no matter where I cut the bar, unless it is at one of the far ends, the app counts it right.</p>	<p>App keeps me informed about my response performance</p> <p>How the object- bar, is moving</p> <p>The bar lets me know if my response was right or wrong</p>	<p>Supportive</p> <p>Behavioral</p> <p>Functional</p>

Figure 6. Coding excerpt from the Fraction Ninja app (Interactive Elementary©) illustrating how the axial code category Features emerged from the field notes, open codes, and axial codes.

Alternatively, the Base Ten Blocks Manipulative app by Braining Camp also includes a virtual manipulative in which base-ten blocks are the dynamic mathematical object (see Figure 8). The features of this app include snapping, which allows units to be snapped into groups of tens and rods of ten to be snapped into hundreds, linking, which connect the symbolic representation to the dynamic mathematical object (see Figure 8). Although these apps shown in Figures 7 and 8 might both be described as base-10 block apps, the features of the virtual manipulatives within them may afford very different learning experiences.

Physical sets of base-10 blocks may have different colors and sizes, or be made

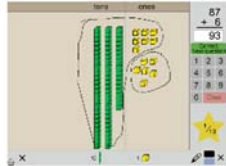
Virtual Manipulative	Field Notes	Open Codes
Base ten blocks within the Base Ten Blocks Math app	<p>Researcher: open app up to a blank mat. Looks like I am supposed to model the addition problem in the top right corner.</p>	User interacts base ten blocks
	<p>Memo: Where are the blocks?</p> <p>Researcher: Found the tiny blocks at the bottom of the mat.</p>	Number correct shown in yellow star
	<p>App: The blocks get bigger when you put them on the mat.</p>	Pen to circle groups
	<p>Researcher: slid 8 tens bars and 13 units on to the mat.</p>	Units do not snap into groups
	<p>App: units do not group together to make a ten</p>	
	<p>Researcher: used a pen to circle the groups of 10 and the left over units</p>	
	<p>App: no response to circling</p>	
	<p>Researcher: entered the sum in the calculator.</p>	
	<p>App: number correct shown in yellow star.</p>	
	<p>Memo: looks I have 9 left. How do I get the next problem?</p>	

Figure 7. Excerpt from the field notes and open codes for the Base Ten Blocks Math app (Tap Fun Inc. ©) that led to the axial code categories of features and dynamic mathematical object.





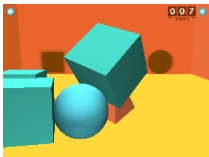

from different materials such as wood, plastic, or foam, but the affordances of these physical manipulative objects will be very similar. On the other hand, virtual manipulatives within apps that contain base-10 blocks as a dynamic mathematical object may afford users completely different mathematical experiences when they contain different features. Therefore, the features of a virtual manipulative app play a central role in how the app will be experienced by the user.

Result 2: Types of dynamic mathematical objects. The second major result was that there are three different types of dynamic mathematical objects. These are: (1) an

analysis resulted in three final axial code categories, identifying the three types of dynamic mathematical objects. These types of dynamic mathematical object codes helped to answer Research question 1 by further describing and identifying subcategories of the dynamic mathematical objects. Figure 9 shows several virtual manipulatives and the field notes that led to the coding that led to the categorization of the three types of dynamic mathematical objects. The figure presents field note excerpts, open codes, axial codes, and axial code categories from nine different observations. Observations like these led to the development of the axial code categories: physical object representation, commercial manipulative representation, and technologically generated representation.

Physical object representations. Dynamic mathematical objects can represent real physical objects. These physical objects could vary from buttons, to elephants, to blocks of ice. As long as the objects represent a mathematical concept they are considered dynamic mathematical objects. For example, in the Bugs and Numbers app by Little Bit Studio, users sort buttons on to a conveyor belt. In the Slice Fractions app by Ululab user cut the chains supporting blocks of ice to represent the correction fractional quantities and allow their mammoth to progress through the game. In the Hungry Guppy app users combine bubbles containing dots to feed a fish the correct quantity.

Commercial manipulative representations. Dynamic mathematical objects often represent a commercially available manipulative such as place value cards, base-ten blocks, pattern blocks, or fraction bars. The Place Value app is an example of this type of dynamic mathematical object. The virtual place value cards that are the dynamic mathematical object in this app may be purchased printed on paper, plastic tiles, or

Virtual Manipulative	Selection from Field Notes	Open Codes	Axial Codes	Axial Code Category
Train Station from Bugs and Numbers app 	<p>Researcher: set the hands on the analog clock to match the time shown on the digital clock.</p> <p>App: a train whistles and puff behind the analog clock</p>	analog clock	represents a physical clock	physical object representation
Store from Bugs and Numbers app 	<p>Researcher: The app opens and candies swirl around the screen.</p> <p>App: A hand appears and points to a candy and then a pipe.</p> <p>Memo: I know to sort the candy into the pipes.</p>	candies to sort	represent real candy	physical object representation
Level 2 from Motion Math Wings app 	<p>Researcher: Next my bird flies to a new set of arrays. There are 10 displayed on the left and 8 on the right. I intentionally fly over the wrong array.</p> <p>App: My bird bounces back like he has run into a glass window.</p> <p>Memo: This give me more time to look at the arrays</p>	arrays to quantify	represents a physical array	physical object representation
Garage from Bugs and Numbers app 	<p>Researcher: the buttons star filling up the box or tray.</p> <p>Memo: I am not sure what to do.</p> <p>Researcher: I tap the button with the 1 on it and it falls onto the black conveyor type belt.</p>	buttons to sort	represent real buttons	physical object representation
3D Shape Sorter from the Geometry Montessori app 	<p>Researcher: the shapes are coming faster and faster now that I have sorted many of them correctly. They are piling up and blocking the holes in the wall.</p> <p>App: The screen is filled with 3D shapes. The activity ends.</p>	3D shapes to sort	like a real shape sorting box	commercial manipulative representation
Pattern Blocks from the Pattern Blocks app 	<p>Researcher: Opened the app and triangle grid paper appears. I dragged out a double hexagon.</p> <p>Memos: These shapes look like physical pattern block manipulatives, but the colors are different.</p> <p>App: Hexagon snaps to grid</p>	pattern blocks	like pattern block sets you can order	commercial manipulative representation

(figure continues)




Virtual Manipulative	Selection from Field Notes	Open Codes	Axial Codes	Axial Code Category
Level 2 from the Fraction Ninja app 	<p>Researcher: The wooden bar is wiggling at the top of the hut.</p> <p>App: 1/5 showing on the roof of the hut.</p> <p>Researcher: I intentionally swipe up right in the middle of the bar.</p> <p>App: The bar falls to the ground around the ninja.</p>	bar to chop	not possible in real life	Technologically generated representation
Level 2 from the City Skate app 	<p>App: We need the groups of 2. “This is 2 and this is 2.”</p> <p>Memo: showing 2 in a horizontal line and a vertical line.</p> <p>App: Gracie skateboard clicks on the sidewalk as she rolls. The first group is a group of 2.</p>	Groups of items to grab from the air	not possible in real life	technologically generated representation
Attribute Blocks from the Hands-On Math Attribute Blocks app 	<p>App: Rectangle blocks only printed above the workspace.</p> <p>Memo: I didn’t see the words at first. It would be great if there were audio directions too.</p> <p>Researcher: I chose all of the obvious rectangles from the attribute blocks surrounding the workspace.</p> <p>App: No response</p> <p>Memo: I initially wanted to see if the app counted squares as rectangles. It does.</p>	attribute blocks	like attribute blocks you can purchase	Technologically generated representation

Figure 9. Field note and coding from nine different observations that led to the development of the axial code categories physical object representation, commercial manipulative representation, and technologically generated representation.

wooden tiles. Teachers often create paper versions of these cards for students. Now this commercially available physical manipulative is available as a virtual manipulative within an app. Another virtual manipulative within an app that contains a commercial manipulative representation is the Number Frames app by Clarity Innovations. In this app the dynamic mathematical objects are ten frames and colored chips like those that are made of plastic and are commercial available. The division board virtual manipulative

within the Division Board app by MontessoriTech is another example of a commercial manipulative representation. This virtual manipulative includes the division board, the beads, the cup, and the group markers included in the physical version.

Technologically generated representations. There are some dynamic mathematical objects in virtual manipulative apps that have no physical equal in the real world. In the app Motion Math: Fractions by Motion Math, users tilt the iPad in order to make the ball land on the number line at the correct location for the fraction indicated. There is no physical equal to this experience. Dropping a ball on a large physical number line would be possible, but the ball and number line in this app change and interact in ways that are only possible in the virtual environment. The Like Terms virtual manipulative within the Algebra Touch app contains another example of a technologically generated representation. In this virtual manipulative users combine like terms by physically moving the terms next to each other and then tapping the operation sign between them. This process can be shown as steps within a diagram, but physically moving and combining terms this way is possible only in a virtual environment. A final example is the touch addition virtual manipulative within the Understanding Math Addition and Subtraction app by APP Media. Within this virtual manipulative, users touch the screen with the number of fingers represented by the first addend. A floating ball then appears where each touch was made. Next, the user touches the screen with the number of fingers represented by the second addend and additional floating balls appear. The user can then see the quantity represented by the addition sentence. This experience has no physical equal.

Result 3: Feature categories. The third major result was that there are eight categories of features. The programmable features of a virtual manipulative determine how the dynamic mathematical object(s) behaves, functions, or is supported. During coding and analysis Step 2, the researcher determined that there are eight categories and many subcategories of virtual manipulative features (see Figure 10). The detailed

Features: Major Categories	Features: Subcategories
mellTask features	Open-ended task Single task Multiple progressive tasks Multiple adaptive tasks
Transition features	Stacked transition Tracked transition distribution Untracked transition distribution
Mathematical features	Appropriate dynamic mathematical object Error free Properly sequenced tasks
Interaction features	Direct interaction Indirect interaction Auditory interaction cues Visual interaction cues Text interaction cues Supportive mapping Hindering mapping
Reward features	Immediate rewards Delayed rewards
Timing Features	Untimed Implicit timing Explicit timing with a numerical timer Explicit timing with a graphical timer Explicit timing with moving objects
Constraint Features	Object constraints Movement constraints Numerical constraints Order constraints Snapping constraints
Feedback Features	Auditory feedback cues Visual feedback cues Text feedback cues Immediate feedback Delayed feedback Requested feedback

Figure 10. Feature major categories and feature subcategories that emerged from the field notes and codes.

emergence of these feature codes and categories and the apps associated with them are presented within each feature section. The emergence of these categories and subcategories occurred in much the same way that the types of dynamic mathematical objects emerged during Phase 2. Because the number of categories and subcategories developed during this portion of the study and are so extensive, the open and axial coding process for each code will not be shared however, the process of emergence for the first feature category task features will be presented and serve as an example of how the feature categories and subcategories emerged for all of the features.

Task features. Virtual manipulatives within apps may be open-ended or task-based. The task the user must complete is determined by the task features that are present or absent within the virtual manipulative. The category of task features emerged from the open and axial coding processes. As each app observation was coded using open coding, phrases like “the user can move the objects to create anything she would like,” and “the user is required to...” appeared often. Axial codes were then assigned to groups of like open codes. As axial codes were grouped and described two axial codes: open-ended and task-based emerged. Both of these codes described the type of activity or task the user participated in within the virtual manipulative. Thus, task features became the axial code category. An excerpt of task codes from several different apps is presented in Figure 11.

Open-ended. Open-ended virtual manipulatives within apps are tools to be used as determined by the user. Open-ended virtual manipulatives within apps could be compared to a physical manipulative pulled from the shelf. The tool sits ready, but the uses of the manipulative are open to the user to determine. An example of an open-ended virtual

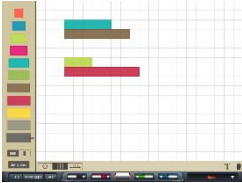

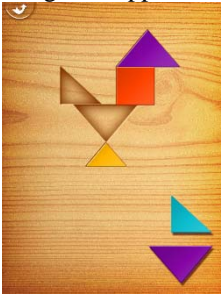
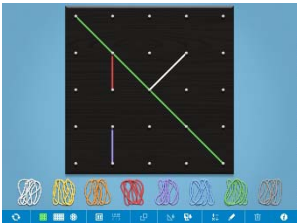
Virtual Manipulative	Field Notes	Open Codes	Axial Codes
<p>Number Rods from Number Rod app</p> 	<p>Researcher: App opens up to grid paper. can move rods from the left had side to the grid paper. There are no preset tasks to do complete.</p>	<p>Experiment with object</p>	<p>open-ended</p>
<p>Fair Shares from the Breakfast Time with Gracie and Friends app</p> 	<p>App: The app tells me to pass out 4 plates.</p> <p>Researcher: I pass them out.</p> <p>App: The app counts 1,2,3,4 and labels each plate with a number.</p> <p>App: Says, “Cut the watermelon into equal pieces.”</p> <p>Researcher: I do what the app says and cut the watermelon into fourths. Can’t do diagonal cuts. I put the watermelon on plates.</p>	<p>Complete specific activities</p>	<p>task-based</p>
<p>Level 2 with the My First Tangrams apps</p> 	<p>App: A wooden puzzle appears. The pieces to complete the puzzle are shown on the bottom of the screen.</p> <p>Researcher: I need to put the puzzle pieces in the shaded areas shown.</p>	<p>One task to complete</p>	<p>task-based</p>
<p>Geoboard with the Geoboard by The Math Learning Center app</p> 	<p>Researcher: I first see a dot grid that is 5 units by 5 units. I choose a color and connect a red virtual elastic from one dot to another. Then I connect a blue.</p> <p>Memo: The app isn’t giving me a shape to build or a problem to solve.</p>	<p>No tasks given by the app</p>	<p>open-ended</p>

Figure 11. Field notes and codes leading to the emergence of the task feature types.

manipulative is the Fraction Circles virtual manipulative found in the Fraction

Manipulatives app by Braining Camp (see Figure 12). This virtual manipulative includes fraction circles and segments as dynamic mathematical objects and many supporting features. The field notes below illustrate that the use of these tools is up to the user, there are no preset tasks.

Task-based. Task-based virtual manipulatives within apps include at least one task for the user to complete. Within Level 1 of the My First Tangrams app by Alexandre Minard, users complete the single task of filling in the Tangram puzzle. While this task requires several movements, there is only one task. Each puzzle within the app is a different virtual manipulative (see Figure 13). The field notes below in the figure illustrate that this virtual manipulative is task-based.

Transition features. Transitions happen when a new task is presented within a virtual manipulative. Transitions can be stacked or distributed. If the transitions are stacked new experiences are layered over old experiences. For example, in the Cut, Paste, and Figure I virtual manipulative within the Matific app the user uses the scissors to cut


Virtual Manipulative	Field Notes
	<p>Researcher: A work mat appears with parts of fraction circles on the left. The pieces are displayed with the largest at the top and the smallest at the bottom,</p> <p>Memo: The pieces feel like they are going to tip over. They would be better displayed with the 1 whole on the bottom.</p> <p>Researcher: I drag a $\frac{1}{2}$ piece on to the work mat.</p> <p>App: The piece gets slightly larger.</p> <p>Researcher: I keep dragging fraction pieces onto the work mat because I have no specific task to accomplish.</p>

Figure 12. Excerpt of field notes from the observation of the Fraction Manipulatives app (Braining Camp©), that led to the open-ended axial code category.

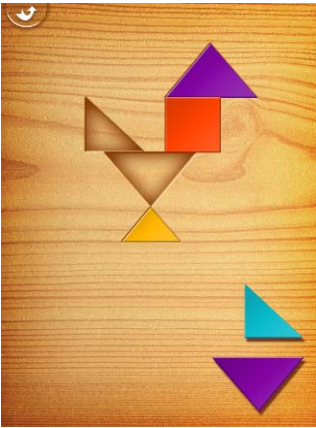
Virtual Manipulative	Field Notes
	<p>Researcher: I have two more pieces to place until I am finished with this puzzle.</p> <p>Memo: I wonder if I will get sparkles when I place the last piece or what will happen....</p> <p>Researcher: I placed the last two triangles that are already oriented correctly.</p> <p>App: a clapping sound is played and stars fall from the top of the screen.</p> <p>Researcher: I have to push the arrow in the top left corner to return back to the menu and select another puzzle to complete.</p>

Figure 13. Excerpt from the field notes taken during the observation of the Tangram virtual manipulative in the My First Tangrams app (Alexandre Minard ©), illustrating a task-based virtual manipulative

the shapes into triangles (Figure 14). After the first shape is cut it is taken from the workspace and a new shape is placed there. These tasks layer over one another and seem to take place in the same space.

If transitions are distributed the tasks seem to move to a new region. For example, in the Motion Math Wings app the user flies the avatar, a bird that has lost all of his colored feathers, towards the largest representation (see Figure 15). If the bird flies to the correct representation it appears to through the dynamic mathematical object and on to the next set. In this virtual manipulative tasks take place in different regions, so they are distributed. The excerpt from the field notes taken during the observation of this app illustrate the distributed tasks.

In the Motion Math Wings app the distributed transitions are untracked, meaning that the user cannot track back to previous tasks. Once a task is completed the user moves

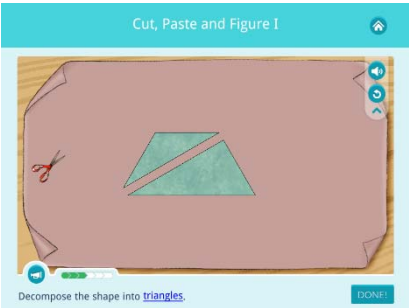
Virtual Manipulative	Field Notes
	<p>App: Use the scissors to cut the shape into triangles.</p> <p>Researcher: I pick up the scissors by tapping and holding on them. I touch the scissors on one corner of the shape.</p> <p>App: A line appears connecting the corner I am touching and the one across from it.</p> <p>Researcher: I slide the scissors along the line.</p> <p>App: The shape splits into two triangles.</p> <p>Researcher: I push the DONE button</p> <p>App: Sparkles explode from the shape and a new shape appears were the first one was.</p>

Figure 14. Excerpt from the field notes taken during the observation of the Matific app (Slate Science©), illustrating layered tasks.


Virtual Manipulative	Field Notes
	<p>Researcher: Slowly flying an array on the left showing 4 and an array on the right showing 2. I tilt the iPad and fly left.</p> <p>App: I hear a ding and chirping. The numerical value of both array shows up. My bird flies faster to the next set of arrays.</p> <p>Researcher: This set is trickier because they are spaced out differently and the quantities are close.</p> <p>App: Displays 8 on the left and 10 on the right.</p> <p>Researcher: I fly towards the 10. Chirping. Dinging and faster again on to the next task a little ways on in the sky.</p>

Figure 15. Excerpt from the field notes taken during the observation of the Motion Math Wings app (Motion Math©), illustrating distributed tasks.

on to a new area, but cannot go back. In other virtual manipulatives within apps users are able to track back and look at previous tasks. Often this is to help inform their decisions or actions on the new tasks. When going back to previous tasks is possible, the distributed tasks are tracked.

Mathematical features. Several mathematical features also help to describe the virtual manipulative. The presence of these features can be determined by the following questions: (1) Is the dynamic mathematical object appropriate for the mathematical topic? (2) Is the mathematics taught within the virtual manipulative error-free? and (3) Are the mathematics tasks within the virtual manipulative properly sequenced?

Appropriate dynamic mathematical object. If the dynamic mathematical object is appropriate for the mathematical topic it matches the mathematical outcomes of the topic. For example, if the task is to divide a group of objects into two equal groups, the dynamic mathematical object would need to be capable of being divided into two equal groups. If the virtual manipulative includes 5 puppies as dynamic mathematical objects, the appropriate dynamic mathematical object feature would be lacking because 5 puppies cannot be divided equally into two groups and partial puppies would not be a reasonable solution.

Mathematically accurate. If the virtual manipulative has the feature of being mathematically accurate, there are no mathematical errors within it and the mathematical object is, “faithful to the underlying mathematical properties of that object in the virtual environment” (Moyer-Packenham, Salkind, & Bolyard, 2008). The task is worded correctly, the labels and dynamic labels within the virtual manipulative are correct, feedback that contains mathematical information is correct, and the dynamic mathematical object functions in a way that is consistent with mathematics principles.

Properly sequenced tasks. If the virtual manipulative has the feature of properly sequenced tasks, the tasks within the virtual manipulative follow a logical order. For

example, if the objective of the virtual manipulative is to teach users to add numbers within 10, smaller quantities would come first. If the user encounters the task of adding $8 + 4$, $8 + 5$, $8 + 7$, and then moves to equations of $x + 1$, these tasks would not be properly sequenced.

Interaction features. Users can interact with apps using many touch and multi-touch gestures. Basic single-touch gestures include swiping up and down, swiping side-to-side, tapping, double-tapping, dragging, flicking, and tracing. Common multi-touch gestures include pinching to zoom, twisting, and pinching to resize. Users can also interact with apps by moving, tilting, or shaking the iPad which take advantage of the iPad's gyroscope and accelerometer. In some cases, the user can select what interaction type to use. For example, the Pink Tower app by Mobile Montessori includes a menu prior to accessing each virtual manipulative. The menu allows the user the option of tapping on the blocks to move them or dragging them to their correct positions (see Figure 16). The field note excerpt illustrates the interaction type choice for this app.

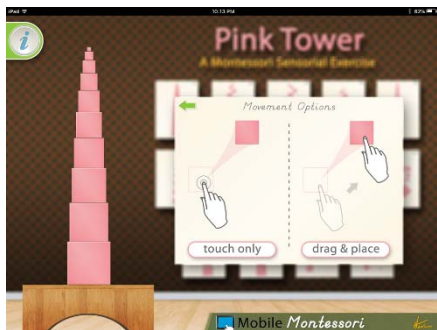
Virtual Manipulative	Field Notes
 <p>The screenshot shows the 'Pink Tower' app interface. On the left is a stack of pink blocks. A central menu titled 'Movement Options' is overlaid, showing two options: 'touch only' with a hand icon pointing to a block, and 'drag & place' with a hand icon dragging a block. The app title 'Pink Tower' and 'A Montessori Sensorial Exercise' are visible at the top. The developer name 'Mobile Montessori' is at the bottom.</p>	<p>Researcher: I select a task card from the menu.</p> <p>App: A menu appears asking me to choose an interaction method. I can choose “touch only” or “drag and place.”</p> <p>Memo: I have used this app many times before and this hasn't been an option. What a great update in the latest version!</p>

Figure 16. Excerpt from the field notes taken during the observation of the Pink Tower app (Mobile Montessori©), illustrating an interaction type selection menu.

Interaction mapping. Modes of interaction can either support or hinder a cognitive task. Modes of interaction that support a task are mapped in a way that corresponds to the user's natural way of moving a similar physical object or that matches an abstract idea about a concept. For example, the Talking Abacus app includes an abacus that moves and functions like its physical counterpart. To move the beads, users touch the bead and push up or push down. Speed on an abacus depends on groups of beads moving together and the ability to interact with more than one column of beads at a time. This app includes these modes of interaction that match how a physical abacus behaves. Had the app been programmed to allow only one bead to move at a time, this would have hindered the task. Alternatively, the Red Rods virtual manipulative within the Intro to Math app by Montessorium requires that users order rods of different lengths from longest to shortest. Unfortunately, the mode of interaction hinders the task. Users cannot stack the red rods like they might in a physical situation, the red rods must be moved by sliding one red rod through another.

Interaction cues. Interaction cues help the user know how to interact with the app. These features can include color changes, flashing elements, pulsing elements, a hand demonstrating the action, a hand pointing to the interaction site, a highlighted area, arrows, and audible instructions. The Claw virtual manipulative within the Bugs and Numbers app includes an interaction feature (see Figure 17). This feature is a hand that demonstrates what to do. The hand picks up a coin from the bottom of the screen and inserts it into the coin slot on the game machine. The field note excerpt in Figure 17 illustrates interaction cues.


Virtual Manipulative	Field Notes
	<p>App: Pennies roll out of the right hand corner. \$0.03 is flashing in digital display.</p> <p>Memo: I am not sure what to do.</p> <p>App: hand appears. It points to one of the pennies and then to the black round circle in the top right hand corner.</p> <p>Researcher: I now see that the black circle says insert.</p>

Figure 17. Excerpt from the field notes taken during the observation of the The Claw virtual manipulative within the Bugs and Numbers app (Little Bit Studio©) illustrating an interaction feature.

Feedback features. Feedback features are clues the app provides following a user response that let the user know about the accuracy of their response or how to proceed. Essentially, feedback is the apps way of communicating with or responding to the user. Feedback can occur after a correct or incorrect response and can be auditory, visual, immediate, delayed, or requested. Although feedback might be rewarding to the user, feedback features are different than rewards features, because feedback features relate directly to the mathematical aspects of the task, whereas reward features relate to _____.

Auditory feedback. Auditory feedback is anything the user hears following a response that relates to the mathematical task. This feedback could tell the user if they are correct or incorrect, or give them a clue about the task. This includes sounds following a correct response such as a crowd cheering, applause, or ascending chimes. It also includes sounds indicating an incorrect response such as a loud beep, a bang, a roar, or descending tones. Auditory feedback also includes spoken words such as, “Perfect,” “Clever,” “You did it,” and “Oops!”

Visual feedback. Visual feedback is anything the user sees following a response that relates to the mathematical task. There are three types of visual feedback: (1) a dynamic mathematical object change or action, (2) an additional representation, and (3) a visual cue.

Dynamic mathematical object change or action. Visual feedback can be given as a dynamic mathematical object change or action that relates to the mathematical task. This includes the dynamic mathematical object shaking after being selected to indicate an incorrect answer, a dynamic mathematical object flying across the screen to indicate a correct response, a dynamic mathematical object changing to gray and white after being selected to indicate an incorrect response, or the dynamic mathematical object changing to include more details (e.g., partitions or markers on a number line). An example of visual feedback as a dynamic mathematical object action can be found in the Counting Caterpillar app by Bellamon (see Figure 18). In this app the user feeds the flying aphid


Virtual Manipulative	Field Notes
	<p>App: Let's count by 2's.</p> <p>Memo: A caterpillar head is waiting on the branch and wiggling back and forth (no body yet. A little strange...)</p> <p>Researcher: I dragged the aphid with the 2 on his back to the caterpillar's mouth.</p> <p>App: The caterpillar's body is now made up of 2 segments, a 1 segment and a 2 segment. The 2 is bolded. The caterpillar is now also walking down the branch. The aphids are flying around the screen.</p> <p>Researcher: I fed the caterpillar and now he has two more body parts. 3 and 4.</p> <p>Memo: The caterpillar is a number line that highlights the skip counting numbers and changes with each aphid.</p>

Figure 18. Excerpt from the field notes taken during the observation of the skip counting virtual manipulative within the Counting Caterpillar app (Bellamon©) illustrating visual feedback as a dynamic mathematical object action.

with the correct number on it to the caterpillar in order to skip count by 2, 5, or 10. If the correct aphid is placed in front of the caterpillar, the aphid is eaten and becomes a section of the caterpillar. If the wrong aphid is placed in front of the caterpillar, the caterpillar will not eat it. The field note excerpt illustrates a dynamic mathematical object change.

Additional representation. Visual feedback can also be the addition of a representation meant to help the user give the correct response on the mathematical task attempt. Additional representations include the addition of a more detailed version of the dynamic mathematical object, the addition of numerals, and the addition of a second model. In the Montessori 1st Operations app by Edoki Academy users are shown an addition problem along with the red and blue number rods that illustrate each addend. The user then selects the number rod from the bottom half of the screen that represents the sum of the two numbers. After the user places the rod representing the sum, the app places the two addend rods on end next to the sum rod so that the user can compare the two. The field note excerpt highlights the addition of a representation (see Figure 19).

Visual cue: Visual feedback can also be basic visual cues that relate to the mathematical task meant to let the user know if the response was correct or incorrect. These can be sparkles, flashing elements, check marks, confetti, balloons, animations, or frowny faces. For example, in the Todo Telling Time app when the user sets the correct time in the Schedule virtual manipulative, confetti flies through the air and a short animation plays on the left to show what might be happening at that time of day (see Figure 20).

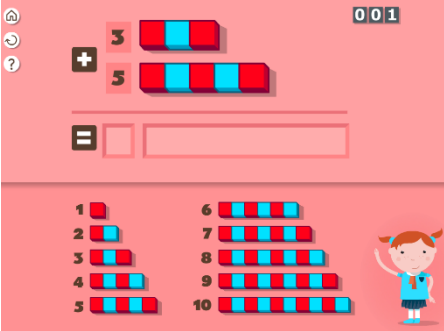
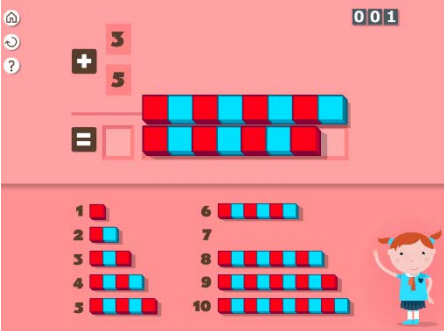
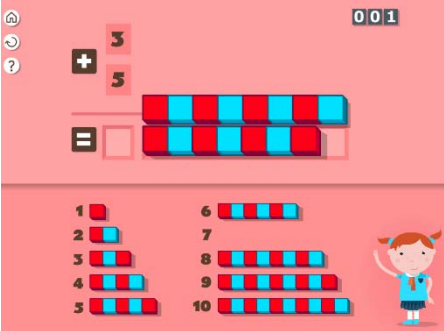
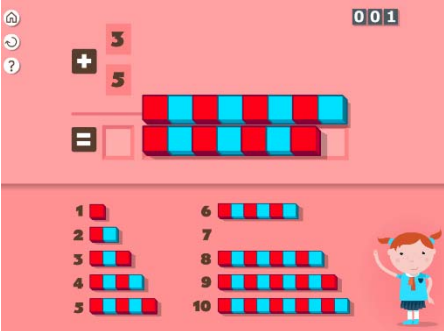
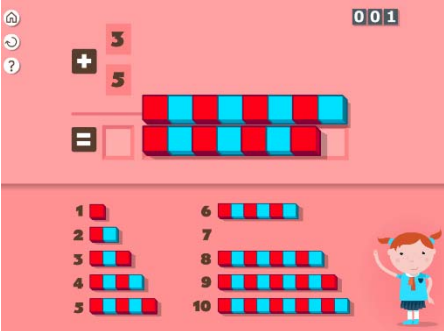
Virtual Manipulative	Field Notes
	<p>App: Can you solve this addition? $4 + 2$ equals. Researcher: tried to slide the rods down the sum space.</p>
	<p>App: rods bounced back to their original places by the addends. Researcher: I slide the 4 rod up to the sum space</p>
	<p>App: the addend rods lined up end to end and moved down by the sum space. Researcher: Because the rods were lined up next to each other I could easily compare the rods and see that four was less than the sum of the rods. I dragged the 6 bar to the sum space.</p>
	<p>App: Addend number rods dropped down again. Researcher: The amounts matched.</p>
	<p>App: Yellow sparkles appear.</p>

Figure 19. Excerpt from the field notes taken during the observation of the adding virtual manipulative within the Montessori 1st Operations app (Edoki Academy©) illustrating visual feedback as an additional representation.

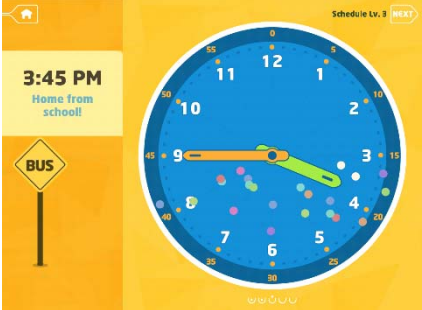
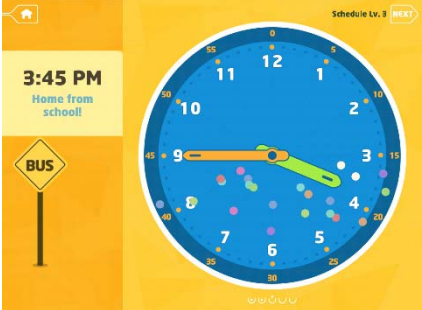
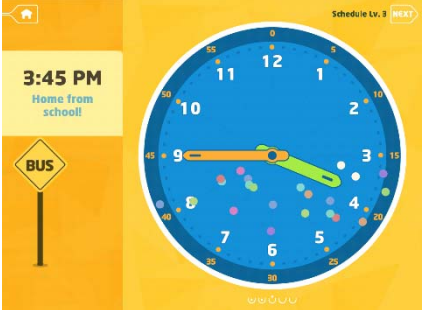
Virtual Manipulative	Field Notes
	<p>Researcher: I read in the left hand bar that 11:00am is playtime. I set the hour hand on the 11 and then the minute hand on the fifteen. I know I got it right because of what the app does.</p>
	<p>App: Confetti pops out of the middle of the clock. Chine. “11:00am. Dogging to the ball.” Then an animation of a boy throwing the ball to that dog plays in the left hand corner.</p>
	<p>Memo: So motivating! I can’t wait to see what the next animation is.</p>

Figure 20. Excerpt from the field notes taken during the observation of the schedule virtual manipulative within the Todo Telling Time app (Enuma©) illustrating visual cues.

Text feedback. In addition to visual feedback, virtual manipulatives within apps can include text feedback. Text feedback is any written word that appears after a user response as a cue for the user indicating if he or she was correct or incorrect, or to give them a clue about the mathematical task. Text feedback can include, “Try again,” “Oops,” “A little higher,” or “Perfect.” For example, in the Pizza Party app by Playpower users share different foods among different numbers of party guests. If the party guests receive the incorrect fraction, the words “try again” appear over their heads. However, if the guests each receive the perfect size piece, the word, “Perfect,” appears over their heads (see Figure 21).

Feedback timing. Auditory, visual, and text feedback can all be programmed to occur at different times. Feedback can be immediate, delayed, or requested. Feedback



Virtual Manipulative	Field Notes
	<p>App: Give Cal half of the pizza is written at the bottom of the screen. One alien sits in front of an empty plate. A square pizza is already cut into $\frac{1}{4}$ pieces.</p> <p>Researcher: Dragged a slice to plate. Clicked done.</p>
	<p>App: The alien gets a mad look, shakes from side to side, and the word “Wrong,” appears over his head.</p> <p>Researcher: touch and drag knife to pastries. Split by slicing in $\frac{1}{2}$ length wise. Drag $\frac{1}{2}$ to each guest.</p> <p>App: Hearts fly up from the plates and the word “Perfect” appears over each the aliens head. New task.</p>

Figure 21. Excerpt from the field notes taken during the observation of the Level 4 virtual manipulative within the Pizza Part app (Powerplay©) illustrating visual cues.

timing includes the appearance of feedback and not the time a user has to complete a task.

Immediate feedback. Immediate feedback occurs immediately following the user's response. It is automatically and gives the user instant information about their response or how to proceed. The Counting Caterpillar app, the Montessori 1st Operations app, and the Todo Telling Time app mention earlier in the feedback section both provide immediate feedback for correct responses.

Delayed feedback. Delayed feedback occurs after some time has passed following the user's response. Usually this type of feedback occurs if there are more than one object that need to be manipulated in order to complete a task. For example, in the Measure This app by Clever Goats Media users are asked to arrange a set of objects in order from longest to shortest. No feedback is given as each item is moved to the correct position, but auditory feedback ("You did it!") is given once all of the items are in the correct position (see Figure 22).

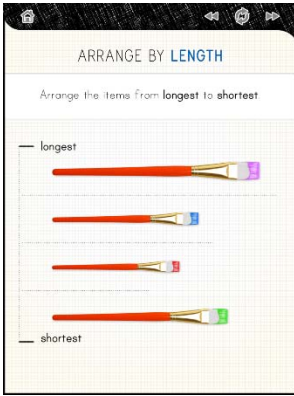
Virtual Manipulative	Field Notes
	<p>App: arranged the items from longest to shortest written at the top of the screen. 4 paintbrushes are positioned horizontally on the screen.</p> <p>Researcher: slid the bottom brush to the second position</p> <p>App: nothing</p> <p>Researcher: switched the bottom two brushes</p> <p>App: says "You did it!" new task</p>

Figure 22. Excerpts from the field notes taken during the observation of the Arrange by Length virtual manipulative within the Measure This app (Clever Goats Media©) illustrating delayed feedback.

Requested feedback. Requested feedback is feedback that is not given to the user until it is requested. The user request could be pushing a “check” or “done” button. In the Pizza Party app shown earlier in the Feedback section, feedback must be requested by the user after they are done sharing food with the party guests by clicking the green “done” button on the right hand side of the screen. Another example can be found in the Fruity Fractions app (see Figure 23). Users label the fraction pieces by dragging the labels to the fruit and then request feedback about their work by clicking the green checkmark button at the bottom of the screen.

Reward features. Some virtual manipulatives within apps include reward features. Reward features are the incentives the user receives (e.g., candy, points, or tickets) that do not directly relate to the mathematics. Within a virtual manipulative, reward features may include points, coins, digital stickers, avatar changes or

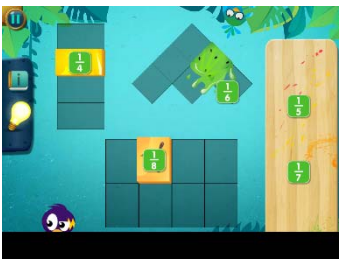
Virtual Manipulative	Field Notes
	<p>App: Changes to screen with 3 fractional pieces to be labeled.</p> <p>Memo: there are no written or audio directions, probably because this is Level 9 and I should know what to do by now.</p> <p>Researcher: Dragged 1/5 label to the kiwi stick (wrong intentionally)</p> <p>App: Fruit squishes and gets juicy. No feedback</p> <p>Researcher: Dragged the 1/8 labels and the 1/4 labels to the correct fruit pieces. Click the check button.</p> <p>App: blue X appears by incorrect label and an information panel pops out on the left</p>

Figure 23. Excerpt from the field notes taken during the observation of the Level 9 virtual manipulative within the Fruity Fractions app (Lighthouse Learning©) illustrating requested feedback.

enhancements, additional parts or pieces to be used within the virtual manipulative, and extra tries or “lives.”

Rewards are necessary to get users initially interested in a virtual activity like those included in this study (Chiou, 2008). However, more rewards do not always lead to increased user attitude and engagement with an app. If users feel they have freedom over their decisions within a virtual environment then the less-is-more theory applies (Chiou, 2008, p. 216). This theory suggests that while rewards are crucial in building user interest, large amounts of rewards are actually not motivating and lead to users having a poorer attitude toward the virtual experience.

Rewards can be impacted by the speed at which the user performs, how accurately a user performs, or the number of tries it takes for the user to give a response. Although rewards are often given at different times throughout the experience with a virtual manipulative, they are different from timing features that limit the time users are given to interact with the virtual manipulative and feedback features that focus more specifically on the mathematics being learned in the app.

The Planet 2 Place Value virtual manipulative within the Math Planet Grade 5 app by Playpower Labs includes coins as a reward feature (see Figure 24). The user earns coins to spend in the avatar shop each time a correct response is given. If the user responds quickly, before the timer in the upper left hand corner runs out, extra coins are earned. Reward features are different from feedback features, because they do not give the user direct feedback about their mathematical response, but are incentives for answering correctly or attempting the task.

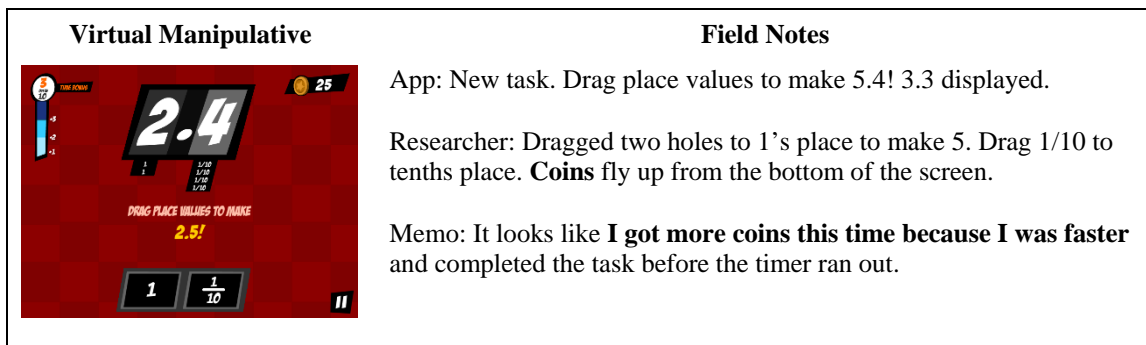


Figure 24. Excerpt from the field notes taken during the observation of the Planet 2 Place Value virtual manipulative within the Math Planet Grade 5 app (Playpower Labs©) illustrating reward features.

Reward features are often part of elaborate reward systems. Users may work with several virtual manipulatives within an app to earn similar rewards. Part of the app, outside of the virtual manipulatives, may be a store or scenario where users spend reward coins to buy digital items for their avatar or stick digital stickers in a virtual sticker book.

Constraint features. Constraint features are programmable virtual manipulative features that restrict users in some way. This study revealed five types of constraints: (1) object constraints, (2) movement constraints, (3) numerical constraints, (4) order constraints, and (5) snapping constraints.

Object constraints. If a virtual manipulative contains object constraints some objects are grayed out or unusable during a task. For example, in the Train Station Telling Time virtual manipulative within the Bugs and Numbers app by Little Bit Studio the minute hand is grayed out and immovable during the first tasks (see Figure 25). These tasks require the user to model time to the hour on the analog clock. By constraining the minute hand users can focus on the placement of the hour hand.

Movement constraints. If a virtual manipulative includes movement constraints,


Virtual Manipulative	Field Notes
	<p data-bbox="667 321 1385 380">Memo: tried moving the minute had, but it is grayed out. Probably because I have been given a task to the hour.</p> <p data-bbox="667 415 1385 474">App: hand pointing to the analog clock and the hour hand. Pulsing arrow on the hour hand.</p> <p data-bbox="667 510 1385 531">Researcher: moved the hour hand counter clockwise to 3:00.</p>

Figure 25. Excerpt of field notes taken during the observation of the Train Station virtual manipulative in the Bugs and Numbers app (Little Bit Studio©) illustrating object constraints.

users are restricted from moving objects in certain ways. The restricted movements could be slides, rotations, or resizing. For example, in the Lab Measurements virtual manipulative within the Bugs and Numbers app by Little Bit Studio users must measure pieces of trash using a ruler and then select the correct measurement (see Figure 26). The object being measured and the ruler include constraint features. Users may not move the object being measured and the ruler must be moved side to side and stays in one vertical plane. These types of constraints let users focus on the measuring task, instead of the possible object movements.

Numerical constraints. If a virtual manipulative includes numerical constraints, users are limited to a range or set of numbers that they may work with. Numerical constraints define the different virtual manipulatives within the Montessori Numbers app by L'Escapadou. Within this app, users choose a numerical constraint and the appropriate virtual manipulative is accessed (see Figure 27). If the Quantity 10 to 99 virtual manipulative is chosen, users are limited to tasks that include modeling numbers 10 to 99.

Order constraints. If a virtual manipulative includes order constraints,


Virtual Manipulative	Field Notes
	<p>App: hot dog to measure and ruler set horizontally</p> <p>Memo: I know that students will want to touch all of the hot dog pieces to see if they do anything and try to rotate the ruler around. Everything on the screen is “locked” from rotating. the only object I can move is the ruler back and forth (left to right).</p> <p>Researcher: intentionally put the ruler in the wrong place measuring 2 to 8.</p>

Figure 26. Excerpts from the field notes taken during the observation of the Lab virtual manipulative within the Bugs and Numbers app (Little Bit Studio©) illustrating movement constraints.


Virtual Manipulative	Field Notes
	<p>Researcher: chose numerals from quantity 10 to 99.</p> <p>App: Text bubble tell me what the menus are and what to do. A finger is pointing the numeral spaces. There are 2. I can only build numbers from 10 to 99.</p> <p>Researcher: Dragged 4 to 1's place. No feedback. I have to place two digits, but there is not room for three</p>

Figure 27. Excerpts from the field notes taken during the observation of the Quantity 10 to 99 virtual manipulative in the Montessori Numbers app (L'Escapadou©) illustrating numerical constraints.

interactions with objects within the virtual manipulative are restricted to a certain order.

For example, in the Candy Count app by YuuZoo, users must first sort the candies into the jars, count the number of candies in each jar, identify the jar with the most items, identify which jar has the fewest items, and then compare the quantities within each jar.

The same dynamic mathematical object is used over and over again with the virtual manipulative, but the interactions with the objects are constrained to a particular order.

Snapping constraints. If a virtual manipulative includes snapping constraints, then objects within the virtual manipulative snap to a grid or correct position if moved within

a close proximity. For example, in the Number Rods app by Braining Camp, the number rods snap to the grid lines to make comparing the rods easy and accurate. In this particular app the snapping constraints are applied vertically and horizontally when the full grid is selected (see Figure 28).

If the number line grid is selected the snapping constraint is only applied when moving the rods horizontally (see Figure 29). In addition, the grid can be completely turned off so that no snapping constraints are applied.

Timing features. Tasks within a virtual manipulative can be timed or untimed. If the tasks are timed, the user has a set amount of time to interact with the virtual manipulative. While timers do give users information about how much time is allowed, they are different from feedback features that inform the user about how well they are performing mathematically.

When timing features are present, timing can either be explicit or implicit. If the timing is implicit the user is unsure if they are being timed and unsure how much time remains. If the timing is explicit the user knows they are being timed and can determine

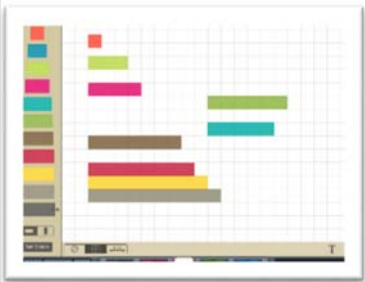
Virtual Manipulative	Field Notes
	<p>Researcher: select grid paper and drag bars to the work mat to compare them. Orange, green, and pink. I try to misalign them</p> <p>Memo: snap to the grid. I can still line them up so the ends are not even, but they snap to the units so they are easier to measure.</p>

Figure 28. Excerpts from the field notes taken during the observation of the Number Rods app (Braining Camp©) illustrating the vertical and horizontal snapping constraints.

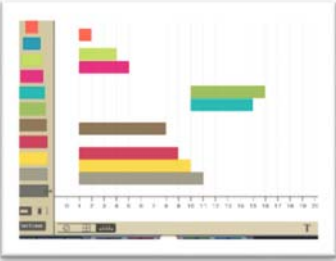
Virtual Manipulative	Field Notes
	<p>Researcher: turned off the grid by clicking a button in the bottom right hand corner.</p> <p>App: The workspace is all white</p> <p>Researcher: moved bars around freely. No snapping.</p> <p>Memo: This would make comparing errors much more likely.</p>

Figure 29. Excerpts from the field notes taken during the observation of the Number Rods app (Braining Camp©) illustrating free movement of the objects.

how much time remains. Explicit timing features include numerical timers, graphical timers, and moving objects that require the user to respond within a certain time limit.

A numerical timer includes numerals that show how much time is remaining. For example, the virtual manipulative Decimals on a Number Line Grade 5 within the app Math Pop Pro has a numerical timer in the upper left hand corner (see Figure 30). This timer lets the user know how much time is left before the bonus points are no longer available. It does not indicate the end of the task, but does encourage the user to move quickly.

A graphical timer is a graphical representation of how much time remains. Graphical timers are often circles or bars that change color or gradually disappear as time runs out. An example of a graphical timer can be seen in the Estimating Fractions virtual manipulative within the Chicken Coop Fractions app by eChalk (see Figure 31). This particular timer is a red circle in the top center of the app. It indicates how long the user has to move the nest before the chicken lays the egg and flings it towards the correct point on the number line.


Virtual Manipulative	Field Notes
	<p>Researcher: I let the timer run out so see what would happen. I still have two bubbles left to place on the numberline.</p> <p>App: timer ran out, nothing happens</p> <p>Memo: I still have to complete the task.</p>

Figure 30. Excerpt from the field notes taken during the observation of the Math Pop Pro app (Playpower Labs©) illustrating a numerical timer.

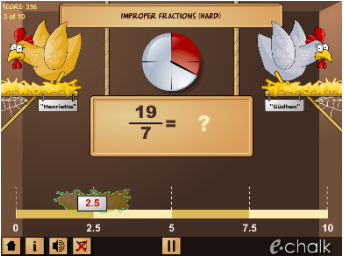
Virtual Manipulative	Field Notes
	<p>Memo: the chickens are clucking and shaking. I am moving the basket back and forth. The red timer is moving quickly. Only a tiny slice of the circle is left (less than $\frac{1}{4}$).</p> <p>Researcher: take my finger off of the basket.</p> <p>App: position reads .25. Chickens lay an egg. Basket catches it. Whew!</p>

Figure 31. Excerpts from the field notes taken during the observation of the Chicken Coop Fractions app (eChalk©) illustrating a graphical timer.

Moving objects can also act as timers within a virtual manipulative. The moving objects may either be the avatar or other objects. In the level 2 virtual manipulative within the Gracie and Friends City Skate app users must make Gracie jump to collect the groups of three (see Figure 32). Gracie speeds along on her skateboard and the user tap or double tap to make Gracie jump or super jump to collect groups before she rolls past them. In this way the user is forced to complete the task of collecting groups at a rate determined by the app.


Virtual Manipulative	Field Notes
	<p>App: here comes another group of threes.</p> <p>Memo: I have to double jump at the right time to get it. I am moving slowly, so I have plenty of time.</p> <p>Researcher: I double tap, but Gracie was too far down the sidewalk. I didn't have time to get this group.</p>

Figure 32. Excerpt from the field notes taken during the observation of the Level 2 Gracie and Friends City Skate app (First 8 Studios at WGBH©) illustrating a moving avatar timer.

In the 3D Shape Sorting virtual manipulative within the Montessori Geometry app by Les Trois Elles Interactive, 3D shapes appear and must be sorted into the correct hole in the wall (see Figure 33). The objects act as a timer and fill up the space if the user does not sort fast enough. When the sorting space is full of shapes, the level ends.

Result 4: Within one virtual manipulative there can be one or multiple mathematical dynamic objects. The fourth major result was that within one virtual manipulative there can be one or multiple dynamic mathematical objects. These dynamic objects may or may not be linked. This result emerged from virtual manipulative coding that contained multiple dynamic mathematical object codes. One example is the Eight virtual manipulative within the Numberland app. This virtual manipulative includes 33 dynamic mathematical objects. This includes four sets of linked objects a set of 8 red flowers, a set of eight white flowers, a group of 8 tropical birds, and a set of 8 objects position. The last dynamic mathematical object is one single object a blue interactive eight, hanging from a tree. A coding excerpt from this virtual manipulative is shown in Figure 34 to illustrate the field notes and open codes that led to the axial code category of multiple dynamic mathematical objects.

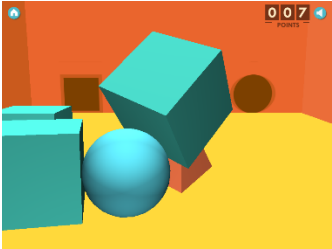
Virtual Manipulative	Field Notes
	<p>Researcher: the shapes are coming faster and faster now that I have sorted many of them correctly. They are piling up and blocking the holes in the wall.</p> <p>App: I was too slow. The screen is filled with 3D shapes. The activity ends.</p>

Figure 33. Field notes from the observation of the 3D Shape Sorting virtual manipulative in the Montessori Geometry app (Les Trois Elles Interactive©) illustrating objects as timers.


Virtual Manipulative	Field Notes	Open Code
	<p>App: 8 red flowers. Researcher: I touch them and they count in sequence from 1 to 8. Memo: Once counted a flower cannot be counted again until all red flowers have been touched.</p>	<p>Physical flowers</p>
	<p>App: 8 white flowers. Researcher: Touch the flowers out of order. When each flower is touched they count in sequence from 1 to 8.</p>	<p>Like physical flowers</p>
	<p>App: 8 birds. Researcher: When I touch each bird I hear the count in sequence from 1 to 8. I touch the 1st bird again and the sequence starts over with 1.</p>	<p>Represent birds in trees</p>
	<p>Researcher: touched the snake App: 8 objects missing from the picture and now need to be put back Researcher: move first object. No app counting this time Memo: my mind immediately counts 1 because of all the other counting that has happened.</p>	<p>Represent puzzle pieces</p>
	<p>App: A blue number 8 is hanging from a looped rope in the tree. Researcher: I touched it. App: The 8 disappears and then is drawn on the screen.</p>	<p>Interactive digit- can only be static in real life</p>

Figure 34. Coding excerpts from the Eight virtual manipulatives within the Numberland app, illustrating that one virtual manipulative can contain multiple dynamic mathematical objects.

While this virtual manipulative includes an unusually large number of dynamic mathematical objects, it illustrates that one virtual manipulative can contain multiple dynamic mathematical. Within a virtual manipulative dynamic mathematical objects may be single objects that are not linked by their features in any way to another object, such as the blue eight in the Numberland app, or the dynamic mathematical object may only object present. An example of a virtual manipulative with only one dynamic mathematical object is the clock with The Talking Teaching Clock app. This is the only dynamic mathematical object present.

Unlinked dynamic mathematical objects. When a virtual manipulative contains multiple mathematical dynamic objects, the objects can be linked or unlinked. If the objects are unlinked the manipulation of one object does not affected any other objects in any way. An example of unlinked dynamic mathematical objects can be found in the Dragon Shapes app by Lumio. The virtual manipulatives within this app are shapes that need to be rotated to fit into designated puzzle areas. Each shape is a dynamic mathematical object that functions independently and is not linked to another object. Each shape must to rotated individually and the rotation of one shape does not affect the rotation or state of any other shape.

Linked dynamic mathematical objects. Multiple dynamic mathematical objects within one virtual manipulative may also be linked. Linked objects are objects that affected one another when one of them is manipulated. In the Numberland app presented earlier in this section, an auditory feedback feature connects the eight red flowers. When one red flower is touched it affects the number of the next red flower. Another example

of multiple linked objects can be found in the Grouping virtual manipulative within the Understanding Math Time Tables app. In this virtual manipulative, users sort balls into boxes to form the correct number of groups in order to model the multiplication sentence given. As users sort the balls, each box displays the number of balls in the box. In this way the balls and the box are dynamically linked. If too many balls are sorted into one box, the task ends and the balls are sorted by the app to show how the sorting is done. In this virtual manipulative the balls are dynamically linked to one another and simultaneously respond to incorrect sorting.

Result 5: Varying relationships can exist among the dynamic object and features within one virtual manipulative. The fifth major result was that dynamic mathematical objects and features within a virtual manipulative can have varying relationships. This relationship among the components of a virtual manipulative is symbiotic. When combined within a virtual manipulative, these components become interdependent and their relationship affords the user an experience that is possible in no other way. In fact, without features the object would be static.

However, the relationship among the dynamic mathematical object and features within a virtual manipulative is not always proportional. In some instances, the dynamic mathematical object(s) are almost the sole focus of the virtual manipulative, with very few features noticeable to the user. An example of a virtual manipulative with this type of relationship is the Pattern Blocks app by Brainiac Camp. The open codes including information about the dynamic mathematical object and the open codes including information about the features are shown in the figure below to illustrate the focus on the

dynamic mathematical object in this virtual manipulative (see Figure 35). Because the focus of the virtual manipulative is the dynamic mathematical object, users are potentially more focused on the mathematics than they would be if the relationship among the features was more equal.

Alternatively, in some virtual manipulatives the features are the focus. For example, in the odd numbers virtual manipulative within the Mathmateer app the feature the open codes are shown in Figure 36 to illustrate the focus on features within this virtual manipulative.

Because this virtual manipulative contains so many features the features may actually become the focus of the interaction instead of the dynamic mathematical object and the mathematical concept it represents. The Mathmateer app includes a rocket ship open codes greatly outnumber the dynamic mathematical object open codes. A sample of

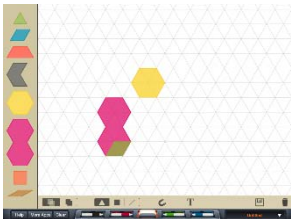
Virtual Manipulative	Open Codes: Dynamic Mathematical Object	Open Codes: Features
	<ul style="list-style-type: none"> Move blocks by dragging Rotate by typing rotate button Change color by tapping ink drop and selecting from 8 presets Duplicate by tapping the copy button Shapes snap to grid if magnet icon is solid Toggle between transparent and solid shapes Remove shapes from the workspace by dragging them to the trash can. 	<ul style="list-style-type: none"> Toggle between triangle grid paper, a coordinate plan, or a blank work mat. Tap the “T” button to insert a text field Tap the disk icon to save the workspace

Figure 35. Open codes pertaining to the dynamic mathematical object and the features within the Pattern Blocks app (BrainingCamp©), illustrating the focus on the dynamic mathematical object in this virtual manipulative.

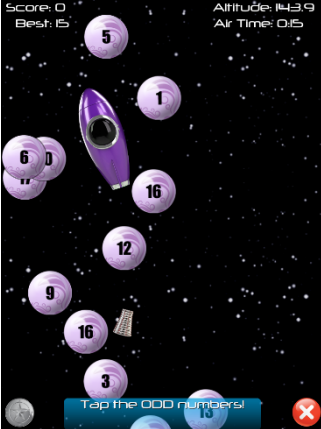
Virtual Manipulative	Open Codes: Dynamic Mathematical Objects	Open Code: Features
	<p>Numbers presented in floating planets or stars</p> <p>If the number within the planet or star is odd, the object sparkles</p> <p>If the number within the planet or star is even, the object disappears in smoke.</p>	<p>Press the launch button to launch the rocket</p> <p>Rocket launches and rocket boosters fall off</p> <p>Better boosters can be purchased with points</p> <p>Altitude shown in top right hand corner</p> <p>Air time shown in the top right hand corner</p> <p>Flight score in top left hand corner</p> <p>Best overall score in top left hand corner</p> <p>Silver, bronze, or gold medal earned for each flight</p>

Figure 36. Field note and coding excerpts from the observation of the Mathmateer app (Dan Russell-Pinson©), illustrating a virtual manipulative with a focus on features.

that the user flies into space to perform a math mission. While the rocket is in space the user touches the correct space items. Dynamic mathematical objects are the space items, but in relation to all of the features this app contains the dynamic mathematical objects have a small role. The app contains many different reward features including a score, medals, time, altitude height, and a rocket ship parts store. The math missions are timed and usually between 30 and 40 seconds. The focus of the apps is the features- obtaining rewards, building a rocket ship, and seeing how long it will fly. In order to do this the user must ultimately interact with the dynamic mathematical object for brief periods.

Result 6: Virtual manipulatives can appear in different ways and have varying relationships within one educational app. The sixth major result was that virtual manipulatives can appear in different ways and have varying relationships within

one educational app. There are three ways virtual manipulatives can appear within an educational app and two types of relationships that can exist among virtual manipulatives in an education app.

The three ways virtual manipulatives can appear within an education app are as a single virtual manipulative, as multiple similar virtual manipulatives, and as multiple varied virtual manipulatives. This result began to emerge when the virtual manipulatives within the Motion Math Zoom app were observed. There were 23 different levels, and an intro level where the user could learn to interact with the app. The researcher first observed level 2. In this level the dynamic mathematical object was a whole number in a bubble and a dynamic numberline that included whole numbers from 0 to 17. The task was to pop the bubble over the correct place on a number line. Features of level 2 included direct interaction features, numerical constraint features, supportive mapping features, and optional timing features.

Following this observation, the researcher observed level 15 within the same app. In this level the dynamic mathematical object was again a bubble, but it contained a decimal number to the hundredths place. The task was to place the decimal number on a number line labeled 0 to 1 and partitioned into tenth sections. To do this the user had to pinch to zoom into the number line to reveal the hundredths partitions. The mathematical task and features were noticeably different in level 15 and level 2. During Phase 3 the researcher compared the codes for these two observations and determined that although each level contained a bubble that needed to be popped and placed on a number line, the differences in the features of each level made them different virtual manipulatives. This

process of comparing one level or activity within a virtual manipulative to another level or activity within the same virtual manipulative was repeated six more times during the study and resulted in the emergence of three ways virtual manipulatives can appear within an educational app, as single virtual manipulatives, multiple similar virtual manipulatives, and multiple varied virtual manipulatives.

Virtual manipulatives that appear as *single virtual manipulatives* within an app make up the whole app. There are no activities outside of the virtual manipulative. An example of a single virtual manipulative app is the Hundreds Board app by Rubber Chicken Apps. When users open the app, the virtual manipulative immediately appears. All app experiences happen within this virtual manipulative.

Virtual manipulatives can also appear as *multiple similar virtual manipulatives*. When virtual manipulatives appear as *multiple similar virtual manipulatives*, more than one virtual manipulative is present, but the differences between the virtual manipulatives are due to slight feature changes, not changes in the dynamic mathematical object. For example, within the Motion Math Zoom app by Motion Math each level contains a number line, but the number line is programmed with different features that give the user a slightly different experience. Within this app the 23 different levels are 23 different, but similar virtual manipulatives.

In apps where virtual manipulatives appear as *multiple varied virtual manipulatives* more than one virtual manipulative is present and the manipulatives contain distinctly different objects. An example of this ways of appearing is evident the Bugs and Numbers app by Little Bit Studio. In this app there are 18 different virtual

manipulatives including a train station where users work on time concepts, a pizzeria where users work on fractions, and a button store where users sort, count, and tally. These virtual manipulatives are all completely different with completely different dynamic mathematical objects.

Two types of relationships that can exist among virtual manipulatives in an education app. When an app contains multiple virtual manipulatives, the relationship between the virtual manipulatives can be a flexible relationship or an ordered relationship. This result emerged from the analysis that occurred during Phase 3 Step 2 from field notes about how the researcher navigated around the app and the inability to access to particular levels that were locked.

In apps that contain a *flexible relationship* all of the virtual manipulatives within the app are available to the user and may be accessed at any time and in any order. The Bugs and Numbers app includes this type of relationship. From the time the app is downloaded the user may choose any virtual manipulative to work with from the map (see Figure 37) at any time. The app simply keeps track of how many tasks have been completed within each virtual manipulative. The field notes, open codes, and axial codes illustrate how the axial code of flexible relationship emerged.

Alternatively, in apps that contain an *ordered relationship* access to more difficult levels or different activities is based on the completion of previous levels or in-app prerequisites. An example of an app that contains an ordered relationship is the Motion Math Zoom app. Figure 38 presents an excerpt of the field notes and open codes that led to the axial code category Order Relationship.


Virtual Manipulative	Field Notes	Open Code	Axial Code
	<p>Researcher: clicked explore button.</p>	<p>Can choose where to begin in the app</p>	<p>Access VMs in any order</p>
	<p>Memo: yeah! It looks like I get to pick what activity to do next. None of them are locked.</p>	<p>Can choose any activity</p>	<p>Access VMs at any time</p>
	<p>Researcher: click on “Store”</p>	<p>Can go in any order</p>	
	<p>App: New activity opens up.</p>		

Figure 37. Excerpts from the field notes and coding for the Bugs and Numbers app (Little Bit Studio ©) that illustrate how the axial code category of flexible relationship emerged.


Virtual Manipulative	Field Notes	Open Code	Axial Code
	<p>Researcher: click the pause button in the top left hand corner.</p>	<p>Levels are locked</p>	<p>Have to access VMs in set order initially.</p>
	<p>App: Menu appears</p>		
	<p>Researcher: click the list icon App: List of levels appear.</p>	<p>Have to do one level to progress to another level</p>	<p>Have to pass one level to get the next</p>
	<p>Memo: I still have many levels that are locked. I'd like to see what the tasks are in the challenge levels are, but I haven't unlocked them yet.</p>	<p>No way to skip to the level needed</p>	

Figure 38. Excerpts from the field notes and coding of Motion Math Zoom app (Motion Math©) that illustrate how the axial code category of ordered relationships emerged.

Coding and Analysis Step 3

Step 3 included a panel of three experts who each member-checked three virtual manipulative educational apps and provided feedback about the codes and categories created to describe the app. The majority of the feedback from these three experts centered on improving the descriptions of the mathematics feature codes and improving distinctions between the feedback features, reward features, and timing features.

Examples of the types of feedback provided by the expert panel included comments asking why the researcher had chosen the specific coding words she had. For example, the dynamic mathematical object type, “physical object representation,” was listed as “physical representation.” The panel member recommended I use the longer version of the terminology to avoid confusion between the code and an actual physical representation. The panel member’s feedback informed the analysis by helping to clarify the distinction between feature types. One panel member coded all of the reward features as feedback features. An explanation was needed about why these two feature categories were separate. This was added to the analysis. The process of member-checking with the panel members resulted in a more refined list of axial codes and axial code categories.

The six results that emerged from Phase 3 are extensive and detailed. These results that answered research question 1 where study result 1: there are two components of virtual manipulatives within apps: dynamic mathematical object and features, study result 2: there are three different types of dynamic mathematical objects, study result 3: there are eight categories of features, and study result 4: within one virtual manipulative there can be one or multiple dynamic mathematical objects. The results that answered research question 2 were result 5: varying relationships can exist among the dynamic mathematical object and features within one virtual manipulative, and result 6 virtual manipulatives can appear in different ways and have varying relationships within one educational app.

Phase 5: Evaluation Tool Prototype Development

The conceptual framework that was developed during Phase 4 of this study, in order to answer research question 3, was presented in the first section of this chapter. For this reason, Phase 4 will not be discussed again here although it did occur immediately following Phase 3.

In this study the conceptual framework developed during Phase 4 led directly to the organization and elements included in the evaluation tool prototype developed during Phase 5. The evaluation tool prototype was developed during Phase 5 to answer research question 4, which focused on how the conceptual framework could aid in developing a tool to evaluate educational apps.

The major sections of the evaluation tool prototype are based on the conceptual framework and include: (1) general app information, (2) dynamic mathematical object, (3) virtual manipulative specifics, (4) task features, (5) transition features, (6) mathematics features, (7) interaction features, (8) reward features, (9) timing features, (10) constraint features, (11) feedback features, and (12) optional descriptive information. A copy of the evaluation tool prototype prompts appears in Appendix C. However, the evaluation tool prototype was also developed in an electronic format within Qualtrics so that it could adapt to user responses. The electronic format summarizes user input and then links this information to potential affordances and potential environment types made possible by the app components and features.

The aim in the development of the evaluation tool prototype was that it could easily and effectively be used by educators and researchers who do not possess a detailed

understanding of mathematics apps or virtual manipulative to identify a “good app” for their purposes. To make this aim possible the researcher created unambiguous prompts for the evaluation tool prototype that would detect the dynamic mathematical objects and features of a virtual manipulative. The questions on the evaluation tool prototype were worded so that they did not include questions that called for judgments about the app. For example, instead of asking an individual to rate the timing features within the virtual manipulative on a scale of 1 to 10, the tool presents a series of prompts (see Figure 39). The user first selects an answer to Prompt 1. If the user chooses “b,” the rest of the timing feature prompts are skipped. If the user chooses “a” they move on to Prompt 2. If the user answers “b” they skip the third question, but if the user answers “a” they are shown Prompt 3. Responses to this series of prompts provides the evaluation tool prototype with enough information to determine if the virtual manipulative includes timing features, what type of timing features the virtual manipulative includes, and if the timing is implicit or explicit.

Evaluation Tool Prototype- Timing Features Section				
	Prompt	Choice A	Choice B	Choice C
1	<i>The virtual manipulative...</i>	a) requires a response within a certain time limit	b) does not include timed responses	
2	<i>Can the user determine how much time remains?</i>	a) yes	b) no	
3	The timing is done using...	a) a digital timer	b) a graphical timer	c) moving objects that require the user to respond within a certain time frame

Figure 39. Timing prompts from the evaluation tool prototype.

The evaluation tool prototype contains a similar series of prompts for each of the 11 major sections to determine the nature of the virtual manipulative and the app being evaluated. The questions and sections of the evaluation tool prototype were purposefully ordered based on significance and the natural flow of an app observation. After over 100 app observations, the researcher determined that this order was the most complementary to the interaction that a user would likely have with an app. Prompts about the app in general and the dynamic mathematical object are first because if there is no dynamic mathematical object in the app and there are no virtual manipulatives in the app, then completing the rest of the prompts would be impossible. The rest of the sections in the evaluation tool prototype follow based on how the researcher proceeded with the observation of the apps. Once the researcher's attention was on the dynamic mathematical object it seemed natural to begin thinking about the virtual manipulative the dynamic mathematical object was situated in and then the features of the virtual manipulative. Optional descriptive information is collected at the end of the survey because gathering this information can be cumbersome and does not affect the evaluation of app components.

Logic is also in place within the tool to expedite the evaluation process. This logic allows users to skip questions, based on previous answers that do not apply. For example, if users indicate that there is no feedback given after a correct response, the rest of the prompts pertaining to correct response feedback are skipped. Alternatively, if the user indicates that tasks within the virtual manipulative increase in difficulty as the user answers correctly they are presented with more prompts that drill-down on the specifics

of this process.

Gathering information about the virtual manipulative features themselves provides all of the information necessary to adequately describe the components and features of apps that contain virtual manipulatives, the relationships among components, the relationships among the virtual manipulatives, the features that may be present in the app, and the type of app environment. When the user completes filling out the survey, the evaluation tool prototype summarizes the information that has been entered by the user. The results of the survey provide a summary about what type of features are included, how they might be helpful, how they might be hindering, and which environment type they are likely to be a part of. By examining these results teachers can tell if the app they have evaluated will likely be a good fit for their purposes and researchers will have a way to describe and compare the components of apps.

Phase 6: Evaluation Tool Prototype Use

During Phase 6 the researcher evaluated the 25 apps reserved for testing using the evaluation tool prototype developed during Phase 5. This was done in order to help answer research question 4. Phase 6 allowed the researcher to apply the results of the analysis to new and novel apps in order to identify how the conceptualization of the virtual manipulative apps and the research results were confirmed by new data and how they may be lacking. Returning to the data once again to improve the analysis is in line with the constant comparative method and grounded theory.

The educational apps evaluated during this phase were selected at the onset of the

study and reserved for this phase. None of them were used during Phases 1 through 5, but proportionally represented all of the app selection criteria. The researcher evaluated each app using the evaluation tool prototype and also repeated the coding and analysis steps described in Phase 3. Field notes and memos were taken when evaluation tool prototype elements were lacking. In these cases, the evaluation tool prototype was immediately updated.

The majority of the changes to the evaluation tool prototype pertained to the functionality of the tool, not the content. Adaptive questions were refined so that questions that were not needed based on previous responses did not display. Additionally, the order of several questions was changed so that the adaptive nature of the tool could function more effectively. Also, during this phase the researcher programmed one prototype of the dynamic results that could result when a user evaluates an app using the evaluation tool prototype.

Following the researcher's analysis of the 25 evaluation apps, one member of the expert panel was trained to use the evaluation tool prototype and then asked to review three of the 25 evaluation apps. This external independent review and double-coding provided data about the usefulness of the tool and helped the researcher to determine whether or not the tool was transferable to a user and to educational apps beyond those used to develop it. At the conclusion of Phase 6, the data gathered through researcher evaluations, the panel member evaluations, and the researcher's final reflections concerning the study were used to inform final improvements in the evaluation tool prototype.

CHAPTER V

DISCUSSION

The invention and rapid adoption of touchscreen mathematics apps as tools for learning has dramatically changed the potential avenues and routines of learning mathematics. This change requires educators and researchers to consider what the anatomy of a “good math app” is so that these apps can be selected for use in the classroom and for inclusion in research. The purpose of this exploratory qualitative study was to expand the explanations of and build theory about educational apps that contain virtual manipulatives.

Results of the study are based on detailed analyses of over 100 educational apps that contain virtual manipulatives. This analysis led to the development of a conceptual framework and an evaluation tool prototype for evaluating apps. The discussion of the results has seven sections. The first four sections are based on the four research questions and the results that emerged to answer each one. The remaining sections include implications of the study, limitations of the study, and suggestions for future research.

Question 1: App Components, Description, and Categorization

Question 1 focused on the components of virtual manipulative apps, how they could be described, and how they could be categorized. Three study results emerged to answer research questions 1. Each of these results is discussed, interpreted, and situated within the literature in this section.

Result 1

Result 1 was that that based on the apps included in the study there are two major components of virtual manipulative educational apps. These components are dynamic mathematical objects and features. A dynamic mathematical object is the central component within a virtual manipulative that the user manipulates, meant to be a representation for a mathematical concept. The features of a virtual manipulative determine how the dynamic mathematical object behaves, functions, and is supported. This means that dynamic mathematical objects and features are the essential components of the virtual manipulative apps included in this study and that variations in these components afford users completely different experiences.

This result is in line with the definition of a virtual manipulative set forth by Moyer et al. (2002) stating that a virtual manipulative is “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p. 373). Although the virtual manipulatives within apps examined in this study are not Web-based, an essential component of virtual manipulative educational apps are the dynamic mathematical objects. The other essential component—features—are not explicitly mentioned in the original definition, but are directly addressed in a recent publication in which Moyer-Packenham and Bolyard (2016) revisit the definition of a virtual manipulative. These researchers point out that, because the original definition of a virtual manipulative did not include direct reference to the features, some have interpreted this to mean that a virtual manipulative is simply the 2D inscription of the object. Moyer-Packenham and Bolyard’s updated definition

makes it explicit that a virtual manipulative includes both a dynamic object and its programmable features. The definition now states that a virtual manipulative is “an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge” (Moyer-Packenham & Bolyard, 2016, p. 16). The dynamic object can only be dynamic and useful to the user when it is coupled with its features (Kirby, 2013). The results of this dissertation study, showing that the dynamic mathematical objects and their features are essential and interdependent components of virtual manipulative educational apps, support this assertion.

The results, confirming that the virtual manipulatives situated in educational apps in this study are comprised of two main components: dynamic mathematical objects and features, show that touch-screen virtual manipulatives are similar to virtual manipulatives designed for mouse-driven interaction. This connection is important because it provides an opportunity to directly apply what is known about mouse-driven virtual manipulatives to research about virtual manipulatives within touchscreen apps. Although virtual manipulatives within apps designed for touchscreen devices are accessed and interacted with in different ways than virtual manipulatives designed for mouse-driven devices, because the basic components of both virtual manipulatives are the same, this will allow the research community to use mouse-driven virtual manipulative research as a basis for future research on virtual manipulative educational apps on touch-screen devices.

Result 2

Result 2 was that there are three different types of dynamic mathematical objects

that emerged in this study. The three different types of dynamic mathematical objects that emerged are (1) an object meant to represent a commercially available physical manipulative (commercial manipulative representation); (2) an object meant represent a real physical object (physical object representation); and (3) an object that is only possible virtually (technologically generated representation). The different types of dynamic mathematical objects have not been categorized in the literature before this study. Therefore, this result adds to the literature by providing a way to categorize and describe the dynamic mathematical objects in virtual manipulative apps.

While features of virtual manipulative apps and other virtual tools have been given much attention in the literature and are the main focus of several evaluation tools, the literature on dynamic mathematical objects themselves is limited. In fact, when evaluating virtual manipulatives, the dynamic object itself is often completely overlooked. In a study by Kay and Knaack (2008), teachers used a feature evaluation tool to assess virtual manipulatives before using them in their classrooms. Students who experienced the manipulatives then evaluated their experience. While learning was correlated with each of the features included in the evaluation tool, the dynamic objects the students interacted with were not considered. By overlooking the dynamic mathematical objects students use, learning is solely attributed to features. Future research is needed to link learning to different dynamic mathematical objects as well as their features.

Result 3

A major finding of this study was that there are eight categories of VM features

based on the apps reviewed in this study. These categories are task features, transition features, mathematical features, interaction features, reward features, timing features, constraint features, and feedback features. Each feature category includes specific programmable features that may be present within a virtual manipulative. Several different virtual manipulative feature descriptions exist in the literature. For example, Sedig and Liang (2006) developed a framework of 12 mouse-driven virtual manipulative interactivity factors that affect specific cognitive processes. This framework included five factors that aligned with the results in this study. Alignment between features reported in the literature and those identified in this study are discussed below.

Constraint features. According to Sedig and Liang (2006), constraints restrict certain actions in an effort to focus and direct learner's cognitive processes. The results of this study support the existence of constraint features in virtual manipulative educational apps and the definition set forth by Sedig and Liang. In this study, constraint features were further subcategorized into object constraints, movement constraints, numerical constraints, and order constraints. Constraint features most likely contribute to the affordance category, Focused Constraint, that emerged from a meta-analysis comparing virtual manipulatives with other instructional treatments (Moyer-Packenham & Westenskow, 2013). Future research is needed to link these constraint features to the affordances they may offer users.

Feedback features. The second feature, feedback features, that emerged from this study were also similar to a feature developed by Sedig and Liang (2006). As data were being simultaneously collected and analyzed in this study, as per grounded theory

methodology, the researcher returned to the literature to gain clarity concerning feedback. During this search several of the feedback subcategory labels developed by Sedig and Liang matched up exactly with the results emerging from this study. These subcategories were immediate feedback, delayed feedback, and requested feedback. In addition to these subcategories, three additional feedback subcategories emerged: auditory feedback, visual feedback, and text feedback which adds to the literature on this category. In a study by Johnston and Moyer-Packenham (2012), preservice teachers were asked to self-identify evaluation criteria for virtual manipulatives. Feedback emerged as one of the criteria valued by the pre-service teachers illustrating their importance as virtual manipulative features.

Feedback features of virtual manipulatives were also the focus of a study by Paek et al. (2011). These researchers claimed that feedback features were the main difference between virtual and physical manipulatives because feedback helps learners link their actions with the dynamic mathematical objects to the abstract symbols they represent. The study showed that visual and audio feedback features impacted students learning more than the input method used to manipulate the virtual manipulative, illustrating the importance of thoroughly understanding these features and their presence within virtual manipulative apps. Future research is needed to examine the impact of text feedback on student learning.

Interaction features. The third features to emerge from the study were interaction features. Interaction features in this study include supportive mapping, hindering mapping, interaction cues, direct interactions, and indirect interactions. In the

feature evaluation tool designed by Kay and Knaack (2008) those virtual manipulatives considered to be interactive had a significant correlation with student performance illustrating the importance of these features. By further subcategorizing these features, research can be done to link student performance to particular interactivity features.

The interactivity subcategories of supportive mapping and hindering mapping refer to how aligned an interaction technique is to a particular task. This concept aligns with and supports the conceptual gestural mapping research done by Segal (2011). Segal's research shows that the more "mapped" a required gesture is to a concept, the more meaningful it is, and the more effective it is for learning. Mapping is partially described by Sedig and Liang (2006) as part of epistemic appropriateness. Interactivity features could be particularly important when researching virtual manipulative educational apps because of the diverse interaction opportunities users have when using touchscreen devices. Mouse-driven virtual manipulatives are limited to the indirect interactions of clicking and dragging. Touchscreen devices allow users to interact in a variety of direct ways that include a variety of gestures. Future research is needed to understand how these interaction features affect student learning.

Timing features. Another feature to emerge from this study was timing features. Virtual manipulative experiences can be untimed, timed explicitly, or timed implicitly. Timing as a feature of gaming is a well-known concept, but research about timing within virtual manipulatives is limited. In a study of gaming and how gaming features could be used in the development of education games, timing features were identified by students as being an important aspect (McFarlane, Sparrowhawk, & Heald, 2002). Students

reported that, within education games, they often had time limits in which to accomplish a task as compared to other games in which the experiences were not timed. Timing features that emerged in this study point to the importance of determining how timing benefits or does not benefit student learning.

Reward features. Reward features emerged as a feature of virtual manipulatives within apps in this study. While reward features are considered feedback features by some (Sedig & Liang, 2006), because there were such extensive reward features observed in the apps in this study, the researcher named reward features as a separate category. While research on reward features of virtual manipulatives within apps is limited, reward features within games are readily recognized. For example, King, Delfabbro, and Griffiths (2010) developed a psychological taxonomy of video game structural characteristics to explain how video game playing behavior may be influenced by structural characteristics. One of the influential characteristics identified was reward features. The subcategories of reward features named during this study aligns with the description of rewards in the King et al. study. In this way the emergence of reward features adds to the research concerning virtual manipulatives within apps and highlights their similarity with games that could be examined through future research. It has also been shown that reward features directly influence player motivation (King et al., 2010). The implication of reward features within virtual manipulatives is that these features could be an important factor to consider in studies on motivation and learning.

Task features. Task features that emerged in this study may determine what activities the user must complete within a virtual manipulative. These tasks may be open-

ended, a single task, multiple progressive, or multiple adaptive. Little research exists concerning tasks within virtual manipulatives within apps so this result adds to the literature. Open-ended tasks allow the user freedom to explore the object in a way not determined by the virtual manipulative itself. In a study by McFarlane, Sparrowhawk, and Heald (2002) students reported that this type of experience in a game was more enjoyable than completing predetermined tasks. Open-ended tasks may also be linked to the affordance category, Creative Variance (Moyer-Packenham & Westenskow, 2013) by affording user the opportunity to have more creative choice. Single-tasks may focus users on one particular concept and be linked to the affordance category, Focused Constraint. Virtual manipulatives within apps that contain multiple tasks may be more motivating to users and be linked to the affordance category, Motivation. Future research is needed on the task features themselves and the affordances the task features may offer users.

Transition features. Transition features emerged as another feature in the results of this study. Transitions occur between tasks within a virtual manipulative. In this study three subcategories of transition features emerged. These are stacked, untracked distribution, and tracked distribution. These features emerged from the study as descriptive open codes, but creating synced axial codes proved difficult. When the researcher returned to the literature, the features developed by Sedig and Liang (2006) described a similar feature to the emerging results in this study. While it is not likely that transitional features are linked to student achievement, because they are simply what happens between tasks, they do effect the look and feel of the virtual manipulative and may potentially influence motivation (Moyer-Packenham & Westenskow, 2013).

Mathematical features. Mathematical features of virtual manipulatives within apps also emerged in this study. These features include dynamic mathematical object appropriateness, mathematical accuracy, and properly sequenced tasks. Because the purpose of a virtual manipulative is to present opportunities for constructing mathematical knowledge, mathematical features may have the biggest impact on student learning. These features are the only features identified in the study that required knowledge beyond simple recognition to evaluate. Teachers and researchers must be able to determine, based on their own knowledge for teaching mathematics, if the dynamic mathematical object is appropriate for a specific learning objective, if the app is mathematically accurate, and if the tasks are sequenced properly.

In a study by Cayton-Hodges et al. (2015), iPad apps were evaluated based on mathematical features. The study reported that the content appeared to be mostly accurate, but that sometimes-conscious design efforts were made to sacrifice accuracy for ease of use or to focus user responses. Future research could examine how these types of sacrifices influence misunderstandings or possibly contribute to inaccuracies in mathematical learning.

Result 4

Result 4 was that dynamic mathematical objects within a virtual manipulative can have varying relationships. Analyses in this study revealed that virtual manipulatives within apps may include one or more dynamic objects and that these dynamic objects may be linked or not linked. Linked dynamic mathematical objects are connected by features. These feature may help the user transition from the use of one object to another

or cause the objects to change simultaneously with one another as the user manipulates one of them. On the other hand, a virtual manipulative may contain many dynamic objects that are not linked, but function independently. Result 4 adds to the literature and expands the visual representation portion of the virtual manipulative definition to include objects instead of just one object. This allows research to be done on each type of dynamic mathematical object within a virtual manipulative individually as well as collectively. It may be possible that one dynamic mathematical object within a virtual manipulative is affecting learning while another is not. It may also be possible that multiple linked dynamic mathematical objects are impacting learning. Research has been done pointing to the positive effects of linking between objects and numerical representation (Botzer & Yerushalmy, 2008) however, research remains to be done on the effects of linking dynamic objects.

Summary

Research question 1 focused on the components that comprise the anatomy of virtual manipulative educational apps and how can these components can be described and categorized. The results that emerged have implications for future researcher and show that virtual manipulatives are composed of dynamic mathematical objects and features, that there are three types of dynamic mathematical objects, that there are eight categories of features, and that dynamic mathematical objects within a virtual manipulative can have varying relationships.

Question 2: Relationships Among App Components

Research question 2 focused on the relationships among the components within educational apps that contain virtual manipulatives. Two results emerged to answer research question 2. Both of these results are discussed, interpreted, and situated within the literature in this section.

Result 5

Result 5 of the study was that dynamic mathematical objects and features within a virtual manipulative can have varying relationships. The relationship between dynamic mathematical objects and features is symbiotic. In fact, in this study a dynamic mathematical object could not be dynamic without behavior and function features. This study showed that the relationship among features and the dynamic object within a virtual manipulative is not always equal. In some virtual manipulatives within apps, the focus is on the dynamic mathematical object and its behavior and function features. Within these virtual manipulatives there are few support features such as timing features and reward features that may take away focus from the dynamic mathematical object. However, it is also possible that within a virtual manipulative the support features are the focus.

This result means that teachers and researchers must consider the learning objective and if the balance among these features is optimal for that learning objective. The features must make learning the priority of the virtual manipulative, while still being interesting enough that the user wants to continue to engage with the app. Flow, or the state of consciousness in which users are so absorbed in an activity that they show high

performance without being aware of their surrounding environment (Finneran & Zhang, 2005), is the goal of all game design. Achieving flow through a balance of features within an educational tool can be challenging (Annetta, 2010). Future research is needed to determine how best to balance the relationship between features for different purposes.

Result 6

Result 6 of the study was that virtual manipulatives can appear in different ways and have varying relationships within one educational app. Virtual manipulatives in this study appeared in three ways: as single virtual manipulatives, as multiple similar virtual manipulatives, and as multiple varied manipulatives. These ways of appearing are new to the literature because no known study has examined or considered the distinctly different objects within one app to be different manipulatives. Current studies examine apps at the app level and consider then entire app to be one experience. This result allows for future research to be conducted at the virtual manipulative level.

There are two types of relationships that emerged in this study that can exist among virtual manipulatives in education apps. These relationships can be flexible, meaning that the user is free to access all of the virtual manipulatives within an educational app from the moment of download and in any order desired. These relationships can also be ordered, meaning that the player must complete one activity or level in order to unlock another activity or level.

These two types of relationships are discussed in different ways throughout the literature. Sedig and Liang (2006) classify flexible relationships as part of a characteristic of a group of features called flexibility, meaning the flexibility to select an interaction

that meets the needs or preferences of the user. The teachers within the Johnston and Moyer-Packenham (2012) study grouped these relationships with motivation. They felt that moving from level to level was motivational for students. As suggested in other studies, different types of virtual manipulative relationships may be appropriate or motivating for different reasons. Future research is needed to determine the effect these relationship types have on students' experiences with virtual manipulatives within apps.

Summary

Research question 2 focused on the relationships among components of virtual manipulatives within educational apps. Two results emerged to answer this question. Result 5: varying relationships can exist among the dynamic mathematical object and features within one virtual manipulative, and result 6: virtual manipulatives can appear in different ways and have varying relationships within one educational app.

Question 3: Conceptual Framework

Question 3 focused on how the anatomy of virtual manipulative educational apps could be conceptualized. To answer this question, a conceptual framework was developed based on the result of components and relationships among app components that emerged during this study. Conceptualization of the virtual manipulative within an app (Figure 1) is meant to visually express the intertwining nature of dynamic mathematical objects and features that swirl in specific combinations to afford users different experiences (Gibson, 1977) and that there may be multiple virtual manipulatives within an app.

No other conceptual framework exists to explain the anatomy of virtual manipulative educational apps. With this framework, future research is needed to link specific features with learning. This framework allows apps to be compared at the feature level and learning to be assessed at the feature level. As a result, features eventually linked to learning may be specifically sought out by educators. Eventually researchers may be able to ascertain an app's potential to offer learning experiences based on the features it contains.

The conceptual framework also allows future research to link affordance categories with specific features. Based on affordance theory (Gibson, 1977), and the observations done in this study, it is likely that affordances are a result of specific features, combinations of features, and the relationships among features. This means that affordance categories such as creative variation, efficient precision, focused constraint, and simultaneous linking (Moyer-Packenham & Westenskow, 2013) can be studied in terms of the features that comprise them. In addition to studying particular apps and attributing learning to certain perceived affordances, apps may be studied based on the features that make these affordances available.

Question 4: Evaluation Tool

Question 4 focused on how a conceptualization of virtual manipulative educational apps could aid in the evaluation of these apps. To answer this question an evaluation tool prototype was designed based on the conceptual framework developed during the study. The evaluation tool prototype is a tool for researchers and educators to

use when selecting virtual manipulative educational apps for research or inclusion in the classroom and for researchers to continue to refine. The tool consists of questions that aim at detecting the dynamic mathematical object(s) and features of virtual manipulatives within an app. The evaluation tool prototype created in this study uses unambiguous questions so that subjective impressions of the app are less likely to interfere with evaluating the components. The evaluation tool prototype is dynamic and adjusts to user input. For example, if a user indicates that there are no feedback features within the app, the detailed questions about feedback are skipped. By answering a series of questions within the tool, educators and researchers can easily tell what dynamic mathematical object and features make up the virtual manipulative, what the relationships are among the components, how many virtual manipulatives make up an app, and the relationship among the virtual manipulatives without having to have an extensive background in apps or virtual manipulatives.

The development of this tool was relevant to the appeals of the educational and research communities for a description of the anatomy of learning apps and a way to evaluate them (Byers & Hadley, 2013). In addition, the tool developed during this study improves the current state of app evaluation in several different ways. First, the evaluation tool prototype is research based. Its development is grounded in the detailed analysis of over 100 educational apps that contain virtual manipulatives. Second, the tool is specific to virtual manipulatives within educational apps delivered on iPads. There are currently many websites and individuals dedicated to reviewing educational apps. These sources vary greatly in their focus and priorities and offer several different tools to help

educators identify “good” apps. This level of specificity within the prototype tool developed in this study allows the user to evaluate virtual manipulatives within apps in a detailed way that is not possible with other available tools. Third, other widely used evaluation tools, base evaluations and recommendations upon the evaluator’s impression of app affordances (Schrock, 2011; Walker 2010). For example, questions such as “How user friendly is the app?” or “How motivated are students to use the app?” lead users to make judgment calls and base their evaluation on their impressions. The tool developed in this study evaluates the components of the app and then links these components to possible affordances and environment type that may be made available by these components.

Limitations

As with all studies, there were limitations that affected the results of this study. The limitations of the study were: researcher bias in the participant-as-observer role, the relatively small number of virtual manipulative apps that were observed, and that only virtual manipulative apps from the iTunes store were included. The researcher recognizes that there is no such thing as an unbiased observer or observation. Although the observer-as-participant is a long-standing and common role for researchers conducting fieldwork, this approach does have its limitations. Because all of the apps were viewed through the researcher, the lens of her personal experiences and her mathematics background affected what she saw, how she viewed, and how she interacted with the virtual manipulative apps. Although some bias may have been reduced because these observations were

human-to-technology interactions, as opposed to human-to-human interactions, the apps were still viewed and interpreted through the lens of the researcher. Because this study was designed as an exploratory study to initially investigate and build theory concerning the anatomy of educational apps that contain virtual manipulatives, it is expected that the results, framework, and tool will be refined through use in future research.

Because little research exists on virtual manipulative apps, it is possible, even following the detailed review of 100 apps, that there are virtual manipulative app features that have not yet emerged and descriptions that could be refined. Thousands of educational apps are available within iTunes, but it is unknown how many of these apps include virtual manipulatives and how representative a sample of 100 is of this group. For example, of the 137 apps the researcher downloaded based on their descriptions, only 100 actually contained virtual manipulatives. It is possible that an app containing a virtual manipulative that was not included in the study could have influenced the results. Additionally, new apps are made available every day. These new apps will need to be considered and compared to the framework on a continuing bases to ensure the accuracy of the framework.

Only apps developed for the iPad and available in iTunes were included in the study. This was due to the technology the researcher had access to, the large number of apps available in iTunes, and the detailed app descriptions within iTunes that allowed the researcher to initially identify apps for download that seemed to contain virtual manipulatives. Only apps appropriate for children ages 3 to 10 were included in the study. This was because of the researcher's interests and expertise and because there are

many more apps for this age group than for children age 11 and above. It is possible that if apps developed for other touchscreen devices and from other app stores, or apps for children age 11 and above were included, different features would have emerged. However, including a more diverse group of apps was out of the scope of this study.

Implications

The results of this study have implications for both educators and researchers. In this study virtual manipulative apps were examined, components defined, and relationships among components described. Now that language for communicating about the anatomy of virtual manipulative apps has been established and the features of virtual manipulatives have been identified, future studies could include examining virtual manipulatives within mathematics apps based on the dynamic object or particular features. By studying apps in this way, apps with certain dynamic objects or those that include certain features could possibly be linked to student achievement. This study makes it possible for researchers to organize research around specific features categories and to study how students respond to these features, if they use them, or if they choose not to. Selecting “good apps” for use in the classroom is a difficult challenge for teachers. With limited funding and often limited experience reviewing apps, teachers are left to select apps based on the recommendations of bloggers, review websites, distribution store ratings, and distribution store descriptions. However, these sources rate apps based on varying focus and priorities. The results of this study could be used by educators to identify and evaluate the components of educational apps when selecting them for use

with children.

Additionally, the evaluation tool prototype could be used to evaluate the apps being considered for download or purchase by classroom teachers or educational leaders. Before paying for school- or district-wide license, apps could be evaluated to ensure they meet the learning objectives of the classroom teacher or the educational leader. The evaluation tool prototype was designed specifically for those who may not have extensive experience with educational apps or with virtual manipulatives.

The purpose of this exploratory study was to expand the explanations of and build theory about educational applications containing virtual manipulatives so that researchers would have language and framework whereby to describe virtual manipulative apps. This study stems from a need for this language, framework, and an evaluation tool that emerged from the researcher's research experiences prior to this study. With this language, framework, and tools in place there are many implications and possibilities for future virtual manipulative app research.

Suggestions Future Research

The first suggestions for future research proposed by the researcher is validating and refining the evaluation tool prototype. Although the evaluation tool prototype emerged directly from the results of the study and the conceptual framework that was developed, it has yet to undergo the validation process. Future research could be done to validate the tool itself, refine the questions included in the tool, and to develop dynamic tool results based on the prototype developed during this study.

The second suggestion for future research is the design of studies in which the virtual manipulative app conceptual framework developed in this study is used to describe the components of virtual manipulative apps. Research involving children and how they learn on iPads is beginning to emerge (Cohen et al., 2011; Paek et al., 2011). Applying the framework in order to select apps for research study or to describe the components children interact with would add to the literature.

The third suggestion for future research is the design of studies linking specific virtual manipulative app components and features to learning. It has been hypothesized that features of apps effect learning (Risconscente, 2011), but prior to this study, a detailed framework and description of these features did not exist. It is now possible to design research studies to investigate specific components with virtual manipulative apps and the learning that may result from user engagement with them.

The fourth suggestion for future research is the design of studies linking specific features with affordance categories (Moyer-Packenham & Westenskow, 2013). It emerged during the study that affordances are not themselves components of virtual manipulative apps, but the result of features and combinations of features with the app. Research linking these features and combinations of features to specific affordances would be valuable.

Conclusion

“What are “good” mathematics apps?” is a question often asked by educators. However, few research- or theory-based studies have been conducted that provide a

sufficient response to this question. “Good” mathematics apps cannot be easily identified without an understanding of the components and features that make up the app. This exploratory study attempted to expand the explanations of and build theory about educational applications containing virtual manipulatives by better describing components and features of the apps in order to provide language, a conceptual framework, and an evaluation tool prototype for evaluation and research.

The results of this study indicated that the anatomy of educational apps containing virtual manipulatives is made up of one or more virtual manipulative and that each virtual manipulative within the app is comprised of two components: dynamic mathematical objects and features. There are three types of dynamic mathematical objects: physical object representations, commercial manipulative representations, and technologically generated representations and eight categories of features. These categories directly informed the development of the conceptual framework. There are also three different types of relationships that exist within virtual manipulative apps: among the virtual manipulatives with an app, among the dynamic mathematical objects within a virtual manipulative, and among the features within a virtual manipulative.

The conceptual framework illustrates the components of educational apps and the relationships among these components. This framework is significant because prior to this study there was no theory- or research-based framework available for describing virtual manipulative apps. The framework is useful for teachers and researchers because it can be used to consider and communicate about virtual manipulative apps.

The categories, relationships, framework, and evaluation tool prototype that

emerged in this study advance the literature on educational apps and virtual manipulatives. Future research involving the evaluation tool prototype, the application of the framework to children's interactions with virtual manipulative apps, connections between virtual manipulative app components and student achievement, and connections between virtual manipulative app features and affordances will contribute to the emergent categories, relationships, and framework as well as validate and refine the evaluation tool prototype.

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APPENDICES

Appendix A

App Assignment to the Development or Testing Group

Award Recipients: Development Group			
	App	Virtual Manipulative	Publisher
1	Math Planet Grade 5	Planet 2 Place Value	Playpower Labs
2	Dragon Shapes	Level 2	Lighthouse Learning
3	Counting Caterpillar	Medium count by 2,5, and 10	Bellamon
4	Dragon Box Elements	Triangulum	We Want to Know AS
5	Oh No! Fractions	addition (bad mapping)	Curious Hat
6	Teachley Multiply Mt. Multiplis	Level 3	Teachley
7	Montessori Math Multiplication	Toolbox > tables	Edoki Academy
8	Todo telling time	Schedule	Enuma
9	Curious Ruler	Ruler	Curious Hat
10	Hungry Guppy	Dots> 4 and 5	Motion Math
11	Little Digits 1,2,3	Little Digits	Cowly Owl
12	Busy Shapes	Shape Sorting	Edoki
13	Motion Math Zoom	Level 2	Motion Math
14	Bugs and Numbers	Garage Count to 100	Little Bit Studio
15	Gracie and Friends Breakfast Time	Level 2	First 8 Studios at WGBH
Award Recipients: Testing Group			
1	Gracie and Friends Park Play	Apple Sharing	First 8 Studios at WGBH
2	Bugs and Numbers	Train Station Telling Time	Little Bit Studio
3	Montessori Numberland	123- Number 4	Edoki Academy
4	Battle Station	Skipper Fractions, Level 1	Playpower
5	Math Planet Grade 5	Planet 3 Bubble Pop	Playpower

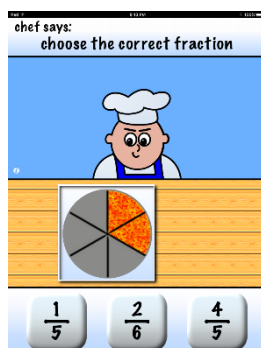
Recommended by Online Reviewer: Development Group			
	App	Virtual Manipulative	Publisher
1	Algebra Touch	Like Terms	RegularBerry
2	Doodle Math Numbers	Together or Take Away (train)	Carsten Studios
3	Fruity Fractions	Level 9	Lighthouse Learning
4	Thinking Blocks Multiplication	Models	Math Playground
5	TallyTots	#2 or #5	Spinlight Studios
6	Understanding Math- Addition and Subtraction	touch	app Media UG
7	Understanding Math- Addition and Subtraction	create and model a sentence	app Media UG
8	Math Pop Pro	Grade 5 Decimals on the Numberline	Playpower Labs
9	Cyberchase 3D builder	Level 2 (shape folding)	PBS Kids
10	Gracie and Friends Birthday Café	Level 4	First 8 Studios at WGBH
11	Monkey Math School Sunshine	no levels, includes many VMs	THUP Games
12	Montessori Division Board	Division Board	MontessoriTech
13	Montessori Geometry	3D Shape Sorter	Les Trois Elles
14	Montessori 1st Operations	Discover odd and even	Edoki Academy
15	Pink Tower	Card 6	Mobile Montessori
Recommended by Online Reviewer: Testing Group			
1	Mathmateer	Odd Number	Dan Russell-Pinson
2	Abacus Adventure	Level 7	EverQuiz
3	Doodle Math Numbers	Put things in Order (fridge magnets)	Carsten Studios
4	Pizza Party	Level 2	PowerPlay
5	My First Tangrams for iPad	Level 1	Alexandre Minard

Popularity: Development Group			
	App	Virtual Manipulative	Publisher
1	Fraction Ninja	Adventure Mode- Level 2	Interactive Elementary
2	Place Value App	Place Value Cards	MontessoriTech
3	Hundreds Board App	Hundreds board	Mobile Montessori
4	Base Ten Blocks Math App	Base Ten Blocks	Tapfun, Inc.
5	Base Ten Blocks Manipulative	Base Ten Blocks	Braining Camp
6	Motion Math Fractions	Easy Setting- Level 2	Motion Math
7	Fraction Manipulative	Fraction Circles	Braining Camp
8	Attribute Blocks	Attribute Blocks	Hands-On Math
9	Intro to Math	Red Rods	Montessorium
10	Bugs and Numbers	The Claw	Little Bit Studio
11	Matific	Grade 3- Square it Up	Slate Science
12	Montessori Numbers	Quantity 10 to 99	L'Escapadou
13	Candy Count app	Counting Candy	YuuZoo
14	Number Rods	Number Rods	Braining Camp
15	Hundreds Board	Hundreds board	Rubber Chicken Apps
16	Numberline Frog	Numberline	Brian West
17	Montessori Bead Skip Counting	Skip Counting Beads	MontessoriTech
18	Angle Asteroids	Level 2	Playpower Labs
19	Motion Math Wings	Pre-Multiplication Island 2	Motion Math
20	Bugs and Numbers	Store: Sorting, Counting, and Tallying	Little Bit Studio
21	Pattern Blocks	Pattern Blocks	Braining Camp
22	Chicken Coop Fractions	Estimating Fractions- Improper Fractions hard	Lumpty Learning
23	Numberline	Whole Numbers Level 2	ShiXian Li
24	Tiny Chicken Learns Math	Chicken Bounce > Addition	TaptoLearn Software
25	AstroMath	Level 4- Primes of 2 and 5	EnsenasSoft
26	Stamp Game	Stamp Game	MontessoriTech
27	Place Value	Regroup	Little Monkey Apps
28	Visual Fractions, Decimals, and Percentages	Grid- One Unit	Esa Helttula
29	Geoboard	Geoboard	Hands-on Math
30	Tens Frame	Tens Frames	Rubber Chicken Apps
31	Friends of Ten	Show me...	Little Monkey Apps
32	Montessori Counting Board	Counting Board	Grasshopper Apps
33	MonteCalc	Addition Activity	ApptoLearn
34	Montessori Bead Facts Plus Minus	Addition 0 to 9	MontessoriTech

35	Number Rack	Number Rack	Math Learning Center
36	Money Pieces	Number Pieces	Clarity Innovations
37	Numberline	Numberline	Clarity Innovations
38	Understanding Math- Times Tables	array	app Media UG
39	Geometry Montessori	3D	Edoki Academy
40	My First Weighing Exercises	Mode 1	Alexandre Minard
Popularity: Testing Group			
1	Montessori Numberland	Playbox- balls	Edoki Academy
2	Place Value and Rounding	Rounding Coach	AppTutor
3	Number Pieces	Number Pieces	Clarity Innovations
4	Bugs and Numbers	Lab Measurements	Little Bit Studio
5	ladybird maths	Sharing	Aleesha Kondys
6	Telling Time	Set to the Clock to the Minute	InfoSoft
7	Matific	Grad 1- That's the Ticket I	Slate Science
8	Equivalence Tiles	Equivalence Tiles	k12.com
9	Visual Multiply	tables	Esa Helttula
10	Visual Fractions, Decimals, and Percentages	Circles- Several Units	Esa Helttula
11	Number Frames	Number Frames	Clarity Innovations
12	Geometry Montessori	Find the Next Shape	Edoki Academy
13	Understanding Math- Times Tables	groups	app Media UG
14	Gracie and Friends City Skate	Level 3	First 8 Studios at WGBH
15	Measure This	Arrange by Length	Clever Goat
16	Jungle Geometry	What is the length- Level 2	Andrew Short
17	Fingu	Level 3	Image and Form International AB
18	Hundreds Board Extensions: Roman Numerals	Roman Numerals	Rantek
19	NumberLine 2	Numberline	Todd Bowden
20	Matific	Grade 1- Cut, Paste, and Figure I	Slate Science

Appendix B

Excerpt from a Non-Virtual Manipulative App Observation

Pizza Fractions App**Field Notes and Memos**

Pizza divided into slices. Some pizza missing.

Some pizza missing

User selects numerical representation.

If the correct numerical representation is selected, the app shows, "good job!"

The user shakes the iPad to get a new fraction task.

Open Codes

Circle fraction representation

Identify the fraction

Interaction with button to select answer.

Written words let the user know the response is correct

Interacts by shaking

Axial codes

Object-commercial manipulative representation

Mathematical concept

Interaction

feature

feature

Appendix C

Evaluation Tool Prototype Prompts

VM App Evaluation Tool

General App Information

Q1 App Name

Q2 App Publisher

Q5 Version

Q4 Cost

Dynamic Object Information

A virtual manipulative contains a dynamic object and features that support the use of this object. The dynamic object and its features work in tandem to provide a specific user experience. The user's experience with a dynamic object can vary greatly depending on the features associated with it. This section is about the DYNAMIC OBJECT.

Q5 What is the virtual object that the user interacts with? (Examples are blocks, buttons, a clock, an array, a number line, bubbles, etc. Occasionally two or more dynamic objects may be present.)

Q6 How can you manipulate this object?

1. Directly (by tapping, dragging, rotating, resizing, etc.)
2. Indirectly (using controls such as buttons, arrows, or a number pad)

Virtual Manipulative Specifics

Q7 Apps frequently contain more than one experience. Each experience is most often either its own level or app section. How would you describe the experiences within this app?

3. there is one experience
4. there are multiple similar experiences (i.e. levels)
5. there are multiple varied experiences (i.e. in one experience you manipulate blocks and in another experience you count coins)

Q8 Navigation between experiences within the app is...

1. based on advancement or achievement (levels are unlocked when certain requirements are met)
2. open (all experiences are available to the user and may be accessed at any time in any order)

Q9 Which specific experience (virtual manipulative) within the app will you be evaluating? (please list give the VM's name, the level name, or any other way in which the VM is designated within the app).

Task Features

Q10 Within the VM...

3. there are no predetermined tasks
4. there are one or more tasks for the user to complete

If "there are no predetermined tasks" is selected, Then Skip To End of the Task Features section

Q11 Describe the task(s) within the virtual manipulative (i.e. "to fly the bird towards the greatest quantity," "to make the analog clock hands represent the same time as the digital clock," or "there is no assigned task."

Q12 The virtual manipulative includes a PROGRESSION.

5. Yes. Tasks within the virtual manipulative automatically increase in difficulty as the user answers correctly.

If "Yes" is selected, these sub-options appear.

6. The progression takes places as tasks change within the virtual manipulative
7. The progression takes places as the user is moved to new level of the virtual manipulative
8. No. All tasks with the virtual manipulative are similar

If "No" is selected, Then Skip To End of the Task Features section

Q13 The virtual manipulative is ADAPTIVE.

9. Yes. Tasks within the virtual manipulative become easier or more difficult depending on the the user's answers.

If "Yes" is selected, these sub-options appear.

10. Tasks adapt and become more difficult.
11. Tasks adapt and become easier
12. No. Tasks within the virtual manipulative stay the same.
13. No. Tasks within the virtual manipulative increase in difficulty in a predetermined-determined way.

If "No" is selected, Then Skip To End of the Task Features section

Transition Features

Display this question only if "within the VM there are one or more tasks for the user to complete" is selected on question 10

Q14 How do transitions between tasks occur in the virtual manipulative?

14. Stacked (the new experience "lays" over the previous experience)

15. Untracked distribution (the experience seems to move to new region, but the user cannot track back to previous experiences)
16. Tracked distribution (the experiences seems to move to a new region and the user can track back to previous experiences)

Mathematical Features

Q15 Does the representation match the mathematical topic?

17. Yes
18. No
19. Somewhat

Q16 Are the mathematics taught within the virtual manipulative error-free?

20. Yes
21. No
22. Somewhat

Q17 Are the mathematics taught within the virtual manipulative properly sequenced?

23. Yes
24. No
25. Somewhat

Interaction Features

Q18 What mode(s) of interaction are used within the virtual manipulative?

26. tapping
27. double tapping
28. dragging
29. flicking
30. tracing
31. swiping down/up
32. swiping left/right
33. tilting
34. shaking
35. pinching
36. twisting
37. resizing

Q19 Can the user control the mode of interaction? (i.e. choose tapping or dragging)

- 38. Yes
- 39. No

Q20 The mode(s) of interaction...

- 40. support the cognitive task (i.e. a user drags to the right to find numbers of higher value)
- 41. hinder the cognitive task (i.e. a user must drag down to find number of higher value)
- 42. are neutral (user interaction do not support or hinder the cognitive task)

Q21 The user knows what to do when he/she opens the virtual manipulative because: (select all that apply)

- 43. cues are integrated within the virtual manipulative itself
- 44. instructions are located in a separate area (such as a help menu or tutorial)

Display This Question only if “cues are integrated within the virtual manipulative itself,” is selected in Q21.

Q22a What interaction cues does the virtual manipulative include? (Interaction cues help the user understand how to interact with the app. These are different that feedback cues that occur in response to an action or user response).

- 45. color changes
- 46. flashing elements
- 47. pulsing elements
- 48. hand demonstrating action
- 49. hand pointing to interaction site
- 50. highlighted area
- 51. arrow(s)
- 52. audible instructions
- 53. there are no interaction cues

Display This Question only if “instructions are located in another area,” is selected in Q21.

Q22b The instructions located in a separate area are: (select all that apply)

- 54. text-based and require reading
- 55. video-based
- 56. audio-based (instructions are given audibly)
- 57. tutorial-based (there is an optional or required experience where users ARE SHOWN how to interact with the app)

58. interactive practice (there is an option or required experience where users ARE SHOWN and PRACTICE how to interact with the app)

Reward Features

Q23 Are rewards given to the user within the app? (These could be points, digital stickers, etc. that are collected each time the user has an experience with the virtual manipulative.)

59. Yes
60. No

If “No” is selected, then skip to the end of the Reward Features section

Q24 What type of rewards are given to the user?

61. digital stickers
62. points
63. coins
64. digital animals/pets/insects
65. avatar or character changes/enhancements
66. additional parts or pieces to be used within the experience

Q25 Are the rewards impacted by the speed at which the user performs?

67. Yes
68. No

Q26 Are the rewards impacted by the number of tries it takes the user to give a correct response?

69. Yes
70. No

Timing Features

Q27 The virtual manipulative...

71. requires a response within a certain time limit
72. is not timed

If “is not timed” is selected, then skip to the end of the Timing Features section.

Q28 The timing is done using...

73. a digital timer (a timer that includes numbers)
74. a graphical timer (a dynamic pie chart or bar indicating time)
75. moving objects that require the user to respond within a certain time limit

Constraint Features

Q29 What type of constraint(s) does this virtual manipulative include?

- 76. object constraints (some objects are grayed out or unusable during a task)
- 77. boundary constraints (the maximum size of an object is limited)
- 78. movement constraints (i.e. users are restricted from rotations)
- 79. algebraic constraints (users are limited to a range or set of numbers that they may work with)
- 80. order constraints (users must interact with objects in a certain order)
- 81. snapping constraints (objects snap to a grid or the correct position if moved within a close proximity)
- 82. there are no constraints

If “there are no constraints” is selected, then skip to the end of the Constraint Features section

Q30 Which constraints may be turned on and off by the user?

- 83. object constraints
- 84. boundary constraints
- 85. rotational constraints
- 86. algebraic constraints
- 87. order constraints
- 88. snapping constraints
- 89. none

Feedback Features

Q31 Feedback features are cues that let the user know how the interaction is progressing. Feedback is the app's way of communicating with or responding to the user.

Q32 After a CORRECT RESPONSE feedback is...

- 90. a sound ("perfect!", cheering sound, etc)
- 91. a dynamic object action or change (object changes color, changes position, size change, etc)
- 92. a visual cue (sparkles, flashing lights, check mark, etc)
- 93. an additional representation (i.e. a correct model is chosen so the corresponding numerical value is shown)
- 94. there is no feedback
- 95. text ("awesome," "five in a row!")

If “there is no feedback” is selected, then skip to Q34

Q33 After a CORRECT RESPONSE feedback is...

- 96. immediate (happens immediately after the response)
- 97. delayed (i.e. after all required responses have been submitted)
- 98. can be requested (i.e. a button can be pushed for feedback)

Q34 After an INCORRECT RESPONSE feedback is...

- 99. a sound ("oops!", "try again," negative noise, etc)
- 100. a dynamic object action or change (shaking, color change, etc)
- 101. a visual cue (frowning face, flashing, etc)
- 102. text ("try again")
- 103. there is no feedback

If "there is no feedback" is selected, then skip to the end of the Feedback Features section

Q35 After an INCORRECT RESPONSE feedback is...

- 104. immediate
- 105. delayed
- 106. can be requested

Q36 Can the amount or type of feedback be changed by the user?

- 107. Yes
- 108. No

Optional Descriptive Information

The following questions marked with **stars are optional.

Q37 * Are "Freemium" version available? (A free limited version is available for download so that users can try basic levels and functions before purchasing the app).

- 109. Yes
- 110. No
- 111. The app is completely free
- 112. Unsure

Q38 * This app is available in (choose all that apply)

- 113. iTunes
- 114. Google Play
- 115. Amazon App Store

Q39 * App store customer rating for current version (in stars)

_____ Number of Stars

Q40 * iTunes age ranking

- 116. 4+
- 117. 9+
- 118. 12+
- 119. 17+

Q41 * App is appropriate for which age group(s) as determined by the app developer?
(listed in the app store description)

- 120. not specified
- 121. pre-K
- 122. Kindergarten
- 123. First Grade
- 124. Second Grade
- 125. Third Grade
- 126. Fourth Grade
- 127. Fifth Grade
- 128. 2 years
- 129. 3 years
- 130. 4 years
- 131. 5 years
- 132. 6 years
- 133. 7 years
- 134. 8 years
- 135. 9 years
- 136. 10 years
- 137. 11
- 138. 12
- 139. 12+

Q42 * CCSSM addressed as determined by the app developer (listed in the app store description)

- Standard 1
- Standard 2
- Standard 3
- Standard 4
- Standard 5
- Standard 6
- Standard 7

Q43 * Are in-app purchases advertised to users?

140. Yes

141. No

If “No” is selected, then skip to Q45

Q44 Where are these purchases advertised?

142. within the virtual manipulative

143. within an outside area (information menu, help menu, parent area, etc.)

Q45 ** Are links to social media present?

144. yes

145. no

If “no” is selected, then skip to Q47

Q46 Where are these links located?

146. within the virtual manipulative

147. within an outside area (information menu, help menu, parent area, etc.)

Q47 *What groups or individuals recommend this app?

Q48 *What awards has this app received?

Q49 *Does the app include a teacher/parent info area?

148. Yes

149. No

Q50 *Does the app include teacher/parent controls?

150. Yes

151. No

Q54 *Does the app include assessment reports for individual users?

152. Yes

153. No

Appendix D

Virtual Manipulative by Common Core State Standards for Mathematics Strand

Award Recipients: Development Group			
	App	Virtual Manipulative	CCSSM
1	Math Planet Grade 5	Planet 2 Place Value	5.NBT
2	Dragon Shapes	Level 2	2.GA
3	Counting Caterpillar	Medium count by 2,5, and 10	K.CC
4	Dragon Box Elements	Triangulum	4.G
5	Oh No! Fractions	addition	4.NF
6	Teachley Multiply Mt. Multiplis	Level 3	3.NBT
7	Montessori Math Multiplication	Toolbox > tables	3.NBT
8	Todo telling time	Schedule	1.MD
9	Curious Ruler	Ruler	2.MD
10	Hungry Guppy	Dots> 4 and 5	K.CC
11	Little Digits 1,2,3	Little Digits	K.CC
12	Busy Shapes	Shape Sorting	K.G
13	Motion Math Zoom	Level 2	1.NBT
14	Bugs and Numbers	Garage Count to 100	1.NBT
15	Gracie and Friends Breakfast Time	Level 2	3.NF
Award Recipients: Testing Group			
1	Gracie and Friends Park Play	Apple Sharing	3.NF
2	Bugs and Numbers	Train Station Telling Time	1.MD
3	Montessori Numberland	123- Number 4	1.NBT
4	Battle Station	Skipper Fractions, Level 1	4.NF
5	Math Planet Grade 5	Planet 3 Bubble Pop	5.NF

Recommended by Online Reviewer: Development Group			
	App	Virtual Manipulative	CCSSM
1	Algebra Touch	Like Terms	5.OA
2	Doodle Math Numbers	Together or Take Away (train)	1.NBT
3	Fruity Fractions	Level 9	4.NF
4	Thinking Blocks Multiplication	Models	3.NBT
5	TallyTots	#2 or #5	K.CC
6	Understanding Math- Addition and Subtraction	touch	2.NBT
7	Understanding Math- Addition and Subtraction	create and model a sentence	2.NBT
8	Math Pop Pro	Grade 5 Decimals on the Numberline	5.NBT
9	Cyberchase 3D builder	Level 2 (shape folding)	3.G
10	Gracie and Friends Birthday Café	Level 4	3.NF
11	Monkey Math School Sunshine	no levels, includes many VMs	1.NBT
12	Montessori Division Board	Division Board	3.NBT
13	Montessori Geometry	3D Shape Sorter	1.G
14	Montessori 1st Operations	Discover odd and even	1.NBT
15	Pink Tower	Card 6	K.G
Recommended by Online Reviewer: Testing Group			
1	Mathmateer	Odd Number	2.OA
2	Abacus Adventure	Level 7	2.NBT
3	Doodle Math Numbers	Put things in Order (fridge magnets)	K.CC
4	Pizza Party	Level 2	3.NF
5	My First Tangrams for iPad	Level 1	K.CC

Popularity: Development Group			
	App	Virtual Manipulative	CCSSM
1	Fraction Ninja	Adventure Mode- Level 2	4.NF
2	Place Value App	Place Value Cards	3.NBT
3	Hundreds Board App	Hundreds board	1.NBT
4	Base Ten Blocks Math App	Base Ten Blocks	1.NBT
5	Base Ten Blocks Manipulative	Base Ten Blocks	1.NBT
6	Motion Math Fractions	Easy Setting- Level 2	3.NF
7	Fraction Manipulative	Fraction Circles	3.NF
8	Attribute Blocks	Attribute Blocks	1.G
9	Intro to Math	Red Rods	K.G
10	Bugs and Numbers	The Claw	1.NBT
11	Matific	Grade 3- Square it Up	3.NBT
12	Montessori Numbers	Quantity 10 to 99	2.NBT
13	Candy Count app	Counting Candy	K.CC
14	Number Rods	Number Rods	K.CC
15	Hundreds Board	Hundreds board	1.NBT
16	Numberline Frog	Numberline	1.NBT
17	Montessori Bead Skip Counting	Skip Counting Beads	2.OA
18	Angle Asteroids	Level 2	4.G
19	Motion Math Wings	Pre-Multiplication Island 2	2.OA
20	Bugs and Numbers	Store: Sorting, Counting, and Tallying	K.CC
21	Pattern Blocks	Pattern Blocks	K.G
22	Chicken Coop Fractions	Estimating Fractions- Improper Fractions hard	5.NF
23	Numberline	Whole Numbers Level 2	2.NBT
24	Tiny Chicken Learns Math	Chicken Bounce > Addition	2.OA
25	AstroMath	Level 4- Primes of 2 and 5	
26	Stamp Game	Stamp Game	2.NBT
27	Place Value	Regroup	2.NBT
28	Visual Fractions, Decimals, and Percentages	Grid- One Unit	4.NBT
29	Geoboard	Geoboard	3.G
30	Tens Frame	Tens Frames	K.CC
31	Friends of Ten	Show me...	K.OA

32	Montessori Counting Board	Counting Board	K.CC
33	MonteCalc	Addition Activity	1.OA
34	Montessori Bead Facts Plus Minus	Addition 0 to 9	1.OA
35	Number Rack	Number Rack	1.NBT
36	Money Pieces	Number Pieces	1.MD
37	Numberline	Numberline	1.NBT
38	Understanding Math- Times Tables	array	3.NBT
39	Geometry Montessori	3D	1.G
40	My First Weighing Exercises	Mode 1	K.MD
Popularity: Testing Group			
1	Montessori Numberland	Playbox- balls	K.G
2	Place Value and Rounding	Rounding Coach	4.NBT
3	Number Pieces	Number Pieces	2.NBT
4	Bugs and Numbers	Lab Measurements	1.MD
5	ladybird maths	Sharing	1.NBT
6	Telling Time	Set to the Clock to the Minute	1.MD
7	Matific	Grad 1- That's the Ticket I	1.NBT
8	Equivalence Tiles	Equivalence Tiles	3.NBT
9	Visual Multiply	tables	3.NBT
10	Visual Fractions, Decimals, and Percentages	Circles- Several Units	4.NF
11	Number Frames	Number Frames	K.CC
12	Geometry Montessori	Find the Next Shape	K.G
13	Understanding Math- Times Tables	groups	3.NBT
14	Gracie and Friends City Skate	Level 3	K.CC
15	Measure This	Arrange by Length	K.MD
16	Jungle Geometry	What is the length- Level 2	1.MD
17	Fingu	Level 3	K.CC
18	Hundreds Board Extensions: Roman Numerals	Roman Numerals	---
19	NumberLine 2	Numberline	4.NBT
20	Matific	Grade 1- Cut, Paste, and Figure I	1.G

CURRICULUM VITAE

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Education**Ph.D. March 2016**

Education, Utah State University, Logan, UT
 Specialization: Curriculum and Instruction
 Emphasis: Mathematics Education and Leadership

M.Ed. May 2005

University of Phoenix, Salt Lake City, Utah
 Masters of Arts in Education, Curriculum and Technology

B.S. December 2002

Weber State University, Ogden, Utah
 Bachelors of Science in Elementary Education
 Elementary Reading Endorsement

B.S. May 2002

University of Phoenix, Salt Lake City, Utah
 Bachelors of Science in Business Management

Employment History**Curriculum Specialist: Elementary Mathematics, Dual Immersion, and Gifted and Talented Education (2017-present)****Weber School District, Ogden, UT**

Responsibilities include overseeing the elementary mathematics curriculum, resources, and professional development for 700 elementary teachers, overseeing and coordinating the Spanish and Mandarin Chinese dual immersion programs, and overseeing all gifted and talented programs and resources. Additional responsibilities include grant writing, collaborating with Utah State Board of Education directors, and technical development of online curriculum resources.

Director, Elementary Mathematics Teachers Academy (2014-2017)

www.online.usu.edu/teachmath

School of Teacher Education and Leadership, College of Education and Human Services

and Regional Campuses and Distance Education

Utah State University, Logan, UT

Responsibilities include Academy recruitment, marketing, and teaching customized courses face-to-face and in the Canvas online course environment. They also include collaborating with the Center for Innovative Design concerning technical development and issues.

University Teacher, Elementary Education Program (2012-2017)

School of Teacher Education and Leadership, College of Education and Human Services

Utah State University, Logan, UT

Responsibilities include teaching undergraduate courses in the elementary education program, collaborating with other instructors, developing course syllabi, revising courses based on course evaluation feedback, creating assessments using online course management system (Canvas), and supervising students during practicum placements.

Graduate Research Assistant (2012 – 2014)

School of Teacher Education and Leadership, College of Education and Human Services

Utah State University, Logan, UT

Provided research assistance to Dr. Patricia Moyer-Packenham. This assistance included helping with project genesis and implementation, grant writing, reviewing articles for inclusion in a research group database, coding, collaborating on papers, and collaborating on presentations.

Chinese Immersion Program Teacher (2011-2012)

Uintah Elementary School

Weber School District, Ogden, UT

Provided half-day language arts instruction for Chinese immersion students. Planned and organized mathematics instruction to be delivered by a cooperating Hon-Bon teacher from Taiwan to these same students.

Elementary Classroom Teacher (2002 – 2011)

Uintah Elementary School

Weber School District, Ogden, UT

Taught all subjects in 6th grade (1 year), 4th grade (3 years), and 3rd grade (5 years). Was the Web Master for school website and the school technology committee chair.

District Technology Integration Specialist (2007 – 2012)

Technical Services Department

Weber School District, Ogden, UT

Responsibilities included evaluating classroom technologies, curriculum integration strategies. Helped to adopt district supported technologies, worked with technology vendors, and made purchasing information available to administrators. Trained teachers district wide on new technologies when classroom purchases are made by administrators.

- Managed biweekly *iLead* courses- a series of technology training courses made mandatory for administrators and district personnel. Select trainers, manage registration, configure training room technology, and personally instruct twice per year.
- Developed, directed, and taught at the WSD BrainBlast Conference- a 2-day hands-on technology conference held each August for 300 educators and 50 administrators chosen by lottery in WSD. Over 600 apply annually to attend.

Awards and Professional Recognition

- Fredrick Q. Lawson Fellowship, January 2015
- Teaching Assistant of the Year award, Emma Eccles Jones College of Education and Human Services, Utah State University, April 2014
- Teaching Assistant of the Year award, School of Teacher Education and Leadership, Utah State University, April 2014
- Utah State University Graduate Student Senate Enhancement Award, April 2014
- Honorable Mention, Oral Research Presentation, *A Loss to Explain: Autoethnographically*, Utah State University Research Week, 2013
- “I Love Teaching Award” Weber School District, Ogden, UT, April 2010
- “Apple for the Teacher” Award Nomination, Ogden, UT, 2008, 2009, 2010, 2012
- University of Phoenix Graduation Speaker, Salt Lake City, UT, May 2005
- Weber School District “Rookie” Teacher of the Year, Weber School District, Ogden, UT, 2002-2003

Publications

Journal Articles

Shumway, J. F., Moyer-Packenham, P. S., Baker, J. M., Westenskow, A., Anderson-Pence, K. L., Tucker, S. I., **Boyer-Thurgood, J.**, & Jordan, K. E. (2016). Using open-response fraction items to explore the relationship between instructional modalities and students’ solution strategies. *International Journal of Education in Mathematics, Science and Technology*, 4(2), 112-132. doi:10.18404/ijemst.20845

Moyer-Packenham, P.S., Bullock, E.K, Shumway, J.F, Tucker, S.I, Watts C.M., Westenskow, A., Anderson-Pence, K.L., Maahs-Fladung, C., **Boyer-Thurgood,**

J. M., Gulkilik, H., Jordan, K. (2016). The role of affordances in children's learning performance and efficiency when using virtual manipulative mathematics touch-screen apps. *Mathematics Education Research Journal*.

Westenskow, A., **Boyer-Thurgood, J. M.**, & Moyer-Packenham, P. S. (2015). A window into mathematical support: How parents' perceptions change following observations of mathematics tutoring. *Journal of Research in Childhood Education*. 29:4, 458-475. doi: 10.1080/02568543.2015.1073816

Conference Publications (Refereed)

Moyer-Packenham, P. S., **Boyer-Thurgood, J. M.**, Legler, N., Larsen, K. (2014, March). The Elementary Mathematics Teachers Academy: An individualized online CCSS mathematics professional development. *Proceedings of the Society for Information Technology and Teacher Education International Conference 2014 (SITE)*, (pp. 2271-2276).

Boyer-Thurgood, J. M., Tucker, S. I., Mejia, J. A., & Norman, P. (2014, January). The socio-cultural importance of writing and sharing autoethnographic research. *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE)*, (pp. 1115-1116), Honolulu, Hawaii, ISSN# 1541-5880.

Boyer-Thurgood, J. M., Moyer-Packenham, P. S., Tucker, S. I., Anderson, K. L., Shumway, J. F., Westenskow, A., Bullock, E., & The Virtual Manipulatives Research Group at Utah State University (2014, January). Kindergarteners' strategy development during combining tasks on the iPad. *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE)*, (pp. 1113-1114), Honolulu, Hawaii, ISSN# 1541-5880.

Moyer-Packenham, P. S., Anderson, K. L., Shumway, J. F., Tucker, S. I., Westenskow, A., **Boyer-Thurgood, J. M.**, Bullock, E., Mahamane, S., Baker, J. M., Gulkilik, H., Maahs-Fladung, C., Symanzik, J., Jordan, K. & The Virtual Manipulatives Research Group at Utah State University. (2014, January). Developing research tools for young children's interactions with mathematics apps on the iPad. In *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE)* (pp. 1685-1694). Honolulu, Hawaii, ISSN# 1541-5880.

Tucker, S. I., Moyer-Packenham, P. S., **Boyer-Thurgood, J. M.**, Anderson, K. L., Shumway, J. F., Westenskow, A., Bullock, E. & The Virtual Manipulatives Research Group at Utah State University. (2014, January). Literature supporting investigations of the nexus of mathematics, strategy, and technology in second-graders' interactions with iPad-based virtual manipulatives. In *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE)* (pp. 2338-2346). Honolulu, Hawaii, ISSN# 1541-5880.

Manuscripts Under Review

Boyer-Thurgood, J. M. (2012). *Place Value “App”titude: Using Mathematical Tools and iPad Technology to Learn Place Value*. Under review.

Shumway, J., Westenskow, A., Moyer-Packenham, P. S., Anderson-Pence, K. L., Tucker, S. I., & **Boyer-Thurgood, J. M.**, (under review). Comparing students’ errors: The relationship between different representational modalities and students’ fractional understanding. Under review.

Research

Research Interests

- Children’s interactions with mathematical representations
- Development of virtual manipulatives for touchpad interfaces
- Online mathematics teacher coursework and professional development

Research Activities

VM BRAIN Patterns: Virtual Manipulatives: Brain Research on Activation and Investigation of Neural Patterns. (2012-present)

- Assisted in developing task sequence for pilot study
- Conducted participant interviews in the NIRS brain imaging lab

Captivated! Young Children’s Learning Interactions with iPad Mathematics Apps. (2012-present)

- Reviewed over 50 apps for inclusion in the study
- Met with app programmers to arrange free use of apps and app design changes
- Selected iPad apps for inclusion in the study
- Created three interview sequences participants of different grade levels
- Conducted preliminary study with children from three age groups
- Arranged for and led pilot study including 20 children and 4 researchers
- Conducted study interviews
- Developed qualitative learning progressions for Kindergarten subitizing and combining app sequence
- Coded video data

TIME Clinic Data Collection and Analysis (2012-2014)
Utah State University, Logan, UT

- Code, analyze, and synthesize data from a parent survey concerning experiences at the TIME Clinic
- Interview and transcribe telephone interviews with parents of students who participated in the TIME Clinic during the summer session.

Virtual Manipulatives Research Group (2011-present)
Utah State University, Logan, UT

- Meet frequently with an interdisciplinary group of professors and doctoral students
- Develop and conduct research on projects concerning virtual manipulatives
- Assisted in writing papers for publication

Grants Funded

Research Grants

Project Director (\$145,981). *K-6 Mathematics e-Learning Professional Development Partnership.* (funded December 2015). Utah State Office of Education. Project goal: Increase the mathematical content and pedagogical knowledge of K-6 teachers using online learning modules and courses (with PI – Sheri Heiter, Weber School District).

Graduate Research Assistant (\$20,000). *Captivated! Young Children's Learning Interactions with iPad Mathematics Apps.* (2013-2014). Utah State University, Vice President for Research RC Funding. Project goal: build theory and knowledge about the nature of young children's ways of thinking and interacting with virtual manipulatives using touch-screen mathematics apps on the iPad. (with Principal Investigator Patricia Moyer-Packenham, Co-PI Cathy Maahs-Fladung, and the Virtual Manipulatives Research Group).

Travel Grants

Graduate Student Professional Conference Awards. (\$1,115: \$700 Department of Teacher Education and Leadership and \$415 Center for Women and Gender). (2014). Travel Funding awarded for presentations at the Hawaii International Conference on Education, Honolulu, HI.

Graduate Student Professional Conference Awards. (\$750 Department of Teacher Education and Leadership). (2013). Travel Funding awarded for presentation at the SSMA Annual Conference in San Antonio, TX.

Professional Presentations

International Presentations

Moyer-Packenham, P. S. & **Boyer-Thurgood, J. M.** (2014, March). *The Elementary Mathematics Teachers Academy: An Individualized Online CCSS Mathematics Professional Development*. Brief Paper Session, Society for Information Technology and Teacher Education Conference, Jacksonville, Florida.

Boyer-Thurgood, J. M., Moyer-Packenham, P. S., Tucker, S. I., Anderson, K. L., Shumway, J. F., Westenskow, A., & Bullock, E. (2014, January). *Kindergartener's Strategy Development during Combining Tasks on the iPad*. Paper Session, Hawaii International Conference on Education, Honolulu, Hawaii.

Boyer-Thurgood, J. M., Tucker, S. I., & Mejia, J. A. (2014, January). *The Socio-Cultural Importance of Writing and Sharing Autoethnographic Research*. Workshop Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Moyer-Packenham, P. S., Shumway, J. F., Westenskow, A., Tucker, S. I., Anderson, K. L., **Boyer-Thurgood, J.**, & Bullock, E. (2014, January). *Young Children's Mathematics Interactions with Virtual Manipulatives on iPads*. Paper Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

Tucker, S. I., Moyer-Packenham, P. S., **Boyer-Thurgood, J. M.**, Anderson, K. L., Shumway, J. F., Westenskow, A., & Bullock, E. (2014, January). *The Nexus of Mathematics, Strategy, and Technology in Second-Graders' Interactions with an iPad-Based Virtual Manipulative*. Paper Session, Hawaii International Conference on Education (HICE), Honolulu, Hawaii.

National Presentation

Moyer-Packenham, P. S., Shumway, J. F., Tucker, S. I., **Boyer-Thurgood, J. M.**, Hunt, J. & Bullock, E. (2014, April). *Young Children's Mathematics Interactions with Virtual Manipulatives on iPads*. Paper Presentation, National Council of Teachers of Mathematics Research Conference, New Orleans, Louisiana.

Invited Presentations

Tucker, S. I., **Boyer-Thurgood, J. M.**, Mejia, J. A., & Norman, P. (2014, April). *Experiencing Autoethnography*. Invited Presentation, TCORE 123C: The Autoethnographic Self. Instructor: Dr. Rich Furman. University of Washington – Tacoma.

Moyer-Packenham, P. S., & **Boyer-Thurgood, J. M.** (2014, March). *The Elementary Mathematics Teachers Academy: An Individualized Online CCSS Mathematics Professional Development*. Utah Association of Mathematics Teacher Educators, Brigham Young University, Provo, UT.

State Level Presentations

Boyer-Thurgood, J. M. (2014, October). *Early Geometry Concepts: The Theory, the Practice, and of Course the Apps*. Utah Council of Teachers of Mathematics Annual Conference, Davis County, UT.

Boyer-Thurgood, J. M. (2014, October). *Introduction to the Elementary Mathematics Teachers Academy*. Utah Council of Teachers of Mathematics Annual Conference, Davis County, UT.

Boyer-Thurgood, J. M. (2012, October). *Technology Techniques: Quick, Motivational, and Effective SMART Board and iPad Based Mathematics Instruction Strategies*. Utah Council of Teachers of Mathematics Annual Conference, American Fork High School, American Fork, UT.

University Presentations

Boyer-Thurgood, J. M., Tucker, S. I. & Mejia, J.A. (2014, January). *Experiences with Autoethnography*. Invited Presentation, EDUC 7780: Qualitative Methods II. Instructor: Dr. Sherry Marx. Logan, Utah.

Tucker, S. I., **Boyer-Thurgood, J. M.**, Mejia, J. A., & Norman, P. (2013, April). *A Loss to Explain, Autoethnographically*. Research and Methodology Session, Utah State University Research Week, Logan, Utah.

School District Presentations

Legislative Review of Immersion Programs, Uintah Elementary School, Ogden, Utah. (2012, April). Legislative representatives and elementary principals involved in funding or overseeing attended a meeting to review the UT Chinese Immersion Program. I was the keynote speaker asked to share the benefits of and concerns about the Chinese Immersion program as implemented in our elementary school.

Weber School District Office, Ogden, Utah. *The State of Classroom Technology*. (2010, April). Presented to the Weber School District Foundation concerning available classroom technologies and their implementation and plans for future implementation in Weber School District.

Weber School District Office, Ogden, Utah. *The Effectiveness of Small Group Based Differentiated Reading Instruction*. (2009, October). Presented the results of an action research project conducted in my classroom.

Weber School District Office, Ogden, Utah. *Effective Blog Development and Use in the Elementary Classroom*. (2009, March). Presented my classroom blog and blogs created by those I mentored as a way of improving communication with parents and extending the classroom experience for students.

State Service

Utah Council of Teachers of Mathematics Board Member and Web Master (2011-2017)
West Jordan, UT

- Responsible for gathering all data related to UCTM and mathematics happenings in the state of Utah and making them available on the UCTM website at www.utahctm.org
- Responsible for yearly conference participant, presenter, and exhibitor registration
- Responsible for maintaining the UCTM membership list and sending out periodical e-mail communications to the membership.
- Social networking committee chair, responsible to schedule, create, and post information to Facebook, LinkedIn, Twitter, and the UCTM Blog.

Community Council Member, Uintah Elementary, Ogden, UT, 2003-2011 and 2014-present

- Identified and addressed critical needs of students at Uintah Elementary
- Oversaw purchase of math manipulatives for school library
- Implemented inventory system to track Community Council Inventory

Community Council Chair, Uintah Elementary, Ogden, UT, 2008-2009 school year

- Carried out phase three of Uintah's E-Learn initiative- including SMART Response systems, projector installations, and document cameras in every classroom
- Adopted the KwaWaToTo school in Kenya, Africa as a sister school- commitment to raise \$9,000 per year to feed the 200 African students 1 daily meal.
- Completion of a guided reading library
- Continued funding for a school reading specialist

University Service

Hillcrest Elementary School, Logan, Utah. *Classroom Technology Basics*. (2012, February). Invited to present during a student teaching seminar concerning classroom technology, specifically SMART Boards and iPads. The use of these technologies as motivational tools, content management tools, and lesson enhancement tools was presented discussed.

Utah State University, Logan, Utah. *Fractions and Technology*. (2012, October). Invited guest speaker Eled 4060. Jessica Shumway

Utah State University, Logan, Utah. *Fractions and Technology*. (2012, October). Invited guest speaker Eled 4060. Katie Andersen

Utah State University, Logan, Utah. *Fractions and Technology*. (2012, October). Invited guest speaker Eled 4060. Stephen Tucker

Professional Affiliations

- Utah Council of Teachers of Mathematics (2012-present)
- National Council of Teachers of Mathematics (2012-present)
- Utah Association of Mathematics Teacher Educators (2013-present)
- School Science and Mathematics Association (2013-present)
- Society for Information Technology and Teacher Education (2014-present)