Utah State University

DigitalCommons@USU

All Graduate Theses and Dissertations

Graduate Studies

5-1996

Analogical Problem Solving: An Exploratory Analysis of the Facilitating Effects of Type of Training, Analog Type, and Level of **Expertise on Spontaneous Transfer**

Dune E. Ives Utah State University

Follow this and additional works at: https://digitalcommons.usu.edu/etd



Part of the Psychology Commons

Recommended Citation

Ives, Dune E., "Analogical Problem Solving: An Exploratory Analysis of the Facilitating Effects of Type of Training, Analog Type, and Level of Expertise on Spontaneous Transfer" (1996). All Graduate Theses and Dissertations. 6084.

https://digitalcommons.usu.edu/etd/6084

This Thesis is brought to you for free and open access by the Graduate Studies at DigitalCommons@USU. It has been accepted for inclusion in All Graduate Theses and Dissertations by an authorized administrator of DigitalCommons@USU. For more information, please contact digitalcommons@usu.edu.



ANALOGICAL PROBLEM SOLVING: AN EXPLORATORY ANALYSIS OF THE FACILITATING EFFECTS OF TYPE OF TRAINING, ANALOG TYPE, AND LEVEL OF EXPERTISE ON SPONTANEOUS TRANSFER

by

Dune E. Ives

A thesis submitted in partial fulfillment of the requirements for the degree

οï

MASTER OF SCIENCE

in

Psychology

Approved:

UTAH STATE UNIVERSITY Logan, Utah

1996

Copyright © Dune Ives 1996

All Rights Reserved

ABSTRACT

Analogical Problem Solving: An Exploratory Analysis of the Facilitating Effects of Type of Training, Analog Type, and Level of Expertise on Spontaneous Transfer

by

Dune E. Ives, Master of Science Utah State University, 1996

Major Professor: Dr. Lani Van Dusen

Department: Psychology

Research on analogical problem solving has delineated several factors that impact one's ability to spontaneously generate a correct solution strategy to a target problem. These factors include, but are not limited to, type of analogy provided to subjects (i.e., partial versus complete), the level of isomorphism between analogies and target problems, and the solver's level of analogical problem-solving expertise

Recently, researchers have begun to focus on providing solvers with direct instruction on analogical problem-solving processes and strategies in an effort to augment analogical problem-solving ability. The most common type of instruction (i.e., teachergenerated) involves providing direct instruction on problem-solving processes and strategies without input from the solver. A second type of instruction (i.e., learnergenerated) that has gained some attention in the literature but has not yet been tested in

the realm of analogical problem solving involves learners actively participating in developing analogical problem-solving strategies while being guided by the instructor.

Using an experimental design, the present study examined the differential effects of type of analogue (i.e., partial versus complete), level of expertise (i.e., novice versus expert), and type of training (teacher-generated, learner-generated, or no training) on spontaneous generation of correct solution strategies to two target problems.

Findings indicate that solvers, regardless of training group or ability level, were better able to solve the target problem to the complete analogies than the target problem to the partial analogies, $\chi^2(1, \underline{N} = 116) = 18$, $\underline{p} < .001$; $\underline{d} = .85$. Moreover, there was no advantage for expert solvers to participate in problem-solving training. However, when examining novice solvers, findings indicate that direct instruction on problem-solving processes and strategies resulted in better performance when solving the partial analogy than did no instruction ($\underline{d} = .61$). Also, active participation in the learning process resulted in better performance when solving the partial analogy than did no instruction ($\underline{d} = .80$).

Limitations of the study, implications for educators, and recommendations for future studies are provided.

(116 pages)

DEDICATION

I would like to dedicate this thesis to my late grandmother, Delores Green, who I know would have enjoyed this moment far more than I will ever be able to.

ACKNOWLEDGMENTS

I would like to thank Dr. Lani Van Dusen for giving me the opportunity to challenge myself with such a complex research endeavor. I would also like to thank Dr. Van Dusen and my committee members, Drs. Xitao Fan, Tamara Ferguson, and Deborah Hobbs, for all their assistance, words of encouragement, and relentless support throughout the course of this project.

To Eric Gee, Bill Luce, David Spach, Shane Stowell, Dennis Willie, and Tressa Woodmancy, thank you for helping me keep my sanity throughout this journey that never seemed to end and for donating your invaluable time to such a worthy cause!

To all of my friends who have stuck by my side during the many crazed moments: I know at times I must have seemed as though I were missing in action or off in bizarro world thinking of "fun" new ways I could entice undergraduates to participate in this once-in-a-lifetime study. For your loyalty, laughter, and love, I will be forever in your debt.

A thank you could not be complete without recognition to my parents, for it was my mother and father who provided their enduring faith in my ability, steadfast support, and sincere patience through thick and thin. Thank you for believing I could make it!

I am especially grateful to my lifelong partner, Chad, for his loving sense of humor that I could not have lived without, his amazing patience and grace, and his ability to make everything bad seem good and do-able. Just one more to go and then it's "Onward Ho!"

Dune Erin Ives

CONTENTS

		Page
ABSTRACT		iii
DEDICATION		V
ACKNOWLEDGME	NTS	vi
LIST OF TABLES		viii
LIST OF FIGURES .		ix
GLOSSARY OF TEI	RMS	X
PROBLEM STATEM	MENT	1
LITERATURE REV	IEW	4
THE STUDY		24
RESULTS AND DIS	SCUSSION	36
CONCLUSION		47
REFERENCES		52
APPENDICES		61
Annendix A	Participant Consent Form	62
Appendix B.	Demographic Information Questionnaire	
Appendix C.	Problem-Solving Ability Test	66
Appendix D.	Participant Training Sign-Up Sheet	70
Appendix E.	Teacher-Generated Training Protocol	
Appendix F.	Learner-Generated Training Protocol	
Appendix G.	Control Group Protocol	
Appendix H.	Training Analogies.	
Appendix I. Appendix J.	Testing Analogies	
Appendix J.	TOTALIONSHIP DOLYTOON I TOOTOHIS WING I HIGHOSAGS	

LIST OF TABLES

Table		Page
1	Number and Percentage of Subjects Producing Correct Solution Strategies to the Testing Problems	38
2	Number and Percentage of Subjects Correctly Solving the Testing Problems According to Training Group	40
3	Number and Percentage of Subjects Correctly Solving the Target Problems According to Training Group Membership and Problem-Solving Ability	43

LIST OF FIGURES

Figure		Page
1	Relationships between problems presented during group sessions and the testing problems	30

GLOSSARY OF TERMS

The following terms and definitions will be helpful to the reader when examining this document. Some replication of definitions may be found in the text where it was deemed necessary to provide the reader with a clear understanding of the study.

Complete problem - a problem that provides the learner each of the following: initial goal state, final goal state, objects, constraints, and correct solution strategy.

Partial problem - a problem that provides the learner at least one but not all of the following: initial goal state, final goal state, objects, constraints, and correct solution strategy.

<u>Isomorphic</u> - two or more problems that are isomorphic to each other have an analogous initial goal state, final goal state, objects, constraints, and correct solution strategies.

<u>Nonisomorphic</u> - two or more problems that are nonisomorphic to each other may be analogous to each other in all but one of the following: initial goal state, final goal state, objects, constraints, and correct solution strategies. The degree of isomorphism between problems is determined by the total number of analogous factors.

PROBLEM STATEMENT

It has been posited that people develop expertise based on an analogical problem-solving process, that is, they assess novel situations based on previous experiences, apply analogous solutions, and establish more well-developed schemata in the process. In the educational setting, this would be a description of the ideal student. Unfortunately, for most this scenario is not the case (e.g., Pierce, Duncan, Gholson, Ray, & Kamhi, 1993). Because of the lack of ability of students to spontaneously solve novel problems, it is necessary for educators to train students in engaging in analogical reasoning.

The identification of this gap between the ideal and realistic student is the impetus for research on analogical problem solving in the domains of cognitive and educational psychology. This research has been extremely widespread with regard to factors that may influence analogical transfer, that is, the successful application of an analogous solution on a target problem. There are several important and influential factors in successful analogical problem solving, but the most effective appear to be the use of hints, use of visual aids, adequate incubation, a high level of isomorphism between problems, retrieval of well-defined schemas, and an expert level of problem-solving ability (Dreistadt, 1969; Gick & Holyoak, 1980, 1983; Hess & Klecha, 1990; Holyoak & Koh, 1987; King-Johnson, 1992; Klauer, 1989; Novick, 1988; Novick & Holyoak, 1991; Wickelgren, 1974).

Based on these findings, researchers, especially those in educational domains, have begun to study the effects of training students to become more effective problem solvers (Mayer, 1985, 1987). Thus, they are attempting to provide the teacher with appropriate information on how to improve their students' problem-solving abilities, build tighter associations between analogous problems, and develop assimilative schemata (Ross, 1989; Solomon, 1991; Zook, 1991). Previous research in this area has found that low problem-solving-ability students may benefit from conceptual stimuli such as examples, analogies, mediators, and cues (Kazanas & Chawhan, 1975), and that training students to retrieve appropriate schema does lead to better recall (Brooks & Dansereau, 1983) and greater facilitation of positive transfer (Robins & Mayer, 1993).

Although several studies have witnessed positive transfer, much of this research has focused primarily on "teacher-generated" training with multiple isomorphic problems (Pierce et al., 1993). This process, referred to as eliminative induction (Black & Rollins, 1982), has more recently been combined with self-monitoring (Osman & Hannafin, 1992) by the subject. In self-monitoring, the subject is trained in employing, monitoring, and evaluating problem-solving strategies. Hence, research on problem-solving strategies is progressing toward mediation of cognitive processes by students; however, even in these new approaches the main methods of problem solving are presented didactically. Thus, as suggested by educational researchers (Kohn, 1993), problem-solving training is beginning to incorporate student participation, but this training still focuses more on training outcomes rather than the process that occurs during the training.

A more useful measure of impact of student input may come from studying the effectiveness of students generating their own analogical problem-solving processes. Evidence exists in cognitive psychology of the benefits received when students are afforded the opportunity to participate in their learning process (Kohn, 1993), that is, student learning and motivation increase. Likewise, perhaps increases in student participation in developing their own training, for example, deciding what is important to focus on, may facilitate greater spontaneous analogical transfer.

The present study was aimed at reducing the discrepancy between what is known about the prevalence of positive transfer with teacher-generated training and what is not known about the relative benefits of including students in developing their own problemsolving strategies. This research has practical utility for educators at the college level in that it may yield important information on how best to aid college students in the problemsolving process. Moreover, it will lend much to the discovery process on how to bridge the gap between novice and expert problem solvers.

LITERATURE REVIEW

Introduction

In everyday life, individuals tend to view problems as extensions of, or similar situations to, previous scenarios they have solved either successfully or unsuccessfully. Consider novice chess players. When individuals first begin playing chess, they have no information on which to base their opening, middle game, and closing moves. However, as they increase the number of games they play, they are able to evaluate moves based on previous similar performances. Upon doing so, they either choose to execute the same move or select another one. Through this process, they add to their repertoire of moves and may eventually become expert chess players. In essence, this is analogical problem solving, that is, the application of a solution or outcome from a similar situation to the present situation (Keane, 1988; Reeves & Weisberg, 1993; Rumelhart, 1989; Seel & Hoops, 1993).

Cognitive and educational psychology researchers have acknowledged the potential benefits of using analogies in problem solving. Among such benefits is increasing a learner's ability to learn how to think about problems differently (Catrambone & Holyoak, 1989) and how to combine new and previously learned information into more well-developed schemata (Ross, 1984; Zook, 1991). Using these schemata, problem solvers may be more apt to make inferences and increase their understanding about current

events or problems (Solomon, 1991; Stein, Way, Benningfield, & Hedgecough, 1986). Although these benefits are realized, it is unclear exactly how problem solvers utilize analogies to solve problems and ultimately produce better schema (Hesse & Klecha, 1990). Ross (1984) noted that "for people unfamiliar with a domain, even if a rule is remembered perfectly, it may not be applied successfully" (p. 374). It is out of this deficiency that problem-solving studies incorporated the use of analogues in their designs (Spencer & Weisberg, 1986).

In its most basic form, analogical problem solving consists of deriving from memory (retrieval) a solution from a previously encountered problem (the base problem or analogue) and using it (mapping) to solve a novel problem (termed the target problem) (Antonietti, 1991; Clement, 1988; Gick & Holyoak, 1980, 1983; Klauer, 1989; Novick & Holyoak, 1991; Okagaki & Koslowski, 1987; Reeves & Weisberg, 1993; Seel & Hoops, 1993; Spencer & Weisberg, 1986; Wickelgren, 1974; Zook, 1991). More specifically, this analysis involves assessing both problems for their surface and structural similarities, including their initial and final goal states, and using their similarities and differences in determining the most appropriate solution strategy to satisfy the current problem's goals (e.g., Okagaki & Koslowski, 1987).

Similarly, analogical transfer refers to the retrieval and mapping of an analogous solution (Klauer, 1989; Novick & Holyoak, 1991; Reeves & Weisberg, 1993), but goes one step further by assessing how "successful" these processes are. Three types of transfer have been identified in the existing literature: (a) positive transfer, (b) negative transfer, and (c) spontaneous transfer (Clement, 1988; Gick & Holyoak, 1980; Novick,

1988; Ross, 1987). The similarity of base and target problems and the application of base solutions determine what type of transfer has occurred (Novick, 1987). That is, a solution derived from an analogous base problem and used correctly would yield positive transfer. More specifically, stating that positive transfer has occurred indicates that the target problem was correctly solved using an analogous base problem's solution. Conversely, if a solution from a nonanalogous, or irrelevant, base problem is applied to the target problem, negative transfer may result. Spontaneous transfer is a subset of positive transfer and refers to the point at which a subject derives the correct solution. If the correct solution is derived before a hint to use an analogous problem is given, spontaneous transfer has occurred. However, if the correct solution is derived after a hint is given, positive transfer is the result. The majority of studies have focused their efforts on determining whether positive or spontaneous transfer has occurred. The present study did not incorporate the use of hints or other assistance in its design, and as such focused on the prevalence of spontaneous transfer only.

Processes Involved in Analogical Transfer

Although the semantics used in describing analogical problem solving vary from study to study, five processes appear to cut across studies: (a) goal identification, (b) representation of base and target propositions/relations, (c) schema induction and adaptation (retrieval), (d) mapping (application), and (e) learning (Clement, 1988; Funkhouser, 1990; Gentner, 1989; Gholson, Eymard, Morgan, & Kamhi, 1987; Gick & Holyoak, 1980; Holyoak & Thagard, 1989; Keane, 1985; Mayer, 1985; Needham &

Begg, 1991; Novick, 1988; Novick & Holyoak, 1991; Pierce et al., 1993; Reeves & Weisberg, 1993; Ross, 1984; Vosniadou, 1989; Zook, 1991). It is important to note that these processes are not viewed as mutually exclusive, that is, problem solvers will move through each of these processes in no structured order per se. Clement (1988) has suggested that solution generation is more of an evolutionary process, that is, the subject does not always use insight in the generation, but rather s/he moves between all problem-solving processes in an attempt to transform or reject the analogous problem. Each of these processes will be briefly defined below.

Goal Identification

Goal identification refers to the learner's ability to identify the primary objectives of the problem. Keane (1985) suggested that problem solvers must attend to "(a) the role the object concept plays in the domain (agent, object, instrument, etc.), (b) any attributional overlap between parallel objects in both domains, and (c) 'functionally relevant attribute' information" (p. 450). Understanding these aspects of both the base and target problems may in fact be the most important first step in one's ability to generate a correct solution (Gick & Holyoak, 1980). Those studies that have employed analogies with similar goal states have increased the learner's identification of correct goals and facilitation of positive transfer (Okagaki & Koslowski, 1987).

Also important is the discrepancy between the goals of the problem solver and the goals of the problem (Gentner, 1989). Ideally, the learner's interpretation of the problem's goals and the actual problem's goals will be isomorphic in nature. If, however, the solver

evaluates the problem with an entirely different focus than is intended, the solver may access an inappropriate schema, thereby reducing the probability that positive transfer would be obtained.

Problem Representation

After the initial and final goal states of the problem have been identified, it is important for the problem solver to be able to attend to the appropriate information in the base and target problems (Black & Rollins, 1982; Bransford & Stein, 1984; Reeves & Weisberg, 1993; Ross, 1987). In essence, this means that "learners must be able to disregard the superficially dissimilar characteristics of two or more analogous domains and attend to the common relational structure" (Zook, 1991, p. 42). Problem representation is necessary to make an accurate and efficient comparison of the analogues and retrieve appropriate analogue solutions (Mayer, 1985; Novick & Holyoak, 1991; Ross, 1984).

Failure to do so has been found to lead to a decrease in positive transfer (Keane, 1985; Silver & Marshall, 1990).

Schema Induction

Schema induction, also referred to as retrieval and solution generation, has been noted to be the most arduous of all the processes (Novick & Holyoak, 1991). Schema induction entails making connections between previously learned information and the goals of the present problem and retrieving this previously learned information (Funkhouser, 1990; Orlich, 1992). Clement (1988), in a study on how physics experts

problem solve, found that there are three ways to generate a solution: (a) generation from a formal principle, (b) generation via a transformation (of the target problem), and (c) generation via an association (use of a previously learned problem and solution).

Subsequent studies have found that subjects who produce and utilize well-developed schema, through a comparison of base and target structural relations and are able to adapt their solution to fit the target problem, tend to experience more positive transfer (Gick, 1981; Keane, 1985; Klauer, 1989; Novick & Holyoak, 1991; Ross, 1984; Seel & Hoops, 1993).

Moreover, the degree to which the target problem cues the solver on the appropriate schemata will have an impact on the prevalence of transfer (Barton, 1989; Catrambone & Holyoak, 1989). In a study on the effects of using instructional analogies to aid the process of solution production, Zook (1991) found that the most useful analogies are those that either "1) activate pre-existing schemata or 2) facilitate the induction of new schemata" (p. 48).

It is from these findings that some researchers have posited that the development and retrieval of these schemata will lead to an increase in expertise (Novick & Holyoak, 1991).

Mapping

Once the solver has retrieved or constructed an appropriate solution, s/he must apply it to the target problem (Novick & Holyoak, 1991; Vosniadou, 1989) through a mapping process. Mapping gives the problem solver information on how to use the

analogue's solution. This process can be facilitated by eliminating any irrelevant information specific to the target or base problems (Black & Rollins, 1982), which may lead to clear identification of the structural similarities of both problems (Ross, 1987). Conversely, hindrances may include "a) surface and structural dissimilarity between analogs, b) impoverished representation of base domain knowledge, c) overextension of the base domain, d) learner misperception of the analogy's instructional purpose, or e) inexperience in mapping procedures" (Zook, 1991, p. 61). Mapping failures may also stem from the learner's inability to generate a solution that meets the criteria of the target problem's goals (Keane, 1985).

Learning

The final stage in analogical problem solving occurs when the solver adds the new solution strategy to his/her existing schemata (Bransford & Stein, 1984; Holyoak & Thagard, 1989; Novick & Holyoak, 1991). This process does not occur by itself alone. Rather, it is a product of the first four stages in this "evolutionary" process of analogical problem solving in which an association between similar base and target problems is made. Once this association is made, the solver retrieves information from the base problem, maps it onto the target problem, generates a solution for the target problem, and adds the new solution to his/her existing long-term memory base. Thus, learning has occurred, which allows the solver to progress toward becoming expert in problem solving.

Facilitation of Positive Transfer

The study of analogical problem solving has delineated several factors that mediate a learner's ability to successfully transfer a base solution to the target problem. This section will examine the level of impact each of these factors has on spontaneous transfer. For clarity, these factors will be divided into two categories: (a) type of analogy and (b) problem solver characteristics. Not all of the factors that impact analogical transfer are discussed in this paper. Rather, only those factors that have been shown or suggested to have the greatest impact on spontaneous transfer have been included in this study and are presented below. See Holyoak (1985) or Zook (1991) for a more comprehensive look at factors that facilitate positive transfer.

Type of Analogy Factors

Researchers have employed several manipulations of analogy formats in problemsolving studies. The most prevalent include, but are not limited to, problems that vary on surface or structural similarities, complete versus partial analogies, and the use of multiple analogies.

Structural Versus Surface Similarities

Perhaps the factor that accounts for the greatest impact on positive transfer is the similarity between surface and structural features shared by the base and target problem. Surface features speak to how the solver represents problems based on their content (e.g., objects, terms) (Vosniadou, 1989), and structural features indicate the similarity between

two problems based on their propositions or relations that may influence goal attainment (Holyoak, 1985; Novick, 1988). As Holyoak and Koh (1987) stated:

Spontaneous analogical transfer is more likely to occur when the target problem shares multiple features with the source analogue. Both salient surface features, which do not impede achievement of the critical solution, and deeper structural differences, which involve the nature of the solution constraints, have an impact on transfer. (p. 338)

Essentially, the amount of structural and surface similarity that exists between the target and analogical problems and the ability of the problem solver to notice these similarities will increase or decrease the probability of obtaining positive transfer (Antonietti, 1991; Gick, 1981; Gick & Holyoak, 1980, 1987; Gick & Paterson, 1992; Hesse & Klecha, 1990; Okagaki & Koslowski, 1987; Orlich, 1992; Ross, 1987; Solomon, 1984; Stein et al., 1986). This probability is further influenced by whether a problem solver focuses primarily on surface features, structural features, or a combination of both. When a problem solver focuses on the surface similarities between problems, he/she often will incur difficulty in accessing the appropriate schema and producing the target problem's solution (Holyoak, 1985; Holyoak & Koh, 1987; Novick, 1988; Ross, 1984; Zook, 1991). This is due in part to the absence of relevant and vital information that is uncharacteristic of surface features (Novick, 1988), as well as the tendency of the solver to become fixated on the irrelevant surface features, thereby being unable to identify and use the propositional relations between the base and target problems that are essential for correct solution generation (Zook, 1991). Related to this is the finding that when structural similarities are unclear to the solver, s/he tends to rely more on surface similarities in the mapping process (Ross, 1987).

Conversely, identification of structural similarities yields a higher rate of positive transfer (Zook, 1991), as does attention given to both surface and structural similarities (Orlich, 1992). To illustrate this, Holyoak and Koh (1987) found that when base and target analogs have a high amount of surface and structure similarity, subjects are better able to produce the correct target solution: 69% of subjects in the high surface/high structure group spontaneously produced the correct solution, whereas only 13% of subjects in the low surface/low structure group spontaneously produced the correct solution. Researchers have suggested that problem solvers would benefit from ignoring sometimes distracting surface similarities and by focusing on structural features of the target and base problems to aid in their retrieval and mapping processes (Holyoak & Koh, 1987; Zook, 1991).

Complete Versus Partial Analogies

The base analogy can also vary with regard to what information it provides the solver, in comparison to what information is required to solve the target problem. A problem may include the initial and final goal states of the problem, objects, constraints, and the analogue's solution. A complete analogy will provide all of this information. Conversely, a partial analogy provides the solver with one of the aforementioned parts, but not all (Antonietti, 1991). For example, solvers may be presented with an analogy's solution and not know exactly how that solution was obtained, or they may receive the initial and final goal states and not know how to solve the analogue. The base analogue

may also include one but not all of the objects or constraints from the target problem.

Thus, it may be similar but not isomorphic.

In a study designed to determine the facilitating effects of using partial versus complete analogies, Antonietti (1991) found that subjects who received a complete analogy yielded a higher rate of positive transfer than did subjects receiving only partial analogies (p < .05). Moreover, when subjects were presented with two partial analogies in which the sequence of information provided paralleled the sequence of information given in the complete analogy, positive transfer rates were equal to the complete analogy group.

Multiple Analogies

The use of multiple analogies in problem-solving studies has facilitated greater positive transfer (Antonietti, 1991; Catrambone & Holyoak, 1989; Gick & Holyoak, 1980, 1983) than the use of one analogy or no analogy. Despite this finding, positive transfer will only be found if these analogies are similar in structure to the target problem (Gick & Holyoak, 1980), thereby increasing the learner's ability to delineate relevant features in the problems (Gholson et al., 1987). To better understand this, consider a study conducted by Catrambone and Holyoak (1989) that presented two groups with either two isomorphic or one isomorphic and one nonisomorphic analogue. Subjects in the two-isomorphic group produced the correct solution more often than did those in the second group (71% versus 41%, respectively, p < .01). Thus, presenting subjects with multiple isomorphic analogies can aid solvers in the transfer process because it affords subjects more practice with

analogical problem-solving processes, and multiple analogies may provide subjects with additional information that, if retrieved, will assist them in solving the target problem.

Problem Solver Characteristics

Not all solvers bring with them the same amount of domain or general knowledge to a learning situation. The amount and kind of knowledge they possess, as well as their ability to adequately retrieve and map this information, can greatly influence the probability of obtaining positive transfer. In the literature, problem solver characteristics have most often been discussed in terms of expert and novice problem solvers (e.g., Novick, 1988). As such, this study will utilize the same distinction.

The most salient finding with regard to novice and expert problem solvers is that expert solvers yield a higher rate of positive transfer (Chi & Van Lehn, 1991; Jausovec, 1993; King-Johnson, 1992; McVey, 1993; Novick, 1988; Novick & Holyoak, 1991; Pirolli, 1991; Van Lehn, Jones, & Chi, 1992; Zook, 1991). The reason for this lies in the tendency of novice solvers to focus more time and effort on irrelevant features (Chi & Van Lehn, 1991; Marr & Sternberg, 1986) and on surface features of the base and target problems (Brown, 1989; Chi, Feltovich & Glaser, 1981; Novick & Holyoak, 1991; Pirolli & Bielaczyc, 1989), whereas experts focus more time on relevant and both structural and surface similarities (Ross, 1984; Silver, 1979) and are better able to derive important information needed to solve the target problem (Chi et al., 1981; Chi & Van Lehn, 1991; Pirolli & Bielaczyc, 1989). Expert/gifted solvers have also been found to use more than one method of solution generation, tend to use more analogies than do novice/average

solvers in this process, Z(N=30)=2.15, p<.03 (Jausovec, 1993, experiment 1), and can better assess their level of understanding of an example (Chi & Van Lehn, 1991). Thus, it appears that experts bring with them prior experience in problem-solving situations and the ability to reason inductively.

Also related to this is the amount of knowledge a problem solver has in a particular domain. Studies, most of which focus on the domains of physics and mathematics, have found that a learner's prior knowledge may facilitate positive transfer (Clement, 1988; Novick, 1988; Solomon, 1984, 1991; Stein et al., 1986). Clement (1988) found that rather than using the association method, physics experts are more apt to generate a solution by way of transformation, which requires one to use more domain intuition than novices may have in their repertoire. Additionally, Novick (1988) explored differences among expert and novice problem solvers on a problem-solving test requiring mathematical knowledge. 1 It was found that experts experience more positive transfer than do high novices and low novices, 54%, 29%, and 22%, respectively, F(1,65) = 6.46, p < .02. Notwithstanding, the amount of impact expertise has is determined by the solver's ability to access that knowledge (Stein et al., 1986) as previously discussed. As would be expected, experts generally have more prior domain knowledge and are better able to access that information, which may lead to higher rates of positive transfer (Seel & Hoops, 1993).

¹ Level of expertise was determined by scores on the math section of the SAT. Subjects scoring between 500 and 650 were classified as novice and subjects scoring between 690 and 770 were designated experts.

Despite the knowledge regarding what will facilitate positive transfer, alarming evidence exists to suggest that the prevalence of a low rate of transfer is not uncommon in cognitive and educational psychology studies, even in research in a wide assortment of domains (Brown, 1989; Pierce et al., 1993). In an attempt to curtail potentially debilitating effects on transfer, researchers have employed the use of problem-solving training in their studies.

This training is targeted not only at increasing individual's problem-solving ability (Catrambone & Holyoak, 1989; Mayer, 1985; Reed, Dempster, & Ettinger, 1985; Robins & Mayer, 1993; Ross, 1989), but also at establishing their "independence and self-sufficiency" (Osman & Hannafin, 1992, p. 90) in the learning process. Many researchers contend that without explicit instruction, problem solvers are unable to accurately represent problems (Keane, 1985; Reed et al., 1985), retrieve the appropriate schemata (Klauer, 1989), and may not know exactly how to map or adapt this retrieved information (Gick & Holyoak, 1987; Royer, 1979; Schoenfeld, 1979).

Researchers further suggest that by training subjects in the analogical problem-solving processes, they will be better able to think of problems in new ways (Catrambone & Holyoak, 1989; Chi & Van Lehn, 1991; Mayer, 1987), identify similarities between base and target problems (Klauer, 1989), and consider several possible solutions to target problems (Bloom & Broder, 1950; Mayer, 1987; Stein et al., 1986). Perhaps more importantly, problem solvers may become critical thinkers when selecting and applying

potential solutions (Bransford & Stein, 1984; Osman & Hannafin, 1992), and, as a result, their academic achievement (Loewenthal & Pons, 1987) and learning (Ross, 1989) may increase. An effective training program may include identifying relevant similarities in practice analogies (Catrambone & Holyoak, 1989; Cramond, Martin, & Shaw, 1990; Stein et al., 1986), use of multiple practice analogies (Chi & Van Lehn, 1991; Kazanas & Chawhan, 1975; Klauer, 1989; Ross, 1984; Seel & Hoops, 1993; Wickelgren, 1974), and active participation by the problem solver in each of the analogical transfer processes (Black & Rollins, 1982; Lewis & Anderson, 1985; Pierce et al., 1993). Each of these elements may aid the solver in future accessing of pertinent information, and, ultimately, positive transfer.

Types of Analogical Problem-Solving Training

Analogical problem-solving training can be broken down into two categories: (a) teacher-generated training and (b) learner-generated training. Teacher-generated training refers to training developed by the teacher or researcher and presented to the problem solver in a didactic teaching style. Conversely, learner-generated training is one that is developed and guided by the teacher, but is directed by the solver through a discovery teaching process. Subtypes of training within each of these categories, as well as research findings on these methods, will be discussed in the following section.

Teacher-Generated Training

The most prevalent type of analogical problem-solving training is teacher- or

researcher-generated. This training primarily involves direct instruction on problem-solving strategies (Black & Rollins, 1982; Brooks & Dansereau, 1983; Brown, Campione, & Barclay, 1979; Cramond et al., 1990; de Volder & de Grave, 1989; Fong, Krantz, & Nisbett, 1986; Funkhouser, 1990; Gabel & Sherwood, 1983; Mayer, 1985; Schoenfeld, 1979), problem types (Robins & Mayer, 1993; Seel & Hoops, 1993), base and target problem representations (Catrambone & Holyoak, 1989; Cramond et al., 1990; Mayer, 1985), problem-solving processes (Bloom & Broder, 1950; Funkhouser, 1990), and the use of multiple practice problems (Cramond et al., 1990; Stein et al., 1986) and similar base and target problems (Gholson et al., 1987).

This type of training has also incorporated the use of hints or suggestions to evaluate the similarities of base and target problems, thereby focusing the learner on relevant features only and yielding a higher rate of positive transfer (Orlich, 1992; Stein et al., 1986). In fact, most analogical transfer research has found that without a suggestion to utilize previously learned information, problem solvers are unable to produce the target solution (Catrambone & Holyoak, 1989; Gick, 1981; Gick & Holyoak, 1980, 1983, 1987; Hesse & Klecha, 1990; Reeves & Weisberg, 1993; Solomon, 1984). To illustrate, Gick and Holyoak (1980) found that 92% of the subjects who were given a hint to use the general analogy were able to produce the convergence solution to the radiation problem (see Appendices I and J for a full description of these problems), while only 20% of the subjects who were not given a hint were able to do so.

Schoenfeld (1979) purported the necessity to train students in general problemsolving skills rather than specific strategies, because of the need for problem solvers to be able to generalize strategies, that is, schemata, from one novel problem to the next. This training would introduce the solver to the strategy and provide direct instruction on when and when not to use it. In a study designed to increase subjects' problem-solving skills, Schoenfeld (1979) trained subjects on five problem-solving strategies, which included the use of diagrams and analogies. Findings suggest that "when problem-solving strategies are identified and taught, and when students [access] them, the impact on the students' problem-solving performance is substantial" (p. 185). More specifically, subjects who were trained solved a greater number of target problems than those who were not trained ($x_{trained} = 3.25$; $x_{control} = 1.67$, p < .05).

Funkhouser (1990) identified the need to include both translation (problem representation) and schema training in problem solving. In a related study on whether or not structural schema training improves processing and recall of scientific text, Brooks and Dansereau (1983, experiment 1) provided subjects 6 hours of instruction on using a schematic aid (a.k.a. DICEOX) followed by three practice sessions that involved memorizing the DICEOX schema, organizing their notes from reading material according to DICEOX, and then utilizing DICEOX as a retrieval tool when taking exams over their practice material. This study found that subjects can be trained to effectively use a structural schema as an aid in processing and recalling scientific text material at the level of main ideas: mean number of main ideas recalled: $x_{trained} = 14.29$, SD = 5.79; $x_{control} = 8.81$, SD = 4.69), F(1,28) = 8.37, p < .01. Findings suggest that training such as those described here facilitate a higher level of performance by reducing errors in problem representation and schema induction.

Learner-Generated Training

Several researchers have acknowledged the potential benefits of utilizing students in their own learning process. In a recent article, Kohn (1993) argued for the inclusion of students in the learning process to increase the meaningfulness of learning and subsequently the motivation and persistence to participate in their education. Likewise, Seel and Hoops (1993) argued for a guided discovery teaching method when teaching for transfer. This would afford students the opportunity to generate their own analogies while at the same time providing students assistance in developing the appropriateness of these analogies. Currently, there is no research which has used such training in analogical problem solving. There have been studies that focus on self-monitoring² (Brown et al., 1979; Cramond et al., 1990; Osman & Hannafin, 1992; Pierce et al., 1993), but these do not target student generation of appropriate steps to take when solving a problem. Rather, as in teacher-generated training, self-monitoring generally involves teachergenerated instruction on the employment and monitoring of problem-solving strategies. Although these studies have yielded results showing learner's decreased reliance on external assistance and subsequent increase in level of self-sufficiency, they do not directly impact student success in analogical problem-solving performance.

Even though self-monitoring training has been purported to be the most effective training method in developing the learners' skills in retrieval, mapping, monitoring, and evaluating problem-solving strategies (Osman & Hannafin, 1992), in reality, it is not much

² Self-monitoring has also been referred to as self-control, but for consistency both will be labeled self-monitoring in this paper.

different from the more traditional teacher-generated training in augmenting subjects' problem-solving ability. Rather than incorporating active student participation in generating the problem-solving process, self-monitoring didactically presents solvers with these processes.

Learner-generated training should include an introduction to what the analogical problem-solving method is, followed by multiple practice problems to which students would be asked to make comparisons, that is, notice any similarities or differences, and think about what steps they might take in solving the problem. Finally, the teacher/researcher would guide the students in further developing these analogical problem-solving steps.

Summary

Studies on the impact specific factors have on analogical transfer in problem solving have delineated several characteristics of the problem solver and analogy types that either facilitate or hinder transfer. Among these, the similarity between base and target problems and identification of relevant relations are the most facilitative. Similarly, the learners' ability to retrieve, map, and adapt appropriate schema will negate the need for hints or other cues given by the researcher.

Training programs have been developed to incorporate these key factors and thereby assist problem solvers in analogical problem solving. All of these programs have been focused on teacher-generated training, which includes explicit instruction on

appropriate strategies and problem types, hints, and use of multiple practice problems to increase the learners' experience with problem solving. Findings indicate that training that instructs solvers on elements involved in problem solving and that utilizes the solver in this process yields more positive transfer. However, in the greater education domain, emphasis has begun to be placed on the importance of utilizing the solver in generating training in which the problem solver directs the focus of the training and is merely guided by the teacher. However, aside from self-monitoring by the problem solver, no evidence has of yet been found on the existence and effectiveness of such a learner-generated training in analogical problem solving. Using learner-generated training may have positive effects on analogical problem solving, but more research in this area is needed.

THE STUDY

Purpose of the Study

Widespread study on the impact of using analogues in problem solving has afforded researchers and educators alike much information regarding how best to help students reach "expert" status. At present, researchers have begun to examine the effectiveness of training subjects in problem-solving skills. In addition, there is evidence in areas such as cognitive and educational psychology that when the student is involved in developing that training, training is obtained that is more meaningful and that results more often in spontaneous transfer. However, there are no studies that have investigated the impact of including the student in generating problem-solving steps in problem-solving training (learner-generated training).

The purpose of this study was to investigate the relative effectiveness of teachergenerated and learner-generated training on spontaneous transfer between complete and
partial isomorphic problems. Based on findings from previous studies, several factors
have been identified as likely to influence spontaneous transfer. Each of these factors was
included in this study to determine their relative effects on spontaneous transfer. The
purpose of the present study was to answer several research questions:

1. Are there differences in problem-solving performance as a result of type of analogy (partial versus complete)?

- 2. Does providing training to students improve students' analogical problemsolving ability?
- 2a. Is analogical problem-solving training more effective than no training in improving students' analogical problem-solving ability?
- 2b. Is teacher-generated training more effective than learner-generated training in improving students' analogical problem-solving ability?
- 3. Do expert and novice problem solvers benefit differently from various types of training?

Method

Design

This study used an experimental design and was primarily exploratory in nature. The dependent variable used to measure problem-solving performance was accuracy of response to two problem-solving exercises. The independent variables included level of problem-solving ability (expert or novice), type of analogy (partial or complete), and training group (teacher-generated training, learner-generated training, or no training).

Subjects

Subjects were 58³ undergraduate Psychology 101 students attending Utah State

³ Sixty subjects were randomly selected to participate in this study, but after completing the training session two novice teacher-generated subjects dropped out of school and were unable to complete the post-test.

University. The average age was 20 (range = 18 to 34), the average GPA was 3.14 (range = 1.50 to 4.00), and 21 of the subjects were male. Psychology 101 was selected primarily to provide a broad spectrum of college students representing a variety of majors. Participation was sought on a volunteer basis, and after being tested for problem-solving ability (see below), subjects were divided into either expert problem solver or novice problem solver groups. Within these groups, subjects were assigned to one of three training groups (teacher-generated, learner-generated, or control) using a systematic random sampling technique until the target number was reached for each group ($\underline{N} = 20$; 10 expert and 10 novice).

Instruments/Measures

Problem-Solving Ability Instrument

To test the problem-solving ability of subjects, a sample of problems (see Appendix C) was administered to subjects at the time of recruitment. The problems selected have been used in numerous studies with subjects similar to the ones used in the present study (e.g., Antonietti, 1991; Gick & Holyoak, 1983; Simon & Hayes, 1976). Subjects were required to solve three problems, and solutions were scored as either correct or incorrect. Subjects who solved two or three problems correctly were labeled as expert problem solvers, and those who solved zero or one problem correctly were labeled as novice problem solvers. These problems were pretested on 15 Psychology 101 undergraduate students at Utah State University who were presumed to be similar to those

who were used in this study. Pretesting revealed no ceiling or floor effects and that subjects understood the problems.

Training Protocols

In order to train subjects for the present study objectives, three training protocols were developed from existing studies on problem-solving factors such as use of hints, use of partial versus complete problems, training in strategies and types of problems, and use of multiple analogies. The first training protocol, used with the teacher-generated group, is presented in Appendix E. This training protocol provided subjects with a highly structured discussion of problem-solving strategies and methods, and did not allow for any subject interaction unless subjects needed clarification. The second training protocol, used with the learner-generated group, is presented in Appendix F. This training also provided subjects with a structured discussion of problem-solving strategies, but the discussion of problem-solving methods was highly dependent on active student interaction where students developed their own problem-solving criteria and steps. Although not a "training" protocol per se, a third protocol, used with the control group, was designed to provide the same level of exposure to problems as the other two groups without providing any direct training on the problem-solving process. The control group protocol is presented in Appendix G.

All protocols were pretested with separate samples of 20 Psychology 101 students who are presumed to be similar to subjects used in this study. At the conclusion of each training session, subjects were asked to identify what the objectives of the training were

and to identify any areas where presentation of material could be improved.⁴ As a result of the pilot-testing, no revisions to the protocols were necessary.

Training Problems

The final instruments used in this study were the training and testing problems, found in Appendices H and I, respectively. While one of the problems was developed by the present author, most have been used in numerous studies with subjects similar to those in this study (Duncker, 1945; Gholson et al., 1988; Gick & Holyoak, 1980, 1983; Reed, Ernst, & Banerji, 1974; Wickelgren, 1974). Permission, either verbal or written, was obtained for the use of each problem. These problems included the parade problem, the general problem, the poker problem, the missionaries and cannibals problem, the book burners and book lovers problem, the farmer's dilemma, the men and boys problem, the wine merchants problem, the Trump suit problem, the Renshaw problem, and the marching band problem.

Each of the aforementioned problems can be described as either being complete or partial problems. The complete problems provide both the initial and final goal states, objects and constraints, and a solution strategy. Conversely, in this study, a partial problem provides everything but the solution strategy. Additionally, each problem was either isomorphic or nonisomorphic to other problems. Isomorphic problems are completely analogous to each other, whereas in nonisomorphic problems there is some element of the two problems that is incongruous. For example, the missionaries and

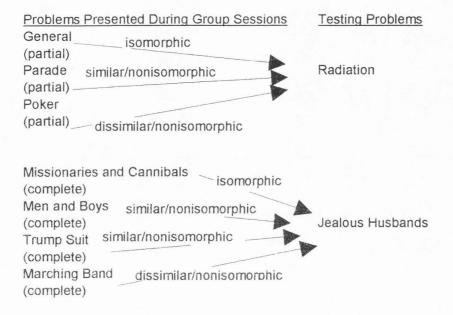
⁴ Participants correctly identified the training objectives, which suggested that the protocols were effective in helping them to understand the material presented to them and that it was pitched at the correct level.

cannibals problem is isomorphic to the book burners and book lovers problem in that they have similar initial and final goal states (i.e., in both problems one must find the best route to get six people across a river), the same objects (one boat and six people--three "good" and three "bad"), the same constraints (the boat can only hold two people, one person must be in the boat for each crossing, and the "bad people" cannot outnumber the "good people" at any time), and share the same solution strategy.

In contrast, the cannibal problem is considered to be nonisomorphic to the farmer's dilemma. In the farmer's dilemma, the initial and final goal states are the same, but there are a differing number of object types (four instead of two) and different constraints (the man must be in the boat at all times and the fox cannot be left alone with the goose nor can the goose be left alone with the corn at any time), and they share somewhat different solution strategies (see Appendix H).

Testing Problems

During the final testing session, subjects were provided with two problems: the radiation problem and the jealous husbands problem. As seen below in Figure 1, the radiation problem is isomorphic to the parade and general analogies and nonisomorphic to the poker analogy provided during the training group sessions. As discussed in the procedure section, these three analogies were presented as <u>partial</u> analogies, and inclusion of the radiation problem in the testing session was designed to test the facilitating effects of partial analogies on retrieval of information relevant to the solution generation for the target problem.



<u>Figure 1.</u> Relationships between problems presented during group sessions and the testing problems.

The jealous husbands problem is completely isomorphic to the missionaries and cannibals and book burners and book lovers analogies, similar but nonisomorphic to the farmer's dilemma, men and boys, wine merchants, and Trump suit analogies, and dissimilar as well as nonisomorphic to the Renshaw and marching band analogies presented in the group sessions. As discussed in the procedure section, these analogies were completely solved during the training sessions. Thus, the jealous husbands problem was presented during the testing session to test the facilitating effects of complete analogies on retrieval of information relevant to solution generation for the target problem.

Testing problems were scored for the correctness of their solutions. Each subject received a score of either zero (an incorrect solution was derived) or one (a correct solution was derived) for each testing problem.

Data from the testing session were scored and coded by the primary researcher and an undergraduate research assistant. Any information that could identify subjects or their group membership was removed before scoring to eliminate experimenter bias.

Approximately 10% of the data were scored by both raters and compared for interrater reliability. This process was repeated until a level of .80 was reached.

Procedure

Recruitment

Psychology 101 students were recruited for this study by being offered extra credit points. Once they agreed to participate, each subject was administered the problemsolving ability test to determine his/her level of expertise and was then randomly assigned to one of three training groups, teacher-generated training, learner-generated training, or no training--the control group. All participants were informed of training and testing times.

Training

The teacher-generated and learner-generated groups attended a group training session on problem solving. The control group attended a group session in which all of the same problems used in the teacher-generated and learner-generated training sessions were presented, but no training on their solutions was provided. Expert problem solvers were trained separately from novice problem solvers to prevent potential intimidation factors and a subsequent lack of participation by novices.

Orientation of trainers. Training was conducted by two graduate students in the Research and Methodology Program at Utah State University for the teacher-generated and learner-generated sessions, and by one undergraduate psychology student at Utah State University for the control session. These trainers were selected based on the similarity in their demeanor and teaching styles. The primary researcher facilitated an extensive and individual orientation with each trainer on familiarity with the protocols, teaching style, and training objectives. Moreover, trainers were required to train 10 Psychology 101 students (from a different section in which the sample was obtained) to familiarize them with the sequence of events in the training sessions. This orientation did not include a discussion of the study's purpose in an effort to avoid any biasing effects that may occur. Thus, trainers were blind to the study's purpose.

Teacher-generated and learner-generated training sessions. Subjects participated in a 2-hour training session focused on discussions of types of analogical problems and problem-solving strategies and mastery of strategies. For both groups, the training began with an overview of problem-solving strategies (i.e., trial and error, working backward, working forward) in which three problems (i.e., the parade problem, the general problem, and the poker problem) and their solution strategies were presented but not solved (i.e., they were presented as and remained partial problems).

⁵ Here, demeanor is used to refer to one's approach to students, tone of voice, and mannerisms in general, while teaching style refers to how the trainers present course material to their students. All of the selected trainers approach students in a helpful yet unobtrusive way, are relatively soft spoken (although not to the point where students are unable to understand them), are quite respectful of students' needs and opinions, have previous teaching experience, and can utilize both traditional didactic as well as discovery teaching styles. These similarities were determined through personal contact with each trainer in a teaching situation.

The remainder of the training focused on using analogical problem-solving methods. For the teacher-generated group, this practice included: (a) being presented with pairs of problems (one complete and one partial, or two partials), (b) being instructed on the comparisons that can be made and found between the two problems (i.e., isomorphic versus nonisomorphic, complete versus partial), (c) being trained on the sequence of steps that should be taken when solving a problem, (d) solving each partial problem, and (e) participating in a recap of the steps that were used to derive the correct solution.

For the learner-generated group, this practice included: (a) being presented with the same pairs of problems as used in the teacher-generated session, (b) having the students generate comparisons (e.g., similarities and differences) between these problems, (c) having a discussion of these comparisons leading to students generating their own analogical problem solving processes, (d) solving the problems, and (e) participating in a recap of the steps each student used to derive their solutions. Both teacher and learner-generated training procedures were repeated until all eight problems were presented, four of which were solved thereby making them complete problems.

Although the sequence of steps was similar in the teacher-generated and learner-generated sessions, the main difference existed in who generated the comparisons between problems and analogical problem-solving strategies. In the teacher-generated group, the instructor generated the comparisons and the analogical problem-solving strategies, while in the learner-generated group, the subjects generated the comparisons as well as the analogical problem-solving strategies to be used. The exact procedure the trainers

followed can be found in Appendix E for the teacher-generated training and Appendix F for the learner-generated training groups. This training occurred approximately 1 week after initial recruitment.

Control group session. Although not a training session, control group subjects were also exposed to all the same problems used during the teacher-generated and learner-generated training sessions, but were not specifically trained on how to solve these problems. Subjects were given adequate time⁶ to solve the cannibal, farmer's dilemma, wine merchants, and Renshaw's problems and were presented with the correct solutions to each of these problems. For the remaining three problems (i.e., the general, parade, and poker problems), subjects were not given an adequate amount of time⁶ in which to derive a solution, and they were not exposed to the correct solutions, thus resulting in control subjects having been exposed to three partial and four complete problems. The protocol for this session is included in Appendix G. This training occurred approximately 1 week after initial recruitment.

Evidence of treatment fidelity. Each training session was videotaped in order to evaluate whether trainers presented the training elements as outlined in the protocols, and if subjects engaged in assisting one another in determining the appropriate problem-solving steps and correct solution strategies for practice problems. Upon the conclusion of the data collection, each videotape was examined by the primary researcher, and it was

⁶ During pilot testing, it was determined that for the problems deemed to become complete, 5 minutes was an adequate amount of time for most subjects to derive the correct solution, and for the problem deemed to remain partial, one-and-a-half minutes was not enough time to solve the problem. Thus, this helped to ensure that the problems either became complete or remained partial as discussed in the procedure section.

determined that, as instructed, the trainers did follow the training protocols and at no time did any subject assist another subject in completing the training activities.

Testing

Approximately 1 week after training, all subjects were brought back and individually tested using two problems (i.e., the radiation problem and the jealous husbands problem). Correct solution generation to the target problems could be facilitated by students transferring to the target situation knowledge of problems and solution strategies generated during the training group and control group sessions.

During the testing sessions, subjects were given 20 minutes to solve each problem, and if this time expired before a solution was derived, time was called and the problem was scored as incorrect.

RESULTS AND DISCUSSION

Given the ordinal nature of the "correctness" solution data, chi-square analyses were conducted to test the null hypothesis that the group difference may have occurred by chance. Because significance testing is heavily dependent on sample size, and its results may not be informative about the magnitude and practical significance of the group differences, effect size measures were also obtained based on the chi-square analysis results so that this additional information would be available. In translating the chi-square analysis results to effect size measures, although several different types of effect size measures are available, the effect size measure of standardized group mean difference, as discussed by Glass, McGaw, and Smith (1981), was used in the present study, mainly for the reason that it is easy to understand and also widely used in meta-analyses. In translating the chi-square analysis results to the effect size measures of standardized group mean difference, three procedural steps, as discussed in Cohen (1988), were implemented as follows to obtain Cohen's d:

$$r = \phi = \sqrt{\frac{\chi^2}{N}}$$
 (p. 223) $t = r\sqrt{\frac{N-2}{1-r^2}}$ (p. 76) $\underline{\mathbf{d}} = t\sqrt{\frac{1}{n_a} + \frac{1}{n_b}}$ (p. 67)

Interpretations of the effect size analyses were based on Cohen's (1988) observations of social and behavioral sciences, in which he determined that an effect size of .2 was small, an effect size of .5 was medium, and effect sizes of .8 or greater were large. Findings are presented below according to each research question.

Differences in Problem-Solving Performance as a Result of Type of Analogy (Partial Versus Complete)

The jealous husbands problem is isomorphic to multiple <u>complete</u> base analogues presented and practiced with during the training sessions and can be entirely solved by employing information from its analogues. Given this, it was hypothesized that if students focused on the relevant aspects of the complete problem and attempted to access relevant schema during the posttest session, they would retrieve and adapt the information from the analogues and in doing so would generate a correct solution strategy.

On the other hand, the radiation problem was isomorphic to two out of the <u>partial</u> base analogues presented but not completely solved during the training sessions. As such, it was hypothesized that if solvers attended to the information from the partial analogues, during the posttest session they would retrieve and adapt some of the information presented to them and might generate a correct solution strategy; however, given the limited amount of information and practice time provided, this would be more difficult than it would be for the complete target problem.

The chi-square analysis of the correctness of solutions generated for the radiation versus jealous husbands problems revealed a statistically significant difference, $\chi^2(1, \underline{N} = 116) = 18$, $\underline{p} < .001$. As shown in Table 1, the greatest number of correct solutions was generated for the complete problem.

Thus, the findings support the aforementioned hypotheses in that solvers,

Table 1

Number and Percentage of Subjects Producing Correct Solution Strategies to the Testing

Problems

Problem type	Correctness of Solution Strategies		
	Number correct	Percent correct	
Problem related to	7	12	
partial analogies			
Problem related to	28	48	
complete analogies			

regardless of training group membership or ability level, generated 36% more correct solution strategies for the complete testing problem than they did for the partial testing problem (<u>d</u> = .85). This finding parallels other studies, which suggest that subjects presented with a complete isomorphic analogy (e.g., Antonietti, 1991; Holyoak & Koh, 1987) are more likely to produce a correct solution strategy to the target problem. Furthermore, presentation and practice with multiple isomorphs (e.g., Catrambone & Holyoak, 1989; Gick & Holyoak, 1980;1983) will also result in an increase of spontaneous transfer.

Differences in Problem-Solving Performance as a Result of Type of Training

(Teacher-generated, learner-generated, or control)

Due to the differences found between the complete and partial problems, training group results were examined by type of analogy. Table 2 shows the number and percentage of subjects who correctly solved the test problems according to training group.

Partial Problem (Radiation Problem)

A chi-square analysis of group differences for the partial problem revealed no statistically significant differences, $\chi^2(2, \underline{N}=58)=.25$, $\underline{p}<.88$. This finding suggests that training did not yield an increase in performance levels on the partial problem. Thus, there were no advantages to having problem-solving training.

This finding is contrary to previous research (e.g., Gick & Holyoak, 1980;
Schoenfeld, 1979), which suggests that providing subjects with training on when to use solution strategies and in general problem-solving skills through a primarily didactic training program should augment spontaneous transfer. Moreover, it has been postulated that inclusion of students in a guided discovery training should yield an increase in spontaneous transfer (Seel & Hoops, 1993) over subjects who receive didactic or no training. It should be noted that this study did differ from previous studies in that no hint was provided during testing to access previously learned information relevant to correct solution generation of the target problem. Also, subjects may not have been exposed to a long enough intervention which may have contributed to a lack of attention to the

Table 2

Number and Percentage of Subjects Correctly Solving the Testing Problems According to

Training Group

	Correctness of Solution Strategies		
Problem type	Number correct	Percent correct	
Problem related to partial an	alogies		
Learner-generated	3	15	
Teacher-generated	2	11	
Control	2	10	
Problem related to complete	analogies		
Learner-generated	10	50	
Teacher-generated	8	44	
Control	10	50	

material. Finally, there was no check in place for student understanding and incorporation of what was taught during the training session. It may have been that students were only completing the training session to receive extra credit points rather than to increase their problem-solving ability, which resulted in a lack of attention to and understanding of the information presented to them.

Complete Problem (Jealous Husbands Problem)

As with the partial problem, no statistically significant differences were found among training groups, $\chi^2(2, \underline{N} = 58) = .15$, $\underline{p} < .92$, for the complete problem. Here, as can be seen in Table 2, the performance of the three groups paralleled findings on the partial problem, which suggests that training had no impact on ability to perform on the complete problem.

Despite this finding, it should be noted that previous studies (e.g., Gick & Holyoak, 1980) have found that only 20% of solvers are able to spontaneously generate a correct solution strategy. However, in this study 48.3% of solvers were able to spontaneously generate a correct solution strategy for the complete target problem. More importantly, 50.0% of control subjects were also able to solve the complete target problem, which may indicate that multiple exposure to complete analogies may be equally or more important in spontaneous transfer than is training for complete target problems. As such, practice with multiple isomorphs appears to have facilitated correct solution generation.

Differences in Expert and Novice Solver's Problem-Solving

Performance as a Result of Type of Training

Again, due to the differences found between the complete and partial problems, separate analyses were performed for each target problem. The mean results for experts and novices within training group are provided in Table 3.

Partial Problem (Radiation Problem)

When examining expert training groups no statistically significant difference, $\chi^2(2,\underline{N}=30)=.54,\,\underline{p}<.75,\,\text{was found.}\,\,\text{Moreover, effect size analyses between training}$ groups found negligible differences. An effect size of .00, $\chi^2(1,\underline{N}=20)=.00,\,\underline{p}<1.00,$ was found when comparing teacher-generated and learner-generated groups and .28 when comparing teacher-generated and control groups, $\chi^2(1,\underline{N}=20)=.40,\,\underline{p}<.53,\,\text{as well as}$ learner-generated and control groups, $\chi^2(1,\underline{N}=20)=.40,\,\underline{p}<.53.$ These findings suggest that expert solvers may have ignored problem-solving training rather than incorporating partial analogy information and learning problem-solving skills that may have assisted them in generating the correct solution strategy to the partial problem.

The chi-square analysis of novice training groups also revealed no statistically significant difference, $\chi^2(2,\underline{N}=28)=3.03$, $\underline{p}<.21$. Although not statistically significant, as seen in Table 3 novices benefited differentially from different types of training. Here, novices in the learner-generated group provided more correct solution strategies (20.0%) than did those in the teacher-generated group (12.5%), and both training groups outperformed the control group (0.0%). This trend is further explicated through effect size analyses. When comparing the teacher-generated and learner-generated groups, a small effect size of .19, $\chi^2(1,\underline{N}=18)=.18$, $\underline{p}<.67$, was found; however, comparisons between teacher-generated and control groups, $\chi^2(1,\underline{N}=18)=1.69$, $\underline{p}<.19$; $\underline{d}=.61$, and learner-generated and control groups, $\chi^2(1,\underline{N}=20)=2.99$, $\underline{p}<.08$; $\underline{d}=.80$, revealed medium to large effect sizes. Thus, it appears that novice solvers

Table 3

Number and Percentage of Subjects Correctly Solving the Target Problems According to

Training Group Membership and Problem-Solving Ability

	Level of proble	Level of problem-solving ability		
	Novice	Expert Number correct $ \underline{N}(\%) $		
Problem type	Number correct <u>N</u> (%)			
			artial problem	
Learner-generated	2 (20)	1 (10)		
Teacher-generated	1 (13)	1 (10)		
Control	0 (0)	2 (20)		
omplete problem				
Learner-generated	5 (50)	5 (50)		
Teacher-generated	4 (50)	4 (40)		
Control	4 (40)	6 (60)		

benefited more from receiving teacher-generated and learner-generated training than solvers receiving no problem-solving training.

These findings are substantiated by previous research on novice learners and on

inclusion of subjects in guided discovery training versus didactic and no training. With regard to problem-solving ability, previous research suggests that novice problem solvers focus more time and effort on irrelevant features (e.g., Chi & Van Lehn, 1991) and on surface features of the base and target problems (e.g., Novick & Holyoak, 1991). Thus, as evidenced by the 0% correct solution strategies generated by the control group, without direct instruction on problem-solving skills and processes, novice learners would be unable to access the relevant information and generate a correct solution to the target problem. In fact, previous research has suggested that providing subjects with training on when to use solution strategies and in general problem-solving skills through a didactic (e.g., Gick & Holyoak, 1980; Schoenfeld, 1979) or a guided discovery (Seel & Hoops, 1993) training program should augment spontaneous transfer in comparison to subjects who receive no training. And although not statistically significant, this was the case with the teacher-generated group doing 12.5% better and the learner-generated group doing 20% better than the control group.

These trends provide support for the assertion that direct instruction on problemsolving processes and active participation in the learning process may facilitate novice problem solvers' abilities to overcome barriers they experience while solving problems. The lack of statistical support may have been a factor of the limited sample size.

Complete Problem (Jealous Husbands Problem)

When examining type of training for the expert solvers, no statistically significant differences were found, $\chi^2(2, \underline{N} = 30) = .80$, p < .67, and effect size analyses revealed

negligible differences when comparing teacher-generated and learner-generated groups, $\chi^2(1, \underline{N} = 20) = .20$, $\underline{p} < .65$; $\underline{d} = .19$, and learner-generated and control groups, $\chi^2(1, \underline{N} = 20) = .20$, $\underline{p} < .65$; $\underline{d} = .19$. However, an effect size of .39 was found when comparing teacher-generated and control groups, $\chi^2(1, \underline{N} = 20) = .80$, $\underline{p} < .37$, with the control group providing more correct solution strategies. Previous research (e.g., Stein et al., 1986) has suggested that expert solvers have a naturally higher ability to access previously learned information and that this ability enables expert solvers to spontaneously generate correct solution strategies. As with the partial problem, and as further indicated by the percent correct in Table 3, there may have been no advantage for expert solvers to participate in problem-solving training most likely due to the experts' high problem-solving ability.

When examining type of training group for the novice solvers, no statistically significant differences were found, $\chi^2(2, \underline{N}=28)=.25$, $\underline{p}<.88$, and effect size analyses found negligible differences when comparing teacher-generated and learner-generated groups, $\chi^2(1, \underline{N}=18)=.00$, $\underline{p}<1.00$; $\underline{d}=.00$, teacher-generated and control groups, $\chi^2(1, \underline{N}=18)=.18$, $\underline{p}<.67$; $\underline{d}=.19$, and LG and control groups, $\chi^2(1, \underline{N}=20)=.20$, $\underline{p}<.65$; $\underline{d}=.19$. Although not as evident as with the partial problem, when solving the complete target problem, novices did perform better when they received some form of training on problem-solving processes versus no training. Novices in the learner-generated and teacher-generated groups provided more correct solution strategies (50.0%) than did those in the control group (40.0%).

Moreover, unlike previous studies (e.g., Gick & Holyoak, 1980, 1983) which saw rates of spontaneous solution generation of only 20%, this study yielded a relatively high

rate (48%) of spontaneously generated correct solution strategies with the target problem for the complete analogies. This finding suggests that presentation and practice with multiple, complete isomorphs may greatly facilitate problem-solving ability.

Similar to the partial problem, these findings support previous research on the inclusion of solvers, especially novice solvers, in guided discovery and didactic training. However, as previously noted, this study yielded a low percentage of spontaneously generated correct solution strategies for the partial problem (12%), especially among novice solvers (10.7%). In comparison to the complete target problem, it appears that problem-solving training, whether didactic or discovery, becomes more essential for novice than for expert solvers when they are presented with partial analogies.

In summary, when examining percentage of correct solution strategies produced for both target problems by training group membership, statistically significant differences were not found. This lack of statistical significance could be due more to experts performing at a high enough level to "cancel out" any effects that may be found among novices rather than to a lack of training group differences. In fact, the data presented above provided some evidence for this supposition.

Moreover, when the target problem was related to partial analogies, there was an advantage for both learner- and teacher-generated groups among novice solvers. Thus, for novice solvers, presentation and practice with multiple isomorphs need to be coupled with direct instruction on problem-solving skills as well as active participation in the learning process in order to augment spontaneous transfer.

CONCLUSION

Perhaps the most salient finding of this study is that solvers, regardless of problem-solving ability or training group membership, were able to spontaneously generate more correct solution strategies for the target problem to the complete analogies. This suggests that presentation and practice with multiple isomorphs augments the learner's ability to correctly solve the target problem. Also, this study supports previous research on the ability of expert learners to spontaneously generate a correct solution strategy without participating in problem-solving training.

In addition, although not statistically significant, novice solvers in the learnergenerated group had an advantage when solving the target problem to the complete
analogies. However, when the target problem was isomorphic to partial analogies, there
was an advantage found for both the teacher- and learner-generated groups. These
findings suggest that direct instruction on problem-solving skills and active participation in
the learning process may facilitate novice problem solvers' abilities to overcome barriers
they experience while solving problems, especially when solving target problems
isomorphic to partial analogies.

Limitations of the Study

Although this study yielded only one statistically significant difference, it is important to consider several limitations that may have contributed to the lack of

differential treatment effects found in this study. The first limitation to consider is the small sample size. With a larger sample size, the trend found among novice solvers in training groups may have reached statistical significance. This supposition stems from the robustness of the effect size analysis of novice training group performance on the target problem to the partial analogies. This analysis revealed a trend among novices to perform differentially depending on training group membership, with an advantage going to learner-generated training.

Another limiting factor may have been the duration of the intervention. Because the intervention was composed of only 2 hours of instruction, its coverage of problemsolving strategies and amount of practice provided may not have been sufficient to induce change in behavior. With more exposure to and practice with various problem-solving strategies, solvers may have become more familiar and comfortable with the processes involved in problem solving and this may have facilitated subjects' performance levels.

In addition to a limited intervention, voluntary participation and the absence of consequences for lack of participation (e.g., a poor grade in their Psychology 101 course) may have resulted in limited attention and motivation toward learning the material being taught to the subjects. It is unknown just how prevalent inattention was because there was no check in place for student understanding and incorporation of what was taught during the training sessions.

A final concern is the amount of difference in the level of difficulty between the partial and complete target problems, that is, was one problem more difficult to solve than the other, thereby resulting in fewer correct solution strategies. Previous research using

the partial and target problems used in this study found relatively low rates of spontaneous transfer. In addition, one of the pretest problems was similar in nature to the complete problem and novices were unable to solve this problem, although later in the testing session they were successful. These two sources of evidence combine to support the contention that differences found in the number of correct solution strategies between the partial and complete problems may not be due to one problem having a higher level of difficulty; rather, the difference is most likely attributable to the amount of information provided to subjects during the training session. Recall that the partial problem is isomorphic to only one partial analogy, whereas the complete target problem is isomorphic to two complete analogies. Thus, subjects had more prior knowledge about the complete target problem, which may have increased their likelihood of generating a correct solution strategy to the complete target problem.

Implications of Findings

The results of this study suggest that after participating in problem-solving training, novice problem solvers may be able to generate a correct solution strategy spontaneously, that is, without the assistance of the researcher. Thus, this study provides support not only for direct instruction, but also for the incorporation of the problem solver in actively participating in the development of his/her own problem-solving strategies. Perhaps researchers will follow this lead and develop new ways of increasing spontaneous transfer so that solvers can become more self-sufficient and better critical thinkers. In addition, teachers who employ the methods advocated in this study may also witness an

increase in their students' academic performance as a direct result of augmenting self-sufficiency and critical thinking. That is, students may become less reliant on their teachers for assistance, better able to think of difficult problems in new ways, and better able to derive the correct answer to difficult problems more often.

Future Studies

This study suggests that future directions in analogical problem solving should include more intense interventions. For example, subjects need to be given more exposure to and practice with multiple problem-solving techniques, more exposure to the partial analogies (perhaps putting them nearer to the end so that problem solvers would be more likely to remember them), and more instruction on how to use partial analogies when performing analogical problem solving. This exposure may provide solvers additional resources to use when solving problems, thereby resulting in a higher occurrence of correct solution strategies. This would afford subjects more practice time while in the instructional setting, as well as more time to reflect on the information and perhaps use it in everyday life situations. Also, researchers should conduct a modified version of this study with samples from broader populations to afford researchers more information about the validity of claims made within this study and to increase the study's generalizability.

Finally, it is hoped that the findings presented here regarding the ability of expert problem solvers to spontaneously generate a correct solution strategy for a target problem to which they were only presented partial analogies will spark interest in examining strategies used by experts. These strategies may begin to unlock the mystery behind

processes involved in analogical problem solving as well as in other realms of problem solving.

REFERENCES

- Antonietti, A. (1991). Effects of partial analogies on solving an ill-defined problem.

 Psychological Reports, 68, 947-960.
- Barton, J. M. (1989). Schema induction and analogical transfer with a mathematics insight problem. <u>Dissertation Abstracts International</u>, 49(7-A), 1735.
- Black, M. M., & Rollins, H. J., Jr. (1982). The effects of instructional variables on young children's organization and free recall. <u>Journal of Experimental Child Psychology</u>, 33, 1-19.
- Bloom, B. S., & Broder, L. J. (1950). <u>Problem-solving processes of college students: An exploratory investigation</u>. Chicago: University of Chicago Press.
- Bransford, J. D., & Stein, B. S. (1984). The ideal problem solver: A guide for improving thinking, learning, and creativity. New York: Freeman.
- Brooks, L. W., & Dansereau, D. F. (1983). Effects of structural schema training and text organization on expository prose processing. <u>Journal of Educational Psychology</u>, <u>75(6)</u>, 811-820.
- Brown, A. L. (1989). Analogical learning and transfer: What develops? In S.

 Vosniadou & A. Ortony (Eds.), <u>Similarity and analogical reasoning</u> (pp. 369-412). Cambridge, England: Cambridge University Press.
- Brown, A. L., Campione, J. C., & Barclay, C. R. (1979). Training self-checking routines for estimating test readiness: Generalization from list learning to prose recall.

- Child Development, 50, 501-512.
- Catrambone, R., & Holyoak, K. J. (1989). Overcoming contextual limitations on problem-solving transfer. <u>Journal of Experimental Psychology: Learning</u>, <u>Memory, and Cognition</u>, <u>15</u>(6), 1147-1156.
- Chi, M. T., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. <u>Cognitive Science</u>, 5, 121-152.
- Chi, M. T., & Van Lehn, K. A. (1991). The content of physics self-explanations. The Journal of The Learning Sciences, 1(1), 69-105.
- Clement, J. (1988). Observed methods for generating analogies in scientific problem solving. Cognitive Science, 12, 563-586.
- Cohen, J. (1988). <u>Statistical power analysis for the behavioral sciences</u> (2nd ed.). Hillsdale, NJ: Erlbaum.
- Cramond, B., Martin, C. E., & Shaw, E. L. (1990). Generalizability of creative problem solving procedures to real-life problems. <u>Journal for the Education of the Gifted</u>, <u>13(2)</u>, 141-155.
- de Volder, M. L., & de Grave, W. S. (1989). Schema training in problem-based learning. <u>Teaching and Learning in Medicine</u>, 1(1), 16-20.
- Dreistadt, R. (1969). The use of analogies and incubation in obtaining insights in creative problem solving. <u>The Journal of Psychology</u>, 71(2), 159-175.
- Duncker, K. (Ed.). (1945). On problem solving. <u>Psychological Monographs</u>, <u>58</u> (5, Whole No. 270).

- Funkhouser, C. (1990). Mathematical problem solving: A review of the literature.

 California State University at San Bernardino. (ERIC Document Reproduction Service No. ED 324 217)
- Gabel, D. L., & Sherwood, R. D. (1983). Facilitating problem solving in high school chemistry. <u>Journal of Research in Science Teaching</u>, 20(2), 163-177.
- Gentner, D. (1989). The mechanisms of analogical learning. In S. Vosniadou & A. Ortony (Eds.), Similarity and analogical reasoning (pp. 200-241). Cambridge, England:

 Cambridge University Press.
- Gholson, B., Eymard, L. A., Morgan, D., & Kamhi, A. G. (1987). Problem solving, recall, and isomorphic transfer among third-grade and sixth-grade children. <u>Journal of Experimental Child Psychology</u>, 43(2), 227-243.
- Gick, M. L. (1981). Analogical reasoning and schema formation. <u>Dissertation Abstracts</u>

 <u>International</u>, 42(2-B), 808.
- Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. <u>Cognitive Psychology</u>, 12, 306-355.
- Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer.

 <u>Cognitive Psychology</u>, 15, 1-38.
- Gick, M. L., & Holyoak, K. J. (1987). The cognitive basis of knowledge transfer. In S.M. Cormier & J.D. Hagman (Eds.), <u>Transfer of learning: Contemporary research and applications</u> (pp. 9-46). San Diego, CA: Academic Press.
- Gick, M. L., & Paterson, K. (1992). Do contrasting examples facilitate schema

- acquisition and analogical transfer? <u>Canadian Journal of Psychology</u>, <u>46</u>(4), 539-550.
- Hesse, F. W., & Klecha, D. (1990). Use of analogies in problem solving. Computers in Human Behavior, 6, 115-129.
- Holyoak, K. J. (1985). The pragmatics of analogical transfer. In G.H. Bower (Ed.), <u>The psychology of learning and motivation</u> (Vol. 19, pp. 59-87). New York:

 Academic Press.
- Holyoak, K. J., & Koh, K. (1987). Surface and structural similarity in analogical transfer. Memory and Cognition, 15, 332-340.
- Holyoak, K. J., & Thagard, P. (1989). Analogical mapping by constraint satisfaction.

 Cognitive Science, 13, 295-355.
- Jausovec, N. (1993). The influence of ability on analogical transfer. Ricerche di Psicologia, 17(2), 37-50.
- Kazanas, H. C., & Chawhan, A. R. (1975) Effects of two treatments on cognitive achievement of students varying in problem-solving ability. <u>Journal of</u> <u>Educational Research</u>, 68(7), 269-273.
- Keane, M. T. (1985). On drawing analogies when solving problems: A theory and test of solution generation in an analogical problem-solving task. <u>British Journal of</u> <u>Psychology</u>, 76, 449-458.
- Keane, M. T. (1988). Analogical problem solving. West Sussex, England: Horwood.
- King-Johnson, D. A. (1992). Using analogies to form conceptual models to facilitate transfer. Contemporary Educational Psychology, 17(1), 1-7.

- Klauer, K. J. (1989). Teaching for analogical transfer as a means of improving problem-solving, thinking and learning. <u>Instructional Science</u>, 18, 179-192.
- Kohn, A. (1993, September). Choices for children: Why and how to let students decide.

 Phi Delta Kappan, 8-20.
- Lewis, M. W., & Anderson, J. R. (1985). Discrimination of operator schemata in problem solving: Learning from examples. <u>Cognitive Psychology</u>, 17, 26-65.
- Loewenthal, G. G., & Pons, M. M. (1987). Comparison of three methods for improving verbal analogies solution. Forum for Reading, 18(2), 53-61.
- Marr, D. B., & Sternberg, R. J. (1986). Analogical reasoning with novel concepts:

 Differential attention of intellectually gifted and non-gifted children to relevant and irrelevant novel stimuli. Cognitive Development, 1, 53-72.
- Mayer, R. E. (1985). Implications of cognitive psychology for instruction in mathematical problem solving. In E.A. Silver (Ed.), <u>Teaching and learning</u> mathematical problem-solving: multiple research perspectives (pp. 123-138). Hillsdale, NJ: Erlbaum.
- Mayer, R. E. (1987). Learnable aspects of problem solving: Some examples. In D.E.

 Berger, K. Pezdek, & W.P. Banks (Eds.), <u>Applications of cognitive psychology:</u>

 <u>problem solving, education, and computing</u> (pp. 109-122). Hillsdale, NJ:

 Erlbaum.
- McVey, M. D. (1993). Analogical transfer: Are there performance differences among high-ability students? Dissertation Abstracts International, 53(7-A), 2301-2302.

- Needham, D. R., & Begg, I. M. (1991). Problem-oriented training promotes spontaneous analogical transfer: Memory-oriented training promotes memory for training.

 Memory and Cognition, 19(6), 543-557.
- Novick, L. R. (1987). Analogical transfer in expert and novice problem solvers.

 <u>Dissertation Abstracts International</u>, 47(9-B), 3991.
- Novick, L. R. (1988), Analogical transfer, problem similarity, and expertise. <u>Journal of Experimental Psychology: Learning, Memory, and Cognition</u>, 14(3), 510-520.
- Novick, L. R., & Holyoak, K. J. (1991). Mathematical problem solving by analogy.

 <u>Journal of Experimental Psychology: Learning, Memory, and Cognition, 17</u>(3),
 398-415.
- Okagaki, L., & Koslowski, B. (1987). Another look at analogies and problem solving.

 <u>Journal of Creative Behavior, 21(1), 15-21.</u>
- Orlich, F. (1992). Analogical transfer: Access and metacognition. <u>Dissertation Abstracts</u>

 <u>International</u>, 52(8-B), 4500.
- Osman, M. E., & Hannafin, M. J. (1992). Metacognition research and theory: Analysis and implications for instructional design. <u>Educational Technology Research and Development</u>, 40(2), 83-99.
- Pierce, K. A., Duncan, M. A., Gholson, B., Ray, G. E., & Kamhi, A. G. (1993).

 Cognitive load, schema acquisition, and procedural adaptation in nonisomorphic analogical transfer. <u>Journal of Educational Psychology</u>, 85(1), 66-74.
- Pirolli, P. (1991). Effects of examples and their explanations in a lesson on recursion: A production system analysis. <u>Cognition and Instruction</u>, 8(3), 207-259.

- Pirolli, P., & Bielaczyc, K. (1989). Empirical analyses of self-explanation and transfer in learning to program. In <u>Proceedings of the annual conference of the cognitive science society</u> (pp. 450-457). Hillsdale, NJ: Erlbaum.
- Reed, S. K., Dempster, A., & Ettinger, M. (1985). Usefulness of analogous solutions for solving algebra word problems. <u>Journal of Experimental Psychology:</u>

 Learning, Memory, and Cognition, 11(1), 106-125.
- Reed, S. K., Ernst, G. W., & Banerji, R. (1974). The role of analogy in transfer between similar problem states. <u>Cognitive Psychology</u>, 6, 436-450.
- Reeves, L. R., & Weisberg, R. W. (1993). On the concrete nature of human thinking:

 Content and context in analogical transfer. <u>Educational Psychology</u>, 13(3-4),

 245-258.
- Robins, S., & Mayer, R. E. (1993). Schema training in analogical reasoning. <u>Journal of Educational Psychology</u>, 85(3), 529-538.
- Ross, B. H. (1984). Remindings and their effects in learning a cognitive skill. <u>Cognitive Psychology</u>, 16, 371-416.
- Ross, B. H. (1987). This is like that: The use of earlier problems and the separation of similarity effects. <u>Journal of Experimental Psychology: Learning, Memory, and Cognition</u>, 13(4), 629-639.
- Ross, B. H. (1989). Remindings in learning and instruction. In S. Vosniadou & A.

 Ortony (Eds.), <u>Similarity and analogical reasoning</u> (pp. 438-469). Cambridge,
 England: Cambridge University Press.

- Royer, J. M. (1979). Theories of the transfer of learning. <u>Educational Psychologist</u>, 14, 53-69.
- Rumelhart, D. E. (1989). Toward a microstructural account of reasoning. In S.

 Vosniadou & A. Ortony (Eds.), <u>Similarity and analogical reasoning</u> (pp. 298-312). Cambridge, England: Cambridge University Press.
- Schoenfeld, A. H. (1979). Explicitly heuristic training as a variable in problem solving performance. <u>Journal for Research in Mathematics Education</u>, 10, 173-187.
- Seel, N. M., & Hoops, W. (1993). Analogies and transfer, or analogical transfer?

 <u>Ricerche di Psicologia, 17(2), 105-130.</u>
- Silver, E. A. (1979). Students' perception of relatedness among mathematical verbal problems. <u>Journal for Research in Mathematics Education</u>, 10, 195-210.
- Silver, E. A., & Marshall, S. P (1990). Mathematical and scientific problem solving. In B.F. Jones & L. Idol (Eds.), <u>Dimensions of thinking and cognitive instruction</u> (pp. 265-290). Hillsdale, NJ: Erlbaum.
- Simon, H. A., & Hayes, J. R. (1976). The understanding process: Problem isomorphs.

 <u>Cognitive Psychology</u>, 8, 165-190.
- Solomon, I. (1984). Analogical transfer and 'functional fixedness' in the science classroom. <u>Journal of Educational Research</u>, 87(6), 371-377.
- Solomon, I. (1991). Effects of task context and domain knowledge on analogical transfer of science knowledge. <u>Dissertation Abstracts International</u>, 52(5-A), 1696.

- Spencer, R. M., & Weisberg, R. W. (1986). Context-dependent effects on analogical transfer. Memory and Cognition, 14, 442-449.
- Stein, B. S., Way, K. R., Benningfield, S. E., & Hedgecough, C. A. (1986). Constraints on spontaneous transfer in problem-solving tasks. Memory and Cognition, 14 (5), 432-441.
- Van Lehn, K., Jones, R. M., & Chi, M. T. (1992). A model of the self-explanation effect.

 Journal of the Learning Sciences, 2(1), 1-59.
- Vosniadou, S. (1989). Analogical reasoning in knowledge acquisition. In S. Vosniadou & A. Ortony (Eds.), <u>Similarity and analogical reasoning</u> (pp. 413-437). Cambridge, England: Cambridge University Press.
- Wickelgren, W. A. (1974). How to solve problems. San Francisco: Freeman.
- Zook, K. (1991). Effects of analogical processes on learning and misrepresentation.

 <u>Educational Psychology</u>, 3(1), 41-72.

APPENDICES

Appendix A. Participant Consent Form

Informed Consent Form

which is conducted by Dune I involve approximately two (2) credit points toward my cours	I,				
solve, and my involvement will to these problems. There are can expect from my participat how to become a better proble problem solving ability, will re thereby assuring anonymity of At the conclusion of m experiment. I understand that	Il consist of reading verbal no discomforts, stresses, con include information or em solver. The results of the emain confidential and will my responses. The results of t	to investigate how people problem problems and providing solutions or risks involved. The benefits I how well I problem solve and my participation, including be identified only by number code given an explanation of the any time either by phone or in ent questions about the research			
and my rights as a participant.	now for answers to pertin	ent questions about the research			
print your name here	today's date	daytime telephone number			
sign your name here	e-mail address	evening telephone number			

Dune Ives Rm. 495 Psychology Department Education Building Utah State University Logan, Utah 84322-2810 (801) 797-3391 Appendix B. Demographic Information Questionnaire

Demographic Information Questionnaire

For the following questions, please either write in a response in the space provided or check the appropriate box (please select only one answer for each question).

Q1.	Name	
	(last, first, middle initial)	
Q2.	Gender	
	☐ Male ☐ Female	
Q3.	Year in school	
	☐ Freshman (0-44 credits) ☐ Sophomore (45-89 credits) ☐ Junior (90-134 credits) ☐ Senior (135 + credits) ☐ Other (please specify)	
Q4.	Current age	
Q5.	Current total GPA	
Q6.	Total ACT score:	
Q7.	Is English your first language?	
	☐ yes ☐ no →→→ first language	(please specify)
Q8.	Social Security number:	

Appendix C. Problem-Solving Ability Test

Three Monsters

Three extraterrestrial monsters were holding three crystal globes. Because of the quantum-mechanical peculiarities of their neighborhood, both monsters and globes come in exactly three sizes with no others permitted: small, medium, and large. The medium-sized monster was holding the small globe, the small monster was holding the large globe; and the large monster was holding the medium-sized globe. Since this situation offended their keenly developed sense of symmetry, they proceeded to transfer globes from one monster to another so that each monster would have a globe proportionate to his own size. Monster etiquette complicated the solution of the problem since it requires: 1) that only one globe may be transferred at one time, 2) that if a monster is holding two globes, only the larger of the two may be transferred, and 3) that a globe may not be transferred to a monster who is holding a larger globe. By what sequence of transfer could the monsters have solved this problem?

Solution:

The small monster passes the large globe to the large monster. The medium monster passes the small globe to the small monster. The large monster passes the large globe to the medium monster, and then passes the medium globe to the small monster. The medium monster passes the large globe to the large monster. The small monster then passes the medium globe to the medium monster. Now, the large monster has the large globe, the medium monster has the medium globe, and the small monster has the small globe.

The Cord Problem

Suppose you are in a room where two cords are hung from the ceiling. The two cords are of such a length that when you hold one cord in one hand, you cannot reach the other cord. Your task is to tie the ends of these cords together. To help you in this task, you may use any of the objects listed below, which are also in this room: poles, clamps, pliers, extension cords, tables, chairs. How will you complete this task?

Solution:

Tie a weight to one cord and swing it so it becomes a pendulum. The other cord can then be brought toward the center, the swinging cord can be caught as it approaches the midpoint, and then the two cords can be tied together.

The Artificial Lake

An engineer plans the construction of an artificial lake to produce electric energy. According to his first plan, a unique wide canal collects water coming from a valley and conveys it into the lake. However, the engineer realizes that the capacity of the lake is not sufficient to produce the needed amount of energy. However, it is not possible to widen the lake because there are no free areas around it. What could the engineer do to increase the capacity of the lake?

Solution:

To avoid this mishap, the engineer elaborates a second plan. According to this plan, the lake must be deepened so that it contains a higher amount of water.

Appendix D. Participant Training Sign-Up Sheet

Training Session Date Time Place		
Trainer		
	Participant Training Sign-up Sheet	
Name 1.	Telephone Number	e-mail address
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		

Appendix E. Teacher-Generated Training Protocol

Teacher-Generated Protocol

[Begin this training session by having each participant write down their name and telephone number on the sign-up sheet--they must do this!!]

I. Introduction Hello and thank you for agreeing to participate in our problem-solving study. My name is ______ and I will be your trainer today. Let me begin first by telling you a little about myself. I am in the Research and Evaluation Methodology Ph.D. program here at Utah State University, and am currently in my ____ year of the program. My concentration is _____ and _____. Dune Ives, the project coordinator asked me to give you a few pointers on how to become better problem solvers. As you may have been told, this training is the first of our two-part study. The second part will consist of testing your problem solving ability in a separate section for

which you should have already signed up. I have a list of times and days you are signed

up for testing and will read through it at the end of our session today.

First, let me give you a brief overview of what I will be covering today. As I said before, I will teach you about specific types of problems and problem solving strategies, and then I will put your knowledge to the test and let you practice what I have taught you. Are there any questions before we get started? [allow only a couple of minutes for questions and answers] Great, let's get started.

II. Problem Solving Strategies

There are several ways to solve problems. The first method is called **trial and error**. In this method, you attempt to solve the problem using one method, evaluate your efforts, and either solve the problem, try a new method, or stop trying because you have run out of methods. Let's work through an example that can be solved through trial and error. **[put parade problem up on the overhead and read it out loud--make sure students do not generate any solutions and shout them out]** First, you might suggest that the general put the parade on television. Upon evaluation of this solution, you discover that this is impossible since the fortress was surrounded by farms and villages and the chances of them having tv's is very slim. Okay, so how about having the army walk on very high stilts in the parade so that everyone could see them above the trees? No? You're right, this may not be the best solution. So you see how trial and error works? You find one approach, try it, and when it no longer works go on to the next approach or stop trying because you have exhausted your methods.

A second problem solving strategy is called **working backwards**, in which you identify your goal state, that is, what the problem will look like when it is solved. You continue to work backward from this state until you have reached the beginning. Here is an example of a problem that might require using working backwards. [put poker problem up on the overhead and read it out loud--make sure students do not

generate any solutions and shout them out] In this problem, you should first identify what the goal is--to find out what the original stake of each player was and begin to work backwards from the last game--game number three and continue until you have figured out what happened in the second and first games.

Another strategy is called **working forward** in which you either break the problem into smaller sub-goals and provide a means of solving each sub-goal, or you identify the initial state of the problem (what information you have) and its final goal state (what the problem should end up looking like) and work to decrease the difference between the two. Again, let's look at an example of a problem in which you might use working forward. **[put general problem up on the overhead and read it out loud--make sure students do not generate any solutions and shout them out]** Here, we know that the general must attack the fortress with his entire army and that only small bodies of men could pass over the planted mines safely. Thus, we have identified the initial state of the problem. We also know that the final goal is to determine what the general must do to capture the fortress without detonating the mines, and, as a result, blowing up the road and destroying many neighboring villages. Next, we would begin to decrease the differences between where the general is and where he must go with his army. Or, we could break up the problem into subgoals--the first being sending his men over the mines safely and the second being attacking the fortress--and begin by solving each subgoal.

Finally, one of the most effective strategies is problem solving by analogy. In this case, you would search for solutions to situations similar (that is, analogous) to the one you are presented with, and test whether this solution would be appropriate for the present problem. There are steps you should consider when solving by analogy. To illustrate how these steps work, let's take a look at the book burners and book lovers problem. [hand out problem, put it up on the overhead, and read it out loud. The first step is to identify the goal of the current problem you are trying to solve, which again is the information you need to understand to solve the problem. In this case, the goal is to determine the simplest schedule of crossings that would permit all book burners and book lovers to cross the river without losing any books. Next, you would want to search for and retrieve from your memory problems similar to the one you are attempting to solve. Then, you would want to compare the objects and constraints in your current problem and the problem you retrieved. This allows you to determine how similar the problems are. After comparing the problems, you would want to retrieve the solution or strategy used in the similar problem, your analogy, and finally you would apply this solution or strategy to your current problem and evaluate it for its appropriateness.

Let's suppose you had already been exposed to the next problem, the missionaries and cannibals problem. [hand out problem, put it up on overhead, and read it out loud] If you were asked to solve this problem and you did so by the analogical problem solving method, the most helpful analogy would be one that was similar to your problem, and in which both the goal state and the strategy or solution were provided to you. The cannibal problem does exactly that. Thus, your first step in solving the book burners problem would be to identify the goal of the problem. Take a minute to write down the goal state on the piece of paper labeled "book burner." [give them about one minute to

complete this task] Doing this, you find that the goal state is to determine how to get all six people across the river in such a way that all the books are left in tact. Next, you would compare the objects in the book burner and cannibal problems. Take a minute to do this now. [give them about two minutes to complete this task] By doing so, you find that there are three book burners and three book lovers just as there are three cannibals and three missionaries, the boat can only hold two people in each problem, and the book burners and cannibals can never outnumber the book lovers and missionaries. The third step would be to retrieve the strategy or solution from the cannibal problem, and finally apply it to the book burners problem. Let's see if this works. Using the paper labeled "book burners," take a few minutes and try to solve the book burners problem. [give them 10 minutes to solve the problem] Okay, let's see how your solution compares to the correct solution to the book burners problem. [put up book burner solution and read it out loud] So, we see that using the cannibal problem would be an appropriate analogy to the book burner problem.

There are several factors to consider when solving by analogy. The first factor is whether the analogy is complete or partial. A complete problem is one that provides you with the initial and final goal states, objects and constraints, strategies, and a solution. The cannibal problem we just encountered is a complete problem. If the cannibal problem had not provided us with all this information it would have been called a partial problem. Both types of problems can be used as analogies, but the most helpful are the complete problems. As you can probably guess, it would have been more difficult to solve the book burner problem using the cannibal problem as an analogy had it not provided a solution, but just knowing the general strategy would have been helpful.

Next, we evaluate two problems based on whether they are isomorphic or non-isomorphic to each other. By isomorphic, I mean that they have similar goals, objects, and constraints. This is really the same as representing or comparing two problems for their similarities as we did with the cannibal and book burner problems. In fact, these two problems are isomorphic to each other. Let's take a look at how we might represent these problems. [put up Correspondence A overhead] As you can see, the problems have similar objects (book burners/cannibals and book lovers/missionaries), problem goals (get all book burners/cannibals and book lovers/missionaries across the river), and constraints (boat can only carry two people and book burners/cannibals cannot outnumber book lovers/missionaries). So, the missionary and cannibals and book burners and book lovers problems are isomorphic to each other.

Now, read through this next problem [hand out farmer's dilemma problem, put it up on overhead, and read it out loud]. At first glance, this problem may look similar to the book burners and cannibals problems in that the farmer has to transport a fox, goose, and corn across the river to his house, his boat can only carry two items at a time, and the fox cannot be left alone with the goose nor can the goose be left alone with the corn. Actually, it is different from the first two problems. [put up Correspondence B overhead] Here we see that there are three objects in both the farmer's dilemma (fox, goose, corn) and book burners problem (book burners, book lovers, books), but only two in the cannibal problem (missionaries, cannibals), the problem goals are the same

(transport everything from one side of the river to the other side) but involve a fewer number of things to transport across the river (four versus six), and the constraints are slightly different (the farmer must be in the boat at all times, the fox cannot be left alone with the goose and the goose cannot be left alone with the corn, whereas in the first two problems no one specific person had to remain in the boat at all times, the cannibals couldn't outnumber the missionaries and the book burners couldn't outnumber the book lovers). This third problem is considered to be non-isomorphic to the first two problems. More specifically, it looks similar but has different objects and constraints. This same problem, the farmer's dilemma, also has similarities to other problems, however, it is more isomorphic to this next problem, the men and boys problem. Ihand out problem, put it up on the overhead, and read it out loud] [when finished put up Correspondence C overhead! In this problem, there are 11 objects instead of three, the problem goals are slightly different (men and boys problem involve more objects to transport across the river and you need to determine the number of time the boat must cross the river), and the constraints are slightly different (the boat can carry either one man or the two boys). Thus, while these are more isomorphic to each other, they are considered to be nonisomorphic. Having an isomorphic analogy is going to be more helpful to you when solving a problem by the analogical strategy, but depending on the similarity between two problems a non-isomorphic analogy may also be somewhat helpful.

Using the farmer's dilemma problem as an analogy, let's try to solve the men and boys problem. The first step would be to determine what our final goal state is, which in this case is to determine the number of trips it would take to cross the river and successfully transport all 11 persons across the river. Next, you would represent both the problem and an appropriate analogy, in this case the farmer's dilemma as we have just done. [put up Correspondence C overhead again] After this step you would retrieve the farmer's dilemma solution strategy and apply it to the men and boys problem. [put up farmer's dilemma solution on the overhead] Using the paper labeled "men and boys", take a few minutes to solve the men and boys problem. [give them 7 minutes to solve the problem] Okay, let's look at the correct solution to the men and boys problem. [put up men and boys solution on the overhead and read it through] So, the farmer's dilemma problem would be an appropriate analogy to the men and boys problem even though they are not completely isomorphic in nature.

So, when solving a problem, you would first want to retrieve a complete isomorphic problem and see if it helps you solve your current problem. However, if you don't have a complete isomorph, you would then want to try a complete non-isomorphic problem. Finally, you may find that a partial isomorph would be helpful to you when solving your current problem.

Let's see how using a partial yet isomorphic problem may assist you. Let's try solving this next problem, the Trump Suit problem using its corresponding partial and isomorphic analogue, the Wine Merchants problem. [hand out Trump problem and Wine Merchants problem, put up on the overhead, and read through each] First, take a minute to read through each problem while noticing similarities between their problem goals, objects, and constraints, and then attempt to solve the Trump suit problem

using the piece of paper labeled "Trump suit." [give them 7 minutes to solve the Trump suit problem] Okay, let's go through these together. You probably noticed that both problems involve similar problem goals (to travel through water to the rich man's dwelling with the requested item before sunset), objects (one rich man and two other men), and similar constraints (the water proves to be a barrier to the travel). Let's see how your solution compares to our correct solution strategy. [put up Trump suit problem solution on the overhead and read through]. While this is our correct solution, there are several solutions that may work. Also, even though the Wine Merchants problem did not provide a solution strategy, it may have been helpful to you in determining the best strategy to use based on what you think may have worked with the Wine Merchants.

The second problem to be solved is the marching band problem and its corresponding isomorphic problem is the Renshaw problem. [hand out the marching band problem and the Renshaw problem, put up on the overhead and read through both problems out loud Take a minute to read through each problem while noticing similarities between their problem goals, objects, and constraints, and attempt to solve the marching band problem using the piece of paper labeled "marching band." [give them 10] minutes to solve the problem! Okay, let's go through these together. You probably noticed that both problems involve similar problem goals (arranging objects into rows or groups so that there were the same number of objects in each row or group or that there were no objects left over), object groups (an undetermined number of plants and band members), and similar constraints (an extra person and two extra plants that were not originally taken into consideration). So to summarize, the band members are like plants, the rows and columns of band members are like kinds of plants, and the number of band members per row or column is like the number of plants of each kind. Seeing these similarities, you could simply apply the strategy from the Renshaw problem to see if it is appropriate and adequately solves the marching band problem. [put up overhead of marching band problem solution As you can see, you must apply the formula of finding the least common multiple of the divisors from the marching band problem (12, 8, and 3), which equals 24, 48, 72, 96, etc., add the constant remainder of 1 (extra band member) to each LCM (25, 49, 73, 97, etc.) and find the LCM which can be evenly divisible by 5 (the fourth suggestion on number of band members per row). This LCM is 145. So, as with the problems we have already been exposed to, the marching band problem can be solved by using the solution from the Renshaw problem.

One final point is that analogues come by recalling previously seen problems and they are not necessarily always given to us. So the first best step, after identifying the problems' goal, is to ask yourself "have I ever seen a problem like this before?"

You all did a really good job at following the analogical problem solving process. First, you determined the problems' goal, identified any similarities and differences between two problems, you attempted to apply the analogical strategy and solution to the problem you were solving, you modified the solution if necessary, and, finally, you correctly solved the problems.

III. Conclusion

That concludes the first part of our problem-solving study. I want to thank each of you for participating. As I said before, I have a list of the times and days you signed up to complete the second and final portion of the study. Remember, it is extremely important that each of you participate in the second part, so please look over these days and times to make sure you can attend. In addition to this list, I have a piece of paper for each of you that lists the day and time you agreed to be tested. As Dune may have told you, you will receive _____ extra credit points when you complete the study, so not only are you helping us out--you receive something in return!

One final note, during the next few weeks it is vital that you do not discuss this training, or the testing you will complete, with anyone in this group or anyone else in your Psychology 101 course. Again, thank you for coming. If there any no more questions you are free to go after you have picked up your paper. [collect all handouts at this time and let them leave only after they have confirmed their testing date and time with you and you have given them their individual confirmation slips]

Appendix F. Learner-Generated Training Protocol

Learner-Generated Protocol

[Begin this training session by having each participant write down their name and telephone number on the sign-up sheet--they must do this!!]

I. Introduction
Hello and thank you for agreeing to participate in our problem-solving study. My
name is and I will be your trainer today. Let me begin first by
telling you a little about myself. I am in the Research and Evaluation Methodology Ph.D.
program here at Utah State University, and am currently in my year of the program.
My concentration is and Dune Ives, the project
coordinator asked me to give you a few pointers on how to become better problem
solvers. As you may have been told, this training is the first of our two-part study. The
second part will consist of testing your problem solving ability in a separate section for
which you should have already signed up. I have a list of times and days you are signed
up for testing and will read through it at the end of our session today.
First, let me give you a brief overview of what I will be covering today. As I said
before, I will teach you about specific types of problems and problem solving strategies,
and then I will put your knowledge to the test and let you practice what I have taught you.

Are there any questions before we get started? [allow only a couple of minutes for

II. Problem Solving Strategies

questions and answers | Great, let's get started.

There are several ways to solve problems. The first method is called **trial and error**. In this method, you attempt to solve the problem using one method, evaluate your efforts, and either solve the problem, try a new method, or stop trying because you have run out of methods. Let's work through an example that can be solved through trial and error. [put parade problem up on the overhead and read it out loud] First, you might suggest that the general put the parade on television. Upon evaluation of this solution, you discover that this is impossible since the fortress was surrounded by farms and villages and the chances of them having tv's is very slim. Okay, so how about having the army walk on very high stilts in the parade so that everyone could see them above the trees? No? You're right, this may not be the best solution. So you see how trial and error works? You find one approach, try it, and when it no longer works go on to the next approach or stop trying because you have exhausted your methods.

A second problem solving strategy is called **working backwards**, in which you identify your goal state, that is, what the problem will look like when it is solved. You continue to work backward from this state until you have reached the beginning. Here is an example of a problem that might require using working backwards. [put poker problem up on the overhead and read it out loud] In this problem, you should first identify what the goal is--to find out what the original stake of each player was and begin

to work backwards from the last game--game number three and continue until you have figured out what happened in the second and first games.

Another strategy is called **working forward** in which you either break the problem into smaller sub-goals and provide a means of solving each sub-goal, or you identify the initial state of the problem (what information you have) and its final goal state (what the problem should end up looking like) and work to decrease the difference between the two. Again, let's look at an example of a problem in which you might use working forward. **[put general problem up on the overhead and read it out loud]** Here, we know that the general must attack the fortress with his entire army and that only small bodies of men could pass over the planted mines safely. Thus, we have identified the initial state of the problem. We also know that the final goal is to determine what the general must do to capture the fortress without detonating the mines, and, as a result, blowing up the road and destroying many neighboring villages. Next, we would begin to decrease the differences between where the general is and where he must go with his army. Or, we could break up the problem into subgoals—the first being sending his men over the mines safely and the second being attacking the fortress—and begin by solving each subgoal.

Finally, one of the most effective strategies is problem solving by analogy. In this case, you would search for solutions or strategies to situations similar (analogous) to the one you are presented with, and test whether this solution or strategy would be appropriate for the present problem. Normally, you would do this by using problems you have been exposed to before, but for right now I will provide you with an analogous problem. For the remaining problems you will be given, I want you to practice problem solving using the analogical method. First, I am going to hand out to you a pair of problems-- the book burners book lovers and the cannibals and missionaries problems. [hand out each problem to every student and put up on overhead] Take just a minute to read through both problems. [read each problem out loud] Okay, on the piece of paper labeled "book burners" write down anything you notice about these problems. For example, try to determine how similar the problems are, what their goal states are, the types of objects they include, etc. [hand out paper to each student and give students 3 minutes to complete this task] Great, let's have everyone tell the whole group one thing that they noticed about these problems. First, what similarities did you notice between these problems' goal states? How about the objects they used in the problems? Did you notice any similarities between constraints in the problems? How about the amount of information each problem provided you with--any differences or similarities? We call these two problems isomorphic to each other. Someone describe what I mean by isomorphic. Do you think the solution from the cannibals problem would work with the book burners problem? Why? Why not? [write responses on a blank overhead and recap the goals of each problem, any comparisons with regard to objects and constraints that were made, any discussion around whether they could use the cannibals problem to solve the book burners problem. On the same sheet of paper, write down what steps you might take if I asked you to solve the book burners problem. For example, what is the first step you would take, the next step, and so on. [give students 3 minutes to complete this task Okay, let's see what you came up with as

appropriate steps to take. [first, go around room asking about how they solved the problem--what was their first step, last step--and then go back around the room and elicit their solutions to the problem--write down steps on a blank overhead and recap them in the order: goal identification, problem representation, retrieval of analogous solution, application of analogous solution] Now, take a minute to solve the book burners problem. [give 10 minutes to solve the problem] How did everyone do? [elicit response from everyone] How do your answers compare to the correct solution to the book burners problem? [put up correct solution on overhead and read out loud]

Okay, it looks like you are getting the hang of this, let's try this again with another problem--the farmer's dilemma. [hand out farmer's dilemma problem to each student] On the piece of paper labeled "farmer's dilemma" write down anything you notice about this problem in comparison to the book burners and cannibals problems. For example, try to determine how similar the problems are, what their goal states are, the types of objects they include, etc. [hand out paper to each student and give students 3 minutes to complete this task! Great, let's have everyone tell the whole group one thing that they noticed about these problems. First, what similarities did you notice between these problems' goal states? How about the objects they used in the problems? Did you notice any similarities between constraints in the problems? How about the amount of information each problem provided you with--any differences or similarities? We refer to the farmer's dilemma as being non-isomorphic to the book burners and cannibals problems. Someone describe what I mean by non-isomorphic. Do you think the solution from the farmer's dilemma would work with the book burners problem? Why? Why not? write responses on a blank overhead and re-cap the goals of each problem, any comparisons with regard to objects and constraints that were made, any discussion around whether they could use the cannibals problem to solve the farmer's dilemma problem! Now, take a look at the fourth problem-the men and boys problem. [hand out analogue, put analogue on overhead, and read out loud | Using the same process as before, and using the "farmer's dilemma" paper again, write down anything you notice about this problem in comparison to the farmer's dilemma. For example, try to determine how similar the problems are, what their goal states are, the types of objects they include, etc. [give students 3 minutes to complete this task] Okay, what did you find? First, what similarities did you notice between these problems' goal states? How about the objects they used in the problems? Did you notice any similarities between constraints in the problems? How about the amount of information each problem provided you withany differences or similarities? Do you think the solution from the farmer's dilemma would work with the men and boys problem? Why? Why not? [go around the room and make sure you ask every student to provide some response--write down on blank overhead as before and recap if they determined the goals of each problem, any comparisons with regard to objects and constraints that were made, any discussion around whether they could use the farmer's dilemma problem to solve the men and boys problem On the same sheet of paper, write down what steps you might take if I asked you to solve the men and boys problem. Again, what would be your first

step, second step, and so on. [give students 3 minutes to complete this task] Okay, let's see what you came up with as appropriate steps to take. [first, go around room asking about how they solved the problem--what was their first step, last step--and then go back around the room and elicit their solutions to the problem--write down steps on a blank overhead and recap them in the order: goal identification, problem representation, retrieval of analogous solution, application of analogous solution]

Now, take a minute to solve the men and boys problem. [give 7 minutes to solve the problem] How did everyone do? [elicit response from everyone] Did the farmer's dilemma solution strategy help you solve the men and boys problem? Even though the farmer's dilemma may not be isomorphic, its solution strategy can be useful when attempting to solve the men and boys problem. How do your answers compare to the correct solution to the men and boys problem? [put up correct solution on overhead and read out loud]

Now let's try to solve two additional problems. The first problem is the Trump suit problem and its corresponding analogy--the wine merchants problem. [hand out Trump problem and Wine Merchants analogue On the paper labeled "Trump suit" write down what steps you should take to solve the Trump suit problem. Again, what would be your first step, second step, and so on. [give students 3 minutes to complete this task! Okay, let's see what you came up with as appropriate steps to take. [first, go around room asking about how they solved the problem--what was their first step, last step-write down steps on a blank overhead and recap them in the order: goal identification, problem representation, retrieval of analogous solution, application of analogous solution] Now, take a minute to solve the Trump suit problem using the piece of paper labeled "Trump suit." [give 7 minutes to solve the problem] How did everyone do? [elicit response from everyone] How do your answers compare to the correct solution to the Trump suit problem? [put up correct solution on overhead and read out loud] Great, you are all getting the idea. Let me ask you a question about this process--what do you think you did that follows good analogical problem solving? [have each student respond What do you think that maybe didn't follow good analogical problem solving? [have each student respond] Now, was the Wine Merchant problem useful to you in solving the Trump Suit problem? Why? Why not? What would have made it more useful? [have each student respond]

The last problem to be solved is the marching band problem and its corresponding analogy is the Renshaw analogue. [hand out the marching band problem and the Renshaw analogue, read through both problems out loud] Try the problem solving steps you have generated and try to solve the marching band problem using the piece of paper labeled "marching band." [give students 10 minutes to complete this exercise] Now let's see how each of you did. [first, go around room asking about how they solved the problem--what was their first step, last step--and then go back around the room and elicit their solutions to the problem] Now, compare your solution to the actual solution. [put up marching band solution on the overhead and read out loud] Great, you are all getting the idea.

You all did a really good job at following the analogical problem solving process. Now, let's write the steps on the board that you should take when solving a problem by the analogical problem solving method. You tell me what would be the first step.... the second step.... third step.... fourth step.... and the final step..... [put all this information on the board or blank overhead and the students should arrive at: First, you determine the problems' goal, identify any similarities and differences between two problems, you attempt to apply the analogical solution strategy to the problem you are solving, you modify the solution if necessary, and, finally, you correctly solve the problem.] Remember, analogues come by recalling previously seen problems and they are not necessarily always given to us. Most of the time we will have to retrieve a previously encountered problem from memory when attempting to solve a new problem.

III. Conclusion

That concludes the first part of our problem-solving study. I want to thank each of you for participating. As I said before, I have a list of the times and days you signed up to complete the second and final portion of the study. Remember, it is extremely important that each of you participate in the second part, so please look over these days and times to make sure you can attend. In addition to this list, I have a piece of paper for each of you that lists the day and time you agreed to be tested. As Dune may have told you, you will receive _____ extra credit points when you complete the study, so not only are you helping us out--you receive something in return!

One final note, during the next few weeks it is vital that you do not discuss this training, or the testing you will complete, with anyone in this group or anyone else in your Psychology 101 course. Again, thank you for coming. If there any no more questions you are free to go after you have picked up your paper. [collect all handouts at this time and let them leave only after they have confirmed their testing date and time with you and you have given them their individual confirmation slips]

Appendix G. Control Group Protocol

Control Group Protocol

[Begin this training session by having each participant write down their name and telephone number on the sign-up sheet--they must do this!!]

1. Introduction
Hello and thank you for agreeing to participate in our problem-solving study. My
name is and I will be your trainer today. Let me begin first by
telling you a little about myself. I am in the Research and Evaluation Methodology Ph.D.
program here at Utah State University, and am currently in my year of the program.
My concentration is and Dune Ives, the project
coordinator asked me to give you a few pointers on how to become better problem
solvers. As you may have been told, this training is the first of our two-part study. The
second part will consist of testing your problem solving ability in a separate section for
which you should have already signed up. I have a list of times and days you are signed
up for testing and will read through it at the end of our session today.

First, let me give you a brief overview of what we will be doing today. Today I am going to give you a series of seven problems and have you attempt to solve them. Are there any questions before we get started? [allow only a couple of minutes for questions and answers] Great, let's get started.

II. Practice

First, let me tell you how this will work. I will time you as you attempt to solve the problems and may call time before you complete a problem. On occasion, I may provide you with the completed answer to help you think about these problem solving strategies. It is important that when solving each of these problems you can only use the items listed in the problems themselves. [hand out 7 labeled pieces of paper and one pencil to each participant Please use the blank paper provided to you to write down your solutions to the problems that correspond to the label at the top of the paper. The first problem I want you to solve is the general problem, so please find the paper labeled general problem. [hand out problem, put up on the overhead, and read through out loud] Go ahead and begin. [call time after 1 minute] Here is a second problem, the book lovers and book burners problem. [hand out problem, put up on the overhead, and read through out loud] Please begin solving this problem. [call time after everyone has finished or when five minutes has passed, whichever comes first] Okay, for those of you who did not have enough time to finish this problem let me give you the answer. [put up book lover problem solution] Now try the parade problem. [hand out problem, put up on the overhead, and read out loud] Begin. [call time after 1 minute] The next problem is the men and boys problem. [hand out problem, put up on the overhead, and read out loud] Begin solving this problem. [call time after everyone has finished or when five minutes has passed, whichever comes first]

Okay, for those of you who did not have enough time to finish this problem let me give you the answer. [put up men and boys problem solution] Lets have you try the poker problem this time. [hand out problem, put up on the overhead, and read out loud] Go ahead and start. [call time after 1 1/2 minutes] Okay, now how about the Trump Suit problem. [hand out problem, put up on the overhead, and read out loud] Begin. [call time after everyone has finished or when five minutes has passed, whichever comes first] For those of you who were not able to derive a solution, here is the correct solution for that problem. [put up Trump Suit problem solution] Finally, let me have you try one more problem—the Marching Band problem. [hand out problem, put up on the overhead, and read out loud] Please begin. [call time after five minutes] Okay, for those of you who did not have enough time to finish this problem let me give you the answer. [put up Marching Band problem solution]

III. Conclusion

That concludes the first part of our problem-solving study. I want to thank each of you for participating. As I said before, I have a list of the times and days you signed up to complete the second and final portion of the study. Remember, it is extremely important that each of you participate in the second part, so please look over these days and times to make sure you can attend. In addition to this list, I have a piece of paper for each of you that lists the day and time you agreed to be tested. As Dune may have told you, you will receive _____ extra credit points when you complete the study, so not only are you helping us out--you receive something in return!

One final note, during the next few weeks it is vital that you do not discuss this training, or the testing you will complete, with anyone in this group or anyone else in your Psychology 101 course. Again, thank you for coming. If there any no more questions you are free to go after you have picked up your paper. [collect all handouts at this time and let them leave only after they have confirmed their testing date and time with you and you have given them their individual confirmation slips]

Appendix H. Training Analogies

The Book Burners and Book-Lovers Problem

Three book burners and three book wielding book lovers are on a river bank. The book burners and book lovers need to cross over to the other side of the river. They have for this purpose a small rowboat that will hold just two people (and several books). There is one problem, however. If the number of book burners on either river bank exceeds the number of book lovers on that bank, the book burners will destroy the books of the book lovers. How can all six people get across to the other side of the river in a way that guarantees that they all arrive there with the books intact? It is assumed that all passengers on the boat unboard before the next trip, and at least one person has to be in the boat for each crossing.

Solution:

Begin with either one book lover and one book burner, or two book burners, traveling to the right bank. Then leave one book burner and have either the book lover or the second book burner travel back across to the left bank and pick up the remaining book burner (leaving the book lover on the other side). The two book burners would then travel to the right bank dropping off one book burner for a total of two book burners on the right bank. The third book burner would travel to the left bank, get out of the boat and let two book lovers travel back to the right bank. One book lover would stay at the right bank and one book burner already on the right bank would travel back with the second book lover. Now, you have one book lover and one book burner at the right bank. Next, the book burner would get out of the boat and let the two book lovers return to the right bank. These book lovers would de-boat and let the book burner on the right bank take the boat back to the left bank. Now there are three book lovers on the right bank and three book burners on the left bank. Two book burners would travel to the right bank, one would stay and either the second would travel back to the left bank or one book lover would travel to the left bank and pick up the remaining book burner and travel back to the right bank. Now there are three book burners and three book lovers on the right bank.

The Cannibal Problem

Three missionaries and three cannibals having to cross a river at a ferry, find a boat but the boat is so small that it can contain no more than two persons. If the missionaries on either bank of the river, or in the boat, are outnumbered at any time by cannibals, the cannibals will eat the missionaries. Find the simplest schedule of crossings that will permit all the missionaries and cannibals to cross the river safely. It is assumed that all passengers on the boat unboard before the next trip and at least one person has to be in the boat for each crossing.

Solution:

Begin with either one missionary and one cannibal, or two cannibals, traveling to the right bank. Then leave one cannibal and have either the missionary or the second cannibal travel back across to the left bank and pick up the remaining cannibal (leaving the missionary on the other side). The two cannibals would then travel to the right bank dropping off one cannibal for a total of two cannibals on the right bank. The third cannibal would travel to the left bank, get out of the boat and let two missionaries travel back to the right bank. One missionary would stay at the right bank and one cannibal already on the right bank would travel back with the second missionary. Now, you have one missionary and one cannibal at the right bank. Next, the cannibal would get out of the boat and let the two missionaries return to the right bank. These missionaries would deboat and let the cannibal on the right bank take the boat back to the left bank. Now there are three missionaries on the right bank and three cannibals on the left bank. Two cannibals would travel to the right bank, one would stay and either the second would travel back to the left bank or one missionary would travel to the left bank and pick up the remaining cannibal and travel back to the right bank. Now there are three cannibals and three missionaries on the right bank.

The Men and Boys Problem

Nine men and two boys want to cross a river, using a raft that will carry either one man or the two boys. Assuming that one person must be in the raft for each river crossing, how many times must the boat cross the river in order to accomplish this goal? (A round trip equals two crossings.)

Solution:

First, the two boys cross; then one boy brings the boat back. Next, the man crosses by himself, and finally, the boy on the far bank returns with the boat. To get all nine men across requires that this sequence be repeated eight more times for a total of thirty-six crossings. At that point, the boat will be on the original bank, and only the two boys will remain. They cross together, thus making a total of thirty-seven crossings.

The Farmer's Dilemma Problem

Once a man bought a fox, a goose, and some corn at the market. He wanted to take them to his house, but his house was on the other side of a river that he had to cross. He had a boat, but it would only carry the man and one other thing over to his house at a time. He knew that if he left the fox alone with the goose, the fox would eat it. He also knew that if he left the goose alone with the corn, the goose would eat it. So he had to figure out how to get them all across the river to his house without anything being eaten. What should the man do?

Solution:

First, the man took the goose across the river to his house. The man then went back across the river, and brought the fox back across the river to his house. Next the man took the goose back across the river with him, and made the goose wait near the river bank while he took the corn across the river to his house. The man then went back across the river, found the goose was waiting on the river bank, and took the goose back across the river to his house. Now he had the fox, goose, and corn at his house.

The Trump Suit Problem

One day Donald Trump found that the provisions of caviar on his yacht were severely depleted. Unfortunately, he was also out of gas and could not travel back to shore to buy more for his party that next evening. So he sent out messengers to announce a generous offer. The first person to bring Mr. Trump a canister of caviar would be given 100 shares of stock in Trump Enterprises. However, the offer would expire at sundown.

Two investors heard the news. Each had a Cigarette (a fast, fancy boat) loaded with large canisters of caviar. They both set out for Trump's yacht at once. An hour before sundown they came to a place where the water level was very low. The first investor drove his boat into the shallow water in a desperate attempt to reach the yacht. But the boat was too loaded down with caviar and could not move fast enough to hover across the low water. Consequently, the boat was beached on the sand bar beneath the water's surface and the investor was stranded with 200 pounds of caviar.

Seeing this, the second (and more conservative) investor decided not to make the same mistake. What did the investor do?

Solution:

The second investor, seeing the problems the first investor had, decides to empty out all of the canisters of caviar but one, ties them to the front of his boat in a down and outward manner, and heads toward the same direction as the first investor did. This way, the barrels will be the first things to hit the sandbar and will act as warning signals that the boat is getting too close to shallow water.

The Wine Merchants Problem

One day a rich man found that his wine cellar was empty. So he sent out messengers to announce a generous offer. The first person to bring the rich man a barrel of wine would be given a brick of solid gold. However, the offer would expire at sundown.

Two wine merchants heard the news. Each had a horse-drawn cart loaded with large barrels of wine. They both set out for the duke's palace at once. An hour before sundown they came to a place where the bridge had been washed out by a raging river. The first merchant drove his horses and cart into the flood in a desperate attempt to reach the other side. But the horses were already exhausted and could not fight the current. The cart overturned, and the horses, wine, and driver were washed away. What did the second merchant do?

Solution:

The second merchant tried a different tactic. He poured the wine out of all but one of his barrels, and lashed them together to form a raft; then he loaded the one full barrel, a horse, and himself on top. He set the raft adrift and floated downstream. In a few minutes the raft came to rest on the shore in front of the town where the rich man lived. The merchant disembarked, loaded the wine barrel on the horse, and led it to the rich man's house. He arrived just as the sun was setting, and collected the gold brick as a reward for his efforts.

The Marching Band Problem

Members of the West High School Band were hard at work practicing for the annual Homecoming Parade. First they tried marching in rows of twelve, but Andrew was left by himself to bring up the rear. The band director was annoyed because it didn't look good to have one row with only a single person in it, and of course Andrew wasn't very pleased either. To get rid of this problem, the director told the band members to march in columns of eight. But Andrew was still left to march alone. Even when the band marched in rows of three, Andrew was left out. Finally, in exasperation, Andrew told the band director that they should march in rows of five in order to have all the rows filled. He was right. This time all the rows were filled and Andrew wasn't alone any more. Given that there were at least 45 musicians on the field but fewer than 200 musicians, how many students were there in the West High School Band?

Solution:

Answer = 145.

The Renshaw Problem

Mr. and Mrs. Renshaw were planning how to arrange vegetable plants in their new garden. They agreed on the total number of plants to buy, but not on how many of each kind to get. Mr. Renshaw wanted to have a few kinds of vegetables, and ten of each kind. Mrs. Renshaw wanted more different kinds of vegetables, so she suggested having only four of each kind. Mr. Renshaw didn't like that because if some of the plants died, there wouldn't be very many left of each kind. So they agreed to have five of each vegetable. But then their daughter pointed out that there was room in the garden for two more plants, although then there wouldn't be the same number of each kind of vegetable. To remedy this, she suggested buying six of each vegetable. Everyone was satisfied with this plan. Given this information, what is the fewest number of vegetable plants the Renshaws could have in their garden?

Solution:

Since at the beginning Mr. and Mrs. Renshaw agree on the total number of plants to buy, 10, 4, and 5 must all go evenly into that number, whatever it is. Thus the first things to do is to find the smallest number that is evenly divisible by those three numbers, which is 20. So the original number of vegetable plants the Renshaws were thinking of buying could be any multiple of 20 (that is, 20 or 40 or 60 or 80 etc.). But then they decide to buy 2 additional plants, that they hadn't been planning to buy originally, so the total number of plants they actually end up buying must be 2 more than the multiples of 20 listed above (that is, 22 or 42 or 62 or 82 etc.). This means that 10, 4, and 5 will no longer go evenly into the total number of plants. Finally, the problem states that they agree to buy 6 of each vegetable, so the total number of plants must be evenly divisible by 6. The smallest total number of plants that is evenly divisible by 6 is 42, so that's the answer.

The General Problem

A small country was ruled from a strong fortress by a dictator. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads led to the fortress through the countryside. A rebel general vowed to capture the fortress. The general knew that an attack by his entire army would capture the fortress. He gathered his army at the head of one of the roads, ready to launch a full-scale direct attack. However, the general then learned that the dictator had planted mines on each of the roads. The mines were set so that small bodies of men could pass over them safely, since the dictator needed to move his troops and workers to and from the fortress. However, any large force would detonate the mines. Not only would this blow up the road, but it would also destroy many neighboring villages. It therefore seemed impossible to capture the fortress. What should the general do in order to capture the fortress?

Solution:

However, the general devised a simple plan. He divided his army into small groups and dispatched each group to the head of a different road. When all was ready he gave the signal and each group marched down a different road. Each group continued down its road to the fortress so that the entire army arrived together at the fortress at the same time. In this way, the general captured the fortress and overthrew the dictator.

The Parade Problem

A small country was controlled by a dictator. The dictator ruled the country from a strong fortress. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads radiated outward from the fortress like spokes on a wheel. To celebrate the anniversary of his rise to power, the dictator ordered his general to conduct a full-scale military parade. On the morning of the anniversary, the general's troops were gathered at the head of one of the roads leading to the fortress, ready to march. However, a lieutenant brought the general a disturbing report. The dictator was demanding that this parade had to be more impressive than any previous parade. He wanted his army to be seen and heard at the same time in every region of the country. Furthermore, the dictator was threatening that if the parade was not sufficiently impressive he was going to strip the general of his medals and reduce him to the rank of private. But it seemed impossible to have a parade that could be seen throughout the whole country. What should the general do in order to have the parade seen and heard by everyone throughout the whole country?

Solution:

The general, however, knew just what to do. He divided his army up into small groups and dispatched each group to the head of a different road. When all was ready he gave the signal, and each group marched down a different road. Each group continued down its road to the fortress, so that the entire army finally arrived together at the fortress at the same time. In this way, the general was able to have the parade seen and heard through the entire country at once, and thus please the dictator.

The Poker Problem

Three people play a game in which one person loses and two people win each game. The one who loses must double the amount of money that each of the other two players has at that time. The three players agree to play three games. At the end of the three games, each player has lost one game, and each player has eight dollars. What was the original stake of each player?

Solution:

In the third game, one person lost, and two won. The person who lost doubled the money of those who won. Because everyone has ended up with eight dollars, the two people who won the third game must have had only four dollars after two games. Consequently, the person who lost the third game had to pay out eight dollars to the two winners, so the loser of the third game must have had sixteen dollars after two games had been played. Use the same reasoning to find out what happened in the second and the first games.

Appendix I. Testing Analogies

The Radiation Problem

Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?

Solution:

The doctor could direct multiple low-intensity rays toward the tumor simultaneously from different directions, so that the healthy tissue will be left unharmed, but the effects of the low-intensity rays will summate and destroy the tumor.

The Jealous Husbands Problem

Three jealous husbands and their wives having to cross a river at a ferry, find a boat but the boat is so small that it can contain no more than two persons. Find the simplest schedule of crossing that will permit all six people to cross the river so that none of the women shall be left in company with any of the men, unless her husband is present. It is assumed that all passengers on the boat unboard before the next trip, and at least one person has to be in the boat for each crossing.

Solution:

Begin with either one husband and one wife, or two wives, traveling to the right bank. Then leave one wife and have either the husband or the second wife travel back across to the left bank and pick up the remaining wife (leaving the husband on the other side). The two wives would then travel to the right bank dropping off one wife for a total of two wives on the right bank. The third wife would travel to the left bank, get out of the boat and let two husbands travel back to the right bank. One husband would stay at the right bank and one wife already on the right bank would travel back with the second husband. Now, you have one husband and one wife at the right bank. Next, the wife would get out of the boat and let the two husbands return to the right bank. These husbands would deboat and let the wife on the right bank take the boat back to the left bank. Now there are three husbands on the right bank and three wives on the left bank. Two wives would travel to the right bank, one would stay and either the second would travel back to the left bank or one husband would travel to the left bank and pick up the remaining wife and travel back to the right bank. Now there are three wives and three husbands on the right bank.

Appendix J. Relationship Between Problems and Analogues

Correspondence A

Cannibal problem

Initial state

Goal: transport each cannibal and missionary to the other side of the river

Resources: boat

Constraint: at least one person must be in the boat at all times, boat can only carry two people at a time, cannibals cannot outnumber missionaries or they will eat the missionaries

Solution plan: vary the number of missionaries and cannibals such that cannibals will not outnumber missionaries

Outcome: each cannibal and missionary is transported to the other side of the river

Book Burners problem

Initial state

Goal: transport each book burner and book lover to the other side of the river

Resources: boat

Constraint: at least one person must be in the boat at all times, boat can only carry two people at a time, book burners cannot outnumber book lovers or the book burners will burn the books

Solution plan: vary the number of book burners and book lovers such that book burners will not outnumber book lovers

Outcome: each book burner and book lover is transported to the other side of the river and the books are still intact

Correspondence B

Cannibal problem

Initial state

Goal: transport each cannibal and missionary to the other side of the river

Resources: boat

Constraint: at least one person must be in the boat at all times, boat can only carry two people at a time, cannibals cannot outnumber missionaries or they will eat the missionaries

Solution plan: vary the number of missionaries and cannibals such that cannibals will not outnumber missionaries

Outcome: each cannibal and missionary is transported to the other side of the river

Farmer's Dilemma problem

Initial state

Goal: transport the fox, goose, and corn across the river to the man's house

Resources: boat

Constraint: the boat would only carry two things at a time--the man and one other thing, the fox would eat the goose if left alone with it, the goose would eat the corn if left alone with it

Solution plan: transport the things one at a time in varying orders such that the fox would never be left alone with the goose and the goose would never be left alone with the corn

Outcome: the fox, goose, and corn are all transported across the river to the man's house

Correspondence C

Farmer's Dilemma problem

Initial state

Goal: transport the fox, goose, and corn across the river to the man's house

Resources: boat

Constraint: the boat would only carry two things at a time--the man and one other thing, the fox would eat the goose if left alone with it, the goose would eat the corn if left alone with it

Solution plan: transport the things one at a time in varying orders such that the fox would never be left alone with the goose and the goose would never be left alone with the corn

Outcome: the fox, goose, and corn are all transported across the river to the man's house

Men and boys problem

Initial state

Goal: transport each man and boy to the other side of the river

Resources: boat

Constraint: the boat can carry either one man or two boys at a time

Solution plan: vary the number of trips the men and boys make across the river such that there are never two men, or more or less than two boys attempting to make a crossing at any one time

Outcome: each man and boy is transported to the other side of the river