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A COMPARISON OF TWO METHODS OF TEACHING A REMEDIAL
MATHEMATICS COURSE AT THE COMMUNITY COLLEGE

by

James C. Olsen

A dissertation submitted in partial fulfillment
of the requirements for the degree

of

DOCTOR OF EDUCATION

in

Curriculum Development and Supervision

Approved:

UTAH STATE UNIVERSITY
Logan, Utah

1973

No man can reveal to you aught but that which
already lies half asleep in the dawning of your know-
ledge.

The teacher who walks in the shadow of the
temple, among his followers, gives not of his wisdom
but rather of his faith and his lovingness.

If he is indeed wise he does not bid you enter
the house of his wisdom, but rather leads you to the
threshold of your own mind.

--Kahlil Gibran, 1923

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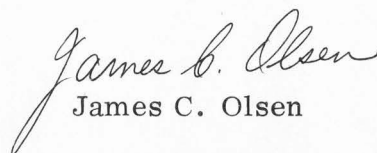

James C. Olsen

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ABSTRACT

A Comparison of Two Methods of Teaching a Remedial
Mathematics Course at the Community College

by

James C. Olsen, Doctor of Education

Utah State University, 1973

Major Professor: Dr. Ross R. Allen

Department: Secondary Education

Problem

This study was concerned with the effectiveness of two different programs for teaching remedial mathematics to community college students. An individualized instruction program, making use of independent study and the small group setting, was compared with the traditional lecture-textbook instruction program to determine if there existed significant differences in (a) the rate of attrition, and (b) mathematics performance. A secondary objective was to determine if significant differences existed between students enrolled in remedial mathematics classes at 9 AM and those enrolled at 12 Noon.

Method

The target population consisted of all students in fourteen community colleges in the Los Angeles area. The sample consisted of all students enrolled in

the 9 AM and 12 Noon elementary algebra classes at Rio Hondo College during the fall semester, 1972. One experimental group and one control group were randomly formed from all the students enrolled at 9 AM. The same procedure was used at 12 Noon.

The posttest-only control group design was utilized in the study. A chi-square test was used to determine if frequency of dropout is associated with being in the control or experimental groups. Mean scores on a mathematics posttest of achievement were analyzed by two-way analysis of covariance to determine if mathematics performance is associated with being in the control or experimental groups, and if mathematics performance is associated with being in the 9 AM or 12 Noon classes.

Results

According to the study, there appeared to be no significant differences, at the .05 level, in either the rate of attrition or mathematics performance of community college students taught remedial mathematics under an individualized instruction program as compared with those students taught under the traditional lecture-textbook instruction program. Also, there appeared to be no significant difference, at the .05 level, in the mathematics performance of community college students enrolled in 9 AM remedial mathematics classes compared to students enrolled in 12 Noon remedial mathematics classes.

Conclusions

As a result of the findings of this study, the following conclusions were drawn.

1. Community college remedial mathematics classes using an individualized instruction program as described in this study did not have significantly fewer dropouts than those classes using the traditional lecture-textbook approach.
2. Community college students enrolled in remedial mathematics courses taught under an individualized instruction program as described in this study did not receive significantly higher scores on a mathematics posttest of achievement than those students taught under the traditional lecture-textbook instruction program.
3. There was no significant difference in mathematics performance of community college students enrolled in remedial mathematics courses taught at 9 AM and those taught at 12 Noon.

The study, even though the results did not show significant differences at the .05 level, proved to be very helpful in planning for further mathematics classes at the community college.

(82 pages)

CHAPTER I

BACKGROUND AND PROCEDURES

Problem Statement

In contemporary society, people of all ages and from all walks of life are attending colleges and universities. Many people are enrolled in degree programs because a college degree has been a criterion of employment for many vocations. Many people are enrolling in courses at the college level to broaden their own backgrounds in certain subject areas. Some people are taking courses just for their own enjoyment. The United States Office of Education (1969) estimated that the number of students enrolled in colleges and universities will increase 169 percent from 1960 to 1977. The community and/or two-year colleges will enroll a large percentage of these people. According to Mensel (1971), the community colleges of California now enroll roughly 10 percent of all the students that enroll in colleges and universities in the United States.

Roueche (1967) reported that 91 percent of the nation's community colleges follow an "open door" admissions policy for all high school graduates and for all persons 18 years of age and older who can profit from the instruction. The "open door" policy is not only increasing enrollments at the community college, but it also is presenting these institutions with an increased number of students whose mathematical backgrounds are deficient. Morgan (1970) estimated that about 50 percent of the students entering the community college fall in this

category. As a result, anywhere from 50 percent to 75 percent of all mathematics classes at the community college are remedial in nature.

According to statistics gathered from ten community colleges in the Southern California area, 25 percent to 60 percent of all the students enrolled in remedial mathematics courses drop these courses before the semester has concluded (Mathematics departments, personal communication, April, 1971). College mathematics instructors and college administrators have suggested that research be initiated to see if ways can be found to alleviate the attrition problem in the remedial mathematics classes. At the 1971 National Council of Teachers of Mathematics Annual Meeting in Anaheim, California, a group of community college mathematics instructors and administrators from the Southern California area voiced their feelings that curriculum revision might possibly be the answer to the attrition problem.

Curriculum revision has been proposed as a potential solution to attrition in higher education by many educators. Williams (1963), commented in regard to cases where students listed lack of success as the cause for their dropping out of school, "... can the failure be attributed to the students' inadequacy or to the inadequacy of the school program?" Edgerton and Sylvester (1966) stated that the problem was that of making the programs fit the trainees rather than trying to select trainees to fit the programs. Canfield (1967) concurred and stated:

Schools are much like hospitals--both being characterized by the diagnosis, treatment, and evaluation of human needs, one for health and the other for education. Schools differ from hospitals in that every student gets essentially the same treatment

method (lecture-textbook), and treatment failures are explained largely on the basis of student (patient) inadequacies. This is a little like saying that our treatments are fine but we keep getting the wrong patients (students). If medical men had failed to persistently study and evaluate their treatments for disease, 'bleeding' could have persisted as a standard treatment routine.

With specific regard to community college programs in mathematics, both Scharf (1964) and Ragland (1969) have indicated that a need for curriculum reform and innovation exists.

Although substantially little curricular change has taken place at the community and/or two-year college level, there is evidence that an individualized instruction program may be a method of instruction that will help to meet the needs of community college students (Roueché and McFarlane, 1970; Shanberg, 1971).

One component of an individualized instruction program that has been supported by many researchers is independent study (Baskin, 1967; Brown, 1968; Dearing, 1965; Hatch, 1964). Another component of an individualized instruction program that has received support is the small group setting (Anderson, 1961; Flanders, 1960; Frandsen, 1969; Hudgins, 1960; Yaker, 1955). Although there is evidence to suggest that both independent study and small group settings are effective teaching methods, very little has been written regarding their simultaneous use in an individualized instruction program.

The problem is, then, the lack of research testing the effectiveness of the simultaneous use of independent study and the small group setting, as compared with the traditional lecture-textbook instruction program, as a means of

reducing the attrition rate of students in remedial mathematics classes at the community college level.

Related Research

Research regarding dropouts and methods of decreasing the attrition rate of college students has been conducted for a number of years. One of the earliest studies was made by Thorndike in 1905. Since that time the range in age and ability of college students has increased along with enrollments; and the ever-increasing number of articles in journals has indicated a growing concern over those who withdraw before completing their education. Although most of the articles have dealt with the high school level, research on the college level is available. Fishman and Pasanella (1960), in reviewing published and unpublished research between 1948 and 1958 on factors related to success in college, found a total of 580 studies. Iffert (1958), in a similar review of research on student withdrawal from college, listed a total of 156 related references, 127 of which had appeared between 1950 and 1957.

In spite of the many studies, the need for more research has been emphasized. Summerskill (1962) stated that economic and administrative approaches to the problem had not explained the causes of attrition and had not significantly reduced dropout rates. Miller (1964), Spector (1966), Roueche (1967), and Demos (1968) echoed the same complaint. Roueche and Boggs (1968) pointed out

that research studies of the community college dropout were urgently needed. They particularly called for data to explain dropouts and formulae for reducing attrition rates. This investigation will attempt to provide one such formula.

This review of selected literature, then, will focus on findings related to the development and validation of an individualized instruction program as an alternative to the traditional instruction program that is used for teaching remedial mathematics at many community colleges. First, the focus will be on curriculum revision for the dropout, and second, on methods of instruction including independent study, the small group setting, and the lecture-textbook approach.

Curriculum revision for the dropout

Researchers have made studies for the purpose of finding ways to help the school adjust to handle the student attrition problem. Many of the findings related to the schools' failure to adapt to the needs of the students. Segel and Schwarm (1957) and Kennedy (1957) stated that lack of adaption to the students' needs resulted in students becoming disinterested in school, failing, and withdrawing from school. Suczek and Albert (1966) and Havighurst and Stiles (1961) concurred that revision of college programs to meet the varied needs of potential dropouts would reduce the frequency of premature terminations.

Studies regarding curriculum and the dropout at the community college level have indicated that community colleges with "open door" admissions policies must either (a) design and implement curricula which will meet the

needs of students whose high school records give evidence of low achievement, or (b) see large numbers of students leave formal education in defeat. According to Dahlin (1967), research indicates that the curricular offerings of the community college must be extended for the lower end of the ability range if all students' needs and interests are to be met and full student potential is to be realized.

Hence, evidence does seem to indicate that curricular innovations are feasible as a means of reducing the attrition rate of community college students.

Methods of instruction

Methods of instruction for optimal learning is an age old topic of discussion. Students, teachers, administrators, boards of education, and even communities have often debated questions like, "What is the 'best' way to teach?" Research indicates that possibly there is no 'best' way for all situations.

Hatch and Bennett (1960), in a review of literature on the effectiveness of college teachers, concluded that no one method has been demonstrated to produce more or better learning than any other. McKeachie (1963) on the other hand, argued that the appropriate teaching method depends upon what particular goals are sought.

According to McKeachie, research studies regarding teaching methods have been conducted over an extensive period of time. Some of these studies have centered on topics such as lecture methods versus discussion method,

distribution of lecture and discussion time, lecture versus automation, and student-centered versus instructor-centered teaching to name a few. An exhaustive review of literature involving all of these various teaching methods was not attempted. The review was centered around research involving independent study and the use of small group discussions, as compared to the lecture-textbook approach.

Independent study. Independent study is a term normally used to describe programs and practices that emphasize student responsibility for their own education. Independent study may occur in a variety of ways. Dearing (1965) cited a number of forms of independent study and then stated, "... it is important that in any discussion of independent study, the nature of the independence under consideration be made explicit."

The idea that colleges should employ programs of independent study is not new. A 1957 report by Bonthius, Davis, and Drushal described a total of 334 independent study programs in 256 institutions. Nearly all of these programs, however, were designed for honors or superior students. But as Baskin (1967) stated:

Independent study as a concept has long been regarded as the prerogative of the superior student in honors or tutorial courses. In its growing use, it is available to all students, and, further, it is available at the beginning rather than the close of the student's college career.

Dearing (1965) concurred and further indicated that independent study is needed for all college students in a particular course rather than reserving these procedures for the superior or abler students only.

Brown (1968) provided a list of 17 arguments in support of independent study. Furthermore, in a list of 21 standards of quality of teaching in colleges (Hatch, 1964), item number six stated, "Quality may be indicated in colleges that are most successful in involving their students in independent study."

Bloom (1968) stated that about 90 percent of all students are between the extremes of those who learn very quickly and those who learn very slowly. If members of this large group are given sufficient time and appropriate instruction, they can learn a subject to a high level of mastery. In fact, Bloom stated that, "If it were not so costly in human resources, providing a good tutor for each student would be the ideal strategy."

John Carroll (1963) has challenged contemporary educators with the view that aptitude is the amount of time required by the learner to attain mastery of a learning task. Gagné (1962) has demonstrated that once a learning unit has begun, it is not the student's general aptitude that predicts best how well he will achieve any particular step. The best predictor is the extent to which he has attained the prerequisites to that step. Thus, the student's readiness and aptitude come to coincide. Aptitude becomes a matter of how long it takes to achieve readiness, rather than whether a student is or ever can become ready.

Traditional concepts treat time as a constant while allowing achievement to act as a variable. Not only is variability in achievement expected, but welcomed, for it serves as the basis for the grading systems. Then variability is attributed to those unchangeable student characteristics--aptitude and motivation. If education's purpose is to see that a certain level of competence is

achieved by each learner, why not set levels of achievement as constant and let time act as a variable?

Skinner (1961) said that by trying to teach more than one student at once we harm both fast and slow learners. Skinner believed that it is the slow learner who suffers the more disastrous consequences. He also pointed out that a student who has not mastered a first lesson is less able to master a second and that a small difference in speed may cumulatively exaggerate the learner's shortcomings and eventually result in an immense difference in comprehension.

Research (Smith, 1939) indicates that students differ in the rate at which they learn. In fact, evidence (Atkinson, 1967) shows that in studies of programs where students can proceed at their own rate, the ratio of time taken by faster learning students to learn given material to the time taken by slower students is about one to five. Herriot (1967) reported that "slow learning" students achieve as well in SMSG mathematics as average students when they are allowed substantially more time than is usually allotted to the mathematics curriculum.

Over a four-year period, Thompson (1941) carried out a series of longitudinal studies that focused on arithmetic and algebra learning at the high school level. Each student in the experimental group worked alone. A test on the material of the unit being studied was administered to each student upon his request. If the student passed the test, he proceeded to the next unit. Failure to pass the test was followed by remedial work and retesting until the unit was

mastered. Thompson concluded that the use of independent study, diagnostic testing, and remediation was a very effective way to teach mathematics. In one study over a ten-week period, the experimental group gained 1.41 years in arithmetic achievement while the control group gained just 0.40 years.

Suppes (1966) has been an assiduous investigator of the inherent differences which exist between individuals in their capacity to learn mathematics. Suppes expressed the belief that the more individualized classroom teaching becomes, the more it will facilitate a speed of learning and a depth of understanding that now seems impossible to achieve.

Evidence, then, does seem to indicate that a successful independent study program of instruction is feasible.

Small group setting. Because the teacher has for so long been considered the generating force of education, we often forget the fact that students learn from each other. Students at almost any age tend to associate and communicate more with each other than with the teacher. This relationship might be used very effectively in teaching in small group settings.

There is much research on group learning. Most of the conclusions from these studies indicate that working in groups is not only feasible but also desirable in both the cognitive and affective domains of learning. Flanders (1960) suggested that if the class is divided into groups small enough to permit spontaneous contact within the groups and freedom to contact other groups, certain types of social-educational objectives can be attained.

The actual number of students per group may be varied depending upon a number of variables. However, groups of three or four have been mentioned by many researchers as a desirable number for effective teamwork. Anderson (1961) reported that groups of three members attained a higher level of achievement than groups of two members, and that groups of two members attained a higher level of achievement than individuals in solving a problem involving anagrams.

With effective interaction within a team, its composite output should exceed that of both the 'average' and the 'best' achievement of comparable individuals working separately. On this criterion, the results have sometimes fallen short of those expected. On a complex verbal judgement problem, Hall, Mouton, and Blake (1963) found that teams were more accurate than the average but not the best of comparable individuals.

Advantages have, however, often been found for teamwork. In research studies regarding groups versus individuals on problem solving tasks, Hudgins (1960), Watson (1928), and Yuker (1955) found that teams of students exceeded the average performance of comparable individuals. Also, Frandsen (1969) indicated that the composite achievement of teams of students will exceed the average performance of comparable students working separately. Moreover, with more practice in teamwork, the advantage of teams over individuals tends to increase. But the advantage of teamwork falls short of the level which interaction among diverse participants should make possible. The composite team scores only approximate the best individual in that team.

Shaw (1932) compared the ability of individuals and groups of students in solving complex problems involving a number of sequential steps--all necessary to obtain the correct solution. Some of her findings were: (a) Small groups solved a higher proportion of the problems than did individuals; (b) the group tended to reject incorrect suggestions of its members and detected errors that individuals were unable to detect; and (c) groups did not make errors as soon as the average individual in the sequential reasoning process.

Group study, then, seems to be an effective method of instruction, especially if students can cooperate and help each other without the competition of the ordinary classroom situation. Learning can be turned into a cooperative process where everyone is likely to gain. In these small groups, the more able student has the opportunity to strengthen his own learning while at the same time helping other students. Hence, the small group setting can be a valuable technique in teaching.

Lecture-textbook approach. For many years the lecture approach has been used as a method of instruction in higher education. Rothstein (1966) attempted to justify the lecture approach not only as a method for transmitting knowledge, but also as a means of displaying how the lecturer himself learns.

Rothstein stated:

As long as the college maintains its traditional character, the lecture remains an appropriate vehicle for the demonstration of, and motivation for, learning. For, besides its being so natural to tell others what we know, it is a method of teaching learning.

Marr, Plath, Wakeley, and Wilkins (1960) compared two groups of students enrolled in an introductory psychology class. One group had assigned readings and one question session per week while the other group read the text and attended four lectures per week. The study, previously cited (Marr et al., 1960), indicated that students taught by the lecture approach scored higher on a posttest of achievement than students taught by the non-lecture approach.

In recent years, however, the effectiveness of the lecture approach has been questioned. Some investigators have indicated that significant differences in levels of performance of students taught under the lecture approach and students taught under the non-lecture approach have occurred after some time has elapsed. In comparing lectures and assigned readings with individual programmed instruction, Rawls, Perry, and Timmons (1966) found no significant difference between methods at the end of the course, but six weeks later the level of performance was significantly higher for the group using programmed instruction.

Probably the greatest criticism of the lecture method has been the passive role of the student. Many educators have suggested that student response and participation are necessary for effective learning to take place. Glaser (1961) said, "Response is the primary object of manipulation in instructional technology." Since this response is often not apparent in a lecture approach, some suggestions have been made to foster it. According to Verner and Dickenson (1967), "If the instructional objective clearly indicates the lecture to be the appropriate technique, it can be enhanced by insuring that . . . [it] is augmented by instructional devices and/or techniques which provide for learner participation."

Palmer and Verner (1959) stated:

In the past half century there have been scores of studies concerned with comparing the efficiency of instructional processes. In comparing the lecture with other techniques the majority of the studies have shown that the lecture is equal to or better than other techniques in terms of immediate recall; however, when measuring delayed recall other techniques prove to be superior to the lecture in most cases.

Hence, evidence does exist that seems to indicate that an all lecture approach is not necessarily the 'best' method of instruction.

In brief, then, the review of related research attempted: (a) to show that curriculum revisions are appropriate for reducing the attrition rate of community college students; (b) to show that an independent study program, in which students work at their own speed until mastery, as compared to the lecture approach, is an appropriate strategy for teaching remedial mathematics to community college students; and (c) to show that the small group setting, in which students receive help from each other and the instructor if needed, as compared to the lecture approach, is also an appropriate strategy for teaching remedial mathematics to community college students.

Purpose and Objectives

Purpose

The purpose of this study was to compare the effectiveness of two different curricular programs for teaching remedial mathematics to community

college students. Since the rate of attrition of students from community college classes is a major concern today, this investigation, hopefully, will be used as a basis for further research in developing successful programs of instruction for community colleges.

Objectives

The first objective of this investigation was to determine the effectiveness of an individualized instruction program, as compared with the traditional lecture-textbook instruction program, as a means for reducing the number of students that drop their remedial mathematics classes at Rio Hondo College.

A second objective of this investigation was to determine the effectiveness of an individualized instruction program, as compared with the traditional lecture-textbook instruction program, as a means for increasing mathematics performance in the remedial mathematics classes at Rio Hondo College.

A third objective of this investigation was to determine if there existed any difference in the mathematics achievement of students enrolled in 9 AM remedial mathematics classes compared to students enrolled in 12 Noon remedial mathematics classes at Rio Hondo College.

Hypotheses

In order to meet the above objectives, the following null hypotheses were tested.

Hypothesis 1. There is no significant difference in the number of students that drop out of remedial mathematics classes at the community college using an individualized instruction program and those classes using the traditional lecture-textbook instruction program.

Hypothesis 2. There is no significant difference in the mean scores on a mathematics posttest of achievement between community college students in remedial mathematics classes taught under an individualized instruction program and those students taught under the traditional lecture-textbook instruction program.

Hypothesis 3. There is no significant difference in the mean scores on a mathematics posttest of achievement between community college students in remedial mathematics classes taught at 9 AM and those students taught at 12 Noon.

Procedures

Population and sample

The target population for this study consisted of students enrolled in fourteen community colleges in the Los Angeles area. The fourteen schools were those used by the Rio Hondo College Board of Trustees whenever any comparative studies are to be made. The Board chose the above mentioned schools because the communities served by these schools were very similar to the Rio Hondo College district. The specific group to which this study attempted to generalize

was all the students enrolled in the elementary algebra classes at these fourteen community colleges.

The accessible population for this study consisted of all students enrolled in Algebra 50 (elementary algebra) at Rio Hondo College during the fall semester, 1972. Ten Algebra 50 classes were offered during the fall semester, each consisting of approximately 30 students. The classes were offered throughout the day and evening with two sections at 9 AM and two sections at 12 Noon.

The Dean of Instruction at Rio Hondo College developed the class schedule for the fall semester. The students selected their own classes according to their own needs. The sample for this study, then, consisted of all the students enrolled at 9 AM and 12 Noon. The basic reasons for choosing these hours were: (a) Some common threats to internal validity, such as maturation, may be lessened with an experimental group and a control group at the same hour; and (b) the assignment of subjects to treatment groups should produce more representative sections when 30 subjects are chosen at random from 60 subjects and assigned to a particular group, rather than a given 30 subjects assigned to a particular group.

Campbell and Stanley (1963) noted that the most adequate all-purpose assurance of lack of initial biases between groups is randomization. The proper choice of a design, one in which true randomization is possible, was, then, the next concern.

Research design

The posttest-only control group design (Campbell and Stanley, 1963) was used in this experiment. The posttest-only control group design, according to

Campbell and Stanley, contains effective controls for all eight forms of internal validity--history, maturation, testing, instrumentation, statistical regression, differential selection, experimental mortality, and interactions between these seven effects. The interaction effect of testing and treatment regarding external validity can also be controlled.

The control group was taught by the traditional lecture-textbook approach. The class consisted of four 50-minute sessions per week with attendance optional. All four sessions were devoted to the lecture approach. Written assignments were given daily. Unit tests (see Appendix A) were administered to all students at one prescribed time (at the end of each unit). There was no opportunity for retesting.

The treatment or experimental group was taught using an individualized instruction program that was a composite of teaching strategies. The class consisted of four 50-minute sessions per week with attendance optional. Two sessions per week, usually Tuesdays and Thursdays, depending upon the unit being studied, were devoted to an instructor-student discussion approach. The remaining two sessions per week, usually Mondays and Wednesdays, were devoted to small group study sessions (usually four members per group) and individualized instruction with instructor help being given only if asked for by the students.

The individualized instruction consisted of students working on their assignments without any outside help, students using cassette players and pre-recorded tapes (see Appendix B) as an aid to learning, and students receiving help from the instructor on a one-to-one basis.

At the beginning of each unit the student was given a sheet of performance objectives for that unit (see Appendix C). The student was told that he would be prepared to take a test on the unit when he was able to answer successfully all of the performance objectives. Each student was given the unit tests (see Appendix A) whenever he thought he had mastered the basic concepts and skills of that unit. Remedial work and retesting were used for those students not achieving a minimum standard score of 70 percent on the unit tests.

Instructors were available from 8 AM to 4 PM, Monday through Friday, to aid students with their problems.

During the first class meeting all of the students enrolled at the 9 AM hour randomly signed a numbered roster. The students that signed their name after an odd number (1, 3, 5, ...) were assigned to section I and those that signed their name after an even number (2, 4, 6, ...) were assigned to section II. The same procedure was used to form sections III and IV at 12 Noon.

Two instructors, one the investigator, taught the experimental and control groups. To help control for teacher variables, each instructor taught one experimental and one control group. Time of day variables were controlled by having one experimental group and one control group taught at 9 AM and the other two taught at 12 Noon. To further randomize selection, one instructor drew his assignment by lot from the following list (Note: I and II refer to 9 AM sections, III and IV refer to 12 Noon sections, X refers to the experimental group, and C refers to the control group.)

- (a) I-X, III-C (b) I-X, IV-C (c) I-C, III-X (d) I-C, IV-X
(e) II-X, III-C (f) II-X, IV-C (g) II-C, III-X (h) II-C, IV-X

The assignment of the other instructor was, then, determined by the first assignment so that one experimental group and one control group met each hour.

The next step was to collect the data of the experiment in such a way that a satisfactory degree of reliability and validity could be achieved.

Data and instrumentation

Information relating to the attrition rate and the mathematics performance of individual students was collected by administering a mathematics posttest of achievement (see Appendix D). The posttest used was developed by mathematics professors from Pierce College, Los Angeles, California. The posttest was administered to the students in both the experimental and control groups. The control group was given the posttest as part of their final examination at the end of the semester. Each student in the experimental group was given the posttest as part of his final examination as soon as he completed all of the assigned units of the course.

At the end of the semester, a tally was made of those students that officially enrolled in the class and attended at least twelve class sessions, but failed to take the posttest. These students were called dropouts. Fortunately, all of the students in the experimental classes completed the course prior to the end of the semester.

In judging the relation between test content and the instructional objectives of the course, the members of the mathematics department of Rio Hondo College indicated that the test exhibits exceptional content validity.

Although no coefficients of reliability were available, the producers of the test stated that student achievement was highly consistent (F. Fleming, personal communication, April 5, 1972). The cutoff scores for the grades of the more than 2,000 students (over 80 classes) that have taken the examination have never fluctuated more than one point. The Kuder-Richardson Formula 21 produced an estimated reliability coefficient of 0.86 for this sample.

Another important ingredient in an experiment of this nature is the content and process by which the data of the study is to be analyzed.

Analysis

Two sources of data were analyzed to determine the effectiveness of the individualized instruction program.

- (a) The official enrollment sheets and the list of students not taking the posttest were used regarding the attrition rate.
- (b) Posttest mathematics scores were used regarding mathematics performance.

The data was analyzed in the following way:

- (a) The number of students dropping their remedial mathematics classes was totaled for both the control and experimental groups.

A chi-square test was then used to determine if frequency of dropout is associated with being in the control or experimental groups. The test was done separately for the two instructors and the results added together.

- (b) The mean scores on the mathematics posttest were determined for the control and experimental groups, and for the 9 AM and 12 Noon groups. These mean scores were then analyzed by two-way analysis of covariance, using the students' School and College Ability Test (SCAT) mathematics scores as a covariate, to determine if mathematics performance is associated with being in the control or experimental groups, and if mathematics performance is associated with being in the 9 AM or 12 Noon classes.

The background of the study and the procedures used in the study have now been formulated. This leads, then, to the findings of the study.

CHAPTER II

RESULTS

Results Relative to Objective One

The first objective of this investigation was to determine the effectiveness of an individualized instruction program, as compared with the traditional lecture-textbook instruction program, as a means for reducing the number of students that drop their remedial mathematics classes at Rio Hondo College. To meet this objective, the following hypothesis was tested.

Hypothesis 1: There is no significant difference in the number of students that drop out of remedial mathematics classes at the community college using an individualized instruction program and those classes using the traditional lecture-textbook instruction program.

To test this hypothesis, a tally was made, at the end of the semester, of those students that were officially enrolled in the class and attended at least twelve class sessions, but failed to take the mathematics posttest of achievement given as part of the final examination (Table 1).

Table 1 presents data comprised of paired observations on two nominal variables (methods of instruction and attrition). A table of this kind is called a 2 X 2, or fourfold, contingency table. With such tables, chi-square provides an appropriate test of independence (Ferguson, 1966). Table 2, then, presents a chi-square analysis for the data presented in Table 1.

Table 1. Number of students, by instructor, in the control and experimental groups that either dropped or tested

	<u>Instructor A</u>			<u>Instructor B</u>			
	Dropped	Tested	Total	Dropped	Tested	Total	
Control	18	11	29	Control	18	11	29
Experimental	15	7	22	Experimental	13	14	27
Total	33	18	51	Total	31	25	56

Table 2. Chi-square analysis for methods of instruction compared to attrition

Source	Degrees of freedom	Chi-square
Instructor A	1	0.20
Instructor B	1	1.10
Total	2	1.30

Using 1 degree of freedom, a chi-square value of 3.84 was required for significance at the .05 level. Using 2 degrees of freedom, a chi-square value of 5.99 was required for significance at the .05 level. Since the analysis of data yielded chi-square values less than those required for significance at the .05

level, the null hypothesis cannot be rejected from this data. Hence, according to this study, the individualized instruction program, as defined by the researcher, does not appear to be significantly more effective, as a means for reducing the number of students that drop their remedial mathematics classes at the community college, than the traditional lecture-textbook instruction program.

Results Relative to Objectives Two and Three

A second objective of this investigation was to determine the effectiveness of an individualized instruction program, as compared with the traditional lecture-textbook instruction program, as a means for increasing mathematics performance in the remedial mathematics classes at Rio Hondo College. To meet this objective, the following hypothesis was tested.

Hypothesis 2: There is no significant difference in the mean scores on a mathematics posttest of achievement between community college students in remedial mathematics classes taught under an individualized instruction program and those students taught under the traditional lecture-textbook instruction program.

A third objective of this investigation was to determine if there existed any difference in the mathematics achievement of students enrolled in 9 AM remedial mathematics classes compared to students in 12 Noon remedial mathematics classes at Rio Hondo College. To meet this objective, the following hypothesis was tested.

Hypothesis 3: There is no significant difference in the mean scores on a mathematics posttest of achievement between community college students in remedial mathematics classes taught at 9 AM and those students taught at 12 Noon.

To test these hypotheses, a mathematics posttest of achievement was administered to each of the students in both the control and experimental groups. The students in the control group were given the posttest as part of their final examination at the end of the semester. The students in the experimental group were given the posttest as part of their final examination as soon as they completed all of the required units of the course. Although some students in the experimental group completed all of the units prior to the final examination, they elected to take the posttest with the other members of their class. Two or three students from the experimental group were working at a pace slower than that needed to complete the course by the end of the semester. With approximately three weeks remaining in the semester, these students were informed, again, that they could take the posttest whenever they completed all of the unit work. However, if these students wanted to come in for extra help on a one-to-one basis with the instructor, they might be able to complete all of the course work and take the posttest with the other members of their class. This extra help was provided for in the experimental program by the researcher's definition of individualized instruction. All of the students mentioned elected to take advantage of the extra help and were able, then, to take the posttest with the other members of their group.

In perusing hypotheses 2 and 3, it became apparent that a 2 X 2, or two-way factorial experiment existed. Analysis of variance and analysis of covariance are both statistical methods that can be used to analyze the data in factorial experiments. However, the choice of which method to use often depends upon whether complete randomness of selection of subjects for the experiment can be achieved. Analysis of variance is an appropriate method when complete randomness in the selection of subjects is possible. In the researcher's experiment, the students selected their own classes according to their own needs; hence, complete randomness was not achieved between the 9 AM group and the 12 Noon group. To try to control or adjust for the effects of the time variable, and permit, thereby, a valid evaluation of the outcome of the experiment, the investigator opted to use analysis of covariance.

The choice of a covariate common to each subject in the experiment was quite difficult to make. In the community college, with its "open door" policy, very little background information is available regarding each student. Fortunately, a School and College Ability (SCAT) mathematics score, or its equivalent, was available, and, hence, was used as a covariate.

In analyzing the data of the experiment, then, the mean scores, adjusted on the posttest, were determined (Table 3). Although apparent differences existed in the adjusted mean scores, especially between the 9 AM and 12 Noon groups, a complete analysis of covariance was completed to determine if the differences were significant (Table 4).

Table 3. Adjusted mean scores on the posttest

Time of day class offered	Method of instruction		Row means
	Control	Experimental	
9 AM	36.14	37.75	36.95
12 Noon	39.47	40.59	40.03
Column means	37.81	39.17	

Table 4. Analysis of covariance for time of day class is offered compared with the method of instruction

Source of variance	Degrees of freedom	Mean squares	F ratios
Method of instruction	1	18.00	0.40
Time of day	1	94.87	2.09
Interaction	1	0.56	0.01
Experimental error	38	45.34	

Regarding hypothesis 2, using 1 degree of freedom for method of instruction and 38 degrees of freedom for experimental error, an F ratio of 4.10 was required for significance at the .05 level. Since the analysis of the data yielded

an F ratio less than that required for significance at the .05 level, the null hypothesis cannot be rejected from this data. Hence, according to this study, the individualized instruction program, as defined by the researcher, does not appear to be significantly more effective, as a means for increasing mathematics performance in the remedial mathematics classes at the community college, than the traditional lecture-textbook instruction program.

Regarding hypothesis 3, using 1 degree of freedom for time of day and 38 degrees of freedom for experimental error, an F ratio of 4.10 was required for significance at the .05 level. Since the analysis of the data yielded an F ratio less than that required for significance at the .05 level, the null hypothesis cannot be rejected from this data. Hence, according to this study, there appears to be no significant difference in mathematics achievement of students enrolled in 9 AM remedial mathematics classes compared to students enrolled in 12 Noon remedial mathematics classes at the community college.

According to this study, then, there appears to be no significant differences, at the .05 level, in either the rate of attrition or mathematics performance of community college students taught remedial mathematics under an individualized instruction program as compared with those students taught remedial mathematics under the traditional lecture-textbook instruction program. Also, there appears to be no significant difference, at the .05 level, in the mathematics performance of community college students enrolled in 9 AM remedial mathematics classes. However, some interesting observations, opinions, and suggestions, some of which are discussed in Chapter IV, emerged as a result of this study.

CHAPTER III
CONCLUSIONS AND RECOMMENDATIONS

According to the findings of this particular study, it seems that the following conclusions can reasonably be drawn.

1. Community college remedial mathematics classes using an individualized instruction program as described in this study did not have significantly fewer dropouts than those classes using the traditional lecture-textbook approach.
2. Community college students enrolled in remedial mathematics courses taught under an individualized instruction program as described in this study did not receive significantly higher scores on a mathematics posttest of achievement than those students taught under the traditional lecture-textbook instruction program.
3. There was no significant difference in mathematics performance of community college students enrolled in remedial mathematics courses taught at 9 AM and those taught at 12 Noon.

After reading the conclusions, however, the researcher hopes that the individualized instruction program as described in this study would not be completely discarded. With some modifications it seems as if the experimental

program could become very operative. A brief discussion of such revisions is presented in the next chapter.

During the course of this study valuable information regarding future innovations and revisions was gathered in discussions with the other instructor involved in the study and with many of the students in the experimental group. A brief summary of these deliberations is presented in the next chapter. However, as a result of these discussions and the study in general, the following recommendations seem appropriate.

1. There is a need for more research regarding what influences students to drop out of community colleges. Then, specifically, why do they drop out of remedial mathematics classes? Is the method of instruction actually a contributing factor as to whether a student does or does not drop out? Answers to these and similar questions, hopefully can be attained through further research.
2. There is a need for more research regarding small group learning. What methods should be used to facilitate optimum learning in small group settings? How should the groups be organized? How active a role should the instructor take in the group learning process? How structured should the sessions be in the group learning process? These, also, are a few of the questions regarding group learning that need answers.

Again, the fact that significant differences did not exist between the experimental groups and the control groups does not mean that further research should not be initiated regarding improvement of methods of instruction with respect to community college remedial mathematics programs. On the contrary, more research is needed in the near future, since the attrition rate has not been appreciably reduced and mathematics performance has not been significantly increased.

This leads, then, to a final discussion, which includes some interesting observations, opinions, and suggestions made by the different subjects involved with the study. It is hoped that this information can be utilized in further research toward the development of successful programs of instruction for teaching remedial mathematics to community college students.

CHAPTER IV

DISCUSSION

The study, even though the results did not show significant differences at the .05 level, proved to be very helpful in planning for further mathematics classes at the community college.

The researcher gathered information regarding the experiment through discussions with the other instructor involved in the study and with many of the students in the experimental group (both dropouts and nondropouts). Many interesting and valuable observations, opinions, and suggestions were presented--some of which follow.

The students in the experimental group, for the most part, had no previous experience in an individualized instruction program; hence, some students seemed unable to function effectively in a semistructured approach to learning. Some of the students seemed to lack the self-discipline needed for an independent study program of this type. They often did not attend class. Since no specific daily assignments were due, they tended to procrastinate. The less capable students in this situation soon became discouraged and dropped out.

Some of the students in the experimental group seemed to lack self-direction. This was apparent on group session days when some of the groups were unable to function effectively.

A few of the more capable students often elected not to attend class on group session days since they already understood the material being covered. This limited the number of so-called "helpers". Some of the more capable students hesitated to give help unless asked, and some of the less capable students hesitated to ask for help.

On a more positive side, most of the students in the experimental groups, that completed the course, found the individualized instruction program to be extremely favorable as a method for learning mathematics. A number of the students that dropped out of the experimental groups also commented that the reasons for dropping had nothing to do with the method of instruction. In fact, many of these students said this was the first time they had ever enjoyed studying mathematics, and they planned to enroll in the same class next semester. This response was seldom given by those dropping out of the control groups. The students also requested that the same type of program be initiated and maintained in other mathematics courses at the community college. They liked the idea of working problems together where cooperation and immediate reinforcement can be achieved. Instruction on a one-to-one basis was extremely helpful for many students also.

Probably the one item that was mentioned as being the most important by most of the students was the performance objective sheets distributed at the beginning of each unit. The students considered the objective sheets invaluable, as the performance objectives gave the students a sense of direction.

The investigator talked with students from both the control and experimental groups that enrolled in the next level mathematics course during the following semester. The students from the experimental group expressed the feeling that they seemed to have retained so much of the material from the elementary algebra course. Learning mathematics seemed to be much easier for them as a result of their training in the experimental group.

As an outgrowth of the observations and opinions expressed by the researcher, his colleague, and the students, a number of suggestions regarding future mathematics instruction have been formulated.

One such suggestion was that more research be initiated regarding effective methods of group instruction--mainly with respect to organization and motivation of the students toward small group learning. At the outset of the course, the instructor should play a very active role in the motivation and organization of the group sessions, becoming more passive as the groups begin to function more effectively.

Specific individuals should be assigned to the groups at the beginning of the course. Group membership should be changed weekly for the first five to seven weeks so that the students can become acquainted with numerous members of the class. After approximately seven weeks, groups can be formed according to the sections on which they are presently working. It would be advisable to try to have at least one of the more capable students in each group.

In the experimental classes students were completely free to do what they wanted within groups. It was suggested that the group sessions be more structured. A specific group project, in the form of a handout sheet that was to be completed and returned by each group during each group session, could be assigned. This, also, should help to lessen the students' need for self-direction.

Distribution of performance objective sheets to all of the students at the beginning of each unit should be continued. As previously mentioned, the students in the experimental groups considered the performance objective sheets to be their primary aid in learning mathematics.

Many of the observations, opinions, and suggestions presented have already been implemented into the mathematics curriculum at the community college. During the spring semester, 1973, the researcher followed up this study by testing some of these suggestions on a college arithmetic class, two elementary algebra classes, and two calculus classes. Some of the resulting experiences are outlined below.

1. In a calculus class, a set of unit performance objectives were presented to each student about one week prior to the unit examination. The class session prior to the unit examination was then devoted to this objective sheet. The students worked together in small groups during the first part of the hour. During the latter part of the hour, the entire class, by groups, discussed

the objectives. The students involved in the class voiced their approval of this approach time and time again. Class attendance at these sessions was extremely high.

2. In an elementary algebra class specific groups were formed early in the semester. The group membership was changed often so that students became acquainted with other students in the class quite readily. On group sessions days, a specific handout sheet, with problems from the unit being discussed, was given to each group. The assignment was to be completed and handed in by each group during the class session. The students involved in this technique were very enthusiastic about it.
3. In both the elementary algebra class and the calculus class mentioned above, the attrition rate was approximately 10 percent less than in the other similar classes in which the particular technique of instruction was not utilized. Also the mathematics achievement of the students in these classes was significantly higher than of those students not having been given the treatment. This does not mean that the results are due, necessarily, to the treatment; however, it tends to indicate that.

Hopefully, then, further research can be initiated, with the help of this study, toward the development of successful instruction programs for teaching remedial mathematics to community college students.

This particular study fortified an age old conviction that learning is truly an individualized experience. People learn in many different ways and at varying rates of speed. At the community college, then, provisions should be made for these individual differences whenever possible.

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APPENDIXES

Appendix A

Unit Tests

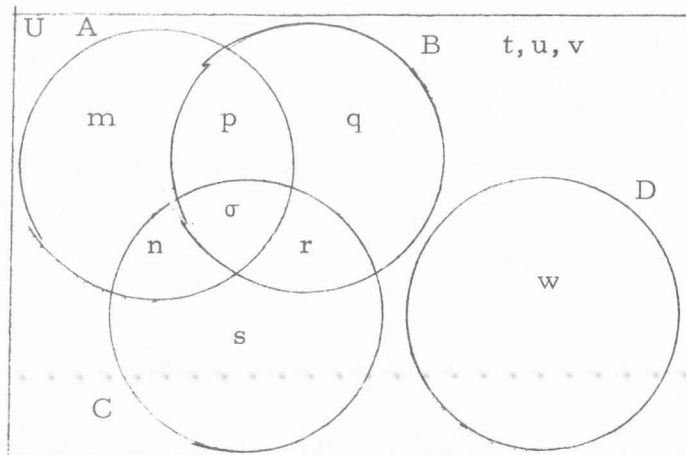
3 points per question or parts thereof.

Let $U = \{a, b, c, d, e\}$, $A = \{c, d\}$, $B = \{a, c, d\}$, and $C = \{a, e\}$

- ① List all subsets of C.
- ② Replace each comma with \in or \subset . (a) d, B (b) A, B
- ③ Replace the comma with \cup or \cap . (a) $B, C = \{a, c, d, e\}$
(b) $A, C = \phi$
- ④ List the elements in each set. (a) $A \cup C'$ (b) $B \cap C$ (c) $B \cap \phi$

- ⑤ How are S and T related if: (a) $S \cup T = \phi$ (b) $S \cap T = T$
- ⑥ Let U equal any universal set. Complete: (a) $\phi' \cup \phi = \underline{\hspace{2cm}}$
(b) $U' \cap U = \underline{\hspace{2cm}}$

Given:



- ⑦ List the elements of the indicated set: $(A \cup B \cup C)'$

(8) Assume U is the set of all females in a given class; A = set of all females with blue eyes; B = set of all females with blond hair; C = set of all females under 5 ft. 3 in. tall; and D = set of all females over 5 ft. 8 in. tall.

List all the characteristics that you know regarding:

(a) female t

(b) female r

(9) Apply the distributive property and rewrite without parentheses.

$$2(a + 6)$$

(10) Specify the cardinality of: $\{3, 4, 5, +, -, \div\}$

(11) Replace the comma with $<$, $=$, $>$: 6, 3

(12) List the numbers in the following set: $\{x \mid 2 \leq x \leq 5, x \in W\}$

(13) What is the basic numeral for $\frac{6}{0}$?

(14) Write 60 in prime factor form.

(15) Find the solution set; assume $x \in W$. (a) $x + 2 = 8$ (b) $3 \cdot x = 8$

(16) List the elements of the set: $\{x \mid 0 < x < 4, x \in W\} \cap \{y \mid y \geq 2, y \in N\}$.

(17) If $x, y \in W$, how are x and y related if $x - y$ equals a natural number?

(18) Graph the following on a number line: $\{x \mid 3 \leq x < 6, x \in W\}$

(19) Is the following set closed under addition? $\{1, 3, 5, \dots\}$. If

your answer is yes, give an example; if it is no, give a counterexample.

(20) Same question as No. 19 except is it closed under multiplication?

(21) List the prime numbers between 6 and 14.

(22) Given: $A = \{13 - 2k \mid 1 < k \leq 3, k \in N\}$. List the elements of the set.

10 pts. (23) I'm as tired of making this exam as you are of taking it, so have

10 points on me. Question: Why are you taking Algebra 50?

Algebra 50 - Test - Unit 2 - Form A

3 pts per question
plus 1 point free.

If $a, b, c \in J$, name the axiom or theorem that justifies each statement.

- ① $a + b = b + a$ ② If $a = b$, then $a \cdot c = b \cdot c$
-

- ③ What is the additive inverse of -13 ?

- ④ Replace the comma with $<$, $=$, or $>$: $0, -(-2)$
-

Do the indicated operations. Simplify and write the answer as a basic numeral, if it exists.

- ⑤ $6 - 4$ ⑥ $-3 - 5$ ⑦ $(-3)(2)$ ⑧ $\frac{0}{6}$
 ⑨ $\frac{-12}{2}$ ⑩ $9 - 6 - 5$ ⑪ $(-3)(-5)$ ⑫ $(-4)(-2)(5)(0)$
-

Find the solution set

- ⑬ $x + 2 = 10, x \in J$ ⑭ $3 \cdot X = 8, x \in J$
 ⑮ What is the identity element for addition in the set of integers?
 ⑯ Write the multiplicative inverse of $\frac{2}{3}$.
 ⑰ If $x \in Q$, for what value(s) of x is $\frac{x+1}{x+1}$ not equal to 1?
-

Write each fraction as a basic numeral in lowest terms.

- ⑱ $\frac{-21}{+7}$ ⑲ $\frac{-3}{0}$
-

Write each expression as a basic fraction or basic numeral, $xy \in J$.

- ⑳ $\frac{X}{7} + \frac{3}{7}$ ㉑ $\frac{2}{3} \div \frac{3}{2}$ ㉒ $\frac{3}{4} - \frac{5}{6} + \frac{2}{9}$
 ㉓ $\frac{X}{2} \div \frac{Y}{2}$ (State all possibilities) ㉔ $\frac{-6}{5} \cdot \frac{-10}{-21}$
-

Find the solution set

(25) $5 \cdot X = 9 \quad x \in \mathbb{Q}$

(26) If $X, Y \in \mathbb{Q}$, under what conditions is $X - Y$ a negative number?

Express as a basic numeral or basic fraction.

(27) $\sqrt{\frac{16}{25}}$

(28) $\frac{\sqrt{16} - \sqrt{36}}{\sqrt{9}}$

(29) Can $|X \cdot Y| < X \cdot Y$? Why or why not?

(30) Under what conditions will $\frac{X}{Y} < 0$, assuming $X, Y \in \mathbb{J}$, and $Y \neq 0$?

State all conditions.

Write the missing numerator so that the second fraction will be equal to the first. $X, Y \neq 0$.

(31) $\frac{7}{8} = \frac{\quad}{24}$

(32) $\frac{5 \cdot X}{4 \cdot Y} = \frac{\quad}{12 \cdot Y}$

(33) Pick any counting number between 0 and 11 (i. e., 1-10 inclusive).

Multiply it by 2. Add 8. Subtract 6. Divide by 2. Subtract the number you started with. Multiply by 25. Take the square root (positive square root). Since your answer is 5, put it down.

Algebra 50 - Test - Unit 3 - Form A

3 pts per question
plus 1 free pt.

$$\text{Let } U = \{-\sqrt{3}, 0, 1/2, -4/5, 7, \sqrt{16}, -4, 5\}$$

① List the members in the subset of U as follows: $\{x \mid x < 0, x \in H\}$

② Draw the graph of: $\{x \mid 1 \leq x < 3, x \in R\}$

③ True or False: $Q \cap H = R$

Find the solution set in R of each equation.

④ $\sqrt{x} = 3$ ⑤ $\sqrt{x} + 2 = 0$ ⑥ $x - \sqrt{25} = 7$

True or False:

Given: $N = \{\text{natural nos.}\}$, $Q = \{\text{rational nos.}\}$, $H = \{\text{irrationals}\}$,
 $W = \{\text{whole nos.}\}$, $J = \{\text{integers}\}$, & $R = \{\text{real nos.}\}$

⑦ $N \subseteq J$ ⑧ $J \cup N = J$ ⑨ $R \cap H = H$ ⑩ $H \cup J = R$

⑪ $Q \cap Q' = \emptyset$

⑫ Solve for x (i. e., find the solution set in R): $\sqrt{x+3} - 4 = 2$

⑬ Write $60x^2y^3$ in completely factored form.

⑭ Give the name and degree of the polynomial: $2x^3 - 3x + 1$

⑮ Write $\frac{3^2 \cdot (2+11)}{3} - \frac{16}{4} + 3$ as a basic numeral.

⑯ If $P(x) = 3x^2 - 2x + 7$, what is $P(-2)$?

⑰ Find the zeros of $(x+1)(x-3)$.

Do the indicated operations; simplify whenever possible.

⑱ $(2x^2 - 6x + 5) - (x^2 - 3x - 3)$ ⑲ $(2x^2y)(-3xy^3)$

⑳ $\frac{-12x^2y^4}{-3xy^4}$, $x, y \neq 0$

- (21) Express .000347 in scientific notation.
- (22) Simplify: $-3 - [-6 - (x + 2)] + (x - 2)$.
- (23) Simplify: $\frac{(2^0)(3^{-1})(4^2)}{(3)(4)}$
- (24) Find the product of: $(2x - 3)(2x - 3)$
- (25) Find the quotient: $\frac{6x^2 - 5x - 6}{3x + 2}$
-

Factor completely, if possible. If not factorable, write prime.

- (26) $2x + 6$ (27) $x^2 + 2x - 8$ (28) $8x^2 - 80x + 72$ (29) $x^2 - 9$
- (30) $x^2 + 4x + 4$ (31) $x^2 + 3$ (32) $9x^2 + 3x + 2$
- (33) $55x^2y + 77xy - 66y$
-

1-14 6 pts. each

15 12 pts.

16 4 pts.

Reduce to lowest terms and state the restrictions on the variables.

① $\frac{x^2 + 5x + 6}{x + 3}$

② $\frac{x^2 - 6x - 7}{x^2 + 2x + 1}$

Write each expression as a single fraction in lowest terms.

③ $\frac{3x^2 - 4}{x^2 - 4} - \frac{x^2}{x^2 - 4} - \frac{4}{x^2 - 4}$

④ $\frac{5x + 10}{2x - 6} \cdot \frac{6x - 18}{3x + 6}$

⑤ $\frac{2}{x^2 - x - 6} + \frac{3}{x^2 - 9}$

⑥ $\frac{x^2 - 4}{x^2 - 5x + 6} \div \frac{x^2 + 3x + 2}{x^2 - 2x - 3}$

⑦ Simplify and reduce to lowest terms:

$$\frac{2 - \frac{2}{x}}{x - \frac{1}{x}}$$

Solve the following equations, i. e. find the solution set, over the set of reals.

⑧ $3(2x + 5) = 21$

⑨ $x - 6 = 1 - 4(x - 2)$

⑩ $\frac{3}{x} + \frac{5}{x} = \frac{-2}{3}$

⑪ $\frac{3x}{4} - \frac{3x + 5}{6} = 1$

⑫ Solve for x: $3x = ax - 4$

⑬ $\frac{2x + 1}{5} \leq 1$, $x \in \mathbb{R}$ solve the inequality.

⑭ Graph the solution set of the inequality of problem 13.

(15) How many quarts of a 30% salt solution must be added to 12 qts of a 10% salt solution to obtain a 15% salt solution? Show all work.

(16) Have 4 free points - I can't remember the question, but the answer is "yes."

Algebra 50 - Test - Unit 5 - Form A

Given: $\{(x, y) \mid y = 3x - 1, x \in \{1, 2, 3\}\}$

1-6 5 pts. each
7-9 10 pts. each
10-11 15 pts. each
12 10 pts.

- ① Is the above relation a function?
- ② State the domain of the above relation.
- ③ State the range of the above relation.
- ④ List the set of ordered pairs for the relation.

- ⑤ If $f(x) = 2x^2 - 3x$, find $f(-1)$.
- ⑥ What is the slope of the graph of $2x + 3y - 6 = 0$?
- ⑦ Write the equation of the line which passes through the point $(2, -3)$ and has a slope of 2.

Find the solution set for the following systems of linear equations.

⑧
$$\begin{cases} 2x - y = 4 \\ 3x + y = 6 \end{cases}$$

⑨
$$\begin{cases} 3x = 2y - 7 \\ 15 + 4y - 6x = 0 \end{cases}$$

- ⑩ A man blends flour worth 8 ¢ per pound with sugar worth 30 ¢ per pound to obtain 10 pounds of a mixture worth 19 ¢ per pound. How many pounds of sugar were used? Show all work.

- ⑪ Graph the solution set of the following system:

$$\{(x, y) \mid y > 2x\} \cap \{(x, y) \mid y \leq x + 1\}$$

- ⑫ Have 10 free points. Who was buried in Grant's tomb?

Algebra 50 - Test - Unit 6 - Form A

Simplify each expression. Assume all variables are positive real numbers.

① $\sqrt{64}$

② $\sqrt{36x^2}$

③ $-\sqrt{8x^2}$

④ $\sqrt{\frac{4}{9}}$

⑤ $\frac{2}{\sqrt{2}}$

⑥ $6\sqrt{x} - 4\sqrt{x}$

⑦ $\sqrt{27} + \sqrt{48} - \sqrt{300}$

⑧ $\sqrt{x^2y} + \sqrt{4x^2y}$

⑨ $\frac{6 - \sqrt{12}}{2}$

⑩ $\sqrt{2}\sqrt{8}$

⑪ $\frac{1}{4 - \sqrt{2}}$

⑫ $(\sqrt{2} - 1)(\sqrt{2} + 2)$

⑬ $\frac{\sqrt{2x^3}\sqrt{8x^2}}{\sqrt{3x}}$

⑭ $\sqrt{16 + 9}$

⑮ $(\sqrt{2})^6$

Solve the following equations.

⑯ $x^2 - 25 = 0$

⑰ $3x^2 - 5x = 0$

⑱ $3x^2 + 7x - 6 = 0$

⑲ $x^2 + 2x - 1 = 0$

⑳ $2x^2 - 6x + 3 = 0$

Appendix B

Information Regarding Pre-recorded Tapes

The pre-recorded tapes used in this project were developed through the University of California, Los Angeles, by a group of mathematics educators from the Southern California area. Information regarding the tapes can be obtained by contacting Mr. Gerald E. Bruce, mathematics department chairman, Rio Hondo College, Whittier, California.

Appendix C

Unit Performance Objective Sheets

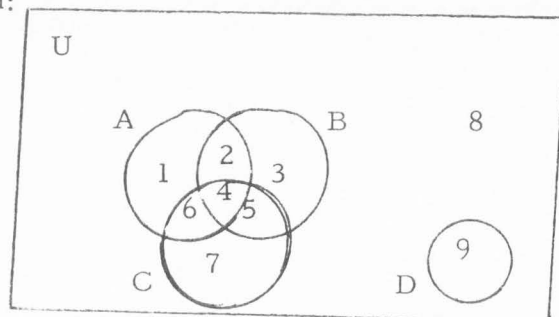
You should be able to answer the following questions.

Let $U = \{a, b, c, d\}$, $A = \{a, b, c\}$, $B = \{d\}$, and $C = \{b, c\}$

- ① List all the subsets of A.
- ② List the elements of: 1) $A \cap B$ 2) $B \cap C$ 3) $B \cup C$ 4) $A' \cap C'$,
5) $A' \cup \phi$ 6) $B' \cup A'$ 7) $C' \cap B$ 8) $(A \cup B)'$
- ③ Replace each comma with \in or \subseteq : 1) b, A 2) C, A
- ④ Replace each comma with \cap or \cup : 1) $A, C = \{b, c\}$ 2) $B, C = \{b, c, d\}$

- ⑤ State the conditions on sets M & N if: 1) $M \cap N = \phi$ 2) $M \cup N = N$

- ⑥ Given:



- 1) List the elements in $(A \cup B)'$.
- 2) What characteristic properties does $\{4, 6\}$ possess? Include those that it does not possess.

- ⑦ "Bill took 6th out of 20 people running the first annual 3 mile run at Muddy Gap, Wyoming." 1) List the cardinal numbers.
2) List the ordinal numbers.
- ⑧ Know the meaning of $>$, $=$, and $<$.
- ⑨ List the members of the following set: $\{x \mid 3 < x \leq 7, x \in W\}$
- ⑩ Write 330 in prime factored form.
- ⑪ Find the solution set of: 1) $x+3 = 11, x \in N$ 2) $5 \cdot x = 12, x \in W$
- ⑫ Graph the following using a number line: $\{x \mid 2 \leq x \leq 4, x \in W\}$

- ⑬ Are the following sets closed under: a) addition b) multiplication
- 1) $\{0, 1, 2\}$ 2) even natural nos. 3) odd natural nos.
- 4) whole numbers
- ⑭ Write the prime numbers between 10 and 20.
- ⑮ Write the composite numbers between 12 and 18.
- ⑯ What is the cardinality of: $\{2 \cdot k + 3 \mid 1 \leq k < 4, k \in \mathbb{W}\}$

Algebra 50 - Objectives - Unit 2 - Ch. 3 & 4

You should be able to answer the following questions.

If $a, b, c \in J$, name the axiom or theorem that justifies each statement.

- ① $a=a$. ② If $a = b$, then $b = a$. ③ If $a = b$ and $b = c$, then $a = c$.
 ④ If $a = b$, $ac = bc$. ⑤ $a + -a = 0$ ⑥ $a \cdot \frac{1}{a} = 1$, $a \neq 0$. ⑦ $a \cdot 0 = 0$
 ⑧ $a + b \in J$.

Also statements involving any of the other axioms or theorems in Ch. 3 & 4

- ⑨ Find the additive inverse and multiplicative inverse of: -3 and $\frac{a}{b}$
 ⑩ Replace the comma with $>$, $=$, or $<$: a) $|6|, |-6|$ b) $0, -(-3)$

Do the indicated operations. Simplify and write the answer as a basic numeral, if it exists.

- ⑪ $-6-3$ ⑫ $6-7$ ⑬ $(-3)(-2)$ ⑭ $(-3)(+2)$ ⑮ $(0)(6)$ ⑯ $\frac{0}{4}$
 ⑰ $\frac{4}{0}$ ⑱ $\frac{-12}{-3}$ ⑲ $-6-1-5+8$ ⑳ $(3)(-5)(4)(0)$
 ㉑ Assuming $x \in Q$, for what value(s) of x is $\frac{1}{x-1}$ undefined?

Write each expression as a basic numeral or basic fraction. $x, y \in J$.

- ㉒ $\frac{3}{5} + \frac{4}{5}$ ㉓ $\frac{x}{7} - \frac{y}{7}$ ㉔ $\frac{2}{7} - \frac{3}{5} + \frac{1}{10}$ ㉕ $\frac{2}{5} \div \frac{5}{2}$ ㉖ $\frac{x}{2} \div \frac{y}{4}$

- ㉗ Express $\sqrt{\frac{1}{25}}$, $\sqrt{\frac{81}{100}}$, and $-\sqrt{\frac{100}{4}}$ as a basic numeral or a basic fraction.

Write the missing numerator so that the second fraction will be equal to the first. $x, y \neq 0$.

- ㉘ $\frac{2}{3} = \frac{?}{12}$ ㉙ $\frac{5x}{8x} = \frac{?}{32x}$ ㉚ $\frac{0}{7} = \frac{?}{21}$ ㉛ $\frac{1}{y} = \frac{?}{-5y}$

Problems 44-56 on page 61; problems 47-62 on pages 65 and 66; and problems 47-56 on page 70.

Algebra 50 - Objectives - Unit 3 - Ch. 5 & 6

You should be able to answer the following questions

Let $U = \{-\sqrt{3}, 5, -1/5, \sqrt{16}, 40, 0, -1, \sqrt{7}\}$

① List the members of the subset of U as follows:

a) $\{x \mid -3 \leq x \leq 3, x \in H\}$

b) $\{x \mid x \geq 0, x \in Q\}$

② Draw the graphs of: a) $\{x \mid -3 \leq x < 5, x \in R\}$

b) $\{x \mid 0 < x \leq 5, x \in J\}$

Find the solution set in R for each equation.

③ $\sqrt{x} = 6$ ④ $\sqrt{x} + 8 = 0$ ⑤ $\sqrt{16} - \sqrt{9} = x$ ⑥ $\sqrt{x+3} - 6 = 1$

Let $N = \{\text{natural nos.}\}$, $W = \{\text{whole numbers}\}$, $J = \{\text{integers}\}$,

$Q = \{\text{rationals}\}$, $H = \{\text{irrationals}\}$, and $R = \{\text{reals}\}$.

True or false

⑦ $Q \subset R$ ⑧ $Q \cap H = \phi$ ⑨ $H \cup Q = R$ ⑩ $N \cap W = N$ ⑪ $Q' \cap H = Q$

Simplify

⑫ $\frac{(x^{-1})(x)^{-2}}{x^{-3}}$

⑬ $3 - [-3 - (x-1) + 4] - (x-2)$

⑭ $\frac{(2^0)(2^{-1})(2^1)}{(3^{-1})(3^0)}$

⑮ Write $120x^4y^3$ in completely factored form.

⑯ Give the name and degree of the polynomial: $6x^3 - 4x$

⑰ Write $\frac{6^2 - 6}{3} - \frac{12}{3} + 1$ as a basic numeral.

⑱ If $P(x) = 2x^3 - 3x + 1$, find $P(1)$, $P(-1)$, & $P(0)$.

⑲ Find the zeros of $(x-6)(x+1)(x-3)$.

⑳ Express .0000312 in scientific notation.

Do the indicated operations and simplify.

⑳ $(3x^2 - 6x + 4) - (2x^2 - 6x - 8)$ ㉑ $(-6x^2y^3)(-2xy^2)$ ㉒ $\frac{-6x^2y^3}{-2xy^2}, x, y \neq 0$

②4 Find the product: $(2x + 1)(3x - 1)$.

②5 Find the quotient: $\frac{4x^2 + 4x + 1}{2x + 1}$

Factor completely, if possible. If not factorable, write prime.

②6 $x^2 + 13$ ②7 $x^2 - 9$ ②8 $2x + 8$ ②9 $x^2 - 7x + 10$ ③0 $x^2 + x - 20$

③1 $x^2 + x - 56$ ③2 $9x^2 - 72x + 144$ ③3 $39x^2 - 65x - 26$

③4 $55x^2y + 77xy - 66y$

Algebra 50 - Objectives - Unit 4 - Ch. 7 & 8

You should be able to answer the following questions

- ① Reduce to lowest terms and state the restrictions on the variable:

$$\frac{x^2 + 8x - 9}{4x^2 - 4}$$

Write each expression as a single fraction in lowest terms.

② $\frac{3}{x^2 - 1} + \frac{5}{x^2 - 3x + 2}$ ③ $\frac{3x^2}{x^2 - 1} + \frac{7x^2 - 4}{x^2 - 1} - \frac{4x^2 + 2}{x^2 - 1}$

④ $\frac{x^2 - 4}{x^2 - x - 6} \cdot \frac{x^2 - 6x + 9}{x^2 - 3x + 2}$ ⑤ $\frac{x^3 - x^2}{4x + 12} \div \frac{x^2 - x}{3x + 9}$

⑥ Simplify and reduce to lowest terms: $\frac{x - \frac{4}{x}}{1 + \frac{2}{x}}$

Solve the following equations, i. e. find the solution set over the set of real numbers.

⑦ $8x - 7 = 9$ ⑧ $(x+1)^2 = x^2 + 7$ ⑨ $\frac{1}{x} + \frac{2}{x} = \frac{3}{x}$

⑩ $2 + \frac{x+5}{x-2} = \frac{7}{x-2}$ ⑪ $6(x-1) = 4(2x+1)$ ⑫ $\frac{2}{3} - \frac{1}{x+1} = 2$

⑬ Solve for x: $ax + 6 = x - 1$

⑭ $\frac{2x+3}{5} \geq \frac{-9}{5}$, $x \in \mathbb{R}$ Solve the inequality

⑮ Graph the solution set of the inequality of problem 14.

⑯ One powder is worth \$.50 a pound and a second is worth \$.20 a pound. How many pounds of each should be mixed to form 60 pounds of a mixture worth \$.30 a pound?

Problems 9, 11, 15, and 17 of Exercise 8.8, pages 203-206

Algebra 50 - Objectives - Unit 5 - Ch. 9 & 10

You should be able to answer the following questions.

Solve each equation explicitly for y in terms of x .

① $x + y = 10$

② $3x = \frac{y-1}{y}$

③ $3x + 2y = 7$

GIVEN: $\{(x, y) \mid y = 3x + 1, x \in \{0, 2, 4\}\}$

- ④ Is the above relation a function?
 ⑤ State the domain of the above relation.
 ⑥ State the range of the above relation.
 ⑦ List the set of ordered pairs for the relation.
-

⑧ State the domain of the function defined by: $y = \frac{1}{x-4}$.

- ⑨ If $f(x) = 2x^2 - 5x + 1$, find $f(-3)$.
 ⑩ Find the slope of the line passing thru $(2, -1)$ and $(5, 3)$.
 ⑪ What is the slope of the graph of $3x - 4y = 12$.
 ⑫ Write the equation of the line that has the same slope as $2x + 3y + 6 = 0$ and passes thru $(-3, -4)$.
-

Find the solution set for the following systems of linear equations:

⑬ $\begin{cases} x + y = 4 \\ x - y = 0 \end{cases}$

⑭ $\begin{cases} x + 2y = 4 \\ 2x + 4y = 8 \end{cases}$

⑮ $\begin{cases} 2x - y = 6 \\ 6x - 3y = 6 \end{cases}$

⑯ $\begin{cases} 9y = 7x + 3 \\ 8 + 2x = -y \end{cases}$

-
- ⑰ How many pounds each of a 50% copper alloy and a 75% copper alloy must be mixed to obtain $87 \frac{1}{2}$ pounds of a 60% alloy?
-

- ⑱ Graph the solution set of the following system:

$$\{(x, y) \mid y \leq 2x + 1\} \cap \{(x, y) \mid y > 3 - x\}$$

Algebra 50 - Objectives - Unit 6 - Ch. 11 & 12

You should be able to solve the following problems

Simplify each expression. Assume all variables are positive real numbers.

① $\sqrt{81}$

② $\sqrt{25x^4}$

③ $-\sqrt{27x^2}$

④ $\sqrt{\frac{9}{100}}$

⑤ $\frac{4}{\sqrt{2}}$

⑥ $2\sqrt{x} - 5\sqrt{x}$

⑦ $2\sqrt{20} - \sqrt{80} + \sqrt{45}$

⑧ $\sqrt{4xy^3} - \sqrt{xy^3} + 2\sqrt{xy^3}$

⑨ $\frac{6 - \sqrt{18}}{3}$

⑩ $\sqrt{2}\sqrt{32}$

⑪ $\frac{2}{4 - \sqrt{2}}$

⑫ $(\sqrt{5} - 2)(\sqrt{5} + 3)$

⑬ $\frac{\sqrt{5x} \sqrt{4x^5}}{\sqrt{40x^2}}$

⑭ $\sqrt{25 + 9}$

⑮ $(\sqrt{5})^6$

Solve the following equations.

⑯ $x^2 - 81 = 0$

⑰ $6x^2 + 8x = 0$

⑱ $12x^2 + 4x - 5 = 0$

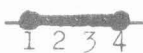
⑲ $x^2 - 3x - 4 = 0$

⑳ $4x^2 + 16x - 3 = 0$

Appendix D

Mathematics Posttest of Achievement

Instructions: Put the letter, corresponding to the correct answer, in the answer column. There is one and only one correct answer for each question. Do not write on this test sheet. Scoring will be the number correct minus one-fourth the number wrong. All the variables represent real numbers unless otherwise indicated.

- If $A = \{-3, -1, 1, 3\}$ and $B = \{0, 1, 2\}$, then $A \cap B =$
 a) $\{-3, -1, 0, 1, 2, 3\}$ b) $\{1\}$ c) $\{0, 2\}$ d) $\{-3, -1, 3\}$ e) none of these
- The prime factored form of 90 is
 a) $(9)(10)$ b) $(2)(45)$ c) $(2)(3)(3)(5)$ d) $(3)(3)(10)$ e) none of these
- The additive inverse (opposite) of "x", $x \in \mathbb{R}$, is
 a) $-x$ b) \sqrt{x} c) 1 d) $\frac{1}{x}$ e) none of these
- If $x > 0$, then
 a) $|x| > -x$ b) $|x| < x$ c) $|x| > x$ d) $-|x| > x$ e) none of these
- If $x > y > 0$, which of the following is false?
 a) $y + x > 0$ b) $y - x > 0$ c) $(y)(x) > 0$ d) $\frac{y}{x} > 0$ e) none of these
- The quotient $\frac{24y^5}{12y^5}$ represents the same number as
 a) $\frac{1}{2}$ b) $\frac{1}{2}y^{10}$ c) 2 d) $2y^{10}$ e) none of these
- Assuming "x" is a rational number, for what value(s) of "x" is $\frac{x+3}{x+5}$ undefined?
 a) -3 b) 3 c) 5 d) -5 e) none of these
- $\sqrt{9} + \sqrt{16} =$
 a) 7 b) 5 c) 12 d) $\frac{3}{4}$ e) none of these
- $\frac{x}{2} \div \frac{1}{2} =$
 a) $\frac{x+1}{2}$ b) $\frac{1}{x}$, $x \neq 0$ c) $\frac{x}{2}$ d) x e) none of these
- The graph of $\{x \mid 1 < x < 4, x \in \mathbb{R}\}$ is
 a)  b)  c)  d)  e) none of these
- If $P(x) = \frac{1}{4}x - 2$, then $P(8) =$
 a) 4 b) 30 c) 2 d) $-\frac{3}{2}$ e) none of these

12. $-(-x-(y+z)) =$
 a) $-x-y-z$ b) $x+y+z$ c) $x-y+z$ d) $x-y-z$ e) none of these
13. If $x, y \neq 0$, then $\frac{-x^3}{x} - \frac{-yx}{y} =$
 a) $-x^2 - x$ b) $2 + x$ c) $x^{-2} + \frac{x}{y^2}$ d) $-x^2 + x$ e) none of these
14. If $y = -5$, then $y^0 =$
 a) 0 b) -5 c) 1 d) 5 e) none of these
15. $2.18 \times 10^{-4} =$
 a) 2,180 b) 0.000218 c) 0.00218 d) 21,800 e) none of these
16. $x^2y^2 - 2xy - 15 =$
 a) $(xy+5)(xy+3)$ b) $(xy+5)(xy-3)$ c) $(xy-5)(xy+3)$ d) $(xy-5)(xy-3)$
 e) none of these
17. $(2x^2 + 3x - 1) - (-3x^2 - 2x + 2) =$
 a) $-x^2+x+1$ b) $5x^2+x+1$ c) $-x^2+5x-3$ d) $5x^2+5x+1$ e) none of these
18. $(2x^2y^3)(-3xy^4) =$
 a) $-6x^2y^7$ b) $-6x^3y$ c) $-6x^3y^7$ d) $-6xy^7$ e) none of these
19. $3-(-(2-x)+1)-(1-x) =$
 a) 3 b) $-2x-1$ c) $2x-1$ d) $5-2x$ e) none of these
20. $(2x+4)(3x-1) =$
 a) $6x^2+14x-4$ b) $6x^2-10x-4$ c) $6x^2+10x+4$ d) $6x^2+10x-4$ e) none of these
21. The complete factorization of $9x^2-16$ is
 a) $(9x-4)(9x+4)$ b) $(3x-4)(3x-4)$ c) $(3x+4)(3x+4)$ d) $(3x-4)(3x+4)$
 e) none of these
22. If $x \neq 3, -3$, then $\frac{x+3}{x-3} - \frac{x-3}{x+3} =$
 a) $\frac{12x+18}{x^2-9}$ b) $\frac{2x^2+18}{x^2+9}$ c) $\frac{12x}{x^2-9}$ d) $\frac{18}{x^2+9}$ e) none of these
-
23. If $a \neq 0$, $b \neq 0$, and $a+b \neq 0$, then $1 - \frac{1}{\frac{a}{1+\frac{b}{a}}}$
 a) $\frac{a-1}{a+b}$ b) $\frac{-a^2}{b}$ c) $\frac{-1}{b}$ d) $\frac{a}{b}$ e) none of these

24. $\frac{x^2-x-6}{x^2+2x-15} \div \frac{x^2-4}{x^2-25} =$
 a) $\frac{(x-3)(x+5)}{(x+3)(x+2)}$ b) $\frac{(x+2)(x-5)}{(x-3)(x-2)}$ c) $\frac{(x-5)}{(x-2)}$ d) $\frac{(x-2)}{(x-5)}$ e) none of these
25. The solution set of $4(x-2)-3x = 5x+4$ is
 a) $\{-2\}$ b) $\{3\}$ c) $\{-1\}$ d) $\{-3\}$ e) none of these
26. The solution set of $2a+3bx = -bx+10a$, where $a, b \neq 0$, is
 a) $\{32ab\}$ b) $\left\{\frac{2a}{b}\right\}$ c) $\left\{\frac{a}{2b}\right\}$ d) $\left\{\frac{4a}{b}\right\}$ e) none of these
27. The solution set of $\frac{2x+4}{3} < 0$, $x \in \mathbb{R}$, is
 a) $\{-2\}$ b) $\{x|x < -2, x \in \mathbb{R}\}$ c) $\{x|x > -2, x \in \mathbb{R}\}$ d) $\{x|x < -\frac{1}{2}, x \in \mathbb{R}\}$
 e) none of these
28. The length of a rectangle is 2 ft. more than the width. The perimeter of the rectangle is 44 ft. Its width is
 a) 20 ft. b) 12 ft. c) 10 ft. d) 6 ft. e) none of these
29. The solution set of $(x-3)(x+2)-3 = 1+x^2$ is
 a) $\{-10\}$ b) $\{10\}$ c) $\{-2\}$ d) $\{2\}$ e) none of these
30. How many quarts of a 20% acid solution must be added to 10 qts. of a 30% acid solution to obtain a 24% acid solution?
 a) 12 qts. b) 14 qts. c) 15 qts. d) 16 qts. e) none of these
31. The domain of the function $\{(x,y)|y = 1-2x, x \in \{0, 1, 2\}\}$ is
 a) $\{-1, 0, 1\}$ b) $\{0, 1, 2\}$ c) $\{1, -1, -3\}$ d) $\{-\frac{1}{2}, 0, \frac{1}{2}\}$
 e) none of these
32. If $f(x) = 3x^2-1$, then $f(-1) =$
 a) 2 b) -4 c) 8 d) 5 e) none of these
33. The slope of the graph of $3x-2y = 4$ is
 a) $-\frac{2}{3}$ b) $-\frac{3}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$ e) none of these
34. The graph of $\{(x,y)|y = 2\}$ is a straight line parallel to the
 a) y-axis b) x-axis c) the line $y = x$ d) the line $y = -x$
 e) none of these
35. Consider the system $\begin{cases} 3x+2y = 9 \\ 6x+4y = 5 \end{cases}$
 a) the equations are dependent b) the system has exactly one solution
 c) the equations are inconsistent d) the system has exactly two solutions
 e) none of these

36. The solution set of the system $\begin{cases} 2x+y = 12 \\ x-2y = 1 \end{cases}$ is
 a) $\{(2,5)\}$ b) ϕ c) $\{(5,2)\}$ d) $\{(x,y) \mid x-2y = 1\}$ e) none of these
37. The graph of $\{(x,y) \mid x < 0 \text{ and } y < 0\}$ is
 a) quadrant I b) quadrant II c) quadrant III d) quadrant IV
 e) none of these
38. $(\sqrt{7y})(\sqrt{8y}) =$ (assume $y > 0$)
 a) $56y$ b) $y^2\sqrt{56}$ c) $2\sqrt{14y}$ d) $2y\sqrt{14}$ e) none of these
39. $\sqrt{75} - 3\sqrt{48} =$
 a) $-3\sqrt{27}$ b) $-2\sqrt{3}$ c) $-7\sqrt{3}$ d) $17\sqrt{3}$ e) none of these
40. $\frac{1}{3-\sqrt{5}} =$
 a) $\frac{3+\sqrt{5}}{4}$ b) $\frac{3-\sqrt{5}}{14}$ c) $\frac{3+\sqrt{5}}{14}$ d) $\frac{3-\sqrt{5}}{4}$ e) none of these
41. The solution set of $2x^2-18 = 0$ is
 a) $\{3\}$ b) $\{9, -9\}$ c) $\{9\}$ d) ϕ e) none of these
42. The solution set of $2x^2-3x = 0$ is
 a) $\left\{\frac{3}{2}\right\}$ b) $\left\{0, \frac{3}{2}\right\}$ c) $\left\{\frac{2}{3}\right\}$ d) $\left\{0, \frac{2}{3}\right\}$ e) none of these
43. The solution set of $x^2+4x-8 = 0$ is
 a) ϕ b) $\{-2+2\sqrt{3}, -2-2\sqrt{3}\}$ c) $\{2+2\sqrt{3}, 2-2\sqrt{3}\}$ d) $\{-2+3\sqrt{2}, -2-3\sqrt{2}\}$
 e) none of these
44. The value of $(x)(x+1)(x-3)(x+4)$, when $x = 1$, is
 a) 0 b) 5 c) -20 d) 6 e) none of these
45. $6ab-3ab+5ab =$
 a) 8 b) $8ab$ c) 14 d) $14ab$ e) none of these
46. If $a = 1$, $b = 2$, and $c = 0$, then $\frac{a^2+a-b}{2a+c}$ is
 a) undefined b) 0 c) 4 d) 1 e) none of these
47. $x(1-x) - 2\left(\frac{1}{2}x - \frac{1}{2}x^2\right) =$
 a) $-2x^2$ b) $-2x$ c) $2x^2$ d) 0 e) none of these
48. $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3}) =$
 a) -5 b) $-\sqrt{5}$ c) -1 d) 5 e) none of these

49. Bill can paint a house alone in 6 days. Bob can paint the same house alone in 4 days. How many days will it take to paint the house if they both work together?
- a) 2 days b) $2\frac{1}{4}$ days c) $2\frac{2}{5}$ days d) $2\frac{1}{2}$ days e) none of these
50. Have one on me! I don't remember the question but the answer is "e". Good luck in the future. It's been fun working with all of you. Stop by and say hello when you get a chance.

VITA

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