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STATISTICAL METHODS ON RISK MANAGEMENT OF EXTREME EVENTS

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ABSTRACT

STATISTICAL METHODS ON RISK MANAGEMENT OF EXTREME EVENTS

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The goal of the dissertation is the investigation of financial risk analysis methodologies, using the schemes for extreme value modeling as well as techniques from copula modeling.

Extreme value theory is concerned with probabilistic and statistical questions related to unusual behavior or rare events. The subject has a rich mathematical theory and also a long tradition of applications in a variety of areas. We are interested in its application in risk management, with a focus on estimating and forecasting the Value-at-Risk of financial time series data. Extremal data are inherently scarce, thus making inference challenging. In order to obtain good estimates for risk measures, we develop a two-stage approach: (1) fitting the GARCH-type models at the first stage to describe the volatility clustering and other stylized facts of financial time series; (2) using the extreme value theory based models to fit to the tails of the residuals. Additionally, the performance measures provide information in terms of the compar-

ison of the two-stage semi-parametric approach with the parametric methodologies, through robust backtesting.

Copula is a particular branch of probability theory, with which, given sufficient data, we can separate the marginal behavior of individual risks and their dependence structure from a multivariate random variable. Linear correlation is widely used to model dependence but has limitations as a measure of association and thus we opt to use copulas to analyze the dependence structure and build models for our different problems arising in risk management. For this part of the dissertation, we take a look at different copula families, highlight for some when they are most appropriate to use for a particular application, discuss some of their drawbacks as diverse scenarios occur in different risk management models, and explore the possibility of developing the copula modeling to reflect the complicated dependence structure of portfolios.

PREFACE

Sections of this Ph.D. thesis have been submitted for publication. They are as follows:

Chapter 1, Section 1.1[89].

Chapter 1, Section 1.2[91].

Chapter 1, Section 1.3[92].

Chapter 2, Section 2.1.

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INTRODUCTION

In the last decade risk management has become a major discipline in Finance. It is studied in different fields within finance: financial econometrics, mathematical finance or financial engineering. Financial risk management is the process by which the financial risks are identified, assessed, measured, and managed in order to create economic value. The main concern of risk management is analyzing the causes and consequences of negative events for investor interests. In addition, one of the more promising research areas in finance recently is the development of financial instruments and investment strategies that allow one hedging from negative events.

As financial markets have expanded over recent decades, the risk management function has become more important. Risk can never be avoided. More generally, the goal is not to minimize risk, it is to take smart risks. Some risks can be measured reasonably well. For those, risk can be quantified using statistical tools to generate a probability distribution of profits and losses. Other risks are not amenable to formal measurement but are nonetheless important. Risk that can be measured can be managed better. Investors assume risk only because they expect to be compensated for it in the form of higher returns. To decide how to balance risk against return, however, requires risk measurement.

Since uncertainty is intrinsic to the definition of random variable and is usually described by the variance. Risk however entails something more not captured by the variance. Risk in this situation comes from very low or high forecasted values. Centralized risk management tools such as Value-at-Risk (VaR) were developed in the early 1990s. They combine two main ideas. The first is that risk should be measured at the top level of the institution or the portfolio. This idea is not new.

It was developed by Harry Markowitz (1952)[66], who emphasized the importance of measuring risk in a total portfolio context. A centralized risk measure properly accounts for hedging and diversification effects. It also reflects the fact that equity is a common capital buffer to adsorb all risks. The second idea is that risk should be measured on a forward-looking basis, using the current position.

VaR raise the issue of defining risk as something occurring in the tails of the distribution of the random variable and entailing the knowledge of its probability distribution. It uses quantiles, which requires us to pay attention to discontinuities and intervals of quantile numbers.

Definition 0.1. Given $\alpha \in [0,1]$ the number q is an $\alpha-$**quantile** of the random variable X under the probability distribution $\mathbb{P}$ if one of the three equivalent properties below is satisfied:

a. $\mathbb{P}(X \leq q) \geq \alpha \geq \mathbb{P}(X < q)$,

b. $\mathbb{P}(X \leq q) \geq \alpha$ and $\mathbb{P}(X \geq q) \geq 1-\alpha$,

c. $F(q) \geq \alpha$ and $F(q^-) = \lim_{x \to q^-} F(x) \leq \alpha$, where F is the cumulative distribution function of X.

We formally define VaR in the following way.

Definition 0.2. Given $\alpha \in [0,1]$, and a reference instrument r, the Value-at-Risk $\mathbf{VaR}_\alpha$ at level α of the final new worth X with distribution $\mathbb{P}$, is the negative of the quantile q_α^+ of X/r, that is

$$VaR_\alpha = -\inf\{x|\mathbb{P}(X \leq x \times r) > \alpha\}.$$

It is interesting however the statisticians and econometricians vision of risk. It boils down to measuring the variance of the random variable describing the event. This is only true if the probability distribution is known and the only unknown is the variance. Consider the example of a normal distribution $\mathcal{N}(\mu, \sigma^2)$ where μ is unknown but σ is known. The knowledge of σ is not sufficient to know the uncertainty neither the risk. The probability distribution is not known, but a set of possible distribution functions. This unusual example reflects the ambiguity of knowing only the variance. In the particular example of analyzing financial returns it is a common hypothesis to assume the expected value of the returns to be zero, and then makes sense to think of the volatility as measuring the risk. Nevertheless this example derived from financial econometrics needs of another assumption. The prices of the financial instrument are assumed to follow a log-normal distribution, and in consequence the returns distribution is assumed normal.

There is a handful of econometric techniques for estimating the risk under these assumptions. The focus is in the estimation and modeling of the volatility process. The standard methodology is estimation from the historical distribution where the volatility is considered constant, and all the observations have the same weight in estimating the variance. Instead, if some dynamics is observed in the data, a more adequate estimator for the volatility is some exponential smoothing technique where the most recent observations have more protagonism than past observations. GARCH models, which was introduced in Engle and Bollerslev (1986)[14], followed this philosophy. There are minor modifications of this model reflecting different stylized facts of the financial data. Examples of these models are Exponential GARCH, Threshold GARCH, and Asymmetric Power ARCH regarding the leverage effect, IGARCH where describing infinite variance, Fractionally Integrated GARCH, Hyperbolic GARCH and Fractionally Integrated Power ARCH regarding the long memory effect, etc.

More sophisticated forms of measuring risk in this setting are given by the implied volatility and the realized volatility. Implied volatility is derived from option pricing and in consequence from Black-Scholes formula, Black and Scholes (1973)[13]. The prices are supposed to follow a geometric Brownian Motion. Other volatility measure founded on stochastic differential equations is the realized volatility. The expression for volatility builds on the theory of continuous-time arbitrage-free price processes and the theory of quadratic variation.

All of the above different methodologies to quantify risk fail if the distribution of returns is far from the Gaussian assumption. This fact is gaining popularity within the academics and practitioners that have raised the need of a more realistic modeling of the distribution of returns, and of the analysis of risk. The focus moves from a measure for the dispersion of the data to a measure that describes the probability in the tails. The risk underlying the financial sequence is renamed as downside risk since it is associated to negative outcomes that are usually represented in the left tail of the distribution of returns. It is worth mentioning the upside risk due to the existence of hedging instruments that are designed to compensate values in the left tail and can yield negative outcomes when the returns take on large positive values. The interest of risk managers is found in estimating the distribution of the data, in particular the distribution in the tails. The results found in Gnedenko (1943)[39] derived from the distribution of the sample maximum are the basis of a new and exciting area in Statistics involving the analysis of the extreme values of random sequences and the distribution in the tails. This area is denominated Extreme Value Theory (EVT) and is the theoretical basis and statistical toolkit for the techniques developed in this thesis.

The foundations of the theory were laid by Fisher and Tippett (1928)[34] and (1943)[39], who demonstrated that the distributions of the extreme values of an independent and identically distributed sample from a cumulative distribution function F, when adequately rescaled, can converge towards one out of only three possible distributions. The crucial element of this finding is that the type of asymptotic distribution of extreme values does not depend on the exact cumulative distribution function F of returns. The precise form of F can thus be ignored and a non-parametric or a semi-parametric method can be used to estimate VaR. This is important, given that the whole tail of the distribution of returns is unknown and that, although financial time series usually exhibit skewed and fat-tailed distributions, there is no complete agreement on what distribution would fit them best.

In principle, EVT-based estimates of VaR should be more accurate and reliable than the usual ones because EVT concentrates directly on the tails of the distribution. This avoids a major flaw of parametric approaches, i.e. that their estimates are somehow biased by the credit given on the central part of the distribution, thus underestimating extremes and outliers, which are precisely that is of interest when calculating VaR. The third and final reason why EVT is especially promising in risk measurement is that it allows each of the two tails of the distribution to be tackled independently, in a flexible approach that takes the skewness of the underlying distribution into account.

These three main advantages of an EVT approach to risk management are summarized as "letting the tails speak for themselves". This is a very fitting description, as risk management focuses primarily on avoiding large unexpected losses and sudden crashes rather than on long sequences of medium-sized losses.

Nowadays, the most popular application of EVT to finance is for the estimation of VaR and Expected Shortfall (ES), which takes into account the whole tail of the distribution and also possesses the properties required for a coherent risk measure as defined by Artzner et al. (1999)[6]. But it is not the only possible one, Rocco (2012)[75] presents a critical survey of all main financial applications.

Here we give a brief presentation of the main theoretical underpinnings of EVT. For a thorough presentation of the theory, we refer the reader to the specialist literature, such as Beirlant et al. (2006)[10], Coles (2001)[27], de Haan and Ferreira (2007)[31].

Given an unknown distribution F, EVT only models the tails of F, without making any specific assumption concerning the centre of the distribution. There are two different parametric approaches to EVT. The two parametric approaches differ as to the meaning that they assign to the notion of 'extreme value'.

Considered N independent and identically distributed random variables $X_i, i = 1, 2, \cdots, N$, representing positive losses and denote by their distribution F. The first parametric approach, the block maxima method, divides a given sample of N observations into m subsamples of n observations each (n-blocks) and picks the maximum M_K $(K - 1, 2, \cdots, m)$ of each subsample, a so-called block maximum. The set of extreme values of F is then identified with the sequence $(M_k)_k$ of block maxima and the distribution of this sequence is studied. The main result of EVT is that, as m and n grow sufficiently large, the limit distribution block maxima belongs to one of three different families. Which one it belongs to depends on the behavior of the upper tail of F, whether it is power-law decaying, exponentially decaying, or with upper bounded support. The three asymptotic distributions of block maxima can be written in a unified manner by means of the generalized extreme value (GEV) distribution, a parametric expression depending on a real parameter, known as the shape parameter, that we denote by ξ. The three cases just mentioned correspond, respectively, to $\xi > 0$ (Fréchet case), $\xi = 0$ (Gumbel case) and $\xi < 0$ (Weibull case).

The second parametric approach, the threshold exceedances method, defined extreme values as those observations that exceed some fixed high threshold μ. This method models the distribution of the exceedances over μ, that is to say, the random variables $Y_j = X_j - \mu$, calculated for those observations X_j that exceed μ, i.e. such that $X_j > \mu$. The main result of EVT following this approach is that as the threshold μ tends to infinity, the distribution of the positive sequence $(Y_j)_j$, appropriately scaled, belongs to a parametric family, the generalized Pareto distribution (GPD), whose main parameter is the same shape parameter ξ as the corresponding GEV distribution.

Any of these approaches to EVT entails choosing an adequate cut-off between the central part of the distribution and the upper tail, i.e. a point separating ordinary realizations from extreme realizations of the random variable. When working with threshold exceedances, the cut-off is induced by the threshold μ, while in the block maxima method, it is implied by the number m of blocks. This is a very problematic aspect of the statistical methods of EVT, as the estimated value of the shape parameter can vary considerably depending on the chosen cut-off. Indeed, there is a trade-off between bias and variance of the estimated of the shape parameter ξ. For instance, with threshold exceedances, if μ is set too low, many ordinary data are taken as extreme, yielding biased estimated. By contrast, an excessively high threshold gives scant extreme observations, too few to obtain efficient estimates. In both cases, the resulting estimates are flawed and may lead to erroneous conclusions when assessing risk.

An optimal cut-off cannot be selected once and for all as it depends on the time series at hand. The literature suggests three main ways to cope with this issue:(a) employing graphical methods, Hill plots, that display the estimated values of ξ as a function of the cut-off in order to find some interval of candidate cut0off points that yields stable estimates of ξ; (b) making Monte Carlo simulations and then choosing the cut-off that minimizes a statistical quantity, yielding a trade-off between bias and variance of the estimates; (c) implementing algorithms, based for instance on the bootstrap method, that endogenously pick out the cut-off best suited to the data at hand.

Another important issue raised by the practical implementation of EVT is that for the theory to work, the data must be independent and identically distributed, whereas most financial time series do not satisfy this requirement. Therefore, using

EVT without properly considering the dependence structure of the data yields incorrect estimates, possibly resulting in unexpected losses or in excessively conservative positions.

Two main approaches are usually employed to take data dependence into consideration. (a) If the time series is strictly stationary, then an additional parameter can be estimated, the extremal index, which accounts for the clustering of extremal values due to dependence. (b) Alternatively, the dependence structure can be explicitly modeled, fitting some GARCH-type model to the data. If the standardized residuals exhibit a roughly independent and identically distributed (i.i.d.) structure, EVT can then be applied to them rather than directly to the data. This is the same as implementing a two-stage procedure that filters the data with econometric tools and is suited to deal with conditional heteroskedasticity before applying EVT methods.

The latter approach works well when using EVT for estimating quantile-based measures of risk, such as VaR or ES, and it seems to be sufficiently robust to yield good estimates even when the GARCH type model is mis-specified to some extent.

Finally, when EVT is applied to some data, the very choice of the dataset may be an issue owing to the dichotomy inscribed in the theory: on the one hand, EVT requires a lot of data as its results are asymptotic, but, on the other hand, it necessarily encounters a scarcity of data because it concentrates on the tails of the distribution and extreme events are by definition, rare. Several practical remedies to this antinomy are found in empirical studies, such as using high frequency data, expanding the time window as much as possible, jointly modeling extreme values from both the upper and the lower tail, or pooling different data series in a single one.

In Chapter 1, we applied and improved the EVT cased methodologies to estimate and forecast VaR for different financial assets.

From a theoretical point of view, when studying extremes of multivariate time series, the dependence between the extreme values of the different components plays a crucial role. The common notion of correlation, which is useful for the normal distribution, is often inadequate to explain the dependence between extremes of multivariate time series. Pearson correlation, which is the most common measure of dependence, is neither a good measure of dependency in cases where the extreme realizations are important. This has resulted in the introduction of copulas method, which has become rapidly developed and has bought the attention in various fields as a way to overcome the limitations of classical dependence measures as exemplified by the linear correlation.

Copula theory was first developed in Sklar (1959)[83]. It is a powerful tool as it does not require any assumptions on the selection distribution function and it allows the risk manager to decompose any n-dimensional joint distribution function into n marginals and a copula. In the field of finance, the two major phenomena account for the rise of copula modeling are the lack of normality in returns and the dependence between extreme values of various assets. The oldest research group is that of Paul Embrechts. As early as 1999, Embrechts, McNeil and Straumann[33] were using the concept of copula to alert readers of Risk Magazine to the pitfalls of correlation. The papers by Embrechts and his collaborators on the use of copulas in managing financial risks are by far the most numerous and cited. They culminated in 2015 with the publication of the book by McNeil, Frey, and Embrechts (2015)[68].

Copulas are defined as functions that links univariate marginals to form multivariate distributions. A real advantage of using copula functions for the description of dependence structures consists in the ability to combine different types of marginal distributions into a joint risk distribution. At the same time, the joint distribution created using copulas can have a dependence structure described by more than a simple correlation matrix.

Definition 0.3. A function $C : [0,1]^n \to [0,1]$ is a n-dimensional **copula** if it satisfies the following properties:

(1) For all $u_i \in [0,1]$, $C(1, \cdots, 1, u_i, 1, \cdots, 1) = u_i$. (2) For all $u_i \in [0,1]$, $C(u_1, \cdots, u_n) = 0$ if at least one $u_i = 0$.

(3) C is grounded and n-increasing.

Hence a n-dimensional copula is a joint distribution function defined on $[0,1]^n$ with standard uniform marginal distributions. According to Sklar's well-known theorem, copulas allows the dependence structure of a joint distribution to be disentangled from its marginal behavior.

Theorem 0.4. ***Sklar's theorem***: *Given a d-dimensional distribution function G with continuous marginal cumulative distributions $F_1, \cdots, F_d$, then there exists a unique n-dimensional copula $C : [0,1]^d \to [0,1]$ such that for $x \in \mathbb{R}^n$*

$$G(x_1, \cdots, x_n) = C(F_1(x_1), \cdots, F_n(x_n)).$$

Moreover, if $F_1, \cdots F_n$ are continuous, then C is unique. Sklar's theorem is a fundamental result concerning copula functions.

If F is a univariate distribution function then the generalized inverse of F is defined as

$$F^{-1}(t) = \inf\{x \in \mathbb{R} : F(x) \geq t\}$$

for all $t \in [0,1]$, and using the convention $\inf\{\emptyset\} = \infty$.

Corollary 0.5. Let G be an n-dimensional distribution function with continuous marginals $F_1, \cdots, F_d$ and an n-dimensional copula C. Then for any $u \in [0,1]^n$,

$$C(u_1, \cdots, u_n) = G(F_1^{-1}(u_1), \cdots, F_n^{-1}(u_n)).$$

Note that without the continuity assumption, this relation may not hold. The copula links the quantiles of the two distributions rather than the original variables, so one of the key properties of a copula is that the dependence structure is unaffected by a monotonically increasing transformation of the variables. The definition and invariance properties suggest that we interpret a copula associated with $(x_1, \cdots, x_n)$ as being the dependence structure. This makes particular sense when all the F_i are continuous and the copula is unique; in the discrete case there will be more than one way of writing the dependence structure.

Dynamic modeling of dependence between financial risks is crucial to achieving consistent calibration through time to market data, as well as to dynamic hedging of there risks. In this thesis, the concordance between extreme values of random variables is of interest. Such a dependence measure is essentially related to the conditional probability that one index exceeds some value given that another exceeds some value. If such a conditional probability measure is a function of the copula, then it too will be invariant under strictly increasing transformations.

In the case of a bivariate series, the coefficient of upper tail dependence measures the conditional probability of one component of the series exceeding a given quantile, provided that the other component exceeds the same quantile, as this quantile tends to one. If the coefficient of upper tail dependence equals zero, the two components of the bivariate time series are asymptotically independent; otherwise, they are asymptotically dependence. The extremal dependence structure is typically different from the dependence we find at the centre of the distribution, since asymptotically independence can occur even if the components of the distribution are not linearly independent.

In general, a statistical problem for copulas could be decomposed into two steps: the identification of marginal distributions and the definition of an appropriate copula function. We refer to the first step as modeling the marginal distributions and second as modeling the dependence structure. We describe and implement the copula based two step methodologies to discover the dynamic tail dependence of financial assets in Chapter 2.

CHAPTER 1

EXTREME VLUE THEORY IN RISK MANAGEMENT

1.1 Calendar Effects in AAPL Value-at-Risk

This study investigates calendar anomalies: day-of-the-week effect and seasonal effect in the VaR analysis of stock returns for AAPL during the period of 1995 through 2015. The statistical properties are examined and a comprehensive set of diagnostic checks are made on the two decades of AAPL daily stock returns. Combing the Extreme Value Approach together with a statistical analysis, it is learnt that the lowest VaR occurs on Fridays and Mondays typically. Moreover, high Q4 and Q3 VaR are observed during the test period. These results are valuable for anyone who needs evaluation and forecasts of the risk situation in AAPL. Moreover, this methodology, which is applicable to any other stocks or portfolios, is more realistic and comprehensive than the standard normal distribution based VaR model that is commonly used.

1.1.1 Background

Into the maelstrom of digital revolution came a greatly innovative digital company: Apple Inc.. The company designs, manufactures, and markets mobil communication, media devices, personal computers, and portable digital music players, and sells a variety of related software, services, accessories, networking solutions, and third-party digital content and applications. America's favorite pastime used to be baseball, but during the last couple of years, that has changed. The new American pastime has become getting long Apple Inc. stock (NASDAQ:AAPL) any way that you can, and wait for the profits to accumulate. While the above saying may not be absolute true, increasing numbers of investors to AAPL trading, make the current topic about AAPL risk in a certain time frame is indeed worthy to be studied thoroughly.

For a rational financial decision maker, expected returns constitute only one part of the decision making process. Another part that must be taken into consideration is the volatility or risk of returns. Therefore, understanding the risks and volatility involved in stock investing is essential. It is helpful to know whether there are variations in the risk of stock returns by the day-of-the-week as well as during different seasons. If investors can identify a certain pattern in the risk, then it would be easier to make investment decisions based on both returns and risk. It is also important to know whether a high stock performance is associated with a correspondingly high risk taking behavior. For example, there have been extensive studies of the relation between aggregate volatility and expected returns of the market, see Campbell and Hentschel (1992)[18], Campbell (1996)[17], and Guo and Whitelaw (2003)[42]. Uncovering certain volatility or risk patterns in returns might also benefit investors in option pricing, portfolio optimization, and risk management.

It is well known that the financial institutions with significant amounts of trading activity are vulnerable to extreme market movements. Hence risk quantification, i.e. estimations of probabilities of large losses in financial markets, has become a primary concern for regulators and also for internal risk control. Ideally, the best and most informative risk measure of financial vulnerability is given by the whole tail of the loss distribution. A popular method of risk measurement is the Value-at-Risk (VaR), which is defined as the loss level that will not be exceeded with a certain confidence level during a certain period of time. The VaR was firstly used as an internal management tool by a number of banks after the 1987 crash, then improved by J.P. Morgan who designed its RiskMetrics System in 1994. It has emerged as one of the most used risk measures in the financial industry, mostly because of its simplicity and intuitive interpretation. Details can be found on the homepage of MSCI. To make the risk measurement coherent, the quantity of Expected Shortfall (ES) is also widely used. The ES of an asset or a portfolio is the average loss, given that VaR has been exceeded. Thus, it is also called conditional value at risk. The advantage of ES is that it is not only sensitive to the shape of the loss distribution in the tail of the distribution, but also possesses the properties required for a coherent risk measure as defined by Artzner (1999)[6].

Hence, the goal of this section is to characterize the VaR of AAPL relative returns. Based on investigations of the day-of-the-week effect and seasonal effect in extreme event risk, we also provide valuable and applicable analysis for investors who are interested in Apple Inc. stock. The major obstacle to this investigation is a viable measure of tail risk over time. Ideally, one would directly construct a measure of aggregate tail risk dynamics from the time series of stock returns in analogy to dynamic volatility estimated from a GARCH model. But dynamic tail risk estimates are infeasible in a univariate time series model due to the infrequent nature of extreme events. In this section, by using the Extreme Value Theory, we not only overcome this problem, but also analyze the week effect as well as the seasonal effect based on our computation of the small quantile of VaR. However, we should be aware of various layers of uncertainty, which include the parameter uncertainty, model uncertainty, and data uncertainty, in extreme value analysis. In a sense, it is never possible to have enough data in an extreme value analysis.

Here we first examine certain statistical properties of the time series of stock returns, including stationarity, correlations as well as non-normal distributions. Thereafter, we apply the extreme value analysis on the tested AAPL returns sample set. The calendar effect in stock market returns includes day-of-the-week effect, weekend effect, January effect, and holiday effect, etc. It has been widely studied and investigated in finance literature. Studies by Cross (1973)[29], and Rogalski (1984)[76] demonstrate that there are differences in distribution of stock returns for each day of the week. Studies by Baillie and DeGennaro (1990)[7], Berument and Kiymaz (2001)[12] posit that day-of-the-week effect has an impact on stock market volatility. In recent years, another stream of research has considered seasonality in stock returns and volatility, see Saunders (1993)[79], Bouman and Jacobsen (2002)[16], Hirshleifer and Shumway (2003)[47], Kamstra, Kramer and Levi (2003)[57], and Cao and Wei (2005)[19], etc. These studies generally report that calendar anomalies are present in both returns and volatility equations in the stock market. None of these studies, however, test for the possible existence of day-of-the-week and seasonal variation in stock return VaR. Empirical findings in this section show that both the day of the week effect and seasonal effect are present in the AAPL VaR. In the empirical results of the day-of-the-week effect on AAPL tail risk, we observe the lowest VaR of AAPL returns on Fridays and Mondays. We also find that the lower VaR occur on Q1 and Q2 during the test period. AAPL VaR and SPY VaR were compared, the AAPL was found to have its own personality.

1.1.2 Data exploration and statistical analysis

1.1.2.1 Data description

In this study, we examine the daily AAPL stock price activity over the twenty-year period, July 3, 1995 to July 2, 2015. The collection of AAPL daily adjusted closing price was from Yahoo Finance. The adjusted closing price is used to develop an accurate track record of the stock's performance.

Further, use the negative log return to examine extreme losses of the stock. Let p_t denote the adjusted closing price of a stock on day t, then the daily percentage change on the day is defined by

$$r_t = -100 \log \frac{p_t}{p_{t-1}} = 100 \log \frac{p_{t-1}}{p_t}. \tag{1.1}$$

The reason for using the negative returns is that we are mainly interested in the possibility of large losses rather than large gains.

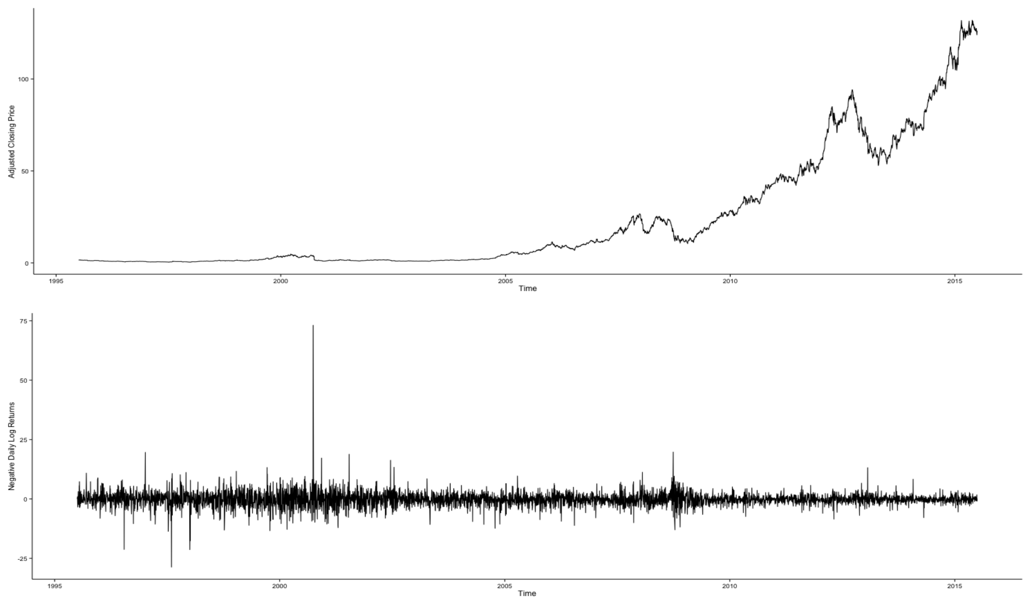

Figure 1.1: Time plots of AAPL from 1995-7-3 to 2015-7-2.

Fig.1.1 shows the time plots of adjusted closing price and negative daily log returns of AAPL stock from July 3, 1995 to July 2, 2015. The upper plot shows that AAPL stock price has skyrocketed over 100 times since 2005. The lower plot shows AAPL negative daily log return time series. We also observe that there are more pronounced peaks than one would expect from Gaussian data. Table 1.1 summarizes the basic statistical characteristics of the whole AAPL stock negative daily log return series. Note that the expected AAPL log returns during the test period is 0.09. The skewness and kurtosis measures are highly significant, and those indicate substantial departures from normality. Since the possibility of time-varying variance and non-

Table 1.1: Summary statistics of the AAPL returns from 1995-7-3 to 2015-7-2

Mean	Range	Std dev	Skewness	Kurtosis	Observations
-0.09	(-28.69, 73.12)	3.05	2.55	70.82	5035

normal behavior have been noticed, we provide formal tests to check the stationarity and normality of the return process.

1.1.2.2 Test for stationary property

The invariance of statistical properties of the return process in time corresponds to the stationarity hypothesis that the joint probability distribution of the returns does not change when shifted in time. It is not obvious whether AAPL returns verify this property in calendar time since financial time series data often have non-stationary behaviors, such as trends, and cycles. Here we use the KPSS test[61], to verify the hypothesis of weak stationarity, i.e. time invariance of the mean value and the autocorrelation function of AAPL returns.

Proceeding in the spirit of Kwiatkowski, Phillips, Schmidt and Shin (1992)[61], we assume that the series $\{r_t\}_{t=1}^{T}$ can be decomposed into the sum of a deterministic trend, a random walk and a stationary error. We express this symbolically by writing

$$r_t = \beta t + \alpha_t + \epsilon_t, \tag{1.2}$$

where the constant β is the trend; ϵ_t is assumed to be stationary; and α_t is a random walk, i.e.

$$\alpha_t = \alpha_{t-1} + u_t.$$

Here $\{u_t\}$ is a white noise series with zero mean and variance σ_u^2.

The hypothesis for the KPSS test is

$$H_0 : \sigma_u^2 = 0 \;\; vs \;\; H_1 : \sigma_u^2 \neq 0.$$

Assume $e_t = r_t - (\hat{\beta} t + \hat{\alpha}_t)$ as residuals of the regression of r_t on an intercept and time trend, $S_t = \sum_{i=1}^{t} e_i$, $t = 1, 2, \cdots, T$, as the partial sum process of the residuals. The KPSS test statistics is

$$KPSS = \frac{T^{-2}\sum_{t=1}^{T} S_t^2}{s^2}, \tag{1.3}$$

where s^2 is a consistent estimator of the long-run variance of α_t.

The rejection rule is that if the value of the $KPSS$ statistic in Eq.(1.3) exceeds the critical values estimated in [61], or the p-value is less than or equal to the significance level α, we reject H_0.

Table 1.2: Tests for the AAPL negative daily log returns from 1995-7-3 to 2015-7-2

	Null Hypothesis H_0	Stats	p-value	Test Result
KPSS test for stationary	The series is stationary around a straight line time trend	0.08	0.1	Accept H_0, the series is stationary.
	The series is stationary around a constant.	0.21	0.1	
Shapiro-Wilk test for normality	The series come from a normally distributed population.	0.86	2.2e-16	Reject H_0, the series does not come from a normal distribution.
Ljung-Box test for correlation	$\rho_1 = \rho_2 = \cdots = \rho_5 = 0$	16.455	0.005658	Reject H_0, the series is not autocorrelated.
	$\rho_1 = \rho_2 = \cdots = \rho_{10} = 0$	30.34	0.0007536	
	$\rho_1 = \rho_2 = \cdots = \rho_{15} = 0$	42.303	0.0002018	

As shown in Table 1.2, for the null hypothesis which claims that the series follows a straight line time trend with stationary errors, i.e. $\beta \neq 0$ in Eq.(1.2), the p-value is 0.1 and the corresponding $KPSS$ statistic is 0.080433. In addition, for the null hypothesis that the series is stationary around a constant rather than a trend with stationary errors, i.e. $\beta = 0$ in Eq.(1.2), the p-value is 0.1 and the corresponding $KPSS$ statistic is 0.20611. In conclusion, the KPSS test result indicates that the considered AAPL returns is stationary from July 3, 1995 to July 2, 2015.

1.1.2.3 Test for normality

In studying the financial time series, one common assumption is that the process follows normal distribution. However, it is barely true in the real stock return series. Our study shows that the AAPL stock returns are not normally distributed. We begin by forming a QQ-plot of the AAPL negative daily log returns sample set against the normal distribution, in order to confirm that an assumption of normality is unrealistic, and that the innovation process has fat tails or is leptokurtic – see Fig.1.2.

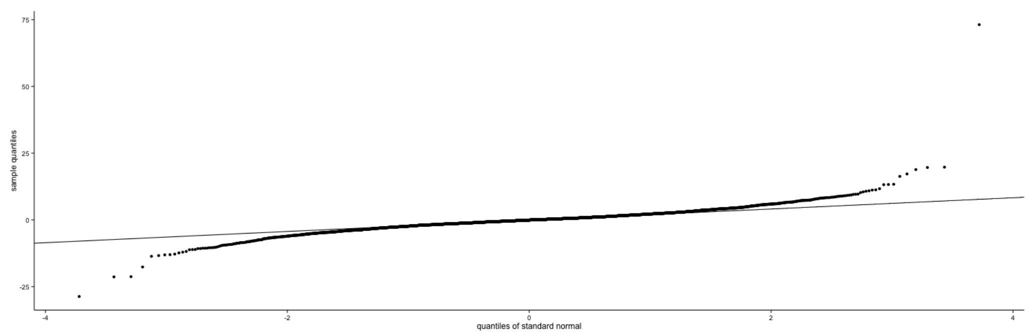

Figure 1.2: QQ-plot of AAPL returns from 1995-7-3 to 2015-7-2 against normal distribution.

We also use the Shapiro-Wilk test[81], which has been demonstrated as one of the most powerful normality tests by Razali and Wah (2011)[74], to verify an empirical fact that the AAPL returns do not have the normality property. The Shapiro-Wilk test utilizes the null hypothesis principle to check whether the series $\{r_t\}_{t=1}^T$ comes from a normally distributed population. The Shapiro-Wilk test statistic is defined as

$$W = \frac{(\sum_{t=1}^T a_t r_t)^2}{\sum_{t=1}^T (r_t - \bar{r})^2}, \tag{1.4}$$

where r_t is the t-th order statistic; $\bar{r}$ is the sample mean; $(a_1, a_2, \cdots, a_T)$ are the weights[1]. The value of W lies between zero and one. Small values of W lead to the rejection of normality whereas a value of one indicates the normality of data. We reject the null hypothesis if the p-value of the test is less than the predetermined significance level.

Apply the Shapiro-Wilk test on the considered AAPL returns, we get the Shapiro-Wilk statistic $W = 0.86$. The p-value is less than 2.2e-16, see Table 1.2. Hence, we reject the null hypothesis at the significant level 1% and conclude that the AAPL returns are not normally distributed during above time period.

1.1.2.4 Test for correlations

In the finance literature, testing for zero autocorrelations has been used as a tool to verify the efficiency of the market hypothesis. Since applying extreme value theory on a data set suggests that the time series are highly uncorrelated with a common cumulative distribution function, we need to check the correlations of the AAPL returns.

We begin by considering the autocorrelation function of a time series $\{r_t\}$. The correlation between r_t and its past values r_{t-l} is called the lag-l autocorrelation of $\{r_t\}$ and is commonly denoted by ρ_l. Under the weakly stationary assumption, we assume ρ_l is a function of l only, i.e.

$$\rho_l = \frac{Cov(r_t, r_{t-l})}{\sqrt{Var(r_t)Var(r_{t-l})}} = \frac{Cov(r_{l+1}, r_1)}{Var(r_1)} = \frac{\gamma_l}{\gamma_0}, \tag{1.5}$$

where the property $Var(r_t) = Var(r_1) = \gamma_0$ for a weakly stationary series is used.

[1] $(a_1, a_2, \cdots, a_T) = \frac{\mathbf{m}^T V^{-1}}{(\mathbf{m}^T V^{-1} V^{-1} \mathbf{m})^{1/2}}$; $\mathbf{m} = (m_1, m_2, \cdots, m_n)^T$, $m_1, m_2, \cdots, m_n$ are the expected values of the order statistics of independent and identically distributed random variables sampled from the standard normal distribution, and V is the covariance matrix of those order statistics.

For a given sample of returns $\{r_t\}_{t=1}^{T}$, let $\bar{r} = (\sum_{t=1}^{T} r_t)/T$ is the sample mean. The lag-l sample autocorrelation of $\{r_t\}$ can be represented as:

$$\hat{\rho}_l = \frac{\sum_{t=l+1}^{T}(r_t - \bar{r})(r_{t-l} - \bar{r})/(T - l + 1)}{\sum_{t=1}^{T}(r_t - \bar{r})^2/(T - 1)}, \quad 0 \leq l \leq T - 1. \tag{1.6}$$

If a time series is not autocorrelated, then the estimates of $\hat{\rho}_l$ will not be significantly different from 0.

Fig.1.3 shows the sample autocorrelation coefficient $\hat{\rho}_l$ plotted against different lags l (measured in days), along with the 95% confidence band around zero for AAPL negative daily log returns, for the period July 3, 1995 to July 2, 2015. The dashed lines represent the upper and lower 95% confidence bands $\pm\frac{1.96}{\sqrt{T}}$, where the time length for our AAPL returns is $T = 5036$ days. Fig.?? shows a small autocorrelation in AAPL daily log price changes. Even in the cases where the autocorrelations are outside the confidence bands, the autocorrelation coefficients are quite small, less than 5%.

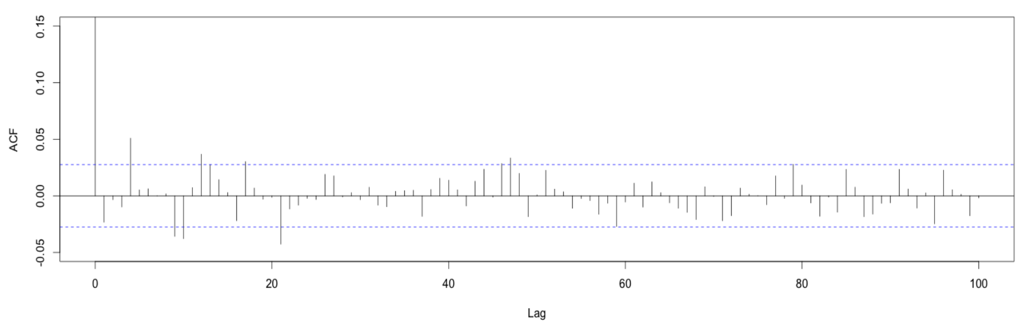

Figure 1.3: Sample autocorrelation coefficients up to 100 lags for AAPL returns from 1995-7-3 to 2015-7-2.

Besides using the graphical plot to check autocorrelation, we also apply a formal statistic test: the Ljung-Box test by Ljung and Box (1978)[62], which checks serial correlation of the time series. The null and alternative hypothesis of the Ljung-Box test is

$$H_0 : \rho_1 = \cdots = \rho_m = 0 \ vs \ H_1 : \rho_i \neq 0 \text{ for } i \in \{1, 2, \cdots, m\}$$

As Ljung and Box[62] proposed, under the assumption that $\{r_t\}_{t=1}^{T}$ is an i.i.d. sequence with certain moment conditions, the modified Portmanteau statistic is defined as

$$Q(m) = T(T+2)\sum_{l=1}^{m}\frac{\hat{\rho}_l^2}{T-l}. \tag{1.7}$$

It is asymptotically a chi-squared random variable with m degrees of freedom, i.e. $Q(m) \sim \chi_m^2$ under the null hypothesis H_0. In the definition of $Q(m)$, T is the sample size, $\hat{\rho}_l$ is the sample autocorrelation at lag l, and m is the number of lags being tested.

The decision rule is to reject H_0 if $Q(m) > \chi_{1-\alpha,m}^2$ for significance level α, where $\chi_{1-\alpha,m}^2$ denotes the $100(1-\alpha)$th percentile of a chi-squared distribution with m degrees of freedom. Also, one should reject H_0 if the p-value of $Q(m)$ is less than or equal to the significance level α.

The test result in Table 1.2 confirms that the AAPL returns does not have strong serial correlations during the test period. The p-values of lag 5, lag 10 and lag 15 Ljung-Box test for AAPL returns are all less than significant level 1%.

Based on the statistical analysis for AAPL negative daily log returns, we discovered that the AAPL returns is a stationary, uncorrelated time series, yet is not normally distributed. Some computations of VaR are based on the assumption that the series $\{r_t\}$ is normally distributed, or has t-distribution, see [63][5][11]. That is the main reason why these study can use volatility to estimate VaR. However, the real time series $\{r_t\}$ may not follow any known distributions, such as the normal or t-distribution. To overcome the difficulty of $\{r_t\}$ having an unknown distribution, we compute the VaR of AAPL returns by applying Extreme Value Theory, which avoid making assumptions of the distribution of $\{r_t\}$.

1.1.3 Methodology

While exposure to risk can be summarized as a single number by estimating the VaR, which is defined by Jorion[55] as " the worst expected loss over a great horizon within a given confidence level ", it is crucial to have an accurate estimate on VaR. Following the approach by Longin (1999a,b) [65][64], and Ruey S. Tsay (2007)[85], we introduce the statistical principles behind VaR as well as the VaR estimation methodology in this section.

1.1.3.1 VaR of a time series

VaR is the amount that might be lost in a portfolio of assets over a specified time period T with a specified small failure probability α, usually set as 0.01 or 0.05. Suppose a random variable X characterizes the distribution of negative returns of a portfolio over a certain time horizon T, the right-tail α-quantile of the portfolio is then defined to be the VaR_α such that

$$Pr(X \leq \mathrm{VaR}_\alpha) = 1 - \alpha. \tag{1.8}$$

The VaR_α is the largest value for X such that the probability of a loss over the time horizon T is no more than $1 - \alpha$. Although the parameters T and α are arbitrarily chosen, the analysis in this study does not refer to the process of choosing the two parameters of VaR which were considered to be $T = 1\text{day}, \alpha \in \{0.01, 0.05, 0.1\}$.

The crux of being able to provide an accurate estimate for VaR is in estimating the cutoff return VaR_α. Studies of VaR are essentially concerned with the estimation of the cumulative distribution function (CDF) of portfolio negative returns and/or its quantile, especially the upper tail behavior of the loss CDF. Therefore, the CDF of $\{X_t\}$ is the focus of econometric modeling. Different methods for estimating the CDF give rise to different approaches to VaR estimation.

1.1.3.2 Extreme Value Theory approach to Value-at-Risk

In this section, we further estimated the upper tail behavior of the AAPL returns CDF by using the extreme value approach. Extreme Value Theory (EVT) is experiencing a boom in the financial field, especially with respect to its application to the market risk. Its appearance as a popular instrument for estimating VaR can be explained as a consequence of two factors. On the one hand, the assumption of the normality of financial markets does not reflect the reality of the situation. As a consequence, the VaR estimation methods which are based on the normality assumption underestimates the risk. Historical or Monte Carlo simulation methods arise as alternative methods. But given the difficulties and the inefficiencies of these methods, EVT is sought out as a new solution.

The mathematical foundation of EVT is based on the class of extreme value limit theories, originally posited by Fisher and Tippett (1928)[34] and later derived rigorously by Gnedenko (1943)[39]. The central result in EVT is that the extreme tail of a wide range of distributions can approximately be described by the Generalized Pareto distribution (GPD), which is derived by Smith (1989)[**?**], Davison and Smith (1990)[30].

For a random variable X, we first fix some high threshold μ and consider the distribution of excess values $Y = X - \mu$, which is defined as:

$$F_{\mu}(y) = Pr(X - \mu \leq y | X > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)}, \tag{1.9}$$

where F is the underlying distribution of X, F_{μ} is the conditional excess distribution function. In fact, Pickands (1975)[72] introduced the GPD as a two parameter family of distributions for exceedance over a threshold.

Extreme Value Theory. *Assume $\{X_t\}$ is a sequence of stationary, uncorrelated random variables with distribution F. For any $\mu > 0$, let F_{μ} be the conditional excess* *distribution function, for random variables defined in (1.26) with* $Y_t = X_t - \mu$*. Let* $\omega_F = sup\{x : F(x) < 1\}$*, then*

$$\lim_{\mu \to \omega_F} F_\mu(y) = H_{\sigma_\mu,\xi}(y)$$

where $H_{\sigma_\mu,\xi}(y)$ *is called GPD, specified as*

$$H_{\sigma_\mu,\xi}(y) = 1 - \left(1 + \xi \frac{y}{\sigma_\mu}\right)_+^{-1/\xi}. \tag{1.10}$$

The parameters of GPD are the scale parameter σ_μ *and the shape parameter* ξ*.*

Although we may not know the distribution of each individual random variable X_t, EVT specifically describes the tail distribution. The tail fatness of a distribution is reflected by the shape parameter:

- $\xi < 0$ refers to thin tails;
- $\xi = 0$ implies that the kurtosis is 3 as for a standard normal distribution;
- $\xi > 0$ implies fat tails.

Therefore, the shape parameter measures the speed with which the distribution's tail approaches zero. The fatter the tail, the slower the speed and the higher the shape parameter. Using GPD, EVT models the right tail of the distribution, i.e. the returns in excess of a threshold. Because we are interested in extreme loss, the EVT analysis is developed on negative stock returns. As tested in previous section that AAPL returns are stationary and serially uncorrelated, its VaR is analyzed by using EVT.

In the literature, an optimal threshold is selected by employing graphical methods, the mean excess plot[2] and Hill plot[3]. The mean excess plot for threshold exceedance

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[2]Details about the mean excess plot are described in Davison and Smith (1990)[30].

[3]Technical details about Hill plot can be found in Hill (1975)[45].

is a diagnostic plot drawn before fitting any model and can therefore give guidance about what threshold to use. One difficulty with this method is that the sample mean excess plot typically shows very high variability, particularly at high thresholds. This can make it difficult to decide whether an observed departure from linearity is in fact due to the failure of the GPD or is just sample variability. As an alternative approach to choose threshold, the Hill plot has some advantages. It displays the estimated values of the shape parameter ξ as a function of the cut-off threshold, so that one can easily find some interval of candidate cut-off points that yields stable estimates of the shape parameter ξ. Here we use both approaches to choose a reasonable threshold, see Figure 1.4.

According to the research of Hosking and Wallis (1987)[49], for the shape parameter $\xi > -0.5$, it is shown that maximum likelihood regularity conditions are fulfilled and that maximum likelihood estimates $\{\hat{\xi}_n, (\hat{\sigma}_\mu)_n\}$ based on a sample of n excesses are asymptotically normally distributed. Therefore, we choose to use the parametric approach, maximum likelihood method (MLE) to estimate the two parameter in GPD, which are the shape parameter ξ and the location parameter σ_μ.

Next, we make explicit the relationship between excess value and a observed return series $\{r_t\}$. Assume that $\{r_t\}$ have distribution F, and a high enough threshold μ is given. We define the number of exceedance of the threshold μ within $\{r_1, \cdots, r_n\}$ as:

$$N_\mu = card\{t : r_t > \mu, t = 1, \cdots, n\}.$$

Then the conditional excess distribution function can be presented as:

$$F_\mu(y) = Pr(r_t - \mu \leq y | r_t > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)}.$$

Denote $\overline{F}_\mu(y) = 1 - F_\mu(y)$, then

$$\overline{F}_{\mu}(y) = Pr(r_t - \mu > y | r_t > \mu) = \frac{\overline{F}(\mu + y)}{\overline{F}(\mu)},$$

which is equivalent to

$$\overline{F}(\mu + y) = \overline{F}(u)\overline{F}_{\mu}(y).$$

Consequently, the estimators of $\overline{F}(u)$ and $\overline{F}_{\mu}(y)$ can be written as:

$$\widehat{\overline{F}(u)} := \frac{1}{n}\sum_{i=1}^{n} I(X_i > \mu) = \frac{N_{\mu}}{n},$$
$$\widehat{\overline{F}_{\mu}(y)} := 1 - H_{\hat{\sigma}_{\mu},\hat{\xi}}(y) = \left(1 + \hat{\xi}\frac{y}{\hat{\sigma}_{\mu}}\right)_{+}^{-1/\hat{\xi}},$$

where $\hat{\xi}$ and $\hat{\sigma}_{\mu}$ are (maximum likelihood) estimators of the shape parameter ξ and location parameter σ_{μ}. Therefore the tail estimator can be written as

$$\widehat{\overline{F}(\mu + y)} = \frac{N_{\mu}}{n}\left(1 + \hat{\xi}\frac{y}{\hat{\sigma}_{\mu}}\right)_{+}^{-1/\hat{\xi}}. \tag{1.11}$$

This relationship between probabilities allows us to obtain VaR for the original asset return series $\{r_t\}$. More precisely, for a specified small probability α

$$\alpha = Pr(r_t > \mu + y) = \overline{F}(\mu + y),$$

where the α-th upper tail quantile VaR of $\{r_t\}$ is $\mu + y$. Consequently, for a given small probability α, one can check that the VaR of holding a long position in the asset underlying return $\{r_t\}$ is

$$\mathrm{VaR}_{\alpha} = \begin{cases} \mu + \frac{\hat{\sigma}_{\mu}}{\hat{\xi}}\left((\frac{n}{N_{\mu}}\alpha)^{-\hat{\xi}} - 1\right), & \hat{\xi} \neq 0 \\ \mu + \hat{\xi}\ln(\frac{n\alpha}{N_{\mu}}), & \hat{\xi} = 0 \end{cases} \tag{1.12}$$

We preferred to use the extreme value approach, or named GPD approach in this study to tail estimation mainly for three reasons. One is that in finite samples of the order of points from typical return distributions, EVT quantile estimators are more efficient than the historical simulation method. Second, considering the fact that most financial returns series are asymmetric, the EVT approach is advantageous over models which assume symmetric distributions such as t-distributions, or generalized error distribution. Third, comparing with Hill method which is designed specifically for the heavy tail ($\xi > 0$) data, the EVT approach to VaR has larger applicability since it also applicable to light tail ($\xi = 0$) cases or even short tail ($\xi < 0$) cases.

1.1.4 VaR analysis of AAPL and SPY

In this section, one-day-ahead VaR forecasts are adopted along with the 5%, 1% and 0.1% level of significance in the empirical investigation. In order to make a comparison, we use AAPL as well as SPDR S&P 500 ETF (AMEX:SPY) daily negative log returns to compute the VaR and related statistical properties. The daily AAPL negative log returns data set is introduced in section 2.1. The SPDR S&P 500 ETF is the first and most popular ETF in the U.S.. It tracks one of the most popular indexes in the world, the S&P 500 Index. The objective of the SPY is to duplicate as closely as possible, before expenses, the total return of the S&P 500 Index. Since the performance of SPY is thought to be representative of the stock market as a whole, we compare VaR between AAPL returns and SPY returns to find the characteristics of AAPL. The daily SPY negative log returns are examined for the period of July 3, 1995 to July 2, 2015, which is the same test period as AAPL returns. We also ran Shapiro-Wilk test and Ljung-Box test in SPY returns and found no evidence against the non-normal and non-correlated assumptions for the series.

Before applying the extreme value approach to VaR on our data sets, it is necessary to choose a specific threshold, confining the estimation to those observations that are above the given threshold. As mentioned in section 1.1.3, we chose the threshold through graphical procedures: Mean Excess plot and Hill plot. Fig.1.4 shows the Hill

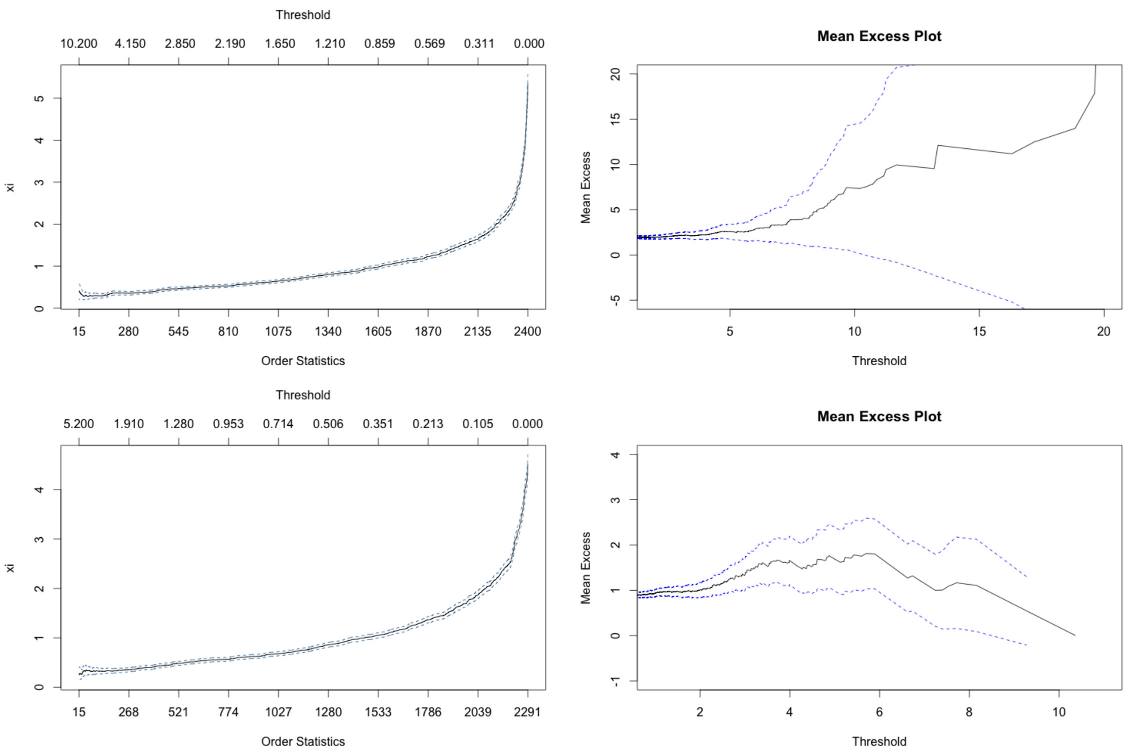

Figure 1.4: Hill plots (left) and mean excess plots (right) for the AAPL returns (top) and SPY returns (lower) with 95% asymptotic confidence bounds (dotted line) based on the normal asymptotics of the estimator, depending on different threshold values μ.

plots and Mean Excess plots, with 95% confidence bands, for the AAPL returns and SPY returns respectively. Since for the generalized pareto distribution, a possible choice of threshold is given by the value, above which the empirical mean excess value is approximately linear. The right-hand plots of Fig.1.4 indicate a reasonable choice for AAPL returns where threshold should around 5, and SPY returns threshold should around 2. The Hill estimator estimates the shape parameter ξ in the GPD model as a function of the N_μ exceedances upper order statistics in the return sample. The estimate is taken in the N_μ–region where the plot does not change much. For the AAPL returns, $N_\mu = 280$, with corresponding threshold $\mu = 4.153575$; for SPY returns, $N_\mu = 268$, with corresponding threshold $\mu = 1.911077$ would be reasonable.

Table 1.3: GPD parameter estimators and one-day-ahead VaR forecasts for AAPL and SPY returns.

Negative daily log returns from 1995-7-3 to 2015-7-2	AAPL	SPY
Threshold μ	4.153575	1.911077
Exceedances N_μ	280	268
Shape parameter ML estimator $\hat{\xi}$	0.2619561	0.2411802
Scale parameter ML estimator $\hat{\sigma}_\mu$	1.6145412	0.7537094
VaR($T = 1$ day, $\alpha = 5\%$,)	4.327371	1.958427
VaR($T = 1$ day, $\alpha = 1\%$)	7.65061	3.463027
VaR($T = 1$ day, $\alpha = 0.1\%$)	15.64864	6.935867

Table 1.3 contains the empirical results on the AAPL and SPY daily negative log returns for the whole sample period using a total of 5035 observations. The upper part of Table 1.3 contains the threshold values and the corresponding exceedances values as well as the maximum likelihood GPD parameter estimates used in the construction of tail estimators of AAPL and SPY negative daily log returns from July 3, 1995 to July 2, 2015. The shape parameter estimates of the right tail are 0.2619561 and 0.2411802 for AAPL and SPY returns, respectively, which indicate that the AAPL returns show fatter tails than the SPY returns. Those values and estimators enable us to estimate the upper 5%, 1% and 0.1% quantile of the AAPL and SPY negative daily price changes. As is obvious from the estimation of the quantile by means of extreme value theory in this table, the AAPL returns VaR are much larger, even more than two times, than SPY returns VaR. Therefore, AAPL exhibits a more downside risk than SPY.

In order to visualize the model (1.12) accuracy, we backtest the extreme value approach on the AAPL and SPY returns and show the fitness summary in Fig.1.5 and Fig.1.6. At the top panel of Fig.1.5 and Fig.1.6, the probability density function of the empirical distribution and the log probability density function of the empirical distribution are all plotted along with the estimated GPD. The scatterplot and QQ-plot of residuals are at the lower panel. Based on those plots, we find that the

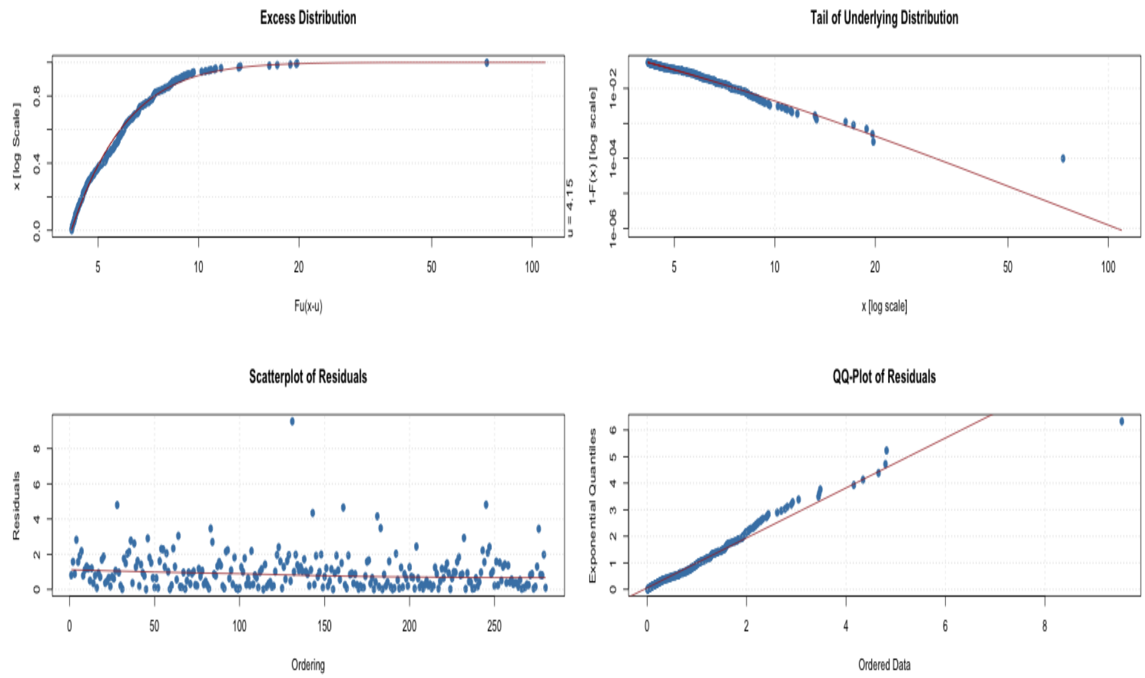

Figure 1.5: GPD tail estimates fitness summary of the AAPL returns distribution and the innovations distribution.

estimates fit the given AAPL and SPY returns quite well, even in the far end tail. It confirms that the assumption of an underlying heavy tailed distribution is well in line with the data. In this context, the corresponding estimate of the upper 5%, 1% and 0.1% quantile of the VaR seems very plausible.

After modeling the distribution of AAPL and SPY returns and computing all the necessary quantiles, we proceed to the determination of VaR. Using a 2520 day (approximately 10 years) rolling window, we apply an iterative procedure of the EVT based model (1.12) to predict the 1-day ahead, 5%, 1% and 0.1% VaR for the period July 3, 2005 to July 2, 2015. The moving window design starts with the estimation of the VaR model using in-sample period data to predict the 1-day ahead VaR estimate. Then, we move the in-sample period forward by one period to iterate the estimation and prediction. The whole process keeps running forward step by step until the end of the entire data set. Before applying the procedure, we choose the corresponding 95-th

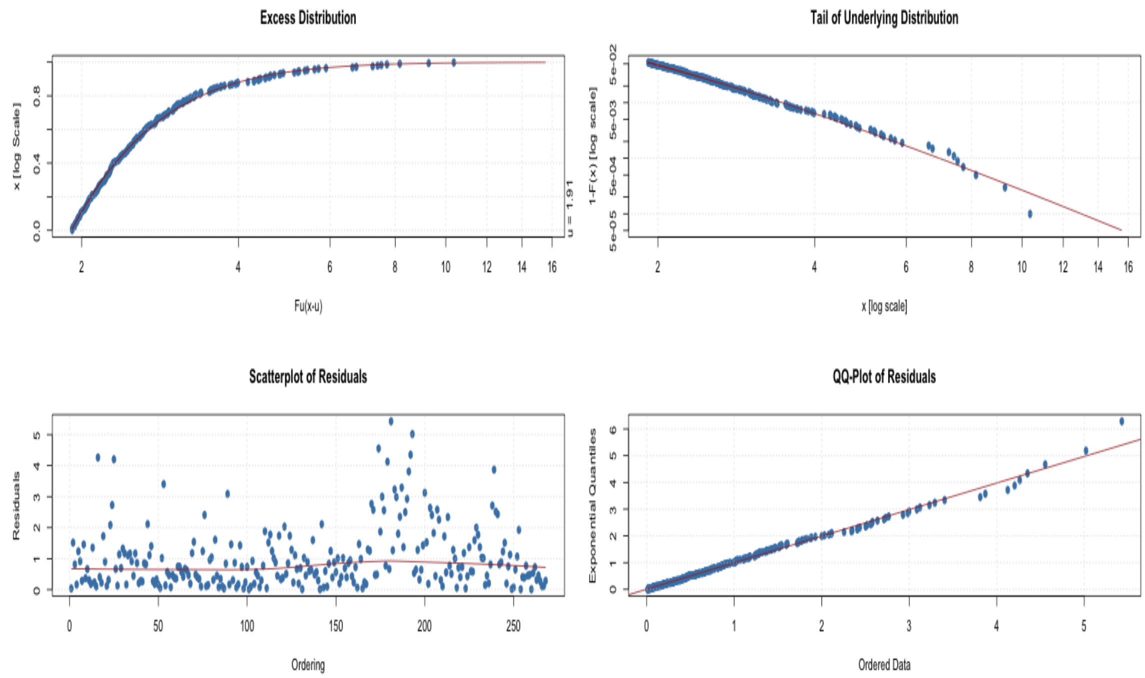

Figure 1.6: Diagnostic plots for GPD fit to SPY daily negative log returns.

sample quantile as the threshold of each in-sample period data. This yields a total of 2516 out-of-sample VaR forecasts for AAPL and SPY returns, respectively. The results obtained for VaR along with the negative log returns of the AAPL and SPY are shown in the following figures. From Fig.1.7, we find that the AAPL returns VaR slightly increased during the financial crisis of 2008. During the middle of 2010 to 2013, it had a decreasing trend. Since then, the AAPL returns VaR seems stable. If we take SPY returns VaR as a comparison, except the increasing during the financial crisis of 2008, we can see that it has been stable for the last decade.

1.1.5 Day-of-the-week effect on AAPL Value-at-Risk

Notice that the definition of VaR is based on the upper tail of a loss function. The reason we use the negative returns is that loss occurs when the returns are negative for a long financial position. We write the whole sample set as $\{r_t\}$ ={AAPL Negative daily log returns from July 3, 1995 to July 2, 2015}.

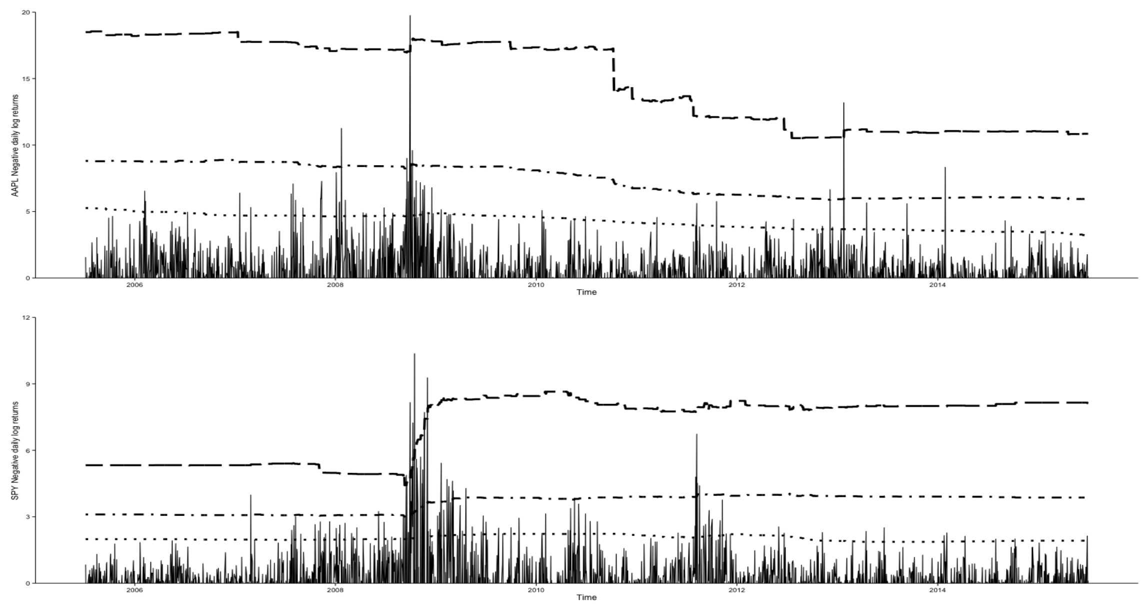

Figure 1.7: 1-day-ahead, 5%(dotted), 1%(dotdash) and 0.1%(longdash) VaR of AAPL (upper) and SPY (lower) returns from 2005-7-6 to 2015-7-2.

To formally test the timing and existence of weekly patterns, we divide the whole data set $\{r_t\}$ to five subsets by day-of-the-week, which is written as:

$$\{d_{it} r_t\}, \quad i = 1, 2, 3, 4, 5,$$

where d_{it} are dummy variables such that if day t is a Monday $d_{1t} = 1$, if $d_{1t} = 0$ remove the data; if day t is a Tuesday $d_{2t} = 1$, if $d_{2t} = 0$ remove the data, etc. The five subsets are the AAPL return time series for Monday through Friday respectively. The basic statistical characteristics of the five return series are calculated and shown in Table 1.4. The AAPL mean returns is calculated to observe differences of expected returns during the week. The hypothesis of equal expected returns for each trading day of the week is rejected for the testing period. Our results show that the highest returns occur on Mondays and the lowest returns occur on Fridays, which have a negative average return. The standard deviation indicates that the trading risk of Fridays is the highest among that of the week. There is less fluctuation in Mondays and Tuesdays return,

Table 1.4: Summary statistics of the AAPL returns by day-of-the-week

	Mean	Range	Std dev	Skewness	Kurtosis	Obs
Mon.	-0.20	(-13.02, 19.75)	2.89	0.51	5.78	949
Tue.	-0.08	(-17.64, 13.25)	2.82	-0.17	3.18	1032
Wed.	-0.12	(-28.69, 18.84)	2.98	-0.26	12.17	1034
Thu.	-0.14	(-21.27, 13.19)	3.03	-0.54	4.52	1012
Fri.	0.10	(-21.36, 73.12)	3.47	8.91	194.73	1008

but there are large fluctuations in Fridays return. The table also reports skewness and kurtosis for the return series of each weekday. The distribution of Mondays and Fridays return are positively skewed while the distribution of all other sample return are negatively skewed, indicating that they are nonsymmetric. Furthermore, Fridays and Wednesdays return exhibit high levels of kurtosis, indicating that these distributions have thicker tails than a normal distribution. These initial findings show that the day-of-the-week returns are not normally distributed, they are skewed and leptokurtic. Moreover, on average, an investor buying stock on Friday afternoon and then sell it by Monday afternoon may make more profit. Next, we examine the

Table 1.5: Normality and independency tests of the AAPL returns by day-of-the-week

	W	Q(5)	Q(10)	Q(15)
Mon.	$<2.2e$-16	0.2963	0.5304	0.2641
Tue.	2.713e-15	0.7911	0.08139	0.1224
Wed.	$<2.2e$-16	0.8355	0.7908	0.5826
Thu.	$<2.2e$-16	0.5063	0.8421	0.2699
Fri.	$<2.2e$-16	0.06514	0.1182	0.08148

normality and independency of the day-of-the week return series. Table 1.5 reports p-values of the Shapiro-Wilk test and the Ljung-Box Q statistics for the AAPL returns at 5-, 10- and 15- day lags. We use the Shapiro-Wilk test to test the normality of every subset. W here is the Shapiro-Wilk test statistic in Eq.(1.4). The normality test result shows that the p-value of every weekday subset is far less than significant level 1%. It indicates that none of these AAPL return subsets has normal behavior during the test period. For all series, the Ljung-Box test is applied to test the serial correlation. $Q(m)$ in the table is the Portmanteau statistic in Eq.(1.7). The Ljung-Box test result confirms that none of the AAPL return subsets has serial correlations. Rather than using a single value for lag m, we choose three different lags $m = 5, 10, 15$ to test the correlation of each series. Even the minimum p-value of each series is greater than the usual significant level 5%. Therefore, we can approximately view all AAPL day-of-the-week return series as stationary, independent, non-normal distributed series. Based on the above tests, in order to test the weekly effects on AAPL VaR, we apply the VaR estimation approach introduced in section 2 to capture the day-of-the-week effect on AAPL returns VaR.

We apply the extreme value approach to the negative daily log returns of AAPL day-of-the-week series from July 3, 1995 to July 2, 2015. Table 1.6 and Table 1.7 summarize some estimation results of the shape parameter ξ and VaR. We applyed the maximum likelihood method to estimate parameters of the generalized Pareto distribution for AAPL returns and we calculated the upper 5% quantile VaR based on Eq.(1.12). Table 1.6 shows the shape parameter estimates for the day-of-the-

Table 1.6: Estimators of GPD parameters and forecast VaRs of AAPL returns by day-of-the-week

	ξ	σ_μ	μ	N_μ	$\mathrm{VaR}_{0.05}$
Mon.	0.2688498	1.6041546	3.795551	60	4.184124
Tue.	0.01042914	1.76975174	4.031559	69	4.546602
Wed.	0.3107167	1.5597188	3.871468	68	4.317635
Thu.	-0.01453561	1.95606434	4.032778	68	4.609665
Fri.	0.3825541	1.4247496	3.367018	71	3.888713

week excess returns respectively and corresponding VaR estimates. In Table 1.6, the threshold μ was chosen via the Hill Plot of the AAPL returns. Around the exceedances N_μ selected by Hill plot, the estimates of the shape parameter are stable for the extremes. Based on the estimated AAPL upper 5% quantile VaR, we found that Friday and Monday VaR is smaller than that of the rest of the week.

To further investigate systematic weekday differences for AAPL VaR, we also estimate the upper 5% quantile VaR via the same threshold $\mu = 4.032778$, which is the highest threshold in Table 1.6. Using the same threshold allows a better comparison of the day-of-the-week VaRs. The results in Table 1.7 are mostly consistent with

Table 1.7: Estimators pf GPD parameters and VaR of AAPL returns by day-of-the-week via the same threshold $\mu = 4.032778$

	ξ	σ_μ	N_μ	$\text{VaR}_{0.05}$
Mon.	0.2725663	1.6566470	51	4.153487
Tue.	0.01126478	1.76707300	68	4.521224
Wed.	0.3821768	1.4131813	64	4.347029
Thu.	-0.01453561	1.95606434	68	4.609665
Fri.	0.4554803	1.4688451	47	3.931803

the previous findings in Table 1.6. VaRs due to the different tail shapes and the tail fatness of distributions are reflected by the shape parameter ξ. The shape parameter measures the speed with which the distribution's tail approaches zero. The fatter the tail, the slower the speed and the higher the shape parameter. From the results in Table 1.6 and 1.7, the right tail fatness of Friday excess returns is the highest while that of Thursday is the smallest. The most interesting feature of the results is that the day-of-the-week effect on AAPL VaR is examined. Low Friday and Monday VaR and high Thursday VaR are observed for the AAPL returns.

In capturing the character of AAPL, we present the estimated VaR of SPY returns in Table 1.8, which were obtained via the same approach as applied on AAPL returns. The S&P 500 Index is composed of five hundred selected stocks in which AAPL weights 3.88% of total assets. By comparing the VaR of AAPL and SPY, we were able to better capture the characteristic of AAPL. Table 1.8 shows that during the test period July 3, 1995 to July 2, 2015, the upper 5% quantile VaR of SPY Tuesday

Table 1.8: Estimators of GPD parameters and VaR of SPY returns by day-of-the-week

	ξ	σ_μ	μ	N_μ	$\text{VaR}_{0.05}$
Mon.	0.2346827	1.0624234	1.677897	63	1.989292
Tue.	0.1305324	0.6817181	1.665352	68	1.856927
Wed.	0.3743073	0.5765108	1.780462	65	1.918262
Thu.	0.1189783	0.8888867	1.777264	65	2.003223
Fri.	-0.06331137	0.91646749	1.714381	64	1.93167

and Wednesday returns are smaller than that of the rest of the week. As the top one holding stock of SPY, AAPL naturally has a positive correlation with SPY. However, based on above results, we find the day-of-the-week effect on AAPL VaR and SPY VaR are different. Moreover, the day-of-the-week VaRs of SPY are much more stable and smaller than that of AAPL. The interesting finding about AAPL is that there is comparatively a high mid-week risk and a low Monday and Friday risk is observed during the test period.

There are many reasons that may cause the day-of-the-week effect on AAPL VaR. Possible explanations for the day-of-the-week effect include the dividends effect, weekend effect and trading activity effect. Apple usually pay its shareholders quarterly dividend on Thursday. It may cause lower trading activity on the following Friday. Due to the positive correlation between trading activity and returns, the trading activity during the middle of the week averagely is higher than that of Monday and Friday. In addition, options expiration can influence the overall market as well as specific equities, especially on the last trading day before expiration. AAPL Weeklys option are listed to provide expiration opportunities every week. Weeklys are typically listed on Thursdays and expire on Fridays[4]. Weeklys options can provide opportunities for investors to implement more targeted buying, selling or spreading strategies, which may be the reason why the AAPL returns has an increasing trend during Thursday

[4] Weeklys are not listed if they would expire on a 3rd Friday or if a Quarterly option would expire on the same day

to Friday. Further research about the exact reasons of weekly effect on AAPL VaR is needed.

1.1.6 Seasonal Effect on AAPL VaR

Despite finding a weekly pattern in AAPL, one should stress that seasonal effect is by far more relevant in determining stock performance because a three-month period on a financial calendar acts as a basis for the reporting of stock earnings and the paying of dividends. In order to investigate systematic quarterly effects on the stock AAPL, we divide the sample data $\{r_t\}$ into the following four groups:

$$\{r_t | t \in Q_i\}\,, \quad i = 1, 2, 3, 4,$$

which are referred to the four quarters AAPL returns. A quarter refers to one-fourth of a year and is typically expressed as Q. Basic tests to examine the seasonal pattern in AAPL returns are carried out next. Table 1.9 contains the summary statistics

Table 1.9: Summary statistics of AAPL returns by quarter

	Mean	Range	Std dev	Skewness	Kurtosis	Obs
Q1	-0.14	(-21.36, 19.62)	2.98	-0.25	5.7	1227
Q2	-0.04	(-12.09, 16.30)	2.58	-0.23	3.11	1264
Q3	-0.10	(-28.69, 73.12)	3.65	6.10	130.62	1272
Q4	-0.07	(-13.37, 17.21)	2.87	-0.08	3.33	1273

for the four quarter AAPL returns. During the trading period from July 3, 1995 to July 2, 2015, all quarters have positive average return. The first quarter Q1 has the largest average return. Significantly a large range and standard deviations are observed for the Q3 return. The AAPL Q2 return has the smallest standard deviation and average returns than those of the rest seasons. Moreover, the kurtosis indicates the Gaussian behavior of Q2 return since the kurtosis of Q2 return is around 3. Table 1.10 presents that all four quarter AAPL returns are not normally distributed since

Table 1.10: Normality and independent test p-value of the AAPL returns by quarter

	W	Q(5)	Q(10)	Q(15)
Q1	<2.2e-16	0.01275	0.001784	0.001235
Q2	<2.2e-16	0.3114	0.2341	0.07385
Q3	<2.2e-16	0.1665	0.1343	0.01003
Q4	<2.2e-16	0.6867	0.7337	0.2699

all the p-value of the Shapiro-Wilk test are less than 2.2e-16, which are less than the significant level 1%. The correlation test results indicate that quarterly AAPL returns are uncorrelated because the smallest p-value among three different lags correlation tests for each group is smaller than the significant level 1%. While for AAPL Q1 returns, the 10 lag and 15 lag Ljung-Box tests p-value indicate the serial correlation of the data set during the test period. However, the 5 lag Ljung-Box test p-value is greater than the significant level 1%, which indicate that there is only weak correlation of AAPL Q1 returns. Therefore, we still can process the four datasets as non-normal, independent time series.

Table 1.11: Estimated GPD parameters and VaR of AAPL returns by quarter

	ξ	σ_μ	μ	N_μ	$\text{VaR}_{0.05}$
Q1	0.1582695	1.6388243	4.110644	77	3.40719
Q2	0.1856602	1.0737989	3.724132	84	4.037854
Q3	0.4833337	1.6498127	3.931378	79	4.308532
Q4	-0.009111332	2.049748142	4.038558	82	4.557201

Table 1.12: Estimated GPD parameters and VaR of AAPL returns by quarter via the same threshold $\mu = 4.110644$.

	ξ	σ_μ	N_μ	$\text{VaR}_{0.05}$
Q1	0.1582695	1.6388243	77	3.40719
Q2	0.2627277	0.9750694	63	4.107555
Q3	0.4524895	1.8346705	64	4.302533
Q4	-0.04550122	2.23903661	75	4.492442

Next, we implement the same extreme value approach to VaR on the four seasonal AAPL returns. Table 1.11 and 1.12 tell us the same story. Irrespective of our choice to use the different thresholds by Hill plot on the four groups or applying the same threshold, the shape parameter ξ of Q3 returns is the largest which indicates the fattest tail behavior. The upper 5% quantile VaR of AAPL returns is increasing as the seasons go by in a year. The first season Q1 upper 5% quantile VaR is the smallest and the fourth season Q4 upper 5% quantile VaR is the largest among four seasonal AAPL returns.

Before giving any explanations of the seasonal effect on AAPL VaR, we take SPY returns as a comparison again to see the characteristic of AAPL. Table 1.13 contains

Table 1.13: Estimated GPD parameters and VaR of SPY returns from 1995-7-3 to 2015-7-2 by quarter.

	ξ	σ_μ	μ	N_μ	$\text{VaR}_{0.05}$
Q1	-0.01303372	0.81464133	1.819322	76	1.445435
Q2	0.1257745	0.5497238	1.614129	78	1.693531
Q3	0.2794518	0.7088729	1.956663	80	2.124614
Q4	0.3808598	0.8443904	1.887993	75	2.030969

estimated upper 5% quantile VaR of SPY returns by quarter. When comparing results in Table 1.13 and Table 1.11, the difference between AAPL VaR and SPY VaR is that the VaR of SPY third quarter returns is the highest while the VaR of AAPL fourth quarter returns is the highest. Moreover, the upper 5% quantile VaR of seasonal SPY returns is more stable and twice smaller than that of seasonal AAPL returns. It is immediately apparent from the above results of test period returns that the seasonal effect on AAPL VaR is different from that on SPY VaR. More importantly, we captured the comparatively high risk in Q4 and low risk in Q1 for AAPL returns during the test period.

Possible explanations for the seasonal effect on AAPL VaR include the tax-motivated trading, economic and political announcements dates concentrated in one part of the season. For instance, Apple often releases its new products, like iPhone, iPad or iMac, during July to November. It may cause large AAPL stock vibration to occur subsequently.

Overall, our findings have implications for investors, financial institutions, and futures exchanges. For conservative investors who would prefer lower risk, they can choose to trade during the lower VaR period to avoid potential high loss. The methodology of extreme value approach to VaR can also be used in other stock or asset returns.

1.2 Calendar Effects Analysis of Americas Indexes

We apply an approach for estimating VaR describing the tail of the conditional distribution of a heteroscedastic financial return series. The method combines quasi-maximum-likelihood fitting of AR-GARCH model to estimate the current mean as well as volatility, and EVT to estimate the tail of the mean and volatility adjusted standardized return series. We employ the approach to investigate the existence and significance of the calendar anomalies: seasonal effect and day-of-the-week effect in Americas Indexes VaR. during the period of 2006-7-17 through 2015-11-13. We also examined the statistical properties and made a comprehensive set of diagnostic checks on the one decade of considered Americas Indexes returns. Our results suggest that the lowest VaR of considered Americas Indexes negative log returns occurs on the fourth season among all seasons. Moreover, comparatively low Wednesday VaR is captured among all weekdays during the test period.

1.2.1 Background

In today's financial world, the large increase in the number of traded assets in the portfolio of most financial institutions has made the measurement of market risk a primary concern for regulators and for internal risk control. Following the Basle Accord on Market Risk (1996) every bank in more than 100 countries around the world has to calculate its risk exposure for every individual trading desk, banks are also required to hold a certain amount of capital as a cushion against adverse market movements. VaR has become the benchmark risk measure. In a mathematician's view, VaR is simply a quantile of the profit-and-Loss distribution of a given portfolio over a prescribed holding period. The importance of VaR is undoubted since regulators accept this model as a basis for setting capital requirements for market risk exposure.

In this section, we discover the calendar anomalies in Americas equity market movements, which including the seasonal effect and the day-of-the-week effect on Americas Indexes returns. The calendar effect in stock market returns includes day-of-the-week effect, weekend effect, January effect, and holiday effect, etc. It has been widely studied and investigated in finance literature. Studies by Cross (1973)[29], and Rogalski (1984)[76] demonstrate that there are differences in distribution of stock returns for each day of the week. Studies by Baillie and DeGennaro (1990)[7], Berument and Kiymaz (2001)[12] posit that day-of-the-week effect has an impact on stock market volatility. In recent years, another stream of research has considered seasonality in stock returns and volatility, see Saunders (1993)[79], Bouman and Jacobsen (2002)[16], Hirshleifer and Shumway (2003)[47], Kamstra, Kramer and Levi (2003)[57], and Cao and Wei (2005)[19], etc. These studies generally report that calendar anomalies are present in both returns and volatility equations in the stock market. None of these studies, however, test for the possible existence of day-of-the-week and seasonal variation in stock return VaR. Hence, the goal of this section is to characterize the VaR of Americas Indexes returns. Based on investigations of the day-of-the-week effect and seasonal effect in extreme risk, we also provide valuable and applicable analysis for equity market investors. The major obstacle to this investigation is a viable measure of tail risk over time.

We are concerned with tail estimation for those considered financial return series. Our basic assumption, whose validation is examined in this section, is that returns follow a stationary time series model with stochastic volatility structure. The presence of stochastic volatility implies that returns might dependent over time. Therefore, we consider to model the return distribution as the conditional return distribution where the conditioning is on the current volatility and mean. Although VaR only deal with extreme quantiles, disregarding the centre of the distribution, estimation of the extreme quantile is not an easy task. As one wants to make inference about the extremal behavior of a portfolio, there is only a very small amount of data in the tail area of a sample set. Furthermore, exploration even beyond the range of the data might be wanted. Statistical methods have been developed which are based only on that part of the sample that carries the information about the extremal behavior. In this study, we need a method that not only based on the smallest or largest sample values, but also includes a probabilistic argument concerning the behavior of the extreme sample values. This leads to a semi-parametric method, which is based on extreme value theory, may prove to be an effective tool for obtaining reliable estimates.

In the field of probability, it is widely used to study the distribution of extreme realizations of a given distribution function, or stochastic processes that satisfy suitable assumptions. The foundations of the theory were laid by Fisher and Tippett (1928)[34] and Gnedenko (1943)[39], who demonstrated that the distributions of the extreme values of an independent and identically distributed sample from a cumulative distribution function, when adequately rescaled, can converge towards one out of only three possible distributions. Unfortunately, most financial time series are not independent, but exhibit some very delicate temporal dependence structure. In this study, we capture it by a fully parametric method, which is based on an econometric model for volatility dynamics and the assumption of conditional normality, AR-GARCH model. We use AR(1)-GARCH(1,1) model and quasi-maximum-likelihood estimation to obtain estimates of the conditional mean and the conditional volatility. Statistical tests and exploratory data analysis confirm that the standardized returns, i.e. mean and volatility adjusted returns, do form approximately i.i.d. series. If we only use GARCH model to estimate VaR, the assumption of conditional normality does not seem to hold for real data. Thereafter, we use threshold methods from EVT to estimate the distribution of the standardized returns. EVT is a well known technique in many fields of applied sciences including risk management, insurance and engineering. Numerous research studies surfaced recently which analyze the extremes in the financial markets due to currency crises, stock market turmoils and credit defaults. The behavior of financial series tail distributions has, among others, been discussed in McNeil and Frey (2000)[67], Longin (1999)[65] and (2000)[64], and Ameli and Malekifar, 2014[5]. An estimate of the conditional return distribution is now easily constructed from the estimates of the conditional mean and volatility as well as the estimated distribution of the standardized returns. We learned the central idea of the dynamic two stage extreme value process from McNeil and Frey (2000)[67], to forecast daily VaR with historical data in a moving window. This approach reflects two stylized facts exhibited by most financial return series – stochastic volatility, and the non-normal behavior of conditional return distributions.

1.2.2 Data Exploration and Statistical Analysis

1.2.2.1 Data Description

Our sample covers the period from July 17, 2006 to November 13, 2015. Five different Americas Indexes, namely, the S&P 500, Financial Select Sector SPDR ETF, NASDAQ-100 Technology Sector, Dow Jones Utility Average, and Dow Jones Transportation Average, are used to characterize the performance of specific sectors of the market. The S&P 500 is an American national index composed of large capitalization stocks. It represents the overall performance of the stock market. The Financial Select Sector SPDR ETF tracks the overall S&P Financial Select Sector Index. The NASDAQ-100 Technology Sector is an equal weighted index based on the securities of the NASDAQ-100 Index that are classified as Technology according to the Industry Classification Benchmark classification system. The Dow Jones Utility Average is a stock index from Dow Jones Indexes that keeps track of the performance of 15 prominent utility companies. The Dow Jones Transportation Average is a U.S. stock market index from S&P Dow Jones Indices of the transportation sector, and is the most widely recognized gauge of the American transportation sector. The collection of those indices' daily adjusted closing price were from Yahoo Finance. The adjusted closing price is used to develop an accurate track record of the stock's performance.

Further, use the daily negative log return to examine extreme losses of the stock. Let p_t denote the adjusted closing price of a stock on day t, then the daily percentage change on the day is defined by

$$r_t = -100 \log \frac{p_t}{p_{t-1}} = 100 \log \frac{p_{t-1}}{p_t}. \tag{1.13}$$

Fig.**??** shows the time plots of adjusted closing price and negative daily log returns

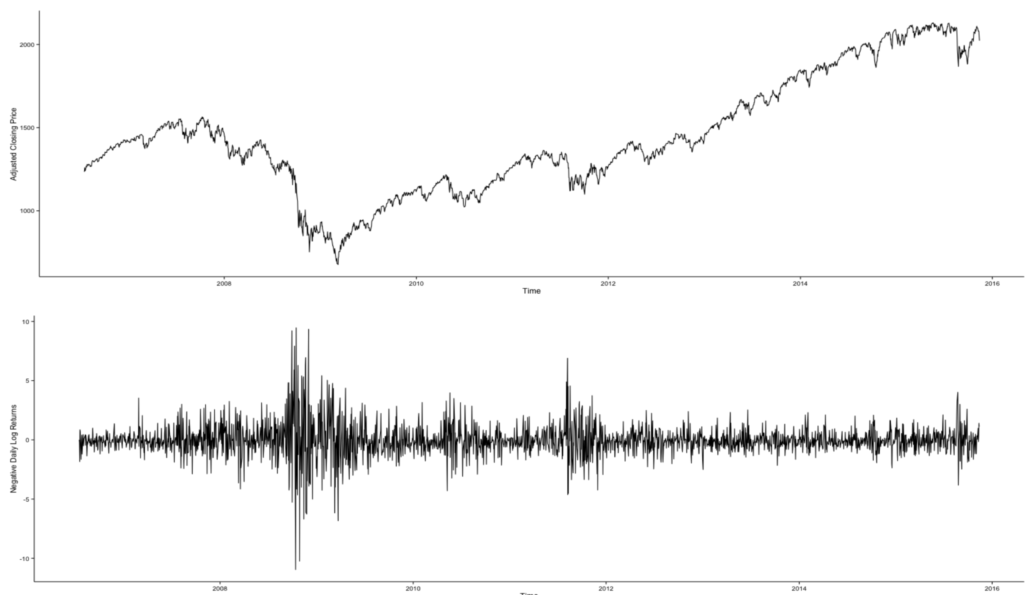

Figure 1.8: Time plots of Standards and Poors index from 2006-7-17 to 2015-11-13.

of S&P 500 from July 17, 2006 to November 13, 2015. From the lower plot, we observe that daily log returns of the index show clear evidence of volatility clustering. That is, periods of large returns are clustered and distinct from periods of small returns, which are also clustered. If we measure such volatility in terms of variance, then it is nature to think that variance changes with time, reflecting the clusters of large and small returns. We also observe that there are more pronounced peaks than one would expect from Gaussian data. Since the possibility of time-varying variance and non-normal behavior are noticed in Fig.**??**, we provide formal tests to check the stationarity, normality, and independency of those log return series.

1.2.2.2 Statistical Tests of Stationarity, Normality and Independence

Table 1.14: Stationarity and normality tests on the five Americas Indexes returns from 2006-7-17 to 2015-11-13

Data[5]	S&P	XLF	NDXT	DJU	DJT
Observations	(2350)	(2350)	(2398)	(2351)	(2350)
KPSS Test for time series level stationarity					
$KPSS$	0.17128	0.28586	0.072364	0.081087	0.1073
p-value	0.1	0.1	0.1	0.1	0.1
Shapiro-Wilk Test for time series normality					
W	0.87934	0.82416	0.94139	0.88601	0.9479
p-Value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16

The KPSS rest results on the five America Indexes negative daily log returns from July 17, 2006 to November 13, 2015 are shown in Table 1.14, all p-values are greater than the significant level 5%. Therefore, we accept the null hypothesis and conclude that the five return series are stationary during the test period.

To confirm that an assumption of normality is unrealistic, and that the innovation process is leptokurtic, we begin by forming a QQ-plot on the S&P 500 negative daily log returns against the normal distribution – see Fig.1.9.

Thereafter, we use the Shapiro-Wilk test[81] to verify an empirical fact that the five America Indexes negative daily log return series do not have the normality property. Applying the Shapiro-Wilk test on the five America Indexes negative daily log returns from July 17, 2006 to November 13, 2015, we show the test result in Table 1.14.

[5]The five Americas Indexes data sets are: S&P 500 (GSPC), Financial Select Sector SPDR ETF (XLF), NASDAQ-100 Technology Sector (NDXT), Dow Jones Utility Average (DJU), and Dow Jones Transportation Average (DJT).

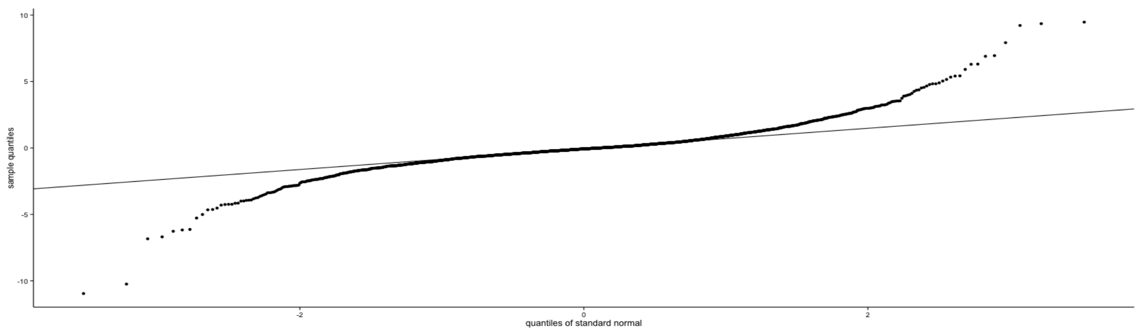

Figure 1.9: Quantile-quantile plot of S&P 500 returns from 2006-7-17 to 2015-11-13 against the normal distribution.

Because all p-values are less than 2.2e-16, we reject the null hypothesis and conclude that all the five return series are not normally distributed during the test time period.

Except the verified stylized fact of the fat tail distribution, we explore the correlations for the returns and their squared values. Fig.1.10(a) shows the sample au-

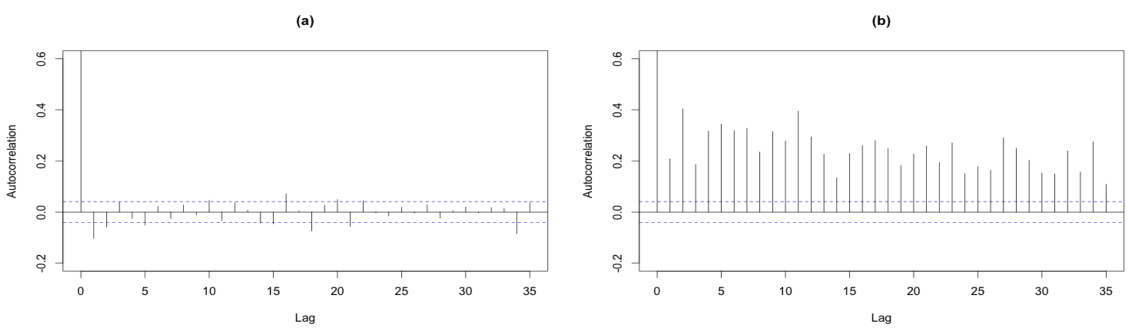

Figure 1.10: Sample autocorrelations of (a) returns and (b) squared returns of the S&P 500 from 2006-7-17 to 2015-11-13.

tocorrelation coefficient $\hat{\rho}_l$ plotted against different lags l (measured in days), along with the classical 95% significance bands around zero for S&P 500 negative daily log returns, for the period July 17, 2005 to November 13, 2015. The dashed lines represent the upper and lower 95% confidence bands $\pm\frac{1.96}{\sqrt{T}}$, where the time length for our S&P 500 returns is $T = 2350$ days. A stylized fact that absence of autocorrelation for the daily price variations is illustrated in Fig.1.10(a). The series of S&P 500 returns displays small autocorrelations, making it close to a white noise. However, the S&P 500 squared returns are strongly autocorrelated, see Fig.1.10(b). This property is not incompatible with the white noise assumption for America Indexes returns, but shows that the white noise is not strong.

Based on the statistical analysis for the five America Indexes negative daily log return series from July 17, 2006 to November 13, 2015, we discovered that those America Indexes returns are stationary, and uncorrelated time series, yet are not normally distributed and the squared returns are strongly correlated. Those properties illustrate the difficulty of daily price returns modeling. Any satisfactory statistical model for daily returns must be able to capture the main stylized facts, including the leptokurticity, the unpredictability of returns, the existence of positive autocorrelations in the squared returns, and the conditional heteroscedasticity. Some computations of VaR are based on the assumption that the series $\{r_t\}$ is normally distributed, or has t-distribution, see reference [69][5][11][51]. That is the main reason why these study can use volatility to estimate VaR. However, the real time series $\{r_t\}$ may not follow any known distributions. To overcome the difficulty of a return series $\{r_t\}$ having an unknown distribution, we compute the VaR of America Indexes returns by the Extreme Value Theory, which avoid making assumption about the distribution of $\{r_t\}$.

1.2.3 Methodology

Following the approach by Longin (1999a,b)[65][64], and McNeil and Frey (2000)[67], we use a two-stage approach to estimate the VaR of the five America Indexes negative daily log return time series.

(1) Fit a AR(1)-GARCH(1,1) model to the returns and use a pseudomaximum-likelihood approach to estimate parameters. Use the fitted model to standardize the raw returns to a strict white noise process, i.e. independent, identically distributed

process with zero mean and unit variance.

(2) Use EVT to model the tail of the marginal distribution of the standardized returns, and use this EVT model to estimate VaR.

1.2.3.1 Standardization – Estimating σ_t and μ_t

Since stock returns have heavy-tailed and/or outlier-prone probability distributions, we use GARCH models to deal with both the conditional heteroskedasticity and the heavy-tailed distributions of American Indexes returns. we consider the reason for outliers may be that the conditional variance is not constant, and the outliers occur when the variance is large. In fact, GARCH processes exhibit heavy tails even if the innovations is Gaussian. Nonetheless, many financial time series have tails that are heavier than implied by a GARCH process with Gaussian innovations. To handle such data, one can assume that, instead of being Gaussian white noise, the innovations is an i.i.d. white noise process with a heavy-tailed distribution. Therefore, we assume the standardized, i.e. mean and variance adjusted American Index returns series is an i.i.d. white noise process with a generalized Pareto distribution.

Let $\{r_t\}_{t=T-n+1}^{T}$ be a strictly stationary time series representing the negative daily log return on a financial asset price. We fix a constant memory n so that at the end of day T our data consist of the last n negative daily log returns $\{r_{T-n+1}, \cdots, r_{T-1}, r_T\}$. We assume that the dynamics of $\{r_t\}_{t=T-n+1}^{T}$ to be a realization from a AR(1)-GARCH(1,1) process, which are given by

$$\begin{cases} r_t &= \mu_t + \sigma_t z_t, \\ \mu_t &= \phi_0 + \phi r_{t-1}, \\ \sigma_t^2 &= \omega + \alpha(r_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2, \end{cases} \tag{1.14}$$

where the innovations z_t are a strict white noise process with zero mean, unit variance, and marginal distribution function F; $w > 0, \alpha > 0$, and $\beta > 0$; the conditional mean

$\mu_t|\mathcal{F}_{t-1}$, and the conditional volatility $\sigma_t|\mathcal{F}_{t-1}$ are measurable, $\mathcal{F}_{t-1}$ is the information about the return process available up to time $t-$.

This model is fitted using the quasi-maximum-likelihood estimation (QML) method, which assumes normal distribution and uses robust standard errors for inference. It means that the likelihood for a GARCH(1,1) model with normal innovations is maximized to obtain parameter estimates $\{\hat{w}, \hat{\alpha}, \hat{\beta}\}$. While this amounts to fitting a model using a distributional assumption we do not necessarily believe, the QML method delivers reasonable parameter estimates. Bollerslev and Wooldridge (1992)[15] proved that if the mean and the volatility equations are correctly specified, the QML estimates are consistent and asymptotically normally distributed.

From Eq.(1.14), we get estimates of the conditional mean $\{\hat{\mu}_{T-n+1}, \cdots, \hat{\mu}_{T-1}, \hat{\mu}_T\}$ and the conditional volatility $\{\hat{\sigma}_{T-n+1}, \cdots, \hat{\sigma}_{T-1}, \hat{\sigma}_T\}$ of $\{r_t\}_{t=T-n+1}^{T}$. To check the

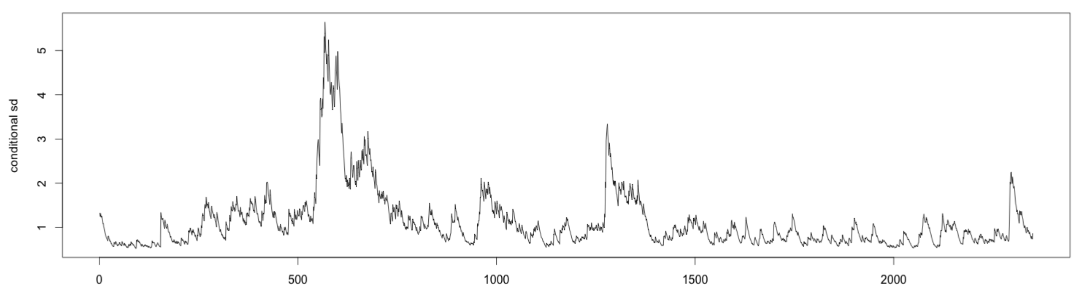

Figure 1.11: Estimation of the conditional standard deviation derived from AR(1)-GARCH(1,1) model of the S&P 500 returns.

adequacy of the model and to use in next stage of the approach, we calculate the standardized returns

$$\{z_{T-n+1}, z_{T-n+2}, \cdots, z_T\} = \{\frac{\hat{r}_{T-n+1} - \hat{\mu}_{T-n+1}}{\hat{\sigma}_{T-n+1}}, \frac{\hat{r}_{T-n+2} - \hat{\mu}_{T-n+2}}{\hat{\sigma}_{T-n+2}}, \cdots, \frac{\hat{r}_T - \hat{\mu}_T}{\hat{\sigma}_T}\}$$

The standardized returns should be i.i.d. if the fitted model is tenable. In Fig.1.12, we plot the sample autocorrelation of the standardized S&P 500 returns as well as the squared standardized S&P 500 returns. As shown in Fig.**??**, while the raw returns are clearly not i.i.d., this assumption may be tenable for the standardized returns.

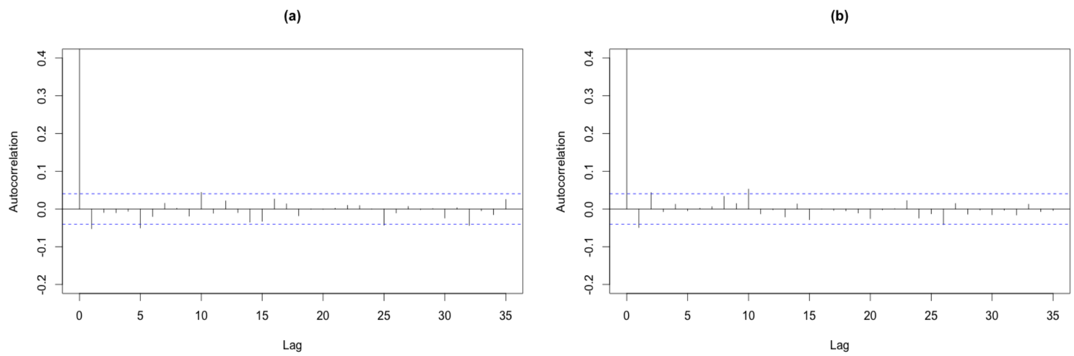

Figure 1.12: Sample autocorrelations of (a) standardized returns and (b) squared standardized returns of the S&P 500.

We end the standardization stage by calculating estimates of the conditional mean and variance for day $T+1$, which are the 1-step forecasts

$$\begin{aligned} \mu_{T+1} &= \hat{\phi}_0 + \hat{\phi} r_T, \\ \sigma^2_{T+1} &= \hat{\omega} + \hat{\alpha}(r_T - \hat{\mu}_T)^2 + \hat{\beta}\sigma^2_T. \end{aligned} \tag{1.15}$$

1.2.3.2 VaR Estimation – Extreme Value Approach

In the second stage, we estimate the upper tail behavior of the cumulative distribution function of the standardized returns F by extreme value theory. Extreme Value Theory is experiencing a boom in the financial field, especially with respect to the application to the market risk measure VaR. Its appearance as a popular instrument for estimating VaR can be explained as a consequence of two factors. On the one hand, the assumption of normality of financial markets does not reflect the reality of the situation. As a consequence, the VaR estimation methods which based on the normality assumption provide wrong estimates. Historical or Monte Carlo simulation methods arise as alternative methods. But given the difficulties and "slowness" of these methods, Extreme Value Theory (EVT) as a new solution is sought. On the other hand, although VaR can be calculated with simulation methods, it still has limitations, so this measure needs to be complemented with others. We present it as a way of solving the problem of fat tails when calculating VaR.

For a random variable X, we first fix some high threshold μ and consider the distribution of excess values $Y = X - \mu$ as

$$F_{\mu}(y) = Pr(X - \mu \leq y | X > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)}, \tag{1.16}$$

where F is the underlying distribution of X, F_{μ} is the conditional excess distribution function. Pickands (1975)[72] introduced the GPD as a two parameter family of distributions for exceedances over a threshold. More precisely, he proved that for a large class of underlying distribution functions F, the conditional excess distribution function $F_{\mu}(y)$, as $\mu \to \omega_F = sup\{x : F(x) < 1\}$, is well approximated by

$$F_{\mu}(y) \approx H_{\sigma_{\mu},\xi}(y)$$

where $H_{\sigma_{\mu},\xi}(y)$ is called GPD, specified as

$$H_{\sigma_{\mu},\xi}(y) = 1 - \left(1 + \xi \frac{y}{\sigma_{\mu}}\right)_{+}^{-1/\xi}. \tag{1.17}$$

The parameters of GPD are the scale parameter σ_{μ} and the shape parameter ξ.

EVT describes specifically at the distribution of the standardized returns in the tails. The tail fatness of the distribution is reflected by the shape parameter: the case when $\xi < 0$ means thin tails, $\xi = 0$ means the kurtosis is 3 as for the standard normal distribution; while $\xi > 0$ implies fat tails, which is the case of interest in our study. Therefore, the shape parameter measures the speed with which the distribution's tail approaches zero. The fatter the tail, the slower the speed and the higher the shape parameter is. Since almost all returns in EVT assume that the returns are i.i.d., the analysis was developed on the standardized returns which, in many cases, could be reasonably assumed to be i.i.d.. Because we are interested in extreme negative returns, we use EVT to model the right tail of the distribution, i.e. the standardized returns in excess of a high threshold.

It is necessary to choose a specific threshold to confine the estimation to those observations that are above the given threshold. However, it is difficult to apply threshold based methods because of the lack of a clear-cut criterion for choosing the threshold. If the threshold is chosen too low, the GPD may not be a good fit to the excesses over the threshold, and consequently there will be a bias in the estimates. Conversely, if the threshold is too high, then there are not enough exceedances over the threshold to obtain reliable estimates of the extreme value parameters, and consequently, the variances of the estimators will be high. In this section, an optimal threshold is selected by employing graphical methods, known as the Hill plot and the mean excess plot. The Hill plot displays the estimated values of shape parameter ξ as a function of the cut-off threshold in order to find some interval of candidate cut-off points that yields stable estimates of the shape parameter ξ. Technical details about Hill plot can be found in Hill (1975)[45]. The mean excess function is the mean of exceedances over a threshold. If the underlying distribution of those exceedances follows a GPD, then the corresponding mean excess must be linear in the threshold. Details about the mean excess plot are described in Davison and Smith (1990)[30]. Fig.1.13 shows the Hill plot and the mean excess plot of the negative daily S&P 500 log returns. A threshold 1.968748, with 132 exceedances, seems to be reasonable for the S&P 500 returns.

We have seen that the GPD contains two parameters, shape parameter ξ and location parameter σ_μ. They can be estimated by using either parametric or non-parametric methods. From the research of Hosking and Wallis (1987)[49], for the tail index $\xi > -0.5$, it can be shown that maximum likelihood regularity conditions are

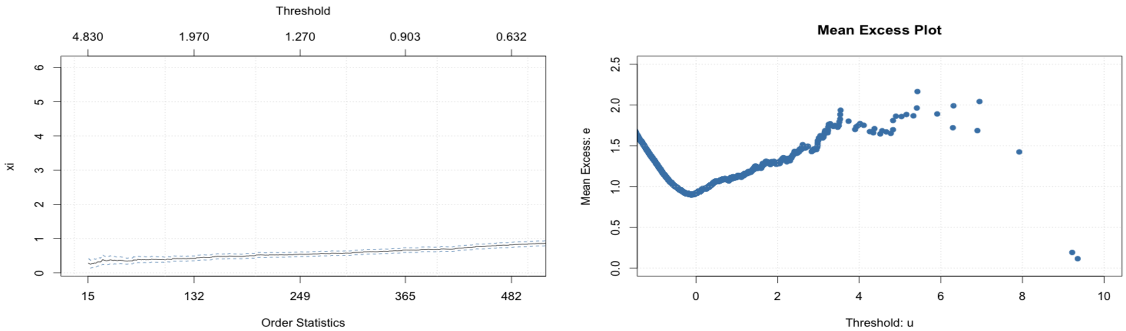

Figure 1.13: Hill plot and Mean Excess Plot of the S&P 500 returns from 2006-7-17 to 2015-11-13.

fulfilled and that maximum likelihood estimates $\{\hat{\xi}_n, (\hat{\sigma}_\mu)_n\}$ based on a sample of n excesses are asymptotically normally distributed. Therefore, we use MLE to estimate parameters in GPD.

Next, we make explicit the relationship between excess value and the standardized return series, denoted as $\{z_t\}$. We may use the following relationship to estimate the VaR of the standardized asset returns $\{z_t\}$. Assume that $\{z_t\}$ are i.i.d. random variable with CDF F, and a high enough threshold μ is given. Define

$$N_\mu = card\{t : z_t > \mu, t = 1, \cdots, n.\}$$

Then

$$F_\mu(y) = Pr(z_t - \mu \leq y | z_t > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)}$$

i.e.

$$\overline{F}_\mu(y) = Pr(z_t - \mu > y | z_t > \mu) = \frac{\overline{F}(\mu + y)}{\overline{F}(\mu)},$$

which is equivalently to

$$\overline{F}(\mu + y) = \overline{F}(u)\overline{F}_\mu(y).$$

Then, the estimators of $\overline{F}(u)$ and $\overline{F}_\mu(y)$ can be written as:

$$\widehat{\overline{F}(u)} = \frac{1}{n}\sum_{i=1}^{n} I(X_i > \mu) = \frac{N_\mu}{n},$$

$$\widehat{\overline{F}_\mu(y)} = 1 - H_{\hat{\sigma}_\mu,\hat{\xi}}(y) = \left(1 + \hat{\xi}\frac{y}{\hat{\sigma}_\mu}\right)_+^{-1/\hat{\xi}},$$

where $\hat{\xi}$ and $\hat{\sigma}_\mu$ are maximum likelihood estimators of shape parameter ξ and location parameter σ_μ. Therefore the tail estimator can be written as:

$$\widehat{\overline{F}(\mu + y)} = \frac{N_\mu}{n}\left(1 + \hat{\xi}\frac{y}{\hat{\sigma}_\mu}\right)_+^{-1/\hat{\xi}}. \tag{1.18}$$

This relationship between probabilities allows us to obtain VaR for the original asset return series $\{r_t\}$. More precisely, for a specified small probability α such that

$$\begin{aligned}\alpha &= Pr(r_{T+1} > v) = Pr(z_{T+1} > \frac{v - \mu_{T+1}}{\sigma_{T+1}}) = Pr(z_{T+1} - \mu > \frac{v - \mu_{T+1}}{\sigma_{T+1}} - \mu | z_{T+1} > \mu) \\ &= \overline{F}_\mu(\frac{v - \mu_{T+1}}{\sigma_{T+1}} - \mu) = \overline{F}(\frac{v - \mu_{T+1}}{\sigma_{T+1}})/\overline{F}(\mu),\end{aligned}$$

the α-th upper tail quantile VaR of $\{r_t\}$ is v. Consequently, for a given small probability α, one can check that the VaR of holding a long position in the asset underlying return $\{r_t\}$ is

$$\text{VaR}_\alpha = \begin{cases} \left(\mu + \frac{\hat{\sigma}_\mu}{\hat{\xi}}\left((\frac{n}{N_\mu}\alpha)^{-\hat{\xi}} - 1\right)\right) * \sigma_{T+1} + \mu_{T+1}, & \hat{\xi} \neq 0 \\ \left(\mu + \hat{\xi}\ln(\frac{n\alpha}{N_\mu})\right) * \sigma_{T+1} + \mu_{T+1}, & \hat{\xi} = 0 \end{cases} \tag{1.19}$$

We favor the extreme value approach, or the GPD approach in this study to tail estimation mainly for three reasons. One is that in finite samples of the order of points from typical return distributions, EVT quantile estimators are more efficient than the historical simulation method. Moreover, considering the fact that most financial returns series are asymmetric, the EVT approach is advantageous over models which assume symmetric distributions such as t-distributions, GARCH distribution family.

In addition, comparing with Hill method which is designed specifically for the heavy tail ($\xi > 0$) data, the EVT approach to VaR has larger applicability since it also applicable to light tail ($\xi = 0$) cases or even short tail ($\xi < 0$) cases.

1.2.4 Empirical Results and Calendar Effect Analysis

We backtest the approach on the five Americas Indexes historical series of negative daily log returns: the the Standard and Poors index S&P 500, the Financial index SPDR ETF, the Technology index NASDAQ-100, the Utility index Dow Jones Utility Average, and the Transportation index Dow Jones Transportation Average. As introduced in Section 2, we excerpt all five indexes' adjusted closing price from July 17, 2006 to November 13, 2015.

To backtest the approach, we first estimate σ_t and μ_t and use it to standardize the daily negative log returns $\{r_t\}$. The reason we use the negative returns is that loss occurs when the returns are negative for a long financial position. We show the estimation results of AR(1)-GARCH(1,1) model for Americas Indexes negative daily log returns in Table 1.15. After getting the standardized returns $\{z_t\}$, we apply the second stage to test the calendar effect on Americas Indexes returns.

1.2.4.1 Seasonal Effect on Americas Indexes VaR

Because a three-month period on a financial calendar acts as a basis for the reporting of stock earnings and the paying of dividends, the seasonal effect is a vital factor in determining stock performance. To identify the existence of seasonal effect on Americas Indexes returns, we divide the each Americas Index's standardized returns $\{z_t\}$ into the following four subsets:

$$\{z_t | t \in Q_i\}, \quad i = 1, 2, 3, 4,$$

Table 1.15: Estimation results of AR(1)-GARCH(1,1) model for Americas Indexes returns from 2006-7-17 to 2015-11-13

Parameter	S&P	XLF	NDXT	DJU	DJT
Mean equation AR(1)					
ϕ_0	-0.069493	-0.037231	-0.082928	-0.042878	-0.060223
ϕ	-0.058278	-0.079456	-0.015441	-0.029874	-0.016125
Variance equation GARCH(1,1)					
ω	0.025183	0.027637	0.026652	0.018436	0.028343
α	0.114753	0.128632	0.078668	0.097303	0.077237
β	0.867333	0.867966	0.908884	0.889307	0.910656
LogLikelihood	-3342.299	-4189.893	-4062.424	-3268.324	-4117.66
p-Value of Standardised Residuals Tests					
Shapiro-Wilk Test	0	1.92972e-15	6.597162e-11	9.209965e-13	7.049808e-10
Ljung-Box Test Q(10)	0.1749702	0.02211829	0.4162687	0.6369799	0.3831675
Ljung-Box Test Q(15)	0.1254892	0.03006419	0.4009677	0.8425757	0.2452127
Ljung-Box Test Q(20)	0.2380792	0.06046306	0.4091626	0.9131188	0.1832375
Ljung-Box Test Q(10) for squared residuals	0.02556951	0.4572453	0.7371797	0.06534186	0.08447315
Ljung-Box Test Q(15) for squared residuals	0.05896544	0.4860042	0.8815682	0.1705194	0.2434914
Ljung-Box Test Q(20) for squared residuals	0.1596955	0.5549062	0.7223037	0.3516701	0.2387464
Information Criterion Statistics[6]					
AIC	2.848765	3.570121	3.392347	2.784623	3.508647
BIC	2.861025	3.582381	3.404404	2.796879	3.520907

which are referred to the four quarters Index returns. A quarter refers to one-fourth of a year and is typically expressed as Q. Table 1.16 provides a summary of descriptive statistics for the considered return series.

Table 1.16 reports skewness and kurtosis for the standardized return series of each quarter. In statistics, skewness and kurtosis, which are normalized third and fourth central moments of a process, are often used to summarize the extent of asymmetry and tail thickness. For the normal distribution, kurtosis is 3. We observe that distributions of all four seasons' standardized returns are positively skewed, indicating that they are nonsymmetric. Further, except for the S&P 500 Q1 standardized returns, all kurtosis are less than 3, indicating that those series have distributions with tails that

Table 1.16: Descriptive Summary Statistics of Americas Indexes Seasonal Standardized Returns

Summary Statistics	S&P	XLF	NDXT	DJU	DJT
Observations n					
Q1	550	550	565	550	550
Q2	569	569	579	569	569
Q3	626	626	639	626	626
Q4	604	604	614	605	604
Skewness					
Q1	0.81	0.6	0.45	0.42	0.32
Q2	0.56	0.34	0.16	0.45	0.4
Q3	0.53	0.27	0.35	0.35	0.21
Q4	0.23	0.03	0.16	0.19	0.24
Kurtosis					
Q1	3.15	2.97	1.19	1.3	1.23
Q2	0.56	0.68	0.33	0.54	0.47
Q3	1.32	1.5	0.95	1.7	0.59
Q4	0.73	0.87	0.39	0.85	0.49
Shapiro-Wilk normality test p-Value					
Q1	1.021e-10	1.959e-09	2.88e-06	3.952e-06	3.669e-05
Q2	1.37e-07	0.0004483	0.001367	1.454e-05	0.0001729
Q3	3.42e-09	1.923e-07	3.879e-05	2.109e-06	0.003301
Q4	0.0001126	0.005276	0.06336	0.006695	0.02605

are thinner than those of the normal distribution. This indication of non-normality is also supported by the Shapiro-Wilk test, which rejects the null hypothesis of a normal distribution at 5% significance level. Based on the Ljung-Box test results fromTable 1.15 and the Shapiro-Wilk test results from Table 1.16, we consider all five Americas Indexes seasonal standardized returns are i.i.d. and non-normally distributed.

Thereafter, we apply the extreme value approach to the considered seasonal standardized return series. Table 1.17 summarizes estimation results of the shape parameter ξ, scale parameterσ_μ from fitted GPD model for the standardized Americas Indexes returns as well as 0.95 quantile VaR and 0.99 quantile VaR for the original considered negative daily log returns. To better investigate systematic seasonal differences for Americas Indexes VaR, we first find a proper threshold for each seasonal series by Hill plot and then choose the highest one as the common threshold for all four seasonal series. Using the same threshold allows a better comparison of the quarterly VaRs.

Table 1.17: Results from Fitted GPD for Standardized Returns & Estimates for VaRs of Negative Daily Log Returns

Standardized Return (Threshold μ)	S&P (1.651907)	XLF (1.658118)	NDXT (1.595358)	DJU (1.630336)	DJT (1.590404)
Shape Parameter ξ					
Q1	0.01263226	0.08324439	0.2507826	-0.2261632	-0.05614722
Q2	-0.4155444	0.04520885	-0.8514776	-0.2794260	-0.06843322
Q3	-0.4418723	-0.1682089	-0.0387090	0.1329915	-0.2223172
Q4	-0.1534931	0.1375718	-0.1974187	0.06310129	-0.1744764
Scale Parameter σ_μ					
Q1	0.77437179	0.77968703	0.4372119	0.9268999	0.68909844
Q2	0.9013567	0.53571577	0.9237556	0.7847136	0.61659084
Q3	1.2207484	0.8877799	0.6161069	0.6354024	0.7442377
Q4	0.6340782	0.4532491	0.6423728	0.49605948	0.6551974
Exceedances N_μ					
Q1	37	36	44	31	39
Q2	44	46	45	44	43
Q3	41	44	48	35	46
Q4	34	33	35	33	32
95% quantile VaR ($\text{VaR}_{0.05}$) of original negative daily log returns					
Q1	1.61934	1.947586	2.00777	1.996732	1.908087
Q2	1.739842	2.000061	2.166308	2.252225	1.921836
Q3	1.694324	2.036937	2.059414	1.951028	1.947265
Q4	1.474158	1.759062	1.864537	1.917086	1.691029
99% quantile VaR ($\text{VaR}_{0.01}$) of original negative daily log returns					
Q1	2.797445	3.453345	3.147227	3.459343	3.032853
Q2	2.562514	3.002238	2.820178	3.334415	2.909932
Q3	2.857624	3.333057	3.172462	3.341999	2.947104
Q4	2.301884	2.665381	2.879703	2.932556	2.675514

From Table 1.17, we find that for S&P 500, at a quantile level of 95%, the smallest estimated VaR among all seasons is 1.474158 for the Q4 returns; at a quantile level of 99%, the smallest estimated VaR is 2.301884 for the Q4 returns as well. This is, in the fourth season, with the AR(1)-GARCH(1,1)-GPD model, we are 95% confidence that the expected overall Americas equity market value would not lose more than 1.474158% for the worst case scenario; we are 99% confidence that the expected market value of the S&P 500 would not lose more than 2.301884%. Similar interpretations can be made for the other Americas Indexes.

In comparison of all four seasonal returns, it is also interesting to note that our model produced the smallest VaR in the fourth season, at the 95% quantile level for all five America Indexes. While at the 99% quantile level, except for the NASDAQ-100 technology index, the four seasonal VaRs exhibits analogous characteristics as observed from 95% quantile VaRs under different Americas Indexes seasonal returns. Moreover, given the quantile levels, the corresponding VaR estimates for S&P 500 seasonal returns are less than the rest Indexes seasonal returns. It indicates that the trading risk of S&P 500 is the smallest among all five Americas Indexes.

Our findings have important implications for investors and financial institutions. For example, for conservative investors who would prefer lower risk, they can choose to trade during the lower VaR period or trade lower risk stocks to avoid potential high loss.

1.2.4.2 Day-of-the-Week Effect on Americas Indexes VaR

To formally test the timing and existence of weekly patterns, we divide the whole standardized returns $\{z_t\}$ to five subsets by day-of-the-week, which is written as:

$$\{z_t | d_{it} = 1\}, \quad i = 1, 2, 3, 4, 5,$$

where d_{it} are dummy variables such that if day t is a Monday $d_{1t} = 1$, if $d_{1t} = 0$ remove the data; if day t is a Tuesday $d_{2t} = 1$, otherwise remove the data, etc. The five subsets are the considered Indexes weekly returns for Monday through Friday respectively.

The basic statistical characteristics of the five return series are calculated and shown in Table 1.18.

Table 1.18: Descriptive Summary Statistics of Americas Indexes Weekly Standardized Returns

Summary Statistics	S&P	XLF	NDXT	DJU	DJT
Observations n					
Monday	442	442	468	442	442
Tuesday	480	480	483	481	480
Wednesday	483	483	485	483	483
Thursday	473	473	479	473	473
Friday	471	471	482	471	471
Skewness					
Monday	0.41	0.24	0.22	0.17	0.41
Tuesday	0.77	0.68	0.31	0.47	0.12
Wednesday	0.04	0.27	0.08	-0.1	0.08
Thursday	0.18	0.03	0.38	0.47	0.47
Friday	0.65	0.43	0.49	0.83	0.53
Kurtosis					
Monday	1.22	1.15	1.28	2.04	0.8
Tuesday	3.1	3.14	0.88	0.62	0.3
Wednesday	0.47	1.01	0.61	0.41	0.57
Thursday	0.94	0.78	0.44	0.77	0.52
Friday	1.04	1.1	0.41	1.82	1.69
Shapiro-Wilk normality test p-Value					
Monday	2.07e-06	0.0008976	0.0004619	1.911e-05	0.0007413
Tuesday	2.242e-09	1.94e-09	0.001296	9.745e-05	0.5958
Wednesday	0.002344	0.001305	0.005957	0.4103	0.00485
Thursday	3.495e-08	0.006136	0.0004881	8.066e-05	0.0001246
Friday	8.661e-08	8.494e-05	6.554e-06	2.965e-09	4.802e-06

Except the Dow Jones Utility Average Wednesday standardized returns, the distribution of the rest weekday standardized returns are slightly positively skewed, indicating that they are nonsymmetric. The kurtosis of the S&P 500 and the SPDR ETF Tuesday standardized returns show Gaussian property while the rest of the week returns are substantially departure from normal distribution. The Shapiro-Wilk test results also indicates that normal distribution is not a realistic assumption for the weekly standardized returns for considered Indexes.

Next, we apply the VaR estimation approach to capture the day-of-the-week effect on Americas Indexes VaR and show the result in Table 1.19. Given Americas Indexes and the 99% quantile level, we captured the comparatively low risk in Wednesday. Among the five Americas Indexes, the number of exceedances is comparatively small in Monday and Friday.

1.2.5 Conclusion

With the empirical analysis of this section we demonstrated how we can use a GARCH-EVT approach to model VaR for short term forecasting. The dynamic EVT method has the advantage of dynamically reacting to changing market conditions which is useful in getting better VaR forecasts. We apply the two stage approach on five Americas Indexes return series. Empirical findings in this section show that both seasonal effect and day-of-the-week effect are present in Americas Indexes returns. We captured the comparatively low VaR in Q4 and Wednesday for considered returns during the test period.

Overall, our findings have implications for investors, financial institutions, and futures exchanges. For example, for conservative investors who would prefer lower risk, they can choose to trade during the lower VaR period to avoid potential high loss. The dynamic EVT approach to VaR can also be used in other stock or asset returns. Finally, it has significant value for investors and regulators in terms of an in depth analysis of the equity market.

Table 1.19: Results from Fitted GPD for Standardized Returns & Estimates for VaRs of Negative Daily Log Returns

Standardized Return (Threshold μ)	S&P (1.685563)	XLF (1.597324)	NDXT (1.616329)	DJU (1.498996)	DJT (1.654929)
Shape Parameter ξ					
Monday	-0.490609	0.08324439	0.3182840	0.2423261	-0.4065323
Tuesday	0.09756212	0.1224898	0.0999698	-0.1381534	-0.1377236
Wednesday	-0.1117621	0.1075196	-0.2249917	-0.1726082	-0.2138769
Thursday	-0.3322698	-0.1846136	-0.3958165	0.06310129	-0.1077049
Friday	-0.5091703	-0.1816384	-0.1592645	-0.174639	-0.09100887
Scale Parameter σ_μ					
Monday	1.239719	0.77968703	0.3634457	0.4779636	1.0962016
Tuesday	0.65606741	0.7608726	0.5040591	0.6698328	0.6560630
Wednesday	0.5055943	0.4589546	0.6706803	0.5467823	0.6069613
Thursday	-0.3322698	0.7758208	0.9214631	0.49605948	0.6291783
Friday	1.1695172	0.8252605	0.5823002	1.037172	0.87551725
Exceedances N_μ					
Monday	25	30	29	26	22
Tuesday	33	36	33	37	27
Wednesday	30	35	27	44	40
Thursday	38	39	39	33	35
Friday	24	28	39	32	17
95% quantile VaR ($\text{VaR}_{0.05}$) of original negative daily log returns					
Monday	1.574378	1.9127	1.886357	1.844478	1.714618
Tuesday	1.63393	1.994884	1.976933	2.085929	1.802911
Wednesday	1.537115	1.838576	1.875154	2.123752	2.034037
Thursday	1.827274	2.054521	2.272032	1.949974	1.981222
Friday	1.456647	1.802248	2.10879	2.121393	1.406727
99% quantile VaR ($\text{VaR}_{0.01}$) of original negative daily log returns					
Monday	2.784977	3.423013	2.848478	3.020582	3.117782
Tuesday	2.733362	3.554699	3.044567	3.180491	2.810845
Wednesday	2.214732	2.758804	2.914875	2.956329	2.836568
Thursday	2.776619	3.134498	3.335698	2.981338	2.945308
Friday	2.642806	3.025973	3.009405	3.77907	2.867233

1.3 The Dynamics of Precious Metal Markets VaR

The data analysis of the metal markets has recently attracted a lot of attention, mainly because the prices of precious metal are relatively more volatile than its historical trend. A robust estimate of extreme loss is vital, especially for mining companies to mitigate risk and uncertainty in metal price fluctuations. This section examines the VaR and statistical properties in daily price return of precious metals, which include gold, silver, platinum, and palladium, from January 11, 2000 to September 9, 2016. An advanced two stage approach which combining GARCH-type models with Extreme Value Theory is implemented. In the first stage, the conditional variance is modeled by different rolling univariate GARCH-type models (GARCH, EGARCH and TGARCH) under the GED error assumption in the returns of precious metal markets and compare the same with other well-known models. In the second stage, Extreme Value approach is applied to capture the tail behavior of distribution for the extracted standardized residuals. In comparison with the dynamic VaRs of these precious metals, we find that gold has the most steady and the highest VaRs, followed by platinum and silver; on the other hand our results show that palladium has the most volatile VaRs. The backtesting result confirms that our approach is an adequate method in improving risk management assessments and hedging strategies in the high volatile metal markets.

1.3.1 Background

Commodity markets have been highly volatile in recent years due to many factors, such as political unrest, extreme weather conditions, introduction of new financial innovations, and international inflation. In this study, we mainly focus on the risk analysis of precious metal markets because of the following reasons. (1) Precious metals play important roles in portfolio selection and management; (2) Investors have more belief and faith in metal markets as compare to stock market because metals act as a retainer of economic value and best hedging against inflation in economy; (3) Precious metals have high liquidity in market. Therefore, at the time of economic crisis people can convert their jewelry, currency, bars etc., into hard cash; (4) The prices of precious metal are relatively more volatile than its historical trend due to the introduction of new financial innovations, such as futures, options, exchange-traded funds, and changes in demand and supply. During the periods of uncertainty caused by the global financial crises, certain precious metals may serve as important hedge assets against inflation. Hillier, Draper and Faff (2006)[46] examined the weak-form efficiency of precious metals markets and concluded that precious metals have low correlations with stock index returns. Consequently, precious metals are important components of investment portfolios for individuals and institutions, due mainly to their effectiveness as a safe haven. In particular, there are a number of studies focusing on gold, not only for its role as a hedge in portfolio diversification, but also for its unique characteristic that are comparable to a monetary univariate (see, e.g. Goodman (1956)[41], Jaffe (1989)[52], Baur and Lucey (2010)[9]). Silver is also widely used, both as a financial instrument for inclusion in investment portfolios since it has been considered as an intrinsic store of wealth, and a valuable industrial commodity. For other precious metals, such as platinum, which is the rarest precious metal, as well as palladium, their unique physical properties make them very desirable industrial metals, especially for jewelry and automotive industries. Hence, quantification of the risk in precious metal markets is fundamental in designing risk management strategies. Yet, quantitative literature about the characteristics of metal markets risks are insufficient.

To measure market risk, VaR which is the maximum loss of a portfolio such that the likelihood of experiencing a loss exceeding that amount over a specified risk horizon, is undoubted a suitable measurement, since regulators accept this quantity as a basis for setting capital requirements for market risk exposure. The development of more robust approaches in estimating VaR is thus crucial. In this section, we aim to take the advantage of probability and time series theory to improve estimations of appropriate underlying distributions, to capture extreme tails of the profit and loss distribution; and as a result, improve the estimation of VaR. Although VaR only characterizes the extreme quantiles while disregarding the center of the distribution, estimation of the tail is not an easy task. As one wants to make inferences about the extremal behavior of a portfolio, there is only a very small amount of data in the tail area of a sample set. Advanced methods and tools are needed to enable us to explore beyond the range of the limited data set.

A common approach to model VaR is using GARCH (Generalized AutoRegressive Conditional Heteroskedasticity)-type models to estimate volatility and correlations. It is similar in spirit to RiskMetrics, who demonstrates the behavior of the daily volatility estimator produced by a GARCH(1,1) volatility model with normal disturbances. Although the standard GARCH model is able to encompass volatility clustering as well as the leptokurtic behavior in the tails of the distribution of the underlying financial return time series, it cannot model asymmetries of the volatility with respect to the sign of past shocks. Bad news which is identifies by a negative sign in the standard GARCH model, has the same influence on the volatility as good news. The so called Leverage Effect can be modeled using extensions to the GARCH, such as a threshold GARCH (TGARCH) or exponential GARCH (EGARCH). However, there are still limitations to apply GARCH models since it based on the assumption of error distribution.

In the last two decades, the Extreme value theory has experienced a boom in the financial field, especially with respect to the application to measure VaR. The EVT methods are attractive because of the following features: they are based on a profound probability theorem; they offer a parametric form for the distribution of tail events yet requires no knowledge of the original distribution. Moreover, extreme value approach has been applied to depict the dynamic VaR of metal markets. Tolikas (2008)[84] compared the EVT-based VaR estimates with those generated by traditional methods. He showed that when the focus is on the extreme tails of stationary, uncorrelated time series, the EVT methods can be particularly useful. Unfortunately, empirical results show that most financial time series are correlated, some even exhibit asymmetric and/or long memory structure.

To remedy those shortcomings of both methods, we contain a development of GARCH theory and the application of different, symmetric and asymmetric models, to predict the volatility of metal returns, accompanied with the theory of EVT to estimate VaR. First, we estimate the conditional mean and volatility, then standardize the time series to make them stationary and uncorrelated. Because the presence of stochastic volatility implies that returns might have volatility clustering, time dependent, heteroskedastic and leverage effect behavior, we capture the conditional mean using autoregressive moving average (ARMA) model, and the conditional volatility using univariate GARCH-type models. The standardized residuals will be confirmed stationary and uncorrelated by statistical hypothesis tests. Second, we use threshold methods of EVT to estimate the tail distribution of the standardized residuals.

To date, VaR has been applied in metal markets to measure risk, and several authors have accomplished commendable research. In order to offer a comparative view, we summarize the key findings of major studies in the related literature in Table 1.20, which demonstrates that both GARCH-type models and extreme value approach are widespread tools used in the literature to analyze volatility and VaR in metal markets. We go beyond previous research by (i) considering daily spot price of four precious metal during the latest sixteen years, which includes the period around September 11, 2001, the beginning of the Iraq war in 2003, the global financial crisis of 2008, and the 2016 Brexit referendum; (ii) integrating the linear symmetric/ asymmetric GARCH-type models with EVT to estimate VaR, which is confirmed a more advanced approach to VaR by backtesting result. While some results speak in favor of nonlinear GARCH models, for instance, Chkill, Hammoudeh and Nguyen (2014)[23] found that the FIAPARCH model performs best in predicting VaR. In this study, we choose to exclude nonlinear GARCH-type models to estimate the conditional volatility for two reasons. First, there is a trade-off between model flexibility on the one hand and interpretability and complexity on the other. Based on Chkill's research, the difference between the performance in estimating conditional volatility of FIAPARCH model with other linear GARCH models are very small, while the FIAPARCH model has more parameters to estimate which increases uncertainty as well as decreases the interpretability. Second, our data are tested to exhibit certain stylized features, including volatility clustering and asymmetry. Moreover the lack of long memory feature in metal markets convinces us that comparing to the FIAPARCH model, the linear GARCH model would be better suited in modeling the considered return series. (iii) To evaluate objectively whether the VaR model is adequate, two statistical backtest methods are used in our paper, following Kupiec (1995)[60], Christoffersen and Pelletier[24].

1.3.2 Data exploration and statistical analysis

This study analyzes the precious metal markets risk on gold, silver, platinum, and palladium daily spot price from January 11, 2000 to September 9, 2016. Since the gold, silver, platinum and palladium price auctions take place in London on a daily basis. All of these prices are internationally regarded as the pricing mechanism for a variety of precious metal transactions and products. We thus collect daily gold P.M. fixing price and silver fixing price from LBMA[7], as well as platinum P.M. fixing price and palladium P.M. fixing price from LPPM[8]. All the four metal price based in U.S. Dollars.

To develop an accurate track record of asset performance, the initial price data are transformed into daily log-returns. Let p_t denotes the metal price on day t, then the daily log returns on day t is defined by

[7]LBMA: London Bullion Market Association.

[8]LPPM: LBMA platinum and palladium price data.

Table 1.20: Previous research about analyzing market risk of metals

Studies	Purposes	Data	Methodology	Main findings
Tully and Lucey (2007) [86]	The paper investigates the applicability of the asymmetric power GARCH model (APGARCH) and its nested variants to the gold market.	Monthly observations of gold, both cash and futures prices, and a set of macroeconomic variables, from 1984 to 2003.	APGARCH	Confirm that the US dollar is the main macroeconomic variable which influences gold.
Hammoudeh and Yuan (2008) [44]	The paper examines the volatility behavior of three strategic commodities: gold, silver and copper in presence of oil and interest rate shocks.	Daily time series for the closing three-month futures prices of oil, gold, silver and copper, and for the US three month Treasury bill rates from Jan. 2, 1990 to May 1, 2006.	GARCH, CGARCH, EGARCH	Monetary policy and to leaser extent the oil shocks have calming effects on precious metals but not on copper if the T bill rate is used. Crises heighten metal volatility.
Tolikas (2008) [84]	To describe the distribution of the extreme minima for daily returns of a wide set of markets; to assess whether the EVT approach can be useful for risk measurement purposes by deriving VaR estimates.	Daily closing prices of the CAC-DS index and the CRB index over the period 1977 to 2006, and daily prices for the German 10 year benchmark bond index from 1980 to 2006.	GEV and generalized logistic distributions	When the focus is on the really ruinous events, the EVT methods can be particularly useful since they produce VaR estimates that outperform those derived by the traditional methods at high confidence levels.
Hammoudeh, McAleer, and Malik (2011) [43]	This paper examines volatility and correlation dynamics in price returns of gold, silver, platinum and palladium, and explores the corresponding risk management implications for market risk and hedging.	Daily returns based on closing spot prices for the four precious metals(gold, silver, platinum, and palladium) from Jan. 4, 1995 to Nov. 12, 2009.	RiskMetrics, GARCH, GARCH-FHS	Portfolio managers engaged in precious metals should calculate VaR using GARCH-t as it will yield fewer violations, though with lower profitability.
Chaithep et al. (2012) [21]	This paper focuses on risk evaluation of gold price return and the tail distribution of extreme events in gold price returns.	Daily gold price from Jan. 1, 1985 to Aug. 31, 2011	Extreme Value Approach, Generalized Extreme Value (GEV model)	Reveals that value of gold price return when modeled after Generalized Extreme Value is that a maximum tomorrow's loss is 6.5461% at the significant of 99 percent confidence interval.
Chkili et al. (2014) [23]	To explore the relevance of asymmetry and long memory in modeling and forecasting the conditional volatility and VaR of four widely traded commodities (crude oil, natural gas, gold, and silver).	Daily spot and three-month futures prices of WTI, Henry Hub natural gas, gold and silver from Jan. 7, 1997 to Mar. 31, 2011.	GARCH, IGARCH, EGARCH, RiskMetrics, and FIGARCH, FIAPARCH, HYGARCH	The FIAPARCH model is the best suited for estimating the VaR forecasts. This model also gives the lowest number of violations under the Basel II Accord rule.
Chinhamu et al. (2015) [50]	To improve current assumptions of appropriate underlying distributions to capture extreme tails, and improve the estimation of VaR and Expected Shortfall.	Monthly gold prices from Jan.1969 to Oct. 2012	Generalized Pareto Distribution (GPD model)	EVT provides effective means of estimating tail risk measures, such as VaR and Expected Shortfall, which is confirmed by backtesting procedures.

$$r_t = 100 \log \frac{p_t}{p_{t-1}} = 100 \times (\log p_t - \log p_{t-1}). \tag{1.20}$$

Since gold has been considered as a financial indicator and also has an influence on other precious metals, we could capture some metal markets historical tendency through taking a glance at gold prices. Fig.**??** provides the time series plots of daily spot gold prices as well as log-returns. As can be seen in Fig.**??**, the lower figure

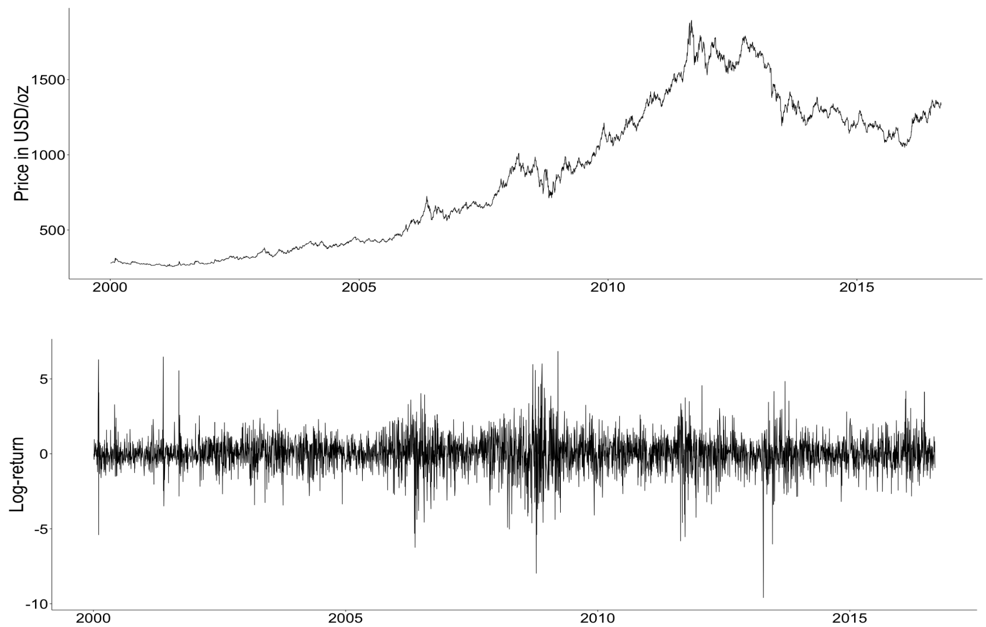

Figure 1.14: Time series plots of gold daily prices and daily log-returns from 2000-1-11 to 2016-9-9.

indicates heteroscedasticity and volatility clustering behavior. Also, there are more isolated pronounced peaks than one would expect from Gaussian series. The statistical results of the four precious metal returns are shown in Table 1.21.

As demonstrated in Panel A of Table 1.21, the mean of these return series are low, whereas the corresponding standard deviation are substantially high. Palladium has the highest standard deviation, while gold has the lowest. Meanwhile, compared with the standard normal distribution with skewness 0 and kurtosis 3, we conclude that each return has a leptokurtic distribution with fat tail. The indication of non-normality is also supported by the Shapiro-Wilk test in Panel B, which rejects the null hypothesis of a normal distribution at all levels of significance. We also examine the stationary property for the returns by means of KPSS test. Results are reported in Panel B, which indicates that the null hypothesis of weak stationarity for all returns

Table 1.21: Descriptive statistics and hypothesis tests results for the four precious metal prices daily log-returns

	Gold	Silver	Platinum	Palladium
Panel A: Descriptive statistics				
Num.	4218	4219	4220	4220
Mean	0.02987	0.03076	0.00952	-0.00585
Std. dev.	1.14	2.09	1.48	2.16
Maximum	6.84	18.28	8.43	15.84
Minimum	-9.6	-18.69	-17.28	-17.86
Skewness	-0.29	-0.55	-0.7	-0.33
Kurtosis	5.35	9.83	8.92	5.28
Panel B: Hypothesis tests results				
KPSS	1.6858e-229 (0.1)	0.12013 (0.1)	0.33771 (0.1)	0.10912 (0.1)
Shapiro-Wilk	0.9437∗ (<2.2e-16)	0.91238∗ (<2.2e-16)	0.93077∗ (<2.2e-16)	0.94044∗ (<2.2e-16)
LB-Q(5)	3.3998 (0.6386)	37.507∗ (4.739e-07)	6.9491 (0.2245)	39.766∗ (1.664e-07)
LB-Q(10)	13.137 (0.2161)	41.187∗ (1.046e-05)	8.2851 (0.601)	43.584∗ (3.91e-06)
LB-Q_S(5)	280.49∗ (<2.2e-16)	589.2∗ (<2.2e-16)	432.53∗ (<2.2e-16)	421.57∗ (<2.2e-16)
LB-Q_S(10)	469.21∗ (<2.2e-16)	857.11∗ (<2.2e-16)	621.48∗ (<2.2e-16)	599.79∗ (<2.2e-16)

The KPSS test[61] corresponds to the test statistic for the null hypothesis of weak stationarity, i.e. time invariance of the mean value and the autocorrelation function, in the distribution of sample returns.
The Shapiro-Wilk test[81] utilizes the null hypothesis principle to check whether the series come from a normally distributed population. As Razali and Wah (2011)[74] demonstrate, the Shapiro-Wilk test is one of the most powerful formal normality tests.
The Ljung-Box statistics, Q(n) and Q_S(n), check for serial correlation of the return series and the squared returns up to the n-th order, respectively.
p-values are reported in parentheses; ∗ indicates rejection of the null hypothesis at the 1% significant level.

are not rejected at the 1% significant level. To check the autocorrelation of the returns, the Ljung-Box (LB for short) test is applied for returns at lag 5 and 10, Q(5) and Q(10), and squared returns $Q_S(5)$ and $Q_S(10)$. Although partial sample returns (gold and platinum) support for the null hypothesis of no serial autocorrelation, the Ljung-Box test results for squared return series confirms that all sample return series have short memory. Therefore, the unpredictability of returns is evidenced through the autocorrelation effect in squared returns indicate volatility clustering.

Base on the statistical analysis for precious metal price return series, we conclude that these metal returns are stationary, non-normally distributed, and have short memory. Those properties illustrate the complication of estimating the distribution of the returns series. To overcome these difficulties, we use the methodology incorporate GARCH type models with EVT to accommodate the stylized facts exhibits in the metal markets. By taking advantage of EVT, the VaR can be evaluated using generalized Pareto distribution (GPD) of extreme events, without the need of exploring the full distribution of $\{r_t\}$.

1.3.3 Methodology

1.3.3.1 Estimating μ_{t+1} and σ_{t+1} using ARMA - GARCH-type model

Let $\{r_t\}_{t=T-n+1}^{T}$ be a time series representing the daily log-return of metal price. We fix a constant memory n so that at the end of day T our data consist of the last n daily log-returns $\{r_{T-n+1}, \cdots, r_{T-1}, r_T\}$. Since the existence of the volatility clustering and leptokurtosis in precious metal returns, we assume that the conditional mean of $\{r_t\}_{t=T-n+1}^{T}$ follows a autoregressive moving average model ARMA(p,q), and the conditional volatility follows a univariate GARCH-type model, which is given by

$$\begin{cases} r_t &= \mu_t + \sigma_t z_t, \\ \mu_t &= \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_j \\ \sigma_t^2 &\sim \text{GARCH-type model} \end{cases} \tag{1.21}$$

where the innovations z_t are white noise process with zero mean, unit variance, and marginal distribution F; the conditional mean $\mu_t = \mathbb{E}(r_t|\mathcal{F}_{t-1})$, and the conditional volatility $\sigma_t = \text{Var}(r_t|\mathcal{F}_{t-1})$, $\mathcal{F}_{t-1}$ is the historical information about the return process available up to time $t-1$.

Bollerslev (1986)[14] developed the generalized ARCH, or GARCH, to capture the time-vary volatility, which relies on modeling the conditional variance as a linear function of the squared past innovations. The conditional variance of the standard GARCH(1,1) is defined as

$$\sigma_t^2 = \omega + \eta(r_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2, \tag{1.22}$$

where $\omega > 0, \eta > 0, \beta > 0$, and $\alpha + \beta < 1$ which reflects the duration of the return volatility.

However the standard GARCH model has a drawback, as it fails to describe the leverage effect in the volatility of metal price returns. Leverage effect means that the volatility tends to increase dramatically following bad news, and to increase moderately or even to diminish following good news. The threshold GARCH, or TGARCH (Zakoian (1994)[88]), and the similar GJR-GARCH (Glosten, Jagannathan, and Runkle (1993)[38]), which define the conditional variance as a linear piecewise function, is applied to discuss this leverage effect topic. The conditional variance of TGARCH(1,1) can be depicted as

$$\sigma_t^2 = \omega + \eta(r_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2 + \gamma(r_{t-1} - \mu_{t-1})^2\mathbb{I}_{\{r_{t-1}-\mu_{t-1}>0\}}, \tag{1.23}$$

where $w > 0, \eta \geq 0$, and $\beta \geq 0$. Due to the use of switching condition $\mathbb{I}_{\{r_{t-1}-\mu_{t-1}>0\}}$, the influence of return increase and decrease on the conditional variance are characterized distinctly, as long as $\gamma \neq 0$.

Another popular model proposed to capture the asymmetric effects is Nelson's (1991)[71] exponential, or EGARCH model. The EGARCH(1,1) is defined as

$$\ln \sigma_t^2 = \omega + \eta[|z_{t-1} - \mathbb{E}(|z_{t-1})] + \beta \ln \sigma_{t-1}^2 + \gamma z_{t-1}, \tag{1.24}$$

where η depict the leverage effect. In contrast to the GARCH model, no restrictions need to be imposed on the model parameters since the logarithmic transformation ensures that the forecasts of the variance are non-negative.

In this study, we fit model (1.21) by means of quasi-maximum-likelihood estimation method (QML), under the assumption that the innovations $\{z_t\}$ from the above univariate GARCH-type models follows a generalized error distribution (GED) (Nelson, 1991 [71]). It means that the likelihood for a return series $\{r_t\}_{t=T-n+1}^{T}$ in model (1.21) with GED innovations $\{z_t\}$ is maximized to obtain parameter estimates $\{\hat{\phi}_0, \hat{\phi}, \hat{\omega}, \hat{\alpha}, \hat{\gamma}, \hat{\beta}\}$. The main reason we choose QML estimation is that according to Bollerslev and Wooldridge (1992)[15], the QML estimates are consistent and asymptotically normally distributed, if the mean and the volatility equations are correctly specified. We choose GED to estimate the residual series based on the statistical results which indicate these metal price returns have leptokurtic and fat tail distribution which do not in accordance with the commonly used Gaussian or Student t distribution.

The probability density function of GED is given by:

$$f(z_t) = \frac{k \exp\{-\frac{1}{2}|\frac{z_t}{\lambda}|^k\}}{\lambda 2^{(k+1)/k}\Gamma(1/k)}, \; k \geq 0,$$

where $\lambda = \left(2^{-\frac{2}{k}} \frac{\Gamma(1/k)}{\Gamma(3/k)}\right)^{\frac{1}{2}}$, $\Gamma(\cdot)$ denotes the Gamma function. k is the tail-thickness parameter. In particular, if $k = 2$, z_t is standard normally distributed; $k < 2$ indicates its tail is fatter than that of the standard normal distribution; while $k > 2$ indicates a thinner tail.

For each GARCH-type model mentioned above, we can calculate the estimates of conditional mean $\hat{\mu}_{t+1}$ and conditional variance $\hat{\sigma}^2_{t+1}$ for day $t+1$, which are the 1-step forecasts of day $t, t = T-n+1, \cdots, T$. Subsequently, estimation has been carried out using GARCH-type models, based on the GED, for VaR of returns in precious metal markets.

$$VaR_{\alpha,t} = -\hat{\mu}_t + z_\alpha \hat{\sigma}_t, \tag{1.25}$$

where z_α is the left α-quantile of the GED distribution which is used for the residual series of GARCH-type model.

However, in our methodology, we implement ARMA - GARCH model to the original return series for getting the standardized residuals. From Eq.(1.21), we get estimates of the conditional mean $\{\hat{\mu}_{T-n+1}, \cdots, \hat{\mu}_{T-1}, \hat{\mu}_T\}$ and conditional volatility $\{\hat{\sigma}_{T-n+1}, \cdots, \hat{\sigma}_{T-1}, \hat{\sigma}_T\}$ of $\{r_t\}_{t=T-n+1}^T$. The residual series therefore can be formulated as

$$\{z_{T-n+1}, \cdots, z_T\} = \left\{ \frac{\hat{r}_{T-n+1} - \hat{\mu}_{T-n+1}}{\hat{\sigma}_{T-n+1}}, \cdots, \frac{\hat{r}_T - \hat{\mu}_T}{\hat{\sigma}_T} \right\},$$

which should be uncorrelated and stationary if the fitted model is tenable. In the implementation stage, the best fitted GARCH-type model will be chose based on the goodness of fit measure Akaike information criterion (AIC).

1.3.3.2 Estimating VaR using Extreme Value Theory

For convenience of interpretation, we produce all analogous results for negative residuals by taking into account the relation

$$\min\{z_{T-n+1}, \cdots, z_T\} = \max\{-z_{T-n+1}, \cdots, -z_T\}.$$

As we are interested in extreme negative returns, we use EVT to model the residuals' left tail behavior, which is equivalent to model the right tail of the distribution for the corresponding negative residuals.

From the negative residuals, we estimate the upper tail behavior of its cumulative distribution function (CDF) using EVT. For a random variable X, we fix a high threshold μ and consider the distribution of excess values $Y = X - \mu$ as

$$F_\mu(y) = Pr(X - \mu \leq y | X > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)}, \tag{1.26}$$

where F is the underlying CDF of X, F_μ is the conditional excess distribution function. Pickands (1975)[72] introduced the GPD as a two parameter family of distributions for exceedances over a threshold. More precisely, for a large class of underlying distribution functions F, the conditional excess distribution function $F_\mu(y)$, as $\mu \to \omega_F = sup\{x : F(x) < 1\}$, can be well approximated by

$$F_\mu(y) \approx H_{\sigma_\mu, \xi}(y),$$

where $H_{\sigma_\mu, \xi}(y)$ is called GPD, which is specified as

$$H_{\sigma_\mu, \xi}(y) = 1 - \left(1 + \xi \frac{y}{\sigma_\mu}\right)_+^{-1/\xi}. \tag{1.27}$$

The parameters of GPD are the scale parameter σ_μ and shape parameter ξ.

EVT describes specifically at the tail of distributions. The tail fatness of the distribution is reflected by the shape parameter: $\xi < 0$ indicates thin tails; $\xi = 0$ means the kurtosis is 3 as for the standard normal distribution; $\xi > 0$ implies fat tails. Therefore, the shape parameter ξ measures the speed with which the distribution's tail approaches zero. The fatter the tail, the slower the speed and the higher ξ is. Because EVT begins with the assumption that the sequence of variables are i.i.d., the analysis in this study was developed on the residuals, which would be much more reasonable than assuming the original returns are i.i.d. sequence.

It is necessary to choose a specific threshold to confine the estimation to these observations that are above the given threshold. In this study, an optimal threshold is selected by employing graphical methods, known as the Hill plot and the mean excess plot. The Hill plot[45] displays the estimated values of shape parameter ξ as a function of the cut-off threshold in order to find some interval of candidate cut-off points that yields stable estimates of ξ. The mean excess function[30] is the mean of exceedances over a threshold. If the underlying distribution of these exceedances follow GPD, then the corresponding mean excess must be linear in the threshold.

Hosking and Wallis (1987)[49] proved that, for shape parameter $\xi > -0.5$, the maximum likelihood regularity conditions are fulfilled and that maximum likelihood estimates $\{\hat{\xi}_n, (\hat{\sigma}_\mu)_n\}$ based on a sample of n excesses are asymptotically normally distributed. Hence, we estimate the shape parameter ξ and location parameter σ_μ using maximum likelihood estimation.

Next, we make explicit the relationship between excess value of negative residuals, denoted as $\{z_t\}$, and the original return series. Assume that $\{z_t\}$ are i.i.d. random variable with CDF F, and a high enough threshold μ is given. Define the number of exceedances as N_μ such that

$$N_\mu = card\{t : z_t > \mu, t = 1, \cdots, n\}.$$

Then

$$F_\mu(y) = Pr(z_t - \mu \leq y | z_t > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)},$$

i.e.

$$\overline{F}_\mu(y) = Pr(z_t - \mu > y | z_t > \mu) = \frac{\overline{F}(\mu + y)}{\overline{F}(\mu)}.$$

The estimators of $\overline{F}(u)$ and $\overline{F}_\mu(y)$ can be written as:

$$\widehat{\overline{F}(u)} = \frac{1}{n}\sum_{i=1}^{n} I(X_i > \mu) = \frac{N_\mu}{n},$$

$$\widehat{\overline{F}_\mu(y)} = 1 - H_{\hat{\sigma}_\mu, \hat{\xi}}(y) = \left(1 + \hat{\xi}\frac{y}{\hat{\sigma}_\mu}\right)_+^{-1/\hat{\xi}},$$

where $\hat{\xi}$ and $\hat{\sigma}_\mu$ are maximum likelihood estimators of ξ and σ_μ. Thereafter the tail estimator is

$$\widehat{\overline{F}(\mu + y)} = \frac{N_\mu}{n}\left(1 + \hat{\xi}\frac{y}{\hat{\sigma}_\mu}\right)_+^{-1/\hat{\xi}}.$$

This relationship between probabilities allows us to obtain VaR for the original asset returns $\{r_t\}$. For a specified small probability α such that

$$\begin{aligned}\alpha &= Pr(r_{T+1} < v) = Pr(-z_{T+1} > -\tfrac{v - \mu_{T+1}}{\sigma_{T+1}}) \\ &= \overline{F}\big(\mu + (-\tfrac{v - \mu_{T+1}}{\sigma_{T+1}} - \mu)\big),\end{aligned}$$

the lower tail α-th quantile VaR of $\{r_t\}$ is v. Consequently, for a given small probability α, one can check that the VaR of holding a long position in the asset with underlying return $\{r_t\}$ is

$$\text{VaR}_{t,\alpha} = \begin{cases} -\Big(\mu + \frac{\hat{\sigma}_\mu}{\hat{\xi}}\big((\frac{n}{N_\mu}\alpha)^{-\hat{\xi}} - 1\big)\Big) \times \sigma_{t+1} + \mu_{t+1}, \hat{\xi} \neq 0, \\ -\big(\mu + \hat{\xi}\ln(\frac{n\alpha}{N_\mu})\big) \times \sigma_{t+1} + \mu_{t+1}, \hat{\xi} = 0. \end{cases} \tag{1.28}$$

1.3.4 Backtesting

Based on the market risk amendment by the Basel Committee (1996), qualifying financial institutions have the freedom to specify their own model to compute their VaR. The procedure of backtesting thus becomes crucially important for regulators to assess the quality of the models.

Consider a time series of daily portfolio returns, r_t, and a corresponding time series of VaR forecasts, $VaR_{t,\alpha}$ with promised coverage rate α, such that ideally

$$\Pr(r_t < VaR_{t,\alpha}) = \alpha.$$

Define the hit sequence of $VaR_{t,\alpha}$ violations as

$$I_t(\alpha) = \begin{cases} 1 & \text{if } r_t < VaR_{t,\alpha} \\ 0 & \text{else.} \end{cases}$$

As stressed by Christoffersen (1998)[25], VaR forecasts are valid if and only if the violation sequence $\{I_t\}$ satisfies the following two hypotheses:

(i) The unconditional coverage hypothesis: the probability of an return exceeding the VaR forecast must be equal to the coverage rate

$$\Pr(I_t(\alpha)) = 1) = \mathbb{E}(I_t(\alpha)) = \alpha.$$

(ii) The independence hypothesis: VaR violations observed at two different dates for the same coverage rate must be distributed independently.

We thereafter implement two backtesting tests, the Kupiec's unconditional coverage test and the Christoffersen and Pelletier's duration-based test of independence, for the GARCH-VaR and GARCH-EVT-VAR model evaluation.

1.3.4.1 Kupiec's unconditional coverage test

Kupiec likelihood ratio unconditional coverage test (Kupiec, 1995[60]) exploits the fact that an adequate model is supposed to have its proportion of violations of VaR estimates close to the corresponding tail probability level. Assume the sample size is n and days of failure is N, then the frequency of failure is N/n. Subsequently, for the null hypothesis that the expected proportion of violations is equal to α, i.e. $H_0 : N/n = \alpha$, Kupiec (1995)[60] proposed a proper likelihood ration test.

Under the null hypothesis, the statistic function

$$LR = 2\ln\left((1-\frac{N}{n})^{n-N}(\frac{N}{n})^{N}\right) - 2\ln\left((1-\alpha)^{n-N}\alpha^{N}\right)$$

is asymptotically distributed according to a chi-square distribution with one degree of freedom, i.e. $LR \sim \chi^2(1)$. Consequently, one rejects the null hypothesis if the p-value of the unconditional coverage test is less than the predetermined significance level.

1.3.4.2 A Duration-based test of independence

Denote D_i the duration between two consecutive violations as

$$D_i = t_i - t_{i-1},$$

where t_i denotes the date of the i-th violation. Under the null hypothesis that the risk model is correctly specified, the no-hit duration should have no memory and a mean duration of $1/\alpha$ days. Hence, the duration variable $\{D_i\}$ follows a geometric distribution given by

$$f_{geo}(D;\alpha) = \alpha(1-\alpha)^{D-1}. \tag{1.29}$$

A more convenient representation of the same information is given by transforming the geometric probabilities into a flat function. The hazard rate defined as

$$\lambda(D_i) = \frac{\Pr(Di = d)}{1-\Pr(D_i < d)}, \tag{1.30}$$

where $\lambda(D_i)$ could be written explicitly as

$$\frac{(1-\alpha)^{d-1}\alpha}{1-\sum_{j=1}^{d-2}(1-\alpha)^{j}\alpha} = \alpha,$$

collapses to a constant after expanding and collecting terms.

Exploiting Eq.(1.29), Christoffersen and Pelletier (2004)[24] proposed the first duration-based test. They used under the null hypothesis the exponential distribution, which is the continuous analogue of the geometric distribution with a probability density function, defined as:

$$f_{exp}(D;\alpha) = \alpha exp(-\alpha D). \tag{1.31}$$

The most powerful of the two alternative hypotheses they consider is that the duration follow a Weibull distribution where

$$f_{Weibull}(D;a,b) = a^b b D^{b-1} exp(-(aD)^b). \tag{1.32}$$

As the exponential distribution corresponds to a Weibull with a flat hazard function, i.e $b = 1$, the test for the independence hypothesis (Christoffersen and Pelletier, 2004[24]) is then simply:

$$H_0 : b = 1.$$

When $b < 1$, the hazard, i.e., the probability of getting a violation at time D_i given that we did not up to this point, is a decreasing function of D_i. One rejects the hull hypothesis if the p-value less than the predetermined significance level.

In conclusion, Kupiec's unconditional coverage test [60] checks whether the amount of expected versus actual exceedances given the tail probability of VaR actually occur as predicted, while the conditional coverage test of Christoffersen [24] is a joint test of the unconditional coverage and the independence of the exceedances. In this section, both the joint and the separate unconditional test will be reported in the following section since it is always possible that the joint test passes while failing either the independence or unconditional coverage.

1.3.5 Empirical results and discussion

By observing the autocorrelations in section 1.3.2, we found the unpredictability and volatility clustering behavior in the considered precious metal returns. To filter out the autocorrelations of considered metal log-returns, the autoregressive AR(1) model is used here since AR(1) is singled out according to the censored orders of autocorrelation and partial autocorrelation functions graphs through numerous trails. Whereas the four metal returns has significant volatility clustering, so a GARCH-type model needs to be adopted. Because of the fat tail of the return, the GED is carried out to depict the residual of the GARCH-type model. Hence, GARCH(1,1), TGARCH(1,1) and EGARCH(1,1) models are developed so as to further investigate the leverage effect of the precious metal returns. According to the minimum AIC value and the need to describe the asymmetric volatility, a AR(1) - GED based EGARCH(1,1) model is singled out for the four metal daily log-returns, whose estimation results as well as estimation results from the other two model are stated in Panel A of Table 1.22.

As can be seen from Panel A of Table 1.22, GED degree parameter k of all return series on any of the three models are all less than 2, which confirms the fact that the tails of these metal returns are thicker than that of Gaussian distribution. According to the parameter estimation result from the conditional variance EGARCH(1,1) equation, we found that the leverage effects coefficient γ are all positive and significant at any significant level which means that good news generates more volatility than bad news for precious metal markets. Moreover, the asymmetric volatility behavior is the most significant in Palladium while the least significant in gold. The coefficient estimators β of $\ln \sigma_{t-1}^2$ in the EGARCH(1,1) conditional variance model are all greater than 0.95, which indicates that over 95% of current variance shock can still be seen in the following period. Therefore, the volatility clustering in those metal returns are clear, and the decay of the volatility shock are quite slow.

After choosing the most appropriate conditional variance model for each metal log-returns, the standardized residual series from the AR(1) - GED based EGARCH(1,1) model can be retrieved. In Fig.1.15, we plot the negative standardized residuals of gold price return series from AR(1) - GED based EGARCH(1,1) model. We thereafter

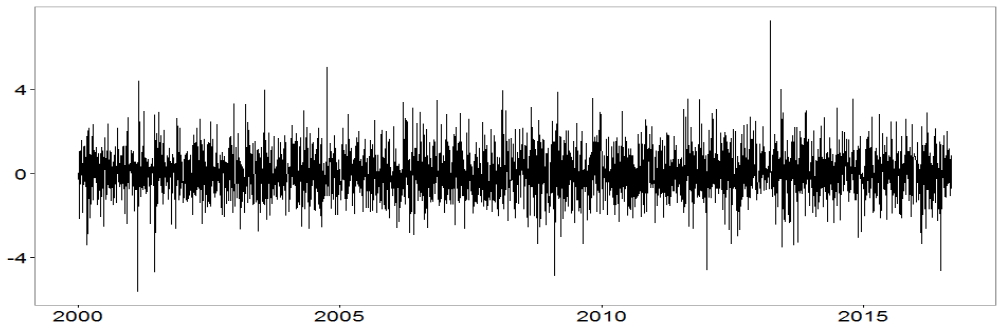

Figure 1.15: Time series plot of the negative standardized residuals for gold daily returns from 2000-1-11 to 2016-9-9.

investigate the normality and autocorrelation function for the negative standardized residuals, and show the result in Panel B of Table 1.22. The result of the Shapiro-Wilk normality test confirms that none of these negative standardized returns follow normal distribution. Examining the result of Ljung-Box test for the negative standardized residual series and the squared residuals from AR(1) model, it can be seen that most of the autocorrelation coefficients fall within the 99% confidence interval. Therefore, one can deduce that there are no longer autocorrelation in these non-normally distributed residual series. The autocorrelation correlograms of the negative standardized residuals are plotted in Fig.1.16 which confirms that although the original gold price return time series are autocorrelated, the standardized residuals are memoryless. All of this indicates that the AR(1) - GED based EGARCH(1,1) model has fitted the four precious metal returns very well.

Then we apply the extreme value approach as stated in section 1.3.3.2 on the negative standardized residuals to estimate VaR of the four precious metals. First, we choose a proper threshold for each metal residual series using Hill plot and mean excess

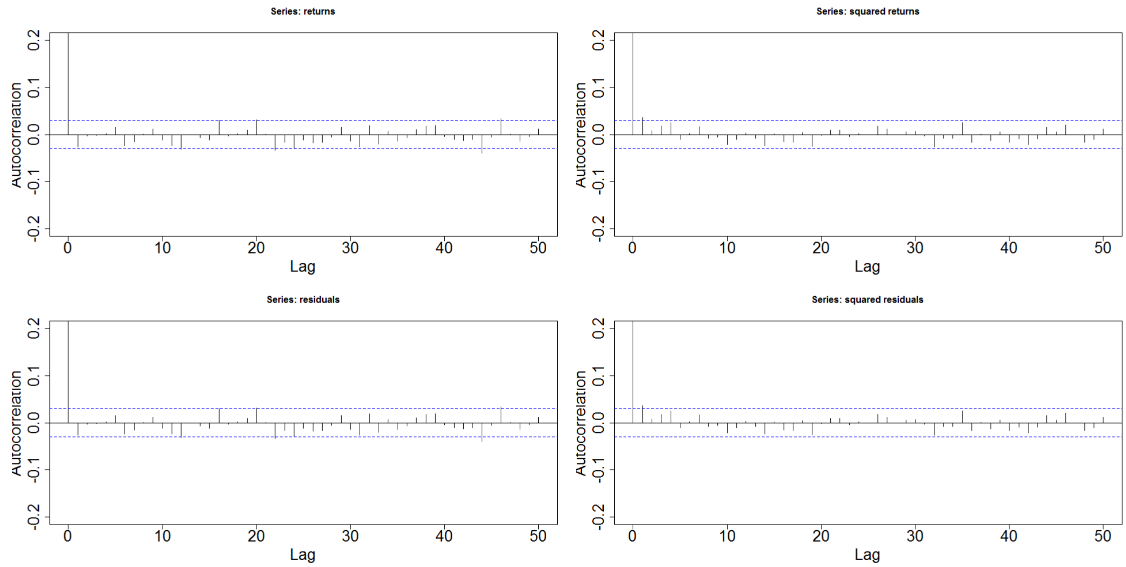

Figure 1.16: Correlograms for the gold price returns and their squared values, as well as for the standardized residuals and squared residuals.

plot. Fig.1.17 shows the Hill plot and mean excess plot of gold negative standardized residuals, which indicates that a threshold with around 50 exceedances, is reasonable for the gold residual series. The selected threshold values and the corresponding

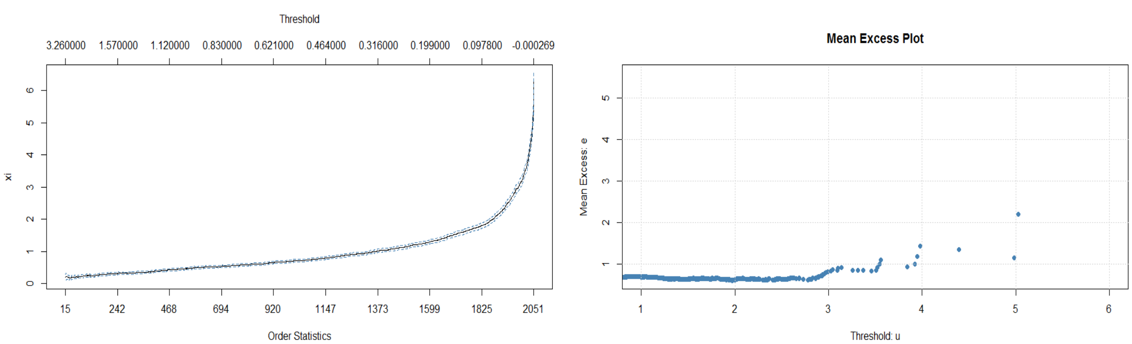

Figure 1.17: Hill plot and Mean Excess Plot of gold negative standardized residuals from 2000-1-11 to 2016-9-9

number of exceedances N_μ are reported in Panel A of Table 1.23. Then, we fit these excess values to a GPD model and use the maximum likelihood estimation to determine the shape parameter ξ and location parameter σ_μ, which are also included in Panel A of Table 1.23.

According to the Panel A of Table 1.23, the estimated shape parameter ξ are positive suggesting that the left tail of standardized residuals for gold, silver and platinum are characterized by heavy-tail distributions. Meanwhile, for palladium, the negative $\hat{\xi}$ indicates that the left tail of palladium standardized residuals are characterized by a light-tail distribution. In comparison of the four precious metal VaRs, Panel B and C of Table 1.23 provides the one-day ahead estimates of VaR for each returns, at various quantiles levels. The table presents the forecasts constructed from the fitted GPD model on the standardized negative residuals, which is shown is Eq.(1.28), and these are contrasted against the estimates drawn directly from the traditional GARCH model, see Eq.(1.25).

At a quantile level of 99.5% , the estimated VaR from our GARCH-EVT approach is 3.2283881 for losses. This means that we are 99.5% confidence that the expected market value of gold would not lose more than 3.2283881% for the worst case scenario, within one-day duration. Similar interpretations can be made for the considered GARCH-VaR model. The reason we choose the AR(1) - GED based EGARCH(1,1) model as the representative of traditional GARCH models to VaR is that EGARCH(1,1) model is identified as the most proper conditional variance model for the four metal returns. In comparison of different models to VaR, it is also interesting to note that the GARCH-EVT produced lower VaR forecasts than the EGARCH model, at any quantile levels for any metal price return series. It indicates that our methodology, which first adopt GARCH-type models to forecast volatility and then concentrate on the tail distribution of standardized residuals, is more realistic and comprehensive than the commonly used GARCH to VaR model.

To investigate further about the dynamics of VaR for the four precious metal, we use a moving window to estimate the one-day-ahead 0.5% quantile VaRs using our GARCH-EVT approach and show it in Fig.1.18.

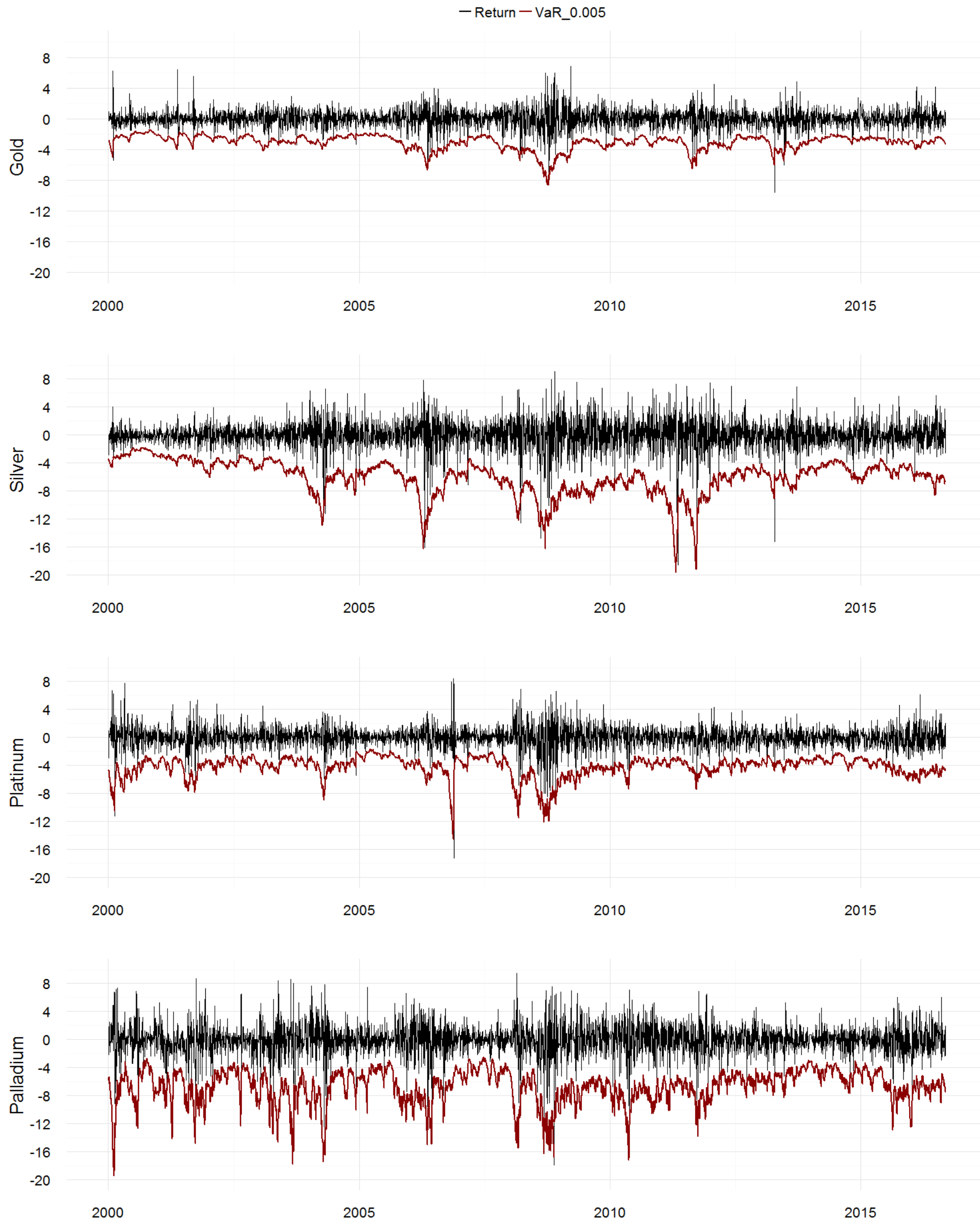

Figure 1.18: Downside 0.005 quantile VaRs of gold, silver, platinum, and palladium (from top to bottom).

In comparison of dynamic VaRs for the four precious metals, averagely speaking, gold has the most steady and the highest VaR, then is platinum and silver, while palladium has the most volatile and the lowest VaR. As a valuable asset in terms of investment, gold has served as the most stable monetary standards which might be a reason that gold has the most steady VaR comparing to other precious metals. The VaRs for palladium are the most volatile one mainly due to the demand and supply of it is highly unstable since palladium is much rarer than other precious metals. It is interesting to note that, according to the comparison graphs Fig.1.18, silver has been more volatile than platinum. Moreover, there are several factors contributing to VaR de-escalations in metal markets. Firstly, in the period that financial markets crash, such as around 2008, investors lose confidence on equity market and prefer to invest those assets that do not have heavy liability or unpredictability such as precious metals. Conversely, when the financial market thriving or there is a bull run in U.S. stocks, the demand for gold will decline, such as around 2013, most of the investors moved out of gold and into equities because there was a string of strong economic data soothed worries about wealth preservation and encouraged investors to seek greater gains in equities. Gold's losses at 28% in 2013, neatly match the percentage gains in U.S. equities, illustrating how funds flowed from one asset class and into the other. Secondly, precious metal trading can offset the potential movement of real value in the short-term market against international inflation and US dollar oscillations. Thirdly, the oil price is a main macroeconomic variables that influence the precious metal market. Fig.1.18 hence also depict several significant VaRs historical jumps in the latest sixteen years.

To show the validation result our GARCH-EVT approach to VaR, Table 1.24 provides the backtesting results of VaR estimates, where the level of VaR ranging among 0.5%, 1% and 5%. Both the Kupiec test and Christoffersen test suggest that the VaR estimates from our asymmetric GARCH-EVT approach cannot be rejected.

Therefore, the GED based EGARCH model combined with EVT approach does very well in predicting critical loss for precious metal markets. Our findings reveal that models considering for some stylized facts such as leptokurtic, volatility clustering and asymmetry in the financial time series behavior enhance the VaR predicting.

1.3.6 Conclusion and future work

In this section, we introduce an extension of the two stage approach by McNeil and Frey (2000)[67]. We extend the GARCH model to GED based GARCH-type models by taking into account major stylized facts into the price return volatilities of precious metal markets. Our findings reveal that precious metals are characterized by fat tail distributed, volatility clustering and leverage effect behavior. After illustrating the GED based GARCH-type models combined with EVT methodology, we then implemented it in predicting one-day ahead VaRs for precious metal markets and compared it with VaRs from GARCH-type model directly. We also compared VaRs from our GARCH-EVT approach with that from GARCH based VaR model, The comparative analyses with the well-known GARCH-based VaR models were included as well. Moreover, we compared the dynamic VaRs in gold, silver, palladium and platinum and found that gold has the most steady VaRs, while palladium has the most volatile VaRs. At the end, the backtesting results confirms that our approach performs exceptional.

Since a combination of our results with analysis about the multivariate dependence structure between four precious metals may prove very useful in the context of an investor's optimal portfolio choice, we will return to these, copulas application in estimating multivariate dependence structure, in future publications. Moreover, we are also interested in quantifying spillover effects of extreme price movements from one precious metal to other precious metals. Based on our results, CoVaR (Adrian and Brunnermeier (2011)[1]), which is a systemic risk measure, would be used in analyzing this question. By providing the VaR of one precious metal price conditional on the fact that another precious metal price is experiencing extreme movements as measures by its own VaR, CoVaR captures spillover effects in precious metal prices. We have not done so yet. A detailed analysis of this question is left for future research.

Overall, our findings confirm that taking into account volatility clustering and asymmetry in the behavior of precious metal return time series as well as combining filtering process such as extreme value approach is important in improving risk management assessments and hedging strategies.

Table 1.22: Estimation and hypothesis test result of AR(1) - GARCH-type models for four precious metal daily log-returns from 2000-1-1 to 2016-9-9

Parameter	Gold	Silver	Platinum	Palladium
Panel A: Parameter estimation and information criterion statistics				
AR(1) - GED based GARCH(1,1) model				
ϕ_0	0.032112 (0.01124)	0.007650 (0.69950)	0.034659 (0.05210)	0.022264 (0.38095)
ϕ	0.010066 (0.46697)	-0.069080 (0.00000)	0.026840 (0.08754)	0.058940 (0.00007)
ω	0.016303 (0.00294)	0.011118 (0.02539)	0.041797 (0.00006)	0.113815 (0.00002)
η	0.052498 (0.00000)	0.054522 (0.00000)	0.093124 (0.00000)	0.133045 (0.00000)
β	0.0934726 (0.00000)	0.944478 (0.00000)	0.888573 (0.00000)	0.850146 (0.00000)
k	1.211903 (0.00000)	1.318597 (0.00000)	1.351262 (0.00000)	1.191177 (0.00000)
AIC	2.8515	3.9304	3.3205	4.0727
AR(1) - GED based TGARCH(1,1) mode				
ϕ_0	0.029930 (0.04181)	0.009773 (0.63482)	0.036904 (0.03693)	0.027338 (0.11242)
ϕ	0.009053 (0.53166)	-0.068119 (0.00001)	0.027271 (0.07388)	0.060296 (0.00001)
ω	0.015212 (0.00285)	0.011672 (0.02255)	0.040867 (0.00008)	0.116066 (0.00001)
η	0.040201 (0.00046)	0.062613 (0.00000)	0.101645 (0.00000)	0.162129 (0.00000)
β	0.937073 (0.00000)	0.944359 (0.00000)	0.889090 (0.00000)	0.847365 (0.00000)
γ	0.021170 (0.11379)	-0.015951 (0.19605)	-0.016083 (0.41100)	-0.050227 (0.05322)
k	1.214966 (0.00000)	1.317588 (0.00000)	1.352807 (0.00000)	1.191175 (0.00000)
AIC	2.8514	3.9305	3.3208	4.0722
AR(1) - GED based EGARCH(1,1) model				
ϕ_0	0.023192 (0.10257)	0.003407 (0.87198)	0.037506 (0.04652)	0.025043 (0.33152)
ϕ	0.007056 (0.61582)	-0.065504 (0.00004)	0.027154 (0.086327)	0.062081 (0.00032)
ω	0.000650 (0.66099)	0.005104 (0.00016)	0.010462 (0.00002)	0.044633 (0.00017)
η	-0.022538 (0.02696)	-0.006479 (0.48258)	0.008270 (0.509879)	0.022015 (0.17214)
β	0.990753 (0.00000)	0.995171 (0.00000)	0.979532 (0.00000)	0.965677 (0.00000)
γ	0.113033 (0.00005)	0.120380 (0.00000)	0.189213 (0.00000)	0.260041 (0.00000)
k	1.224575 (0.00000)	1.328313 (0.00000)	1.353979 (0.00000)	1.197389 (0.00000)
AIC	**2.8436**	**3.9203**	**3.3174**	**4.0647**
Panel B: Hypothesis tests on negative residuals				
Shapiro-Wilk	0.97862 ($< 2.2e-16$)	0.9784 ($< 2.2e-16$)	0.97503 ($< 2.2e-16$)	0.97815 ($< 2.2e-16$)
LB - Q(5)	3.7988 (0.5787)	7.7768 (0.169)	7.9392 (0.1596)	19.85* (0.001333)
LB - Q(10)	8.2653 (0.6029)	11.015 (0.3564)	9.7096 (0.4663)	23.057 (0.01054)
LB - Q_s(5)	9.8461 (0.07972)	23.025* (0.0003339)	5.3307 (0.3769)	4.5388 (0.4747)
LB - Q_s(10)	13.253 (0.2099)	28.794* (0.001346)	5.6133 (0.8466)	8.9894 (0.5331)

Note: p-values are reported in parentheses; the Akaike information criterion (AIC for short) is a measure of the relative quality of statistical models.

Table 1.23: Parameter estimation results from fitted GPD and estimates for one-day ahead VaR

	Gold	Silver	Platinum	Palladium
Panel A: results from fitted GPD model for negative standardized residuals				
μ	2.358336	2.469984	2.125745	2.699225
N_μ	50	55	95	52
$\hat{\xi}$	0.2060037	0.1758788	0.2008195	-0.1175530
	(0.2020086)	(0.1546582)	(0.09845623)	(0.1527495)
$\hat{\sigma_\mu}$	0.5383237	0.6859990	0.5596442	0.7239877
	(0.1318199)	(0.1397956)	(0.07874522)	(0.1488937)
Panel B: estimates for 1-day ahead VaRs from the GARCH-EVT approach				
$\text{VaR}_{T+1,0.005}$	-3.2283881	-4.3744174	-5.3888110	-6.7105573
$\text{VaR}_{T+1,0.01}$	-2.7632794	-3.6741353	-4.5493432	-5.9440062
$\text{VaR}_{T+1,0.05}$	-1.9081735	-2.3425701	-2.9970233	-3.9037074
Panel C: estimates for 1-day ahead VaRs from the AR(1) - GED based EGARCH(1,1) model				
$\text{VaR}_{T+1,0.005}$	-2.954591	-3.53238	-4.905009	-5.272009
$\text{VaR}_{T+1,0.01}$	-2.565507	-3.090641	-4.288894	-4.547062
$\text{VaR}_{T+1,0.05}$	-1.614489	-1.614489	-2.746142	-2.784847

Standard deviation are reported in parentheses; The quantile level α is various in $\{0.005, 0.01, 0.05\}$.

T represents the last day of the time series, which is September 9th, 2016.

Table 1.24: Backtesting results of VaR from the asymmetric GARCH-EVT approach for four precious metal return series from 2000-1-11 to 2016-9-9

	Gold	Silver	Platinum	Palladium
Panel A: Kupiec's unconditional coverage test result				
$LR_{\alpha=0.005}$	4.399908	0.0003865	0.07121597	0.07121597
EE/AE	20/31	21/21	20/22	20/22
Decision	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	8.056199	0.1158002	0.007386259	0.004905377
EE/AE	41/61	42/40	41/41	41/42
Decision	Reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	0.3001823	0.4966088	0.8401481	0.02556351
EE/AE	207/200	210/201	207/195	207/210
Decision	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
Panel B: Christoffersen's duration-based test result				
$LR_{\alpha=0.005}$	0.6083054	0.04492569	0.4389884	0.1929353
Decision	Fail to Reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	0.6782278	0.6003412	0.8752114	0.427504
Decision	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	0.01013684	0.8848006	0.001358113	0.02317906
Decision	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0

LR means the likelihood ratio test statistic with various confidence level α in $\{0.005, 0.01, 0.05\}$.

In Panel A, EE/AE means the ratio of expected exceed (EE) and the actual exceed (AE) of the tested data series.

Decisions are made with 1% significance level.

CHAPTER 2

COPULAS FOR FINANCE

2.1 Conditional dependence among oil, gold and U.S. dollar exchange rates: a copula-GARCH approach

This section investigates the dependence structure among nominal crude oil (WTI), gold, and specific U.S. dollar against four major currencies (Euro, British Pound, Japanese Yen and Canadian Dollar) on a daily basis over the last decade. In order to capture the tail dependence between commodity market and USD exchange rates, we apply both bivariate zero tail and tail copulas, as well as trivariate copulas, combined with the AR-GARCH marginal distribution for gold, oil and exchange rates daily returns. The primary findings are as follows. Firstly, based on the concordance and correlation coefficient, we find that there is a positive correlation between gold and crude oil prices, and a negative dependence between gold and currencies as well as oil and currencies. Secondly, the crude oil price can be viewed as a short term indicator in the exchange rates movement; the crude oil price also can be viewed as a short term descend indicator of gold price, while the gold price is an short term rise indicator of oil price. Thirdly, small degree of conditional extreme tail dependence for all considered pairs are observed. Our results provide useful information in portfolio diversification, asset allocation and risk management for investors and researchers.

2.1.1 Background

As a financial indicator, gold is classed as one of the most important commodities and one of the most stable monetary asset. As a multifaceted metal through the centuries, it has common ground with money in that it acts as a unit of value, a store of wealth, medium of exchange and a hedging instrument. Therefore, gold has always been used as a hedge against inflation, deflation and currency devaluation. Gold also plays an important role with significant portfolio diversification properties. An abundance of research point to the benefits of including gold holdings that leads to a more balanced portfolio (Johnson and Soenen (1997)[54]; Ciner (2001)[26]; Shafiee and Topal (2010)[80]).

Since the international gold and foreign exchange markets are both dominated by the U.S. dollar, the relationship between gold and U.S. exchange rates have received much attention, especially after the international financial crisis. Moreover, the price of oil, another one of the most important commodities, is also dominated in U.S. dollar. The importance of crude oil in global economy will continue during this century as a unique raw material responsible for power generation and lots of derivatives production. Hence, due to its effect on world economic growth and energy costs, the behavior of crude oil price has attracted considerable attention. Also, the oil price and inflation rate are two main macroeconomic variables that influence the gold market.

The above motivations demonstrate the importance in measuring and capturing the stylized facts exhibited in the oil price, gold price and U.S. dollar exchange rates, as well as the relationship among them. In this section, we focus on investigating both the conditional dependence and the extreme comovement of gold, crude oil and U.S. dollar exchange rates on each other using a copula-GARCH approach. The analysis of our study is not merely for risk management and market trading issues, but also for the better regulation of foreign exchange markets.

In recent years, a number of methods have been employed to explore the relationship between gold prices or oil prices with US dollar exchange rate. Sjaastad and Scacciavillani (1996)[82] identified the effect of major currency exchange rates on the prices of gold. A variation in any exchange rate will result in an immediate adjustment in the prices of gold. The power of such phenomenon is also suggested by Capie et. al. (2005)[20] where assessed the role of gold as a hedge against the dollar and concluded that the negative relationship was found between gold prices and the sterling-dollar, yen-dollar exchange rates. Recently, Sari et. al.(2010)[78] examined the co-movement and information transmission among precious metals, oil price, and dollar-euro exchange rate. Joy (2011)[56] applied the dynamic conditional correlations model on 23 years weekly data for 16 major dollar-paired exchange rates and find that the gold has behaved as a hedge against dollar. For the theory on oil prices and dollar exchange rates, Krugman (1983)[59], Golub (1983)[40], and Rogoff (1991)[77] identified the important relation between the oil prices and the exchange rate movements. Using various data set on oil prices and dollar exchange rates over different time period, the extensive evidences on the co-movement between two variables can also be found in literature, see Amano et al. (1998)[4], Akram (2009)[2], Basher (2012)[8] , Wu et al. (2012)[87], and Aloui et al. (2013)[3]. To offer a comparative view, we summarize the key findings of major studies in the related literature in Table 2.1. In this study, we use a Copula - GARCH model to capture the conditional volatility and dependence structures of gold, crude oil and USD exchange rates on each other. To appropriately investigate the behavior of considered assets, AR-GARCH models have been chosen to describe and measure the conditional mean and conditional volatility of returns. The advantage of our method is that we standardize the return series by filtering out the influence of the conditional mean and the volatility using AR-GARCH models; then we apply the copula approach to analyze the tail dependence for the standardized residues. The conditional dependence and tail dependence analysis are based on copula approach with proper marginal distributions. The reason to apply copula based approach to our data is that copulas allow for better flexibility in joint distributions than multivariate normal and Student-t

Table 2.1: Previous research on the interactions among gold prices, oil prices and exchange rates

Studies	Purposes	Data	Methodology	Primary findings
Recent literature on modeling gold prices and USD exchange rates.				
Sari et. al. (2010)	This study examines the co-movements among the prices of metals, oil price, and the exchange rate.	Daily data (1999 - 2007)	The forecast error variance decomposition on impulse response functions	The evidence of a weak long-run equilibrium relationship but strong feedbacks in the short run.
Pukthuanthong and Roll(2011)[73]	The paper investigated relationship between Dollar, Euro, Pound, and Yen.	Daily data (1971 - 2009)	GARCH	The gold price expressed in a currency can be associated with weakness in that currency and vice versa.
Joy (2011)	This paper addresses a practical investment question if the gold act as a hedge against the US dollar.	Weekly data (1986 - 2008)	Multivariate GARCH	The gold has acted, increasingly, as an effective hedge against currency risk associated with the US dollar.
Yang and Hamori (2014)	The paper investigates the dynamic dependence structure between specific currencies(GBP, EUR, JPY) and gold prices	Daily data (2012 - 2013)	Copula - GARCH	Lower and upper conditional dependences between currencies and gold were weaker during the financial turmoil period than normal period
Recent literature on modeling oil prices and USD exchange rates.				
Akram (2009)	The author investigates the contribution of a decline in real interest rates and the US dollar to higher commodity prices.	Quarterly data (1990 - 2007)	Structural VAR model	A fall in the value of the US dollar leads to drive up commodity prices, including crude oil price.
Wu et al. (2012)	The authors examine the economic value of comovement between WTI oil price and U.S. dollar index futures.	Weekly data (1990 - 2009)	Copula - GARCH	The dependence structure between oil and exchange rate returns becomes negative and decreases continuously after 2003.
Basher et al. (2012)	The authors study the dynamic link between oil prices, exchange rates and emerging market stock prices.	Monthly data (1988 - 2008)	Structural VAR model	Positive shocks to oil prices tend to depress emerging market stock prices and the trade-weighted US dollar index in the short run.
Aloui et al. (2013)	The authors study the conditional dependence structure between crude oil prices and U.S. dollar exchange rates.	Daily data (2000 - 2011)	Copula - GARCH	The rise in the price of oil is found to be associated with the depreciation of the dollar.

distributions. In addition, copulas not only capture linear dependence as correlation, but also describe nonlinear dependence of different financial markets. Moreover, since copulas present rich patterns of tail dependence, it helps us to examine changes in the dependence structure during a financial crisis period.

The data we used are daily log returns of gold price, Brent and WTI prices, and specific exchange rates which including U.S. dollar against four major currencies

(Euro, British Pound, Japanese Yen and Canadian Dollar) from March 1, 2006 to March 18, 2016. Since Brent is the reference for about two-thirds of the oil traded around the world, and WTI the dominant benchmark for oil consumed in the United States, daily prices of Brent and WTI are used in this study to represent crude oil market. To investigate the dynamic of conditional dependence among gold, oil and U.S. dollar exchange rates, we first select the most appropriate marginal for each time series asset returns among four types of marginal models. Then, we apply copula models (elliptical and Archimedean copulas) on the standardized residuals to describe the conditional dependence structure between all considered pairs. We select Gaussian copula, Student-t copula, Clayton, Gumbel, BB7 copulas and their rotated versions copulas to compare and contrast with the conditional correlation.

2.1.2 Methodology

2.1.2.1 Marginal Distributions

The complexity of modeling financial time series is mainly due to the existence of stylized facts. After investigating daily log returns of gold value, Brent and WTI prices, and each of the four U.S. dollar exchange rates, the following three properties are concerned in this study. First one is that the price variations generally displays small autocorrelations while the corresponding squared returns or absolute returns are generally strongly autocorrelated. The second is leptokurtosis, which means financial time series tendency to have distributions that exhibit fat tails and excess peakedness at the mean. The third is the volatility clustering that large absolute returns are expected to follow large absolute returns and small absolute returns are expected to follow small absolute returns.

To capture these stylized facts, we use the autoregressive moving average model $ARMA(p, q)$ to quantify the conditional mean and the univariate generalized autoregressive conditional heteroscedasticity model GARCH(1,1) to capture the conditional variance. This modeling approach is advantageous in that it offers the possibility to separately model the margins and association structure of different variables.

Let $\{r_t\}_{t=T-n+1}^{T}$ be the time series representing the daily log return on a financial asset price. Here we fixed a constant memory n so that at the end of day T our data consist of the last n daily log returns $\{r_{T-n+1}, \cdots, r_{T-1}, r_T\}$. Assume the dynamics of $\{r_t\}_{t=T-n+1}^{T}$ be a realization from an $ARMA(p,q)$-GARCH(1,1) process, which are given by

$$\begin{cases} r_t &= \mu_t + \sigma_t z_t \\ \mu_t &= \mu + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_j \\ \sigma_t^2 &= \omega + \alpha(r_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2, \end{cases} \tag{2.1}$$

where the innovations z_t are white noise process with zero mean, unit variance, and marginal distribution function F; $\omega > 0, \alpha > 0$, and $\beta > 0$. The conditional mean $\mu_t = \mathbb{E}(r_t|\mathcal{F}_{t-1})$, and the conditional volatility $\sigma_t^2 = \mathrm{Var}(r_t|\mathcal{F}_{t-1})$ are measurable with respect to $\mathcal{F}_{t-1}$ which is the σ-algebra generated by information about the return process available up to time $t-1$.

The traditional GARCH model assumes a normal distribution for the innovations z_t. However, to capture the leptokurtosis properties for considered return series, we consider various marginal distributions for z_t, which includes normal, skewed normal, Student-t and skewed Student-t distributions. For each considered return series, we specify the marginal distribution by comparing with Akaike information criterion (AIC) under different assumptions of innovation marginal distributions.

2.1.2.2 Copula function

Recently, the study of copula functions have been a popular phenomenon in constructing joint distribution functions and modeling statistical dependence in real multivariate data. Copulas have been applied to many areas including finance[22], actuarial science[35], medical research[32], econometrics[**?**], environmental science[90], just to name a few. Copulas provide flexible representations of the multivariate distribution by allowing for the dependence structure of the variables of interest to be modeled separately from the marginal structure. We here briefly review the multivariate copulas. For the general copula theory, see [70, 53].

A bivariate copula is a joint cumulative distribution function (CDF) on $[0,1]^2$ with standard uniform marginal distributions. More precisely, a bivariate **copula** (or 2-**copula**) is a function $C : [0,1]^2 \mapsto [0,1]$ satisfying following properties:

(i) $C(u,0) = C(0,v) = 0$, for $u, v \in [0,1]$,

(ii) $C(u,1) = u, C(1,v) = v$, for $u, v \in [0,1]$, and

(iii) For any $u \leq u', v \leq v'$, $C(u',v') - C(u,v) - C(u',v) + C(u,v) \geq 0$.

Let $(X_1, X_2)^T$ be a 2-dimensional random vector with CDF denoted as $H(x_1, x_2)$, and marginal CDF's $F_1(x_1)$, $F_2(x_2)$. Sklar's theorem [83] states that if the marginals of $(X_1, X_2)^T$ are continuous, then there exist a unique copula C such that

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2)).$$

Copulas can be used to characterize the dependence in the tails of the distribution. Tail dependence is a measure of strength of dependence in the joint lower or joint upper tail of a joint distribution, which are particularly helpful for measuring the probability or the tendency of markets to crash or boom together. Two tail dependence measures defined in terms of the copulas are known as the upper and the lower tail dependence coefficients, respectively. The coefficient of lower tail dependence λ_L quantifies the probability of observing a lower X_2 assuming that X_1 itself is lower. It is defined as

$$\lambda_L = \lim_{u \to 0^+} P(X_2 \leq F_2^{-1}(u) | X_1 \leq F_1^{-1}(u)) = \lim_{u \to 0^+} \frac{C(u,u)}{u}.$$

Similarly, the upper tail dependence measure λ_U is defined as

$$\lambda_U = \lim_{u \to 1^-} P(X_2 > F_2^{-1}(u) | X_1 > F_1^{-1}(u)) = \lim_{u \to 1^-} 1 - \frac{1 - C(u,u)}{1-u}.$$

Most of the commonly used copulas are exchangeable, which requires that the value of the copula is invariant under permutations of its arguments. For some practical situations where one component of the variables influences the other one more than the other way around, exchangeability assumption on copula is not suitable. If the copula is assumed to be exchangeable, then there is a symmetric tail dependence between two random variables, i.e., $\lambda_U = \lambda_L$.

2.1.2.3 Copula models of conditional dependence structure

In this section, we consider two families of copulas: elliptical copulas (Gaussian copula and Student-t copula) and Archimedean copulas (Clayton, Gumbel, and BB7 copulas). These copula models allow us to study the conditional dependence structure and to evaluate the degree of tail dependence.

The normal and the Student-t copulas are constructed based on the elliptically contoured distribution such as multivariate Gaussian or Student-t distributions, respectively. Consider random variables X_1 and X_2 with standard bivariate normal distribution:

$$H_\rho(x_1, x_2) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{t^2 + s^2 - 2\rho st}{2(1-\rho^2)}\right) dtds,$$

where ρ is the Pearson correlation between X_1 and X_2. The marginal distributions of X_1 and X_2 follow standard normal distributions $N(0,1)$ with distribution function Φ. Then, the **Gaussian copula** is defined by

$$C_G(u,v) = H_\rho(\Phi^{-1}(u), \Phi^{-1}(v)),$$

where $\rho \in (-1, 1)$ is the correlation coefficient, and if $\rho = 0$ the Gaussian copula is reduced to be independent copula.

For random variables X_1 and X_2 with standard bivariate Student t distribution,

$$H_t(x_1, x_2; \rho, \nu) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \frac{1}{2\pi\sqrt{1-\rho^2}} \left(1 + \frac{t^2 + s^2 - 2\rho st}{2(1-\rho^2)}\right)^{-\frac{\nu+2}{2}} dtds.$$

We let T_ν denote the standard univariate Student t distribution function with degree freedom ν for the marginals X_1 and X_2 . Then the **Student-t copula** is defined by

$$C_t(u, v) = H_t(T_\nu^{-1}(u), T_\nu^{-1}(v); \rho, \nu), \tag{2.2}$$

where $\rho \in (-1, 1)$ and $\nu > 0$. The Gaussian copula is symmetric and has no tail dependence while the Student-t copula is also symmetric and can capture extreme dependence between variables. The trivariate Gaussian copula and t copula can be defined in similar fashion. Both the trivariate Gaussian copula and t copula associated with the random variables X_1, X_2 and X_3 has a correlation matrix, inherited from the elliptical distributions, and t-copula has one more parameter, the degrees of freedom (df). The correlation matrix in elliptical copulas determines the dependence structure. When we model the conditional dependence among variables in Section 2.1.3.3, we utilize the unstructured correlation matrix for both Gaussian copula and t-copula.

Archimedean copula family, a very popular family of parametric copula, contains the most widely used copulas like, Ali-Mikhail-Haq, Clayton, Frank, Gumbel, and Joe as the nest models [37]. The bivariate Archimedean copula is defined as

$$C(u_1, u_2) = \phi^{[-1]}(\phi(u_1) + \phi(u_2)), \tag{2.3}$$

where $\phi : [0, 1] \to [0, \infty]$ is a continuous strictly decreasing convex function such that $\phi(1) = 0$ and $\phi^{[-1]}$ is the pseudo-inverse of ϕ, i.e.,

$$\phi^{[-1]}(t) = \begin{cases} \phi^{-1}(t) & \text{if } 0 \le t \le \phi(0) \\ 0 & \text{if } \phi(0) \le t \le \infty. \end{cases}$$

The convex function ϕ is called the generator function of the copula C. If $\phi(t) = \frac{1}{\theta}(t^{-\theta} - 1)$, $\theta > 0$, then C defined in Eq.(2.3) is the Clayton copula. If we set $\phi(t) = (-\log t)^{\theta}$, $\theta \ge 1$, C defined in Eq.(2.3) is the Gumbel copula. Furthermore, C defined in Eq.(2.3) is called the BB7 copula when $\phi(t) = (1 - (1-t)^{\theta})^{-\delta}$, $\theta \ge 1$, and $\delta > 0$.

One limitation for the Clayton copula, Gumbel copula, and the BB7 copula is that they only allow the positive association. And in this section, we employ the rotated Clayton, Gumbel, and BB7 copulas to model the negative dependence among variables. Note that the t-copula defined in Eq.(2.2), the Clayton, Gumbel, the BB7 copula are able to capture the tail dependence. Furthermore, the Clayton, Gumbel, the BB7 copula are asymmetric copulas which can be utilized in modeling the asymmetric dependence and asymmetric tail dependence among variables during bear and bull markets.

For an absolutely continuous copula C, the **copula density** is defined to be

$$c(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v}. \tag{2.4}$$

2.1.2.4 Estimation of copulas

In the copula literature there are several commonly used estimation methods, for instance, the maximum likelihood (ML) estimation , the inference functions for margins (IFM)[53], and the maximum pseudo-likelihood (MPL) estimation[36]. The ML and IFM methods require the specification of parametric models for the marginals. In contrast, the advantage of MPL method is that it uses the rank-based estimators for the marginals, thus it is robust against misspecification of the marginal models.

In this section we take the advantage of the MPL method to estimate the proposed class of copulas, as it is not influenced by the choice of the marginal distributions.

Given a sample of m observations $(x_{11}, x_{21}), \ldots, (x_{1m}, x_{2m})$ from a random vector $(X_1, X_2)^T$, and let $C(u, v)$ be the associated copula. We first compute the normalized ranks or the rescaled empirical distributions for the variable X_1 and X_2, which are defined as: $u_i = \frac{r_i}{m+1}$, $v_i = \frac{s_i}{m+1}$, for $i = 1, \ldots, m$, where r_i and s_i are the rank of x_{1i} and x_{2i} among m data points from X_1, X_2, respectively. The pseudo log-likelihood function for the parameters in the copula is

$$\ell^p(\theta) = \log \prod_{i=1}^{n} c(u_i, v_i) = \sum_{i=1}^{n} \log c(u_i, v_i), \tag{2.5}$$

where $c(u, v)$ is the copula density of $C(u, v)$ in Eq.(2.4). We can obtain the maximum pseudo-likelihood estimators (MPLE) for the parameters by maximizing Eq.(2.5) with respect to θ.

To compare the performances of different parametric copula models with continuous response variables, we apply the goodness of fit procedures used by Genest et. al. (2009)[36] and Kojadinovic and Yan(2011)[58]. The goodness of fit test for copulas is obtained from the process comparing the parametric estimate of the copula derived under the null hypothesis with empirical copula,

$$\mathbf{C}_n = \sqrt{n}(C_n(u, v) - C_{\theta^{MPLE}}(u, v)),$$

where C_n is the empirical copula defined by

$$C_n(u, v) = \frac{1}{n} \sum_{i=1}^{n} I(u_i \leq u, v_i \leq v), \qquad u, v \in [0, 1].$$

$C_{\theta^{MPLE}}(u, v)$ is the copula with parameter θ^{MPLE} estimated based on the MPLE of θ. In other words, we want to test

$$H_0 : C \in \mathcal{C}_0 \qquad \text{against} \qquad H_1 : C \notin \mathcal{C}_0$$

The test statistics is computed based on the rank-based versions of the Cramér-Von Mises,

$$\mathbf{S}_n = \int \mathbf{C}_n^2 dC_n = \sum_{i=1}^{n} (C_n(u_i, v_i) - C_{\theta^{MPLE}}(u, v))^2.$$

Large values of the statistics $\mathbf{S}_n$ indicates the lack of fit. We find the p-values associated with test statistics by utilizing the R function 'gofCopula' in copula package[48]. The highest p-value indicates the distance between the estimated and empirical copulas is the smallest which in turn shows the best fit to the data.

2.1.3 Data and Empirical results

2.1.3.1 Data description and stochastic properties

To study the dynamical correlations, risk contagion and portfolio risks among gold price, oil prices, and exchange rates, we select the daily gold price in London bullion market quoted in U.S. dollars per gram, daily closing oil prices in US dollars per barrel of West Texas Intermediate (WTI), and five U.S. dollar (USD) exchange rates over the period from March 1, 2006 to March 18, 2016. As for the exchange rates, we employ the data come from the amount of USD per unit of each of the four major currencies in international trade: Euro (EUR), British Pound Sterling (GBP), Japanese Yen (JPY) and Canadian Dollar (CAD). The data used in this study are all taken from the database of Quandl. Fig.**??** provides the time series plots of daily spot oil prices, gold prices, as well as USD exchange rates.

To develop an accurate track record of asset performance, the initial price data are transformed into daily log-returns. Let p_t denotes the asset price on day t, then the corresponding daily percentage change is defined by

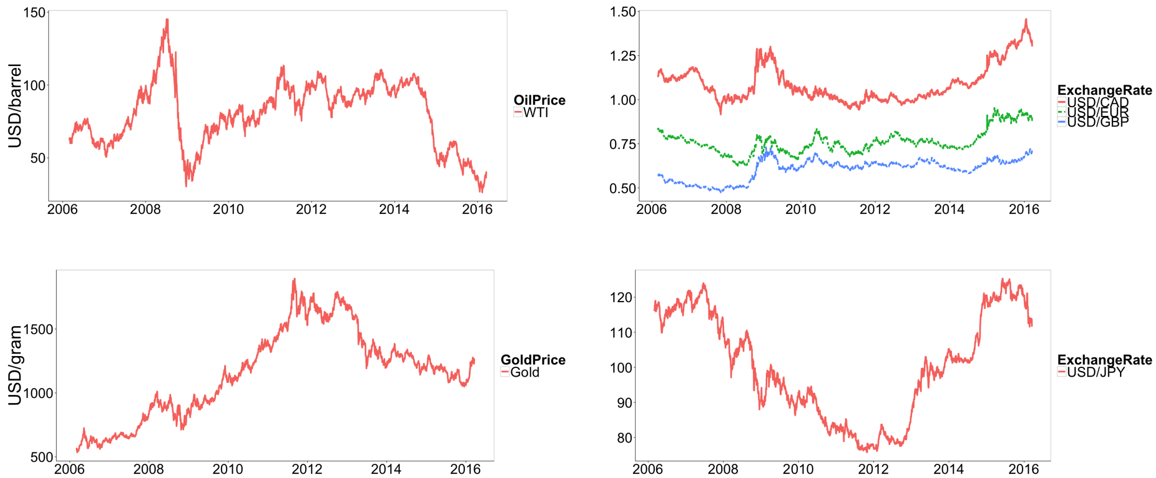

Figure 2.1: Time series plots of oil prices (upper left), gold value (lower left) and USD exchange rates(upper and lower right) from 2006-3-1 to 2016-3-18.

$$r_t = 100 \log \frac{p_t}{p_{t-1}} = 100(\log p_t - \log p_{t-1}).$$

We show the time paths of considered daily log returns in Fig.**??**. According to Fig.**??**, we observe that there are more isolated pronounced peaks than one would expect from the Gaussian series. Besides that, the high instability and volatility clustering behavior are also noticed in all return series. Those return series also exhibit two important price shocks, one is around the 2008 global financil crisis, the other one ranges from 2015 until recently.

Table 2.2 reports the descriptive statistics and distributional characteristics of all return series. As can be seen in Panel A of Table 2.2, the mean of all returns are quite small. As expected, the standard deviation of crude oil returns are larger than that of gold since oil is traded more heavily and actively than gold. Meanwhile, comparing with the standard normal distribution with skewness 0 and kurtosis 3, we confirm that all returns are lightly skewed and exhibit excess kurtosis. Verification of non-normal distributed behavior is from the results of Shapiro-Wilk normality test[81] in Panel B, which rejects the null hypothesis that the series follow a normal distribution at 1%

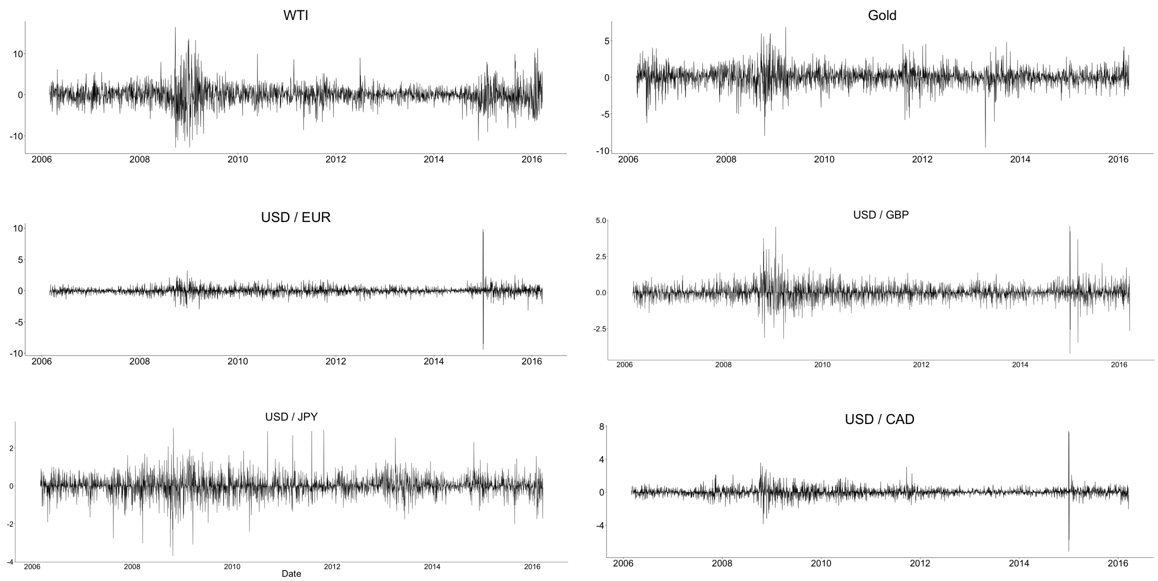

Figure 2.2: Daily returns on crude oil, gold and USD exchange rates from 2006-3-1 to 2016-3-18.

significance level. To check the autocorrelation of those returns, the Ljung-Box test is applied for returns at lag 5 and 10, i.e. Q(5) and Q(10), and squared returns $Q_2(5)$ and $Q_2(10)$. Specifically, Q(10) is the Ljung-Box Q-statistic for the null hypothesis that the series has no autocorrelation up to lag 10. The Ljung-Box test results for both return series and squared return series confirm that all sample returns have significant autocorrelation.

Based on above statistical analysis, we discover that all return series exhibit stationary, non-normally distributed, autocorrelated, and volatility clustering properties, which supports our choice of using the ARMA-GARCH based approach to analyze the conditional mean and conditional volatility for all returns.

2.1.3.2 Marginal distribution specifications and parameter estimations

In order to filter out the autocorrelation of the considered return series, the ARMA model is used in this section. According to the censored orders of autocorrelation and partial autocorrelation function graphs, an AR(1) model is singled out through

Table 2.2: Descriptive statistics and stochastic properties of return series from 2006-3-1 to 2016-3-18

	WTI	Gold	USD/EUR	USD/GBP	USD/JPY	USD/CAD
Panel A: Summary statistics						
Obs.	2532	2623	3661	3661	3661	3661
Min.	-12.83	-9.6	-9.4	-4.23	-3.68	-7.21
Max.	16.41	6.84	9.84	4.6	3.06	7.39
Mean	-0.02	0.03	0	0.01	0	0
Std. dev	2.47	1.26	0.59	0.49	0.48	0.52
Skew	0.14	-0.38	0.46	0.64	-0.19	0.51
Kurtosis	4.68	4.75	67.54	13.04	6.33	39.54
Panel B: Hypothesis tests about stochastic properties (p-value)						
L-B $Q(5)$	361.64 (<2.2e-16)	272.48 (<2.2e-16)	177.14 (<2.2e-16)	18.879 (0.002025)	42.461 (4.752e-08)	51.517 (6.777e-10)
L-B $Q(10)$	400.63 (<2.2e-16)	323.79 (<2.2e-16)	180.92 (<2.2e-16)	32.153 (0.0003776)	51.185 (1.615e-07)	57.071 (1.293e-08)
L-B $Q_2(5)$	870.18 (<2.2e-16)	1188.2 (<2.2e-16)	2896.2 (<2.2e-16)	696.64 (<2.2e-16)	108.88 (<2.2e-16)	2394.5 (<2.2e-16)
L-B $Q_2(10)$	1058.8 (<2.2e-16)	1703.5 (<2.2e-16)	2896.3 (<2.2e-16)	837.62 (<2.2e-16)	183.89 (<2.2e-16)	2400.3 (<2.2e-16)
S-W W	0.27775 (<2.2e-16)	0.15051 (<2.2e-16)	0.76637 (<2.2e-16)	0.8637 (<2.2e-16)	0.9114 (<2.2e-16)	0.79191 (<2.2e-16)

numerous trials. Examine the result of the Ljung-Box test for the residual series of the AR(1) model, it can be seen in Table 2.4 that almost all of the autocorrelation coefficients fall within the given confidence interval as well as their squared values. We hence conclude that the conditional mean of all the considered return series can be well fitted by the AR(1) model.

Whereas the considered return series has significant volatility clustering, the ARCH LM test is carried out for the residual series of the AR(1) model above. The result indicates that GARCH model needs to be adopted since there is high-order ARCH effect. According to the requirements that the AIC value should be relatively small, and model coefficients must be significant and positive, the GARCH(1,1) model is the best when comparison are made among GARCH(1,1), GARCH(1,2), GARCH(2,1) and GARCH(2,2) models. Because of the fat tail of the return, we consider different distributions, including normal, skewed normal, Student-t, and skewed Student-t distributions, for the innovation term z_t. The most appropriate distribution for z_t are chosen based on the information criteria AIC. As reported in Table 2.3, the return series of WTI and Gold can be adequately modeled by GARCH(1,1) model with skewed Student-t distribution; while for the return series of all USD exchange rates, GARCH(1,1) model with Student-t distribution is the most appropriated marginal distribution.

Table 2.3: AIC of GARCH(1,1) model with different innovation distributions for modeling the conditional heteroscedasticity

	WTI	Gold	USD/EUR	USD/GBP	USD/JPY	USD/CAD
norm	9.565341	8.909433	1.462971	1.153586	1.243578	1.191535
snorm	9.352697	8.493306	1.453081	1.149500	1.243839	1.174589
std	6.561036	5.407278	**1.178011**	**0.9509337**	**1.020435**	**0.8782591**
sstd	**6.559500**	**5.405993**	1.178552	0.9514799	1.020910	0.8786730

Thereafter, we apply the AR(1)-GARCH(1,1) model based on correspondingly specified innovation distribution to model the marginal distributions of considered return series. Table 2.4 summarizes the marginal distribution estimation results as well as diagnostic of the residuals. In Panel A of Table 2.4, μ and ϕ are respectively estimates of a constant and an autoregressive coefficient in the conditional mean equation; ω, α, and β are the coefficients of the conditional variance equation (see Eq.(2.1)); while γ is the degree of freedom as well as *skew* represents the skewness parameter of the innovation distributions. We note that for all the return series, the conditional variance term β with values above 0.93, which indicates that conditional variance is majorally past dependent and thus highly persistent over time. Moreover, all the degrees of freedom term γ are statistically significant with positive values, with relatively high value for the oil returns. Panel B of Table 2.4 reports

Ljung-Box $Q(20)$- and $Q_2(20)$-statistics to justify the empirical results of the specified marginal distribution models. According to Panel B, except the USD/GBP and USD/JPY exchange rates, the null hypothesis of no autocorrelation up to lag 20 for standardized residuals and squared standardized residuals are accepted for all the rest return series. Moreover, the test results for the ACFs of the standardized residuals and squared standardized residuals confirms that the standardized residuals are not autocorrelated, which support our model specifications. Thereafter, instead of using raw returns, we use standardized residuals obtained from the GARCH fit to copula estimation.

Table 2.4: MLE result of AR(1)-GARCH(1,1) models for each return series and the descriptive statistics of standardized residual series

	WTI	Gold	USD/EUR	USD/GBP	USD/JPY	USD/CAD
Panel A: Quasi-maximum likelihood estimation of AR(1)-GARCH(1,1) models for returns						
Mean equation						
μ	0.018770	0.034694	-0.0074404	-0.0048215	0.0030268	-0.00065804
ϕ	-0.033442	-0.018618	-0.0115148	0.0434363	0.0615313	0.06941182
Variance equation						
ω	0.024691	0.011855	0.0021825	0.0050164	0.0092501	0.00324397
α	0.064767	0.035795	0.0587397	0.1400125	0.2373281	0.10712361
β	0.932880	0.958417	0.9762747	0.9770070	0.9564173	0.96291750
γ	8.542901	4.346358	2.2506521	2.0852898	2.0987041	2.20365290
skew	0.939485	0.969979	-	-	-	-
Panel B: Ljung-Box test results of standardized residuals ($p-$value)						
L-B	11.56275	27.15204	20.46524	20.37437	38.88932	23.32544
$Q(20)$	(0.9303042)	(0.1310404)	(0.4291852)	(0.4347408)	(**0.0068818**)	(0.2730989)
L-B	21.88527	5.109431	28.26894	152.7406	26.18052	11.86768
$Q_2(20)$	(0.3467685)	(0.9996719)	(0.1031843)	(**0**)	(0.1599324)	(0.9205551)

Then, the copula functions are estimated based on the Pseudo data through the MPL method as described in Section 2.1.2.3. We consider the standardized residuals obtained from GARCH models and transform them into uniform variates. Moreover, we check the rank correlation coefficients for the dependence between the gold prices and oil prices, gold prices and exchange rates, as well as oil prices and exchanges rates, respectively. Table 2.5 reports the Kendall's τ and Spearman's ρ statistics for both the overall sample period and crisis period. We select the crisis period from July 1st, 2008 to June 30th, 2009 because the key trigger event for the global financial crisis on summer 2008, and the real GDP rebound modestly to 1.8 percent growth in 2009 according to the U.S. quarterly national GDP reports[28].

Table 2.5: Correlation estimates of the Kendall's τ and the Spearman's ρ between oil prices, gold price and exchange rates

	Overall sample (March 1, 2006-March 18, 2016)		Crisis period (July 1, 2008-June 30, 2009)	
	Kendall's τ	Spearman's ρ	Kendall's τ	Spearman's ρ
Gold-WTI	0.122	0.177	0.11	0.161
Gold-USD/EUR	-0.23	-0.333	-0.221	-0.328
Gold-USD/GBP	-0.182	-0.265	-0.029	-0.034
Gold-USD/JPY	-0.08	-0.119	-0.058	-0.086
Gold-USD/CAD	-0.181	-0.263	-0.368	-0.526
WTI-USD/EUR	-0.19	-0.274	-0.256	-0.358
WTI-USD/GBP	-0.164	-0.236	-0.231	-0.325
WTI-USD/JPY	0.061	0.09	0.24	0.341
WTI-USD/CAD	-0.261	-0.375	-0.315	-0.442

The monotone property of Kendall's τ and Spearman's ρ indicates the negative association relationship for all pairs between gold and exchange rates, as well as oil and exchange rates except oil to USD/JPY. And we observe the positive Kendall's τ and Spearman's ρ for gold and oil prices as excepted. In overall period, the constant correlations for the gold price and oil prices are positive and range from 0.12 to 0.25, while the constant correlation between the gold price and USD exchange rates are all negative and range from -0.058 to -0.526. Moreover, during the crisis period, the association between the gold price and oil prices, as well as the gold price and the USD/CAD are higher than that of the overall period; while the association between the gold price and the rest currencies are smaller than that of the overall period. It implies that the gold price are more deviated from the oil prices rather than the currencies. However, by comparing the correlations for oil prices with others in overall period and crisis period, we found that both of the comovement between oil prices and gold price, as well as currencies are substantially higher during the crisis period. It indicates that the oil prices are deviated not only by gold price but also the USD exchange rates. Specifically, we conclude that the association between USD/CAD exchange rate with both gold and oil are significant high during the crisis period.

To investigate further on the dynamical correlation between all considered pairs, we provide the dynamical curves which display the changes of association measure (the rolling Kendall's tau) in Fig.2.3. The figures are constructed by the following steps: (1) we start to compute the Kendall's tau by using the standardized residuals including the data between the period March 1st, 2006 and July 1st, 2008; (2)Kendall's tau is then calculated by shifting one data point at a time until the time window reaches up to March 18th, 2016. From the top panel of Fig.2.3, we can see that the association between WTI price and USD/CAD exchange rates are peaked between the year 2010 and 2011(end of crisis period). The bottom panel of Fig.2.3 indicates the normalized WTI price and negative normalized USD/CAD exchange rate. Note that we utilize the negative USD/CAD exchange rates because the negative association between oil prices and exchange rates as shown in Table 2.5. Notice that a rise or fall of WTI price at the time Jan 1st, 2007, July 1st, 2008, Dec 20th, 2008, June 20th ,2014, and Jan 10th, 2016 were followed by similar motion in the USD/CAD exchange rates. This indicates that crude oil (WTI price) is a good short-term indicator in the move in asset prices(USD/CDA exchange rates).

2.1.3.3 Conditional tail dependences

The estimates of the dependence parameters for the copula functions among each pair variables, which includes Normal copula, Student-t copula, Clayton copula, Gumbel copula, and BB7 copula and their rotated versions, are reported in Table 2.6. They are highly significant for almost all pairs of the considered copula functions.

Figure 2.3: Plot of negative dynamic Kendall's tau (the rolling Kendall's tau) between WTI price and USD/CDA exchange rate(top panel), normalized WTI price and negative normalized USD/CDA exchange rate chart(bottom panel).

We also apply the trivariate Gaussian copula and t copula functions to model the conditional dependence among trivariate variables (Gold-WTI-Exchange rates) and their parameter estimations are reported in Table 2.7 . They are highly significant for all combinations of variables for the considered copula functions. By comparing the p-values of the goodness-of-fit test, we found that the trivariate Gaussian copula

Table 2.6: Estimates of the dependence parameters of different copula models

	Normal Copula	Student-t Copula		(rotated) Clayton	(rotated) Gumbel	(rotated) BB7	
Gold-WTI	0.194	0.194	ν=10.718	0.243	1.116	$\theta = 1.054$	$\delta = 0.2134$
	(0.0189)	(0.0204)	(2.6686)	(0.0275)	(0.0156)	(0.021)	(0.0293)
Gold-USD/EUR	-0.363	-0.366	ν=10.951	-0.454	-1.278	$\theta = -1.21$	$\delta = -0.34$
	(0.0163)	(0.0177)	(2.6979)	(0.0309)	(0.0191)	(0.0292)	(0.0342)
Gold-USD/GBP	-0.29	-0.292	ν=10.496	-0.329	-1.211	$\theta = -1.182$	$\delta = -0.225$
	(0.0176)	(0.0191)	(2.4994)	(0.0294)	(0.0176)	(0.0273)	(0.032)
Gold-USD/JPY	-0.191	-0.193	ν=11.013	-0.209	-1.125	$\theta = -1.097$	$\delta = -0.157$
	(0.0189)	(0.0204)	(2.7959)	(0.0269)	(0.0156)	(0.0234)	(0.0285)
Gold-USD/CAD	-0.285	-0.285	ν=13.945	-0.31	-1.203	$\theta = -1.175$	$\delta = -0.215$
	(0.0177)	(0.0189)	(4.4067)	(0.0285)	(0.0174)	(0.0267)	(0.0308)
WTI-USD/EUR	-0.275	-0.279	ν=10.769	-0.33	-1.189	$\theta = -1.128$	$\delta = -0.258$
	(0.0178)	(0.0192)	(2.585)	(0.0293)	(0.0172)	(0.0254)	(0.0319)
WTI-USD/GBP	-0.237	-0.239	ν=13.149	-0.241	-1.165	$\theta = -1.155$	$\delta = -0.155$
	(0.0184)	(0.0196)	(3.878)	(0.0278)	(0.0164)	(0.025)	(0.0296)
WTI-USD/JPY	0.069	0.071	ν=12.157	0.11	1.054	$\theta = 1.001$	$\delta = 0.109$
	(0.0199)	(0.0212)	(3.271)	(0.0238)	(0.0126)	(0.014)	(0.025)
WTI-USD/CAD	-0.36	-0.362	ν=13.647	-0.423	-1.277	$\theta = -1.233$	$\delta = -0.293$
	(0.0164)	(0.0175)	(4.132)	(0.0305)	(0.019)	(0.0297)	(0.034)

The table summarizes the copula estimation results between gold price and oil price, currency exchange rates. The values in the parenthesis represent the standard error of the parameter estimations. The Clayton, Gumbel and BB7 copulas are fitted when the Kendall's value of the pair in Table 2.5 is positive and the 90 degree rotated Clayton, Gumbel and BB7 copulas are fitted when the Kendall's value of the pair in Table 2.5 is negative.

fits the data best for each combinations of variables. In order to capture the tail dependence among each pair variables, we focus on the bivariate copulas next.

From Table 2.6, we see that the dependence between gold and exchange rates, oil and exchange rates are all negative associated except for the WTI-USD/JPY. To choose the most appropriate model for our data, we adopt the goodness-of-fit test described in section 2.1.2.4. We summarize the results of the goodness-of-fit test for considered four copula models in Table 2.8. By comparing the p-values of the goodness-of-fit test, we select the copula which fits the data best for each pair of variables.

Combining the findings of Table 2.6 and 2.8, we see that the dependence between gold and crude oil returns is positive, while the dependence between gold and USD exchange rate returns are negative for all considered pairs. Note that from the es-

Table 2.7: Estimates of the dependence parameters of trivariate copula models and the goodness of fit tests for copulas

	Normal Copula	GOF statistics (p-value)	Student-t Copula	GOF statistics (p-value)
Gold-WTI-USD/EUR	(0.194, -0.363, -0.275)	0.0273	(0.197, -0.365, -0.278) ν=11.744	0.0327
	(0.018) (0.015) (0.016)	**(0.24)**	(0.02) (0.018) (0.019) (1.886)	(0.055)
Gold-WTI-USD/GBP	(0.194, -0.29, -0.238)	0.0296	(0.197, -0.294, -0.233) ν=12.417	0.055
	(0.018) (0.016) (0.017)	**(0.245)**	(0.02) (0.019) (0.02) (2.102)	(0.0004995)
Gold-WTI-USD/JPY	(0.194, -0.192, 0.068)	0.0296	(0.194, -0.19, 0.068) ν=11.58	0.0612
	(0.018) (0.018) (0.018)	(**0.235**)	(0.02) (0.02) (0.021) (1.859)	(0.0004995)
Gold-WTI-USD/CAD	(0.194, -0.285, -0.36)	0.025	(0.198, -0.286, -0.358) ν=13.018	0.0465
	(0.018) (0.017) (0.016)	(**0.33**)	(0.02) (0.019) (0.018) (2.338)	(0.002498)

The table summarizes the copula estimation, the goodness of fit test statistics and p-value results among gold price and oil price, currency exchange rates by using the trivariate Gaussian copula and trivariate t-copula. The values in the parenthesis represent the standard error of the parameter estimations and p-values for the goodness of fit test.

Table 2.8: P-value of the goodness-of-fit test for different copula functions

	Normal Copula	Student-t Copula	(rotated) Clayton	(rotated) Gumbel	(rotated) BB7
Gold-WTI	**0.43**	0.29	0.07	0	0.37
Gold-USD/EUR	0.02	**0.39**	0	0	0.05
Gold-USD/GBP	0.26	**0.34**	0	0.03	0.16
Gold-USD/JPY	0.43	**0.84**	0	0.03	0.4
Gold-USD/CAD	0.4	0.46	0	0.02	**0.48**
WTI-USD/EUR	0.5	**0.85**	0	0	0.04
WTI-USD/GBP	0.56	**0.67**	0	0	0.21
WTI-USD/JPY	0.09	0.54	**0.79**	0.63	0.65
WTI-USD/CAD	0.71	**0.77**	0	0	0

Notes: The largest p-value indicate that the copula fits best for the data.

timation of the dependence parameters for copula models, the dependence between crude oil and USD exchange rates are negative for all considered pairs, except for WTI-USD/JPY. It indicates that a fall in the value of the U.S. dollar leads to drive up gold price, while an increase of crude oil price causes the depreciation of U.S. dollar. Also, the oil price and gold price are positively correlated. When the gold price increases, oil price increases as well.

The negative relationship between oil price and the price of dollar is supported by the historical facts that, for example, the crude oil price rose steadily from 20 per barrel in January 2002 to a high of 147 per barrel in July 2008. It then fell sharply to 32 per barrel in January 2009. On the other hand, since 2002 the US dollar index has behaved in a markedly distinct manner compared to the way it behaved prior to 2002 where it has moved in the totally opposite direction to the price of crude oil. One explanation for the negative relationship between oil and dollar prices, as indicated by [?], is that the oil-exporting countries want to stabilize the purchasing power of their export revenues (US dollar) in terms of their imports (non-US dollar), so in order to avoid losses they may adopt currencies pegged to the US dollar.

Since extreme events may create huge disruptions in dependence structure of markets, tail dependence are very helpful to examine how extreme events affect the correlation during crisis periods. Hence, we use the best fitted copula models selected from the Table 2.8 to capture the extreme dependence of all pairs and reported it in Table 2.9.

Table 2.9: Tail dependence coefficients of the best fit copula

Gold-Brent	$\lambda_L = \lambda_U = 0$		
Gold-USD/EUR	$\lambda_L = \lambda_U = 2.767 \times 10^{-4}$	WTI-USD/EUR	$\lambda_L = \lambda_U = 6.77 \times 10^{-4}$
Gold-USD/GBP	$\lambda_L = \lambda_U = 7.055 \times 10^{-4}$	WTI-USD/GBP	$\lambda_L = \lambda_U = 2.747 \times 10^{-4}$
Gold-USD/JPY	$\lambda_L = \lambda_U = 1.199 \times 10^{-3}$	WTI-USD/JPY	$\lambda_L = 1.834 \times 10^{-3}$, $\lambda_U = 0$
Gold-USD/CAD	$\lambda_L = \lambda_U = 0$	WTI-USD/CAD	$\lambda_L = \lambda_U = 5.618 \times 10^{-5}$

Since the Student-t copula is symmetric, the upper and lower tail dependence coefficients are the same. According to Table 2.9, the tail dependence, i.e., the hypothesis of extreme comovements, between gold and U.S. dollar exchange rates, as well as crude oil and U.S. dollar exchange rates are weak. The strongest extreme comovement with gold is found in USD/JPY. Moreover, the strongest extreme comovement with crude oil markets is in USD/JPY.

2.1.4 Conclusion

This section investigates the dependence structure among gold, nominal crude oil and major U.S. dollar exchange rates from March 1, 2006 to March 18, 2016. Based on a copula-GARCH approach, we examine the conditional dependence structure and the extreme comovement on returns between paris of gold and oil, gold and currencies, as well as oil and currencies. We first apply the AR(1)-GARCH(1,1) model based on different innovation distributions to model the margins. The adoption of this filtering step is motivated by the stylized facts of our financial returns including non-normal distributed, autocorrelation of squared returns and volatility clustering. Then, different copula models are fitted to standardized residuals from the best fitted marginal models. The comparison results of various copula models show that the Student-t copula outperforms other copulas for fitting the conditional dependence structure of all considered pairs.

Empirical results show that (i) each of the analyzed series of gold, oil and currencies returns can be adequately described with the proposed AR(1)-GARCH(1,1) model based on either Student-t or skewed Student-t innovation distributions; (ii) there are positive dependence between gold and oil, negative dependence between gold and currencies, as well as oil and currencies, as indicated by the Kendall's τ and Spearman's ρ concordance, and the correlation coefficient; (iii) there is a small degree of conditional dependence in the extreme tail of all considered pairs; (iv) furthermore, we found that the crude oil price was a good short-term indicator in the move in asset prices like exchange rates. The crude oil price was a short term descend indicator of gold price, and the gold price was an short term rise indicator of oil price. The above findings lead us to conclude that the U.S. dollar depreciation was a key factor in driving up the crude oil price and gold price, while gold market and oil market are positively associated.

Besides the applied contribution, our paper have three main contributions for investors. First, the results of the study provide useful information for investors in asset allocation and portfolio diversification. Second, we show that gold has served as a hedge against fluctuation in the U.S. dollar exchange rates. Moreover, the appreciation of the U.S. dollar are found to coincide with a decrease in crude oil prices. Third, taking into account the extreme comovement between different assets, investors can improve the accuracy of market risk forecasts.

CHAPTER 3

CONCLUSION

This thesis aims at computing accurate estimates for the risk of a portfolio by constructing its conditional loss distribution with a flexible methodology that separates the description of the marginal distributions from the dependence structure.

In Chapter 1, we have presented extreme value theory from a double perspective: on the one hand, the main elements of the probabilistic theory and the statistical methods related to it; on the other hand, their applications to finance.

One part was intended as a critical resume of both the foundations of the theory and its scope and limitations. From a theoretical viewpoint, EVT shows some considerable pros:

(a) it ofers tools, with strong theoretical underpinnings, to model extreme events, which are of great interest in many applications, pertaining to several different fields. In finance, in particular, EVT is especially useful in the context of risk measurement, given the importance of extreme events to the overall profitability of a portfolio;

(b) it provides a variety of such tools, ranging from non-parametric methods to point processes, thus guaranteeing a flexible approach to the modeling of extreme events, that can be adjusted to the particular features of the problem at hand;

(c) the fact that the vast majority of standard distributions, even though displaying considerably different tail behavior, can be equally modeled by EVT also increases flexibility;

(d) furthermore, the flexibility and the accuracy of modeling are enhanced by the fundamental characteristic of EVT, namely its exclusive consideration of the tail of the distribution of the data, disregarding ordinary observations (the centre of the distribution);

(e) and they are also enhanced by the capability of EVT of independently modeling each tail of the distribution;

(f) finally, the availability of parametric approaches allows for projections and forecasting of extreme events.

Some drawbacks emerged as well:

(a) the most problematic one is probably the dependence of the parameters on the choice of the so-called cut-off, i.e., the delimitation of the subsample employed to estimate the extreme quantiles, given that there is not yet complete agreement on how such a choice should be made;

(b) moreover, the basic theory of extreme values assumes that the data are not serially correlated; when this assumption is violated, we have some alternative approaches at hand, but there is no agreement on which of them is the most suitable one;

(c) multivariate EVT is admittedly not as straightforward as its univariate counterpart and can still encounter severe computational limitations, in some applications;

(d) EVT is characterized by an unavoidable trade-off between its asymptotical nature and its interest in extreme events, therefore, the choice and preparation of the data-set can be a crucial step in applying EVT.

Coming to applications, we have only considered financial applications and mainly focused on some of them. The return series for each of the financial assets was modeled using GARCH type methods in order to explain the autocorrelation and time-varying volatility. Then, the innovation series resulted from the GARCH type model is described as a semi-parametric distribution with GPD tails and a kernel-smoothed interior that captures the stylized facts of financial time series. The most important one, both for its role in financial regulation and for the amount of contributions to the research concentrating on it, is the employment of EVT for the estimation of quantile-based risk measure Value-at-Risk. Many papers deliver comparative analyses of the accuracy of different methods for VaR calculation and they agree in indicating EVT as a considerably valuable candidate when calculating VaR at high confidence levels. The great degree of accuracy displayed by EVT-based estimates of VaR for several different markets probably makes the employment of EVT in risk measurement one of the most relevant and better acknowledged contributions of extreme value theory to finance.

Related to the use of EVT for risk management is the role of EVT in asset allocation problems, associated to the concept of "safety first investor". The awareness of the importance of taking into account the risk profile of the investor is permeating the financial practice and, for investors who are particularly interested in avoiding extreme shocks, i.e. huge and rare losses, EVT provides a suitable tool, given its accuracy in modeling such shocks.

Then, portfolio selection naturally entails the consideration of a multivariate setting. In this setting, another important problem is that of systemic risk and the issue of contagion across markets in presence of extreme events. This topic, highlighted by the credit crisis, deserves a particular attention, since the dependence pattern in a multivariate time series can be different in normal times and under stress conditions, i.e. extremal dependence can differ from ordinary correlation. Another fact is that the correlation among the price or volatility behaviors of the financial assets within a portfolio is a crucial dimension for the proper estimation of the VaR amount. However, restrictions on the joint distributions of the financial assets within the portfolio might decrease the performance of the VaR estimation. The joint distribution of the portfolio should be free from any normality assumptions especially if the portfolio is composed of assets from markets where there exists high volatility and non-linearities in the returns. Those facts have an impact on diversification effects and has to be explicitly modeled and taken into account. Copulas offers statistical tools suited to this aim.

Copulas reveal to be a very powerful tool in the finance profession, more especially in the modeling of assets and in the risk management. Nevertheless, the finance industry needs more works on copula and their applications. Even if it is an old notion, there are many research directions to explore. Moreover, many pedagogical works have to be done in order to familiarize the finance industry with copulas.

In Chapter 2, we present the concept of copula and how it could be used in quantitative finance, especially to risk management. We focus on study and modeling of interdependencies between extreme events. Thanks to Sklars Theorem such tasks decompose into the study of the tail behavior of the marginal univariate distributions and of the tail (i.e. corner) behavior of the corresponding copulas. Chapter 2 describes a model for estimating portfolio VaR by the conditional copula-GARCH model, in which the empirical evidence shows that this method can be quite robust in estimating VaR. Copula-GARCH models allow for a very flexible joint distribution by splitting the marginal behaviors from the dependence relation. In contrast, most traditional approaches for the estimating VaR, such as variance-covariance, and the Monte Carlo approaches, of the traditional method shows that the copula model captures the VaR most successfully. The copula method has the feature of flexibility in distribution, which is more appropriate in studying highly volatile financial markets, and which there is lack of in traditional methods.

At the end, we can see those contributions made by EVT to finance are based on the very definition of extreme value theory and Sklar theorem, namely on its capability to accurately model the distribution of extreme events, which are the main concern of modern risk management. Thus, in the end, we come back to the widely quoted motto of DuMouchel we began with, which is key to risk management "Let the tails speak for themselves".

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