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Necessity and contingency in Leibniz.

G. W. Fitch
University of Massachusetts Amherst

NECESSITY AND CONTINGENCY IN LEIBNIZ

A Dissertation Presented

By

GREGORY WERNER FITCH

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GREGORY WERNER FITCH

Approved as to style and content by:

Robert C. Sleigh Jr., Ph.D., Chairman of Committee

Vere C. Chappell, Ph.D., Member

Fred A. Feldman, Ph.D., Member

Frank W. Heny, Ph.D., Member

Vere C. Chappell, Ph.D., Department Head

Department of Philosophy

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Gregory W. Fitch, B.A., Western Washington State College
M.A., University of Massachusetts
Directed by: Robert C. Sleigh Jr., Ph.D.

Among the objections raised against Leibniz's metaphysical views, the problem of contingency is one of the most crucial and difficult problems Leibniz faced. The objection that Leibniz could not allow for contingency in his metaphysical system was pressed on two fronts: first, in connection with Leibniz's views on God; and second, in connection with his analysis of truth. The only book written by Leibniz that was published during his lifetime, the Theodicy, is Leibniz's attempt to reconcile his views on God and contingency. Leibniz's concern over the relation between contingency and his definition of truth can be seen in the first part of the correspondence he initiated with Arnauld. This dissertation is an attempt to find a solution to these problems for Leibniz.

The project, roughly speaking, is presented in three parts. The first part, which is Chapter I, deals with the problem of contingency as it relates to God. I give a brief sketch of the view I later propose for Leibniz, and then see how one can account for God within the conceptual framework given. Various arguments for the necessity of God's choice in creating this world are discussed, and three different ways of conceiving God's role in Leibniz's metaphysical system are considered. While I point out the difficulties with each view, I suggest that one of them is better than the other two. In the end I am forced to conclude that God did create this world of

necessity, but argue that God's lack of freedom does not necessarily rule contingency completely out of his system.

The second part, which consists of Chapters II and III, presents some solutions offered by contemporary philosophers to the "analytic-necessary" problem. This difficulty for contingency in Leibniz arises when we reflect on Leibniz's analysis of truth. Leibniz claims that in every true proposition the concept of the predicate is included in the concept of the subject. This makes all true propositions analytic, and thus necessary. G. Parkinson and N. Rescher suggest ways of resolving this problem for Leibniz, and their views are presented in Chapter II. Both Parkinson and Rescher believe the solution is to be found in Leibniz's views on "infinite analysis", though each has his own approach to the problem. I discuss as clearly as possible their proposed solutions, but find them inadequate in various respects. In Chapter III, I consider B. Mates' interesting new approach to the problem. Mates presents a formal system which he believes incorporates Leibniz's views on possible worlds, and which allows for contingency. Much of what Mates claims seems true, and in Chapter III, I offer support for some of his views. Yet, because certain features of Mates' system appear non-Leibnizian, I suggest that a better account of Leibniz can be given.

Chapters IV and V constitute the third and final part of the project. In Chapter IV, I re-examine the "analytic-necessary" problem in light of what has preceded and argue that in various places, especially the Theodicy and the correspondence with Arnault, Leibniz suggests a way to resolve the problem, while keeping his definition of truth. I argue that Leibniz suggests we understand necessity and contingency in terms of possible worlds and counterparts. With this in mind I present the view more formally in Chapter V. I discuss various formal aspects of the system presented in Chapter V and reply to an objection raised by Mates against the use of counterparts for Leibniz. I conclude by pointing out the relative merits of the system I present.

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ABBREVIATIONS

(OC)	Leibniz, G. W.	_Discourse on Metaphysics_, Correspondence with Arnauld and _Monadology_. George R. Montgomery, Trans. The Open Court Publishing Co. La Salle, 1962.
(P2)		_Leibniz: Logical Papers_. G. H. R. Parkinson, Trans. and Ed. Oxford University Press. London, 1966.
(NE)		_New Essays Concerning Human Understanding_. Alfred Gideon Langley, Trans. and Ed. The Open Court Publishing Co. La Salle, 1949.
(L)		_Philosophical Papers and Letters_, Vol. I and II. Leroy E. Loemker, Trans. The University of Chicago Press. Chicago, 1956.
(Lewis)	Lewis, David K.	"Counterpart Theory and Quantified Modal Logic", _The Journal of Philosophy_, Vol. LXV, No. 5, March 7, 1968.
(M1)	Mates, Benson	"Leibniz on Possible Worlds", _Logic, Methodology, and Philosophy of Science III_, B. van Rootselaar and J.F. Staal, Ed. North Holland Publishing Company. Amsterdam, 1968.
(M2)		"Individuals and Modality in the Philosophy of Leibniz", _Studia Leibnitiana_, Vol. II, 1972.
(P1)	Parkinson, G.H.R.	_Logic and Reality in Leibniz's Metaphysics_. Oxford University Press. London, 1966.
(Res1)	Rescher, Nicholas	_The Philosophy of Leibniz_. Prentice-Hall, Inc. New Jersey, 1967.
(Res2)		"Contingence in the Philosophy of Leibniz", _Philosophical Review_, Vol. LXI. January, 1952.
(R)	Russell, Bertrand	_A Critical Exposition of the Philosophy of Leibniz_. George Allen & Unwin, Ltd. London, 1900.

CHAPTER I

God's role in Leibniz's metaphysical system is a main source of difficulty for Leibniz. B. Russell has suggested that Leibniz's views on God in conjunction with other views Leibniz holds lead him into inconsistencies.[1] In particular, one of the major problems is Leibniz's account of contingency. Leibniz wants to maintain that he can allow for contingency and that he does not fall into what has been called 'the disease of Spinozism'.[2] In this chapter some of the problems about God and contingency will be presented and various ways Leibniz might be able to avoid them will be discussed.

Leibniz's picture of creation was, briefly, that God had in his understanding an infinite number of possible worlds. Among all these possible worlds God found the world which was the best and made it actual.[3] But the situation is actually more complicated, and for better understanding of Leibniz's view of creation, let us first turn our attention to the world as it actually is.

According to Leibniz, for each substance in the world there is a corresponding concept, sometimes called a complete concept or a complete individual concept.[4] These concepts include or contain all the properties that the substance to which the concept corresponds has, had, or every will have. For our purposes, we can view concepts as sets of properties. For example, consider Adam, the first man.[5] Leibniz holds that Adam has a complete individual concept which contains all the properties that Adam has or ever will have. If Adam has the property of having blond hair, then the property of having blond hair is a member of Adam's complete individual concept. For any property Φ, Φ is a member of Adam's complete individual concept if and only if Adam has Φ. Thus Adam's complete individual concept contains all and only those properties which Adam has. This is true for every substance.

An atomic sentence is said to be true just in case the property associated with the predicate is included in the concept associated with the subject. Thus, for example, the sentence, "Adam has blond hair," is true because the concept of blond hair (i.e., the property of having blond hair) is included in the complete individual concept of Adam.[6]

In each complete individual concept there are an infinite number of properties. Moreover, for each substance there is exactly one complete individual concept. Suppose Adam had two distinct concepts. If the concepts are distinct, then they must differ with respect to some property (say) Φ. Either Adam has Φ or he lacks it. If Adam has Φ, then the concept which does not contain Φ could not be Adam's since Adam's concept must have all his properties. If Adam lacks Φ, then the concept which contains Φ could not be Adam's since that concept contains a property which Adam lacks. This is easy to see when we consider this world only, but when we consider all possible worlds the problem becomes more complicated.

For our present purposes it will be assumed that for Leibniz each possible world is a special kind of set of complete individual concepts.[7] The real world differs from the others in that in the real world the concepts are realized (i.e., there is a substance corresponding to each concept) while in other worlds the concepts are not realized. A possible world is not just any set of concepts, but rather a possible world is a "collection of compossibles".[8] Exactly what Leibniz meant by "compossible" is far from clear and different interpretations are possible. Some philosophers have viewed compossibility as a relation between two things.[9] But one can also view compossibility as a predicate of sets.[10] Since there are certain problems involved in viewing compossibility as a relation between two things,[11] we will view compossibility as a property of sets. For the moment we will say that a set of concepts is compossible just in case all the members of the set can be realized together.[12] A possible world is a maximal compossible set of concepts.

Leibniz also believes that each concept in a world "expresses" or "mirrors" that world.[13] Again it is unclear what Leibniz means by "mirrors". The idea is that concepts of the same world are related to each other in such a way as to reflect the existence of each other. For example, the concept of Adam contains the property of being married to Eve. Thus in some way the concept of Adam reflects the existence of the concept of Eve (i.e., Adam could not exist if Eve did not exist since the concept of Adam could not be realized without the concept of Eve being realized). In a similar way the concept of Adam reflects all the concepts which make up this world. Thus, the definition of mirroring would be roughly something like the following: a concept mirrors a world just in case that concept reflects every member of that world. A concept C reflects a concept D only if it is contradictory to suppose that C is realized and D is not realized.[14]

A concept can only be a member of one possible world. If a concept C were in two distinct possible worlds, C would mirror a world W which did not have as a member some concept D which C reflects. Since W is a world it must be a maximal compossible set of concepts. But W can not be a maximal compossible set of concepts, because W lacks D as a member and C can not be realized without D. Thus all the members of W can not be realized together, and hence W is not a possible world. So, through the use of compossibility and mirroring we get the result that a concept is a member of only one possible world.

Viewing possible worlds as maximal compossible sets of concepts will help us better understand God's role in Leibniz's metaphysics. According to Leibniz, God could not affect which sets of concepts were compossible. Thus God did not create possible worlds, but rather found them already formed in his understanding. God decided which of these possible worlds he would realize, if any. God did not decide whether Adam would sin, but decided whether to create Adam who would sin as opposed to realizing other concepts.[15] To return to our picture of the creation, then, God decided among all these compossible sets of concepts which set to realize, and God chose the best.

Leibniz's account of the creation and God's role in his metaphysical system seems on the surface consistent, although there are some obscurities. A closer look, however, reveals certain difficulties for Leibniz.

Leibniz believes that God has the properties of being omnipotent, omniscient, and omnibenevolent. But if God really is omnipotent, omniscient, and omnibenevolent, could God have created any world less than the best of all possible worlds? If not, then some have argued there is no contingency in Leibniz. Whether or not Leibniz could allow for contingency, even with the supposition that God necessarily created this world, is something which will be discussed later. We are now interested in whether God necessarily created this world.

Leibniz does say that God created the best world of necessity, but only in a special sense of necessity. Leibniz tries to make a distinction between two kinds of necessity. One he calls "moral necessity" and the other "metaphysical necessity".[16] Metaphysical necessity for Leibniz is what contemporary philosophers have called necessity or logical necessity. Something is metaphysically necessary just in case its negation (or opposite, as Leibniz puts it) implies a contradiction. In this sense of necessity it is not necessary that God create this world. However, it is morally necessary that God create the best world. Leibniz's notion of moral necessity is not as clear as his notion of metaphysical necessity since he never actually defines it. One can, however, get some idea of what he meant.

Moral necessity for Leibniz is a kind of "hypothetical necessity".[17] Leibniz says something is "hypothetically necessary" when it follows from certain free decrees of God.[18] That is, if God decides a certain thing, then the results of that decision or what follows from it are hypothetically necessary. It is clear that Leibniz does not want to say that Q is hypothetically necessary only if P then Q, where P is some decree of God. This is especially true if we understand the 'if, then' as a material conditional. Leibniz would want to say something stronger, such as, if P entails Q, then Q is hypothetically necessary.

We might put it by saying if it is metaphysically necessary that if P then Q, then Q is hypothetically necessary. Since moral necessity is a kind of hypothetical necessity, to say something is morally necessary is to say that given a certain condition it must occur or it must be true. The condition seems to be one of moral perfection.[19] Thus, for example, to say that it is morally necessary that God do act α is to say that it is metaphysically necessary that if God is morally perfect (or acts according to moral perfection), then God does act α. Leibniz says that while God did necessarily create this world, it is not a necessity which destroys contingency because God's creation was only morally necessary and not metaphysically necessary. God could have done otherwise, in the sense that his doing otherwise does not imply a contradiction.

Unfortunately for Leibniz, it is not at all clear that his distinction between moral and metaphysical necessity removes the difficulties about God. If our analysis of what Leibniz means by moral necessity is correct, then from the fact that it is morally necessary that God created this world and an assumption about the nature of God's properties we can conclude that it is metaphysically necessary that God created this world (or at least the best of all possible worlds). Consider the following argument:

I. (1) It is morally necessary that God create the best of all possible worlds.

(2) It is metaphysically necessary that God is morally perfect.

/.. (3) It is metaphysically necessary that God create the best of all possible worlds.

Premise (1) is true according to Leibniz,[20] (2) seems true in virtue of God's nature and (3) does follow from (1) and (2). We will accept Leibniz's view that (1) is ture, although it is not clear that Leibniz has to hold (1). However, it is not so clear that Leibniz holds (2). In the Discourse, Leibniz says:

> For it would be found that this demonstration of this predicate as belonging to Caesar is not as absolute as are those of numbers or of geometry, but that this predicate supposes a sequence of things which God has shown by his free will. This sequence is based on the first free decree of God which was to do always that which is the most perfect and upon the decree which God made following the first one, regarding human nature, which is that men should always do, although freely, that which appears to be the best. Now every truth which is founded upon this kind of decree is contingent, although certain, for the decrees of God do not change the possibilities of things and, as I have already said, although God assuredly chooses the best, this does not prevent that which is less perfect from being possible in itself. (OC p. 22)

If we take Leibniz literally when he says that God's first free decree was always to act in the most perfect way, then we can see why Leibniz would deny (2). Since Leibniz says that everything based on that decree is contingent, then God's acting in the most perfect way is contingent, and thus it is not metaphysically necessary that God is morally perfect. God's being morally perfect is based on his own free decree to be morally perfect. Thus, the argument is unsound and Leibniz is saved from God's being metaphysically necessitated to create the best of all possible worlds. But the problem for Leibniz is not so easily solved.

In the first place, in order for Leibniz to hold the position suggested above he must give up a traditional view about God, namely, that God by definition is morally perfect as well as omnipotent and omniscient. Traditionally, God has those attributes by definition. But if God is morally perfect because of a free decree he made, then in a metaphysical sense God could have been less than morally perfect. Of course, pointing out that Leibniz's views on God are not in accord with traditional views on God is not a criticism of his view. But it is strange that Leibniz would allow that it is possible that God is not morally perfect. A more important problem for Leibniz is that of reconciling his above account of God's moral perfection with his view of truth.

As mentioned earlier, Leibniz said that a sentence is true just in case the concept of the predicate is included in the concept of the subject. (For the purposes of this discussion, we will ignore some difficulties of this account of truth by assuming all sentences can be put into subject-predicate form.) In those cases where the referent of the subject of a sentence is a human or any other finite substance, we can see how Leibniz's account works. But what about God? That is, what about sentences in which the referent of the subject is God? Are those sentences to be handled in the same way as sentences in which the referent of the subject is a finite substance? There are two strong reasons for believing the answer to be yes. In the first place, Leibniz never makes an exception to his definition of truth, and it is hard to believe that he would make God one. In the second place, Leibniz says that corresponding to every substance there is a complete individual concept and there is no reason to believe that Leibniz thought God was an exception, even though God is an infinite substance. If God has a concept, then there seems to be no reason not to claim that sentences about God are true just in case the concept of the predicate is included in the concept of God. However, if we say this then it seems, as opposed to what Leibniz says, that (2) is true.

If we consider the sentence, "God is morally perfect," as true, then the concept of moral perfection is included in the concept of God. But if the concept of moral perfection is included in the concept of God, then does it not follow that it is metaphysically necessary that God is morally perfect? To determine whether or not this follows, we must briefly consider what Leibniz says about metaphysical necessity.

As mentioned earlier, Leibniz explicitly says that something is metaphysically necessary just in case its negation implies a contradiction. However, there are certain problems if we understand Leibniz as saying this *simpliciter*. The suggestion being presented is that P is necessary if and only if not-P entails a contradiction. But on this view we can show that it is necessary that Adam has blond hair, clearly an unwanted result. The proof of this is something like the following:

(1) For any x, and for any set S, if x ε S, then necessarily x ε S.

(2) If the concept of blond hair ε the concept of Adam, then necessarily the concept of blond hair ε the concept of Adam.

(3) Necessarily the concept of blond hair ε the concept of Adam.

Assume: (4) It is not the case that Adam has blond hair.

(5) It is not the case that the concept of blond hair ε the concept of Adam.

(6) The concept of blond hair ε the concept of Adam.

/.. (7) Q and not-Q.

(8) It is not the case that Adam has blond hair implies Q and not-Q.

(9) It is not the case that Adam has blond hair entails Q and not-Q.

/.. (10) Necessarily Adam has blond hair.

(1) is a truth about sets, and (2) is just an instantiation of (1). Assuming that Adam does have blond hair and given Leibniz's definition of truth, from (2) we can get (3). We then assume Adam does not have blond hair, and using Leibniz's definition we arrive at a contradiction. Given that (3) is a necessary statement, and (8) followed from only our assumption and necessary truths, we can conclude (9). From the definition of necessity suggested and (9) we finally conclude (10). It is obvious that something has gone wrong here. Leibniz clearly wants Adam to have blond hair contingently.

The answer to this problem lies in the narrow view suggested above of Leibniz's notion of metaphysical necessity. While it is true that in some sense Adam can not lack the property of having blond hair, namely in the sense described above, it should not follow that it is metaphysically necessary that Adam has blond hair. Leibniz's notion of metaphysical necessity involves in part Leibniz's use of possible worlds. We want to say that a proposition is metaphysically necessary just in case it is true in all worlds, not just the world that exists. Setting up such an account for Leibniz is something which will be discussed at great length later, as well as opposing points of view on this quention. A complete account of Leibniz's notion of metaphysical necessity is not necessary to discuss the problems about God in Leibniz's metaphysical system. For different accounts presented and discussed in detail, see Chapters II, III, and IV. In Chapter IV it is suggested that for Leibniz to say a sentence of the form 'x is F' is necessary is to say that all the "counterparts" of the concept of x include the concept of F. A "counterpart" of a concept is a concept which contains certain properties which the original concept contains. Intuitively, if the two concepts were realized, then the two substances would be very similar to each other. For example, to say of a sentence about Adam that it is metaphysically necessary is to say that all the counterparts of the concept of Adam in various worlds include the concept of the predicate. This will hold true for all finite substances. But a difficulty emerges when we try to account for God in this conceptual framework.

It makes some sense to talk about the counterparts of the concept of Adam being in various worlds, as we can talk about the concept of Adam being a member of this world. But in God's case it is not so clear that his concept is a member of any world. Talk about counterpart concepts of the concept of God seems, on the face of it, bizarre. There are, as far as we can tell, three plausible ways of considering God in the conceptual framework just set up, but all have difficulties.

We could say that the concept of God is not a member of any world, but somehow exists apart from all worlds (call this view A). We can still say that an atomic sentence about God is true just in case the concept of the subject includes the concept of the predicate. However, we can no longer say, as we did in the case of Adam, that a sentence about God is necessarily true just in case all the counterparts of the concept of God in various possible worlds include the concept of the predicate, since it makes no sense to talk about the counterparts of the concept of God, nor does it make sense to talk about God's concept being a member of a possible world. We might say that God is an exception and the truth conditions for necessary sentences about God are somehow different from those about (say) Adam. But it is not clear which sentences are to count as being about God. Clearly all atomic sentences in which the concept of the subject is the concept of God will be counted as being about God, but what about such sentences as, "There is an all-knowing being," or, "There exists a necessary being"? Even if we could somehow find a way to distinguish sentences about God from sentences not about God, we would still need to decide what the truth conditions for necessary sentences would be. And without the use of possible worlds it is far from clear what they would be.

A second view, (B), and an alternative approach to the suggestion that God's concept is not a member of any world, would be the view that God's concept is a member of every world. Atomic and necessary sentences about God would be treated the same as sentences about Adam. We would thus have a uniform account of truth for all sentences in the language. Consider, for example, the sentence, "God is all-powerful". This sentence will be necessary just in case all the counterparts of the concept of God in various worlds have the property of being all-powerful. In God's case the counterparts of the concept of God will simply be the concept of God. But this also has its difficulties. If you will recall, a possible world is a maximal compossible set of concepts. Moreover, each concept mirrors the world of which it is a member. As argued earlier,[21] no concept can be a member of two worlds, yet on this view we are supposing that God's concept is a member of every world. This is clearly an inconsistency.

The only way we can see to avoid this inconsistency, given the view described above, is to claim that in some way God is an exception. We might wish to claim that there is a basis for making God an exception, namely God is an infinite substance, whereas we mere mortals are only finite substances. The idea would be that the compossibility and mirroring relations are only applicable to finite substance concepts (i.e., concepts such that if actualized, the corresponding substance would be finite) and not to infinite substance concepts (of which there is only one). If we accept this finite-infinite substance concept distinction, then the view does not appear to be inconsistent. However, while not inconsistent, it has some obvious bad results for Leibniz. For example, it turns out that all of God's properties are possessed by him of necessity. Consider any property P that God possesses. Since God has P, P is a member of the concept of God. The concept of God is a member of every world, thus God has P of necessity. In particular, it is metaphysically necessary that God is morally perfect. But this result is exactly the result which started our discussion of Leibniz's view of metaphysical necessity, and which we had hoped to avoid.

A third approach, (C), to the problem would be to say that God does in fact have distinct counterpart concepts in various possible worlds, as does Adam.[22] On this view we suppose that God has certain essential properties (e.g., being all-knowing), but also God has contingent properties which are in some of his counterparts, but not all. We could then say that being morally perfect is a contingent property of God's. We can also hold the Leibnizian view that God exists necessarily. This would be true because the concept of God would have a counterpart in every world. This view avoids a number of problems that the second view must account for. For example, in the second view we had to make God an exception to the principle that a concept is in only one world. But with the view now being suggested, God is not an exception, because the concept of God is only a member of one world, namely this one. The relations of compossibility and mirroring will apply to the concept of God and to the counterparts of the concept of God. This view has the major advantage of being uniform in that God is treated on a par with Adam, or with any other substance. However, this view is not without its problems.

One major difficulty with view C is that it is non-Leibnizian. Leibniz says that possible worlds are found in God's understanding, which is the region of possibles, and that from among them God chose one to create. On this view there seems to be no way of explaining how God, whose concept is a member of only one world, viewed all the possible worlds and picked one to create. The picture of creation that Leibniz presents is that of God standing apart from the possible worlds and viewing them to see which is the best to create. View A seems closest to the text in this respect, view C seems farthest from it, and B somewhere between A and C. The idea that the concept of God has distinct counterparts in every possible world is totally alien to Leibniz, and it seems clear he would reject it. Thus, while view C is an interesting one and it solves a number of difficulties, it is too un-Leibnizian to be acceptable.

The most promising of the three views presented seems to be view B, but if we opt for view B then, at the very least, we are left to deal with the conclusion of argument I, namely, it is metaphysically necessary that God create the best of all possible worlds, much to Leibniz's chagrin. However, this result is not as bad as one might think. Before we pursue this, let us return to the original argument for a closer look.

It does seem curious that Leibniz affirms premise (1) in the argument as opposed to affirming something like:

> (4) It is morally necessary that if God decides to create some world, then God will create the best of all possible worlds.

If we understand moral necessity as suggested, then (4) translates into the following in terms of metaphysical necessity:

> (5) It is metaphysically necessary that if God decides to create some world, then God will create the best of all possible worlds.

In order to logically conclude (3), we would need an additional premise, namely:

(7) It is morally necessary that God decide to create some possible world.[23]

The reason that Leibniz so willingly affirms premise (1) as opposed to something like (4) is that he holds (7), or something like it, to be true. Leibniz says:

> . . . it may be said that God can cause virtue to be in the world without any mixture of vice, and even that he can do so easily. But, since he has permitted vice, it must be that that order of the universe which was found preferable to every other plan required it. One must believe that it is not permitted to do otherwise, since it is not possible to do better. (T p. 197)

Leibniz seems to believe that if God did not create any world at all then God would not be doing what was best.[24] The best possible series of events that could occur would be for God to do exactly as he did. Thus (7) is true. It therefore makes no difference whether Leibniz affirms (1) or (4), since in either case we can conclude (3). But is (3) really that bad for Leibniz?

One might want to distinguish between (3) and something like:

(8) It is metaphysically necessary that God create the actual world;

and,

(9) It is metaphysically necessary that God create this world.

One might want to claim (9) is a bad result, but (3) is not since this world is not necessarily the best of all possible worlds. But for Leibniz (3), (8) and (9) all say the same thing. It is necessary that this world is the best of all possible worlds, hence (3) and (9) say the same thing. The phrase "the actual world" is just another name for this world, thus (8) and (9), and hence (3) say the same thing. Leibniz would reject (3), (8) and (9) for reasons similar to those given for his rejection of (2). But if we adopt the second view of metaphysical necessity suggested, then it seems Leibniz is stuck with (9). God did create this world, thus included in his concept is the property of creating this world. Hence, it is metaphysically necessary that God create this world. As noted before, any property that God has, he has of necessity.

It appears that for Leibniz we have reached the end of the rope. It is metaphysically necessary that God create this world, thus God had no choice but to create this world. This conclusion is bad in itself for Leibniz, but what seems worse is that everything which follows from God's creation is also necessary. What this seems to mean is that all true sentences about this world are necessarily true, and hence Leibniz cannot allow for contingency as his objectors have maintained. But while it is true that God does nothing but of necessity, it is not so clear that we mortals are under the same constraint.

It is not at all clear that it follows from God's creating this world of necessity that (say) it is necessary that Adam has blond hair. (Necessity will be used in the metaphysical sense henceforth.) On the view suggested above, to say that it is necessary that Adam has blond hair is to say that all of the counterparts of the concept of Adam in various possible worlds include the concept of blond hair. Surely this will still be false, and hence it is contingent that Adam has blond hair.

One might think that since God created this world of necessity, this world is the only possible world, other "worlds" being impossible. If this world is the only possible world, then the concept of Adam has a single counterpart in the various possible worlds, namely itself. Thus, it is true that all of the counterparts of the concept of Adam include the concept of blond hair (since there is only one counterpart), and hence it is necessary that Adam has blond hair. This argument presupposes a certain view about what possible worlds are. It assumes that a world is a possible world just in case the world could have been actualized. And this is indeed the way we have been considering possible worlds. But this is not the only way to view possible worlds. Leibniz says, "although God assuredly chooses the best, this does not prevent that which is less perfect from being possible in itself," (OC p. 22). The notion that is important here is that of something "being possible in itself". We can view possible worlds not as worlds which God might create, but rather as worlds which are not contradictory. In order to see how this might work we will have to revise our definition of compossibility.

Let P be the set of all properties $P_1 \ldots P_n$, and C the set of all complete individual concepts $C_1 \ldots C_n$. Let us further suppose we have a first-order language such as the lower predicate calculus.[25] In our language we have a number of predicates $F_1 \ldots F_n$ (let F be the set of all predicates) and constants $a_1 \ldots a_n$ (let A be the set of all constants). Let f be a function from F onto P, and from A onto C, so that for each F_i in F, $f(F_i)=P_i$ for some i, and for each a_i in A $f(a_i)=C_i$ for some i. Let H be the set of all the sentences of our language of the form Φx where Φ is a member of F and x is a member of A. We can now define a function g from C into the power set of the Cartesian product of F and A. For each C_i in C, $g(C_i)$ is the set of ordered pairs (x, y) such that x is a member of F and y is a member of A and $f(y)=C_i$ and $f(x)=P_i$ for each P_i in C_i. We can now define a function h from $g(C_i)$ into H. $h(g(C_i))$ is a set S of sentences of the form Φx and $F_i a_j$ is a member of S if and only if the ordered pair (F_i, a_j) is a member of $g(C_i)$. A set of complete individual concepts $C_1, C_2 \ldots C_n$ is compossible if and only if $h(g(C_1))$ union $h(g(C_2))\ldots$union $h(g(C_n))\ldots$ is consistent. A set of sentences is consistent if and only if it is not the case they mutually entail every sentence. A possible world is a maximal compossible set of concepts. Possible worlds are possible in the sense that they are somehow internally compatible, and not according to whether God could or could not have created them. While it may be impossible that God create any world other than this world, that does not make the worlds themselves impossible. However, it appears that even if we make the distinction between two views of possibility we are still left with the original objection.

On view B God has all of his properties of necessity. In particular, God has the property of being self-identical and Adam existing of necessity. But if God has that property, then it would seem to follow that Adam exists of necessity. Since this seems true of every substance for all the properties it has, there appears to be no contingency. But a closer examination of this argument will reveal that on the view being suggested it is unsound. Let 'a' represent 'Adam', 'g' represent 'God', and '$\Box$' represent 'it is necessary that'. We can symbolize the property in question with the use of abstracts.[26] The argument can be represented as follows:

II. (1) $\Box\ \hat{x}\left[x=x \wedge (Ey)(a=y)\right] g$

(2) $\Box\left(\hat{x}\left[x=x \wedge (Ey)(a=y)\right] g \rightarrow (Ey)(a=y)\right)$

/.. (3) $\Box (Ey)(a=y)$.

The argument is clearly valid, and sound on <u>some</u> interpretations of ' ' and '^'. But on the view being suggested premise (2) is false. The key to understanding premise (2) is realizing that '$\hat{x}\left[x=x \wedge (Ey)(a=y)\right]$' names a property just as 'F' names a property. A modal operator in front of an abstract does not alter the name of the property. Thus the following sentence can be true:

(4) $\Diamond\ \left(\hat{x}\left[x=x \wedge (Ey)(a=y)\right] g \wedge \sim(Ey)(a=y)\right)$

Consider a world W where the concept of Adam lacks a counterpart. In that world it will be true that the concept of God contains the property of being self-identical and Adam existing, and it will be true that the concept of Adam lacks a counterpart. This is the interpretation of (4). If (4) is true, then it is clear that (2) must be false. The point can be put in a different way.

While it is true that $\hat{x}\left[x=x \wedge (Ey)(a=y)\right]g$ is equivalent to $g=g \wedge (Ey)(a=y)$, they are not necessarily equivalent. The reason they are not necessarily equivalent is in the nature of modal operators on this view. When a modal operator preceeds a sentence which contains a constant not included in the name of a predicate, then the sentence is understood as saying something about the counterparts of the concept associated with the constant. However, when the constant occurs in the name of a predicate in a sentence, then adding modal operators to the sentence does not affect the name of the predicate. Thus, $\hat{x}[x=x \wedge (Ey)(a=y)]g$ says something different from $(g=g \wedge (Ey)(a=y))$. Admittedly, it seems strange, even contradictory, to say that in some world God has the property of being self-identical and Adam existing, yet Adam does not exist in that world. One might ask himself, how can it be that God have that property and Adam not exist? The answer is that God could not have that property unless Adam existed, but it is not necessary that Adam exist in every world in order that God have the property in every world. In effect, the property that God has in every world is that of being self-identical and Adam existing in some world. In view of these considerations it seems that for all the problems that view B has, it can allow for contingency.

The fact that God created this world of necessity does create some minor problems for the view being suggested. Intuitively, counterparts of concepts are those concepts God might have realized in place of the concepts he did realize. But if God created this world of necessity, then we can not literally view counterparts this way since God could not have realized any concepts other than the ones he in fact realized. But I do not believe this to be a major difficulty. The problem of what counterparts are is discussed in detail in Chapter V.

To summarize the position being suggested, an atomic sentence Fa is true if and only if the concept of F is included in the concept of a. An atomic sentence Fa is necessary just in case all the counterparts of the concept of a include the concept of F. The concept of God is different than any other concept and it is not subject to the same restrictions that other concepts have. This is in part because the concept of God is the concept of an infinite substance. The concept of God is a member of every world and hence has every property necessarily, including the property of creating this world. But even though God necessarily created this world, it does not follow that all true propositions are necessary. Adam has blond hair contingently because some of the counterparts of the concept of Adam do not include the property of having blond hair.

Leibniz believed that he could avoid the consequence that God necessarily created this world, and his writings reflect his belief. In order to facilitate discussions in the remainder of this dissertation it will be assumed for the most part that God was free in his creation. Since God's necessarily creating this world does not affect the contingency of other sentences, the assumption will not cause any major difficulties. If this becomes important, it will certainly be noted.

As pointed out in the introduction, there are two kinds of objections raised against Leibniz to the effect that he cannot allow for contingency. One deals with the problem of God and has been accounted for in this chapter. The other is what we will call the "analytic-necessary" problem. In the next chapter, two solutions offered for this problem by two different philosophers will be discussed. It will be assumed in that chapter that God is free in his creation of this world.

NOTES FOR CHAPTER I

1. R pp. 38-39.

2. R p. 39.

3. T p. 151.

4. OC pp. 13, 19. Also see p. 55 (this paper).

5. I am assuming that Adam existed.

6. In order to avoid confusion I introduce a standard use for "concept" terminology which will be continued throughout the rest of this paper. A proposition is the bearer of truth and is expressed by a sentence. When I say a sentence is true I mean the proposition expressed by the sentence is true. The terms "subject" and "predicate" refer to parts of sentences. The phrase "the concept of the subject" will refer to the concept associated with the subject of a sentence. The phrase "the concept of the predicate" will refer to the concept associated with the predicate of a sentence, which is a property. "The concept of the subject of a proposition" is to be understood as the concept of the subject of a sentence which expresses the proposition. The concept of a term is the concept associated with the term.

7. I discuss this point in more detail on pp. 57-58.

8. R p. 223.

9. Both Russell and Mates view compossibility this way. See R p. 66, and M1 pp. 511-514. Later Mates' view is discussed in more detail; see pp. 58-62.

10. This is the view I later argue for. See pp. 100-104.

11. See pp. 58-62.

12. Because of certain problems raised later in this chapter, the definition of compossibility will have to be revised.

13. OC p. 109.

14. For a precise, complete account of "compossibility", "mirroring", and "possible world" see pp. 100-104 and the appendix.

15. L p. 414.

16. T pp. 203, 229, 270, 271.

17. T pp. 187, 197, 252.

18. OC pp. 20, 21.

19. T pp. 187, 387. OC p. 22.

20. T pp. 187, 197, 271.

21. See p. 5.

22. This view was suggested to me by Fred A. Feldman.

23. Robert C. Sleigh Jr. pointed out to me that this is the weakest premise possible in order to conclude (3) from (5) and (2).

24. See also T pp. 377, 378, 386, 429.

25. The language being imagined is the following (ALPC):

I. Logical symbols, '$\sim$', '$\rightarrow$', 'v', '(', ')', '$\wedge$', '$\equiv$', '[', ']', '$\hat{}$'.

II. Non-logical symbols, terms:
(i) Constants $a_1 \ldots a_n$
(ii) Variables $x_1 \ldots x_n$
Predicate letters: $F^1_1, \ldots F^1_n, F^2_1 \ldots F^2_n, \ldots, F^n_1 \ldots F^n_n$.

III. Definition of wff: Φ is a wff if and only if
(i) Φ is an n-place predicate followed by n terms, or
(ii) If Φ is a wff then $\sim\Phi$ is a wff, and
(iii) If Φ and ψ are wffs then $\Phi \vee \psi$ is a wff, and
(iv) If Φ and ψ are wffs then $\phi \wedge \psi$ is a wff
(v) If Φ and ψ are wffs then $\Phi \equiv \psi$ is a wff
(vi) If Φ is a wff then $(x)\Phi$ is a wff
(vii) If Φ is a wff then $(Ex)\Phi$ is a wff

Φ is an n-place predicate iff either
(i) Φ is an n-place predicate letter, or
(ii) If Φ is a wff containing n free variables $x_1 \ldots x_n$ then $\hat{x}_1 \ldots \hat{x}_n[\Phi]$ is an n-place predicate (where 'free' is defined in the usual way.

The rules for ALPC are the same as LPC with the following addition:
(1) $\hat{x}_1 \ldots \hat{x}_n [\Phi]\ a_1 \ldots a_n \equiv \Phi^{a_1}/x_1 \ldots {}^{a_n}/x_n$ (where $\psi\alpha/\beta$ is read"α replaces all occurrences of β in ψ").

Example sentence: "Adam is married to Eve" will be translated as the following: $\hat{x}$ [xMe]a, where 'xMy' is 'x is married to y' and 'e' is 'Eve', and 'a' is 'Adam'. $\hat{x}$[xMe]a is understood as expressing the proposition that Adam has the property of being married to Eve.

26. See note 25 above.

CHAPTER II

Some philosophers argue that Leibniz cannot allow for contingency not because of God's lack of freedom, but because of Leibniz's definition of truth. John W. Nason presents a now-familiar criticism of Leibniz based on Leibniz's view of truth. Nason says of Leibniz:

> ... he asserts that all true affirmative propositions are analytic, i.e., they are true because the subject includes the predicate. This is as true, he asserts, of contingent propositions as it is of necessary truths. But if it were true that all true affirmative propositions are analytic, then all such propositions are necessary and there is no contingency. If some propositions are genuinely contingent, they can not be analytic. . . [1]

In this chapter, the views of two contemporary philosophers, G.H.R. Parkinson and N. Rescher, will be considered. Both suggest a way of understanding Leibniz in which Leibniz can avoid the objection Nason and others have raised against him. We will call this objection the "analytic-necessary" problem since the criticism is, in effect, that since Leibniz holds that all true propositions are analytic, it follows that all true propositions are necessary. Parkinson and Rescher offer different solutions to the problem, and it shall be argued here that each solution is in some way inadequate.

G.H.R. Parkinson presents what he believes is Leibniz's solution to the analytic-necessary problem in his book <u>Logic and Reality in Leibniz's Metaphysics</u>.[2] Parkinson says:

> By making use of the notion of an infinite analysis of certain concepts, Leibniz has succeeded in reconciling his view that every truth is either an expressly or implicitly identical proposition with his view that not all truths are necessary. (P1 p. 73)

> He reconciles the two views by saying that to speak of necessary and contingent truths is to speak of our ability or inability to prove that a true proposition is identical. Briefly, every truth is an identical proposition, or reducible to one; a truth is necessary if it is either an identical proposition, or human beings can demonstrate that it is an identical proposition; it is contingent if they cannot but know its truth by other non-deductive means. (P1 pp. 71-72)

> All truths, in his (Leibniz's) view, are either identical propositions or reducible to them; but those which are either identical propositions or reducible to such propositions in a finite number of operations we call 'necessary', and those which require an infinitenumber of operations for their reduction we call 'contingent'.
> (P1 p. 73)

In order to understand the view that Parkinson is trying to present, one should first try to understand some of the expressions Parkinson uses in presenting the view.

Parkinson holds that Leibniz gives two accounts of truth, one in terms of inclusion of the concept of the predicate in the concept of the subject, and the other in terms of what he calls 'identical propositions'. On the first account, to say a proposition is true is to say that the concept of the subject includes or contains the concept of the predicate. The second account is that a proposition is true just in case either it is an identical proposition or it is reducible to an identical proposition. Parkinson points out that for Leibniz an identical proposition is not just a proposition expressed by an identity sentence.

> He (Leibniz) makes it clear, however, that when he speaks of an identical proposition in the present context he has in mind, not only propositions such as 'A man is a man', but also propositions such as 'A white man is white'. In effect, he is using the term'identical' as a synonym for 'tautologous', as he himself implies when he remarks that he calls

> certain truths 'identical' because 'it seems that they do nothing but repeat the same thing, without teaching us anything'. In saying, then, that a true proposition is either an identical proposition or reducible to one, Leibniz means that a true proposition either is or is reducible to a tautology.
> (P1 p. 65)

Parkinson writes inaccurately when he applies the word 'tautology' to a sentence like, "A white man is white". This sentence is not a tautology, as the word is generally used, however it is a logical truth. We should understand Parkinson here to mean by tautology what is ordinarily meant by logical truth. Thus an identical proposition is a proposition expressed by a logical truth.

Parkinson says that Leibniz relates these two accounts of truth, "by saying that in an identical proposition the predicate is in the subject manifestly or expressly, whilst in all other true propositions it is present in the subject in a concealed form (tecte), or implicitly or virtually," (P1 p. 57). Parkinson is claiming, in effect, that the two accounts of truth Leibniz presents are the same. To say that a proposition is true if it is an identical proposition is the same as saying that a proposition is true if the concept of the predicate is included in the concept of the subject expressly. Similarly, to say that a non-identical proposition can be reduced to an identical proposition is the same as saying the concept of the predicate is included in the concept of the subject, but only implicitly or virtually. Given that these two accounts of truth are the same, we are still left to puzzle over how a non-identical proposition is reduced to an identical proposition.

Parkinson tries to explain away our puzzlement by means of an example. He asks us to consider the non-identical proposition expressed by the sentence, "Every man is rational". Since the sentence is non-identical, the inclusion of the concept of the predicate in the concept of the subject is only implicit. Parkinson continues and says, "This inclusion can, as he [Leibniz] remarks, be made explicit by analysis of the concepts or terms of the proposition; in this case, by replacing the term 'man' by the term 'rational animal', giving the proposition, 'Every rational animal is rational,' which Leibniz would call an identical proposition," (P1 p. 58). There is a minor difficulty here. This example can create more problems than it should if one believes that the proposition expressed by, "Every man is rational," is identical to the proposition expressed by, "Every rational animal is rational". In order to avoid problems which are not really relevant to the problem at hand, we assume the sentences express different propositions. But even if we ignore the problem of propositional identity, it seems we have removed the problem only one step further. In order to explain how a non-identical proposition can be reduced to an identical proposition, Parkinson introduces the notion of an "analysis of the concepts or terms of the proposition". It is not that the notion is non-Leibnizian, for Leibniz often talks about performing an analysis of a concept, but it seems just as opaque as the idea of a reduction. Parkinson, however, attempts to clarify it.

He says, "a proposition is 'reduced' by means of the analysis of concepts, i.e. by substitutions made on the basis of definitions. This analysis of concepts, it may be remembered, is analogous to spelling out the letters of a word," (P1 p. 74). But this explanation by Parkinson does not seem to be much help. Consider the example Parkinson gives, where we start with the proposition that every man is rational and by an analysis of concepts end with the proposition that every rational animal is rational. According to Parkinson the analysis in this case is the replacement of the term 'man' by the term 'rational animal'. It is clear that one can not replace the term 'man' in the proposition that every man is rational, since the term does not occur in the proposition. What Parkinson might mean is that if we replace the term 'man' in the sentence "Every man is rational" by the term 'rational animal', the resulting sentence is, "Every rational animal is rational". But while this makes some sense, it hardly seems like performing an analysis on a concept. However, it does indicate how we might reduce one proposition to another proposition. We can say that the proposition that every man is rational can be reduced to the proposition that every rational animal is rational, if the sentence "Every man is rational", which expresses the proposition that every man is rational, is such that when we replace the term 'man' by the term 'rational animal' the resulting sentence expresses the proposition that every rational animal is rational and the concept associated with the term 'man' is identical to the concept associated with the term 'rational animal'. But the problem suggested above be ignored because it seemed irrelevant seems very relevant now. If the concept of man is identical to the concept of rational animal, then it would seem that the proposition that every man is rational is identical with the proposition that every rational animal is rational. And if we are only discussing one proposition, what sense can be made of the claim that a reduction has occurred?

One way to avoid this problem is to understand the reduction as occurring in the language with which we express the propositions. That is, we understand the notion of a reduction of propositions which allows that if we reduce a proposition to a proposition, the propositions need not be distinct. For example, suppose that the proposition that every man is rational is identical to the proposition that every rational animal is rational. We can still say that the proposition expressed by the sentence "Every man is rational" is reduced to the proposition expressed by the sentence "Every rational animal is rational", because the sentence "Every rational animal is rational" can be obtained from the sentence "Every man is rational" by replacement of terms whose concepts are identical. Thus, what we define is a notion of reducibility relative to propositions and sentences. When we talk about propositions being reduced, we mean propositions expressed by certain sentences.

One might object that on this view the relation of reducibility is symmetrical. But for our pusposes, it does not matter if the relation is symmetrical. It is unimportant that it follows that if one can reduce a non-logical truth to a logical truth, then one can reduce a logical truth to a non-logical truth. We are primarily interested in the first step, that is, how one can reduce non-logical truths to logical truths. This view seems to explain this. We can generalize this view in a definition. First, some notation: if Φ is a sentence, then let $P(\Phi)$ be the proposition Φ expresses, if any.

(DFR) $P(\Phi)$ can be reduced to $P(\psi)$ <u>iff</u> Φ contains a term α, and ψ is exactly like Φ except that for each occurrence of α in Φ, ψ contains a term β, and the concept of α is identical to the concept of β.

The idea of an analysis is relevant in the sense that in some way one has to analyze the concept of α and the concept of β in order to determine whether they are identical or not. One analyzes the concept of α by, as it were, spelling out the properties contained in the concept. Parkinson suggests it is, "analogous to spelling our the letters of a word". Perhaps (DFR) is not much help in understanding what an analysis of a concept is, but it does give us an idea of what a reduction is, which is what we wanted in the first place. Thus we can say with some clarity that a proposition is true just in case it is an identical proposition or it is reducible to one. We now turn to the problem of necessary propositions.

Parkinson seems to present two different accounts of necessity for Leibniz. According to one, a proposition is necessary just in case either it is an identical proposition or human beings can demonstrate that it is an identical proposition. On the other, a proposition is necessary if and only if either it is an identical proposition or is reducible to an identical proposition in a finite number of operations. However, even though these two accounts appear to be different, they are essentially the same. In the first account Parkinson wants to understand the word 'can' in the phrase 'human being can demonstrate' in a strict logical sense. Thus, according to Parkinson, any proposition that is reducible to an identical proposition in a finite number of operations is one such that humans 'can' demonstrate that it is an identical proposition. Therefore, the two accounts are really equivalent. A necessary proposition is either an identical proposition or one reducible to an identical proposition in a finite number of operations. A contingent proposition is one that is not an identical proposition and its reduction to an identical proposition would require an infinite number of operations. But unfortunately it appears that under (DFR) all reductions are finite; all require a single operation. If this is the case, then all true propositions are necessary. There are at least two possible ways of attempting to meet this difficulty. One way is to change the definition of reduction to allow in some way the notion of an infinite number of operations. The other way is to attempt a more precise definition of a concept and see if the idea of an infinite number of operations arises there. The latter way appears to involve us in a number of obscurities, thus we will try the former method first. First, some notation to make the definitions more readable. If ψ is obtained from Φ by replacing all occurrences of α in Φ by β, then $S(\psi)=\{\beta\}$.

(DFR1) $P(\Phi)$ can be reduced to $P(\psi)$ in a finite number of operations if there is a series of sentences $\Phi_0, \Phi_1, \ldots \Phi_n$ such that $\Phi_0=\Phi$ and $\Phi_n=\psi$, and for each i, $1 \leq i \leq n$, Φ_{i+1} is obtained from Φ_i by replacing a term α in Φ_i by a term β not a member of $\bigcup_{j\leq i} S(\Phi_j)$, and the concept of α is identical to the concept of β.

(DFR2) $P(\Phi)$ can be reduced to $P(\psi)$ in an infinite number of operations if there is a series of sentences $\Phi \ldots \Phi_n \ldots$ such that

$\Phi = \Phi_0$, and for any i, $1 \leq i$, Φ_{i+1} is obtained from Φ_i by replacing a term α in Φ_i by a term β not a member of $\bigcup_{j \leq i} S(\Phi_j)$, and the concept of α is identical to the concept of β, and ψ is the <u>limit</u> of the series $\Phi_0 \ldots \Phi_n \ldots$

To say that ψ is the limit of the series is to say that for each Φ_i which is a member of the series, Φ_i is closer to ψ than Φ_{i-1} is, and no member of the series is identical to ψ. The idea is that just as 1 is the limit of the series 1/2, 1/2 + 1/4, 1/2 + 1/4 + 1/8, ..., ψ is the limit of the series $\Phi_0 \ldots \Phi_n, \ldots$ The analogy between 1 and ψ is a good one in the sense that Leibniz himself often gives similar kinds of examples in attempting to explain the notion of an infinite analysis. But it is not clear just how analogous the two cases are. It makes sense to talk about 1 being the limit of the series 1/2, 1/2 + 1/4, ..., since the notion in this case is well defined. But in the case of the series $\Phi_0 \ldots \Phi_n \ldots$ it is not clear that there is a limit. To investigate this problem, several definitions are needed.[3] Let $\{S_n\}$ be a sequence with n members. Let $\{S_n\}$ be a sequence with n members. For example if n equals 5, then $\{S_n\}$ stands for the sequence $\langle S_1, S_2, S_3, S_4, S_5 \rangle$ where the subscripts stand for the identity of members and 'S' stands for the kind of objects in the sequence. In the example given 'S_1' may be '1/2', and 'S_2' may be '1/2 + 1/4', and so on. Let '$\geq$' represent the ordering relation of the sequence in question. '$\geq$' might mean 'is greater than or equal to' and in our example we could say '$S_1 \geq S_2$'. An ω sequence is a sequence whose cardinality is equal to that of ω.

DF1: $\{S_n\}$ is an increasing ω sequence if and only if $(n)(S_{n+1} \geq S_n)$

DF2: An increasing ω sequence $\{S_n\}$ is bounded above if and only if $(ES)[(n)\ S_n \leq S]$

We define the notion of a limit for a bounded increasing ω sequence:

DF3: $\lim \{S_n\} n \in \omega =_{df} (\iota x)(n)[S_n \leq x \wedge (y)(S_n \leq y \rightarrow x \leq y)]$

Let $\{\Phi_n\}$ represent the appropriate sequence. It is not clear that $\{\Phi_n\}$ has a limit. In the first place, it is not clear that $\{\Phi_n\}$ is an ω sequence.

Before we can even start we must assume that the language has at least an infinite number of terms. But even if we assume that the set of terms is infinite, we are still not guaranteed that the sequence is infinite. We need a further condition. To avoid needless complexity, suppose Φ_0 only contains one term which can be replaced. Let $R(\Phi_0)$ represent the replacement set for Φ_0 (i.e., the set of all those terms which can replace <u>the</u> term in Φ_0). In order to guarantee that $\{\Phi_n\}$ is infinite, the following condition must hold: $(E):(n)[R(\Phi_n) \not\subseteq_{r \leq n} S(\Phi_r)]$. This condition, in effect, guarantees for us (if it holds) that the replacement set for Φ_n (for any n) has not already been exhausted or used up by the time we reach Φ_n in the series. If (E) holds, then $\{\Phi_n\}$ will be infinite. But a sequence's being infinite is not sufficient for it to have a limit. For one thing, the sequence must be bounded. Is it true that all the members of $\{\Phi_n\}$ are less than ψ? In this context it does not, of course, make any sense to talk about one sentence being less than another sentence. The proposition expressed by ψ is supposed to be an identical proposition, while the members of the sequence do not express logical truths. As we progress along the sequence they become closer and closer to ψ. In order to find out whether ψ really is the limit, we need some definition of the closeness relation. Consider the sequence $\Phi_0, \Phi_1 \ldots \Phi_i \ldots \Phi_j \ldots \psi$.

DF4: Φ_j is closer to ψ than Φ_i if and only if: (i) There exists a sequence such that Φ_i is before Φ_j and ψ is the limit; and (ii) For all sequences $\{\Phi'_n\}$ such that $\Phi_i = \Phi'_0$, then if Φ_j is not in $\{\Phi'_n\}$, then the limit is not ψ.

This definition in effect says that if Φ_j is closer to ψ than Φ_i is, then the only way to approach ψ from Φ_i is through Φ_j. DF4 gives us a precise definition of the relation of closeness, but it also leads one to a question which seems to be the crux of the whole problem. That is, are the rules given for replacement such that it is true that if a sentence occurs later in the sequence then it is closer to the sentence which we want to say is the limit? If the answer is no, then DF4 will never hold and the sequences described can not be said to have limit. The problem is that in the sequence 1/2, 1/2 + 1/4, ... we have a clear sense of what it means to say each succeeding member is closer to the limit, but in the sequence $\{\Phi_n\}$ it makes no difference whether Φ_i or Φ_j occurs later in the sequence (this is assuming the answer to the question posed above is no). If it makes no difference if Φ_i occurs before Φ_j or vice versa, there can be no sense in saying one is closer than the other. It would, thus, make little sense to say the sequence approaches a limit. If the answer to the question posed is yes, then we have a clear idea of what it means to say a non-identical proposition is reduced to an identical proposition (provided the conditions stated earlier are satisfied).[4]

Unfortunately, while the idea expressed in (DFR2) is an interesting one, it will not be of much help for us, since it is unclear that DF4 will ever be satisfied under the rules given for replacement. Consider Φ_i and Φ_{i+1}. Φ_{i+1} is obtained by replacing some term in Φ_i, and the condition of the replacement is that the concepts of the two terms be identical. Provided that Φ_i is not the first member of the sequence, it could have been obtained from Φ_{i+1}. There is nothing in the rules which would lead one to believe that it is possible to arrive at Φ_{i+1} from Φ_i and not Φ_i from Φ_{i+1}. We might try changing the rules of replacement to avoid this problem, but it is not clear that we could change the rules and yet retain an adequate idea of a reduction. Thus, we must conclude that while this is an interesting approach to the problem it does not seem to solve it.

Earlier two alternative approaches to the problem that under (DFR) all reductions are finite were discussed. One was to change the definition of a reduction, and the other was to attempt to clarify what an analysis of a concept is. We have already discussed the first approach and it has been suggested that this will not be of much help. Perhaps if we understand what an analysis of a concept is, we can see how, under Parkinson's suggestion, some sentences turn out to be contingent. Parkinson asks us to consider the contingent sentence, "The sun is now shining", and says, "only if the concept of the subject of the true proposition 'The sun is now shining' is analyzed into a concept of infinite complexity, describing everything in the universe, can it be seen how the concept of the subject includes that of the predicate," (P1 p. 73). Parkinson clarifies the problem somewhat when he indicates that an analysis is an, "analysis of a concept into its component concepts," (Pl p. 75). If we understand Parkinson correctly, he is suggesting that in contingent sentences the analysis of the concept of the subject is such that its 'component concepts' are infinitely complex. One difficulty here is in understanding what Parkinson means by "component concepts". Presumably, one of the 'component concepts' of the concept of man is the concept of animal. Thus, we can say that a 'component concept' of a concept C is a concept which is either a member of C or a subset of C. For example, one component concept of the concept of square is the concept of rectangle, since the concept of rectangle is a subset of the concept of square. One of the component concepts of the concept of the sun is the concept of now shining (where 'now' is a demonstrative referring to a particular time). Parkinson claims that the concept of the sun is a concept of infinite complexity. Even if this is true it is difficult to see why we would have to completely analyze the concept of the sun in order to determine that the concept of now shining is included in it. It would seem more reasonable to believe that since the concept of now shining is included in the concept of the sun we would not have to analyze the concept into a concept of infinite complexity. We would only have to analyze the concept enough to see that the concept of now shining is included in it, which surely is not infinitely complex. Parkinson says, "to explain the possibility of contingent truths Leibniz need only say, without being more specific, that there are certain true propositions whose analysis is infinite," (Pl p. 73). But surely the whole problem here is in trying to understand what it means to say an analysis is infinite.

One person could claim that in order to see that the concept of the predicate is included in the concept of the subject in the sentence, "The sun is now shining," an infinite analysis would be required, while another person could deny this. Neither claim could be shown to be correct. The notion of what an analysis of a concept is is just too obscure to be any help.

We must conclude that Parkinson's attempt to provide for Leibniz an answer to the analytic-necessary problem fails. While the account he presents is Leibnizian in certain respects, it does not clearly answer the objection. Until we can get a clear idea of what an analysis of a concept is, or until we can make sense of the notion of an infinite number of operations in a reduction from one proposition to another, we are simply unable to determine whether Parkinson's account of Leibniz really solves the difficulty. Rescher also tries to solve this problem by using the notion of an infinite analysis. His approach to the problem, however, is somewhat different from Parkinson's.

In explaining his view of Leibniz's theory of contingency, Rescher makes use of three principles: The Principle of Sufficient Reason, The Principle of Identity or The Principle of Contradiction, and The Principle of Perfection or of The Best.[5] According to Rescher, The Principle of Sufficient Reason (PSR), "is a principle asserting that, if a proposition is <u>true</u>, then it is possible to show that its predicate is contained in its subject by means of an analysis or demonstration which need not terminate but may proceed <u>in infinitum</u> (in which case God alone can carry out the analysis fully)," (Res2 p. 27). In other words, it is the principle that all true propositions are analytic.

The Principle of Identity (PI) is that all "finitely" analytic propositions are necessarily true. By a "finitely" analytic proposition Rescher means a proposition such that the concept of the predicate can be shown to be included in the concept of the subject in a finite number of steps. The Principle of Perfection (PP) is the principle that every "infinitely" analytic proposition is contingently true. An infinitely analytic proposition is one such that it would take an infinite analysis in order to show that the concept of the predicate is included in the concept of the subject. Although Rescher's view is somewhat unclear, he basically wants to hold that (PP) is Leibniz's principle of contingence and, in accord with (PP) God selects the best possible world. In order to understand this view we must consider what Rescher says about possible worlds and perfection.

According to Rescher, every "possible substance" is a member of some possible world, and each of these possible substances has a complete concept which involves its entire history and mirrors the world of which it is a member. While every possible substance mirrors the world of which it is a member, different substances in that world have different degrees of "clarity" at a given state. A state is a particular time in the development of a substance. At a given state a substance in a world "perceives" the rest of that world with a certain degree of clarity. "Let us call the degree of clarity with which at a given state a possible substance mirrors its universe its _amount of perfection for that state_," (Res2 p. 29). The amount of perfection a possible substance has is the total amount of perfection it has for all states. The amount of perfection of a possible world is the total amount of perfection of all possible substances which are members of that possible world. According to The Principle of Perfection, "God selects that universe for which the amount of perfection is a maximum."[6] Rescher claims that this enables us to understand the infinite analyticity of contingent truths. To help us understand the connection between The Principle of Perfection and infinite analysis, we first look at Rescher's account of the nature of analysis.

Rescher says:

> The Leibnizian "analysis" of a proposition about a substance consists of two steps:
> 1. To scrutinize the list of properties of the substance that is the subject of the proposition in order to determine what is and what is not included in its complete individual notion.
> 2. To determine whether the properties imputed by the predicate of the proposition to the substance are in fact included in this list (or is a derivative of properties so included). (Resl p. 23)

Rescher's idea seems to be that the analysis of a sentence like, "Adam has blond hair," consists of listing the properties contained in the concept of Adam and checking to see if the property of blond hair is a member of the list. In certain cases Rescher claims that the analysis will be infinite. In particular, he claims that the analysis of a contingent proposition is infinite. He says:

> Since true contingent propositions concern contingent existents ... the concatenation of subject and predicate asserted by them depends on the nature of existence. In this way the principle of contingent existence, the Principle of Perfection, enters into their analysis. It is via this principle and comparison of perfection of an infinite number of possible worlds involved in it, that an infinite process is imported into the analysis of contingent truths ... A truth of fact is

> such that the state of affairs it asserts is one belonging to the best of all possible worlds, hence its analysis, which consists in showing that this is indeed so, requires an infinite process of comparison.
> (Resl pp. 37-38)

Rescher is not speaking quite accurately when he suggests that for Leibniz all true contingent propositions concern existents, with the exception of God. Leibniz says that the laws of nature, in particular the laws of motion, are also contingent, and the concatenation of subject and predicate asserted by them can not be thought to depend on the nature of existence in the way a sentence like, "Adam has blond hair", might be thought to depend. This is a possible problem for Rescher, if his account of contingency excludes such propositions. Rescher suggests here that a sentence is contingently true if the state of affairs it asserts is one belonging to the best of all possible worlds. But the connection between this view and the idea that contingent truths require an infinite analysis is far from clear. Perhaps a better understanding of The Principle of Perfection will clarify matters.

It is obvious that The Principle of Perfection plays an important role in Rescher's account of contingence in Leibniz. Yet Rescher never definitively states this principle. Sometimes he says that it is the principle that every infinitely analytic proposition is contingently true,[8] while other times he says:

> This principle is a formulation of the thesis that, in His decision of creation, God acted in the best possible way; the actual world is that one among the possible worlds which an infinite process of comparison showed to be the best.
>
> The existence of an objective criterion of goodness is a crucial feature of this principle.
> (Resl p. 28)

While these two statements of the principle are in some ways connected, they clearly do not say the same thing. Rescher also says that The Principle of Perfection enters into the analysis of propositions concerning contingent existents. If the principle were simply that every infinitely analytic proposition is contingently true, it would be very difficult to see how it could enter into any analysis of a contingent proposition like Adam has blond hair. As it is, seeing how The Principle of Perfection enters into any analysis is going to be difficult. It would be more reasonable to say that according to Rescher that every infinitely analytic proposition is contingently true follows somehow from The Principle of Perfection. The Principle of Perfection is, then, that God acted in the best possible way when he created this world. What role does this principle play in Rescher's account of contingent truths?

Rescher says that:

> A given proposition concerning a contingent existence is true, and its predicate is indeed contained in its subject, if the state of affairs characterized by this inclusion is such that it involves a greater amount of perfection for the world than any other possible state. (Res2 p. 30)

In her review of Rescher's book, Margaret D. Wilson suggests one way of understanding him.[9] The sentence, "Adam has blond hair," is contingent because God created the best of all possible worlds and in doing so caused the concept of Adam to contain the property of having blond hair. Adam could have lacked the property of having blond hair because Adam's concept need not contain that property. Adam's concept contains the property of having blond hair because God chose the best of all possible worlds, but if God had chosen a different world, then Adam's concept would not contain the property of having blond hair. This way of understanding Rescher would seem to account for contingency in Leibniz in that the proposition that Adam has blond hair is true because God chose the best of all possible worlds, but would have been false had God chosen a different world. Wilson criticizes this view by pointing out that Leibniz tells us that all concepts were completely formed in God's understanding before God decided which world to create. The concept of blond hair is included in the concept of Adam whether or not God chose to create this world. Yet given the above account of Rescher it would seem that the concept of Adam is not completely formed until God decides to create this world, for the concept of blond hair could have or could have not been included in the concept of Adam, depending on which world God decided to create. Since God decided to create this world, the concept of blond hair is included in the concept of Adam. Wilson comments, "This seems to imply that God, by creating the world in accordance with the Principle of Perfection causes certain predicates to be included in certain subject-concepts that would not be included were it not for his decision."[10] This view explicitly contradicts what Leibniz says about complete concepts in, among other places, his correspondence with Arnauld. There Leibniz says that God created a completely determined Adam in the sense that the concept of Adam, before God created Adam, included all the properties Adam would ever have.[11] Thus even if the above view allows for a distinction between necessary and contingent truths, it is not a view that Leibniz would hold or could consistently hold.

While Wilson seems to be correct in her criticisms of the view presented, it seems incorrect to attribute that view to Rescher. While Rescher's statement of his views is very unclear, and hence open to numerous interpretations, he does explicitly say, "The existence of an objective criterion of goodness for possible worlds wholly independent of the will of God is a crucial feature of this principle," (Res1 p. 28; emphasis added). Given Rescher's account of perfection in worlds,[12] it is clear that Rescher considers the worlds totally formed before God makes a choice. Thus, while Wilson's account of Rescher is one way to interpret him, it seems unfair to him.

There is a more plausible way to understand Rescher than the account that Wilson presents. Rescher seems to believe that the contingency of true sentences is closely connected with both The Principle of Perfection and the notion of infinite analysis. At one point Rescher says, "A truth of fact is such that the state of affairs it asserts is one belonging to the best of all possible worlds."[13] Rescher clearly did not mean for this to be a sufficient condition for the contingency of true propositions, since all necessary truths are also such that the state of affairs they assert belongs to the best of all possible worlds. But this condition, combined with the idea that contingent truths are infinitely analytic, allows us to present a reasonable view for Rescher. Let '$S(\Phi)$' be 'the state of affairs expressed by Φ, or the state of affairs asserted by the proposition expressed by Φ'. The view being suggested can be expressed as follows:

(DF1) A sentence Φ expresses a contingent true proposition if and only if $S(\Phi)$ occurs in the best of all possible worlds and Φ is infinitely analytic.

(DF2) A sentence Φ expresses a necessary true proposition if and only if $S(\Phi)$ occurs in the best of all possible worlds and Φ is finitely analytic.

(DF3) A sentence Φ expresses a contingent false proposition if and only if either $S(\Phi)$ does not occur in the best of all possible worlds and Φ is infinitely analytic, or not-Φ is infinitely analytic.

(DF4) A sentence Φ expresses a necessary false proposition if and only if either $S(\Phi)$ does not occur in the best of all possible worlds and Φ is finitely analytic, or not-Φ is finitely analytic.

A sentence Φ is finitely analytic just in case the concept of the predicate is included in the concept of the subject and the analysis of Φ occurs in a finite number of steps. A sentence Φ is infinitely analytic just in case the concept of the predicate is included in the concept of the subject and the analysis of Φ does not occur in a finite number of steps.[14] Rescher's account of the nature of analysis has already been described,[15] and it is being used here as Rescher uses it.

The view explicated by (DF1) through (DF4) appears to correspond with the view that, according to Rescher, saves Leibniz from the objection that God necessarily created the best of all possible worlds. In order to understand Rescher's proposed solution, we must make his distinction between "metaphysical perfection" and "moral perfection". Metaphysical perfection is the amount of potential for existence a thing has, while moral perfection is the amount of "goodness" a thing possesses. According to Rescher, Leibniz believes that God is necessarily perfect, but perfect in the metaphysical sense. Thus it is supposed to follow from God's essence alone that he exists.[16] God is also morally perfect, but his moral perfection is not necessary. Rescher says:

> God's moral perfection (goodness) has a sufficient reason, and this in turn another, _et caetera ad infinitum_; but this sequence of sufficient reasons converges on God's metaphysical perfection. Or, putting this another way, we can say that God's moral perfection is indeed a logical consequence of His metaphysical perfection, but a consequence which no finite deduction suffices to elicit. In this way, as Leibniz insists, the proposition asserting God's moral perfection is contingent; ...
> (Res2 p. 38)

So, according to Rescher the proposition that God is morally perfect is infinitely analytic, and since it is true in the best of all possible worlds, it is contingent. But while the analysis of the proposition that God is morally perfect is infinite, Rescher tells us that it, "converges on God's metaphysical perfection," and is indeed a logical consequence of God's metaphysical perfection. Thus far we have not dealt with the problem of the nature of an infinite analysis, but it is a crucial aspect of the view being suggested.

In describing his account of an analysis, Rescher points out that in certain cases an analysis of a proposition may be nonterminating "Analysis of certain propositions will not result in _explicit_ identities; they are only _virtually_ identical, in that their analysis comes closer and closer to yielding, but never actually yields, an actual identity."[17] In a case like this, the analysis "converges" on some actual identity. But Rescher never explains how an analysis of a proposition can be said to "converge" on an explicit identity (by which he means a logical truth[18]). It is particularly difficult to understand given Rescher's account of the nature of analysis. Suppose an analysis is performed on the proposition that Adam has blond hair. On Rescher's view, one would start listing the properties in the concept of Adam to determine whether or not the property of having blond hair is on the list. Even if the number of properties in the concept is infinite,[19] if Adam has blond hair then the property of having blond hair will, sooner or later, appear on the list. When the property of blond hair does appear on the list, there will only be a finite number of properties before it. Thus we can show that Adam has blond hair in a finite number of operations. This same process could be repeated for any "contingent" true proposition, and hence it appears that all true propositions are necessary. On Rescher's view it is not clear that there are any infinite analyses, let alone one that "converges" on some proposition.

It might be possible to avoid this difficulty by changing Rescher's account of the nature of an analysis to something which would make more sense out of the idea of an analysis converging. However, the only plaudible way to do this seems to be the way discussed earlier in the account of Parkinson, and we have already seen the problems involved in that. There appears to be no complete, coherent way of accounting for contingency in Leibniz in terms of infinite analysis alone. Rescher's account of Leibniz is more obscure than that of Leibniz himself, and even under what seems the most reasonable interpretation of Rescher we do not seem to have a solution to the problem. In the next chapter we will discuss a view of Leibniz presented by

Benson Mates which ignores the notion of an infinite analysis and concentrates solely on Leibniz's view of possible worlds to avoid the analytic-necessary problem.

NOTES FOR CHAPTER II

1. Nason, John W., "Leibniz and the Logical Argument for Individual Substances", Mind Vol. LI, 1942, p. 213.

2. Pl pp. 56-76.

3. To avoid needless complexity, I will only consider increasing sequences.

4. I am indebted to Michael Jubien for helping me understand some of the difficulties involved in this account. Our discussions on this subject were very valuable in formulating this view of Parkinson.

5. See Res1 and Res2.

6. Res1 p. 28.

7. OC p. 108.

8. Res1 p. 27, Res2 p. 32.

9. Wilson, Margaret, Journal of Philosophy, January 11, 1968, pp. 23-27.

10. Ibid., p. 26.

11. OC p. 104.

12. Described on pages 41-42.

13. Res1 p. 38.

14. Not-Φ is infinitely analytic just in case the concept of the predicate is not included in the concept of the subject, and the analysis of not-Φ does not occur in a finite number of steps.

15. See pp. 41-42.

16. Res1 pp. 44-45.

17. Res1 p. 24.

18. Res1 p. 25.

19. I am assuming that the cardinality of the concept of Adam is not greater than the cardinality of ω. If it were greater, then there would be no "list of properties" as Rescher claims.

CHAPTER III

Benson Mates' views on Leibniz on necessity and contingency differ significantly from the views of Rescher and Parkinson. In his explication of necessity and contingency Mates concentrates solely on Leibniz's views of concepts and possible worlds and mentions Leibniz's discussions of infinite and finite analysis only to point out the difficulty in understanding them.[1] Mates also presents his views on Leibniz more formally in his paper "Leibniz on Possible Worlds" in terms of a semantics for a formal language. In this chapter, Mates' semantics will be presented, followed by a discussion of some of the Leibnizian aspects of it, and, finally, some possible difficulties with his system will be considered.

In presenting Mates' semantics it is assumed that we have a modal predicate calculus with identity. The notion of well-formedness is defined in some usual way. We will, following Mates, restrict the predicates to one-place predicates (except identity).

Mates' system is based on the Leibnizian notions of "complete individual concepts", "compossibility", and "possible worlds". "A _complete individual concept_ is a set of simple properties satisfiable by exactly one thing and containing all the simple properties that would belong to that thing if it existed," (M1 p. 254). _Compossibility_ (for Mates) is an equivalence relation which partitions the set of all complete individual concepts into equivalence classes, which are _possible worlds_. There are a denumerably infinite number of possible worlds, each containing infinitely many concepts, also denumerable.[2]

The individual constants and one-place predicates are interpreted as follows:

Let f be a function that maps the set of individual constants onto the set of complete individual concepts (if α is an individual constant, then $f(\alpha)$ is the complete individual concept associated with α). f also maps the set of singulary predicates onto the set of simple properties (if F is a singulary predicate, then f(F) is the simple property associated with F).

Mates then defines the relation <u>true of</u> in the following way: For any sentence Φ, formulas ψ, χ, constants a, b, predicate F (other than identity), variable α, and possible world W, then:

(i) If Φ is Fa, then Φ is true of W <u>iff</u> $f(F) \in f(a)$ and $f(a) \in W$.

(ii) If Φ is a=b, then Φ is true of W iff f(a) is f(b) and $f(b) \in W$.

(iii) If Φ is $\sim\psi$, then Φ is true of W iff ψ is not true of W.

(iv) If Φ is $(\psi \rightarrow \chi)$, then Φ is true of W <u>iff</u> either ψ is not true of W or χ is true of W, or both.

(v) If Φ is $(\alpha)\psi$, then Φ is true of W iff $\psi\alpha/b$ is true of W for every individual constant b such that $f(b) \in W$.[3]

(vi) If Φ is $\Box\,\psi$, then Φ is true of W iff ψ is true of every possible world W'.

A sentence is a necessary truth <u>iff</u> it is true of all possible worlds.

In Mates' system truth is defined "intensionally" as opposed to "extensionally". In an extensional account of truth, constants are assigned to objects from the domain (if assigned at all), and predicates are assigned sets of objects which are called the "extension of the predicate". An atomic sentence is true if the object assigned to the constant (if there is one) is a member of the extension of the predicate.

In an intensional system, predicates are not assigned to sets of objects, but rather they are assigned to properties. Properties are the intension of the predicate, and are taken as primitive. Depending on the system one wanted, constants could then be assigned to sets of properties as in Mates' system. An atomic sentence is true provided that the intension of the predicate is a member of the set assigned to the constant. It is this intensional notion of truth that we find in Leibniz. He says:

> In fact when I consult the conception which I have of all true propositions, I find that every necessary or contingent predicate, every past, present, or future predicate, is involved in the concept of the subject, and I ask no more. (OC p. 117)

> The concept of the predicate is always in the subject of a true proposition. (OC p. 126)

> Always in every affirmative proposition whether veritable, necessary or contingent, the concept of the predicate is comprised in some sort in that of the subject. Either the predicate is in the subject or else I do not know what truth is. (OC p. 132)

If one looks at truth conditions in Mates' semantics, one can see that this view of truth is indeed included. Consider, for example, the sentence, "Adam is the first man". Since Mates also wants to account for certain modal notions (i.e., necessity), all sentences have a truth-value relative to a possible world, but for our present purposes this point is not crucial and we can consider the example sentences relative to this world. "Adam is the first man," is true (of this world) provided that the property of being the first man is a member of (or is contained in) the complete individual concept of Adam, and that concept is a member of this world. If we symbolize the sentence as Fa where a stands for "Adam" and F for "is the first man", Fa is true (of this world) if and only if f(F) (i.e., the property assigned to the predicate F) is a member of f(a) (i.e., the concept assigned to the constant a), and f(a) is a member of this world. Thus, in case of atomic sentences Mates' system agrees with Leibniz in intensionality of truth. However, Leibniz believes that the concept of the predicate is included in the concept of the subject in all true propositions. Leibniz would hold that the proposition that every human is an animal is true just in case the concept of the predicate is included in the concept of the subject. But in this case it is much more difficult to see how his definition is supposed to work.

Consider the two sentences, "Adam is human", and "Every human is an animal". In the first sentence the concept of the subject can be thought of as a set of properties. Leibniz says that for every substance there is a complete individual concept which contains all its properties. He says:

> . . . We are able to say that this is the nature of an individual substance or of a complete being, namely to afford a conception so complete that the concept shall be sufficient for the understanding of it and for the deduction of all the predicates of which the substance is or may become the subject. (OC p. 13)

In the sentence, "Adam is human", the name "Adam" refers to some individual for which there is a complete concept. We can thus talk about the complete concept of Adam which contains all and only those properties which Adam has. Given that the language we are using has the appropriate connections, the concept of the subject of the sentence "Adam is human" is the complete concept of Adam. The concept of the predicate in the sentence "Adam is human" is the property of being human. The sentence is true provided that the complete concept of Adam contains the property of being human.

Leibniz indicates that there are more complete individual concepts than there are or will be substances. This is because there are complete individual concepts which were never realized by God; i.e., God never created a substance which corresponded to them. Mates takes possible worlds to be constituted of these unrealized concepts, rather than "possible substances". But Leibniz often speaks about "possible individuals" or "possible persons", which God never created.[4] Mates suggests that we understand Leibniz as talking about unrealized concepts rather than "possible substances" when he speaks of "possible persons". Mates' suggestion is a good one. Leibniz most frequently refers to "possible persons" when he is discussing God's choice in creation, saying that God chooses to create one individual from among many possible individuals. But Leibniz also refers to God's choice as a selection from among concepts. He says, ". . . I consider the individual concept of Adam as possible when I maintain that among an infinite of possible concepts God has selected a certain Adam," (OC pp. 107-108). This indicates that Leibniz did not distinguish between "possible persons" and what he calls "possible concepts". Leibniz does not believe concepts are possible, in the sense that God could or could not create them as he chose. As he says in his discussion of the complete concept of Adam, "Now, there is no ground for doubting that God can form such a concept, or rather, that he finds it already formed in the region of possibilities, that is to say, in his understanding," (OC p. 111). So, by "possible concept" we should understand Leibniz as meaning a concept which God could _realize_. Leibniz further indicates that by "possible persons" he is referring to concepts and not "possible substances" when he says:

> In order to call anything possible it is enough that we are able to form a notion of it when it is only in the divine understanding, which is, so to speak, the region of possible realities. Thus in speaking of possibles, I am satisfied if veritable propositions can be formed concerning them. (OC p. 131)

A proposition is true provided that the _concept_ of the subject of the proposition contains the concept of the predicate of the proposition. Thus, in order to form true propositions concerning possibles we need only speak of concepts, and not of "possible substances". If Leibniz is satisfied with that, there is no reason why we should not be. Therefore, we will agree with Mates' suggestion and understand Leibniz to be referring to unrealized complete concepts when he speaks of possible individuals.

Given this view of possible individuals it is natural to assume, as Mates suggests, that possible worlds are made up not of individuals, but rather of concepts, and only in the real world are these concepts actualized. Not all concepts are actualized for the reason that not all possibles are, as Leibniz puts it, "compossible". Intuitively, by "compossible" Leibniz means compatible. For example, if concept x contains the properties of being the first man and having red hair, and concept y contains the properties of being the first man and having blond hair, it seems reasonably clear that x and y could not be realized together. In any world there can be only one first man and he can have either red hair or blond hair, but not both. Thus Leibniz holds each world to be a collection of compossible possibles. He says:

> The Universe is only the collection of a certain kind of compossibles; and the actual Universe is the collection of all existant possibles, i.e., of that which form the richest compound. And as there are different combinations of possibles, some better than others, there are many possible Universes, each collection of compossibles making one of them. (R p. 223)

Mates tries to incorporate the Leibnizian idea that possible worlds are simply collections of compossibles in his semantics. Mates says, "Individual concepts are said to be compossible if they are capable of joint realization," (Ml p. 511). Mates later continues:

> One sees, therefore, that the relation of compossibility between individual concepts, unlike that of consistency between sentences or propositions, is transitive; since it is also reflexive and symmetrical it is an equivalence relation. As noted above, the possible worlds are 'maximal' or 'closed' with respect to this relation; so they are just the equivalence classes into which the relation of compossibility partitions the entire class of complete individual concepts. Thus, each such concept belongs to one and only one possible world, and two concepts are compossible if and only if they belong to the same possible world. (Ml pp. 511-512)

Mates appears to be on the right track, but there are some difficulties with his treatment of compossibility. According to Mates, compossibility is a two-place relation among concepts. X is compossible with Y just in case X and Y can be realized together. Mates says the compossibility relation is reflexive and symmetrical, which it clearly is, and also transitive. Initially, it is difficult to see why the relation would be transitive. Surely it could be the case that X and Y can be realized together and Y and Z be realized together, and X and Z can not be realized together. Suppose X and Z both contain the property of being the only man seven feet tall and they differ with respect to some other property. Further suppose that Y contains the property of being six feet tall as well as others. There is no inconsistency in X and Y being realized together, nor in Y and Z being realized together, yet it is clear that X and Z cannot be realized together. Since counter-examples of this type seem so obvious, why does Mates believe compossibility to be transitive and hence an equivalence relation? In a later paper, "Individuals and Modality in the Philosophy of Leibniz", Mates considers this point and says:

> It is blocked by the Leibnizian doctrine that in the actual world and in every other possible world, each concept 'mirrors' or 'expresses' all the other individual concepts in that world. Each individual of the actual world is related to all the others, and every relation is 'grounded' in simple attributes of the things related; the same is true of the other possible worlds as well. (M2 p. 91)

Mates is correct in believing that Leibniz holds that every concept mirrors or expresses the world of which it is a member. Leibniz often says things like, "Now every individual substance of this universe expresses in its concept the universe into which it has entered," (OC p. 109). But this relation of mirroring between concepts is obscure.

Mates believes that the mirroring relation will yield the result that compossibility is an equivalence relation. The idea is that if concept A mirrors concept B, then for any property P that is contained in B it can be shown from A that P is contained in B. For example, in the case suggested above Y might contain the property of being six feet tall and X is the only man seven feet tall. Y would no longer be compossible with Z, since Y cannot be realized without X's being realized, and X and Z are not compossible. Given this idea of mirroring, Mates is right that compossibility will be an equivalence relation which partitions the set of complete concepts into equivalence classes. But at this point we are faced with another difficulty. It is not clear that we should identify possible worlds with these equivalence classes, which is what Mates wants to do.

The problem with this is that while the members of a given equivalence class are pair-wise compossible, the set itself may not be possible (i.e., not all the members can be realized together). Consider the following case: Suppose X contains, among other things, being the only man at place P at time t, Y contains being the only man at place Q at time t when X is the only man at place P at time t, and Z contains being the only man at place Q at time t, where X, Y and Z are distinct concepts. X is compossible with Y and Y is compossible with Z and Z is compossible with X, yet X, Y and Z cannot be realized together. One might say that the possible worlds are just certain subsets of the equivalence classes, but then there is a problem about worlds being "maximal" in the appropriate sense. If worlds are not maximal, then it seems clear that a concept can be a member of two distinct worlds. It is more likely that Mates would say that mirroring handles this problem, and X, Y and Z cannot really be pair-wise compossible. Whether or not the relation of mirroring can do this depends in part on how one defines mirroring, which Mates does not do. However, for present purposes we will assume one can define mirroring in the appropriate way and go on to consider Mates' account of necessary truth.

In order to understand Mates' account of necessary truth we must again briefly look at his account of truth for atomic sentences. As pointed out earlier, a sentence is true relative to a given possible world. The sentence, "Adam has blond hair," is true of a world (say) W_i just in case the concept of blond hair is included in the concept of Adam and the concept of Adam is a member of W_i. The sentence is necessarily true just in case it is true of all possible worlds. It can be easily seen that on Mates' account of Leibniz it is not the case that all true sentences are necessary. Consider the sentence, "Adam has blond hair," and suppose Adam really did have blond hair and he really did exist. The sentence, "Adam has blond hair," is true of this world since the concept of Adam is a member of this world and has the concept of blond hair as a member. However, it will not be true of all possible worlds because the concept of Adam will not be a member of all possible worlds. The reason the concept of Adam is not a member of all possible worlds is that possible worlds are simply the equivalence classes which are partitioned off the set of all concepts by the relation of compossibility. Thus Adam's concept is only a member of one of those worlds, namely this one. Thus if the sentence, "Adam has blond hair," is true of this world, it must be false of all other possible worlds. If the sentence is true of one world and false of all others, it is contingent. This does not mean that all sentences in which the concept of the subject is not a member of a given world are false of that world. Complex sentences can be true of a world even though the concept of the subject is not a member of that world.

Consider the sentence, "Either Caesar crossed the Rubicon or Caesar did not cross the Rubicon." This sentence will be true of all possible worlds (and hence a necessary truth) even though the concept of Caesar is a member of only one world. To see this, consider the world (presumably this one) which has as a member the concept of Caesar. In this world the sentence will be true because Caesar did in fact cross the Rubicon, thus one of the disjuncts is satisfied. The concept of Caesar is not a member of any other world, and thus the other disjunct will be satisfied in those worlds, since in order to satisfy it, it only has to be the case that the concept of Caesar not be a member of the world. Thus the sentence in question does express a necessary truth. However, it should be noted that only complex sentences can express necessary truths; all true atomic sentences are contingent.

Basically, the reason it turns out that all true atomic sentences are contingent is that for an atomic sentence to be true of a world the concept of the subject must be a member of the world in question. Mates treats truth this way in his semantics because of a view he attributes to Leibniz about non-referring names. In his first paper, Mates presents the Leibnizian principle, "Nothing has no properties," and says, "The point is rather that Leibniz's advocacy of this principle amounts in practice to a decision to regard as false every atomic sentence that contains a nondenoting name," (M1 p. 514). However, it is far from clear that Leibniz actually held this view, and the evidence Mates presents to support it is inconclusive.

Mates bases his view that Leibniz believed that all sentences with nondenoting names are false, in part, upon the following passage in Leibniz:

> Hoc autem praesupponit negari omnem propositionem, quam ingreditur terminus qui non est res. Ut scilicet maneat omnem propositionem vel veram vel falsam esse, falsam autem omnem esse cui deest constantia subjecti, seu terminus realis. (C p. 393)

Mates presents the following translation of the above:

> This however presupposes denying every proposition in which there is a term that does not exist. In order, namely, to keep (the principle) that every proposition is true or false, (I consider) as false every proposition that lacks an existent subject or real term. (M2 p. 93)

Parkinson, in his book Leibniz: Logical Papers, translates the exact same passage somewhat differently. His translation reads:

> But this presupposes that every proposition which has as an ingredient a term which is not a thing is denied. So it remains that every proposition is either true or false, but that every proposition which lacks a consistent subject, i.e. a real term, is false. (P2 p. 82)

The differences between the translations of Mates and Parkinson are important ones. In particular, Mates translates "constantia subjecti" as "existent subject" while Parkinson translates it as "consistent subject". This difference in translation is highly significant. If Parkinson is correct, then it seems that Leibniz is going to consider false every atomic sentence in which there is an inconsistent term.[6] By "inconsistent term" we mean a term which has associated with it a concept that is not consistent.[7] Thus, for example, the sentence, "The round square is round," would be false. Yet this does not say anything about terms which do not refer in the real world but do refer in some other possible world.[8] If, on the other hand, Mates is correct in his translation, and we assume that by "existent subject" Leibniz means a subject such that its referent exists in the real world, then Leibniz is saying he will consider false every atomic sentence in which there is a term which does not refer in this world. Mates generalizes this idea and says that if the concept of the subject of an atomic sentence is not a member of the world at which the sentence is being evaluated, then the sentence is false. But is Mates correct in his interpretation of Leibniz, or is Parkinson closer to what Leibniz meant?

The key to understanding Leibniz here seems to be when Leibniz indicates that "constantia subjecti" means the same as "terminus realis".[9] It seems that for Leibniz "real term" means nothing more than "possible term", that is, a term which has a consistent concept. If in fact Leibniz does mean this, then Parkinson's interpretation is the correct one. Moreover, there is other evidence to indicate that Leibniz did not want to say that all atomic sentences without an existent subject are false. In discussing his view of logic as opposed to the view of The Scholastics, he says, "However, I have preferred to consider universal concepts, i.e., ideas, and their combinations, as they do not depend on the existence of individuals," (P2 p. 20).[10] This is part of the reason why Leibniz wanted an intensional account of truth rather than an extensional one. Mates considers Leibniz's holding the principle, "Nothing has no properties," as evidence that Leibniz wanted to consider, "false every atomic sentence that contains a non-denoting name," (M1 p. 514). But given what Leibniz says about nothing, it appears that there is more evidence for Parkinson's position. Leibniz says, "Nihil est quicquid nominari potest, cogitari non potest, nomen sine re, sine mente sonus."[11] That is, "Nothing is that which can be named but can not be conceived; a name without a thing, a sound without a meaning." This suggests that names which have associated with them a concept which can not be realized (i.e., an inconsistent concept) denote nothing. Atomic sentences containing such names will be false. One intuitive way of understanding the relationship between names such as "Adam" and the complete concept of Adam is that the concept is the meaning of the name or the intension of the name. The referent, denotation, or extension of the name is the man Adam. Names such as "Pegasus" which do not refer or have an extension in this world are still considered real terms because they have a consistent intension or meaning. Terms such as "the round square" are not real terms, since the meaning of the term is inconsistent. It seems reasonably clear that when Leibniz speaks of nothing as a name without a thing, the names he has in mind are like "the round square". Thus, when Leibniz says he is going to consider false every proposition which lacks a constantia subjecti, we should understand him as saying that atomic sentences containing inconsistent terms are false.

One of the results of Mates' misinterpretation of Leibniz is the second conjunct in his truth conditions for atomic sentences. By itself, this is not a very powerful objection to Mates' system. However, it leads to certain results in Mates' system which Leibniz would find unacceptable.

There are at least two results of Mates' system that it seems Leibniz would disagree with, and one result of possible disagreement.

They are, (a) the fact that in Mates $(Ex)\Box Fx$ is always false, (b) $\Diamond Fa \rightarrow \Box[(Ex)(x=a) \rightarrow Fa]$ is a necessary truth, and (c) $\Diamond(a \neq a)$ is a necessary truth.[12] It is not clear whether Leibniz would disagree with Mates about the fact that $(Ex)\Box Fx$ is always false. There are places in Leibniz which suggest that he held that certain properties of things are possessed of necessity. For example, he says, "I think that there is something essential to individuals and more than you suppose. It is essential to substances to act, to created substances to suffer, to minds to think, to bodies to have extension and motion."[13] But while he says that there are certain things essential to individuals, it is unclear whether he means particular individuals or individuals in general. The context suggests that he is referring to particulars, but it is inconclusive. When discussing the problem of contingency with Arnauld, he says:

> The other reply is that the sequence, in virtue of which events follow from the hypothesis, is indeed always certain, but that it is not always necessary by a metaphysical necessity, as is that instance which is founded in M. Arnaud's example: that God, resolving to create me, could not avoid creating a nature capable of thought. The sequence is often only physical and presupposes certain free decrees of God. . . . (OC pp. 104-105)

This passage suggests that Leibniz believes he has the property of being capable of thought necessarily. Leibniz says that from the hypothesis, which is that God will create a certain Adam, and hence the world, all the events which follow are certain, but not all are necessary. Some are contingent, and some are necessary, such as Leibniz's being capable of thought. The interesting point here is that Leibniz wants to distinguish between his having the property of being capable of thought and his having other properties, such as the property of having black hair. We suggest that Leibniz believes that it is metaphysically necessary that God, resolving to create Leibniz, could not avoid Leibniz's being capable of thought, yet it is not metaphysically necessary that God, resolving to create Leibniz, could not avoid Leibniz's having black hair. How it is possible that Leibniz could consistently hold this view is discussed in detail in the next chapter, but if he does, then (a) would seem to be something Leibniz would reject.

This objection to Mates is based on one way of reading $(Ex)\Box Fx$, namely, where the sentence is understood as saying there is something in this world which is such that it has a certain property of necessity. One could understand $(Ex)\Box Fx$ via Mates as saying there is something in this world such that in every world it exists and has a certain property. But if we read $(Ex)\Box Fx$ in the latter way, then how will we translate the sentence, "There is something which is such that it has property F of necessity"? Surely this sentence should be translatable into the formal language of a given system. Unless we understand $(Ex)\Box Fx$ as its translation, there appears to be no other way to translate it.

We can show that (b) is true by assuming $\Diamond Fa$. Thus, by assumption Fa is true of some world (say) W_1. Thus $f(F) \varepsilon f(a)$ and $f(a) \varepsilon W_1$. Also $(W)(W \neq W_1 \rightarrow f(a) \notin W)$, since every complete individual concept is in only one world. This is because a world is an equivalence class of complete individual concepts partitioned off the set of all complete individual concepts by the relation of compossibility. $\Box[(Ex)(x=a) \rightarrow Fa]$ says that $(Ex)\ x=a \rightarrow Fa$ is true of every world. $(Ex)(x=a) \rightarrow Fa$ is true of a world just in case either $(Ex)(x=a)$ is not true of that world or

Fa is true. (Ex)(x=a) is not true of a world just in case there is no constant b, such that f(b) is in that world and b=a is true of that world. The only worlds of which b=a is true are those worlds such that f(a) is a member of them. But as we have shown f(a) is a member of only one world, hence for all worlds other than W_1, $(Ex)(x=a) \rightarrow Fa$ is true of them. In W_1, Fa is true, hence $(Ex)(x=a) \rightarrow Fa$ is true of every world. Thus, $\Box[(Ex)(x=a) \rightarrow Fa]$ and finally, $\Diamond Fa \rightarrow \Box[(Ex)(x=a) \rightarrow Fa]$.

(c) also is a necessary truth in Mates' system. Assume there is some W in which (c) is false, i.e., assume $\Box(a=a)$ (for some constant a) is true of W. If $\Box(a=a)$ is true of W, then for every W′, (a=a) is true of W′. In order for a=a to be true of any world, f(a) must be a member of that world. Since f(a) is a member of only one world, there will be some world such that f(a) is not a member of it. Thus, it is false that for every W′, (a=a) is true of W′. The reason I point out that (b) and (c) are necessary truths in Mates' system is because they indicate a difficulty in accepting Mates' system as an appropriate Leibnizian semantics. The difficulty is one of translation and interpretation.

Consider the following sentence:

(1) If it is possible that Adam has black hair, then necessarily if Adam exists, he has black hair.

It seems that Leibniz would want to deny (1) since from the claim that it is possible that Adam has black hair and Adam exists, it follows that Adam in fact has black hair (I am assuming that Adam has blond hair). Leibniz would clearly agree that it is possible that Adam has black hair and certainly wants to claim Adam exists (in the timeless sense), yet he would disagree that Adam in fact had black hair. Making the appropriate assumptions about the predicate letters and constant letters in (b), then it would appear that (b) is the translation into the formal language of (1). One would think that the semantics Mates presents would reflect Leibniz's unacceptance of (1). However, what Mates' semantics reflects is that (b) interpreted in his system is something Leibniz would accept. (b) interpreted in Mates' system says, if the concept of Adam is a member of some world and that concept contains the property of having black hair, then in every world in which the concept of Adam is a member (there is only one such world), the concept contains the property of having black hair. Leibniz would agree with this statement. It should also be pointed out that we cannot conclude that Adam has black hair in Mates' system given our assumptions about Adam (namely that he exists and has blond hair), since $\Diamond$Fa is false in the system. Mates might deny that (b) is the translation of (1), but if he does it becomes unclear whether he can translate (1) at all. At least it is unclear what the translation would be if not (b). A more likely response from Mates would be that this is not a difficulty, since Leibniz would not deny (1). Mates would hold that Leibniz would accept the claim that if Adam does not have black hair, then it is not possible that he have black hair. Mates would base this view on some of the things Leibniz has to say about complete concepts.

For example, Leibniz says:

> . . . if, in the life of any person, and even in the whole universe, anything went differently from what it has, nothing would prevent us from saying

> that it was another person or another possible universe which God had chosen. It would then indeed be another individual. (M2 p. 105)

Mates would take the above quote as evidence for the claim that Leibniz held that if Adam lacked black hair, it is not possible that he have black hair. Yet at many places Leibniz points out that just because Adam has a certain property in his concept, it does not follow that Adam has that property of necessity.[14] However, in order to show conclusively that (b) is a difficulty for a Leibnizian semantics, one must point out a way of understanding Leibniz's views of complete concepts, compossibility, and possible worlds, which makes the above quote consistent with (1).

A way of understanding Leibniz which allows for this possibility is suggested in the next two chapters. The problem with (c) is similar to the problem with (b), although (c)'s being a necessary truth seems to constitute a stronger objection to Mates than (b)'s being a necessary truth.

If we let <u>a</u> in (c) stand for "Adam", then it appears that (c) is the translation of:

(2) It is possible that Adam not be Adam.

Even if Leibniz would not deny (1), it seems he would deny (2). At one time Leibniz tells us that all identical propositions are necessary.[15] He then later adds a condition for the truth of identical propositions. He says:

> As it is agreed that identical propositions themselves can be trusted only in the case of real concepts, so that no truth can be asserted without fear of the opposite except concerning the reality

> of those concepts themselves--at any rate their essential reality, though not their existential reality . . . (P2 p. 82)

Leibniz is saying that identical propositions can be trusted only in the cases where the concepts involved are consistent or possible.[16] But if the concepts involved are "real" concepts, then the identical proposition will be necessary. As mentioned in the discussion of Parkinson,[17] identical propositions for Leibniz are not just propositions expressed by identity sentences. However it is clear that the proposition that Adam is Adam is included among the propositions that Leibniz calls "identical". Thus, it is reasonably clear that Leibniz would say that it is necessary that Adam is Adam, which contradicts (2). It seems unlikely that Mates would claim (c) is not the translation of (2). A more likely response would be for him to deny that (2) is false.

Mates believes that just as all atomic sentences containing a non-referring expression are false, so are all identity sentences containing a non-referring expression. He holds that Leibniz believes identity sentences have "existential import". That is, if an identity sentence is true, then the terms in the identity sentence denote an object. Mates supports this view, in part, by pointing out that Leibniz says, "Thus, if I say of an existing thing, 'A is B', it is the same as if I were to say 'AB is an existent'; e.g., 'Peter is a denier', i.e., 'Peter denying is an existent',"(P2 p. 65).[18] But it is not clear that Leibniz here means what Mates is implying. Leibniz is only discussing sentences in which the subject term refers to some object. This passage does not tell us what Leibniz thought about sentences such as, "Pegasus is Pegasus". The problem that Leibniz is considering is how to handle the problem of existence given his definition of truth. Thus he says, immediately following what is quoted above, "The question here is how one is to proceed in analysing this; i.e. whether the term 'Peter denying' involves existence, or whether 'Peter existent' involves denial--or whether 'Peter' involves both existence and denial, as if you were to say, 'Peter is an actual denier', i.e. is an existent denier; which is certainly true," (P2 p. 65). In the end Leibniz says 'Peter' involves both existence and denial, but what we are interested in is that Leibniz is not claiming that all sentences (and in particular identity sentences) are false unless they have a term which refers to an object.

There is additional evidence to support the belief that Leibniz thinks sentences of the form "a is a" are always true. He says:

> (154) But if someone prefers signs to be used in such a way that AB=AB, whether AB is a thing or not, and that in the case in which AB is not a thing, B and not-B can coincide--namely, per impossible--I do not object. This will have as a consequence the need to distinguish between a term and a thing or entity.
> (155) All things considered, then, it will perhaps be better for us to say that, in symbols at least, we can always put A=A, though nothing is usefully concluded from this when A is not a thing.
> (P2 p. 82)

When Leibniz says in (155) 'A is not a thing' it is clear from what he says in (154) that he means A is impossible. So, what Leibniz is suggesting is that even if the term A has associated with it an inconsistent concept we can still say "A=A". If the term A has associated with it a consistent concept, then the sentence will be true. Almost everything

Leibniz says between (151) and (156) (P2 pp. 81-82) suggests that he holds that "A=A" is true when the concept associated with A is consistent. Mates himself seems to agree to a certain extent when he allows that "Pegasus is Pegasus" is true in some world even though "Pegasus" does not denote any object in any world on his view. But Mates' view seems too restrictive in its account of truth, and sentences like (c) become necessary truths.

Perhaps one should refrain from making any final judgements about Mates' system until the merits of an alternative account can be compared with it. Mates does incorporate many of Leibniz's views in his system, and he does suggest a way to avoid the problem of contingency. In the next chapter we will take a close look at Leibniz's views on these matters and compare our interpretation of Leibniz with that of Parkinson, Rescher and Mates.

NOTES FOR CHAPTER III

1. See M2 pp. 98-99.

2. M1 pp. 524-525.

3. For any formula ψ, variable α, constant b, $\psi\alpha/b$ is the result of replacing all occurrences of α in ψ by occurrences of b.

4. OC p. 80.

5. NE p. 516.

6. I say atomic sentence because we do not want Fa and $\sim$Fa to both be false.

7. A concept is consistent provided that it is a subset of some complete individual concept.

8. A term α refers in a world W just in case the concept associated with α is a member of W.

9. Leibniz also says, "Is every universal negative, then impossible? It seems that it is because it is understood of concepts, and not of existing things; thus if I say that no man is an animal, I do not understand this of existing men alone," (P2 p. 76).

10. For another discussion of what "constantia subjecti" means in this passage see Ishiguro, Hidé, Leibniz's Philosophy of Logic and Language, Cornell University Press, Ithaca, 1972, pp. 128-130. My conclusions on this subject are the same as Ishiguro's.

11. Leibniz, G.W., Samtliche Schriften und Briefe, herausgegeben von der Preussischen Akademie der Wissenschaften zu Berlin, 1923, Volume II, Series 6, p. 487.

12. The symbols 'F' and 'a' stand for any predicate letter and any constant letter in Mates' system respectively. Because there will be no ambiguity between use and mention in discussing these formulas, I have left off the quotation marks which would usually accompany them.

13. NE p. 331.

14. OC pp. 19-20, 125-126.

15. P2 p. 77.

16. For what Leibniz means by "essential reality" as opposed to "existential reality", see P2 pp. 80-81.

17. See p. 29.

18. M2 pp. 94-95.

C H A P T E R I V

In the discussion of the various views that certain philosophers attributed to Leibniz and in the discussion about God, various aspects of Leibniz's position have been mentioned. In this section will be an attempt to present a coherent interpretation of Leibniz which takes into account most of the Leibnizian doctrines discussed. We will try to present as clearly as possible Leibniz's account of necessity and contingency, by considering what Leibniz says in various places and presenting it in a consistent way. Since Leibniz has never written a single major work on the topic, the closest thing coming to that being the Theodicy, we must consider what Leibniz says in his correspondence and in various articles. The two major sources for the view suggested for Leibniz are the correspondence with Arnauld and the Theodicy. These are not the only sources, but they are the major ones being considered. In presenting this view there will be some repetition of material presented in previous chapters, but this does seem necessary to present a complete picture of Leibniz on this topic.

One of the basic views of Leibniz, and one he affirms often, is his definition of truth. Moreover, it is in part his definition of truth which leads one to believe there is no contingency in Leibniz. For, Leibniz says that a proposition is true just in case the concept of the predicate is included in the concept of the subject. In effect, Leibniz holds that every true proposition is analytic.[1] Herein lies the difficulty: if all true propositions are analytic, then surely it follows that all true propositions are necessary, since all analytic propositions are necessary. Yet Leibniz denies that all true propositions are necessary. On the face of it, this position seems inconsistent, yet a close examination of Leibniz's views will reveal there is no inconsistency.

Truth is defined in terms of inclusion; the concept of the predicate being included in the concept of the subject. As mentioned in the preceeding chapter, for Leibniz the concept of the subject can be thought of as a set of properties.[2] If the subject is an individual thing, then the concept is complete. Possible worlds are sets of complete concepts as described in the preceeding section.[3] Leibniz introduces "possible worlds" and the like to explain creation, in part, but they also help in understanding necessity.

In responding to a charge by Arnauld, Leibniz says:

> If what I said be thought over a little it will be found to be evident _ex terminis_: for by the individual concept, Adam, I mean of course a perfect representation of a particular Adam who has certain individual characteristics and is thus distinguished from an infinity of possible persons very similar to him yet for all that different from him (as ellipses always differ from the circle, however closely they may approach it). God has preferred him to these others because it has pleased God to choose precisely such an arrangement of the universe, and everything which is a consequence of this resolution is necessary only by hypothetical necessity and by no means destroys the freedom of God nor that of the created spirits. There is a possible Adam whose posterity is of a certain sort, and an infinity of other possible Adams whose posterity would be otherwise; now is it not true that these possible Adams (if we may speak of them thus) differ among themselves and that God chose only one who is precisely ours? (OC p. 80)

If we ignore for the moment the difficulties presented in Chapter I in connection with God, it seems that Leibniz is saying that what follows from God's decision to create this world is not necessary (in the metaphysical sense). In particular, it is not the case that all true statements about Adam are necessary since there are other "possible Adams" very similar to the real Adam which God might have chosen to realize instead of the real Adam. The idea is that Adam does not have (say) blond hair of necessity because there is a possible Adam which is very similar to the real Adam, and this possible Adam lacks blond hair. This appears to be what Leibniz means when he says, "an infinity of other possible Adams whose posterity would be otherwise," (OC p. 80). Leibniz does indicate that a sentence such as, "Adam has blond hair", is hypothetically necessary. By this Leibniz means that it is necessary (in the metaphysical sense) that if God creates Adam, then Adam has blond hair. But this is hardly surprising, since included in the concept of the real Adam is the concept of blond hair. Given that God is going to realize the concept of the real Adam, the real Adam must have blond hair since it is in his concept. Thus, the reason it is contingent that Adam has blond hair is that there are these "possible Adams" which are similar to the real Adam yet lack blond hair. But does it even make sense to talk about "possible Adams"?

In the preceding section it was argued for that Leibniz's "possible individuals" were unrealized complete individual concepts in other possible worlds.[4] When such terms as "possible persons", or "possible Adams" are used, no more is meant than unrealized complete individual concepts in other possible worlds (except in the case of the "possible Adam" or "possible person" which is in fact actual). Arnauld objects to Leibniz's position that there are an "infinity of other possible

Adams" on the grounds that such a view in conjunction with Leibniz's view of complete individual concepts is inconsistent. Arnauld says:

> Moreover, Monsieur, I do not see how, in taking Adam as an example of a unitary nature, several possible Adams can be thought of. It is as though I should conceive of several possible me's; a thing which is certainly inconceivable. For I am not able to think of myself without considering myself as a unitary nature, a nature so completely distinguished from every other existent or possible being that I am as little able to conceive of several me's as to think of a circle all of whose diameters are not equal. The reason is that these various me's are different, one from the other, else there would not be several of them. There would have to be, therefore, one of these me's which would not be me, an evident contradiction. (OC p. 94)

Arnauld continues and says:

> Is it not clear that . . . since my present me is necessarily of a certain individual nature, which is the same thing as having a certain indivicual concept, it will be as impossible to conceive of contradictory predicates in the individual concept me, as to conceive of a me different from me? (OC pp. 94-95)

Transferring what Arnauld says about himself here to Adam, he seems to be saying something like the following: there is a unique complete concept of Adam. If there are several possible Adams, there must be at least two complete concepts of Adam which are distinct. But, since the complete concept of Adam is distinct from all other concepts, there can not be two complete concepts of Adam. Thus it is not the case that there are several possible Adams. Leibniz responds to this objection by saying:

> . . . in speaking of several Adams I do not take Adam for a determined individual but for a certain person conceived <u>sub ratione generalitatis</u> under the circumstances which appear to us to determine Adam as an individual but which do not actually

> determine him sufficiently. As if we should mean by Adam the first man, whom God set in a garden of pleasure whence he went out because of sin, and from whose side God fashioned a woman. All this would not sufficiently determine him and there might have been several Adams separately possible or several individuals to whom that would apply. This is true, whatever finite number of predicates incapable of determining all the rest might be taken, but that which determines a certain Adam ought to involve absolutely all his predicates, and it is this complete concept which determines the particular individual. (OC pp. 128-129)

Leibniz wants to label possible persons who are very similar to Adam as possible Adams, but he does not want to claim that they are in any sense the same Adam as the real Adam. This becomes clear when Leibniz says, "as if we should mean by Adam the first man, whom God set in a garden of pleasure . . . there might have been several Adams separately possible or several individuals to whom that would apply," (OC p. 129). Thus Leibniz is agreeing with Arnauld that there is only one complete concept of Adam. But there are other complete concepts which are unrealized and which are very similar to the concept of Adam in that they contain a number of the properties that the concept of Adam contains. They do not contain all and only those properties that the concept of Adam contains, otherwise we would be talking about a single concept rather than many concepts. Leibniz refers to these concepts which are similar to, but not identical with the concept of Adam when he talks about "possible Adams".

Some contemporary terminology will now be introduced to avoid continually using the phrase "possible persons very similar to Adam". Hereafter these possible persons will be referred to as "counterparts" of Adam. The use of such terminology is not completely unwarranted, given what Leibniz's position seems to be. One must keep in mind, however, that "counterparts" as used here does not mean exactly the same thing as used in contemporary philosophy. The expression "counterparts of Adam" will be used to refer to those unrealized concepts which are very similar to the concept of Adam; the ones to which it appears Leibniz was referring. Thus the counterpart relation in the Leibnizian sense is a relation between concepts, not individuals. A more standard usage would have the relation between individuals. It seems that Leibniz, through the use of counterparts, can allow for contingency. Before a more detailed account of the use of counterparts by Leibniz in allowing for contingency is given, however, we will consider a more complete picture of the problem facing Leibniz. We will then suggest a way that counterparts can solve the difficulties.

As suggested in the beginning of this section, while Leibniz claimed there are true contingent propositions it is not clear that he can consistently hold that view given his definition of truth. According to Leibniz, all true propositions are analytic. That is, the concept of the subject contains the concept of the predicate in any true proposition. The sentence, "Adam has blond hair", expresses a true proposition provided that the concept associated with the name "Adam" (in some sense the "meaning" of the name Adam" includes the concept associated with the predicate "has blond hair". The proposition that Adam has blond hair is analytic because its truth depends solely on the concepts involved in the proposition.

Since all analytic propositions are necessary, it seems to follow on Leibniz's view that all true propositions are necessary. In order to consistently hold that there are true contingent propositions, Leibniz must either give up his definition of truth, claim that it does not follow from his definition of truth that all true propositions are analytic, or claim that it is not the case that all analytic propositions are necessary. Leibniz clearly did not want to give up his definition of truth, and given that definition of truth there seems no way for him to deny that all true propositions are analytic. But if Leibniz can consistently deny that all analytic propositions are necessary, then he can consistently hold that there are true contingent propositions. However, it is unclear whether Leibniz can consistently deny that all analytic propositions are necessary. Since complete individual concepts are sets of properties, they are defined in extension, or by their members. If one adds a member to a set, then one would have a different set, and if one takes a member away from a set, then one would have a different set. Because sets are defined in extension, they necessarily have the members they have. Since in order for a true proposition to be contingent it must be possible that the proposition not be true, it seems as if there must be a case such that the concept of the predicate is contained in the concept of the subject, yet it is possible that the concept of the predicate not be contained in the concept of the subject. But it is not clear that this is possible, since any particular concept can not change its members. It would be possible if there were more than one concept associated with the subject of a sentence, but Leibniz clearly indicates that there may only be one. Leibniz allows that where the subject of the sentence refers to a substance, we can discuss subsets of the concept of the substance (i.e., conceive of it sub ratione generalitatis[5]). There is only one complete concept of the substance, however, and that is the concept which is associated with the subject of a sentence which is about the substance. Thus, contrary to what Leibniz says, it appears that there are no true contingent propositions.

Leibniz recognizes the difficulty in his system yet still believes that he can allow for contingency. In his paper "On Freedom" he says:

> I found myself very close to the opinions of those who hold everything to be absolutely necessary; believing that when things are not subject to coercion, even though they are to necessity, there is freedom, and not distinguishing between the infallible, or what is known with certainty to be true, and the necessary.
>
> But I was pulled back from this precipice by considering those possible things which neither are nor will be nor have been. For, if certain possible things never exist, existing things cannot always be necessary; otherwise it would be impossible for other things to exist in their place, and whatever never exists would therefore be impossible. For it cannot be denied that many stories, especially those we call novels, may be regarded as possible, even if they do not actually take place in this particular sequence of the universe which God has chosen. (L pp. 404-405)

Here Leibniz suggests that there is contingency because there are possible things which could have existed in the place of the things which actually exist. This becomes even clearer when he says:

> Thus it is obvious that God elects from an infinity of possible individuals those whom he judges best suited to the supreme and secret ends of his wisdom. In an exact sense, he does not decree that Peter should sin or Judas be damned but only that, in preference to other possible individuals, Peter, who will sin--certainly indeed, yet not necessarily but freely--and Judas, who will suffer damnation--under the same condition--shall come into existence, or that the possible concept shall become actual.
> (L p. 414)

Again Leibniz is saying that there is contingency because God could have chosen to realize complete concepts different from the ones which in fact he chose to realize. Thus while the concept of Peter does contain the property of sinning, as well as all the other properties that Peter has, God might have realized a different concept in place of the concept of Peter. Here Leibniz is unclear about the relationship between the concept of Peter and these other "possible concepts" (i.e., concepts that could have been realized), but in his discussion of Adam quoted earlier,[6] Leibniz says it is one of similarity. He tells us that God chose Adam from among possible persons who are "very similar" to Adam. But the most it seems we can conclude from these passages is that the existence of Adam, Peter, and Judas is contingent, and not that the proposition that Peter will sin is contingent. Yet Leibniz wants to say that the proposition that Peter will sin is contingent as indicated when he says, "Peter, who will sin--certainly indeed, yet not necessarily but freely." Even if Peter's existence is contingent, how is it that Peter's sinning is contingent, since the property of sinning is included in the concept of Peter? The answer to this question seems to be contained in what Leibniz says at the end of the Theodicy, his major work on freedom.

After discussing various objections to freedom and contingency for God and individuals, Leibniz decides to present a dialogue. He says about it:

> I thought it would be opportune to quote it in abstract, retaining the dialogue form, and then to continue from where it ends, keeping up the fiction it initiated; and that less with the purpose of enlivening the subject, than in order to

> explain myself towards the end of my dissertation as clearly as I can, and in a way most likely to be generally understood. (T p. 365)

Leibniz continues with the story, which deals with the fate of Sextus, in the following way:

> Jupiter who loves you (she said to him) has commended you to me to be instructed. You see here the palace of the fates, where I keep watch and ward. Here are representations not only of that which happens but also of all that which is possible. Jupiter, having surveyed them before the beginning of the existing world, classified the possibilities into worlds, and chose the best of all . . . I have only to speak, and we shall see a whole world that my father might have produced . . . one may know also what would happen if any particular possibility should attain unto existence . . . you can picture to yourself an ordered succession of worlds, which shall contain each and every one the case that is in question, and shall vary its circumstances and its consequences. But if you put a case that differs from the actual world only in one single definite thing and in its results, a certain one of those determinate worlds will answer you. These worlds are all here, that is, in ideas. I will show you some, wherein shall be found, not absolutely the same Sextus as you have seen (that is not possible, he carries with him always that which he shall be) but several Sextuses resembling him, possessing all that you know already of the true Sextus, but not all that is already in him imperceptibly, nor in consequence all that shall yet happen to him. You will find in one world a very happy and noble Sextus, in another a Sextus content with a mediocre state, a Sextus, indeed, of every kind and endless diversity of forms. (T pp. 370-371)

Leibniz here suggests that it is possible that Sextus have a property he lacks, such as being noble, because there is another Sextus in another possible world which has the property of being noble. Leibniz points out that these various Sextuses in different possible worlds are not identical to the real Sextus but resemble him closely, just as he pointed out that God chose Adam from among possible persons very similar to him but distinct from him. Considering what Leibniz says here about Sextus and what he says elsewhere about Adam and Peter, the view he seems to present is that Adam does not have blond hair necessarily because among an infinity of complete concepts which closely resemble the concept of Adam there is one which lacks the property of blond hair and which could have been realized in place of the concept of Adam. In the case of Sextus, Leibniz says, "You will find in one world a very happy and noble Sextus, in another a Sextus content with a mediocre state, a Sextus, indeed, of every kind and endless diversity of forms," (T p. 371). We can expect to find in one world an Adam with blond hair, and in another world an Adam without blond hair. Since these various Adams are not identical with Adam, yet are very similar to him, it seems that Leibniz believes that they are counterparts of Adam. As pointed out earlier,[7] the counterpart relation for Leibniz is one which holds between concepts and not individuals, as there are only individuals in the real world. Thus we can say that a true proposition is contingent just in case there is one counterpart of the concept of the subject which contains the concept of the predicate and one that does not.

The advantage of this view for Leibniz is great. Leibniz can hold that there is a complete concept of Adam which contains all the properties that Adam possesses. Moreover, an atomic sentence about Adam will be true just in case the concept of Adam contains the concept of the predicate, and hence true propositions expressed by atomic sentences about Adam are analytic. Yet even though all such propositions are analytic, they are not all necessary. Thus, Leibniz can allow for contingency even though all true propositions expressed by atomic sentences are analytic. Leibniz never explicitly states that a sentence is possible because one concept has a property that a counterpart of it lacks. But given what he does say, this view seems strongly suggested.

If we accept this view for Leibniz, there remains a puzzle as to what Leibniz is doing when it appears he analyzes necessity and contingency not in terms of counterparts and possible worlds, but in terms of infinite and finite analysis. If we are to give a complete picture of Leibniz we must be able to give some account of infinite and finite analysis as they relate to necessity. Both Rescher and Parkinson claim that Leibniz introduces these notions, in part, to explain how analytic propositions can be contingent, and Leibniz sometimes speaks as if he is doing this.[8] But if he can allow for contingency through the use of counterparts and possible worlds, there seems little point in introducing further complexity in the notion of an infinite analysis. The key to the solution of the puzzle is that the problem for Leibniz, given his definition of truth, is really two-fold.

If every true proposition is analytic, then not only does it seem to follow that every true proposition is necessary, it also seems to follow that every true proposition is knowable _a priori_. A proposition expressed by a sentence is knowable _a priori_ just in case the truth of the proposition can be known by understanding the meanings of the terms in the sentence (i.e., the concepts associated with the terms) and the logical structure of the sentence. Consider the sentence, "Every man is an animal". We can know the truth of the proposition expressed by this sentence by knowing what 'man' means and knowing what 'animal' means, and by understanding the logical structure of the sentence. In this case the logical structure of the sentence is the form, "Every _ _ _ is _____." We understand this structure when we recognize the form and know the truth conditions for sentences of that form. We know that included in the concept of man is the concept of animal, and that the proposition expressed by the sentence asserts this. Thus we know *a priori* that every man is an animal. On Leibniz's view, *every* true proposition is such that the concept of the predicate is included in the concept of the subject. But if Leibniz is correct, then it seems that every true proposition is knowable *a priori*, since in order to know the truth of a proposition we need only know the concepts involved in the proposition. Consider the proposition expressed by the sentence, "Adam has blond hair." On an intuitive level this proposition does not seem to be knowable *a priori*. It would seem that we would need to know more than just the meanings of the terms in the sentence and the structure of the sentence in order to know the truth of the proposition. Perhaps we might see Adam and note the color of his hair, or we might obtain some authoritative documents indicating that he has blond hair. In any case it would appear that we need some additional evidence in order to know that Adam has blond hair. But on Leibniz's view it seems we know that Adam has blond hair just by understanding the meanings of the terms in the sentence, "Adam has blond hair", since the concept of Adam includes the concept of blond hair. Leibniz's response to this problem is that while all true propositions are in principle knowable *a priori*, we (i.e., human beings) will never know all true propositions *a priori* because we lack certain powers.

Leibniz says he holds two primary truths--the principle of contradiction, and: "The principle that nothing is without reason, or that every truth has its proof *a priori*, drawn from the meaning of the terms, although we have not always the power to attain this analysis," (OC p. 141). But if every truth is in principle knowable *a priori*, how is it that humans can not know all truths *a priori*? The answer to this question is found in the notion of an infinite analysis. Leibniz says:

> In contingent truths, however, though the predicate inheres in the subject, we can never demonstrate this, nor can the propositions ever be reduced to an equation or an identity, but the analysis proceeds to infinity, only God being able to see, not the end of analysis indeed, since there is no end, but the nexus of terms or the inclusion of the predicate in the subject, since he sees everything which is in the series. . .
>
> For us, however, there remain two ways of *knowing* contingent truths. The one is experience; the other, reason. We know by experience when we perceive a thing distinctly enough by our senses; by reason, however, when we use the general principle that nothing happens without a reason, or that the predicate always inheres in the subject by virtue of some reason. (L pp. 407-408; emphasis added)

Leibniz claims that while contingent truths are indeed analytic, we can not demonstrate their truth, since such a demonstration or analysis would have to be an infinite one, hence we can not know them *a priori*. God, on the other hand, while unable to complete the analysis (since it can not be completed) can none the less see that the concept of the predicate is included in the concept of the subject. We *know* contingent truths by experience, though reason does tell us the truth conditions for propositions in general.

It seems that Leibniz introduces the notion of an infinite analysis to explain why humans can not know all analytic propositions a priori. In fact, as we would expect, the truths which we can not know a priori are the contingent truths:

> And there is no truth of fact or of individual things which does not depend upon an infinite series of reasons, though God alone can see everything that is in this series. This is the cause, too, why only God knows the contingent truths a priori and sees their infallibility otherwise than by experience. (L p. 406)

Using the idea of an infinite analysis Leibniz attempts to explain why we do not know all analytic propositions a priori. The problem with this explanation is that Leibniz explains one puzzling fact by something which is even more puzzling, namely an infinite analysis. We have already considered the Rescher and Parkinson accounts of the nature of infinite analysis, but Leibniz himself gives some hints for understanding this notion which seem worth considering at this point.

In explaining the notion of an infinite analysis Leibniz often makes use of mathematical concepts. He says:

> But in proportions the analysis may sometimes be completed, so that we arrive at a common measure which is contained in both terms of the proportion an integral number of times, while sometimes the analysis can be continued into infinity, as when comparing a rational number with a surd; for instance, the side of a square with a diagonal. (L p. 407)

Leibniz wants to make some sort of analogy between the relation between rational numbers to irrational and the relation between infinite and finite analysis. Rational numbers can be expressed by a ratio between two integers, for example 15 can be expressed by 1/2, and .333... by 1/3.

But irrationals cannot be expressed by a ratio between two integers, and have a non-repeating decimal expansion, such as π which is 3.1415 and so on. Thus in some sense irrationals are an infinite series of integers, while rationals are not. Leibniz wants to say that just as π takes an infinite analysis (in some sense), so do truths not knowable *a priori*. However, while one can make some sense of the notion of infinite analysis in mathematics, it is difficult to see how that is to carry over into talk about propositions. An example of a finite analysis will be helpful in understanding the problem.

Suppose we are given that John is a brother and we want to know whether John is male. Leibniz tells us that analyses are carried on by substituting for terms their definitions. We know that "brother" means "male and a sibling". We therefore substitute "male and a sibling" for "brother" in our original sentence, and conclude "John is male and a sibling". From this we can conclude that "John is male", and we have shown in a finite analysis that from the fact that John is a brother it follows that John is male. So far it all makes good sense. However, when we try to apply the same idea to the notion of infinite analysis we encounter some problems. Of course we cannot give an example of an infinite analysis, but even the idea of one seems beyond conception. Consider a contingent truth, (say), "Adam has blond hair." We know that the concept of the predicate is included in the concept of the subject (if the sentence is true, and we are supposing it is), but in order to demonstrate the inclusion, an infinite analysis is required. It seems obvious that by substituting the definition of "blond hair" in the original sentence we get nowhere, thus it must be that we should substitute the definition of "Adam" in order to start the analysis. By the definition of "Adam" Leibniz means the complete concept of Adam, which, as discussed earlier, is to be thought of as a set of properties. Thus, what we are trying to demonstrate is that the property of having blond hair is a member of the set of properties which constitutes the complete concept of Adam. For simplicity, let us call the complete concept of Adam "A", and the property of having blond hair "b". What we are trying to show is whether b is a member of A. A has an infinite number of members, so we might say that in order to show that b is a member of A we would have to list all the members of A, which would be infinite. Thus, to demonstrate that Adam has blond hair requires an infinite analysis, in that it would require a list of all of A's members, which would be infinite. However, there is a slight problem in taking this to be what Leibniz means by infinite analysis, and that is even though it may be true that for any given property we can not decide if it is a member of A or not, if it is a member it will occur on the list which is infinite.

If A is a listable set, then one can construct a machine (say) M such that M will continuously create a list of outputs and for any x if x is a member of A, then x will be output at some time. The point is that even if A is an infinite set and thus we could not list all the members, any particular member of A will occur on the list at some time. It does not follow from this that for any given property we can decide whether it is a member or not, since at any given time if it has not appeared on the list we do not know that it will not appear on the list. If A is listable then it does not appear that it would take infinite analysis to show that b is a member of A. M will start listing the members of A and since b is a member of A, b will occur on the list at some time; when b occurs on the list, only a finite number of members will have occurred before it. It is as if we had a machine programmed to list all the natural numbers in order: the machine will never complete the list, yet for any particular number chosen, the machine will list it in a finite number of outputs. Given this view of concepts (i.e., that they are listable sets), all contingent truths expressible by atomic sentences are knowable a priori.

One might avoid this problem if one takes a different view of what it would be like to attempt to list the members of A. It was suggested that it would be like having a machine trying to list all the positive integers. But, one might suggest it is more like having a machine list in order all the reals between one and four inclusive. In this case, the list would amount to a single number, namely one. It would never be able to list any number after one, since between one and any number after one there are an infinite number of numbers. This way of viewing infinite analysis has a number of advantages. We can explain, in a sense, why it is that we could never demonstrate that b is a member of A. Doing that would be the same as the machine producing the first real number after one; obviously it cannot be done. The case of demonstrating necessary truths would be like the machine producing one on its list. But it has a major disadvantage in that it is hard to see how the relation among the members of Adam's complete concept could be anything like the relation among the reals. That is, it is hard to see how the complete concept of Adam could have the property of "betweenness" as the reals have the property of betweenness. Either way of viewing infinite analysis has its problems.

For our purposes it is not really necessary to solve the problem of defining an infinite analysis, though it certainly would be helpful in presenting a complete account of Leibniz. Even though we cannot present a clear account of infinite analysis, an interpretation of Leibniz explaining why he introduces both possible worlds and counterparts, and finite and infinite analysis, can be suggested. A proposition may be knowable _a priori_ (by us) for Leibniz just in case the truth of the proposition can be demonstrated by us in a finite analysis. A proposition is said to be necessary just in case all the counterparts of the concept of the subject (of a sentence which expresses the proposition) include the concept of the predicate. Using these two notions we can see how Leibniz might avoid the difficulties suggested earlier.

While it is true that every true proposition is analytic, i.e., the concept of the predicate is included in the concept of the subject, it does not follow that they are either all necessary or that we can know all of them _a priori_. Consider the sentence, "Adam has blond hair", and suppose it is true. If it is true, the concept of blond hair is included in the concept of Adam. But in order for it to be necessarily true we must further suppose that all the counterparts of the concept of Adam also include the concept of blond hair. Such a supposition is clearly unwarranted. Moreover, in order for us to claim that we can know _a priori_ that Adam has blond hair, we must suppose that we can show that the concept of blond hair is included in the concept of Adam in a finite analysis. Again, this is a supposition which Leibniz would claim is unwarranted. Thus, for Leibniz the fact that a proposition is analytic does not imply either that it is necessary or that we can know it *a priori*.

One of the difficulties in suggesting this account of necessity and contingency for Leibniz is its apparent lack of precision. Can we make this account more precise using contemporary logical techniques? In other words, can we present a semantics for a formal language as Mates does which avoids the difficulties of Mates' system, has the same good points as Mates' system, yet at the same time incorporates in a more precise way the account of necessity and contingency presented above? In the next chapter a semantics will be presented which meets all of these conditions.

NOTES FOR CHAPTER IV

1. OC pp. 117, 126, 132.

2. See p. 57.

3. See p. 58 f.

4. See pp. 57-58.

5. OC pp. 128-129. Quoted in part on pp. 81-82.

6. See p. 79.

7. See p. 83.

8. See, for example, P2 pp. 75, 63-64.

CHAPTER V

In the preceding chapter it was suggested that Leibniz analyzes truth, necessity and contingency in terms of complete individual concepts, possible worlds, and counterparts. Leibniz, of course, does not present a complete semantics in the sense that he does not provide us with a recursive definition of truth for all formulas of a given formal language. The task at hand is to present a complete semantics for predicate logic plus the modalities in question, which incorporates the account suggested for Leibniz in Chapter IV.

In a paper called "Counterpart Theory and Quantified Modal Logic", David Lewis has suggested a different approach to viewing modal logic, which he calls Counterpart Theory. Given that Leibniz also used the notion of counterparts, Lewis' paper suggests a good approach to a Leibnizian semantics. Lewis does not present a semantics in his paper, but rather provides us with a translation scheme. He presents a way of translating sentences in quantified modal logic to sentences in his Counterpart Theory. We understand the sentences in Counterpart Theory by a number of postulates that Lewis gives as well as by their English readings. Lewis was not trying to account for Leibniz when he formulated this theory, and there are certain Leibnizian ideas not included in Lewis' Counterpart Theory. We now propose to present a counterpart semantics based to a great extent on Lewis' Counterpart Theory but including Leibniz's ideas.

First we need some definitions, postulates, and axioms.[1] A concept is a set of properties. A complete individual concept (cic) is a set of properties such that it is possible that there is one object which has all and only those properties in the set.

Postulates

i) A set K is compossible if and only if K is a non-empty set of cic's, and it is possible that for any X, if $X \varepsilon K$, then X is realized.

ii) A set K is maximal if and only if for any cic X, if $X \not\varepsilon K$, then $K \cup \{X\}$ is not compossible.

iii) A set W is a possible world if and only if W is maximal and W is compossible.

iv) A cic C reflects a cic D only if it is not possible that C is realized and D is not realized.[2]

v) A cic C mirrors a possible world W if and only if for any cic D, if $D \varepsilon W$, then C reflects D.

vi) A set K involves a cic C if and only if K is a non-empty set of cic's and it is not possible that for any X, if $X \varepsilon K$ then X is realized, and C is not realized.

vii) A set K is closed if and only if K is a non-empty set of cic's and for any cic C, if K involves C then C is a member of K.[3]

Axioms

I. Every cic is a member of some possible world.

II. For any possible world W and for any cic C, if C is a member of W then C mirrors W.

Theorems

Th1: Every possible world is closed.

Proof

Assume: (1) There is a possible world W such that W is not closed.

(2) There is a cic C, such that W involves C and $C \notin W$. (Postulate vii)

(3) It is not possible that for all X, if $X \in W$, then X is realized, and C is not realized. And $C \notin W$. (Postulate vi)

(4) W is maximal. (Postulate iii)

(5) $W \cup \{C\}$ is not compossible. ((3), (4) & Postulate ii)

(6) It is not possible that for all X, if $X \in W$, then X is realized and C is realized. ((5) and Postulate i)

(7) It is necessary that for all X if $X \in W$ then X is realized only if C is realized. (From (3))

(8) It is necessary that for all X, if $X \in W$, then X is realized only if C is <u>not</u> realized. (From (6))

(9) It is not possible that for all X, if $X \in W$, then X is realized. (From (7) and (8))

(10) W is not compossible. ((9) and Postulate i)

(11) W is compossible. (Postulate iii)

(12) (10) contradicts (11), thus W is closed. Q. E. D.

Th2: Every cic is a member of one possible world.

By Axiom I every cic is a member of some world, so we will show that every cic is a member of only one possible world.

Assume: (1)	There is a cic C such that $C \varepsilon W$ and $C \varepsilon W'$, and W and W′ are possible worlds and $W \neq W'$.	
(2)	There is a cic D, $D \varepsilon W$ and $D \not\varepsilon W'$ (or $D \not\varepsilon W$ and $D \varepsilon W'$ but since the proof is the same in either case we assume $D \varepsilon W$ and $D \not\varepsilon W'$).	((1) & Postulate iii)
(3)	C mirrors W.	((1) and Axiom II)
(4)	C reflects D.	((2), (3) & Postulate v)
(5)	It is not possible C is realized and D is not realized.	((4) and Postulate iv)
(6)	W′ is closed.	(Th1)
(7)	W′ does not involve D.	(Assume)
(8)	It is possible that for any X, if $X \varepsilon W'$, then X is realized and D is not realized.	((7) & Postulate vi)
(9)	It is possible that if $C \varepsilon W'$, then C is realized;	
(10)	and D is not realized.	(From (8))
(11)	It is possible that C is realized and D is not realized.	(From (10), (1))
(12)	(11) contradicts (5), hence	
(13)	W′ does involve D.	(From (7) through (11))
(14)	$D \varepsilon W'$.	(From (13), (6), Post vii)
(15)	(14) contradicts (2).	Q. E. D.

Thus the axioms and postulates imply that a concept is a member of only one world, which is what we suggested Leibniz holds in Chapter III. These axioms and postulates give us a rather precise conceptual framework to use in developing a more formal account of the suggestion presented in the preceding chapter.

The remaining notion to be clarified, and in some ways the most difficult, is the notion of counterparts. Lewis describes counterparts as follows:

> Your counterparts resemble you closely in content and context in important respects. They resemble you more closely than do the other things in their worlds. But they are not really you. For each of them is in his own world, and only you are here in the actual world. Indeed we might say, speaking casually, that your counterparts are you in other worlds, that they and you are the same; but this sameness is no more a literal identity than the sameness between you today and you tomorrow. It would be better to say that your counterparts are men you *would have been*, had the world been otherwise.
> (Lewis pp. 114-115)

In describing counterparts as he does, Lewis views the counterpart relation to be one among possible objects, rather than a relation among complete individual concepts. Both Lewis and Leibniz indicate that counterparts are things which resemble or are very similar to each other in important respects. However the degree of similarity needed in order to make two things counterparts, or what the important respects are in which they must be similar in order to be counterparts is never clearly defined by either Lewis or Leibniz.

Leibniz says, "in speaking of several Adams I do not take Adam for a determined individual but for a certain person conceived *sub ratione generalitatis* under the circumstances which *appear to us* to determine Adam as an individual but which do not actually determine sufficiently," (OC 129; emphasis added). In talking about several Sextuses he says, "several Sextuses resembling him, possessing all that you know already of the true Sextus, but not all that is already in him imperceptibly," (T p. 371; emphasis added). These two quotes from Leibniz suggest that Leibniz thought the counterpart relation to be somehow a function or a measure of our knowledge of the subject. In the quote about Sextus, Leibniz indicates that in order for something to be a counterpart of Sextus it must have at least all the properties that Sextus is known to have. But Leibniz is very unclear here, and we should not take him too literally. After all, not all people would know exactly the same truths about Sextus, and it seems unfair to attribute to Leibniz the view that the counterparts of Sextus would vary depending on who is talking about Sextus and what he knows about Sextus. For that matter, it seems unfair to attribute to Leibniz the view that the counterparts of Sextus vary as more is known (by anyone) about Sextus. We might say that the counterparts of the concept of Sextus are those concepts which contain at least all of Sextus' "essential properties".

The problem with this idea is that an essential property is usually defined as a property that an object has in all possible worlds. But in a system such as the one being envisioned, an object exists in only one world, so either the object has no essential properties or all of its properties are essential, depending on whether or not one allows it to have properties in worlds in which it does not exist. In either case this notion of essential properties will not be of much help. However, keeping in mind that it appears that Leibniz wants some connection between knowledge and counterparts, there may be a way of defining essential properties which will be of some help in clarifying the notion of counterparts.

We could define essential properties in terms of our a priori knowledge. That is, we could define essential property as follows:

> A property F is essential to a cic X iff humans can know a priori that X includes F.

Thus, for example, the property of being human is essential to the concept of Adam just in case we can know a priori that the concept of Adam includes the property of being human (i.e., we can know a priori that Adam is human). The problem in defining "essential property" this way is that, as noted in Chapter IV, the account suggested for Leibniz of a priori knowledge is itself less than crystal clear. It was suggested that one can know a priori a proposition just in case the truth of the proposition can be demonstrated in a finite analysis. But exactly what a finite or infinite analysis is was left in murky waters which we will not now attempt to cross. For all the lack of clarity in the above definition of essential properties, it will help us to define the counterpart relation.

> DF1: The concept A is a counterpart of the concept B if and only if A contains all the properties which are essential to B.

The most we can get from these definitions of "essential property" and "counterpart" is an intuitive idea of what the counterpart relation is, not a precise notion of it. Moreover, DF1 is not the only plausible way of defining the counterpart relation given that we have a definition of essential property. But before we consider some alternatives to DF1, we will first put to use what we have done and present a counterpart semantics for Leibniz. This will make the consequences of alternative definitions of counterparts more readily apparent.

The individual constants will be $a_1, a_2, a_3 \ldots$, the individual variables $x_1, x_2, x_3 \ldots$, the predicate letters will be all of rank one (except identity) $F_1, F_2, F_3 \ldots$, and the normal logical signs '$\sim$', 'v' and '$\Box$'.

We will say a formula Φ is well-formed iff either Φ is a predicate letter followed by a constant, i.e., $F_i\ (a_i)$ or a predicate letter followed by a variable, i.e., $F_i(x_i)$ or an identity sign flanked by constants or variables, i.e., $a_i = a_j$, $x_i = x_j$, or $a_i = x_j$, or if Φ is well-formed, then:

(i) $\ulcorner \sim\Phi \urcorner$ is well-formed

(ii) $\ulcorner \Phi \vee \psi \urcorner$ (where ψ is well-formed) is well-formed

(iii) $\ulcorner (x)\ \Phi \urcorner$ is well-formed

(iv) $\ulcorner \Box\Phi \urcorner$ is well-formed

A sentence is a closed well-formed formula.

An interpretation is an ordered 6-tuple, <D, G, C, V, f, K> where:

(1) D = the set of all cic's [4]

(2) C(D) = the set of possible worlds

(3) G = the set of properties

(4) f is a function such that:

(i) f is from the set of constants onto D

(ii) f is from the set of predicates onto G

(5) K is the counterpart relation such that:

(i) $(\alpha)(\alpha \varepsilon D \rightarrow K\alpha\ \alpha)$

(ii) $(\alpha)(\beta)[K\alpha\beta \rightarrow (EW_i)(W_i \varepsilon C(D) \wedge \alpha \varepsilon W_i) \wedge (EW_j)(W_j \varepsilon C(D) \wedge \beta \varepsilon W_j)]$

(6) V is a valuation function such that for any sentence Φ, possible world W_i, wffs ψ, χ, variable y, predicate R and constants α, and β

(i) If Φ is $R\alpha$, then $V(\Phi, W_i) = T$ iff $f(R)\ \varepsilon\ f(\alpha)$

(ii) If Φ is $\sim\psi$, then $V(\Phi, W_i) = T$ iff $V(\psi, W_i) \neq T$

(iii) If Φ is $\psi \vee \chi$, then $V(\Phi, W_i) = T$ iff either $V(\psi, W_i) = T$ or $V(\chi, W_i) = T$

(iv) If Φ is $(y)\psi$, then $V(\Phi, W_i) = T$ iff $V[\psi(y/\alpha), W_i] = T$ for all α such that $f(\alpha)$ in W_i.

(v) If Φ is $\alpha = \beta$, then $V(\Phi, W_i) = T$ iff $f(\alpha)$ is $f(\beta)$

(vi) If ψ is such that it contains n distinct constants $\alpha_1 \ldots \alpha_n$ and no others and Φ is $\Box\,\psi$, then

$V(\Phi, W_i) = T$ iff

$(W_j)(\beta_1) \ldots (\beta_n)\{K\ F(\alpha_1)\ f(\beta_1) \ldots K\ f(\alpha_n)\ f(\beta_n) \rightarrow V[\psi(\alpha_1/\beta_1 \ldots \alpha_n/\beta_n)\ W_j] = T\}$

A sentence Φ is a necessary truth iff $\Box\,\Phi$ is true in some world.

A sentence Φ is analytic iff Φ is true in all possible worlds.[5]

In the system presented there are possible worlds which are made up of complete individual concepts. Due to the way possible worlds are defined, no particular cic is in more than one world. In agreement with Leibniz and Mates, truth is defined intensionally, that is, a sentence is true provided that the concept of the predicate is included in the concept of the subject. If we symbolize the sentence, "Adam is the first man," as Fa, then Fa will be true relative to a given world, provided that the concept associated with the predicate F (i.e., f(F)) is a member of the concept associated with the constant a (i.e., f(a)). In this case, presumably, f(F) is a member of f(a).

An atomic sentence, (say) Fa, will be necessary just in case all the counterparts of the concept of a contain the concept of F. An atomic sentence is contingent provided that it is not necessary, yet it is possible. The necessity of sentences, other than atomic ones, is slightly more complicated, unless there are no constants in the sentence. Any sentence which does not contain a constant will be necessary just in case the sentence is true in all worlds, and it will be contingent provided it is true in one world, but not all. For example, consider the sentence (x)Fx. This sentence will be necessary provided it is true of all worlds. In order for (x)Fx to be true of any world, all the complete individual concepts of that world must contain the concept of F. If (x)Fx is necessary, then every complete individual concept contains the concept of F. A complex sentence, (say) $Fa_1 \vee Fa_2$, will be necessary just in case in every world either the counterparts of the concept of a_1 or the counterparts of the concept of a_2 contain the concept of F. Thus, as I suggested for Leibniz, the necessity of a sentence depends upon what properties the counterparts of the concept of the subject contain (at least in those cases where the concept of the subject is a complete individual concept).

In the semantics presented, we have a way to distinguish between analytic and necessary propositions. Not all analytic propositions are necessary. Consider the sentence, "Adam has blond hair," and suppose that it is true. We can symbolize the sentence as Ba. Ba is analytic because in every world f(B) is a member of f(a), even though f(a) is only a member of one world. It is not necessary because, presumably, f(B) will not be a member of all the counterparts of f(a).

An alternative to (6)(vi) is:

(J) If ψ is such that it contains n distinct constants $\alpha_1 \ldots \alpha_n$ and no others and Φ is $\Box\psi$, then

$$V(\Phi, W_i) = T \text{ iff } (W_j)(\beta_1) \ldots (\beta_n)\{(f(\beta_1) \in W_j \wedge \ldots \wedge f(\beta_n) \in W_j$$
$$K\, f(\alpha_1)\, f(\beta_1) \wedge \ldots \wedge Kf(\alpha_n)\, f(\beta_n)) \rightarrow V[\psi(\alpha_1/\beta_1 \ldots \alpha_n/\beta_n), W_j] = T\}$$

One might think that (J) is a more natural account of necessity than (6)(vi) because (J) requires that the counterparts being considered be in those worlds where the sentence is being evaluated. For example, consider the sentence $\Box$Fa. According to (J), $\Box$Fa is true in W_i just in case in every world where f(a) has some counterpart f(β), Fβ is true in that world. When determining the necessity of atomic sentences, (J) and (6)(vi) will yield the same results. The difference between (J) and (6)(vi) appears when we consider molecular sentences. Consider the sentence Fa $\wedge$ Fb. In order for Fa $\wedge$ Fb to be necessary according to (6)(vi), all of the counterparts of f(a) and all of the counterparts of f(b) must contain f(F). But according to (J), Fa $\wedge$ Fb will be necessary just in case those counterparts of f(a) and f(b) which are members of the same world contain f(F). If none of the counterparts of f(a) and f(b) share a world, then Fa $\wedge$ Fb is necessary. Thus, it could turn out on (J) that Fa is not necessary and Fb is not necessary, but Fa ∧ Fb is necessary. This is clearly an unacceptable result and thus for this reason the system presented contains (6)(vi) rather than (J).[6]

In the system presented it is possible that a concept C has two counterparts in some world and it is possible that C be the counterpart of two concepts in a world. This is also true of Lewis' Counterpart Theory, and led Mates to object to a counterpart semantics for Leibniz. Mates discusses the idea of counterparts relative to Lewis' Counterpart Theory, and says:

> As presented by Lewis, the counterpart relation, though always reflexive, need not be symmetric or transitive. Further, it is possible for two or more different things in a given world to be counterparts of a single thing in another world, and it is possible for a single thing in a given world to be a common counterpart of two or more things in another world. These features would block Leibniz from agreeing that 'your counterparts are men you would have been, had the world been otherwise.' For example, he could not agree that there are conceivable conditions under which you would have been A_1 and you would have been A_2 but A_1 would not have been A_2. His theory of identity requires it to be a necessary truth that, given any two individuals, at most one of them is you.
> (M2 p. 111)

From the above, Mates seems to conclude certain things about the nature of the counterpart relation for Leibniz. He says, "Let us agree further with Leibniz that the counterpart relation, whatever other properties it has, must be symmetric and transitive, and also that nothing is a counterpart of anything else in its own world," (M2 pp. 112-113). Then, using what he calls Leibniz's "principle of continuity" he presents what appears to be his strongest argument against the use of counterparts for Leibniz. He says:

> . . . all that is needed for our purposes in the claim that any two concepts from different worlds can be joined by a discrete series of intermediate concepts (from distinct worlds) in which each concept is enough like its predecessor to qualify as a counterpart of it. Then, by the symmetry and transitivity of the counterpart relation, we would have the absurd consequence that every concept in every world would be a counterpart of every other concept in any other world. (M2 p. 115)

Mates' argument against counterparts basically rests on three points: one, an unusual reading of Leibniz's Law of Continuity, which for our purposes we will accept; two, that the counterpart relation is totally defined in terms of similarity; and, three, that the counterpart relation is symmetric and transitive. The last two points are intimately connected in that if either one is lacking the argument will not work. In the system presented, neither one of them holds.

Mates argues that the counterpart relation must be symmetric and transitive for Leibniz as well as that nothing can have a counterpart in its own world besides itself. The reason the counterpart relation must be restricted in this way is that if it is not, then we get certain unintuitive results viewed from a Leibnizian perspective. Mates says that Leibniz would not agree that (say) Adam had two counterparts in the same world. Mates might be right in saying that Leibniz would not like something to have two counterparts in the same world, but his remedy to the problem seems a bit extreme given that he is taking the counterpart relation to be defined totally in terms of similarity. Mates suggests we make the counterpart relation transitive and symmetric and add the following restriction (which is put in terms of the semantics presented above):

(C1) $(W_j)(\alpha)\{(W_j \varepsilon C(D) \wedge \alpha \varepsilon W_j) \rightarrow (\beta)(\beta \varepsilon W_j \rightarrow [K\beta\alpha \rightarrow \beta = \alpha])\}$

While doing what Mates suggests does in fact get us the results Mates wants, it makes any counterpart system based on similarity unintuitive from any point of view, whether it be Leibniz's or not. How does one handle the problem Mates points out if not in terms of transitivity and symmetry? The answer is simply to add the following two conditions:

(C2) $(W_i)(W_j)(\alpha)(\beta)(\delta)\{(W_i \varepsilon C(D) \wedge W_j \varepsilon C(D) \wedge \alpha \varepsilon W_i \wedge \beta \varepsilon W_j \wedge \delta \varepsilon W_j) \rightarrow [(K\beta\alpha \wedge K\delta\alpha) \rightarrow \beta = \delta]\}$

(C3) $(W_i)(W_j)(\alpha)(\beta)(\delta)\{(W_i \varepsilon C(D) \wedge W_j \varepsilon C(D) \wedge \alpha \varepsilon W_i \wedge \beta \varepsilon W_j \wedge \delta \varepsilon W_j) \rightarrow [(K\alpha\beta \wedge K\alpha\delta) \rightarrow \beta = \delta]\}$

Conditions (C1), (C2), and (C3) together insure that if A is a member of W_i and if A has a counterpart in W_j, A has only one counterpart in W_j. They also insure that if A is a member of W_i and A is a counterpart to something in W_j, then A is the only counterpart (in W_i) to that thing in W_j. And, they insure that if a concept is a member of any world, then in that world it is its only counterpart. But the conditions do not imply that the counterpart relation is either transitive or symmetric. Thus, conditions (C1), (C2) and (C3) get the results we want without, it seems, any bad side effects. Moreover, no matter how one defines counterparts, it seems that principles (C1), (C2) and (C3) should hold in a Leibnizian counterpart system. Thus we shall add them to the original system presented under part (5). As far as Mates' objection goes, even if we define the counterpart relation totally in terms of similarity, there is no reason to make the relation transitive and symmetric for Leibniz, provided we add conditions like (C1), (C2) and (C3) to the system. Moreover, even if we did not add

(C1), (C2) and (C3) to the system, Mates' objection would not apply to it.

Mates' objection does not hold against the system presented above because counterparts are not defined totally in terms of similarity. It will not be true that, "any two concepts from different worlds can be joined by a discrete series of intermediate concepts (from different worlds) in which each concept is enough like its predecessor to qualify as a counterpart of it," (M2 p. 115). The reason it is not true is that if we start with two concepts which do not have the same essential properties, there could never be a series between them in which each concept is enough like its predecessor to count as a counterpart of it. All members of the series would have to have the same essential properties, yet by hypothesis the first and the last member do not have the same essential properties. Thus, Mates' objection does not hold against the system being proposed. Moreover, it does not really work against Lewis, since by adding something like (C1), (C2), and (C3) Lewis could avoid it.

There is a slight problem in adding (C1), (C2) and (C3) to the system presented above. Suppose there is a concept C in a world W_i such that in world W_j there are two concepts which contain all the properties essential to C. According to (C2), C can have at most one counterpart in world W_j, yet it appears that on DF1 C has two counterparts in W_j. The problem is, which one of the two concepts in W_j which qualify as counterparts under DF1 is the counterpart of C? We might change the definition of counterparts to avoid this difficulty.

As mentioned earlier, DF1 is not the only way to define the counterpart relation given a definition of essential property. There are at least two other ways of defining it which are worth comparing to DF1.

DF2: The concept A is a counterpart of the concept B if and only if all the properties which are essential to B are essential to A.

DF3: The concept A is a counterpart of the concept B if and only if the properties which are essential to A and the properties which are essential to B are exactly the same properties.

According to DF1, the counterpart relation is reflexive, but not transitive or symmetrical. Under DF2, on the other hand, the relation is reflexive and transitive, but not symmetrical, while under DF3 the relation is reflexive, transitive and symmetrical. We can easily incorporate either DF2 or DF3 into the system presented above by adding the following conditions under part (5):

(for DF2)(iii) $(\alpha)(\beta)(\delta)[(K\alpha\beta \wedge K\beta\delta) \rightarrow K\alpha\delta]$

(for DF3) (iv) $(\alpha)(\beta)(K\alpha\beta \rightarrow K\beta\alpha)$

In order to incorporate DF3 we would add both (iii) and (iv).

Depending on how one defines the counterpart relation, the results of the system will vary. For example, $\Box\Phi \rightarrow \Box\Box\Phi$ is not a result under DF1, but would be under DF2 or DF3. $\Diamond\Phi \rightarrow \Diamond\Box\Phi$ is not a result under DF1 or DF2, but would be if we defined counterpart by DF3.

DF3, the strongest of the three definitions, seems to be the most likely candidate for resolving the problem created by adding (C1), (C2) and (C3) to the system. But on DF3 the difficulty is still there. It is possible that there are two concepts, A and B, in world W_j which have the same essential properties that C has in world W_i. In fact, in this case A and B will be counterparts to each other according to DF3. Intuitively, A in world W_i is a counterpart of B in world W_j, just in case A resembles B more than anything else in W_i. But we do not want the counterpart relation defined totally in terms of similarity relative to worlds, otherwise it could turn out that the concept of a particular tree be a counterpart to the concept of Adam. Perhaps what is needed is a definition of counterpart which combines the ideas of essentiality and similarity for counterparts.

If A and B are both concepts, then '$P_B(A)$' will be the number of properties that A and B share. Thus, $P_B(A) = P_A(B)$.

DF4: A concept A is a counterpart of a concept B if and only if

(i) A and B have the same essential properties, and

(ii) There is a possible world W_i and a possible world W_j such that $A \varepsilon W_i$ and $B \varepsilon W_j$, and $(x)[(x \neq A \wedge x \varepsilon W_i) \rightarrow P_B(A) \rightarrow P_B(x)$ and $(x)\ (x \neq B \wedge x \varepsilon W_j) \rightarrow P_A(B) > P_A(x)]$.

In order to incorporate DF4 into the present system we would add (iv) (mentioned in connection with DF3 above) under part (5). DF4 allows only one counterpart per concept per world, which is what is insured by (C1), (C2) and (C3). DF4 has a slight disadvantage in that in order for A to be a counterpart of B, B must be a counterpart of A. In other words, DF4 makes the counterpart relation symmetrical. This may seem counterintuitive, given that the counterpart relation is defined, in part, in terms of similarity. We can avoid this problem by altering the definition to the following:

DF5: A concept A is a counterpart of a concept B if and only if

(i) A and B have the same essential properties, and

(ii) There is a possible world W_i and a possible world W_j such that $A \varepsilon W_i$ and $B \varepsilon W_j$, and $(x)[(X \neq A \wedge x \varepsilon W_i) \rightarrow P_B(A) > P_B(x)]$, and $(x)(y)\{(x \neq B \wedge y \neq A \wedge x \varepsilon W_j \wedge y \varepsilon W_i) \rightarrow [Px(A) > Px(y)]\}$.

DF5 has an advantage over DF4 in that unlike DF4 it does not imply that the counterpart relation is symmetrical, but also allows only one counterpart per concept, per world. (C1), (C2) and (C3) are all reasonable, given that counterparts are defined by DF5. We need to add nothing to the original system (except (C1), (C2) and (C3)) if we define counterparts by DF5, for the relation so defined is reflexive, but neither symmetrical nor transitive. It is difficult to decide which, if any, of the definitions is the most Leibnizian in its treatment of counterparts. Our own choice is DF5, since DF5 incorporates similarity in the definition, yet does not make the relation symmetrical or transitive. Perhaps we should view DF2 through DF5 as different Leibnizian systems, corresponding to the systems S4, S5, B and T of contemporary modal logic, respectively.[8]

In Chapter III, certain things about Mates' semantics were pointed out and taken to be objections to his system. It would be well to note how the same objections fare against the system being proposed. As it turns out, not one of the alleged difficulties holds in our system. One of the problems was the fact that in Mates $(Ex)\Box Fx$ is always false. This will not occur in the current system, for if 'F' stands for a property which is essential to some concept (in this world), then $(Ex)\Box Fx$ will be true (in this world). Another problem arose in connection with the fact that (b) $\Diamond Fa \rightarrow \Box[(Ex)\ x=a \rightarrow Fa]$ is a necessary truth. (b), interpreted via Mates, is not something which would cause Leibniz any discomfort, and so interpreted is true in our semantics. The problem was that (b) appeared to be a translation of a sentence which Leibniz would not accept, namely, "If it is possible that Adam has black hair, then necessarily if Adam exists, then Adam has black hair." In the system presented, (b) does not turn out to be a necessary truth as a general schema (i.e., it is not a necessary truth for any constant and any predicate). Under DF2 (b) would be a necessary truth for some a and F if the property of F is essential to the concept of a.

Just as (b) is not a necessary truth, neither is (c) $\Diamond(a \neq a)$. In fact, (c) is false in the counterpart semantics, since $\Box(a=a)$ is true for any constant a. But while $\Box(a=a)$ is true for any constant a, necessary identity is not a result of the system. It might seem strange to have a system in which a=a is a necessary truth, but if a=b, then it does not follow that a=b is a necessary truth, but one must remember how formulas are being interpreted in the system. When we say that a=a is necessary, we are only saying that all of f(a)'s counterparts are self-identical, but when we say a=b is necessary (assuming a=b), we are saying all of f(a)'s counterparts are identical, which is false. This is because $\Box(a=b)$ is true (in this world) just in case $(W_i)(\alpha)(\beta)$ if $K\ f(\alpha)f(a) \wedge K\ f(\beta)f(b)$ then $\alpha=\beta$ is true (in W_i). The antecedent of the conditional does not restrict which counterparts of f(a) and f(b) are to be considered. That is, the counterparts being considered do not have to be members of the same world, for reasons mentioned earlier.[9] Thus even though f(a) is identical to f(b), $f(\alpha)$ need not be identical to $f(\beta)$ as they may be members of different worlds. Of course if $f(\alpha)$ and $f(\beta)$ are in the same world, then $f(\alpha)$ will be identical to $f(\beta)$. However, explaining why the system does not yield necessary identity is not a justification for its lacking necessary identity. All things considered it seems necessary identity should be a part of the system. It can be easily added to the system by changing (6)(vi) to the following:

(6)(vi)′ If ψ is such that it contains n distinct constants $\alpha_1 \ldots \alpha_n$ and no others, and Φ is $\Box\psi$, then

$$V(\Phi_1\ W_i)=T \text{ iff } (W_j)(\beta_1) \ldots (\beta_n)\{\{K\ f(\beta_1)\ f(\alpha_1) \wedge \ldots \wedge K\ f(\beta_n)\ f(\alpha_n) \wedge (i)(j)\ [(1 \leq i \leq n \wedge 1 \leq j \leq n) \rightarrow [f(\alpha_i) = f(\alpha_j) \rightarrow f(\beta_i) = f(\beta_j)]]\} \rightarrow V[\psi(^{\alpha}1/\beta_1) \ldots (^{\alpha}n/\beta_n),\ W_j] = T\ \}$$

The addition of necessary identity to the system is not one which yields any unintuitive results. If f(a) is identical to f(b), then surely we would want the counterparts of f(a) to be the same as the counterparts of f(b). Necessary non-identity is not something we would want, and does not follow from (6)(vi)′. If f(a) is distinct from f(b), then it seems it should be left open as to whether they have any of the same counterparts. If f(a) is a member of W_i and f(b) is a member of W_j (where $W_i \neq W_j$), then it should be possible that there is a concept in another world, (say) W_k such that that concept is a counterpart of f(a) and f(b).

The system suggested for Leibniz does capture the Leibnizian account of necessity and contingency presented in Chapter IV. It avoids the objections Mates raises against using counterparts for

Leibniz and at the same time avoids the difficulties that Mates' own system seems to have. Leibniz did not have the use of modern logical techniques when he presented his views on necessity and contingency, and often he was writing only in response to claims other philosophers made, and not trying to present a complete theory. This makes it difficult to apply modern logical methods to Leibniz; one is never completely sure that in using such methods he has in fact captured what Leibniz would have said had these methods been available. Another difficulty in presenting a system such as the one suggested in this chapter is that Leibniz, as most great philosophers, occasionally changed his mind on various philosophical issues throughout his career. What we have attempted to do in a precise way is to present a solution to the problem of contingency for Leibniz, which Leibniz suggests but does not completely work out. The solution we suggest is one which does allow contingency in a very Leibnizian way, yet keeps Leibniz's definition of truth intact. When dealing with a philosopher as great as Leibniz, the most it seems one can do in a project this size is to consider a problem which is central to his philosophy, but does not account for all of his philosophy. We hope to have shown that there is a coherent way of understanding Leibniz which makes a distinction between analytic and necessary truths in terms of counterparts and possible worlds, and hence allows for contingency.

NOTES FOR CHAPTER V

1. In presenting this material I assume that something is possible just in case its opposite is not contradictory if God created freely.

2. This postulate suffers from being non-Leibnizian in important respects. For an attempt at a more Leibnizian treatment of reflection and mirroring, see the appendix.

3. Postulates vi) and vii) were suggested to me by Robert Sleigh Jr., and I am indebted to him for his comments and suggestions on this material.

4. I assume as Mates does that D, C(D), and G are all denumerably infinite.

5. I am indebted to Edmund L. Gettier III for many helpful discussions on Counterpart Theory and formal semantics in general, which in part led to this system.

6. I am indebted to Michael Jubien who pointed out this difficulty in an earlier version of this chapter.

7. I would like to thank Fred A. Feldman, who pointed out this problem in a different version of this chapter. Professor Feldman's comments and criticisms on all aspects of this dissertation have been greatly appreciated.

8. T, S4, S5, and B are presented in G.E. Hughes and M.J. Cresswell, An Introduction to Modal Logic, Methuen and Co., Ltd., London, 1968.

9. See pp. 110-111.

APPENDIX

The Leibnizian idea of reflection and mirroring is actually much stronger than the idea presented on pages 99 - 106, which was sufficient for deriving the results needed there. The postulate of reflection presented on page 101 should be a consequence of the stronger Leibnizian notion. Intuitively, a cic C reflects a cic D if, for any property contained in D, it can be deduced that it is contained in D from C. In this appendix we attempt to present a precise account of the Leibnizian idea of mirroring and reflection using the notions presented on pages 19 and 20 and language ALPC.

Φ is a sentence of ALPC if and only if Φ is a closed wff of ALPC. We introduce a new language, ALPC′ which is just like ALPC, except for the following:

> Φ is a wff of ALPC′ <u>iff</u> either (i) Φ is a one-place predicate of ALPC followed by one term, or (ii) if Φ is a wff of ALPC′, then (x)Φ and (Ex)Φ are wffs of ALPC′. Φ is a sentence of ALPC′ iff Φ is a closed wff of ALPC′.

ALPC′ is just a part of ALPC, but a very interesting part for Leibniz. Let P be the set of all properties, $P_1 \ldots P_n$, and C the set of all complete individual concepts, $C_1 \ldots C_n$. f, g, and h are all functions as defined on pages 19 and 20, where the language being used is ALPC′. Thus the domain of f is the set of all one-place predicates of ALPC. For any set K of cics, S(K) is the set of sentences of ALPC′ associated with K. That is, if $K = \{C_1 \ldots C_n\}$, $S(K) = h(g(C_1))$ union $h(g(C_2)) \ldots$ union $h(g(C_n))$. We can now define "compossibility","mirroring", and "possible world".

Definitions

(DF1) A set K of cics is compossible iff S(K) is consistent.

(DF2) A set K of cics is maximal iff for any cic C, $C \notin K$ then $K \cup \{C\}$ is not compossible.

(DF3) A set W of cics is a possible world iff W is a maximal compossible set of cics.

(DF4) A cic C reflects a cic D iff $(E\alpha)(E\beta)f(\alpha) = C$ and $f(\beta) = D$ and for any ALPC′ wff Φ containing one free variable x, if $\hat{x}[\Phi]\beta \in S(\{D\})$, then there is an ALPC′ wff ψ containing one free variable y such that $\hat{y}[\Phi\ \beta/x \wedge \psi]\alpha \in \mathcal{S}(\{C\})$.

(DF5) A cic C mirrors a possible world W iff for any cic D, $D \in W$ then C reflects D.

Axioms

I. For any cic C, there is a possible world W such that $C \in W$.

II. For any cic C, if $C \in W$ then C mirrors W.

(DF4) is designed to capture Leibniz's idea of reflection. Every concept is supposed to "express" all the other concepts in the same world. I understand Leibniz to be saying that from the concept of (say) Adam, not only can we deduce all of Adam's properties, but we can deduce all of Eve's properties, all of Leibniz's properties, and so on for anyone in the world. If the concept of Adam reflects the concept of Eve and the concept of Eve contains the property of having brown hair, then on (DF4) it follows that the concept of Adam contains Eve's having brown hair. To see this, let f(e) be the concept of Eve, f(a) the concept of Adam, and $f(\hat{x}[BRx])$ the property of having brown hair. The set of sentences associated with the concept of Eve contains, among other things, $\hat{x}[BRx]e$. Given that the concept of Adam reflects the concept of Eve, the set of sentences associated with the concept of Adam will include $\hat{x}[BRe \wedge \Psi]a$ where $f(\hat{x}[\psi])$ is some property that is included in the concept of Adam. The theorems that can be derived from (DF1) through (DF5) and Axioms I and II are analogous to those which can be derived using the definitions, postulates, and axioms presented on pages 99 through 106. For example, one of the theorems will be that each cic is a member of only one world. The proof of this theorem is somewhat complicated but can be shortened by acceptance of a theorem which upon reflection will seem obviously true. First some helpful abbreviations: $L\beta$ (where β is some constant) is the set of all sentences $\hat{x}[\Phi]\beta \in S(\{f(\beta)\})$, where Φ is a wff containing one free variable x. $D\alpha(L\beta)$ (where α and β are constants) is the set of all sentences $\hat{x}[\Phi(\beta) \wedge \psi] \in S(\{f(\alpha)\})$ where $\Phi(\beta)$ is a sentence such that $\hat{x}[\Phi^x/\beta]\beta \in L\beta$ and ψ is a wff containing one free variable x.

Theorem 1: For any set H of sentences of ALPC′ and any constants α and β, if H union $D\alpha(L\beta)$ is consistent, then H union $L\beta$ is consistent.

We leave this theorem unproved, but one can intuitively see why it holds. That is because, in effect, $L\beta$ is a subset of $D\alpha(L\beta)$. Using this theorem we can prove that each cic is a member of only one world.

Suppose not; i.e., suppose there is a cic C such that $C \in W$ and $C \in W'$ where W and W′ are both possible worlds and $W \neq W'$. Since $W \neq W'$, there is at least one cic D a member of one, (say) W, and not the other. C reflects D by AxII and (DF5). Thus, there is a constant, (say) <u>a</u>, such that f(a)=C and a constant, (say) b, such that f(b)=D. Since $D \notin W'$

and W´ is a possible world it follows that W´ ∪ {D} is not compossible. Thus, S(W´ ∪ {D}) is inconsistent, and S(W´) ∪ S({D}) is inconsistent. It also follows that S(W´) union Lb is inconsistent, since, although Lb is a subset of S({D}), for any sentence Φ of ALPC´ in S({D}) there is a sentence equivalent to Φ in Lb. This can be seen if one considers that the only sentences in S({D}) but not in Lb are sentences of the form ⌜Fb⌝ where F is a one-place predicate of ALPC. Yet for each sentence of the form ⌜Fb⌝ in S({D}), a sentence of the form ⌜x̂[Fx]b⌝ will occur in Lb and the two sentences are equivalent. Since C ε W´, Da(Lb) is a subset of S(W´), hence S(W´) union Da(Lb) is consistent. If S(W´) union Da(Lb) is consistent, then S(W´) union Lb is consistent, by Theorem 1. But this contradicts what was said earlier, namely S(W´) union Lb is inconsistent. Thus, every cic is a member of only one world.

The notion of reflection presented here is much more powerful than the one presented on page 101. It also seems more Leibnizian. However, the view is only a suggestion, and not all the details have been worked out. The problem of understanding Leibniz's view of reflection is an interesting one which will certainly have future consideration in the literature on Leibniz.

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