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Jun, Ren
University of Massachusetts Amherst

TWO MODAL PARADOXES AND THEIR SOLUTIONS

A Dissertation Presented

by

JUN REN

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Approved as to style and content by:

Edmund L. Gettier, III

Edmund L. Gettier, III, Chair

Bruce Aune

Bruce Aune, Member

Gareth B. Matthews

Gareth Matthews, Member

Barbara Partee, Member

John G. Robison

John G. Robison, Head
Philosophy

Dedicated to my parents

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ABSTRACT

TWO MODAL PARADOXES AND THEIR SOLUTIONS

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JUN REN, B.A., BEIJING NORMAL UNIVERSITY

Ph.D., UNIVERSITY OF MASSACHUSETTS AMHERST

Directed by: Professor Edmund L. Gettier III

Chisholm's Paradox and the Four Worlds Paradox are paradoxes about the cross-world identity of artifacts that are made of parts. The paradoxes are described as derivable in S5 modal logic from principles concerning the essentiality of the original matter of an artifact and the Tolerance Principle concerning possible changes in the original matter. On one hand, the original matter is essential to the artifact; on the other hand, bare identity or distinctness with respect to the original matter can be inferred by applying the Tolerance Principle in S5.

This dissertation analyzes two solutions that have been proposed. Nathan Salmon developed an Intransitive Accessibility Solution that rejects S5 as the logic for metaphysical modality. We show that Salmon's argument for the intransitivity of metaphysical possibility is unsound. The fundamental problem in Salmon's account is his attempt to derive the mode of metaphysical possibility from the accessibility relation between the possible worlds, which, by the theory of possible worlds that Salmon advocates, has to be determined by metaphysical possibilities with a pre-determined mode. The conclusion of Salmon's argument only reiterates a

premise of the argument. We also discuss Salmon's other defenses, and show that these defenses are unsuccessful.

Graeme Forbes proposes a Counterpart Solution. His solution replaces the standard two-valued semantics by a counterpart semantics with infinitely many degrees of truth-value. Our view is that Forbes' solution is unsatisfactory. Forbes avoids the identity problem by formulating the problem in terms of similarity relation. We argue that the similarity relation must not be a semantic device for representing identity.

Our analysis reveals two versions of tolerance principle that have not been distinguished in literature. The paradoxes are associated with the Strong Tolerance Principle. We argue that the Strong Tolerance Principle is false. The intuition of tolerance is sufficiently described by the Weak Tolerance Principle. Moreover, we argue that the knowing of the possibilities about the origination of an artifact is empirical. The knowledge of the historical background and the origination of the artifact is needed for knowing the possibilities. With this view, S5 as the logic of metaphysical modality can be defended.

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INTRODUCTION

This dissertation discusses the phenomena of paradoxical modal inferences known as "Chisholm's Paradox" and the "Four Worlds Paradox." These modal inferences are about the necessities and possibilities in the origination of a certain artifact. The two paradoxes arise from applying in the inferences of standard S5 modal logic some essentialist principles concerning the essentiality of the original matter of artifacts, together with the so-called "tolerance principle" concerning possible variations in the original matter of artifacts. The essentialist principles express the following intuitions: (i) an artifact could not have been originally constructed from a collection of components sufficiently different from the collection from which the artifact is actually constructed, and (ii) any possible artifact which is constructed from a possible collection of components of a given artifact and shares all other properties (or essential properties) with the given artifact is numerically identical to the given artifact. On the other hand, the tolerance principle used in the modal inferences expresses the intuition that an artifact constructed according to plan ***P*** could have been constructed according to the same plan ***P*** from any collection of components which is qualitatively and quantitatively the same as, but slightly different in its original components from, the actual collection of the artifact. Both the essentialist principles and the tolerance principle are deemed as truths of metaphysical necessity.

Philosophers who participated in the discussion of the two paradoxes all find the tolerance principle inconsistent with the essentialist principles in S4 or S5 modal logic. The tolerance principle used in the modal inferences provides a sufficient condition for inferring possible original collection of components for a given artifact—an artifact could have been constructed according to the same plan from *any* qualitatively and quantitatively identical collection of components which is slightly different in its components from the actual collection of the artifact. According to this tolerance principle, by a certain quantitative measure, one can always determine what is the possible variations in the original matter of the artifact. This tolerance principle is inconsistent, in S4 or S5, with the essentialist principles concerning the original matter of an artifact. The repeated use of the tolerance principle in modal inferences and the transitivity of S4 and S5 together will derive conclusions against essentialism on the issues of identity and distinctness of artifacts.

The argument against essentialism on the issue of identity of artifacts is as follows. Given an artifact made from a certain collection of components **C** according to plan ***P*** in the actual world $\boldsymbol{w_@}$, by the tolerance principle, there is a possible world $\boldsymbol{w_1}$ possible relative to $\boldsymbol{w_@}$ in which the given artifact is made according to plan ***P*** from collection of components $\mathbf{C_1}$ slightly different from **C**. By applying the tolerance principle to $\boldsymbol{w_1}$, there is possible world $\boldsymbol{w_2}$ relative to $\boldsymbol{w_1}$ in which the artifact is made according to plan $\boldsymbol{P}$ from collection of components $\mathbf{C}_2$ which is slightly different from $\mathbf{C}_1$ and shares less components with $\mathbf{C}$ than $\mathbf{C}_1$ does. Repeating this procedure for each $\boldsymbol{w}_i$, one can infer a sequence of worlds such that in each $\boldsymbol{w}_i$ the artifact is constructed from a collection $\mathbf{C}_i$, and as i increases, the components in common between $\mathbf{C}$ and $\mathbf{C}_i$ decrease. One can finally reach a world $\boldsymbol{w}_n$ in which the given artifact is made according to plan $\boldsymbol{P}$ from collection of components $\mathbf{C}_n$ which shares no component with $\mathbf{C}$ and yet, by the transitivity of accessibility in S4 or S5 modal logic, the artifact made from $\mathbf{C}_n$ is identical with the given artifact actually made from $\mathbf{C}$. The acceptance of this identity contradicts the essentialist intuition that an artifact could not have been originally constructed from a collection of components that significantly differs from the collection of components from which the artifact is actually constructed. This is Chisholm's Paradox. The naming of the paradox comes from the recognition that this kind of argument is initially presented in current philosophy by Roderick Chisholm in his article "Identity Through Possible Worlds: Some Questions" published in 1968.[1] The version used in the discussion of Chisholm's Paradox is presented by Graeme Forbes in his paper, "Thisness and Vagueness" published in 1983, where he named this philosophical puzzlement "Chisholm's Paradox."[2]

The argument against essentialism on the issue of numerical distinctness begins by considering two distinct artifacts $\boldsymbol{A}$ in world $\boldsymbol{w}_A$ and $\boldsymbol{B}$ in world $\boldsymbol{w}_B$. Artifacts $\boldsymbol{A}$ and $\boldsymbol{B}$ are assumed to be

[1] Loux [1979], 82.
[2] Forbes [1983], 236-237.

qualitatively identical and made from the same plan $\boldsymbol{P}$, but have different collections of components—$\boldsymbol{A}$ is made from collection C_A whereas $\boldsymbol{B}$ made from collection C_B. Applying the tolerance principle to both $\boldsymbol{A}$ and $\boldsymbol{B}$, one can construct worlds w_{A1} and w_{B1} such that the difference between $\boldsymbol{A}$'s collection C_{A1} in world w_{A1} and $\boldsymbol{B}$'s collection C_{B1} in world w_{B1} is smaller than the difference between C_A and C_B. Repeating this process, one can reach a world w_{Am} in which $\boldsymbol{A}$ is made according to $\boldsymbol{P}$ from collection C_{Am} and a world w_{Bm} in which $\boldsymbol{B}$ is made according to $\boldsymbol{P}$ from collection C_{Bm} such that C_{Am} is C_{Bm}. The two artifacts, $\boldsymbol{A}$ in w_{Am} and $\boldsymbol{B}$ in w_{Bm}, are qualitatively and materially indiscernible, but they are distinct objects—one is $\boldsymbol{A}$ and the other is $\boldsymbol{B}$ by the transitivity of S5 modal logic. The distinctness between $\boldsymbol{A}$ in w_{Am} and $\boldsymbol{B}$ in w_{Bm} contradicts the essentialist intuition that any possible artifact constructed from a possible collection of components of a given artifact and sharing all other properties (or essential properties) with the given artifact is numerically identical to the given artifact. This line of argument is the Four Worlds Paradox. Chisholm in his paper of 1968 notices that an antiessentialist argument on distinctness can be given by the same consideration about tolerable variations as the one involved in the antiessentialist argument on identity. In his paper "Parts as Essential to Their Wholes" published in 1973, Chisholm presented and discussed an antiessentialist argument on distinctness in term of the components of objects.[3] Hugh. S. Chandler in his paper "Plantinga and the Contingently Possible" published in 1976 first presented an

[3] Chisholm [1973], 585-586.

argument for bare distinctness by a model of four worlds, namely, two initial worlds and two qualitatively and materially identical worlds differing only in their relative possibility relation to the two initial worlds.[4] Nathan Salmon provided a more detailed presentation and a more adequate discussion of the Four Worlds Paradox in his paper "How not to Derive Essentialism from the Theory of Reference" published in 1979.[5] Later, Salmon discusses the same problem in Appendix I of his book of 1981, *Reference and Essence*.[6]

There are two solutions developed for solving the paradoxes. One is Salmon's Intransitive Accessibility Solution and the other is Forbes' Counterpart Solution. Salmon and Forbes both hold the view that the essentialist principles and the tolerance principle involved in the paradoxical inferences are all intuitively correct and equally acceptable. More specifically, they both accept the sufficient condition expressed in the tolerance principle. According to this sufficient condition, a further possible matter of a given artifact can be inferred from a known possible matter of the artifact and a certain quantitative measure of "slightly different." Salmon holds that there is a quantitative threshold between tolerable variations and intolerable variations such that any collection of components which varies from the actual collection of components of the given artifact in an amount within the threshold is a possible collection for the origination of the artifact, otherwise, an impossible collection for the artifact. Forbes denies that there is such a definite threshold, but he

[4] Chandler [1976], 107-108.
[5] Salmon [1979], 721-725.
[6] Salmon [1981], 229-240.

holds that a collection sharing more components with the actual collection of the artifact is more possible, and less components, less possible, for the artifact to be originated from. If the variation in a hunk of matter is sufficiently slight, the matter is sufficiently possible for the given artifact to be originated from. Since Salmon and Forbes do not challenge any of the premises of the paradoxical inferences, and since the essentialist principles and the tolerance principle are inconsistent in standard S5 modal logic, there are just two possible approaches for solving the paradoxes: one is to reject S5 as the logic of metaphysical modality, and the other is to adopt or develop some apparatus with which the essentialist principles and the tolerance principle can be rendered consistent in S5. Salmon's Intransitive Accessibility Solution takes the first approach whereas Forbes' Counterpart Solution takes the second one.

Salmon's Intransitive Accessibility Solution views the paradoxes as arising from incorrectly accepting S5 modal logic as the logic of metaphysical modality. Salmon argues that a possible world is defined in terms of possibilities and necessities, and the metaphysically possible worlds of a world ***w*** are determined by the possibilities and necessities contained in ***w***. The necessities and possibilities inferred from the essentialist principles and the tolerance principle in the real world determine that some world is not possible relative to the real world but possibly possible, or possibly possibly ... possible relative to the real world—the relative possibility relation is intransitive. The axiom of S4 modal logic, $\Box P \supset \Box\Box P$, characterizes a transitive accessibility relation between possible worlds, and is inconsistent with the intransitive nature of metaphysical modality. Therefore, the way of solving the paradoxes is to reject S4 and S5 modal logic and accept system T as the correct logic for metaphysical modal reasoning. Salmon's Intransitive Accessibility Solution to the paradoxes is developed in Appendix I of his book, *Reference and Essence*, of 1981.[7] Later, he published two papers to defend his position of rejecting S5, one is "Modal Paradox: Parts and Counterparts, Points and Counterpoints" published in 1986 and the other "The Logic of What Might Have Been" published in 1989.

Forbes' Counterpart Solution has a different view on the nature of the two paradoxes. Forbes holds that in S5 modal logic, the intuition of tolerance is more naturally expressed by the conditional, "if it is possible for a given artifact to be constructed from a collection **C**, then it is possible for the artifact to be constructed from collection **C**′ where **C**′ is slightly different in its components from **C**," than by the necessity, "necessarily if a given artifact is made from collection **C** then it is possible for the artifact to be made from **C**′." The conditional and the necessity are equivalent in S5. Forbes holds that the expression of the intuition of tolerance in the conditional assimilates these two paradoxes to the Sorites Paradox, and the well-known treatment of ordinary Sorites Paradoxes can be applied. This treatment modifies the two-valued semantics by introducing a range of intermediate degrees of truth between the absolute truth and the absolute falsehood. Since it does not make good sense to view the identity relation as intransitive and having degrees, Forbes

7 Salmon [1981], 238-240.

introduces a counterpart relation into the semantics to represent identity. The counterpart relation is based on similarity relation which is intransitive and can be viewed as having degrees. By reformulating the paradoxical inferences in this counterpart and degree-valued semantics, Forbes' Counterpart Solution diagnoses the paradoxical inferences as they commit a "fallacy of detachment." That is the fallacy of inferring by *modus ponens* the consequent of a conditional which has its antecedent true in a higher degree than its consequent. Forbes' Counterpart Solution is first developed in his paper, "Thisness and Vagueness," published in 1983. He later defended his solution in his paper of 1984, "Two Solutions to Chisholm's Paradox", and in Chapter 7 of his book, *The Metaphysics of Modality*, published in 1985.

This dissertation will contain our criticism of the solutions of Salmon and Forbes and a further discussion of the paradoxes. Our first focus will be on Salmon's Intransitive Accessibility Solution. Salmon's solution views Chisholm's Paradox and the Four Worlds Paradox as a denial of the traditional belief that metaphysical modality is characterized by S5 modal logic. According to Salmon's defense of his solution, the intransitiveness of the accessibility relation between possible worlds is shown deductively in his account from a commonly accepted theory of possible worlds, the intuition of the essentiality of the original matter of artifacts, and the intuition of tolerance in the original matter of artifacts. Salmon also defends his position by some other arguments which are supposed to be independent of his Intransitive Accessibility Solution but support his rejection of S5 modal logic. We shall analyze Salmon's arguments and present our reason for thinking that the Intransitive Accessibility Solution is not successfully defended. Our second focus will be on solving the paradoxes and defending S5 modal logic. We shall propose a solution to the paradoxes that is different from both Salmon's solution and Forbes' solution. We shall argue that the particular version of the tolerance principle employed in the paradoxical inferences is false, which we believe to be responsible for the arising of the paradoxes. Our defense of S5 modal logic as the correct logic for metaphysical modality will be based on our views about the nature of the paradoxes and the vagueness of the threshold, and our discussion on the *a posteriori* necessities involved in the problem. Forbes' Counterpart Solution will be examined while we consider the correct solution to the paradoxes.

In Chapter 1 we shall give the initial analysis on the modal principles from which the Paradoxes are deduced. We shall present the paradoxical modal inferences and Salmon's argument of his Intransitive Accessibility Solution.

In Chapter 2 we shall analyze Salmon's reply to the so-called "standard objection." The "standard objection" challenges Salmon on the nature of metaphysical modality that makes transitivity invalid, from which one can be convinced of the correctness of the Intransitive Accessibility Solution. In his reply, Salmon argues that metaphysical modality is a restricted type of modality (there are some worlds that are not metaphysically possible worlds). According to Salmon, metaphysical modality's being characterized by S5 is commonly defended by the unrestrictiveness of metaphysical modality. He argues that since metaphysical modality is neither unrestricted nor least restricted, metaphysical modality's being characterized by S5 is not being successfully defended. On the other hand, if modal quantifiers quantify over all worlds including metaphysically possible ones and metaphysically impossible ones, then the accessibility relation is intransitive. Our analysis of Salmon's reply will first clarify two different senses of "restricted" that philosophers may use for modality. We shall argue that only if metaphysical modality is shown to be restricted in both senses, is Salmon in the position to make the claim that metaphysical modality's being characterized by S5 is not being successfully defended, and that the accessibility relation between metaphysically possible worlds is intransitive. However, we shall show that Salmon did not succeed in providing an argument which is independent of his Intransitive Accessibility Solution and shows that metaphysical modality is restricted in both senses. The key premise of Salmon's argument for the restrictiveness of metaphysical modality relies on the correctness of his Intransitive Accessibility Solution. This is a circular reasoning because Salmon's argument for the restrictiveness is intended to back up his Intransitive Accessibility Solution.

In Chapter 3, we shall continue to examine Salmon's defense of his position. Forbes in rejecting Salmon's Intransitive Accessibility Solution gives a defense of S5 as the logic of metaphysical modality. Forbes argues that metaphysical necessities, *a priori* or *a posteriori*, are fundamentally conceptual truths, and conceptual truths should be held the same in every possible world because all possible worlds are constructed in the same conceptual system. We shall show that this defense of S5 is not sufficient. The logic of metaphysical modality should be S5 if the conceptual system, in which possible worlds are constructed, has the feature of being "absolute essentialist"—the essential properties of an object possessed in a world w are essential to the object *simpliciter*. On the other hand, a sufficient reason to reject S5 consists in showing that an absolute essentialist conceptual system is not acceptable, instead, the conceptual system should be a "relative essentialist" one—the essential properties of an object possessed in world w are essential to the object with respect to the world w. Salmon in his argument of Intransitive Accessibility Solution tries to provide a reason for the unacceptableness of absolute essentialism. He tries to show that the intransitive accessibility relation, or relative essentialism, is a consequence of his theory of possible worlds and the truths of the modal principles (the essentialist principles and the tolerance principle). We shall argue that relative essentialism, or the intransitivity of metaphysical necessity, is actually a presupposition of Salmon's argument of the Intransitive Accessibility Solution, and is by no means a non-trivial consequence of the premises. We shall also examine another argument of Salmon's in attempt to show the falsity of some absolute essentialist principle by means of Leibniz's Law. Our analysis will show that this attempt is also failed. We shall conclude that Salmon's Intransitive Accessibility Solution is a solution to the two paradoxes from a relative essentialist point of view. The plausibility of the

Intransitive Accessibility Solution relies entirely on the plausibility of the relative essentialism. However, no successful defense of relative essentialist position can be found in Salmon's account.

In Chapter 4, we shall begin by examining Forbes' Counterpart Solution. Our view is that Forbes' Counterpart Solution is not a successful solution to the two paradoxes. The Counterpart Solution uses a trans-world similarity relation to represent the trans-world identity relation and uses a range of intermediate truth values to express the decrease of similarity between the actual matter of the given artifact and the hunks of matter in the sequence derived from the tolerance principle. A trans-world identity relation may be indeterminate in some sense in some cases, but we do not think that identity relations, intro-world or trans-world, can be viewed as true in a certain degree or percentage. Thus, the identity relation is either misrepresented or completely removed by the counterparthood. This solution in fact solves the paradoxes and defends S5 by avoiding trans-world identity. We shall provide a different diagnosis to the two paradoxes. In disagreement with Salmon and Forbes, we hold that the particular version of the tolerance principle used in the modal inferences to infer an infinite sequence of possibilities for the given artifact's original construction is an incorrect principle. We call this version the "strong tolerance principle." We shall provide counterexamples for demonstrating the falsehood of the strong tolerance principle. We shall discuss some philosophical views that support our rejection of the strong tolerance principle. Finally, we shall defend the absolute essentialist intuition about the original construction of artifacts, and in turn, the validity of S5 as the logic of metaphysical modality.

CHAPTER 1

MODAL PRINCIPLES, PARADOXES AND SALMON'S INTRANSITIVE ACCESSIBILITY SOLUTION

1.1 Two Essentialist Principles

Essentialism holds the doctrine that certain properties of an object are properties that this object could not fail to have, except by not existing. Thus, the essential properties of an object are properties such that necessarily, if the object exists, it has them. Among essential properties there are some trivial ones, for instance, property of self-identity, property of existing and properties that are *de dicto* truths. Trivial essential properties are essential properties possessed of necessity by every existing thing. The interesting essential properties are non-trivial ones. Non-trivial essential properties are those possessed by a particular thing only or by a particular kind of things only. An individual essence of object x can be defined in terms of non-trivial essential properties of x as follows: an individual essence of object x is a set of properties such that every property in the set is a non-trivial essential property of x and for any object y distinct from x it is impossible that y has every property in the set. We ignore trivial essential properties as they are possessed by everything. In other words, the individual essence of object x composed of non-trivial essential properties of x is the smallest set which necessarily distinguishes x from all other possible objects, and hence it provides a necessary and sufficient condition for the identification of ***x***. There are many discussions about whether there ever exist such individual essences and whether, in what sense, essentialism is true. The paradoxes that concern us, Chisholm's Paradox and the Four Worlds Paradox, may be seen as a challenge to the essentialist belief in individual essences. Specifically, these paradoxes challenge the following two essentialist principles, the Principle (**N**) and Principle (**C**).

(**N**) If a wooden table ***x*** is originally made from a hunk of matter ***y***, and **z** is any hunk of matter whose collection of components is sufficiently different from the collection of ***y***, then ***x*** could not have been originally made from **z**.

(**C**) If a wooden table ***x*** is such that it might have been made originally from a hunk of matter ***y*** according to a certain plan ***P***, then there could not be any table distinct from ***x*** and made from hunk ***y*** according to the same plan ***P***.

Principles (**N**) and (**C**) are in accordance with the essentialist principles stated in Salmon's works when he discusses these paradoxes. Principle (**N**) is in accordance with Principle (III) in Salmon[1986], and Principle (**C**) with Principle (I) in Salmon[1986] as well as Principle (V') in Salmon[1981]. [8]

[8] See Salmon [1981], 211, and Salmon [1986], 75 and 77. Principle (V') says: "If it is possible for a given table x to be originally constructed from a certain hunk of matter y according to a certain plan P, then necessarily any table originally constructed from hunk y according to precisely the same plan P is the very table x and no other." Principle (I) says: "If a wooden table x is such that it might have been the only table originally formed from a hunk of matter z according to a certain plan

Principle (**N**) can be seen as a weakened version of Kripke's well-known thesis, "The Necessity of Origin": "if a material object has its origin from a certain hunk of matter, then it could not have its origin in any other matter."[9] Principle (**N**) is weaker than Kripkean thesis because of a weaker condition that hunk ***z*** is not merely different, but sufficiently different, from table ***x***'s original matter hunk ***y***. However, Principle (**N**) is stronger than the weakest version of Kripkean thesis that a material object could not have its origin in any other matter entirely different from the one from which it actually originates. The adjustment in Principle (**N**) is made for accommodating the intuition about tolerable variations in the original matter of an artifact. This intuition has two aspects. One aspect is that some slight variations in the original matter of a given table are considered as possible—these variations do not affect the identity of the given table. This aspect is expressed by the "Tolerance Principle" which we shall introduce in Section 1.2. The other aspect is embodied in Principle (**N**) that it is metaphysically impossible for the given table to be made from a hunk of matter which is dramatically or completely different from the matter from which the table actually originates.

Principle (**C**) is a strengthened version of another principle called "the Principle of Cross-World Identification," which says that if a material object has its origin from a certain hunk of matter, then

P, then there could not be a table that is distinct from x and the only table formed from hunk z according to plan P." Principle (III) says: "If a wooden table x is the only table originally formed from a hunk of matter y, and z is any hunk of matter that does not sufficiently substantially overlap y, then x is such that it could not have been the only table originally formed from z instead of from y."

[9] Kripke [1972], 114, footnote 56.

there could not be any other object originating from this hunk of matter(assuming that other essential properties hold the same for the material objects in question). Principle (**C**) is a stronger version because it asserts not only that other tables could not be made according to the same plan from table ***x***'s actual original matter, but also that they could not be so made from any of table ***x***'s possible matter. The adjustment in Principle (**C**) is made for the same reason as in the case of Principle (**N**): if certain slight changes in the given table's original matter are possible, then it is sufficient that any table being constructed from a possible matter of the given table according to the same plan is the given table itself and no other.

Principle (**N**) and Principle (**C**) are deemed as metaphysically necessary principles. If these principles are true, they are not only true as a matter of fact, but bound to be true no matter how different the world turns out to be. We may formulate Principle (**N**) and Principle (**C**) as necessary principles in the following way.

(**N***) $\Box\forall \boldsymbol{x}\Box\forall \mathbf{y}\Box\forall \mathbf{z}(\boldsymbol{D}(\mathbf{z},\boldsymbol{y})\supset\Box(\boldsymbol{M}(\boldsymbol{x},\boldsymbol{y})\supset\Box\neg \boldsymbol{M}(\boldsymbol{x},\mathbf{z})))$.

Let ***M***(α, β) mean that wooden table α is originally made from hunk of matter β, where by "α is originally made from β" we mean that β is a collection of components which forms α without any left-over. Let ***D***(δ, β) mean that the collection of the components of δ is sufficiently different from the collection of the components of β. Accordingly, let ***x*** be a wooden table, and ***y*** and **z** be hunks of forming materials for wooden tables. Then (**N***) says: For any possible wooden table ***x***, any possible hunk of matter ***y*** and any possible hunk of matter **z**, if **z** is sufficiently different from ***y***, then if it turns out to be the case that x is made from y, then it could not be the case that x is made from z.

(C*) $\Box\forall x\Box\forall y\Box\forall j((j\neq x)\supset\Box(\Diamond(M(x,y)\&Px)\supset\Box\neg(M(j,y)\&Px)))$.

Let j be a table, and $P\alpha$ means that α is made according to plan P. Then (C*) says: For any possible wooden table x and any possible hunk of matter y and any possible wooden table j, if j is not x, then if it turns out to be the case that x is possibly made from y according to certain plan P, then it could not be the case that j is made from y according to the same plan.

Principle (N) and Principle (C) are also deemed as *a priori* propositions. They are principles from *a priori* philosophical analysis (Kripke) or from reflections upon the (essentialist) concept of thisness for individual artifacts(Salmon).[10] An important feature of these principles is that from these principles and some empirical information, certain necessary *a posteriori* truths are derivable. For example, (N), together with the empirical fact that table α is made from hunk of wood ρ and the assumption that δ is a possible hunk of wood sufficiently different from ρ, entails the proposition that table α could not have been made from δ. Similarly, given the empirical knowledge that table α and table β are distinct and the assumption that hunk σ is a possible hunk of matter for table α, one can infer that table β cannot be made from σ according to the same plan. Though Principles (N) and (C) are deemed *a priori*, the necessary propositions derivable from them are not, because some empirical information is needed for obtaining these propositions.

[10] See Kripke [1971] in Schwartz [1977] on page 88 and Salmon [1981] on pages 263-264.

Notice that Principle (**C**) assumes an unchanged "plan ***P***" for the tables in question. We use the word "plan" in Principle (**C**) to refer to all other non-trivial essential properties of those tables, which include, for example, the design(form, size and configuration) and the particular construction process for the table(the raw material, the processing method, and maybe the maker). The meaning of the word "plan" in (**C**) is in agreement with Salmon's usage of the word "plan" in his Principle (V′) in Salmon[1981] and Principle (I) in Salmon[1986].[11] In Principle (**N**) the condition of being made from the original matter is a necessary condition for a certain table to be itself. In other words, a table's being made from its original matter is a property in the individual essence of the table. In Principle (**C**) that tables in question share the same "plan," namely, they share all other essential properties than the original matter. Principle (**C**) indicates that a table distinct from table α cannot share every non-trivial essential property in the individual essence of α. In other words, possessing every essential property of table α is sufficient for an object to be α.

1.2 The Tolerance Principle

"Tolerance Principle" refers to the principle that expresses the intuition about possible variations in the original matter of artifacts. As we briefly stated in the Introduction, the Tolerance Principle employed in the paradoxical inferences has the feature that we can infer from this principle an infinite sequence of possible

[11] Salmon [1981], 210-211. Salmon [1986], 75.

constructive matter for an artifact such that each successive occurrence in the sequence is sufficiently similar to its immediate predecessor, but shares fewer components with the first occurrence than its immediate predecessor does. Thus, whatever standard one may give to the notion "sufficiently similar," the occurrences in the sequence after some point will not be sufficiently similar to the first occurrence. There are, however, different versions of the Tolerance Principle, and not every version possesses the above feature. We shall in this section try to distinguish between different versions of the Tolerance Principle, and make clear which version possesses the above feature and which does not.

It is said that it is intuitively not plausible to insist that every chip or molecule of a given table is essential to the table such that none of them could have been replaced by any qualitatively identical chip or molecule. This is just to say that *some* replacements of the components in the original matter of an artifact are possible.[12] We think that this intuition is acceptable. In addition, we seem to have the intuition that the changes in artifacts' original matter must be small. Combining these two intuitions we may say that an artifact composed of components could be originated from *some* hunk of

[12] Note that we are talking about the forming matter of a table in the way of talking set-theoretically the collection of components of that matter. We thus ignore the details about where each chip or molecule is put to make the table. When we say that one chip is replaced by another qualitatively identical chip, our notion allows the cases that the replacing chip may or may not be in the same spot of the table as the replaced chip was. Also the intuition stated here is an over simplification. We are not only have the intuition about the tolerable replacement of the chips, some quantitative changes in the collection of components may also be tolerable. For example, some chips may be simply missing from the set or some chips may be simply added to the set. This simplification does not alter the nature of the tolerance and is preferred for the discussion.

matter whose collection of components is slightly different from the one from which it actually originates. (Here we confine the difference only to the case that some components are replaced by numerically different but qualitatively identical components in order to simplify the discussion.) This is the intuition captured in principle (**T**) below. We use the phrase "collection of components" as a set-theoretical notion. When we say that collection A is different from collection B, we mean that A and B are different sets—some member of A is not in B or vice versa.

> (**T**) If a wooden table ***x*** is originally made from a hunk of matter ***y*** according to a certain plan ***P***, then ***x*** might have been made according to the same plan **P** from *a(some)* hunk of matter ***y′*** instead of from ***y***, where ***y′*** has the same quantity and quality as ***y*** but the collection of components of ***y′*** is slightly different from that of ***y***.

(**T**) is the weak version of the Tolerance Principle. This version is given in Salmon[1986] as the first unnumbered principle.[13] Also this version is the only version literally given in Forbes[1985]. The Tolerance Principle is also deemed as a metaphysically necessary principle. Forbes has argued that it is not some special properties of actual artifacts or of the actual world that make it true. Had things

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[13] Salmon [1986], 75. Salmon's first unnumbered principle says: "If a wooden table x is the only table originally formed from a hunk (portion, quantity, bit) of matter y according to a certain plan (form, structure, design, configuration) P, then x is such that it might have been the only table formed according to the same plan P from a distinct but overlapping hunk of matter y′ having exactly the same mass, volume, and chemical composition as y." Also this version of tolerance principle is the only version literally given in Forbes [1985] on page 161.

been different, there would still have been this Tolerance Principle.[14] Salmon also views the Tolerance Principle as an *a priori* necessity which has to do with our concept of artifact. The necessary principle **(T)** may be formulated as:

$$(\mathbf{T^*})\quad \Box\forall \boldsymbol{x}\Box\forall \boldsymbol{y}\Diamond\exists \boldsymbol{y}'(\boldsymbol{S}(\boldsymbol{y}',\boldsymbol{y})\&\Box((\boldsymbol{M}(\boldsymbol{x},\boldsymbol{y})\&\boldsymbol{Px})\supset\Diamond(\boldsymbol{M}(\boldsymbol{x},\boldsymbol{y}')\&\boldsymbol{Px}))).$$

Let $\boldsymbol{y}'$ be a hunk of matter of the same sort as $\boldsymbol{y}$, and let $\mathbf{S}(\delta, \beta)$ mean that the collection of components of δ is slightly different from the collection of the components of β, but δ is qualitatively and quantitatively the same as β. **(T*)** says: for any possible wooden table $\boldsymbol{x}$ and any possible hunk of matter $\boldsymbol{y}$, there is a possible hunk of matter $\boldsymbol{y}'$ such that the collection of components of $\boldsymbol{y}'$ is slightly different from but qualitatively and quantitatively the same as that of $\boldsymbol{y}$, and if it turns out to be the case that $\boldsymbol{x}$ is made from $\boldsymbol{y}$ according to plan $\boldsymbol{P}$, then table $\boldsymbol{x}$ might have been made from $\boldsymbol{y}'$ according to the same plan $\boldsymbol{P}$.

Principle **(T)** only asserts that table $\boldsymbol{x}$ might have been made from *some* sufficiently overlapping matter; therefore, "sufficiently overlapping" is not a sufficient condition for a hunk of matter to be a possible matter of table $\boldsymbol{x}$. For example, given an actual table α, its original matter β, a standard for what counts as slightly different and a hunk of matter β' satisfies the standard, one is unable to determine, by **(T)** and the given premises, whether it is possible for α to be made from β'. One needs some additional information or other reasoning to determine whether β' is a possible matter of table **a**. The property of being "slightly different from α's original matter" in **(T)**

[14] Forbes [1985], 161.

is not a sufficient condition, for a hunk of matter to be a possible matter of table α. It is illegitimate to assert any specific possible matter of table α from **(T)** and α's actual matter. Principle **(T)** seems to me intuitively correct, but this is not the version that was involved in the paradoxical inferences.

The version of Tolerance Principle involved in the paradoxical inferences is the following one. We call it **(ST)** standing for Strong Tolerance Principle.

> **(ST)** If a wooden table x is originally made from a hunk of matter y according to a certain plan P, then x might have been made according to the same plan P from *any* hunk of matter y', instead of from y, where y' has the same quantity and quality as y but the collection of components of y' is slightly different from that of y.

We formulate **(ST)** accordingly as a necessary principle:

(ST*) $\Box\forall x\Box\forall y\Box\forall y'(S(y',y)\supset\Box((M(x,y)\&Px)\supset\Diamond(M(x,y')\&Px)))$.

(ST*) says: For any possible wooden table x, any possible hunk of matter y and any possible hunk of matter y', if the collection of components of y' is slightly different from that of y and y' is qualitatively and quantitatively the same as y, then if it turns out to be the case that x is made from y according to plan P, then it might have been the case that x is made from y' according to the same plan P.

(ST) is a much stronger principle, since it asserts that table x might have been made according to the same plan from *any*, instead of from *some*, slightly different matter. Given an actual table α, its matter β, and a hunk of matter β' sufficiently overlapping but qualitatively and quantitatively the same as β, one can infer from these premises and **(ST)** that table α might have been made according to the same plan from β′. Furthermore, given a standard for what counts as "slightly different," one can infer a sequence of possibilities from **(ST)**. In S4 or S5, all the possibilities inferred in the sequence are real possibilities with respect to the actual table α. The property of "slightly different from table α's original matter" in **(ST)** is a sufficient condition for the inference about table α's possible constructions.

(ST) is the same principle as Principle (II) in Salmon[1986],[15] which is explicitly employed in his presentation of the paradoxical inferences. It is not specified in Salmon[1981] and Salmon[1989] which version of Tolerance Principle is employed, but his discussions indicate that Salmon has **(ST)** in his mind.

Salmon claims that principle (II), namely, our principle **(ST)**, is intuitively and literally true.[16] It is not clear to me that the intuition embodied in **(ST)** is indeed true. We ask what exactly is the intuition in **(ST)**, especially, the intuition underlying the word "any" in **(ST)**. This is a question about the reason that permits the move from **(T)** to **(ST)**. In Salmon[1986], we see another unnumbered principle. (The first unnumbered principle in Salmon [1986] is the weak version of tolerance principle that we stated in **(T)**.) Though I am not certain of

15 Salmon [1986], 77. Principle (II) says: "If a wooden table x is the only table originally formed from a hunk of matter y according to a certain plan P, and y′ is any (possibly scattered) hunk of matter that sufficiently substantially overlaps y and has exactly the same mass, volume, and chemical composition as y, then x is such that it might have been the only table originally formed according to the same plan P from y′ instead of from y."

16 Salmon [1986], 80.

Salmon's own intention for introducing this principle there, we may see this principle as elucidating the intuition in **(ST)**. Let us call this principle **(M_1)**, as being the first principle considered in permitting the move from **(T)** to **(ST)**.

> **(M_1)** "If a wooden table $\boldsymbol{x}$ originally formed from a hunk of matter $\boldsymbol{y}$ is such that it might have been originally formed from a hunk of matter $\boldsymbol{y}'$ according to a certain plan $\boldsymbol{P}$, then for any hunk of matter $\boldsymbol{y}''$ having exactly *the same matter* in common with $\boldsymbol{y}$ that $\boldsymbol{y}'$ has, and having exactly the same mass, volume, and chemical composition as $\boldsymbol{y}'$, $\boldsymbol{x}$ is also such that it might have been originally formed from $\boldsymbol{y}''$ according to the same plan $\boldsymbol{P}$."[17]

(M_1) expresses the following thought: if table α made from hunk β is possible to be made from hunk β', then α is possible to be made from any hunk of matter β'' which shares exactly the same components with β as β' does but differs from β' in the place where β' differs from β with some component which is qualitatively identical with but numerically distinct from the corresponding component of β'. This is to propose that one can use a known possibility as a prototype to infer other possibilities. Suppose that β is composed of $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{10}\}$ and that β' overlaps β except having $\boldsymbol{c}_{11}$ as its component instead of $\boldsymbol{c}_{10}$. Suppose that there is a hunk of matter β'' composed of $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_9, \boldsymbol{c}_{12}\}$ in which $\boldsymbol{c}_{12}$ is qualitatively the same as, but numerically distinct

[17] Salmon [1986], 75.

from, c_{11}. By $(\mathbf{M}_1)$, one can infer that table α is also possible to be made from β''.

Given the understanding of (**ST**) in the sense of $(\mathbf{M}_1)$, (**ST**) still does not have the inferential power needed for the paradoxical inferences. When the phrase "any slightly different matter" in (**ST**) is given in the sense of $(\mathbf{M}_1)$, it means "any matter that differs slightly from α's original matter in the same way as the prototype does." By "the same way" we mean what we just explained about the content of $(\mathbf{M}_1)$. The simple property of "slightly different from table α's original matter" is still not a sufficient condition for determining any possible matter for table α. How can one decide whether a hunk of matter is such a prototype is not shown in $(\mathbf{M}_1)$. One needs additional information or other reasoning to determine whether a hunk of matter, which is slightly different from α's original matter, is a prototypic possible matter of α. Principle (**ST**) in the sense of $(\mathbf{M}_1)$ is just as weak as (**T**) when merely a standard for "slightly different" is assumed, though it does strengthen (**T**) in the aspect that some inferences can be done when a prototype is given. Notice that the inference from a prototype does not automatically produce the kind of sequence of possibilities in which each successor differs from the artifact's original matter more than its immediate predecessor does, provided that the prototypes are not given as such a sequence. However, if one does assume the prototypes as such a sequence, we would say that he probably has in mind some even stronger principle, namely, principle $(\mathbf{M}_2)$. In principle $(\mathbf{M}_2)$, the property of "slightly different from **a**'s original matter" is taken as a sufficient condition for inferring possible matter for artifacts.

($\mathbf{M}_2$) If a wooden table $\boldsymbol{x}$ originally formed from a hunk of matter $\boldsymbol{y}$ is such that it might have been originally formed from a hunk of matter $\boldsymbol{y}'$ according to a certain plan $\boldsymbol{P}$, then for any hunk of matter $\boldsymbol{y}''$ having exactly *the same number of components* in common with $\mathbf{y}$ that $\boldsymbol{y}'$ has, and having exactly the same mass, volume, and chemical composition as $\boldsymbol{y}'$, $\boldsymbol{x}$ is also such that it might have been originally formed from $\boldsymbol{y}''$ according to the same plan $\boldsymbol{P}$.

Principle ($\mathbf{M}_2$) proposes a pure quantitative condition, based on how many components of a given matter differ from the table's actual matter regardless of which components are being replaced, as a sufficient condition for inferring possible matter for the given table. The difference between ($\mathbf{M}_1$) and ($\mathbf{M}_2$) is the following. When inferring possibilities from a prototype according to ($\mathbf{M}_1$), one considers not only how many components of the prototype are changed from the original matter but also which component of the original matter is changed in the prototype. The second required consideration makes the pure quantitative condition not sufficient for inferring possibilities for the table. But principle ($\mathbf{M}_2$) allows us to consider only how many components are changed. Let table α, its actual matter β, $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{10}\}$, and the known possible matter β', $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_9, \boldsymbol{c}_{11}\}$, be the same as described above. Suppose that there is another hunk of matter β''' composed of $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_8, \boldsymbol{c}_{13}, \boldsymbol{c}_{10}\}$ and $\boldsymbol{c}_{13}$

is qualitatively equivalent to $\boldsymbol{c}_9$. From ($\mathbf{M}_1$), we don't know whether table α could possibly be made from β'''. But according to ($\mathbf{M}_2$), this is a possibility since β''' differs from β in as many components as β' does and is qualitatively and quantitatively the same as β'. The intuition embraced in ($\mathbf{M}_2$) is probably this: Other components of table α's original matter are no more essential to the identity of table α than the components that are replaced in β', the given prototype of possible matter of α—if these components are replaced in β' without affecting the identity of table α, the replacement of other components instead of the ones replaced in β' should be viewed the same as long as the amount of the changes remains the same. This intuition views the quantitative condition as sufficient for inferring possible forming matter for artifacts. In principle ($\mathbf{M}_2$), the prototype $\boldsymbol{y}'$ merely serves as a quantitative standard for the tolerable changes in table $\boldsymbol{x}$'s original matter. It is an easy step to drop the prototype and maintain only the quantitative standard. In the case of table α, when a pure quantitative standard is taken as sufficient condition for inferring possible matter for α, a sequence of possible matter of α, in which each successor shares less components with α's original matter β than its immediate predecessor does, can be inferred in S4 or S5 modal logic. We shall see that Chisholm's Paradox and the Four Worlds Paradox are derived in S4 and S5 from (**N**), (**C**) and (**ST**) in the sense of ($\mathbf{M}_2$).

1.3 Two Paradoxes

The argument for Chisholm's Paradox proceeds in the following way. The modal logic employed in the inference is S5. Suppose that we have a wooden table α, in the real world $\mathbf{w}_@$, made from a hunk of matter $\boldsymbol{h}_0$ composed of $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{100}\}$ according to plan $\boldsymbol{P}$. Assume that a hunk of matter differing from $\boldsymbol{h}_0$ by only one component definitely counts as slightly different from $\boldsymbol{h}_0$. By the Strong Tolerance Principle (**ST**) there is a world $\boldsymbol{w}_1$, possible relative to the real world $\boldsymbol{w}_@$, in which α is made according to the same plan $\boldsymbol{P}$ from hunk of matter $\boldsymbol{h}_1$ composed of $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{99}, \boldsymbol{c}_{101}\}$, where $\boldsymbol{c}_{101}$ is qualitatively identical to but numerically distinct from $\boldsymbol{c}_{100}$. Since (**ST**) is a necessary principle, there is a world $\boldsymbol{w}_2$ possible relative to $\boldsymbol{w}_1$ in which α is made according to plan $\boldsymbol{P}$ from a hunk of matter $\boldsymbol{h}_2$ composed of $\{\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{98}, \boldsymbol{c}_{102}, \boldsymbol{c}_{101}\}$ where $\boldsymbol{c}_{102}$ is qualitatively identical to but numerically distinct from $\boldsymbol{c}_{99}$. Suppose that $\boldsymbol{h}_0, \boldsymbol{h}_1, \ldots, \boldsymbol{h}_m, \ldots$, are hunks of matter such that each hunk is different from its immediate predecessor in only one component and, as the subscript increases, the hunk of matter shares fewer components with $\boldsymbol{h}_0$, the original matter of α in $\boldsymbol{w}_@$. Let $\boldsymbol{w}_1, \ldots, \boldsymbol{w}_m, \ldots,$ be a sequence of worlds such that $\boldsymbol{w}_1$ is possible relative to $\boldsymbol{w}_@$ and, for each $i>1$, $\boldsymbol{w}_i$ is possible relative to $\boldsymbol{w}_{i-1}$ and table α is made from $\boldsymbol{h}_i$ in $\boldsymbol{w}_i$. By applying (**ST**) repeatedly for the worlds in the sequence, we eventually reach a world $\boldsymbol{w}_n$, possible relative to $\boldsymbol{w}_{n-1}$, in which α is made according to plan $\boldsymbol{P}$ from hunk $\boldsymbol{h}_n$ which differs from $\boldsymbol{h}_{n-1}$ by one component but shares no component with $\boldsymbol{h}_0$. By transitivity in S5 modal logic, $\boldsymbol{w}_n$ is also possible relative to $\boldsymbol{w}_@$. Thus we arrive at the conclusion that from the standpoint of $w_@$ table α could have been made from hunk h_n according to plan P. But, since h_n shares no component with h_0, the two hunks are definitely sufficiently different. By principle (**N**), we come to the conclusion that from the standpoint of $w_@$ table α could not have been made from hunk h_n according to plan P. Hence, we derived a contradiction from principles (**N**) and (**ST**).

The argument for the Four Worlds Paradox assumes a boundary between possible matter and impossible matter of table α. Suppose that h, the original matter of α in the real world, has one hundred components represented by the set $\{c_1, c_2, ..., c_{100}\}$. Assume that any hunk of matter with the same quantity and quality as h but having more than two components different from h counts as sufficiently different from h. Let the real world be α-world1, and let β-world1 be a world, possible relative to α-world1, in which a distinct wooden table β is made from the hunk of matter h' composed of $\{c_1, ..., c_{96}, c_{101}, c_{102}, c_{103}, c_{104}\}$ according to the same plan P according to which table α is made in α-world1. In virtue of principle (**ST**) and above assumption of the tolerable variations, there is a world, α-world2, possible relative to α-world1, in which α is made according to plan P from a hunk of matter h'' composed of $\{c_1, ..., c_{98}, c_{103}, c_{104}\}$. Also, there is another world, β-world2, possible relative to β-world1, in which β is made according to plan P from $\{c_1, ..., c_{98}, c_{103}, c_{104}\}$, the very same hunk of matter as α's matter in α-world2. Since β-world1 is possible relative to α-world1 and β-world2 is possible relative to β-world1, by the transitivity of S5 modal logic, β-world2 is also possible relative to α-world1. This is to say that from the standpoint of the α-world1, table β, distinct from table α, could have been made according to the same plan ***P*** from the hunk $\{\mathbf{c}_1, \ldots, \mathbf{c}_{98}, \mathbf{c}_{103}, \mathbf{c}_{104}\}$, a possible matter of table α. But by principle **(C)**, we have that from the standpoint of α-world1, table β could not be made according to the same plan ***P*** from the hunk $\{\mathbf{c}_1, \ldots, \mathbf{c}_{98}, \mathbf{c}_{103}, \mathbf{c}_{104}\}$. Thus, we have derived a contradiction from principle **(C)** and the Strong Tolerance Principle **(ST)**.

Salmon has argued that even if principle **(C)** is eliminated from the above argument, an equally paradoxical argument can be constructed by the following consideration. In the above argument of the Four Worlds Paradox there is nothing that requires α-world2 and β-world2 to differ in anyway except the identity of two tables α and β. The notion of materially complete proposition is given as: "A proposition is materially complete if it is a complete enumeration of every particle of matter in the cosmos throughout all of a potential history of the world, as well as a complete specification of all the physical interactions and configurations of all the matter in the cosmos in exact chronological sequence throughout that potential history."[18] For each possible world there corresponds a materially complete proposition. (All materially complete propositions corresponding to the same possible world should be equivalent). Assume that α-world1 and β-world1 are exactly the same except that two tables α and β differ by four qualitatively identical components. Assume also that the other two worlds, α-world2 and β-world2, resemble α-world1 and β-world1 respectively with the only

[18] Salmon [1986], 79.

difference of two components in the tables' original matter. Let p be the materially complete proposition corresponding to α-world2. By **(ST)**, it is possible from the standpoint of α-world1 that p is true and table α is made from h''. $(\Diamond(p \& M(\alpha,h'')))$ But, by the same principle **(ST)** through transitivity, it is also possible from the standpoint of α-world1 that p is true and it is not the case that table α is made from h''. $(\Diamond(p \& \neg M(\alpha,h'')))$ This means that the identity of a table cannot be decided by a complete description of material facts. This conclusion contradicts the supervenience principle: a material object is nothing over and above its matter and form. Salmon holds that the supervenience principle is true for the material objects like tables. He says that the fact that h'' constructs table α, "if it does, is supervenient on a complete possible history of all the matter in the cosmos."[19] According to the supervenience thesis, either p necessarily entails that table α is made from h'', or p necessarily entails that it is not the case that table α is made from h''. $(\Box(p \supset M(\alpha,h'')) \vee \Box(p \supset \neg M(\alpha,h'')))$ Thus the second version of the Four Worlds Paradox.

The Supervenience Principle is a more general principle than principle **(N)** and principle **(C)**. Principles **(N)** and **(C)** are supervenience principles on a particular object—they follow from the general supervenience principle. In the second version of the Four Worlds Paradox, principle **(C)** is replaced by the general supervenience principle.

[19] Salmon [1986], 80.

1.4 Salmon's Account of Possible Worlds

In order to have a clear understanding of Salmon's Intransitive Accessibility Solution to the paradoxes, it is important to explain some crucial concepts and viewpoints of the theory of possible worlds that Salmon endorses, which constitute the theoretical basis for his solution and his defense for his solution.

Salmon holds the following concept of possible worlds: In our pre-philosophical views we think that there are alternative ways things might have gone, or other situations in which things could have been. "Possible worlds" is just a philosopher's name for the different ways things might have been. There are various approaches in construing the ontological nature of possible worlds, namely, what kind of entity the possible worlds are. Salmon holds that possible worlds are actually existing maximal abstract entities that can be instantiated or realized. First, possible worlds are entities constructed in our real world—they are not entities existing spatially and temporally in some place disconnected from the real world. Hence possible worlds are actually existing entities. Secondly, a possible world is not a "world" in the usual sense, that is, it is not a concrete object of the same sort as our real world. A possible world is an abstract entity that exists in the real world. Philosophers who view possible worlds as abstract entities construe possible worlds either as maximal propositions, or as maximal sets of propositions, or as maximal states of affairs, or as a maximal property. For Salmon, a possible world is called a "maximal scenario." Thirdly, possible worlds are maximal entities. In Salmon's words, "a possible world [is] a set of (potential) facts or statements that does not leave any of a very comprehensive range of questions of fact undecided."[20] Lastly, possible worlds are a special sort of abstract maximal entity such that in a certain sense they are possibly to be instantiated or realized.

Based on his modal notion of possible world, Salmon emphasizes on the following three viewpoints.

(1) One must distinguish between the generic notion of a world (a way for things to be) and the modal notion of a possible world (a way things might have been). Salmon points out that not all actual existing maximal abstract entities of the sort (propositions, properties or states of affairs) are possible worlds. As a maximal abstract entity it can be any arbitrary maximal combination of the answers to the questions of fact—it needs not be compossible in any sense and even needs not be logically consistent. "Worlds" is a name for all maximal abstract entities, which is defined by him as "total way-for-things-to-be-even-if-things-could-not-have-been-that-way." A possible world represents "a way for things to be such that things might have been that way."[21] The notion of possibility is crucial to the understanding of the notion of a possible world; one must know

[20] Salmon [1989], 6. Here by proposition Salmon means propositions as Russell conceived. He means to exclude the modal logician's conception of a proposition as a set of possible worlds, which would create a circle in explaining the notions. In saying "any of a very comprehensive range of questions of fact" instead of "any question of fact", Salmon is considering the following two cases: (i) given a certain decision on some statement, some other statements are neither true nor false under this condition, and hence neither these latter statements nor their negations can be included in the set of statements of the world which contains the former statement; (ii) certain meta-facts or facts about possible worlds and sets of facts cannot be included in the set of statement of a world for the reasons concerning cardinality problems. (Note that in holding (i), Salmon adopts the Fregean view: Given that the statement "there is no present King of France" is true, the statement "the present King of France is bald" is neither true nor false.)

[21] Salmon [1989], 12.

what the possibilities are in order to know whether a way for things to be is a way that things might have been.

(2) The possibilities and necessities of a given world determines which world is possible relative to the given world. According to Salmon, possible worlds should always be viewed in a relative sense with respect to a certain world. Whether a world is possible relative to a given world is decided by the possibilities the given world has. Salmon defines the notion of relative possibility or accessibility of worlds as this: a world $\boldsymbol{w}'$ is metaphysically possible relative to a world $\boldsymbol{w}$ if and only if every fact of $\boldsymbol{w}'$ is a metaphysical possibility in $\boldsymbol{w}$ (that is, every proposition that is true according to $\boldsymbol{w}'$ is possible according to $\boldsymbol{w}$). This is the standard definition of relative possibility between worlds given in Kripke's pioneer work on the semantics of modal logic.[22] Salmon gives another definition of accessibility which he claims is equivalent to the first one: $\boldsymbol{w}'$ is metaphysically possible relative to $\boldsymbol{w}$ if and only if every metaphysically necessary fact of $\boldsymbol{w}$ obtains in $\boldsymbol{w}'$ (that is, every proposition that is necessary according to $\boldsymbol{w}$ is true according to $\boldsymbol{w}'$).[23] These two definitions of relative possibility relation describe how possible worlds of a given world are determined by the necessities and possibilities of the given world.

We have some comments on the equivalence of the two definitions of accessibility. As we see it, these two definitions are

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22 Kripke [1963], 70.
23 Salmon[1989], 18.

equivalent only if the worlds in question are all consistent.[24] Suppose that world w' is possible relative to a consistent world w by the second definition that every necessary fact of w obtains in w'. If w' is inconsistent, it may still contain some fact that is not a possibility in w. Thus according to the first definition that w' is possible relative to w if and only if every fact of w' is a possibility in w, w' is not possible relative to world w. For example, assume that it is a necessity of w_i that table α cannot be made from hunk of matter h. World w_k, a world possible relative to w_i according to the second definition, must contain the fact that table α is not made from hunk h. Assume that w_k is inconsistent and contains the fact that table α is made from hunk h. Since the fact that table α is made from hunk h is not a possibility of w_i, w_k is not a possible world of w_i according to the first definition. A similar reasoning can be given when we assume that w is inconsistent but w' is consistent.

(3) There exist impossible worlds. In Salmon's theory of possible worlds, the set of possible worlds with respect to world w represent the ways things might have been according to world w. The other worlds that are not qualified to be in the set of possible worlds of w are ways things cannot be according to w, namely, they are impossible worlds relative to world w. Salmon insists on the actual existence of impossible worlds. The impossible worlds of w possess different modal properties from those possible worlds of w. But the impossible worlds of w and the possible worlds of w are not

[24] When the worlds in question are all inconsistent, the two definitions are also equivalent. But these cases are philosophically uninteresting and logically trivial because an inconsistent world contains everything.

different ontologically—they all maximal abstract entities existing in the real world.

Salmon distinguishes the metaphysically impossible worlds that are only contingently impossible from those that are essentially impossible with respect to a certain world ***w***.[25] The contingently impossible worlds of ***w*** are those that are not possible relative to ***w***, but possibly possible, or possibly possibly possible, or possibly ... possibly possible relative to ***w***. That is, a contingently impossible world according to world ***w*** is a world which is not possible relative to ***w*** but bears an ancestral relative possible relation to ***w***. On the other hand, an essentially impossible world relative to world ***w*** is a world which is not even possibly possibly ... possible to any degree of nested accessibility relation to ***w***. Salmon gives two examples for essentially impossible worlds: "A world according to which Nathan Salmon is Henry Kissinger is such a world, for example, as is a world according to which Nathan Salmon is a Visa credit card account with the Bank of America. Since they are ways-for-things-to-be of a certain sort (viz., such that things necessarily cannot be that way, and necessarily necessarily cannot be that way, and so on), ..."[26] Given that the name "Nathan Salmon" rigidly denotes the person Nathan Salmon and the name "Henry Kissinger" rigidly denotes the person Henry Kissinger, the essentially impossible world in which Nathan Salmon is Henry Kissinger is a logically inconsistent world, since it contains both the statement that Nathan Salmon is Henry Kissinger and the statement that it is not the case that Nathan Salmon is Henry

25 Salmon [1989], 7-8, 24-25.
26 Salmon [1989], 7-8.

Kissinger.[27] The other essentially impossible world described in Salmon's example involves a sortal confusion (that is, the person Nathan Salmon is said to be a Visa credit card account). This world is essentially metaphysically impossible because it violates a certain metaphysical principle.

1.5 The Intransitive Accessibility Solution

Salmon's Intransitive Accessibility Solution holds that (i) the principles **(N)**, **(C)** and **(ST)**, and their multiple necessitations (namely, the necessity of the necessity of ... of the principles), are intuitively and literally true; (ii) the conventionally accepted axiom of S4 modal propositional logic, $\Box P \supset \Box\Box P$, or equivalently, the presumption that metaphysical modal accessibility between worlds is transitive, is illegitimate and must be rejected in its unrestricted form.[28] Salmon has expressed the thought that necessity iteration, namely transitivity, is legitimate for the *a priori* principles like **(N)**, **(C)**, **(T)** and **(ST)**, but fallacious with respect to the kind of necessary *a posteriori* propositions derivable from these *a priori* principles. Since necessity iteration is fallacious with respect to at least some necessary *a posteriori* propositions, an unrestricted version of S4 axiom schema is therefore illegitimate.[29]

In the following we present Salmon's Intransitive Accessibility Solution to the Paradoxes.

27 We shall define logically consistent world in Chapter 2 as a world which does not contain any inconsistent finite part. An inconsistent finite part of a world is a statement the form of which is the negation of a thesis of the logical system in question.
28 Salmon [1986], 80.
29 Salmon [1989], Section X.

Salmon argues that in the case of Chisholm's Paradox, according to Principle **(N)**, there is a threshold somewhere in the sequence of $\boldsymbol{h}_0$, ..., $\boldsymbol{h}_n$ which marks the tolerable variations in table α's original matter with respect to the real world $\boldsymbol{w}_@$.[30] The hunks that precede the threshold are regarded as sufficiently similar to table α's actual original matter $\boldsymbol{h}_0$, and the hunks that succeed the threshold as sufficiently different from $\boldsymbol{h}_0$. Suppose that hunk $\boldsymbol{h}_m$ is the last one in the sequence regarded as sufficiently similar to $\boldsymbol{h}_0$. Any hunk of matter in the sequence after $\boldsymbol{h}_m$ is an impossible matter for α from the standpoint of $\boldsymbol{w}_@$. Given Salmon's view that the necessities and possibilities contained in $\boldsymbol{w}_@$ determine whether a world is possible relative to $\boldsymbol{w}_@$, any world in which table α is originated from $\boldsymbol{h}_{i(i>m)}$ is an impossible world relative to $\boldsymbol{w}_@$, because $\boldsymbol{w}_@$ contains the necessity that necessarily table α cannot made from any hunk $\boldsymbol{h}_{i(i>m)}$. World $\boldsymbol{w}_{m+1}$, which contains table α made from $\boldsymbol{h}_{m+1}$, is the first impossible world in the sequence. Since Principle **(ST)** is an *a priori* necessary principle, according to Salmon's view, **(ST)** holds in every world possible relative to the real world and hence in world $\boldsymbol{w}_m$. By the possibility derivable from **(ST)** in $\boldsymbol{w}_m$, World $\boldsymbol{w}_{m+1}$ is a possible world relative to world $\boldsymbol{w}_m$ which is possible relative to $\boldsymbol{w}_@$. So, $\boldsymbol{w}_{m+1}$ is possibly possible relative to $\boldsymbol{w}_@$. But by the *a posteriori* necessity derivable from principle **(N)** in $\boldsymbol{w}_@$, $\boldsymbol{w}_{m+1}$ is not a genuinely possible world. So $\boldsymbol{w}_{m+1}$ is a possibly possible but genuinely impossible world. Using Salmon's terminology, $\boldsymbol{w}_{m+1}$ is a

30 Salmon points out that whether the threshold of the possible matter of an artifact consists in a sharp cutoff point, or in an indeterminate interval, will not affect the way of presenting the paradoxes and his way of solving the problem. I think that he is right at this point.

contingently impossible world relative to $w_@$. Since w_m is possible relative to $w_@$ and w_{m+1} is possible relative to w_m but w_{m+1} is not possible relative to $w_@$, the relative possibility relation is not transitive. The same reasoning can be given to any world w_{m+i} $(1 \leq i \leq m)$, where table α is made from hunk h_{m+i}. (Worlds w_{m+i} are contingently impossible worlds of the same degree as w_{m+1} is relative to $w_@$.) However, Salmon continues to argue, in the paradoxical inference, the transitivity of S4 and S5 modal logic permits the modal inference from $w_@Rw_m$ and w_mRw_{m+i} to $w_@Rw_{m+i}$. By this inference, all worlds w_{m+i}, which are impossible relative to $w_@$ according to the *a posteriori* necessity derivable from (**N**) in $w_@$, become possible relative to $w_@$ and consequently, table α's being made from hunk h_{m+i} becomes possible from the standpoint of $w_@$. The inferences allowed by transitivity are illegitimate. The relative possibility relation determined by the *a posteriori* necessities and possibilities asserted in (**N**) and (**ST**) in the relevant worlds is non-transitive. Therefore, modal inferences involving Principles (**N**) and (**ST**) must be carried out in a modal logic which is consistent with the intransitive nature of the relative possibility relation. The paradox arises because the alleged possible worlds of $w_@$ inferred by the transitivity of S4 and S5 modal logic are not genuinely possible according to Principle (**N**) in $w_@$. Therefore, S4 and S5 modal logic should be rejected as correct logic for metaphysical modal reasoning concerning artifacts.

In the case of the first version of Four Worlds Paradox, it is assumed that any hunk of matter having more than two components different from table α's original matter $\boldsymbol{h}$ are regarded as sufficiently different from table α's original matter $\boldsymbol{h}$. It is also assumed that β-world1, in which table β is made from $\boldsymbol{h}'$ differing from $\boldsymbol{h}$ by four components, is possible relative to α-world1. According to Principle **(ST)** in α-world1, α-world2, in which table α is made from $\boldsymbol{h}''$ that has two components different from α's matter $\boldsymbol{h}$ in α-world1, is possible relative to α-world1. Likewise, according **(ST)** in β-world1, β-world2, in which table β is made from hunk $\boldsymbol{h}''$ that has two components different from β's matter $\boldsymbol{h}'$ in β-world1, is possible relative to β-world1. Salmon argues that though β-world2 is a possible world relative to β-world1 and β-world1 is a possible world relative to α-world1, β-world2 is an impossible world relative to α-world1 according to Principle **(C)** in α-world1. Hence, from the standpoint of α-world1, β-world2 is a possibly possible but not genuinely possible world. The paradoxical inference takes an illegitimate step in inferring that β-world2 is possible relative to α-world1 from β-world1's being possible relative to α-world1 and β-world2's being possible relative to β-world1. This step is validated by the transitivity of S4 and S5 modal logic. To solve the Four Worlds Paradox, one should reject S4 and S5 as correct logic for modal reasonings concerning artifacts.

In the case of the second version of the Four Worlds Paradox, Salmon argues that the supervenience thesis held in α-world1 prevents β-world2 from being accessible from α-world1. This shows that the accessibility relations determined by the possibilities of these worlds is intransitive. The paradoxical conclusion is drawn from a step allowed by the transitivity of S4 and S5 modal logic. By denying the transitivity, one can only infer that (i) it is possible that $\boldsymbol{p}$ is true and table α is made from $\boldsymbol{h}''$, and (ii) it is possibly possible that $\boldsymbol{p}$ is true and it is not the case that table α is made from $\boldsymbol{h}''$.

We may summarize Salmon's argument in his Intransitive Accessibility Solution as follows.

(1) The metaphysical necessities and possibilities contained in a given world determine which world is metaphysically possible relative to the given world.

(2) The Principles **(N)**, **(C)** and **(ST)** express certain metaphysical necessities of the real world. These principles and their multiple necessitations are true in the real world. The stipulation about the possibility of β-world1 is also a correct one.

(3) The necessities and possibilities inferred from Principles **(N)**, **(C)** and **(ST)** in those involved worlds determine that the metaphysical accessibility relation between the worlds is intransitive.

(4) The correct modal logic employed in the modal inferences involving the three principles must be consistent with the intransitive nature of the accessibility relation between the worlds determined by the three principles.

(5) The axiom of S4 modal logic, $\Box P \supset \Box\Box P$, characterizes a transitive accessibility relation between possible worlds.

(6) Therefore, to avoid the paradoxes is to reject S4 and S5 modal logic and accept system T as the correct logic for metaphysical modal reasoning.

We shall discuss the soundness of this argument in Chapter 3.

CHAPTER 2

ON SALMON'S REPLY TO THE "STANDARD OBJECTION"

2.1 The "Standard Objection" and the Restrictiveness of Metaphysical Modality

In his paper, "Logic of What Might Have Been," Salmon has summed up a "standard objection" to his intransitivity account. The objection, as Salmon presents it, has two parts. In the first part, Salmon says, the objection holds the view that the intransitive accessibility relations are introduced into modal semantics for the purpose of interpreting various restricted types of modalities such as nomological modality. A world $\boldsymbol{w}'$ is nomologically possible relative to a world $\boldsymbol{w}$ if and only if every natural law of $\boldsymbol{w}$ is true in $\boldsymbol{w}'$, and a proposition is nomologically necessary in an arbitrary possible world $\boldsymbol{w}$ if and only if it is true in every world nomologically possible relative to $\boldsymbol{w}$. However, the distinguishing characteristic of metaphysical necessity and possibility is that it is completely unrestricted or the least restricted. Such an unrestrictiveness of metaphysical modality honors S5 modal logic. The second part of the objection, as Salmon puts it, asks for an explanation of the restriction in metaphysical modality to explain why transitivity fails. For example, the failure of transitivity in nomological modality can be clearly understood from the definitions of nomological modality and nomologically possible worlds. If the intransitive accessibility account rejects S4 and S5 in favor of system T as the logic of metaphysical modality, it must provide a clear account of the restriction that indicates the failure of transitivity in metaphysical modality.

According to this description of the "standard objection," the defensive argument for S5 in the first part of the objection can be written as below:

(I) If metaphysical modality is unrestricted, then it is characterized by S5 modal logic.

(II) Metaphysical modality is unrestricted.

(III) Therefore, metaphysical modality is characterized by S5 modal logic.

The key word in this argument is the word "restricted". Because the "standard objection" is presented by Salmon in his own words, we must be clear about what Salmon means by the word "restricted" in order to understand what the objection is as Salmon presents it. Salmon explains the notion of "restriction" as follows.

[Passage (1)]

> ... a proposition is said to be necessary, in the restricted sense in question, with respect to an arbitrary possible world w if and only if it is true in every possible world of such-and-such a restricted sort—the restriction in question depending on some appropriate relation to w.[31]

According to this passage, a certain type of necessity and possibility is a restricted type if and only if it is qualitatively defined so that only a special sort of worlds can be the possible worlds of the type relative to an arbitrarily given world ***w***. In other words, if we talk set-theoretically, we may explain Salmon's notion of "restricted" like

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[31] Salmon [1989], 9.

this: a type of modality is restricted if and only if not every world is a member of the set of possible worlds of the type with respect to an arbitrarily given world ***w***; a type of modality is not the least restricted if and only if the set of possible worlds of the type with respect to ***w*** is a proper subset of the set of possible worlds of some other type with respect to the same world ***w***. We may see this from the following passage in which Salmon argues that metaphysical modality is a restricted but not the least restricted type.

[Passage (2)]

> Another type of modality less restrictive than metaphysical modality is provided by what is sometimes called "logical necessity" and "logical possibility," ... A proposition is logically necessary if its truth is required on logical grounds alone, logically possible if its truth is not ruled out by logic alone ... Thus whereas it is logically necessary that Nathan Salmon is not somebody other than Nathan Salmon, and it is also logically necessary that either Nathan Salmon is a Visa credit card account with the Bank of America or he is not, it is not logically necessary that Nathan Salmon is not a credit card account. Although there is a way things logically could be according to which I am a credit card account, there is no way things metaphysically might have been according to which I am a credit card account. This illustrates the restricted nature of metaphysical modality. Some logically possible worlds must be "ignored." Metaphysical necessity is truth in every logically possible world of a certain restricted sort. ... Just as nomological possibility is a special kind of metaphysical possibility, so metaphysical possibility is a special kind of logical possibility.[32]

Considering necessity of any types which have been discussed by philosophers, we see that they are all qualified as a restricted type in the sense given in Passage (1), that a necessary proposition of a

[32] Salmon [1989], 13-14.

certain type in an arbitrary world w is a proposition true in every possible world of the special sort relative to w. For instance, a nomological necessity of w is true in every nomologically possible world relative to w, a metaphysical necessity of w is true in every metaphysically possible world relative to w, a logical necessity of w is true in every logically possible world relative to w, and so on. Salmon himself admits that in this sense there is no (interesting notion of) completely unrestricted modality.[33]

Immediately after Passage (1), Salmon says the following:

[Passage (3)]

> Such restrictions yield failures of the characteristic S4 principle that any "necessary" truth is necessarily necessary, and even of the characteristic B principle that any truth is necessarily possible.[34]

The impression of Passage (3) is that any of such restrictions yields a failure of the S4 principle and perhaps even the B principle. We notice that Passages (1) and (3) are given when Salmon presents the "standard objection." If the impression is right, Salmon might mean that his opponents hold not only (I) but also (IV) and (V) below:

(IV) If a modality is restricted, then it cannot be characterized by S5 modal logic.

(V) If metaphysical modality is restricted, then it cannot be characterized by S5 modal logic.

We can see in Passage (2) that Salmon himself uses the word "restricted" in the same sense as it is defined in Passage (1). This

33 Salmon [1989], 15.
34 Salmon [1989], 9.

means that in Passage (1) Salmon is aiming to clarify the notion of "restricted" and then use it rather than to present a view and then reject it. Because of this, we have reason to believe that he himself agrees with the views stated in (IV) and (V).

The notion of "restricted" given in Passage (1) is better understood with respect to a model. That is, the phrase "all worlds" means all worlds defined in a model, and the phrase "the type of modality is restricted" means that for any world $\boldsymbol{w}$ in the model, not every world in the model is a possible world relative to $\boldsymbol{w}$ with respect to the type of modality in question. A model containing all worlds is just a special case. When models are considered in general, it is easy to see that (IV) is false. It can be the case that the modality is restricted in the given sense and yet characterized by S5. This is illustrated in the following (LPC+S5) model $\langle \mathbf{W}, \mathbf{D}, \mathbf{R}, \mathbf{V} \rangle$, where $\mathbf{W}$ is a set of worlds, $\mathbf{D}$ is a set of individuals, $\mathbf{R}$ is accessibility relation for the type of modality in question, and $\mathbf{V}$ is a value assignment which (i) assigns to any atomic n-place predicate a set of ordered n-tuples of member of $\mathbf{D}$ for each world in $\mathbf{W}$, denoted by $\mathbf{V}(\phi, \boldsymbol{w}_i)$, and (ii) assigns 1 or 0 to any atomic formula $\phi x_1 \ldots, x_n$ in world $\boldsymbol{w}_i \in \mathbf{W}$ according as $<x_1 \ldots, x_n> \in \mathbf{V}(\phi, \boldsymbol{w}_i)$ or not, and (iii) assigns 1 or 0 to any other wffs of LPC+S5 according to the standard rules.[35] The model is the following:

$\mathbf{W} = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3\}$ and $\mathbf{D} = \{\boldsymbol{a}\}$.

For any atomic n-place predicate ϕ,

$\mathbf{V}(\phi, \boldsymbol{w}_1) = \{\boldsymbol{a}, \ldots, \boldsymbol{a}\}$.

[35] See [Hughes & Cresswell] page 147 for rules about ¬, v, ❑ and ◊, and page 192 for rules about =.

$\mathbf{V}(\phi,w_2) = \{a, \ldots, a\}$,

and $\mathbf{V}(\phi,w_3) = \emptyset$.

By this definition of **V**, ϕ is true of everything, namely, true of a, at w_1 and at w_2, but of nothing at w_3.

Let **L** be the type of necessity for which the accessibility relation for the worlds in **W** is:

$\mathbf{R} = \{\langle w_1,w_2\rangle, \langle w_2,w_1\rangle, \langle w_1,w_1\rangle, \langle w_2,w_2\rangle, \langle w_3,w_3\rangle\}$.

The accessibility relation **R** splits **W** into two equivalence classes, w_1 and w_2 versus w_3. In this model, $\mathbf{V}(\mathbf{L}\forall x\, \phi x, \ldots, x, w_1)=1$. The type of necessity **L** is restricted in the very sense defined in Passage (1)—the necessary proposition $\mathbf{L}\forall x\, \phi x, \ldots, x$ in world w_1 is a proposition true in every world of a restricted sort in **W**, that is, in w_1 and w_2. In other words, not every world of **W** is a **L**-possible world with respect to w_1. The same is true for w_2 and w_3. Yet the model satisfies S5 for **L**: Given $\mathbf{V}(\mathbf{L}\forall x\, \phi x, \ldots, x, w_1)=1$, $\mathbf{V}(\mathbf{M}\forall x\, \phi x, \ldots, x, w_1)=1$ where **M** refers to the possibility of the same type as that of **L**. Also, we have $\mathbf{V}(\mathbf{M}\forall x\, \phi x, \ldots, x, w_2)=1$ and therefore $\mathbf{V}(\mathbf{M}\forall x\, \phi x, \ldots, x \supset \mathbf{LM}\forall x\, \phi x, \ldots, x, w_1)=1$. The same holds for w_2 that $\mathbf{V}(\mathbf{M}\forall x\, \phi x, \ldots, x \supset \mathbf{LM}\forall x\, \phi x, \ldots, x, w_2)=1$. Since there is no world accessible from w_3 in which the proposition $\forall x\, \phi x, \ldots, x$ is true, $\mathbf{V}(\mathbf{M}\forall x \phi x, \ldots, x, w_3)=0$ and hence, $\mathbf{V}(\mathbf{M}\forall x\, \phi x, \ldots, x \supset \mathbf{LM}\forall x\, \phi x, \ldots, x, w_3)=1$.[36]

It seems to me, however, by "all worlds" Salmon does not mean all worlds in an arbitrary model but rather all the abstract maximal entities defined as "total way for things to be even if things

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36 Professor Max Cresswell suggested to me a method of using a mini-model to show that there can be two kinds of modality such that one is more restricted than the other and yet each obeying S5. His method is used in the argument presented here in which we show that there can be a restricted type of modality obeying S5.

could not have been that way."[37] Thus, speaking in term of model, the question is: if **W** in the model is specified as a set of all the "total ways for things to be even if things could not have been that way," then whether it is true that any restricted type of modality in the given sense cannot be characterized by S5 modal logic. We shall consider this question later in Section 2.4.

We consider another sense of restriction. We have said that according to Passages (1) and (3), being restricted in the sense given in Passage (1) causes the failure of transitivity and even the failure of symmetry. According to the statement in Passages (1) and (3), being intransitive or being asymmetric is not part of definition of restriction, but an inevitable consequence brought about by being restricted. We have shown, by the model above, that this alleged causal relation is wrong in general, but we postponed the discussion of a special case to Section 2.4. There is another sense of restriction for modality in which being intransitive (or being asymmetric) is part of definition of restriction. In this sense, a type of modality is restricted if and only if the nature of the type of modality determines that (i) not every world is a possible world of the type with respect to an arbitrary world $\boldsymbol{w}$, and (ii) the accessibility relation of the type is not an equivalence relation. In fact, (ii) entails (i). We put the definition in this way for an easy comparison between two senses of restriction.

The second sense of restriction is probably mostly meant by philosophers. Those philosophers who hold the "standard objection"

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37 This is Salmon's definition of a world given in Salmon [1989], on page 12.

to Salmon's account may actually raise the objection using the second sense of restriction. Salmon did not explicitly distinguish the second sense of restriction, but he is certainly aware of it and also tries to argue for his position in term of it. In his presentation of the "standard objection," when explaining how the restriction in nomological modality fails S5 modal logic, Salmon says the following:

> [Passage (4)]
>
> A proposition is nomologically necessary in an arbitrary possible world w if and only if it is true in every possible world in which all of the laws of nature in w are true. ... a world w′ is ... nomologically possible relative to ... a world w if every natural law of w is true in w′. ... Suppose, for example, that w and w′ are worlds so different in their natural constitution that although every natural law of w is true in w′ (so that w′ is nomologically possible relative to w), some of these natural laws of w are not natural laws in w′ but merely accidental generalizations, while certain other generalizations not even true in w are additional natural laws in w′. Then a natural law of w (which is automatically nomologically necessary in w) that is not also a natural law of w′ will not be true in every world nomologically possible relative to w, and hence will not be nomologically necessarily nomologically necessary in w. Similarly, a proposition that is true in w but violates one of the additional natural laws of w′ will not be nomologically necessarily nomologically possible in w. In this restricted scheme, accessibility between worlds is neither transitive nor symmetric.[38]

According to Passage (4), the definition of nomological necessity and the definition of nomological accessibility relation indicate that the set of nomological necessities and possibilities in a world ***w*** may be different from the set of nomological necessities and possibilities of some world nomologically possible relative to ***w***. It allows the case that there is a nomological necessity *p* of world ***w*** which, though true

[38] Salmon [1989], 8-9.

in a nomologically possible world w' relative to w, is not a nomological necessity in world w'. Thus, p is not nomologically necessarily nomologically necessary in w, and some nomologically possible world of w' is not nomologically possible according to w. We call the nature of a modality of this sort the varying nature. Clearly, it is the varying nature of nomological modality that yields the failure of transitivity and symmetry between nomologically possible worlds. The example of nomological modality in Passage (4) tells us that it is not the question of whether the type of modality is simply restricted in the sense given in Passage (1), but rather the question of how the type of modality is specified, which is relevant to the failure of S4 principle or B principle. In other words, the failure of the S4 principle or the B principle is not a consequence of being restricted in the sense given in Passage (1), but a consequence of being restricted in a particular way that makes the type of modality to bear the varying nature. Let us call the restrictions defined in Passage (1) "restriction-sense1," and the restrictions bringing the varying nature to a type of modality "restriction-sense2."

According to Salmon, the characteristic S5 principle is defended by the unrestrictiveness of metaphysical modality. If we understand the word "unrestricted" in restriction-sense2, then "a type of modality is unrestricted" should mean either that the type of modality is simply unrestricted in restriction-sense1, or that there is no varying nature consisted in the type of modality. But since unrestricted in restriction-sense1 is just a special case of not bearing the varying nature, we may simply say that "a type of modality is unrestricted in restriction-sense2" means that there is no varying nature in the type of modality.

We know that if the set of necessities and possibilities of a type in an arbitrary world ***w*** does not vary from ***w*** to worlds accessible from ***w***, then the accessibility relation is an equivalence relation. A model for this type of modality will contain one or more equivalence classes and in each equivalence class every world is a possible world of the type with respect to any world in the equivalence class. (Being unrestricted in restriction-sense2 entails being unrestricted in restriction-sense1 with respect to the equivalence class.) We also know that such a model fits S5. Thus, the argument which defends S5 in term of unrestrictiveness of metaphysical modality in restriction-sense2 can be written as follows:

(VI) If metaphysical modality is unrestricted in restriction-sense2, then it is characterized by S5 modal logic.

(VII) Metaphysical modality is unrestricted in restriction-sense2.

(VIII) Therefore, metaphysical modality is characterized by S5 modal logic.

Notice that (VI) as well as (IX) and (X) below are trivially true:

(IX) If a modality is restricted in restriction-sense2, then it cannot be characterized by S5 modal logic.

(X) If metaphysical modality is restricted in restriction-sense2, then it cannot be characterized by S5 modal logic.

I believe that being characterized by S5 modal logic is usually defended by the unvarying nature of metaphysical modality. Specifically, in the discussions related to the two Paradoxes, Graeme Forbes has argued for the unvarying nature of metaphysical modality as his reason for favoring S5 to be the logic of metaphysical modality. He defines metaphysical necessities as necessities of conceptual truths, and argues that since conceptual truths do not vary from a given world to the worlds metaphysically accessible from the given world, the accessibility relation between metaphysically possible worlds is an equivalence relation. His argument may be seen as an agreement with the general line of argument (VI)-(VIII).

2.2 Salmon's Reply to the "Standard Objection"

We said earlier that the "standard objection," as Salmon presents it, has two parts. The first part of the objection gives a defensive argument for S5 in term of unrestrictiveness of metaphysical modality, whereas the second part of the objection requires the intransitive accessibility account to provide a definition of metaphysical modality from which the failure of transitivity can be explained.

In rejecting the "standard objection," Salmon tries to establish the point that metaphysical modality is a restricted type. By arguing for this point, he intends to defeat the defensive argument in the first part of the objection, namely, to reject the second premise of the argument, "metaphysical modality is unrestricted," and therefore, to demonstrate that S5 has "never been satisfactorily justified to be the logic of what must be and what might have been."[39] We have investigated two senses of restriction relevant to the defense of S5. In order to defeat the defensive argument of S5 in the first part of the "standard objection," Salmon must at least show that there is no successful defense for metaphysical modality's being unrestricted in restriction-sense2. Merely showing that metaphysical modality is restricted in restriction-sense1 is not enough for him to make his desired claim.

The following passage may be thought as Salmon's answer to the second part of the objection:

> [Passage (5)]
>
> What is the restriction? To worlds that are metaphysically possible. (What else!) When we identify necessity with truth in every possible world, the word "possible" means something there, and what it means place a restriction on the sort of worlds under consideration.[40]

Passage (5) indicates that what metaphysical modality is is itself an answer to the question of why transitivity fails for metaphysical modality. This passage itself is uninformative, but from the context, we may understand Salmon as saying that the intransitiveness results from the restriction of being metaphysically possible. Thus, for either part of the "standard objection," whether Salmon is successful in rejecting the objection depends on whether he can argue for the restrictiveness of metaphysical modality in both

[39] Salmon [1989], 29.
[40] Salmon [1989], 13-14.

senses. In the following, we shall see in details how the restrictiveness of metaphysical modality is argued.

In explaining his opponents' defense of S5 in term of unrestrictiveness of metaphysical modality in restriction-sense1, Salmon says that his opponents confuse the two notions: the notion of a world and the notion of a possible world. (The two notions are explained in Section 1.4. The generic notion of a world is a way for things to be whether or not things might have been that way, whereas the modal notion of a possible world is a way that things might have been.) He says that this confusion comes from the equivocation of the two senses of the Leibnizian terminology of "possible world." In one sense, "possible world" is used to distinguish the notion of a world in the metaphysics of modality from layman's notion of a world: the former is a maximal abstract entity whereas the latter is a physical universe of atoms, molecules and etc.. Thus "possible world" in this sense means the same as Salmon's notion of a world. In the other sense, "possible world" is used to refer to the ways for things to be that conforms to certain constraints concerning what might have been. In this sense, a type of possible worlds is a special kind of world. Thus "possible world" in this sense means the same as Salmon's notion of a possible world. Salmon says that this ambiguity in the phrase of "possible world" is "... the primary source of the idea that metaphysical modality is the limiting case of restricted modalities, that metaphysical necessity and possibility is the unrestricted, and hence the least restricted, type of necessity and possibility."[41] What Salmon means here is that his opponents mistakenly take all worlds, namely, all abstract maximal entities of the sort called by him a "total way-for-things-to-be-even-if-things-could-not-have-been-that-way," to be metaphysically possible worlds and, therefore, metaphysical modality appears to be an unrestricted type of modality in restriction-sense1.

Salmon says that this confusion "has led to the widely accepted myths that the concepts of metaphysical necessity and possibility are defined in terms of, or constructed from, the concept of a possible world."[42] According to Salmon, his opponents give the following definition of metaphysical necessity and metaphysical possibility: "a proposition is [metaphysically] necessary in this unrestricted sense with respect to a possible world ***w*** if and only if it is true in absolutely every possible world whatsoever, no restrictions,"[43] and accordingly, a proposition is (metaphysically) possible in this unrestricted sense with respect to a possible world ***w*** if and only if it is true in at least one possible world. Remember that "possible world" here is supposed to be confused with Salmon's notion of a world. Given this definition of metaphysical necessity and possibility. Salmon says, the characteristic S5 principle that any possible truth is necessarily possible may be easily proved in the so-called "oft-used defense" of S5 modal logic quoted below.

[41] Salmon [1989], 13.
[42] Salmon [1989], 29.
[43] Salmon [1989], 9.

[Passage (6)]

> suppose p is a possible truth, that is, a proposition true in at least one possible world w. Then relative to any possible world w′, without exception, there is at least one possible world in which p is true—namely, w. It follows (given our assumption that p is possible) that it is necessary that p is possible. For in the unrestricted sense of "possible," one possible world in which p is true is all that is required for p to be "possible" relative to any given world w′, with no further restriction as to what sort of world p is true in or how that world is related to w′.[44]

According to Passage (6), the validity of S5 modal logic for metaphysical modality is proved by Salmon's opponents using the following 3-tuple model ⟨**W**, **D**, **V**⟩, in which **W** is the set of all worlds (the ways for things to be whether or not things could have been that way), **D** is the set of all possible individuals, and **V** is the value assignment such that it assigns "true" to the proposition $\Diamond p$ (read as "it is metaphysically possible that p") in an arbitrary world $\boldsymbol{w}_i \in \mathbf{W}$ if and only if there is a world $\boldsymbol{w}_k \in \mathbf{W}$ such that **V** assigns "true" to p in $\boldsymbol{w}_k$, and accordingly, **V** assigns "true" to the proposition $\Box p$ (read as "it is metaphysically necessary that p") in an arbitrary world $\boldsymbol{w}_i \in \mathbf{W}$ if and only if for all world $\boldsymbol{w}_k \in \mathbf{W}$, **V** assigns "true" to p in $\boldsymbol{w}_k$. No specific metaphysical accessibility relation between the worlds in **W** is involved. The deletion of metaphysical accessibility relation according to Salmon's explanation results from taking all worlds to be metaphysically possible worlds.[45]

44 Salmon [1989], 10.

45 Salmon's definition of accessibility relation is presented and explained in Section 1.4, which is in fact a standard definition given in Kripke's work and accepted by majority of philosophers: a world $\boldsymbol{w}'$ is metaphysically possible relative to a world $\boldsymbol{w}$ if and only if every fact of $\boldsymbol{w}'$ is a metaphysical possibility in $\boldsymbol{w}$, or, $\boldsymbol{w}'$ is metaphysically possible relative to $\boldsymbol{w}$ if and only if every metaphysically necessary fact of $\boldsymbol{w}$ obtains in $\boldsymbol{w}'$.

Since every world is treated as a metaphysically possible world, the "oft-used defense" is one which defends S5 by the unrestrictiveness of metaphysical modality in restriction-sense1.

Salmon calls another defense of S5 in term of unrestrictiveness of metaphysical modality "the ostrich approach to metaphysical modality":

[Passage (7)]

> One may choose to ignore ways things could not have been, confining one's sights always and without exception to ways things actually might have been. One may stipulate that a proposition is necessary with respect to an arbitrary possible world w if and only if it is true in every world accessible to the actual world—never mind worlds accessible to w—and likewise that a proposition is possible with respect to an arbitrary possible world w if and only if it is true in at least one world accessible to the actual world. ... One may then ignore accessibility altogether. ... If one confines one's sights to genuinely possible worlds, disavowing the impossible worlds, then metaphysical modality emerges as the limiting case—the "unrestricted" modality that takes account of "every" world--and S5 emerges as its proper logic.[46]

According to passage (7), the validity of S5 modal logic for metaphysical modality is proved by Salmon's opponents using the 4-tuple model, $\langle \mathbf{W}, \mathbf{r}, \mathbf{D}, \mathbf{V} \rangle$, where $\mathbf{r}$ is the real world and $\mathbf{W}$ is a set which contains only the worlds metaphysically possible relative to the real world $\mathbf{r}$. The truth value assignment $\mathbf{V}$ assigns "true" to the proposition $\Diamond p$ in an arbitrary world $\boldsymbol{w}_i \in \mathbf{W}$ if and only if there is a world $\boldsymbol{w}_k \in \mathbf{W}$ such that $\mathbf{V}$ assigns "true" to p in $\boldsymbol{w}_k$; accordingly, $\mathbf{V}$ assigns "true" to the proposition $\Box p$ in an arbitrarily world $\boldsymbol{w}_i \in \mathbf{W}$ if and only if for all world $\boldsymbol{w}_k \in \mathbf{W}$, $\mathbf{V}$ assigns "true" to p in $\boldsymbol{w}_k$. We see

[46] Salmon [1989], 20-22.

that when the worlds in **W** are all consistent worlds, the truth value assignment **V** thus defined is equivalent to the truth value assignment given in Passage (7), where the metaphysical accessibility relation is involved and other elements of the model remain the same. That is, **V** is equivalent to the truth value assignment which assigns "true" to the proposition $\Diamond p$ in an arbitrary world $\boldsymbol{w}_i \in \mathbf{W}$ if and only if there is a metaphysically possible world $\boldsymbol{w}_k$ of $\mathbf{r} \in \mathbf{W}$ such that **V** assigns "true" to p in $\boldsymbol{w}_k$; accordingly, **V** assigns "true" to the proposition $\Box p$ in an arbitrary world $\boldsymbol{w}_i \in \mathbf{W}$ if and only if for all metaphysically possible world $\boldsymbol{w}_k$ of $\mathbf{r} \in \mathbf{W}$, **V** assigns "true" to the proposition p in $\boldsymbol{w}_k$. Given Salmon's description of how metaphysical necessity and possibility are defined in the "ostrich approach", it is easy to see that a possible world of **r** is possible relative to any possible world of **r**, on the other hand, if a world is not possible relative to **r**, it is not possible relative to any possible world of **r**, because the **V** assigns the same set of necessities and possibilities to all the possible worlds of **r**. Thus, the possible worlds of **r** bear an equivalent accessibility relation. The same thing is true for any world that **r** may stand for. This gives the reason for the "ostrich approach" to remove metaphysical accessibility relation from the model.

From Salmon's description, we see that the "ostrich approach" views metaphysical modality as a type that does not have the varying nature and can be correctly represented by a model containing only the equivalence class of all worlds metaphysically possible relative to the real world. Therefore the "ostrich approach" actually defends S5 in term of the unrestrictiveness of metaphysical modality in restriction-sense2.

Our immediate question is who has ever confused the two notions and taken the notion of a possible world to mean the same as the notion of a world, and who has ever defined metaphysical necessity and possibility either in the way described in the "oft-used defense" of S5 or in the way described in the "ostrich approach." Salmon seems to have in mind David Lewis as one who holds the "oft-used defense" of S5 modal logic. For Lewis, the name "worlds" and the name "possible worlds" do refer to the same sort of things, but these things are not the same as Salmon's worlds, nor are they the same as Salmon's possible worlds. It is not fair to say that Lewis confuses the two notions as Salmon defines them. Furthermore, given Lewis' worlds (or possible worlds), the entities of the same sort as the real physical world but spatially and temporally disconnected from the real world, it may be seen as the case that metaphysical necessity is identified with truth in every world and metaphysical possibility with truth in at least one world, and the validity of S5 is proved on a 3-tuple model as described above. But, in doing so, Lewis is not confusing the notion of a possible world with the notion of a world. Whatever the justification is, it can only be found in Lewis' theory of possible worlds. I did not see any respectable philosopher who holds the so-called "oft-used defense" of S5. Nor did I see any respectable philosopher who explicitly or tacitly defines metaphysical necessity and possibility in the way as described in the "ostrich approach." But let us consider this: one can imagine objections to one's own account and reject them for the purpose of emphasizing some important points of one's own account. In any case, we want to see how Salmon rejects the "oft-used defense" of S5 and the "ostrich approach."

Salmon's argument against the "oft-used defense" of S5 modal logic and the "ostrich approach" is based on his three points of view which we explained in Section 1.4: (1) the distinction between the generic notion of a world and the modal notion of a possible world, (2) determining the possible worlds of a world according to the possibilities contained in the world, and (3) the existence of metaphysically impossible worlds.

To reject the "oft-used defense" of S5 modal logic, Salmon argues as follows. When talking about metaphysical modality in possible world discourse, the modal operators, $\Box$ and $\Diamond$, are considered as quantifiers quantifying over all worlds. Namely, the modal operators quantify over not only metaphysically possible worlds relative to an arbitrary world w, and not only metaphysically impossible yet consistent worlds relative to w, but also all worlds including inconsistent worlds which are impossible in any sense. Why should it be the case? Salmon's answer is this: Possible worlds are ways for things to be such that things might have been that way, and impossible worlds (whether consistent or inconsistent) are ways for things to be such that things could not have been that way; though they differ modally, they "both are ways for things to be, and in that sense, ontologically on a par."[47] Salmon's point is that the modal

[47] Salmon [1989], 17.

operators, considered in semantics as quantifiers, must quantify over all entities of a certain ontological kind, and "worlds" is the name for this ontological kind—maximal abstract entities.

Given the distinction between the notion of a world and the notion of a possible world, and given modal quantifiers' quantifying all worlds, it is obvious that for any world w in the set **W** of a model **M** containing all worlds, it is not the case that every world in **W** is a metaphysically possible world relative to w.[48] Therefore, metaphysical modality is a restricted type in restriction-sense1. Salmon concludes: "If worlds include ways things metaphysically cannot be in addition to ways things metaphysically might have been, then the idea that metaphysical necessity corresponds to truth in every world whatsoever is flatly mistaken."[49] According to Salmon's view, in the alleged "proof" of the "oft-used defense" of S5, **W** is a set of all worlds, namely all the "ways-for-things-to-be-even-if-things-could-not-have-been-that-way". Certainly, there are metaphysically impossible worlds relative to an arbitrary consistent world w in **W**—some world in **W** (any metaphysically impossible world of w) is such that not every necessity in world w is a truth of it. By removing the metaphysical accessibility relation, the "oft-used defense" of S5 mistakenly makes all worlds metaphysically possible relative to world w, and hence mistakenly makes metaphysical modality unrestricted in restriction-sense1 and suitable for S5 modal logic.

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48 We shall discuss the problem of metaphysical accessibility relation from an inconsistent world in Section 2.4. The words said here will be correct according to the discussion.

49 Salmon [1989], 15.

For the "ostrich approach," Salmon argues that it "flies in the face of the very meanings of the words 'necessary' and 'possible'."[50] "... the ostrich approach misconstrues the simple modal term "necessary" to mean the modally complex concept of actual necessity, or necessity according to [world r], where [r] is the actual world. Likewise, the ostrich approach misconstrues "possible" to mean actual possibility, or possibility according to [world r]."[51] What Salmon says here is that the "ostrich approach" views metaphysical modality as unrestricted in restriction-sense2 for a wrong reason. Premise (VII) that metaphysical modality is unrestricted in restriction-sense2 in argument (VI)-(VIII) is thus not legitimately established, which means that the metaphysically impossible worlds are illegitimately ignored. Therefore, the conclusion that metaphysical modality is characterized by S5 modal logic can not soundly follow.

We think that Salmon's three viewpoints explained in Section 1.4 are correct. We also agree that if the set **W** of a given model contains all worlds as its members, it is indeed wrong to identify metaphysical necessity with truth in every world of **W** and metaphysical possibility with truth in at least one world of **W**. Moreover, it is wrong to simply take "necessary" to mean "actually necessary" and "possible" to mean "actually possible," where "actually necessary" and "actually possible" are as defined in Passage (6). However, as we said earlier, in order to answer either part of the "standard objection," one needs to show that in neither of the two

50 Salmon [1989], 21.
51 Salmon [1989], 23.

senses is metaphysical modality unrestricted. It seems to me that, by his rejection to the "oft-used defense" of S5 and the "ostrich approach," Salmon cannot succeed in demonstrating this point. Later, we shall argue in Section 2.4 for the following three points, by which we explain why Salmon is not successful.

First, in his rejection of the "oft-used defense" of S5, Salmon holds that modal operators should quantify over all worlds including inconsistent worlds. There may be a consideration that if inconsistent worlds are quantified over, being restricted in restriction-sense1 will give rise to being restricted in restriction-sense2. We shall explain this consideration, and argue that the idea that modal operators quantify over inconsistent worlds is theoretically incoherent with the equivalence of two definitions of accessibility relation(which are discussed earlier in Section 1.4), is dubious in its meaning, and is completely not needed in semantics. Furthermore, we shall argue that Salmon's ontological argument does not work.

Secondly, if modal operators quantify only over consistent worlds, we can show that metaphysical modality's being restricted in restriction-sense1 does not entail its being restricted also in restriction-sense2.

Thirdly, though it is incorrect to change the meanings of metaphysical necessity and possibility as what happens in the "ostrich approach," it does not follow that metaphysical modality is therefore restricted in restriction-sense2. We shall point out that Salmon's argument for metaphysical modality's being restricted in restriction-sense2 relies ultimately on his belief in the truth of the following statement: Some metaphysically impossible worlds of a world w are possible relative to some of the metaphysically possible worlds of w. His examples are the cases that world w_{m+1} in Chisholm's Paradox and β-world2 in the Four Worlds Paradox are metaphysically impossible relative to the real world but possible relative to some metaphysically possible worlds of the real world. However, the truth of the statement and the examples just beg the question. To demonstrate the truth or falsehood of this statement and these examples are what the discussion of the two paradoxes is all about. It is clearly not correct to use them as premises of other argument.

We shall lay some ground work in the next section for the discussion in Section 2.4 and Chapter 3.

2.3 Γ-construction and Henkin Completeness Theorem

The most important feature of Salmon's account of possible worlds is that the necessities and possibilities contained in a world w determine which worlds are possible relative to w. Furthermore, from the two equivalent definitions of relative possibility relation,[52] we can see in Salmon's account a description of what he regards as the possible worlds relative to a given world w, or how the possible worlds of world w are determined by the possibilities and necessities contained in w. We think that these views are correct. Hughes and Cresswell in their book, *An Introduction to Modal Logic*, present a

[52] See Section 1.4 for the two definitions of relative possibility relation and our discussion about the equivalence between them.

Henkin proof for the completeness of modal lower (first-order) predicate calculus(LPC) systems LPC+T+BF, LPC+S4+BF and LPC+S5.[53] The proof shows that we can form a tree-construction Γ which is composed of maximal consistent sets Γ_1, ..., Γ_i, ... relative to a system **S** of modal LPC. A Γ-construction starts by constructing a maximal consistent set, and then expands from the maximal consistent set a set of maximal consistent sets according to certain rules. This expansion applies to any maximal consistent set that has been generated in the Γ-construction. The proof shows that we can then define a model ⟨**W**, **D**, **R**, **V**⟩ of system **S** based on Γ such that **W** of the model can be interpreted as a set of worlds corresponding to the maximal consistent sets in Γ and **R** as the accessibility relation which fits **S**. What interests us is that the rules of Γ-expansion are in accordance with Salmon's idea that possible worlds of a world is determined by the necessities and possibilities contained in the given world, and the relation **R** based on the expansion is in accordance with the definitions of relative possibility relation. In the following we shall explain the Γ-construction of Henkin proof. The modal logical reasoning exhibited in the model based on Γ-construction will help us argue with clarity about the restrictiveness of modality.

The LPC system introduced in the book of Hughes and Cresswell contains as primitive symbols a set of individual variables $\{x, y, z, ...\}$, a set of predicate variables $\{\phi, \psi, \chi, ...\}$ and a set of logical

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53 The proof is given in Hughes & Cresswell [1968], 149-169. BF stands for Barcan formula, $(a)\Box\alpha \supset \Box(a)\alpha$. Barcan formula is not a thesis of system T, nor is it a thesis of S4, but it is a thesis of S5. Barcan Formula is employed in the proof.

constants {$\neg$, $\vee$, $\forall$, (,)}. The formation rules and inference rules are standard.[54] The notion of the consistency of a formula with respect to axiomatic system **S** is as follows: A formula α of a system **S** is said to be consistent with respect to **S** iff $\neg\alpha$ is not a thesis of **S**. That is, the negation of a thesis counts as inconsistent, but every other formula counts as consistent. Moreover, a finite set $\{\alpha_1, \ldots\ldots, \alpha_n\}$ of formulae of **S** is consistent iff $\neg(\alpha_1 \& \ldots\ldots \& \alpha_n)$ is not a thesis of **S**. Finally, if Δ is an infinite set of formulae, Δ is consistent iff it contains no inconsistent finite subset of formulae. Since there is no difference in principle in constructing Γ with respect to any one of the three modal LPC systems, we shall not pick a particular modal system for the Γ-construction, as long as the Γ-construction is consistently formed with respect to exactly one system.

To construct the initial maximal consistent set Γ_1 in the Γ-construction, we begin with a consistent wff α relative to the modal LPC system of the Γ-construction. We add to α the set of all selected E_M-formulae as shown in the proof,[55] which guarantees that the set has the E-property—for every wff of the form $(\exists a)\beta$ in Δ there is also in Δ some wff $\beta[b/a]$, which differs from β only in that wherever β has

[54] For formation rules see Hughes & Cresswell [1968], 133-134.

[55] It is required that every maximal consistent set Γ_i in the system Γ has E-property. Any wff of the form $(\exists a)\beta\supset\beta[b/a]$ is called an E_M-formula with respect to b (b is referred as the replacement variable); and if γ is an E_M-formula with respect to b and δ is any wff not containing free b, then the formula, $\Diamond\delta\supset\Diamond(\delta\&\gamma)$, is an E_M-formula with respect to b. All E_M-formulae which differ only in their replacement variables are said to have the same E_M-form. To ensure that every Γ_i in Γ has the E-property, the proof shows a systematic way of adding to every set Γ_i some E_M-formula of each E_M-form, and the resulting set is proven to be consistent. By including these E_M-formulae, whenever there is a formula $(\exists a)\beta$ in Γ_i, the formula $\beta[b/a]$ is derivable from $(\exists a)\beta$ and the relevant E_M-formula. The proof is given on pages 165-168.

free a, $\beta[b/a]$ has some b, which is free in $\beta[b/a]$ but not free in β. Intuitively, this requirement is that whenever the set Γ_i contains an existential statement for some individual or other to be such-and-such, it must also contain a statement that a particular individual is such-and-such. Finally we extend the set to a maximal consistent one. A maximal consistent set is one that there is no formula which is not already in but can be consistently added to the set. In other words, a set of formulae of the modal system is maximal consistent iff it is consistent and every formula of the modal system not in the set is inconsistent with the set. To extend the set we do the following. We assume that the formulae are arranged in a fixed order $\alpha_1, \alpha_2, \ldots, \alpha_n, \ldots$. Let $\Gamma_{1,0}$ be the set $\{\alpha, \delta_1, \ldots, \delta_n, \ldots\}$, where α is a formula consistent with the modal LPC system of the Γ-construction and each δ is an E_M-formula of a distinct E_M-form. In each of the subsequent steps we shall form a set $\Gamma_{1,i}(i>0)$. If α_1 is consistent with $\Gamma_{1,0}$ (i.e. if $\neg(\alpha \,\&\, \delta_1 \,\&\, \ldots \,\&\, \alpha_1)$ is not a thesis of the modal LPC system), let $\Gamma_{1,1}$ be $(\alpha \,\&\, \delta_1 \,\&\, \ldots \,\&\, \alpha_1)$; otherwise, let $\Gamma_{1,1}$ be $\Gamma_{1,0}$. We form $\Gamma_{1,2}$ analogously by considering α_2. In general, given $\Gamma_{1,n}$, if $\Gamma_{1,n} \cup \{\alpha_{n+1}\}$ is consistent, let $\Gamma_{1,n+1}$ be the union $\Gamma_{1,n} \cup \{\alpha_{n+1}\}$; otherwise, let $\Gamma_{1,n+1}$ be $\Gamma_{1,n}$.

The rules of expansion in Γ-construction is the following: For every wff of the form $\Diamond\beta$ in Γ_i we construct a maximal consistent set Γ_k beginning with $\{\beta\}$. We next add the E_M-formulae $\delta_1, \ldots, \delta_n, \ldots$ to $\{\beta\}$ as indicated in the proof, which ensures that Γ_k has the E-property. Then we add to the set $\{\beta, \delta_1, \ldots, \delta_n, \ldots\}$ every wff, γ, such that $\Box\gamma \in \Gamma_i$. The resulting set, $\{\beta, \delta_1, \ldots, \delta_n, \ldots, \gamma_1, \ldots, \gamma_n, \ldots\}$ is an infinite consistent set.[56] Let the set $\{\beta, \delta_1, \ldots, \delta_n, \ldots, \gamma_1, \ldots, \gamma_m, \ldots\}$ be $\Gamma_{k,0}$, the initial set of Γ_k, and then extend Γ_k to a maximal consistent set in the standard way described earlier.

From the above rules of expansion for Γ, we see the following important features of Γ-construction: (i) for every Γ_i in Γ and for every wff of the form $\Diamond\beta$ in Γ_i there is a subordinate set Γ_k of Γ_i such that $\beta \in \Gamma_k$, and (ii) for any subordinate set Γ_k of Γ_i and for every wff of the form $\Box\gamma$ in Γ_i, $\gamma \in \Gamma_k$. A subordinate set Γ_k of Γ_i is a set expanded from Γ_i according to the rules of Γ-expansion. The construction of a subordinate set of set Γ_i is based on one formula β such that $\Diamond\beta$ is in Γ_i and all formulae γ such that $\Box\gamma$ is in Γ_i.

The quadruple $\langle \mathbf{W}, \mathbf{R}, \mathbf{D}, \mathbf{V} \rangle$ is defined to be a LPC+T+BF (or LPC+S4+BF, or LPC+S5) model. With each Γ_i in Γ we associate an entity $\boldsymbol{w}_i$ and let $\mathbf{W}$ be the set of all such entities. Let $\mathbf{R}$ be the relation such that $\boldsymbol{w}_i \mathbf{R} \boldsymbol{w}_k$ iff Γ_k is either a subordinate of Γ_i or is Γ_i itself for LPC+T+BF model (for LPC+S4+BF model, $\boldsymbol{w}_i \mathbf{R} \boldsymbol{w}_k$ iff Γ_k is a subordinate* of Γ_i, where a subordinate* of Γ_i is either Γ_i itself or a subordinate of Γ_i or a subordinate of a subordinate of Γ_i or ...; for LPC+S5 model, whenever Γ_k is a subordinate* of Γ_i, $\boldsymbol{w}_i \mathbf{R} \boldsymbol{w}_k$ and

[56] The consistency of this set is given by: (i) the lemma that where $\beta, \gamma_1, \ldots, \gamma_n$ are any wffs, if $\{\Diamond\beta, \Box\gamma_1, \ldots, \Box\gamma_n\}$ is consistent, then $\{\beta, \gamma_1, \ldots, \gamma_n\}$ is consistent; and (ii) the proved fact that $\Diamond(\beta \,\&\, \delta_1 \,\&\, \ldots \,\&\, \delta_n) \in \Gamma_i$ for any $n \geq 0$, which is given when introducing the specific way of adding to $\{\beta\}$ the E_M-formulae; and (iii) The definition of consistency of an infinite set: an infinite set is consistent iff every finite subset of it is consistent. Consider any finite subset $\{\beta, \delta_1, \ldots, \delta_n, \gamma_1, \ldots, \gamma_m\}$ of the set $\{\beta, \delta_1, \ldots, \delta_n, \ldots, \gamma_1, \ldots, \gamma_m, \ldots\}$, where $n \geq 0$, $m \geq 0$. We have by (ii) $\Diamond(\beta \,\&\, \delta_1 \,\&\, \ldots \,\&\, \delta_n) \in \Gamma_i$, and it is given that for each γ, $\Box\gamma \in G_i$; hence $\{\Diamond(\beta \,\&\, \delta_1 \,\&\, \ldots \,\&\, \delta_n), \Box\gamma_1, \ldots, \Box\gamma_m\}$ is a finite subset of Γ_i. Since Γ_i is by hypothesis consistent, by (iii), this subset of Γ_i is also consistent. By (i) and the consistency of $\{\Diamond(\beta \,\&\, \delta_1 \,\&\, \ldots \,\&\, \delta_n), \Box\gamma_1, \ldots, \Box\gamma_n\}$, we have that every finite subset $\{\beta, \delta_1, \ldots, \delta_n, \gamma_1, \ldots, \gamma_m\}$ of the infinite set $\{\beta, \delta_1, \ldots, \delta_n, \ldots, \gamma_1, \ldots, \gamma_m, \ldots\}$ is consistent; therefore the infinite set itself is consistent.

$w_k R w_i$). Let **D** be the set of individual variables considered as objects. For the value assignment **V**, we have: for every individual variable a, $\mathbf{V}(a)=a$, for any n-place predicate variable ϕ, $\mathbf{V}(\phi)$ is the set of ordered (n+1)-tuples $\{<x_1, ..., x_n, w_i>, ...\}$ such that $\phi(x_1, ..., x_n)\in\Gamma_i$ (for every $\Gamma_i\in\Gamma$), and for any other wff, **V** assigns a value to it according to the standard rules for $\neg$, $\vee$, $\forall$, and $\Box$.[57]

The completeness theorem proves: Given **W**, **R**, **D** and **V** as defined above, for any wff, β, of LPC+T+BF (or of LPC+S4+BF, or of LPC+S5), and for any $w_i\in\mathbf{W}$, $\mathbf{V}(\beta,w_i)=1$ or 0 according as $\beta\in\Gamma_i$ or not.

We interpret the entities in **W** as consistent worlds, where the notion of a world is used in the sense according to Salmon's definition of a world—a maximal abstract entity, and the notion of being consistent is as defined above. Since a world is an infinite abstract entity, a world is consistent if and only if it does not contain any inconsistent finite part. We interpret the relation **R** of the model as the accessibility or relative possibility relation according to definitions of accessibility that Salmon holds: a world w' is accessible from a world w if and only if every fact of w' is a possibility in w; or equivalently, w' is accessible from w if and only if every necessity of w obtains in w'.

The justification for us to interpret **R** as such is given by the completeness theorem. We can see from the completeness theorem that the syntactical features of Γ-construction have a perfect match in the given notion of accessibility. The second feature, (ii), says that for any subordinate set Γ_k of Γ_i and for every wff of the form $\Box\gamma$ in Γ_i,

57 Hughes & Cresswell [1968], 147.

$\gamma \in \Gamma_k$. In a T model, $\boldsymbol{w}_i \mathbf{R} \boldsymbol{w}_k$. According to Henkin completeness theorem, the value assignment **V** assigns true to γ in world $\boldsymbol{w}_k$ when it assigns true to $\Box\gamma$ in world $\boldsymbol{w}_i$. The relation between $\boldsymbol{w}_i$ and $\boldsymbol{w}_k$ is exactly in accordance with the second definition of accessibility relation—every necessity in $\boldsymbol{w}_i$ is true in $\boldsymbol{w}_k$. Furthermore, since both Γ_i and Γ_k are maximal consistent sets, taking the set $\{\beta, \delta_1, \ldots, \delta_n, \ldots, \gamma_1, \ldots, \gamma_m, \ldots\}$ as the initial set of Γ_k precludes any formula θ being included in Γ_k such that $\neg\Diamond\theta$ is in Γ_i. If $\neg\Diamond\theta$ is in Γ_i, then $\Box\neg\theta$ is in Γ_i. Hence $\neg\theta$ is one of the γ's in the set $\{\beta, \delta_1, \ldots, \delta_n, \ldots, \gamma_1, \ldots, \gamma_m, \ldots\}$. Formula θ can not be consistently added to Γ_k because $\neg(\theta \& \neg\theta)$ is a thesis of any of the three modal axiomatic systems. So Γ has another feature, equivalent to feature (ii), that for every subordinate set Γ_k of Γ_i in Γ, and for every wff $\beta \in \Gamma_k$, there is a wff of the form $\Diamond\beta \in \Gamma_i$. According to Henkin completeness theorem, the value assignment **V** assigns true to β in world $\boldsymbol{w}_k$ only if it assigns true to $\Diamond\beta$ in world $\boldsymbol{w}_i$. This is in accordance with Salmon's first definition of accessibility —every proposition that is true in $\boldsymbol{w}_k$ is possible in $\boldsymbol{w}_i$.

In an S4 model, if $\boldsymbol{w}_i \mathbf{R} \boldsymbol{w}_k$ and $\boldsymbol{w}_k \mathbf{R} \boldsymbol{w}_n$ then $\boldsymbol{w}_i \mathbf{R} \boldsymbol{w}_n$. In the Γ-construction with respect to modal logical system LPC+S4+BF, we have that for any subordinate* set Γ_k of Γ_i and for every wff of the form $\Box\gamma$ in Γ_i, $\gamma \in \Gamma_k$. According to the completeness theorem, the value assignment **V** assigns true to γ for every subordinate* world $\boldsymbol{w}_k$ of $\boldsymbol{w}_i$ when it assigns true to $\Box\gamma$ in world $\boldsymbol{w}_i$. Hence the relation between world $\boldsymbol{w}_i$ and any subordinate* world of $\boldsymbol{w}_i$ is in accordance with the second definition of accessibility. Like what we did for T model, we can show analogously that the relation between world $\boldsymbol{w}_i$

and any subordinate* world of w_i is in accordance with the first definition of accessibility as well.

In an S5 model, for any subordinate* world w_k of world w_i, $w_i\mathbf{R}w_k$ and $w_k\mathbf{R}w_i$. The accessibility relation between worlds in a S5 model is not only transitive but also symmetrical. What is needed to be checked further is that, when w_k is accessible from w_i, whether w_i is also accessible from w_k in the same sense of accessibility. We know that S5 can be obtained by adding to S4 an additional axiom $P \supset \Box\Diamond P$. In a Γ-construction with respect to modal system LPC+S5, by the axiom $P \supset \Box\Diamond P$, for every wff $\theta \in \Gamma_i$, $\Box\Diamond\theta \in \Gamma_i$, and by the rule of expansion and the axiom $\Box P \supset \Box\Box P$, $\Diamond\theta \in \Gamma_k$ for any subordinate* set Γ_k of Γ_i. By Henkin completeness theorem, the value assignment $\mathbf{V}$ assigns true to $\Diamond\theta$ for every subordinate* world w_k of w_i when it assigns true to θ in w_i. This is to say that every fact θ of w_i is a possibility of w_k, which is exactly Salmon's first definition of accessibility from w_k to w_i. Equivalently, for any subordinate* world w_k of w_i and every $\Box\gamma$ in w_k, γ is true in w_i. Assume that $\Box\gamma$ is true in w_k but γ is false in w_i, that is, $\neg\gamma$ is true in w_i. By axiom $P \supset \Box\Diamond P$, $\Box\Diamond\neg\gamma$ is true in w_i. Since w_k is accessible from w_i according to the first definition of accessibility, $\Diamond\neg\gamma$ is true in w_k and hence $\neg\Box\gamma$ is true in w_k. But we have assumed that $\Box\gamma$ is true in w_k. This contradicts the consistency of w_k. Thus the accessibility relation from any subordinate* world w_k of w_i to the world w_i is in accordance with the second definition of accessibility as well.

The construction of Γ forms a tree-structure. Consequently, by Henkin completeness theorem, the worlds in the model defined based on Γ are related in a tree-structure. (A tree-structure is a connected structure with a starting point as its root, and latter occurrences in the structure are expanded or generated from some earlier occurrence according to a certain rule.) The tree of a Γ-construction is infinite, so is any subtree in the Γ-construction. An infinite model defined on the Γ-construction represents the modal space of the type of modality concerned in the Γ-construction. In the model, each subtree rooted in world w_i draws a picture of how modal reasoning of the relevant type goes for world w_i. Let us call the subtree rooted at w_i "the scope of modal reasoning (of the relevant type) for world w_i."

It is not always the case that every world in the scope of modal reasoning of w_i is a genuine possible world of w_i. This is true in the cases when the type of modality in question is characterized by S4 or S5 modal logic, but certainly not true when the logic is T. However if we use the name "a potentially possible world relative to world w_i" to mean a world which bears an ancestral accessibility relation to w_i whether or not it is directly accessible from w_i, then every world in the scope is a potentially possible world of w_i. In other words, a potentially possible world of w_i is a world which can be reached by a continuing modal reasoning started from the necessities and possibilities in world w_i.

In talking about the scope of modal reasoning for a world, there is a consideration arising from the maximality of a world. A world as a maximal abstract entity contains various different types of modality. A Γ-construction is formed with respect to the modal logic which characterizes the type of modality that the Γ-construction concerns. Whichever is the logic of the Γ-construction, the maximality of a world requires that different types of modality be stated correctly in the language of the modal logic of the Γ-construction. This can be done by considering the following two aspects.

First, philosophers often talk about different systems of modal logic that characterize different types of modality. This is a issue about whether necessities and possibilities of a type will vary from a given world to worlds accessible from the given world, and if the necessities and possibilities vary, how they vary. This aspect of a modality is called the mode of the modality. In constructing maximal consistent sets in a Γ-construction, every type of modality must be formulated according to its mode using the language of the logic of the Γ-construction.

The second aspect concerns the strictness of necessities. Among different types of necessity, some type of necessity is stricter, and some is less strict. The strictness is a question about what is taken to be a necessity, or say, by what standard a necessity is qualified. The standard for a stricter type of necessity allows fewer propositions to be necessary while the standard for a less strict type allows more to be necessary. The strictness of a type of necessity is an issue separate from the issue about the mode of a type of necessity. Two types of necessity with distinct strictness may be or may not be characterized by the same modal logical system, depending upon whether they share the same mode. In a Γ-construction, how many subordinate sets that set Γ_i will yield depends upon how many possibilities Γ_i contains. Suppose that modality type A is stricter than modality type B. A possibility of type A may not be a possibility according to type B. It is easy to imagine that, a Γ-expansion from set Γ_i based on possibilities of type A generates more subordinate sets than those generated by a Γ-expansion based on possibilities of type B in set Γ_i, simply because modality type A has more possibilities. Therefore, in constructing Γ, in addition to formulating different types of modality in their correct modes in the language of the logic of Γ, we must also correctly present their strictness within the scope of the modality which is the concern of the Γ-construction.[58]

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[58] With respect to metaphysical necessity, the most commonly mentioned stricter type of necessity is logical necessity, and the most commonly mentioned less strict type of necessity is nomological necessity. Salmon has pointed out in Passage (2) a relation between the three types of necessity: with respect to world $\boldsymbol{w}_i$, the set of necessities of a stricter type are extensionally included as a proper subset in the set of necessities of a less strict type. This is probably true, and we may add to it that the converse holds between the corresponding types of possibility.

We have mentioned three alternative modal LPC systems for Γ-construction, LPC+T+BF, LPC+S4+BF and LPC+S5. Suppose that the concern of a Γ-construction is metaphysical modality. We consider the three types of modality mentioned in the previous paragraph. There will be no worry about necessities of a type stricter than metaphysical necessity and possibilities of a type less strict than metaphysical necessity, since they are also metaphysical necessities and metaphysical possibilities respectively. For the necessities of a less strict type, since many of them are not metaphysical necessities, we must use a distinct symbol for them. Let us say that the symbol is "$^{\#}\Box$". Thus, $^{\#}\Box P$ and $\Diamond\neg P$ are not contradictory. Similarly, since some of the possibilities corresponding to a stricter type of necessity are impossibilities according to metaphysical necessity, we must use a distinct symbol for them as well. Let the symbol be "$*\Diamond$". Thus, $\Box P$ and $*\Diamond\neg P$ can be both added to a set consistently. Combining the concern about the strictness with the concern about the modes, we shall do the following. (Since the contention about the logic of metaphysical modality is between T and S5, we shall consider both cases. Likewise we shall not claim any modal system as the logic of the other two types of modality. As long as they can be characterized by one of the three systems, the following general description is applicable):

In the case that the logic for metaphysical modality is system LPC+T+BF, for a type of modality of mode T other than metaphysical modality, if the type is less strict than metaphysical modality is, then for every necessity, necessary γ, of the type in $\Gamma_{i(i\geq 1)}$, let $^{\#1}\Box\gamma$ be the expression of it where "$^{\#1}$" is a specification of the strictness; for any less strict type of modality in mode S4 or S5 and for every necessity, necessary γ, of the type in Γ'_i, let it be expressed by $^{\#2}\Box\gamma$ or $^{\#3}\Box\gamma$ and let

The considerations about proper representations of the modes and the scopes of distinct types of modality are required by the unconditional maximality of the sets in Γ-construction. But, if discussions concern only one or two types of modality, we can condition the maximality of the modal aspect of worlds on these types of modality. For example, if a discussion is about metaphysical necessity and possibility, we condition the maximality of a world for

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double necessitations of γ with the restricted scope, $^{\#2}\Box^{\#2}\Box\gamma$ or $^{\#3}\Box^{\#3}\Box\gamma$, be included in Γ_i in addition to $^{\#2}\Box\gamma$ or $^{\#3}\Box\gamma$. Possibilities of the corresponding types in T or S4 can be taken care of by consistently increasing the set to a maximal one according to the rule of maximization. That is, they are confined by the necessities of corresponding type included in each maximal set. For every possibility of the corresponding type of S5, possible β, let $^{\#3}\Box\Diamond\beta$ be included in the set, and since this necessity is in mode S5, we also include $^{\#3}\Box^{\#3}\Box\Diamond\beta$ in Γ_i.

For any type of modality of mode T other than metaphysical modality, if it is stricter than metaphysical modality is, then for every possibility, possible β, of the type in Γ_i, let $*^1\Diamond\beta$ be its expression in Γ_i. For any stricter type of modality of S4 or S5, and for every necessity, necessary γ, of the type in Γ_i, let double necessitation of γ, $\Box\Box\gamma$, be included in Γ_i in addition to $\Box\gamma$. Possibilities of the corresponding type in mode S4 can be consistently added to Γ_i with the specification of the strictness of the type, $*^2\Diamond\beta$, similar to the case in which the mode is T. For possibilities, possible β, of the corresponding type in mode S5, let $\Box*^3\Diamond\beta$ and $\Box\Box*^3\Diamond\beta$ be included in Γ_i.

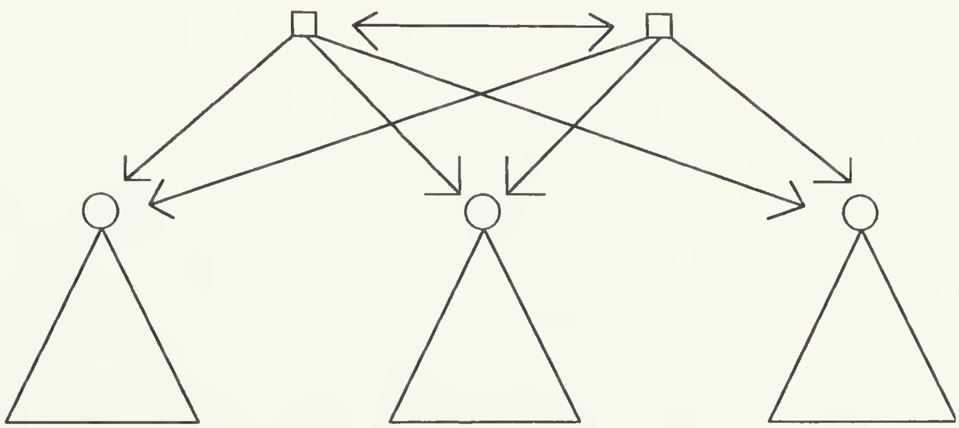
In the case that the logic of metaphysical modality is system LPC+S5, for any type of modality in mode S5 other than metaphysical modality, if it is less strict than metaphysical modality, then for every necessity, necessary γ, of the type in Γ_i, let $^{\#1}\Box\gamma$ be its expression in Γ_i, where "$\#1$" is the specification of the strictness, and let $\Box(^{\#1}\Box P \supset {}^{\#1}\Box^{\#1}\Box P)$ be included in Γ_i. For every possibility, possible β, of the corresponding type of S5 in Γ_i, let $^{\#1}\Box\Diamond\beta$ be its expression in Γ_i. For any less strict type of modality in mode T, and every necessity, necessary γ, of the type in Γ_i, let it be expressed as $^{\#2}\Box$(γ in every subordinate set of Γ_i which is in the scope specified by "#2") in Γ_i. For any less strict type of modality in mode S4, and every necessity, necessary γ, of the type in Γ_i, let $^{\#3}\Box$(γ in every subordinate* set of Γ_i which is in the scope specified by "#3") be its expression in Γ_i. The possibilities of the corresponding type in T or S4 can be taken care of by consistently maximizing Γ_i.

For any stricter type of modality in mode S5, and for every possibility, possible β, of the type in Γ_i, let $\Box*^1\Diamond\beta$ be its expression in Γ_i. For any stricter type of modality in mode T, and every necessity, necessary γ, of the type in Γ_i, let it be expressed as $\Box$(γ in every subordinate set of set Γ_i) in Γ_i. For any type of modality in mode S4 with a larger scope, and every necessity, necessary γ, of the type in Γ_i, let $\Box$(γ in every subordinate* set of set Γ_i) be its expression in Γ_i. The possibilities of the corresponding type in T or S4 can be consistently added to Γ_i with the specification of the scope of the type.

modal facts on metaphysical modality alone. The Γ-construction composed of conditioned maximal sets as such should be equally appropriate for representing metaphysical modal reasoning.

2.4 Our Views about Salmon's Reply

It may be thought that if modal operators quantify over not only consistent worlds but, as Salmon suggested, inconsistent worlds as well, and if the metaphysical accessibility relation between worlds is fixed according to Salmon's definition of accessibility relation, then it is true that S5 is not valid for metaphysical modality. This can be illustrated by the following picture.



(Figure 1)

Let us explain the picture above. An inconsistent world is a world containing contradictions. Since a contradiction entails everything, everything is true in an inconsistent world in the sense that every proposition can be included in the inconsistent world. Hence everything is metaphysically possible according to an inconsistent world. Given the first definition of accessibility relation (the first of the two definitions given in Section 1.4) that a world $\boldsymbol{w}'$ is metaphysically possible relative to a world $\boldsymbol{w}$ if and only if every fact of $\boldsymbol{w}'$ is a metaphysical possibility in $\boldsymbol{w}$, if $\boldsymbol{w}$ is an inconsistent world, then every world is a possible world of $\boldsymbol{w}$ according to the first definition of accessibility relation. However, an inconsistent world itself can only be a possible world of some other inconsistent worlds according to the first definition. The picture represents a model in which **W** is a set of all worlds including both consistent worlds and inconsistent worlds, and **R** is metaphysical accessibility relation according to the first definition. In the picture, a small box represents an inconsistent world, the small circle represents a consistent world, and each triangle represents consistent worlds related by metaphysical accessibility relation (We may think of a triangle as a Γ-construction concerning metaphysical modality that begins from the consistent set represented by the small circle at the top of the triangle). The directed edge from world A to world B represents the relation that B is possible relative to A. In this model, metaphysical modality is a restricted type in restriction-sense1 with respect to any consistent world in the model. That is, not every world in the model is a metaphysically possible world of a consistent world. Whatever is the metaphysical accessibility relation inside each triangle, the picture as a whole does not fit S5 because the metaphysical accessibility relation holds in the direction from inconsistent worlds to consistent worlds but not the other way around—the metaphysical accessibility relation is not symmetric. This might not be what Salmon means when he rejects S5 by arguing that metaphysical modality is restricted in restriction-sense1. Nevertheless, this is how S5 fails to characterize metaphysical modality under the following conditions: (i) modal operators quantify over all worlds including inconsistent worlds, (ii) the accessibility relation is determined according to the first definition of accessibility, and (iii) metaphysical modality is a restricted type in restriction-sense1. According to the above picture, it does seem that if a modality is restricted in restriction-sense1, then it is also restricted in restriction-sense2 because the worlds in the model are not related as equivalence classes—the metaphysical necessities and possibilities vary from a consistent world to an inconsistent world.

The question is whether we should agree with the view that modal operators quantify over inconsistent worlds. We consider the following points.

First, there is a theoretical incoherence. We said in Section 1.4 that Salmon gives two equivalent definitions for the notion of relative possibility. The first definition is: a world w' is metaphysically possible relative to a world w if and only if every fact of w' is a metaphysical possibility in w. This is the definition used above in describing the accessibility relation from an inconsistent world to a consistent world. The second definition is: w' is metaphysically possible relative to w if and only if every metaphysically necessary fact of w obtains in w'. We showed in Section 1.4 that the two definitions are equivalent only if the worlds involved are all consistent. This can be easily reviewed in the above picture. The accessibility relation from an inconsistent world to a consistent world is drawn according to the first definition of accessibility: every fact of a consistent world is a metaphysical possibility of an inconsistent world. Hence every consistent world is metaphysically possible relative to any inconsistent world. But an inconsistent world ***w*** contains both $\Box p$ and $\Diamond\neg p$. When $\neg p$ is true in a consistent world ***w'***, p cannot be true in ***w'***. Thus, according to the second definition of accessibility, the consistent world ***w'*** is not possible relative to the inconsistent world ***w***. In a similar way, we can show that an inconsistent world is not possible relative to a consistent world according to the first definition, but is possible relative to a consistent world according to the second definition. The two definitions of accessibility and their equivalence in determining possible worlds are regarded as standard in the semantics of modal logic. To avoid incoherence one must choose between rejecting the equivalence of the two definitions and rejecting the accessibility relation from an inconsistent world to a consistent world. The former approach involves fundamental changes in the semantics of modal logic, whereas the latter approach will give a reason for not quantifying over inconsistent worlds. In fact, since the two definitions of accessibility are not equivalent with regard to the accessibility relation between consistent worlds and inconsistent worlds, the accessibility relation drawn according to one of the definitions between a consistent world and an inconsistent world is by no means the same relation drawn between two consistent worlds according to both definitions, and hence, it is a convincing reason to reject the accessibility relation between consistent worlds and inconsistent worlds.

Secondly, I cannot imagine any philosophical or modal logical interest in pursuing possibilities in an inconsistent world. Pursuing possibilities in an inconsistent world is logically trivial, and philosophically doesn't make sense. In philosophy, "something is possible" means that the thing in question is in a certain sense instantiable or realizable. What do we mean by saying that something is realizable according to an inconsistent world which is in any sense not realizable?[59]

Thirdly, given that there is no accessibility relation between consistent worlds and inconsistent worlds, one may say that when we consider necessities and possibilities of a certain consistent world, quantifying over inconsistent worlds is harmless because the modal reasoning starting from a consistent world will never reach an inconsistent world. But then quantifying over inconsistent worlds is redundant in semantics.

Fourthly, Salmon's ontological argument doesn't work. Salmon has argued that modal operators should quantify over all worlds, possible worlds and impossible worlds, since they are ontologically the same kind.[60] According to Salmon's definition of a world, worlds as an ontological kind are defined by the property of being abstract and the property of being maximal. We agree to the view that it is

[59] In my view, the indexical sense of "realizable" or "realized" should not apply to inconsistent worlds, since that something is realizable or realized means that something could be true or is true as a whole. A contradiction is something which is always false. If we say an inconsistent world is realizable or realized according to itself, we abandon our logic, and then we are completely out of ground of reasoning.

[60] Salmon [1989], 17 and footnote 11 on page 17.

incorrect to divide worlds into ontological sub-kind by whether they are a possible world of a certain world ***w***. The intuition here is somewhat like the intuition about why we don't divide the ontological kind "cats" into ontological sub-kind by whether a cat is born from a certain mama-cat ***c***. The property of being possible relative to some world and the property of being born from some cat are more general properties. But every (consistent) world is possible relative to some worlds and impossible relative to some other worlds. Therefore, being possible or impossible relative to some world is not a property by which we can determine sub-kind of worlds. Nevertheless we can reasonably divide worlds into ontological sub-kinds by the property of being consistent. Every world is either consistent or inconsistent and cannot be both consistent and inconsistent. An ontological sub-kind of worlds is also an ontological kind. I don't think that being a more general ontological kind than the kind of consistent worlds is a reason for holding the view that modal operator must quantify over all worlds including inconsistent worlds rather than quantifying over consistent worlds. Besides, it is more proper to say that the objects of modal thinking are consistent worlds. Inconsistent worlds possess only trivial modal properties. non-trivial modal reasoning need not take them into account?

The above considerations demonstrate our refutation of Salmon's view that modal operators should quantify over inconsistent worlds. We want to show next that if modal operators quantify over all consistent worlds only, metaphysical modality's being restricted in restriction-sense1 does not imply that it is also restricted in restriction-sense2. In other words, given a model ⟨**W**, **R**, **D**, **V**⟩ in which **W** is a set of all consistent worlds, a modality can be a restricted type in restriction-sense1, that is, not every world in **W** is a possible world of an arbitrary world ***w*** in **W**, but at the same time an unrestricted type in restriction-sense2, that is, the worlds in **W** are related by relation **R** into equivalence classes. In the following we shall show that when **W** of the given model is a set of all consistent worlds, being restricted in restriction-sense1 and being unrestricted in restriction-sense2 are compossible. The same thing should be true when **W** contains less consistent worlds.

To say that a world is consistent is to say that the world does not contain any statement the form of which is a contradiction. Let Ω be an LPC system. A consistent world is also consistent with Ω in the sense that it does not contain any statement the form of which contradicts some thesis of Ω, since the negation of a thesis is a contradiction. A proposition is Ω-logically necessary if and only if it is provable by Ω, and a proposition is Ω-logically possible if and only if it is consistent with Ω (it is not a negation of a thesis of Ω). Salmon has pointed out in Passage (2) a relation between the logical modality and metaphysical modality: "metaphysical possibility is a special kind of logical possibility". We may add to it that, conversely, logical necessity is a special kind of metaphysical necessity.

In the following, we will form a Γ-construction, Γ*, based on Ω-logical necessities and possibilities with a set-up so that the restricted scope of metaphysical modal reasoning is recognizably included as subtrees in Γ^*.[61] We use the symbols "$\Box$" and "$\Diamond$" to denote Ω-logical necessity and Ω-possibility respectively, and the symbols "$^m\Box$" and "$^m\Diamond$" to denote metaphysical necessity and possibility respectively, where "m" specifies the scope of metaphysical modal reasoning. We assume that the logic for Γ^* is S5 because whether a proposition is provable by, or consistent with, the logic Ω does not seem to vary from one Ω-logically possible world to another.[62] We also assume that the mode of metaphysical necessities

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61 Professor Max Cresswell commented on an earlier version of my argument related to the Γ^*-construction. His suggestion simplified my argument.

62 Salmon has expressed his doubt on S5's being the logic of strict logical modality. He says that "the interpretation of the diamond, $\Diamond$, as logical possibility instead of metaphysical possibility could turn $\Diamond\Phi$ into a logical truth for every logically consistent formula Φ." See Salmon [1989], 15. In the footnote following this passage, he seems to hold affirmatively that if it is the propositions but not the sentences that are considered to be possible, then the logical possibility, $\Diamond\Phi$, is itself a truth of pure logic. But he goes on to say that "Even under this construal, however, S5 may not be the appropriate (first-order) propositional logic of logical necessity. The rule of necessitation (...) is inapplicable to such logical validities as 'If Saul Kripke is an anthropologist, then Saul Kripke is actually an anthropologist.'" See Salmon [1989], 14-15, footnotes 7 and 9.

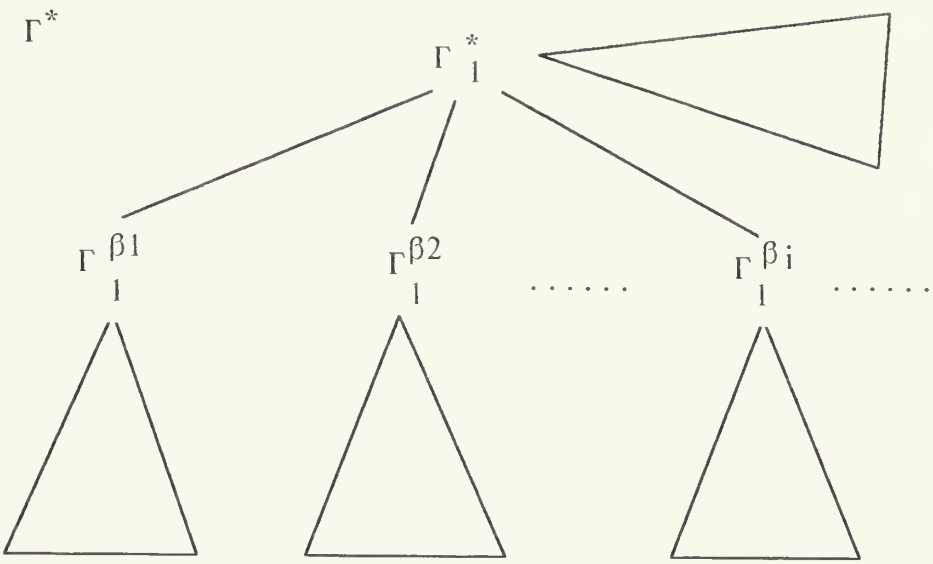
The latter worry, as Salmon indicates, has to do with the logic of indexicals. David Kaplan has shown that some sentence containing demonstratives expresses a true proposition whenever the sentence is uttered, so it is a truth of the logic of indexicals; but the same sentence uttered in most contexts expresses a contingent proposition (that is, the sentence uttered in different context expresses a different contingent proposition), so it is not a metaphysical necessity. See Kaplan [1978], in Salmon & Soames [1988], 66-68. So, the rule of necessitation must treated differently in the logic of indexicals. It seems to me, if this worry was something that we must deal with for our present purpose, then, it would not just be a problem particularly for S5 but a problem for other modal systems as well. But, in my opinion, the worry can be easily avoided by using a demonstrative-free language so that each sentence constantly expresses only one proposition. Then, the rule of necessitation is valid. What we need in the present discussion is a language sufficient for forming maximal consistent sets, and a demonstrative-free language will properly serve this purpose.

Furthermore, the concern about divergence between some sentences and their informational contents—propositions expressed by those sentences—in the states of believing is also separable from the present problem of which modal system is the correct logic for strict logical modality. It is a reasonable assumption that, in forming maximal consistent sets, every sentence used expresses a proposition according to the standard usage of the language, and every equivocation can be eliminated by replacing a non-equivocal phrase for it. So, it seems to me, Salmon's doubts are not real obstacles to the project of constructing a model representing Ω-logical modal space with respect to S5 modal logic.

and possibilities fits S5. We are not claiming that S5 is the correct logic for metaphysical modality; our assumptions are made for showing the compossibility of metaphysical modality's being restricted in restriction-sense1 and being unrestricted in restriction-sense2. We shall limit the maximality of the modal aspect of maximal consistent sets in Γ^* to Ω-logical modality and metaphysical modality.

To begin with, we construct a maximal set, Γ^*_1, in a way similar to the construction of Γ_1 as stated in the previous section. We then construct the rest of Γ^* according to the following two steps. In the first step, for each $\Diamond\beta$ in Γ^*_1, we form a subordinate set, $\Gamma^{\beta i}_1$, beginning with the set $\{\beta, \delta_1, \ldots, \delta_m, \ldots, \gamma_1, \ldots, \gamma_n, \ldots\}$ where each δ is an E_M-formula and for each γ, $\Box\gamma \in \Gamma^*_1$. We then take each $\Gamma^{\beta i}_1$ as the starting maximal consistent set and expand a $\Gamma^{\beta i}_1$-construction from it based on metaphysical necessities and possibilities in $\Gamma^{\beta i}_1$ according to the rules of Γ-expansion. In the second step, we take the original maximal consistent set Γ^*_1 as starting set and form Γ^*_1-construction based on metaphysical necessities and possibilities in Γ^*_1 according to the rules of Γ-expansion. We get a picture in Figure 2. (See next page.)

The construction of Γ^* does not look exactly like the Γ-construction in the previous section. We shall explain that Γ^* is actually constructed according to the rules of Γ-expansion based on Ω-logical necessities and possibilities. Remember, we refer to the whole tree-construction as "Γ^*-construction," and call the subtrees constructed according to metaphysical modality of Γ^*_1, "Γ^*_1-construction," and of $\Gamma^{\beta i}_1$, "$\Gamma^{\beta i}_1$-construction."



(Figure 2)

Let us examine the first level Γ^*-expansion from the set Γ^*_1. In the first step of expansion, for each Ω-logical possibility $\Diamond\beta \in \Gamma^*_1$, we constructed a maximal consistent set based on the truths of β and Ω-logical necessities in Γ^*_1. In the second step (that is, in the expansion from Γ^*_1 for metaphysical modality), we have constructed a maximal consistent set for each ${}^m\Diamond\beta \in \Gamma^*_1$ based on the truths of β and metaphysical necessities in Γ^*_1. Since $\Box\alpha \supset {}^m\Box\alpha$, every Ω-logical necessity is true in the maximal consistent sets expanded in the second step. Since ${}^m\Diamond\beta \supset \Diamond\beta$, the metaphysical possibility β is also a Ω-logical possibility. Therefore, we constructed for every $\Diamond\beta$ in Γ^*_1 (at least) one maximal consistent set beginning with the set $\{\beta, \delta_1, \ldots,$

$\delta_m, \ldots, \gamma_1, \ldots, \gamma_n, \ldots\}$ where each δ is a E_M-formula and for each γ, $\Box\gamma \in \Gamma^*_1$.

In the rest of the Γ^*-construction, for any maximal consistent set Γ^*_m in Γ^*_1-construction, and for every Ω-logical possibility $\Diamond\beta \in \Gamma^*_m$ such that ${}^m\Diamond\beta \in \Gamma^*_m$, we have constructed a maximal consistent set Γ^*_n based on the truths of β and every metaphysical necessity (which include every Ω-logical necessity) in Γ^*_m within Γ^*_1-construction. We know that since the logic of Γ^*_1-construction is S5, the set of metaphysical necessities and possibilities in Γ^*_1 remains the same in every maximal consistent set of Γ^*_1-construction, and hence in Γ^*_m. Since every Ω-logical necessity is a metaphysical necessity, the set of Ω-logical necessities in Γ^*_1 remains the same in every maximal consistent set of Γ^*_1-construction, and hence in Γ^*_m. This in turn determines that the set of Ω-logical possibilities remains the same in every maximal consistent set of Γ^*_1-construction, and hence in Γ^*_m. In the first level Γ^*-expansion from the set Γ^*_1 we have constructed for every $\Diamond\beta$ in Γ^*_1 a maximal consistent set beginning with the set $\{\beta, \delta_1, \ldots, \delta_m, \ldots, \gamma_1, \ldots, \gamma_n, \ldots\}$ Therefore, for every Ω-logical possibility $\Diamond\beta \in \Gamma^*_m$ such that ${}^m\Diamond\beta \notin \Gamma^*_m$, we know that there is maximal consistent set $\Gamma^{\beta i}{}_1$ already constructed in Γ^*-construction based on the truths of β and every Ω-logical necessity in Γ^*_m. Let it be a subordinate set of Γ^*_m. Thus, we have constructed for every $\Diamond\beta$ in Γ^*_m a maximal consistent set beginning with the set $\{\beta, \delta_1, \ldots, \delta_m, \ldots, \gamma_1, \ldots, \gamma_n, \ldots\}$, where each δ is a E_M-formula and for each γ, $\Box\gamma \in \Gamma^*_m$. Since every set $\Gamma^{\beta i}{}_1$ is expanded from Γ^*_1 based on Ω-logical modality of Γ^*_1

according to S5, $\Gamma^{\beta i}{}_1$ contains the same set of Ω-logical necessities and possibilities as the set in $\Gamma^*{}_1$. A similar reasoning can be given to any maximal consistent set $\Gamma^{\beta i}{}_m$ in $\Gamma^{\beta i}{}_1$-construction.

The above also shows that Γ^*-construction as a whole is constructed according to S5 based on Ω-logical modality. This is given by the argument that the same set of Ω-logical necessities and possibilities holds in every maximal consistent set in Γ^*-construction. Thus, Γ^* is a legitimate Γ-construction—it possesses all the important features of a Γ-construction.

We now define a model $\langle \mathbf{W}, \mathbf{M}\ \mathbf{R}, \mathbf{R}^{\mathrm{M}}, \mathbf{D}, \mathbf{V}\rangle$ based on Γ^*-construction. **W** is a set of worlds corresponding to the maximal consistent sets in Γ^*-construction. **M** is a set of sets of consistent worlds $\mathbf{M}^0$, $\mathbf{M}^1$ ... such that $\mathbf{M}^0$ is a set of worlds corresponding to the maximal consistent sets in $\Gamma^*{}_1$-construction, and for each $i > 0$, $\mathbf{M}^i$ is a set of worlds corresponding to the maximal consistent sets in $\Gamma^{\beta i}{}_1$-construction. **R** is the Ω-logical accessibility relation such that for any world $\boldsymbol{w}_i$ and any world $\boldsymbol{w}_k \in \mathbf{W}$, $\boldsymbol{w}_i \mathbf{R} \boldsymbol{w}_k$, that is, **R** is a total relation which relates all worlds in the model. $\mathbf{R}^{\mathrm{M}}$ is the metaphysical accessibility relation such that for any world $\boldsymbol{w}_i$ and any world $\boldsymbol{w}_k \in \mathbf{M}^i$, $\boldsymbol{w}_i \mathbf{R}^{\mathrm{M}} \boldsymbol{w}_k$, that is, $\mathbf{R}^{\mathrm{M}}$ is a total relation with respect to the worlds in $\mathbf{M}^i$. **D** and **V** are as stated in Section 2.3.

The model $\langle \mathbf{W}, \mathbf{M}\ \mathbf{D}, \mathbf{R}, \mathbf{R}^{\mathrm{M}}, \mathbf{V}\rangle$ shows the following three points. First, since Γ^*-construction is an infinite construction based on logical necessities and possibilities, we can claim that every and all maximal consistent sets with respect to Ω are constructed somewhere in Γ^*-construction. Hence, **W** is a set of all consistent worlds with respect to Ω. Secondly, the scope of metaphysical modal reasoning for any arbitrary world $\boldsymbol{w}_i$ in **W** is restricted in the sense that not every world in **W** is a metaphysically possible world relative to $\boldsymbol{w}_i$. Third, the metaphysical accessibility relation is an equivalence relation—every world in **W** is in an equivalence class related by the metaphysical accessibility relation. These three points demonstrate that when modal operators quantify over all and only consistent worlds, it is compossible that metaphysical modality is restricted in restriction-sense1 and unrestricted in restriction-sense2.

Given what we have shown above, Salmon's rejection of S5 relies totally on whether he can convincingly argue that metaphysical modality is restricted in restriction-sense2. We said in Section 2.2 that Salmon's rejection of the "ostrich approach" is not sufficient for this purpose: the rejection of the "ostrich approach" criticizes a particular way of viewing metaphysical modality as unrestricted in restriction-sense2, it does not follow from this rejection that metaphysical modality is therefore restricted in restriction-sense2. However, Salmon holds that in a broad sense, the philosophical practice of the majority of philosophers concerning metaphysical modality all commit the fallacy of the "ostrich approach." Namely, in talking about metaphysical modality, those philosophers all ignored metaphysically impossible worlds of the real world and let modal operators quantify over only metaphysically possible worlds of the real world. He says the following:

[Passage (8)]

Surprisingly, the ostrich approach has nevertheless ascended to the status of orthodoxy. It is precisely the approach followed by my critics. The most obvious sign of the ostrich approach is the explicit denial of impossible worlds, ... Metaphysical modality appears unrestricted because the restriction to metaphysically possible worlds is already built into one's practice concerning which worlds to pay attention to and to quantify over. ... But ignoring impossible worlds does not make them go away, ... by what right do we ignore worlds that are inaccessible? Accessible or not, they're still worlds. Why don't they count?[63]

Salmon's claim in Passage (8) is this: Whatever reason those philosophers may give, ignoring impossible worlds in metaphysical modal reasoning is always fallacious. We know that when possible worlds of a consistent world ***w*** bear equivalent accessibility relation between each of them, there is a legitimate technical reason to ignore impossible worlds of ***w***, since there are no accessibility relation between any possible world of ***w*** and any impossible world of ***w***. Only if metaphysical accessibility is not an equivalence relation, does it make sense to say that ignoring impossible worlds in metaphysical modal reasoning is always fallacious. Clearly there is a gap between the claim made in Passage (8) and what can be demonstrated by Salmon's rejection of "ostrich approach." Then, there must be something crucial to link the reasoning. The answer is not difficult to see. Here is a passage where Salmon explains why impossible worlds must not be ignored.

[Passage (9)]

... as long as there is a possible scenario according to which it is possible for Woody [a wooden table] to have originated from m [an impossible hunk of matter of Woody according to the real world], it is true (in English) to say "It is possible that it is possible that Woody

63 Salmon [1989], 21-24.

originates from m," and one cannot correctly say (in English) "It is necessary that it is necessary that Woody does not originate from m." ... If the possible scenario ... that verify a possibility claim or falsify a necessity claim draw our attention to inaccessible worlds, then we are obliged to pay attention to those inaccessible worlds.[64]

Passage (9) is a repetition of Salmon's Intransitive Accessibility Solution to the Paradoxes. Salmon says that his intransitivity account stems from the following intuition: a particular material artifact, say, Woody, could have originated from a hunk of matter slightly different from its actual original matter. "... by stretching things to the limit, we may select some ... matter m such that, although Woody could not have originated from m, m is close enough to being a possibility for Woody that if Woody had originated from certain matter m' that is in fact possible for Woody ... then it would have been possible for Woody to have originated from m, even though it is not actually possible."[65] This is to say that Salmon's intransitivity account is given on the ground of his belief that it is true that some metaphysically impossible worlds of the real world, like world $\boldsymbol{w}_{m+1}$ in Chisholm's Paradox and β-world2 in Four Worlds Paradox, are possible relative to some metaphysically possible worlds of the real world. In general, Salmon believes that some metaphysically impossible worlds relative to a world $\boldsymbol{w}$ are accessible from some metaphysically possible worlds of $\boldsymbol{w}$. This can be evidently seen in Passage (9). In order for him to say that a possible world of $\boldsymbol{w}$ (a possible scenario relative to $\boldsymbol{w}$) that verifies a possible claim or falsifies a necessary claim draws our attention to an inaccessible world of $\boldsymbol{w}$, Salmon must in the first

64 Salmon [1989], 21.
65 Salmon [1989], 5.

place believe that there truly are possible worlds of ***w*** which contain some metaphysical possibilities that are metaphysically impossible in ***w***.

Our analysis shows that Salmon's reply to the "standard objection" ultimately relies on the correctness of his Intransitive Accessibility Solution, or the truth of his belief in metaphysically impossible but possibly possible worlds. But, it is to the truth of this belief the "standard objection" is raised. We conclude that Salmon's reply to the "standard objection" did not meet the challenge raised by the objection. He fails to offer a convincing account independent of his Intransitive Accessibility Solution that shows the restricted nature of metaphysical modality in restriction-sense2. Hence he provides no real solid answer to the second part of the objection. For the same reason, he cannot successfully defeat argument (VI)-(VIII), the defensive argument for S5 given in terms of being restricted in restriction-sense2. He then cannot effectively reject the first part of the objection. Salmon's reply to the "standard objection" *per se* sounds circular; what remains for us to see is whether the Intransitive Accessibility Solution itself is a convincing account for showing the intransitive nature of metaphysical modality, and this is the task of next chapter.

CHAPTER 3
THE PHILOSOPHICAL POSITION OF SALMON'S INTRANSITIVE ACCESSIBILITY SOLUTION

3.1 Forbes' Defense of S5

In last chapter, we distinguished two senses of the notion of restriction on modality. We defined a restriction on a type of modality in restriction-sense1 with respect to a model of the type of modality as follows: not every world in the model is a possible world of the type relative to an arbitrary world ***w*** of the model. We defined a restriction on a type of modality in restriction-sense2 with respect to a model of the type of modality as follows: the worlds in the model are not related as equivalence classes. The property of being restricted in sense1 and the property of being restricted in sense2 thus stated are formal properties of a modality. These formal properties must be explained by the metaphysical property of what the type of modality in question is. Hence a genuine defense for the restrictiveness or the unrestrictiveness of metaphysical modality can only be given by a metaphysical explanation of what metaphysical modality is.

Graeme Forbes has given a metaphysical defense for the view that S5 is the logic of metaphysical modality (Forbes use the word "broad logical necessity" for metaphysical necessity):

[Passage (10)]

> If we consider substantial philosophical theses whose formulations employ broadly logical possibility and necessity, such as the theses that the members of a set are essential to it or that if it exists, an organism belongs of necessity to the biological kind to which it actually belongs, we see that there is a conceptual character to such claims: establishing them involves investigating the notions of set and set membership, and of kind and subsumption under a kind, and the interconnections of these concepts with the idea of what it is to be an individual thing of the given sort. What the broadly logical necessities are is therefore fundamentally an *a priori* matter, to do with the content of our concepts, even though with the addition of *a posteriori* information, necessary *a posteriori* truths can be inferred.[66]

In Passage (10), metaphysical necessity is explained as fundamentally necessity of conceptual truth. There are *de dicto* metaphysical necessities as well as *de re* metaphysical necessities.[67] Forbes says that "*de dicto* necessities are straightforwardly explicable in terms of the content of concepts, for they are simply definitions, or principles constitutive of some concept's content, or logical consequences of some concept's content, or logical consequences of such principles."[68] *De dicto* necessities are knowable *a priori*. According to Forbes, *a priori de re* necessities (like principles **(N)** and **(C)**) are essentialist principles, while *a posteriori de re* necessities (like those necessities derivable from **(N)** or **(C)**) are essentialist claims about particular individuals. Forbes' argument

66 Forbes [1983], 185.

67 Forbes defines *de dicto* and *de re* formulae as follows: A formula with modal or tense operators is *de re* iff it contains a modal or tense operator R which has within its scope either (1) an in individual constant, or (2) a free variable, or (3) a variable bound by a quantifier not within R's scope. All other formulae with modal or tense operators are *de dicto*. See Forbes [1985], 48-49.

68 Forbes [1985], 231.

about *a posteriori* necessities' being conceptual is mainly made in terms of their being derivable from certain *a priori* necessity: most direct method of establishing a necessary *a posteriori* truth is by inference from a singular *a posteriori* truth and a general *a priori* one, then the source of the necessity in an *a posteriori* truth is still an *a priori* truth. Forbes' conjecture is that no necessary *a posteriori* truth departs from this pattern.[69]

Given this explanation of what metaphysical necessity is, Forbes claims that metaphysical necessities, as fundamentally conceptual truths, must hold in every possible world. Forbes demonstrates his claim by the view that all possible worlds of the real world, as maximal abstract entities, are constructed in the same "conceptual scheme" as the one by which we describe the real world,[70] but he did not provide any more detailed explanation about what this view exactly is. We may have the following intuitive understanding of Forbes' view. First, it is the case of philosophical practice that all possible worlds are constructed in the same interpreted language as the one used in describing the real world. Thus, if $\boldsymbol{w}_@$ is the real world and $\boldsymbol{w}_1$ is a possible world of $\boldsymbol{w}_@$ and sentence P is true in both $\boldsymbol{w}_@$ and $\boldsymbol{w}_1$, then sentence P in $\boldsymbol{w}_1$ expresses the same proposition as the one expressed by P in $\boldsymbol{w}_@$. Secondly, assume that we know the facts of the real world and when describing the real world in the interpreted language we are able to tell whether a sentence expresses a truth. This means that we have the grasp of the boundary for the use of the expressions in the sentence. The boundary for the

69 Forbes [1985], 231.
70 Forbes [1985], 237, footnote 26.

use of an expression is a conceptual truth of the interpreted language that belongs to the very interpretation of the language. Thirdly, since the real world is consistent, the language used to describe the real world should be interpreted consistently. Thus, the conceptual truths of the interpreted language must be systematically related to the establishment of the consistency—this may be called "the conceptual system" of the interpreted language. Fourthly, given what a conceptual system is, we may say that the real world is described in the conceptual system of the interpreted language. Since the possible worlds are constructed in the same interpreted language as the one used to describe the real world, we may say that the possible worlds are constructed in the same conceptual system as the one in which the real world is described. This understanding of Forbes' view, that all possible worlds of the real world are constructed in the same "conceptual scheme" as the one by which we describe the real world, is consistent with the rest of his argument.

According to Forbes, all *a priori* metaphysical necessities (*de dicto* necessities and *a priori de re* necessities) of $w_@$ must hold the same in every possible world of $w_@$. Since *a priori* metaphysical necessities are simply definitions or principles about the content of concept (conveyed by a certain expression of the language), they are part of the conceptual system of the interpreted language and, hence, must hold whenever the interpreted language is used. Thus, every *a priori* metaphysical necessity of $w_@$ is metaphysically necessary in every possible world of $w_@$.

Forbes continues to argue that no *a posteriori* metaphysical necessity of $\boldsymbol{w}_{@}$ can fail in any possible world of $\boldsymbol{w}_{@}$:

[Passage (11)]

Could some *a posteriori* necessary truth, necessary at w*[the actual world], fail at w[a possible world of w*]? Evidently not: the same *a priori* conceptual truths hold at every world, and any *a posteriori* truth *T* necessary at the actual world is so by being true at the actual world and by some conceptual truth's entailing that *T*'s truth makes it necessary. Thus *T* holds at any world accessible to the actual world, so the same conceptual truth will make it necessary at such a world over again; hence we never reach a world where some actual impossibility is true. Since a world is accessible to the actual world provided everything true at it is actually possible, failure of transitivity of accessibility therefore never arises. Similar reasoning settles the question of symmetry, which means that S5 emerges as the correct system.[71]

We may summarize our understanding of Forbes' argument as follows:

(1) *a priori* metaphysical necessities are straightforwardly conceptual truths.

(2) Given that all possible worlds of the real world are constructed in the same interpreted language as the one by which we describe the real world, *a priori* metaphysical necessities, as conceptual truths of the interpreted language, must hold in every possible world.

(3) The general patten of obtaining an *a posteriori* metaphysical necessity $\Box T$ is by inference from a singular *a posteriori* truth and a general *a priori* necessity. So the source of the necessity in an *a*

[71] Forbes [1985], 237, footnote 26.

posteriori metaphysical necessity is an *a priori* conceptual truth.

(4) *a posteriori* truth T hold in every possible world $\boldsymbol{w}$ of the real world. The same *a priori* necessity which makes T's truth necessary in the real world will make T's truth necessary in $\boldsymbol{w}$ over again.

(5) Therefore, S5 is the correct logic for metaphysical modality.

Let us now examine Salmon's response to Forbes' defense of S5. Salmon agrees with Forbes on the first two premises.[72] But he raises an objection to premise (3) of Forbes' argument by proposing a counterexample as quoted in the following passage.

> [Passage (12)]
>
> In fact, not even the conditional "If table $\boldsymbol{a}$ is not originally formed from hunk $\boldsymbol{h}_m$, then it is necessary that $\boldsymbol{a}$ is not originally formed from $\boldsymbol{h}_m$" is *a priori*. ... The necessary *a posteriori* truth that table $\boldsymbol{a}$ is not formed from hunk $\boldsymbol{h}_m$ is thus a counterexample to Forbes' claim concerning the source of necessary *a posteriori* truths. Since the conditional proposition that if $\boldsymbol{a}$ is not formed from $\boldsymbol{h}_m$ then $\boldsymbol{a}$ is necessarily not thus formed is not *a priori*, it cannot be entailed by any conceptual *a priori* truth.[73]

Salmon holds that the epistemological status of the propositions such as "If table $\boldsymbol{a}$ is not originally formed from hunk $\boldsymbol{h}_m$, then it is necessary that $\boldsymbol{a}$ is not originally formed from $\boldsymbol{h}_m$" is neither knowable *a priori* nor knowable *a posteriori*. The hunk of matter $\boldsymbol{h}_m$ is supposed to be the threshold of possible matter for table $\boldsymbol{a}$. Salmon

[72] Salmon [1986], 108-109.
[73] Salmon [1986], 109.

says that since it is "difficult to imagine establishing, by philosophical argument or otherwise, exactly what number $\boldsymbol{m}$ is, i.e., precisely how many molecules of difference from the actual original matter of table $\boldsymbol{a}$ would first result in a new and different table. It seems likely that it is unknowable that table $\boldsymbol{a}$ is necessarily not originally formed from hunk of matter $\boldsymbol{h}_{\mathrm{m}}$."[74]

I do not deny that there may be some unknowable truths, but I do not think this helps provide a counterexample to Forbes' argument at all. As I understand it, Salmon's point is this: though the truths of the necessary *a posteriori* proposition and the given conditional are not knowable either *a priori* or *a posteriori*, they nevertheless must be either true or false; if it happens to be the case that they are in fact true, the truth of the necessary *a posteriori* proposition does not involve any *a priori* factor, or is not entailed by any conceptual principle. We notice that in Salmon's example what is unknowable is the threshold of the tolerance. Even if Salmon is right that the threshold of the tolerance is unknowable whatsoever, the principle stated in (A) below is nevertheless knowable *a priori*:

> (A) "If table $\boldsymbol{a}$ is not originally formed from hunk $\boldsymbol{h}_{\mathrm{m}}$, and $\boldsymbol{h}_{\mathrm{m}}$ is, or is beyond, the threshold of table $\boldsymbol{a}$'s possible forming matter, then it is necessary that $\boldsymbol{a}$ is not originally formed from $\boldsymbol{h}_{\mathrm{m}}$."

(A) expresses an essentialist concept about table $\boldsymbol{a}$. The universal generalization of (A) is an *a priori* essentialist principle. Every instance of the universal generalization of (A) is true because when

[74] Salmon [1986], 113.

the antecedent is true, the consequent is true as well, no matter what value $\boldsymbol{h}_{\mathrm{m}}$ takes.

On the other hand, the universal generalization of the conditional given in Salmon's example:

> (B) "If table $\boldsymbol{a}$ is not originally formed from hunk $\boldsymbol{h}_{\mathrm{m}}$, then it is necessary that $\boldsymbol{a}$ is not originally formed from $\boldsymbol{h}_{\mathrm{m}}$."

is not true because some of its instances are false. Salmon is right in saying that (B) is not knowable *a priori* and does not express any conceptual truth. But for each case that (B) comes out true by $\boldsymbol{h}_{\mathrm{m}}$'s taking a certain value, there is always a corresponding instance of (A) such that the same necessary *a posteriori* proposition, "necessarily table $\boldsymbol{a}$ is not originally formed from $\boldsymbol{h}_{\mathrm{m}}$," which is derivable from (B), is also derivable from (A), together with the same empirical information. Any instance of (A) is knowable *a priori*, and it is clear that we can understand the necessity of the *a posteriori* proposition better from (A) rather than from (B). It is inacurate to claim that the necessary *a posteriori* proposition in question is not entailed by any *a priori* principle, or involves no *a priori* factor just because the same proposition is also derivable from some non-*a-priori* and not-well-informed proposition such as (B). Thus, Salmon's objection to premise (3) does not succeed in refuting Forbes' argument.

The following is another passage of Salmon's, which may be understood as rejecting Forbes' defense of S5 by a different reason—Forbes' argument is inapplicable to the *a posteriori* necessities (derivable from principles (**N**) and (**C**)).

[Passage (13)]

> ... there are examples ... of propositions that are metaphysically necessary yet conceptually *a posteriori*. With respect to these examples, the argument that a priori necessity iterates—the argument that if it is necessary, because *a priori*, that *p*, then it is also necessary that it is necessary that *p*, and so on—is inapplicable. The argument is inapplicable precisely because the examples in question, though necessary, are not *a priori*, and hence not necessary-by-virtue-of-being-*a-priori*.[75]

We see that in Forbes' defensive argument for S5, premise (2) is a rationale for *a priori* metaphysical necessities of the real world to be held in every possible world of the real world, and premise (4) is a rationale for *a posteriori* metaphysical necessities to be held in every possible world of the real world. It is not clear from Passage (13) which of premise (2) and premise (4) is referred to by "the argument" mentioned in the passage. If by "the argument" Salmon means premise (2), Passage (13) is no objection at all to Forbes' defense of S5, since premise (2) is originally not intended to be applied to the cases of *a posteriori* necessity. To make Salmon's rejection more plausible, we may assume that by "the argument" Salmon means premise (4).

Recall that premise (3) states that the a posteriori necessity $\Box T$ is obtained by inference from an *a priori* necessity and a singular *a posteriori* truth. Premise (4) states that the *a posteriori* truth T hold in every possible world $\boldsymbol{w}$ of the real world, and the same *a priori* necessity which makes T's truth necessary in the real world will make T's truth necessary in $\boldsymbol{w}$ over again. Premise (3) and

[75] Salmon [1986], 109.

premise (4) together seem to suggest that *a posteriori* necessities' being held in every possible world is defended by the derivableness of the *a posteriori* necessities in every world. The following are two possible understandings of premise (4) in terms of the derivableness. We found that premise (4) under these understandings is indeed inapplicable to the *a posteriori* necessities derivable from principles (**N**) and (**C**):

(i) One may understand premise (4) as saying that $\Box T$ is derivable in world w from the same *a priori* necessity and the same *a posteriori* truth T. Then the *a priori* necessity has to be of the form $\Box(T \supset \Box T)$. But principles (**N**) and (**C**) are not in that form. Therefore, (4) is inapplicable to the *a posteriori* necessities derivable from principles (**N**) and (**C**).

(ii) One may understand premise (4) as saying that $\Box T$ is derivable from the same *a priori* necessity in world $\boldsymbol{w}$ in the same way as it is derived in the real world. Let T be the proposition "table $\boldsymbol{a}$ is not made from hunk $\boldsymbol{h}_n$," and Q be the proposition "table $\boldsymbol{a}$ is made from hunk $\boldsymbol{h}_1$ and $\boldsymbol{h}_n$ is sufficiently different from $\boldsymbol{h}_1$." $\Box T$ is derived in the real world from (**N**) and the *a posteriori* truth Q. But there is no guarantee that the *a posteriori* truth Q holds in every possible world of the real world. Therefore, (4) is inapplicable to the *a posteriori* necessities derivable from principles (**N**) and (**C**).

I do not know which, if any, of the above understandings of premise (4) is the one that Salmon has in mind when he says that Forbes' argument is inapplicable to certain *a posteriori* necessities. But, from (i) and (ii) above, it seems to me that if it should be the case that *a posteriori* necessities hold in every possible world, the rationale for it must be different from the one given by Passage (12) summarized as premise (4) of Forbes' argument.

3.2 Absolute Essentialism and Relative Essentialism

With the conclusion of the last section, we may ask what could be a defense for the view that *a posteriori* necessities hold in every possible world. Recall that Forbes defines metaphysical necessity as fundamentally conceptual truths. It is not controversial, as Forbes argues in premises (1) and (2), that *a priori* metaphysical necessities (*de dicto* or *de re*), as straightforward conceptual truths, hold in every possible world. The *a posteriori* necessities in question are deemed as *a posteriori de re* metaphysical necessities. An *a posteriori de re* metaphysical necessity differs from any *a priori* necessity because it requires empirical knowledge; in addition, an *a posteriori de re* metaphysical necessity differs from any *de dicto* necessity because it is about an individual, not about a proposition. But, *a posteriori de re* metaphysical necessities are also metaphysical necessities—these necessities are of the same modal type as *de dicto* metaphysical necessity and *a priori de re* metaphysical necessity. One may suggest that if we construe an *a posteriori de re* metaphysical necessity itself as a conceptual truth concerning a certain individual in the real world, then premise (2) of Forbes' argument is applicable to *a posteriori* metaphysical necessities, and hence S5 can be defended. Forbes himself seems to hold this view. Forbes rejects Salmon's accessibility account by saying the following:

[Passage (14)]

> ... the idea of contingent possibility or necessity to which the accessibility theorist is committed hardly makes sense: surely no-one will want to say that a merely possibly possible world would have been possible if our concepts had been different, or if we had had the concepts required to understand the propositions true at that world, which as a matter of contingent fact we do not.[76]

Passage (14) says that the idea of contingent possible world comes from changing concepts from a given world to possible worlds of the given world. We explained in Section 1.5 that in Salmon's account, necessity iteration is valid for *a priori* essentialist principles like (**N**) and (**C**), but invalid for *a posteriori* necessities derivable from *a priori* essentialist principles. The objection in Passage (14) makes sense only if those *a posteriori* necessities are taken to be conceptual truths in Forbes' account. In a context related to Passage (14), Forbes argues that the necessity of an essentialist claim about an individual x comes from certain category concepts (concepts of property or relation) which describe the category to which x belongs and the concept of the individual x's thisness (the concept of x's thisness, according to Forbes, can be articulated as some necessary conditions which may not be jointly sufficient).[77]

In my view, if the *a posteriori* necessities that are derivable from an *a priori* necessity are, as Kripke suggested, of the same type of necessity as those *a priori* ones, and if *a priori* metaphysical necessities are construed as conceptual truths, the suggestion that *a*

[76] Forbes [1983], 185.
[77] Forbes [1985], 234.

posteriori metaphysical necessities are also conceptual truths is reasonable. But, we shall see that even if *a posteriori* metaphysical necessities are viewed as conceptual truths, it is still not enough to sufficiently defend S5. We shall show that given that all metaphysical necessities, *a priori* or *a posteriori*, are conceptual truths, and given that the possible worlds of the real world $\boldsymbol{w}_@$ are constructed in the same interpreted language as the one in which $\boldsymbol{w}_@$ is constructed, there is still a consideration according to which some possible world of world $\boldsymbol{w}$ is impossible relative to $\boldsymbol{w}_@$ but $\boldsymbol{w}$ is possible relative to $\boldsymbol{w}_@$—the transitivity of the relative possibility relation between worlds fails.

We want to first make the following point clear. Salmon emphasizes that the propositions that the intransitive-accessibility account holds to be necessary but not doubly necessary are certain *a posteriori* propositions whose necessity is derived by means of *a priori* modal principles like **(N)** together with certain further information, some of which is not *a priori*—*a priori* principle **(N)** might be used to establish the necessity of table α's not originating from hunk $\boldsymbol{h}_{\mathrm{m}}$, but the fact that α does not thus originate is itself unquestionably empirical and not *a priori*.[78] This may be taken as suggesting that the intransitiveness has to do with the way of how *a posteriori* necessities are established, that is, being *a posteriori* is the reason for those necessities' being intransitive—the need of an empirical information in the inference of an *a posteriori* necessity is

78 Salmon [1986], 109.

responsible for the intransitiveness. We show that this view is incorrect.

Let us recall how an *a posteriori* necessity is derived in the example of Chisholm's Paradox:

(I) $\Box\forall \boldsymbol{x}\Box\forall \boldsymbol{y}\Box\forall \boldsymbol{z}\Box$(If a wooden table $\boldsymbol{x}$ is made from a hunk of matter $\boldsymbol{y}$, and $\boldsymbol{z}$ is any hunk of matter sufficiently different from $\boldsymbol{y}$, then $\Box$(table $\boldsymbol{x}$ is not made from $\boldsymbol{z}$)).

(II) If wooden table α is made from hunk β at the starting world and δ is a hunk of matter sufficiently different from β, then $\Box$(table α is not made from δ).

(III) Wooden table α is made from hunk β at the starting world.

(IV) δ is a hunk of matter sufficiently different from β.

(V) $\Box$(table α is not made from δ).

(I) is an *a priori* modal principle and, according to Salmon, multiple necessitation is true for (I). (II) is an instance of (I), where whatever the mode of the necessity embodied in the consequent of the conditional may be, it is preserved from (I). (III) is the empirical information which makes the antecedent of (II) true. The truth of (IV) is according to the assumption on the threshold of tolerable variations. Nothing in the pattern of the derivation suggests that the empirical discovery, (III), will affect the mode of the necessity in the consequent of (II) and hence give rise to the intransitiveness for the necessity of the necessary *a posteriori* proposition in (V).

We can see from the above that whatever mode of the *a posteriori* necessity stated in (V) is, it is already so in (II) and (I)—whatever mode a particular *a posteriori* necessity has, it comes from the relevant *a priori* conceptual principle. In other words, the mode of the derived *a posteriori* necessity is originally included in the conceptually *a priori* principle as part of the content of the principle. The empirical discovery only helps derive whatever is entailed in the principle. It is not difficult to see that if some *a priori* metaphysical necessity of the form, "if ..., then necessarily ...," contains as part of its informational content an intransitive mode for its inner *a posteriori* metaphysical necessity, and if the *a priori* necessity is reckoned to be true and the inner *a posteriori* necessity can be soundly inferred, the accessibility relation between possible worlds will be intransitive. In the following, we assume that the modal operator "$\Box$" is defined by the axioms of S5, and compare the two different statements:

(N1) $\Box$(if a wooden table α is originally made from a hunk of matter β, and δ is any hunk of matter whose collection of components is sufficiently different from the collection of β, then $\Box$(α can not be originally made from δ)).

(N2) $\Box$(if a wooden table α is originally made from a hunk of matter β, and δ is any hunk of matter whose collection of components is sufficiently different from the collection of β, then $\Box$(relative to the situation in which α is in fact made from β, α can not be originally made from δ)).

The difference between **(N1)** and **(N2)** is that they contain different content for the inner *a posteriori* conceptual truth. **(N1)** can be formulated as:

(N1*) $\Box((\boldsymbol{M}(\alpha,\beta)\&\boldsymbol{D}(\delta,\beta))\supset\Box\neg\boldsymbol{M}(\alpha,\delta))$.

The partial formulation of **(N2)** is:

(N2*) $\Box((\boldsymbol{M}(\alpha,\beta)\&\boldsymbol{D}(\delta,\beta))\supset\Box($Relative to the situation in which α is in fact made from β, $\neg\boldsymbol{M}(\alpha,\delta)))$.

We now show that if the conceptual system of the interpreted language that we use to describe the real world contains *a priori* conceptual truths such as **(N2)**, then it can be the case that the metaphysical accessibility relation between worlds is intransitive, and premise (II) of Forbes' argument is not applicable to the *a posteriori* necessities derivable from these *a priori* necessities.

We may reformulate **(N2)** as follows:

(N2)** $\Box((\boldsymbol{M}(\alpha,\beta)\&\boldsymbol{D}(\delta,\beta))\supset{}^{\boldsymbol{M}(\alpha,\beta)}\Box\neg\boldsymbol{M}(\alpha,\delta))$.

According to **(N1*)**, the *a posteriori* necessity in **(N1*)**, $\Box\neg\boldsymbol{M}(\alpha,\delta)$, holds in every possible world by necessity iteration, but according to **(N2*)**, it is the necessity $\Box$(relative to the situation in which α is in fact made from β, $\neg\boldsymbol{M}(\alpha,\delta)$), not the necessity $\Box\neg\boldsymbol{M}(\alpha,\delta)$, that holds in every possible world by necessity iteration. However, saying that "it is necessary (relative to the situation in which α is in fact made from β, $\neg\boldsymbol{M}(\alpha,\delta)$)" is equivalent to saying that "it is necessary relative to the situation in which α is made from β ($\neg\boldsymbol{M}(\alpha,\delta)$)." The modal operator "${}^{\boldsymbol{M}(\alpha,\beta)}\Box$" stands for "it is necessary relative to the situation in which α is made from β." Since the necessity referred by "$\boldsymbol{M}(\alpha,\beta)\Box$" is a relative one, it carries an intransitive mode—the axiom of S4, $\Box P \supset \Box\Box P$, is invalid for "$\boldsymbol{M}(\alpha,\beta)\Box$."

We now form a Γ'-construction starting with a maximal consistent set, Γ'_1, containing **(N2**)**, **(ST*)** and $\boldsymbol{M}(\alpha,\beta)$, but not **(N1*)**. The logic of Γ'-construction is S5, because our hypothesis is that S5 is valid for all *a priori* metaphysical necessities formulated as "$\Box(......)$." In doing so, all formulations of *a priori* conceptual truths will hold at every maximal consistent set in Γ'. The axiom, $\Box P \supset \Box\Box P$, is invalid for "$\boldsymbol{M}(\alpha,\beta)\Box$." Notice that we are using both "$\Box$" and "$\boldsymbol{M}(\alpha,\beta)\Box$" to refer to metaphysical necessities. In the expansion of a subordinate set Γ'_k of Γ'_i, the initial set of Γ'_k, $\Gamma'_{k,1}$, must include all γ's such that either $\Box\gamma$ or $\boldsymbol{M}(\alpha,\beta)\Box\gamma$ is in Γ'_i. The difference is that every $\Box\gamma$ of Γ'_i is also included in $\Gamma'_{k,1}$ whereas $\boldsymbol{M}(\alpha,\beta)\Box\gamma$'s of Γ'_i are not. This is because when $\Box\gamma$ is in Γ'_i, $\Box\Box\gamma$ is also in Γ'_i by the characteristic axiom of S4, which is not valid for $\boldsymbol{M}(\alpha,\beta)\Box\gamma$'s. The expansion can otherwise proceed normally.

We define a model **M**, composed of **W**, **R**, **D** and **V**, based on Γ'-construction in the way similar to defining a model based on the Γ-construction explained in Section 2.3. We must, however, postpone adding the accessibility relations by transitivity because the complication caused by the modal operator "$\boldsymbol{M}(\alpha,\beta)\Box$." To examine whether the transitive accessibilities do hold, we look at the outcome of the expansion whether a subordinate* world Γ'_k of world Γ'_i in Γ' has the property that every fact contained in Γ'_k is a metaphysical possibility of Γ'_i, or that every metaphysical necessity of Γ'_i is true in

Γ'_k. If it does have this property, we add the accessibility relation from Γ'_i to Γ'_k; otherwise, we do not add the relation between them.

Let the world that corresponds to the starting maximal consistent set of Γ'-construction Γ'_1 be the real world $\boldsymbol{w}'_@$. The necessary principles **(N2**)** and **(ST)** hold in the real world and, by the axiom of S4, hold in every world in **M**. The *a posteriori* necessity ${}^{\boldsymbol{M}(\alpha,\beta)}\Box\neg\boldsymbol{M}(\alpha,\delta)$ can be established in the real world by an inference from **(N2**)** and $\boldsymbol{M}(\alpha,\beta)$. But the axiom of S4 is invalid for this necessity. If in some subordinate world of $\boldsymbol{w}'_@$, $\boldsymbol{M}(\alpha,\beta)$ is still true, the *a posteriori* necessity ${}^{\boldsymbol{M}(\alpha,\beta)}\Box\neg\boldsymbol{M}(\alpha,\delta)$ can be derived in that world again. But since $\boldsymbol{M}(\alpha,\beta)$ is contingent, there must be some subordinate world $\boldsymbol{w}'_i$ of $\boldsymbol{w}'_@$ in which $\boldsymbol{M}(\alpha,\beta)$ is not true, and so ${}^{\boldsymbol{M}(\alpha,\beta)}\Box\neg\boldsymbol{M}(\alpha,\delta)$ is not derivable in $\boldsymbol{w}'_i$. Then, ${}^{\boldsymbol{M}(\alpha,\beta)}\Diamond\boldsymbol{M}(\alpha,\delta)$ can be consistently added to the initial set of $\boldsymbol{w}'_i$. In the expansion at the next level, according to the rule of Γ-expansion, there is at least one subordinate world $\boldsymbol{w}'_k$ of $\boldsymbol{w}'_i$ which contains $\boldsymbol{M}(\alpha,\delta)$—the construction of the maximal consistent set corresponding to $\boldsymbol{w}'_k$ starts from the initial set which contains $\boldsymbol{M}(\alpha,\delta)$. Since $\boldsymbol{w}'_k$ is consistent, $\neg\boldsymbol{M}(\alpha,\delta)$ is not in $\boldsymbol{w}'_k$. The rule of Γ-expansion guarantees that $\boldsymbol{w}'_i$ is possible relative to $\boldsymbol{w}'_@$ and $\boldsymbol{w}'_k$ is possible relative to $\boldsymbol{w}'_i$. But, according to the definition of relative possibility relation, $\boldsymbol{w}'_k$ is not possible relative to $\boldsymbol{w}'_@$ since the necessity ${}^{\boldsymbol{M}(\alpha,\beta)}\Box\neg\boldsymbol{M}(\alpha,\delta)$ holds in $\boldsymbol{w}'_@$ but $\neg\boldsymbol{M}(\alpha,\delta)$ is not true in $\boldsymbol{w}'_k$—the relative possibility relation is intransitive.

In our view, the intransitiveness of the relative possibility relation between the consistent worlds in model **M** is an inherent nature from the conceptual system of the interpreted language in which **M** is given. Namely, the relative possibility relation is intransitive because the worlds in **M** are constructed in an interpreted language the conceptual system of which contains relative conceptual truths such as **(N2**)**. Notice that, corresponding to **(N2**)**, the general principle **(N)** can be re-formulated as:

(N)** $\Box\forall x\Box\forall y\Box\forall z(D(z,y)\supset\Box(M(x,y)\supset^{M(x,y)}\Box\neg M(x,z)))$.

With different empirical fact, different *a posteriori* relative conceptual truth can be inferred from a relevant instance of **(N**)**. Given the assumption of Chisholm's Paradox that a sequence of worlds, $w_@$, w_1, ..., w_n, is such that in $w_@$ table α is made of h_0, and for each $i\geq 1$, table α is made from h_i in w_i, where h_0 and h_i's are distinct hunks of matter, by **(N**)** and the assumption, in each different world in the sequence there holds a different *a posteriori* relative conceptual truth. This may be what Forbes means in Passage (13) that Salmon's account committed the fallacy of adopting different conceptual scheme in each of the worlds. But it seems to me that it is more plausible and more illuminating to construe the cause of the intransitiveness by the relative nature of the conceptual system of the interpreted language in which the worlds are constructed.

In our above argument, all metaphysical necessities, *a priori* or *a posteriori*, are viewed as conceptual truths, and the model **M** is given in the same interpreted language. The argument shows that even if all metaphysical necessities, *a priori* or *a posteriori*, are conceptual truths, and even if the metaphysically possible worlds of an arbitrary world w are constructed in the same interpreted language as the one in which $\boldsymbol{w}$ is constructed, the relative metaphysical possibility relation between worlds can still be intransitive if the conceptual system of the interpreted language contains *a priori* principles like **(N**)**.

It is not difficult to see that if an interpreted language has *a priori* necessities like **(N1)**, instead of those like **(N2)**, as its conceptual truths, then the relative possibility relation between the worlds in a model constructed in this interpreted language will be an equivalence relation. Recall that we let "$\Box$" be defined by the axioms of S5. Principle **(N1)** (formulated as **(N1*)**) asserts its inner *a posteriori* necessity in the same mode, i.e., the mode of S5. Once the *a posteriori* necessity $\Box\neg\boldsymbol{M}(\alpha,\delta)$ is established in a certain world $\boldsymbol{w}$ by an inference from **(N1*)** and $\boldsymbol{M}(\alpha,\beta)$, it holds in every possible world of $\boldsymbol{w}$.

Let us use "$\Box$" and "$\Diamond$" consistently in the mode of S5, and use "$^{R}\Box$" and "$^{R}\Diamond$" in an intransitive mode, where "R" refers to the situation to which the relative necessities and possibilities apply. We then call the *a priori* conceptual truth **(N2)** (formulated as **(N2**)**) a relative essentialist conceptual truth, and call a conceptual system of an interpreted language containing conceptual truths like **(N2)** a relative essentialist conceptual system. Accordingly, we call the *a priori* conceptual truth **(N1)** (formulated as **(N1*)**) an absolute essentialist conceptual truth, and call a conceptual system of an interpreted language containing conceptual truths like **(N1)** instead of **(N2)** an absolute essentialist conceptual system. If a model of metaphysical modality given in an interpreted language with a relative essentialist conceptual system, the relative possibility relation of the model will be intransitive. One the other hand, if a model of metaphysical modality given in an interpreted language with an absolute essentialist conceptual system, the relative possibility relation of the model will be an equivalence relation.

Despite the difference between the two kinds of conceptual systems, a relative essentialist conceptual system and an absolute essentialist conceptual system are all conceptual systems. Forbes' views, that metaphysical necessities are fundamentally conceptual, and that necessity iteration is true for necessary *a priori* propositions, can be held with respect to either conceptual system. But only with respect to an absolute essentialist conceptual system should the logic of metaphysical modality be S5 unrestrictedly. That is, premise (2) of Forbes' argument can apply to *a posteriori* metaphysical necessity only if the relevant conceptual system is an absolute essentialist one. In arguing that S5 is the correct logic for metaphysical modality as a whole, Forbes actually implicitly presupposes an absolute essentialist conceptual system in addition to his premises (1), (2) and (3) of his argument that we summarized in Section 3.1.

We showed that it is not true that if metaphysical necessities are fundamentally conceptual, only an S5-style system is appropriate for representing the logic of metaphysical necessity, but it is true that if metaphysical necessities are fundamentally conceptual and the conceptual system in question is an absolute essentialist one, only an S5-style system is appropriate for representing the logic of metaphysical necessity. Thus the proponents of S5 must argue for their position to favor an absolute essentialist conceptual system over a relative one. On the other hand, a similar reason can apply to Salmon's Intransitive Accessibility Solution. The above analysis reveals that the question about which logical system is correct for metaphysical modality can only be answered from the nature of the conceptual system of the interpreted language that we use to describe worlds. From this point of view, I would say that the nature of the Intransitive Accessibility Solution is such that it solves the Chisholm's Paradoxes and the Four Worlds Paradox by rejecting the absolute essentialist conceptual system and adopting a relative essentialist conceptual system for the language that we use to describe the worlds in question, and therefore taking modal logical system T to be the logic of metaphysical modality. Or, we can simply say that the Intransitive Accessibility Solution is a solution to the two Paradoxes from a relative essentialist point of view. Thus, the plausibility of the solution lies on the plausibility of the relative essentialist conceptual system. We shall analyze, in Section 3.3 and Section 3.4, how successful Salmon is in defending the relative essentialist conceptual system.

3.3 The Argument of Intransitive Accessibility Solution Again

We have the impression that in his Intransitive Accessibility Solution as well as in his paper of 1989, "The logic of what might have been," Salmon intends to have us believe that a relative essentialist conceptual system is not presupposed in his account, instead, the relative essentialist conceptual system is a consequence of his theory of possible world (the one that we presented in section 1.4) and the truths of the three principles (**N**), (**C**) and (**ST**). Recall that the theory of possible worlds that Salmon embraces holds the view that which world is metaphysically possible relative to world w is determined by the metaphysical necessities and possibilities contained in w—the view is clearly expressed in the two equivalent definitions of relative possibility relation: (i) a world w' is metaphysically possible relative to a world w if and only if every fact of w' is a possibility in w, and (ii) w' is metaphysically possible relative to w if and only if every necessary fact of w obtains in w'. More precisely, Salmon's view is this: The three necessary principles (**N**), (**C**) and (**ST**) are true and, because they are *a priori* necessities, they are contained in each of the worlds involved in the modal inferences of Chisholm's Paradox and of the Four Worlds Paradox. Given the theory about how the possible worlds of a world are determined, the modal inferences from the three principles in each of those worlds determine that the accessibility relation is intransitive. Therefore we should adopt a relative essentialist conceptual system and let modal logical system T be the logic of metaphysical modality. This view is explicitly stated in Salmon's argument of the Intransitive Accessibility Solution summarized at the end of Chapter 1.

We explained in the previous section that intransitive relative possibility relation between worlds is a consequence of taking relative essentialist conceptual truths to be metaphysical necessities of the worlds. In our view the intransitive relative possibility relation is explained in terms of adopting a relative essentialist conceptual system. But in Salmon's view the relation is reversed: adopting a relative essentialist conceptual system is explained in terms of the intransitive relative possibility relation. In other words, we hold that the relative possibility relation between the worlds is intransitive because S4 axiom is predetermined as invalid for the *a posteriori* necessities in question; but Salmon holds that S4 axiom should be invalidated for the *a posteriori* necessities in question because the relative possibility relation between the worlds is intransitive. We believe that there are flaws in Salmon's argument. In the following we shall examine the argument of the Intransitive Accessibility Solution summarized at the end of Chapter 1.

The argument is repeated here:

(1) The metaphysical necessities and possibilities contained in a given world determine which world is metaphysically possible relative to the given world.

(2) The principles **(N)**, **(C)** and **(ST)** and their multiple necessitations are true in the real world. The stipulation about the possibility of β-world1 is also a correct one.

(3) The necessities and possibilities inferred from Principles **(N)**, **(C)** and **(ST)** in those involved worlds determine that the metaphysical accessibility relation between the worlds is intransitive.

(4) The correct modal logic employed in the modal inferences involving the three principles must be decided consistently with the intransitive nature of the accessibility relation between the worlds determined by the three principles.

(5) The axiom of S4 modal logic, $\Box P \supset \Box\Box P$, characterizes a transitive accessibility relation between possible worlds.

(6) Therefore, we should reject S4 and S5 modal logic and accept system T as the correct logic for metaphysical modal reasoning.

To consider whether a relative essentialist conceptual system could be a consequence of Salmon's theory of possible world and the truths of the three principles **(N)**, **(C)** and **(ST)**, we let the conclusion (7) follow (1)-(5):

(7) Therefore, we should reject the absolute essentialist conceptual system and adopt a relative essentialist conceptual system.

As one can see, premise (5) expresses a truth independent of other premises. Premise (4) says that the logic of metaphysical modality must be decided consistently with the accessibility relation determined by the three principles. The question is whether the inference from the **(N)**, **(C)** and **(ST)** can determine the accessibility relation without presupposing the logic which is supposedly to be decided after looking at the accessibility relation. In fact, (4) makes sense if the first three premises are all true. Thus whether this argument is a sound one depends on whether the first three premises can be true all together.

The two paradoxes show that the absolute essentialist version of **(N)** and **(C)** cannot be true together with **(ST)**. Suppose that when asserting the truths of all three principles, Salmon actually asserts that the three principles are all true in their relative essentialist version. Given this understanding, premise (2) should be restated that principle **(ST)** and the relative essentialist version of principles **(N)** and **(C)** and their multiple necessitations are true in the real world. Given (2) as such, (1), (2) and (3) can be true together, but Premise (4) is redundant and the conclusion is trivial, as they just reiterate something already given in premise (2).

We notice that, however, in (2) Salmon only asserts that the three necessary principles and their multiple necessitations are true. According to Salmon, the "true multiple necessitations" can definitely apply to *a priori* necessities. But he did not explicitly indicate in premise (2) the mode of the inner *a posteriori* necessities in **(N)** and **(C)**. There are four possibilities to explain premise (2): (i) premise (2) means that the three principles and their multiple necessitations are true and the mode of the inner *a posteriori* necessities is not determined; (ii) premise (2) means that the three principles and their multiple necessitations are true and the mode of the inner necessities may be determined in either way, a mode of S4 or S5, or a mode of T; (iii) premise (2) means that the three principles and their multiple necessitations are true and the mode of the inner necessities is determined in a mode of S4 or S5; (iv) premise (2) means that the three principles and their multiple necessitations are true and the mode of the inner necessities is determined in a mode of T.

We have already dealt with the fourth possibility in previous paragraph. Under the fourth explanation of (2), the argument is sound but trivial. In my opinion, one cannot consistently assert the truths of all three principles in standard semantics without presupposing relative essentialism, therefore, the argument under any of the other three explanations of (2) will not be a sound one. But we want our conclusion that "the argument of the Intransitive Accessibility Solution either presupposes a relative conceptual system or is unsound" to follow from an argument, but not just a simple assertion. We find that, without commenting on the truth of (2) directly, we can still argue that the argument consisting of (1), (2) and (3) is unsound under any of the first three explanations of (2). By doing so we may have an opportunity to reveal some confusions hidden in the argument.

Under the first explanation of premise (2), the argument is either self-contradictory or containing a false premise depending on how premise (1) is understood. Premise (1) expresses the view that possible worlds of world ***w*** are determined by possibilities contained in ***w***. There are two aspects concerning such a determination: in one aspect, it needs to consider what are the metaphysical necessities and possibilities in ***w***, and in the other aspect, it needs to consider what is the mode of the metaphysical necessities and possibilities. Suppose that we have the maximal set of propositions describing a given world and we are to determine which world is possible relative to the given world using the method of the Γ-construction. We determine a minimum set of propositions such that any consistent world that contains this set and a fact which is a possibility of the given world is a possible world relative to the given world. From the rule of Γ-expansion, we know that this minimum set is composed of every γ such that $\Box\gamma$ belongs to the given world. But does this minimum set contain those necessities, $\Box\gamma$'s, themselves? Whichever way one answers this question, he must have a prior understanding about the mode in which the word "necessary" is being used. Hence, the mode of metaphysical necessities must be decided in the given world before the possible worlds of the given world can be determined. Without a definite mode, there is no minimum set that can be definitely given and no possible world of the given world that can be expanded. In other words, philosopher's necessity and possibility are formally defined by a set of axioms of modal logic, and without specifying the set of axioms we don't know exactly what the words "necessary" and "possible" mean, and hence we are not in the position to talk about determining possible worlds by necessities and possibilities. Besides, the maximality of possible worlds requires answers to the questions like whether the necessary proposition, $\Box P$, is itself a necessity of the world. In short, if the viewpoint "possible worlds are determined by necessities and possibilities" makes sense at all, the necessities and possibilities must be fully defined, as explained by the rules of the Γ-expansion: the subordinates are expanded according to the necessities and possibilities of the given world with respect to a set of axioms of a system of modal logic which characterizes the kind of modality.

Suppose that premise (1) means the same as we just explained, and that premise (2) is given the first explanation that the three necessary principles and their multiple necessitations are true but the mode of the inner *a posteriori* necessities is undetermined. Premise (3) says that "the necessities and possibilities inferred from principles (**N**), (**C**) and (**ST**) in those involved worlds determine that the metaphysical accessibility relation between the worlds is intransitive." In this case, (2) and (3) together say the opposite of (1). Premises (2) and (3) are suggesting that possible worlds of a given world can be determined even though the mode of some necessity or possibility is not determined in the given world. If in addition one thinks that a relative essentialist conceptual system can follow from the intransitive accessibility relation, one actually means that sometimes the order of determination is reversed: it is not from the necessities and possibilities to the possible worlds, but is from the possible worlds to the necessities and possibilities.

Let us review Salmon's solution to Chisholm's Paradox. Salmon assumes that hunk $\boldsymbol{h}_m$ is the last one in the sequence regarded as sufficiently similar to $\boldsymbol{h}_0$. Any hunk of matter in the sequence after $\boldsymbol{h}_m$ is an impossible matter for α from the standpoint of $\boldsymbol{w}_@$, thus any world in which table α is originated from $\boldsymbol{h}_{i(i>m)}$ is an impossible world relative to $\boldsymbol{w}_@$, because $\boldsymbol{w}_@$ contains the necessity that table α cannot made from any hunk $\boldsymbol{h}_{i(i>m)}$. World $\boldsymbol{w}_{m+1}$, which contains table α made from $\boldsymbol{h}_{m+1}$, is the first impossible world in the sequence.

Since principle **(ST)** is an *a priori* necessary principle, according to Salmon's view, **(ST)** holds in every world possible relative to the real world and hence in world $\boldsymbol{w}_{\mathrm{m}}$. By the possibility derivable from **(ST)** in $\boldsymbol{w}_{\mathrm{m}}$, world $\boldsymbol{w}_{\mathrm{m+1}}$ is a possible world relative to world $\boldsymbol{w}_{\mathrm{m}}$ which is possible relative to $\boldsymbol{w}_{@}$. So $\boldsymbol{w}_{\mathrm{m+1}}$ is possibly possible relative to $\boldsymbol{w}_{@}$. But by the *a posteriori* necessity derivable from principle **(N)** in $\boldsymbol{w}_{@}$, $\boldsymbol{w}_{\mathrm{m+1}}$ is not a genuinely possible world. So $\boldsymbol{w}_{\mathrm{m+1}}$ is a possibly possible but genuinely impossible world. Since $\boldsymbol{w}_{\mathrm{m}}$ is possible relative to $\boldsymbol{w}_{@}$ and $\boldsymbol{w}_{\mathrm{m+1}}$ is possible relative to $\boldsymbol{w}_{\mathrm{m}}$ but $\boldsymbol{w}_{\mathrm{m+1}}$ is not possible relative to $\boldsymbol{w}_{@}$, the relative possibility relation determined by the *a posteriori* necessities and possibilities asserted in **(N)** and **(ST)** in the worlds is intransitive.

In Salmon's solution it is not specified whether the *a posteriori* necessity derivable from **(N)** in $\boldsymbol{w}_{@}$ also holds in $\boldsymbol{w}_{\mathrm{m}}$, namely, whether this *a posteriori* necessity is itself a necessity in $\boldsymbol{w}_{@}$—this is consistent with our supposition of (2) that the mode of inner *a posteriori* necessity of **(N)** is undetermined. But, by saying that $\boldsymbol{w}_{\mathrm{m+1}}$ is possible relative to $\boldsymbol{w}_{\mathrm{m}}$ but not genuinely possible relative to $\boldsymbol{w}_{@}$, Salmon actually attributes to the real world some modal properties, namely, the properties that the *a posteriori* necessary proposition, $\Box\neg\boldsymbol{M}(\alpha,\boldsymbol{h}_{\mathrm{m+1}})$, which is derivable from the relevant instance of **(N)** in $\boldsymbol{w}_{@}$, is not itself a necessity of $\boldsymbol{w}_{@}$. Given the supposition about premise (2), this practice is in fact to determine some necessity of the real world by the possible worlds of the real world. Therefore, given the assumed meanings of (1) and (2), premises (1), (2) and (3) in Salmon's solution are self-contradictory.

Now let us assume that in Salmon's solution premise (1) means that the possible worlds of a given world can be determined by the necessities and possibilities of the given world without fixing a mode for every (or every kind of) necessity or possibility. In this case, the argument is not self-contradictory but unsound. We have argued that this cannot be done. Premise (1) is false under this interpretation. Thus, under the first explanation of (2), Salmon's argument is either self-contradictory or unsound.

We consider the second possibility: premise (2) means that the three principles and their multiple necessitations are all true, and the mode of the inner necessities may be determined in either way, a mode of S4 or S5, or a mode of T. In this case, premises (1), (2) and (3) together imply that no matter how the mode of the *a posteriori* necessities in **(N)** and **(C)** is determined, the accessibility relation between the worlds is intransitive. This amounts to saying, for example, either that **(N1)**-**(ST)** and **(N2)**-**(ST)** will determine the same set of possible worlds and the accessibility relation between the worlds is intransitive, or that **(N1)**-**(ST)** and **(N2)**-**(ST)** will determine different sets of possible worlds but the accessibility relation in either case is intransitive. By knowing how the content of an *a priori* essentialist principle will affect the accessibility relation between possible worlds (which is discussed in the previous section), we conclude that the two alternatives must be both false. Let us see this with an example.

According to the first alternative, **(N1)**-**(ST)** will determine the same set of possible worlds as the set of possible worlds determined by **(N2)**-**(ST)** But this cannot be the case. Suppose that the determination of possible world in $\boldsymbol{w}_i$ involves **(N1)**-**(ST)**. Any possible world thus determined will contain the *a posteriori* necessity, $\Box\neg\boldsymbol{M}(\alpha,\delta)$. On the other hand, if the determination of possible world in $\boldsymbol{w}_i$ involves **(N2)**-**(ST)** instead of **(N1)**-**(ST)**, there will be some possible worlds of $\boldsymbol{w}_i$ which do not contain this necessity. Therefore, the set of possible worlds determined according to **(N1)**-**(ST)** is not the same set as the one determined according to **(N2)**-**(ST)**.

According to the second alternative, **(N1)**-**(ST)** and **(N2)**-**(ST)** will determine different sets of possible worlds, but the accessibility relations in both cases are intransitive. But that cannot be the case either. A world containing **(N1)**-**(ST)** is inconsistent, and if we use the method of Γ-expansion, the subordinate worlds of the given inconsistent world are also inconsistent.[79] For example, in the case described in Chisholm's Paradox, if **(N1)** is involved, the *a posteriori* necessity $\Box\neg\boldsymbol{M}(\alpha,\boldsymbol{h}_{m+1})$ established in $\boldsymbol{w}_@$ holds in a subordinate world $\boldsymbol{w}_1$ by S4 axiom. On the other hand, $\Diamond\boldsymbol{M}(\alpha,\boldsymbol{h}_{m+1})$ can be inferred from **(ST)** and the fact $\boldsymbol{M}(\mathbf{a},\boldsymbol{h}_1)$ in $\boldsymbol{w}_1$. Therefore $\boldsymbol{w}_1$ is an inconsistent world. If one agrees that the relative possibility relation is defined only on consistent worlds, it is then illegitimate to talk about the relative possibility relation between inconsistent worlds. If one thinks that it makes sense to talk about the relative possibility

79 We discussed in sections 1.4 and 2.4 about the relative possibility relation between a consistent world and an inconsistent world. We reach the conclusion that the two definitions of relative possibility relation will give two incoherent answers to the question of whether a consistent world is a possible world of an inconsistent world. So, we are not going to consider situations like that.

relation between inconsistent worlds (by the same definitions of relative possibility relation), then we cannot see any reason to suppose that some necessity of an inconsistent world w may fail in any subordinate worlds of w—these worlds are also inconsistent and an inconsistent world contains every proposition.

Thus, under the second possible explanation of premise (2), premises (2) and (3) cannot be true together: if (3) is true (2) must be false, and if (2) is true (3) must be false. That is, if the relative possibility relation is intransitive, then it cannot be the case that the mode of the *a posteriori* necessity in (**N**) may be determined either in the mode of S4 or S5, or in the mode of T. On the other hand, if the mode of the inner *a posteriori* necessity in (**N**) may be determined either in the mode of S4 or S5, or in the mode of T, then there is no guarantee that the relative possibility relation is intransitive. Since at least one premise is false, the argument is unsound under the second explanation of (2).

Our discussion has already taken care of the next possibility. In the third explanation, premise (2) says that the three principles and their multiple necessitations are true in their absolute essentialist version. Thus, if (2) is true, (3) must be false, and vise versa.

In summary, under the first three explanations of (2), in no case the three premises, (1), (2) and (3), can be all true—in each case at least one of them is false. So the premises failed to support the conclusions (6) and (7). Under the last explanation of (2), (6) and (7) trivially follow from the premises. But, the relative essentialist conceptual system is already given in the premises of the argument rather than being shown by the argument. Salmon's attempt of deriving relative essentialism in his Intransitive Accessibility Solution has failed. No justification for the relative essentialist version of principles (**N**) and (**C**) can be found in the argument of the Intransitive Accessibility Solution, and hence no non-trivial proof for rejecting S4 and S5 modal logic as the correct logic for metaphysical modal reasoning is exhibited in the Intransitive Accessibility Solution.

In the above, we have taken a detour to show that the Intransitive Accessibility Solution cannot work in whatever possible interpretations of its key premises. Now we point out in a direct way the reason for why this argument cannot work. There is a fundamental confusion in the attempt of deriving relative essentialism from the view that the possible worlds of a given world is determined by the possibilities contained in the given world.

The concept of possible worlds is introduced into philosophy as a semantic tool for representing the meaning of modal expressions. It is unquestionably true that metaphysically possible worlds are determined by metaphysical necessities and possibilities. As we showed earlier, whenever the metaphysically possible worlds of a given world can be determined, it must be the case that the metaphysical necessities and possibilities of the given world and the mode of metaphysical modality are already pre-determined in the given world. In practice, to speak about which world is a metaphysically possible world of the given world, we are always assuming that the metaphysical necessities and possibilities of the given world and mode of the metaphysical modality are already correctly determined. The determination of the possible worlds of the real world $w_@$ according to principles (**N**), (**C**), and (**ST**) and the *a posteriori* necessities and possibilities derivable from the three principles can be carried out only if the mode of these necessities and possibilities is already determined. The Intransitive Accessibility Solution attempts to show the mode of the involved *a posteriori* necessities and possibilities by the outcome of such a determination drawn from these necessities and possibilities. Thus, what is showed by the Intransitive Accessibility Solution cannot but be trivial—this is argued in the discussion about the last explanation of premise (2). Our discussion about other three explanations of premise (2) shows that the proof of intransitive mode for metaphysical modality cannot be otherwise done by the intransitive accessibility account. Such an account cannot by any means explain why the mode of metaphysical modality should be intransitive, or why a relative essentialist conceptual system should be adopted.

3.4 The Supervenience Principle

In the second version of the Four World Paradox, Salmon showed that the conclusion of the modal inference drawn from (**ST**) contradicts the Supervenience Principle: a material object is supervenient on its matter and form. We said earlier in Section 1.3 that the Supervenience Principle is a more general principle than principles (**N**) and (**C**). Salmon holds an affirmative view towards the Supervenience Principle as stated in the following passage.

[Passage (15)]

> ... physical objects are "nothing over and above" their matter and structure, in the sense that a complete accounting of what matter there is in a genuinely possible world, with its causal interconnections and exact configuration through time, atom for atom, quark for quark, must completely and uniquely determine whatever physical facts there are about each of the physical objects such as tables and ships present in the world, ... This principle would require that any two genuinely possible worlds exactly alike at the level of matter and structure must also be exactly alike at least in all their physical-object facts.[80]

William Carter has expressed his puzzlement on Salmon's acceptance of the Supervenience Principle. He argues that the Supervenience Principle and the Strong Tolerance Principle **(ST)**, are not reconcilable. If **(ST)** is reckoned to be true in Salmon's accessibility account, how can the Supervenience Principle be true in the same account. To him, it is incoherent to hold the view that "the essence ... is contingent" while accepting the Supervenience Principle.[81]

Given that metaphysical necessities are conceptual truths, as we said in section 3.2, philosophers may hold opposite views about whether the conceptual system should be an absolute essentialist one or a relative essentialist one. The Strong Tolerance Principle **(ST)** is surely not reconcilable with the absolute essentialist version of the Supervenience Principle, but **(ST)** is reconcilable with the relative essentialist version of the Supervenience Principle. The phrase, "genuinely possible," used in passage (15) suggests that Salmon's

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[80] Salmon [1981].
[81] Carter [1983], 227.

affirmation of the Supervenience Principle is made for the relative essentialist version.

Graeme Forbes comes up a similar objection: "[The Intransitive Accessibility Solution] is internally incoherent, in that it implies the rejection of a principle which is needed to motivate the search for any solution to the Paradox. ... The problem is that it is hard to see why someone comfortable with the distinction between ***u*** and ***v*** should regard the conclusion of Chisholm's Paradox as false in the first place, since the argument for the falsity of the conclusion relies on a certain principle about the concept of identity which the distinction between ***u*** and ***v*** would appear to flout."[82] Here ***u*** corresponds to our α-world2 and ***v*** to our β-world2, which are materially indistinguishable from one another. The principle that Forbes mentioned in this passage is the principle that "numerical distinctions between entities must be grounded in differences between them in intrinsic respects."[83] This is a reductionist principle more general than the Supervenience Principle in our discussion.

Considering the distinction between the absolute essentialist version and the relative essentialist version of the Supervenience Principle, we see that it is in fact coherent for Salmon to be comfortable with the numerical distinction between α-world2 and β-world2, and to accept the relative version of the reductionist principle, and to view the conclusion of the Four Worlds Paradox as false (that is, the accessibility account holds that in no genuinely

[82] Forbes [1983], 182-183.
[83] Forbes [1983], 183.

possible world of the real world a table made from a possible matter of table α according to the same plan is some table other than α).

In my opinion, the question is not whether one can coherently accept the relative essentialist version of the Supervenience Principle (at least with respect to the discussion of the two modal paradoxes). Rather, the question is the plausibility of relative essentialism. According to relative essentialists, the Supervenience Principle can only be asserted relative to a certain modal situation—in different modal situations the specific content of the Supervenience Principle will be different. Therefore, relative essentialists must give up the Supervenience Principle *simpliciter*. To demonstrate the plausibility of the relative essentialist points of view, one must show that the Supervenience Principle *simpliciter* should be repudiated. In the following, we shall examine first how the Supervenience Principle *simpliciter* is repudiated in Salmon's account, and then Salmon's defense for his repudiation of the Supervenience Principle *simpliciter*.

W. Carter challenges the distinction between α-world2 and β-world2. He says, "Since the material configuration and material make-up of world [β-world2] can be stipulated to be precisely the same as that of [α-world2]—since [β-world2] and [α-world2] are indiscernible atom for atom, quark for quark—there is a reason to believe that [β-world2] is [α-world2]. ... given the immensely plausible thesis that 'worlds' that are physically indiscernible are identical."[84] Forbes has expressed a similar view: "... it is the existence of such a ***u***

[84] Carter [1983], 229. We replaced Carter's "Wd" by "β-world2", and "Wc" by "α-world2".

which is in question,"[85] where $\boldsymbol{u}$ is a world in which table α is made of hunk $\boldsymbol{h}_n$ whereas $\boldsymbol{v}$ is a world in which some other table $\boldsymbol{t}$ is made of hunk $\boldsymbol{h}_n$ according to the same plan.

Salmon gives the following reply to these objections.[86] He says that the temptation to identify the worlds like α-world2 and β-world2 by material indiscernibility stems from misconceiving possible worlds as material objects. According to Salmon's definition, worlds are maximal abstract entities, and hence the idea of identifying worlds by material indiscernibility must be rejected. In Salmon's view, the two worlds α-world2 and β-world2 are in fact discernible: though the two worlds are materially and purely qualitatively indistinguishable, they differ in their accessibility relations as well as in which identity facts obtain in them.[87] World α-world2 includes the fact that α is the table formed from hunk $\boldsymbol{h}''$ whereas β-world2 excludes this fact. In β-world2, some table distinct from α is formed from hunk $\boldsymbol{h}''$ according to the same plan. It follows by the principle of Indiscernibility of Identicals that α-world2 and β-world2 are distinct.

The above reply is in fact an escape from the real question. One may still challenge Salmon with the view that the principle of material indiscernibility of identicals should be applicable to the two tables, table α in α-world2 and table β in β-world2, which are material objects qualitatively and materially (in terms of the matter and the distribution of matter) indiscernible. Salmon cannot agree to

[85] Forbes [1985], 183.
[86] Salmon [1986], 107. In footnote 25, Salmon indicates that his reply is made to the objection raised by Carter and Forbes.
[87] Salmon [1986], 107.

the identity of the two tables, since if the two tables are numerically identical, α-world2 and β-world2 will contain exactly the same facts (the same set of propositions). He must say that the two tables are distinct, because they differ in how they relate to the real world—table α in α-world2 is a possible table relative to the real world and table β in β-world2 is a possibly possible but not a genuinely possible table relative to the real world. Thus, he gives up the Supervenience Principle *simpliciter*—a material object is supervenient on its matter and form.

An analogous argument can be made with respect to worlds. Corresponding to the maximal description of the real world there is a physical real world. Similarly, corresponding to each maximal consistent set of propositions that are either possible, or possibly ... possible to the real world, there is a physical entity which will exist if the conjunction of the propositions in the maximal consistent set turns out to be true. By "exist" we mean realization. Salmon uses "worlds" to refer to maximal sets of propositions. We let "*P*-worlds" refer to the physical entities corresponding to Salmon's consistent worlds. Only one *P*-world, namely, the physical real world, actually exists; the other *P*-worlds have potentials to exist. By hypothesis, *P*-α-world2 and *P*-β-world2 are materially and qualitatively exactly the same throughout their entire history. Carter's challenge may be rephrased as: since the material configuration and material make-up of *P*-β-world2 is the same as that of *P*-α-world2—since *P*-β-world2 and *P*-α-world2 are indiscernible atom for atom, quark for quark—there is a reason to believe that *P*-β-world2 is *P*-α-world2, given the immensely plausible thesis that *P*-worlds that are physically indiscernible are identical. Salmon must deny that the two *P*-worlds are numerically identical, for if he does not, the distinction between the two maximal consistent sets of propositions, α-world2 and β-world2, is purely nominal—they are one and the same set of propositions. But if he denies the numerical identity of the two *P*-worlds, he must give up the Supervenience Principle *simpliciter* and claims that the two *P*-worlds are distinct because they differ in how they relate to the real world.

Salmon defends his repudiation of the Supervenience Principle *simpliciter* by demonstrating the following thesis which he numbered (T7):

> (T7) For every ***x*** and every ***y***, if ***x***=***y***, then the fact that ***x***=***y*** is not grounded in or reducible to, qualitative nonidentity facts about ***x*** and ***y*** other than ***x***'s existence, such as facts concerning material origins, bodily continuity, or memory.

By arguing for (T7) Salmon aims to disprove Forbes' reductionist principle and the Supervenience Principle *simpliciter*, which are incompatible with the distinction between α-world2 and β-world2 in his account.[88] Salmon states Forbes' reductionist principle in his own words as follows:

> [Passage (16)]
>
> All facts about the numerical identity or distinctness of a pair of objects, ***x*** and ***y***–including facts of cross-time and cross-world identity and distinctness–are metaphysically

[88] Salmon [1986], 118, footnote 25.

> "grounded in," and "consist in," nonidentity facts about x and y, so that such identity facts do not obtain independently and solely by their own hook but only in virtue of nonidentity facts.[89]

There are two points that we shall make clear before a discussion of Salmon's proof of thesis (T7). First, we must be clear about what is meant by the phrase "grounded in" in Salmon's (T7) and in Passage (16) above. The phrase is Forbes' term. According to Forbes, saying that the identity of a certain object is grounded in such-and-such means that the identity of the object supervenes on such-and-such properties of the object. (Forbes emphasizes that the identity of an object is intrinsically grounded. This view may be restated that the identity of an object is grounded in or reducible to the essential properties possessed by the object.) Since Salmon's purpose of arguing for (T7) is to disprove Forbes' principle, we have reason to believe that he uses the phrase "grounded in" in the same meaning.

Secondly, we need to clarify the meaning of the phrase "qualitative nonidentity facts" in (T7). As commonly understood, qualitative nonidentity facts of a certain individual object are facts about the color and the shape of the object, or about from what kind of matter the object is formed, or about which specie or kind the object belongs to, etc.. Some nonidentity facts about the individual object are viewed as non-qualitative facts such as the fact of the object's existence and the fact about from which particular piece of matter the object is originated. It is not controversial at all that in

89 Salmon [1986], 118, footnote 25.

this sense of "qualitative," two numerically distinct objects may share the same set of qualities, for example, two distinct pencils can be qualitatively indistinguishable. However, I do not think that the phrase "qualitative nonidentity facts" in (T7) is used according to our common understanding of the phrase. If it is, arguing for (T7) will not fit the purpose of disproving Forbes' reductionist principle. Forbes holds that the identity of an object is (intrinsically) grounded in nonidentity facts about the object. These nonidentity facts need not be qualitative when "qualitative" is used according to our common understanding of the word. The phrase "qualitative nonidentity facts" may be given a broader understanding as referring to all facts about the object except the identity of the object. The phrase "qualitative facts about x" in this broad sense may be understood as facts that can be expressed as a property or relation possessed by the individual x. Since every fact about x can be thus expressed, every fact about x is a qualitative fact about x. The fact of x's identity is a qualitative identity fact about x and other facts about x are qualitative nonidentity facts. In this broad sense, the fact of the existence and the fact about the original matter of x are qualitative nonidentity facts about x. The word "qualitative" is thus not important any more. It seems to me that from (T7), Passage (16) and Passage (17) quoted below, it is more appropriate to interpret the word "qualitative" in (T7) as being used in this broad sense. An obvious evidence is that in (T7) Salmon views "existence" as a qualitative non-identity fact. In fact, only if "qualitative" is used in the broad sense can (T7) be counted as a thesis against the

Supervenience Principle *simpliciter*. We shall see that Salmon's argument for (T7) can apply to any nonidentity fact whether or not it is a qualitative fact according to the common understanding of "qualitative."

Salmon's argument for (T7) is succinct as we quote below.

[Passage (17)]

> Whatever x may be, the trivial fact that $x=x$ is not at all grounded in, or reducible to, any facts about x like those concerning x's material origins, x's bodily continuity through time, or x's memory of past experiences. If the fact that $x=x$ is grounded in any other fact about x, it is only grounded in the mere fact that x exists. Thus x has the complex property of being such that the fact that x is identical with it is not grounded in any qualitative nonidentity facts about x other than x's existence. Hence, by Leibniz's Law, for every y, if x and y are one and the very same, then y also has this complex property. Thus, if $x=y$, then the fact that $x=y$ is not grounded in any qualitative nonidentity facts about x (which are also facts about y) other than x's existence. Indeed, since x and y are one and the very same, the fact that $x=y$ is just the fact that $x=x$. Consequently, the fact that $x=y$ must have the property of the fact that $x=x$ that it is not grounded in any qualitative nonidentity facts about x (which are also facts about y) other than x's existence—QED.[90]

There are three premises in Salmon's argument. The first premise is Leibniz's Law: $\Box(x=y\supset(Px\supset Py))$. The second premise is that the fact $x=y$ is the same fact as the fact $x=x$. The third premise is that the trivial fact $x=x$ has the property of being not grounded in any facts about x other than x's existence. We may think of three possible understandings of the third premise. We shall see that under the first two understandings, the argument for (T7) fails to derive the desired conclusion for disproving Forbes' redutionist principle and the

90 Salmon [1986], 112-113.

Supervenience Principle, and that under the third understanding the argument is circular.

We begin by examining Salmon's argument under the first understanding. When regarding the fact $x=x$ as trivial, one is most likely to have in mind the idea that "$x=x$" is a statement of self-identification for object x with respect to any particular context that x is in. (A context consists of an index of time and an index of world.) It is uncontroversial that it is a trivial truth that any object is qualitatively indiscernible from, and numerically identical to, itself with respect to any context. By this understanding of the fact $x=x$, there is indeed a sense in which the truth of the fact $x=x$ is not "grounded in" any other properties of x. That is, the fact that x is qualitatively indiscernible from, and numerically identical to, itself at a particular moment in a particular world is true no matter what other properties x possesses at the moment in the world.

Given this understanding of the third premise in Salmon's argument, we may restate the premise as this: the trivial fact $x=x$ —in the sense that x is qualitatively and numerically identical to itself at any context (at any particular moment and in any particular world)—has the trivial property that the truth of this fact is not grounded in any other properties x possesses at any particular context. Since the fact $x=x$ is a trivial self-identity fact about x with respect to a particular context, if it is indeed true that the fact $x=y$ is the same as the fact $x=x$, then by Leibniz's Law, the fact $x=y$ is also a trivial self-identity fact about x/y with respect to a particular context. ("x/y" means "x" and "y" are interchangeable.) Thus, along the line of Salmon's argument, it follows that the fact ***x*=*y*** has the trivial property that the truth of this fact is not grounded in any other properties that ***x*/*y*** possesses at any particular context.

Given this understanding of the third premise, Salmon's argument for (T7) proves nothing against the Supervenience Principle and Forbes' reductionist principle, simply because the concept of "grounding in" employed in argument is not the same concept as the one used in Forbes' principle. Though the argument reaches the conclusion that the truth of the fact ***x*=*y*** does not depend on other properties ***x*/*y*** possesses, the tricky point is that the conclusion is true simply because an object cannot fail to share the same set of properties with itself in the same context. Thus the question of what properties an object shares with itself in the same context becomes redundant in determining self-identity with respect to the same context. If the phrase "is not grounded in" is explained in this way, the conclusion of Salmon's argument is perfectly compatible with the Supervenience Principle and Forbes' reductionist principle. In fact, the Supervenience Principle and Forbes' reductionist principle are trivially true in the cases of self-identity with respect to the same particular context.

We now consider the second understanding of the third premise of Salmon's argument. There is another way for viewing the fact ***x*=*x*** as trivial. In this case, "***x*=*x***" is understood as "'***x***' in context_1 and '***x***' in context_2 rigidly designate the same individual." Then the fact ***x*=*x*** is trivially true because it is given that the object referred by "***x***" on the left side of "=" is the same object referred by "*x*" on the right side of "=." As Salmon pointed out in his book, *Reference and Essences*, the self-identity of this kind is a trivial consequence of the theory of direct reference, in which proper names and variables are rigid designators that designate the same object in every world (or every context). Surely, given this meaning of $x=x$, there is an understanding in which the trivial fact $x=x$ is not grounded in any fact other than *x*'s existence, namely, the designative rigidity of "*x*" is not determined by any material or qualitative property of the individual designated by "*x*."

The third premise may be restated as this: the trivial fact $x=x$ æin the sense that the "*x*" on the left side of "=" and the "*x*" on the right side of "=" rigidly designate the same individual in every possible world throughout the time—has the property that the co-designation of the "*x*" on the left side of "=" and the "*x*" on the right side of "=" is not grounded in any properties of the individual designated by both "*x*"s. The second premise is that the fact $x=y$ is the same fact as $x=x$. Given the third premise as stated above, the fact $x=y$ should be consistently understood as the fact that "*x*" and "*y*" co-designate the same individual. However, since the fact $x=x$ is trivial, if the fact $x=y$ is indeed the same fact as the trivial fact $x=x$, then by Leibniz's Law, the fact $x=y$ is also trivial, that is, the co-designation of the two names, "*x*" and "*y*," must be trivially given. Thus, Salmon's argument for (T7) with this understanding of the third premise will conclude that the fact $x=y$ has the property that the co-designation of the "*x*" on the left side of "=" and the "*y*" on the right side of "=" is not grounded in any properties of the individual designated by both "x" and "y."

But again, the above conclusion is not desirable for disproving Forbes' reductionist principle and the Supervenience Principle, because the concept of "grounding in" employed in this argument is, as in the earlier case, not the same concept of "grounding in" as the one in Forbes' reductionist principle. The co-designation of the two names, "x" and "y," when trivially given, is independent of the issue whether the identity of an object is reducible to the properties of the object, and of the issue about the limit of changes that an object can endure cross-time or cross-world without becoming something else.

We come to the last understanding of the third premise of Salmon's argument. There is another sense of "grounded in," namely, the reductionist sense employed in Forbes' reductionist principle. In this case, the statement, that the fact $x=x$ has the property of being not grounded in any facts about x other than x's existence, should be understood as that the identity relation between x in $context_1$ and x in $context_2$ is not supervenient on, or not reducible to, any material or qualitative properties of x except x's existence in both contexts. This is to assert that there is no non-trivial essential property that x must possess in order to remain to be itself. However, with this interpretation of the fact $x=x$, the fact $x=x$ is no longer trivial.

We may restate the third premise as this: the fact $x=x$ has the property of being not grounded in any properties of x except x's existence in the sense that x in $context_1$ may not share any property with x in context_2 except x exists in both contexts. Given that the fact $x{=}x$ is the same fact as the fact $x{=}y$, by Leibniz's Law, the fact $x{=}y$ also has the property that it is not grounded in any properties of x/y except x/y's existence in the sense that x in context_1 may not share any property with y in context_2 except x/y's existence in their contexts respectively. This conclusion contradicts the Supervenience Principle and Forbes' reductionist principle. However, with this understanding of the third premise, Salmon's argument for (T7) is a circular argument—the truth of the conclusion is needed for the truth of the third premise.

It is certainly illegitimate to start the argument with the phrase "not grounded in" in either of the first two meanings and conclude the argument with the same phrase in its third meaning—a fallacy of changing concept in a deductive argument. One cannot prove from the redundancy of reducing an object's self-identity in a particular context to the properties the object possesses at the context to the irreducibility of identity in general. Similarly, one cannot prove from a semantical feature of proper names and variables, namely, their designative rigidity, the metaphysical point of view that the identity of object x is not reducible to the properties that x possesses. Especially, if Salmon is right in arguing that essentialism, or absolute essentialism, is not deducible from the theory of direct reference,[91] anti-essentialist or relative essentialist is not deducible from the theory of direct reference either—they are on a par.

[91] Salmon [1981], Part II.

We have reached the following conclusion: Salmon's Intransitive Accessibility Solution is a solution to the two Paradoxes given from a relative essentialist point of view. Our analysis of Salmon's defense on the plausibility of his relative essentialist view shows that none of his attempts has thus far succeeded. We have shown in Chapter 2 that the restrictive nature of metaphysical modality in restriction sense1, which is taken by Salmon as an independent reason from his Intransitive Accessibility Solution to support the intransitive accessibility relation between metaphysically possible worlds, has in fact nothing to do with the mode of metaphysical modality. We have shown in Section 3.3 and this section that no non-trivial proof of the relative essentialist point of view is given in Salmon's Intransitive Accessibility Solution or in his argument for thesis (T7). Contrary to what Salmon hoped for, the theory of possible worlds that Salmon embraces, the theory of direct reference, the trivial truth of self-identity, and Leibniz's Law, play no effective role in the attempt of disproving absolute essentialism or proving relative essentialism.

As I see it, Salmon's acceptance of relative essentialism relies solely on his own metaphysical intuition. In Salmon's account, no substantial defense for the plausibility of relative essentialism, or at least, for preferring a relative essentialist conceptual system to an absolute essentialist one, is successfully given. The question about which modal logical system is the correct logic for metaphysical modality remains open.

CHAPTER 4

RESOLUTION TO THE PARADOXES AND A DEFENSE OF S5 MODAL LOGIC

We showed in the previous chapter that Salmon's rejection of modal logic systems S4 and S5 and his acceptance of system T as the correct logic for metaphysical modality are argued by presupposing relative essentialism: the *a posteriori* essential properties of an artifact are essential to the artifact relative to a certain possible situation (a certain possible world)—no *a posteriori* essential property *simpliciter* can be asserted for an artifact. On the other hand, we also showed that a defense for S5 to be the logic of metaphysical modality ultimately consists in a defense for the absolute essentialist view that the *a posteriori* essential properties of an artifact are essential to the object *simpliciter*. For those who incline to view S5 as the logic of metaphysical modality, there are two tasks: (i) solving the paradoxes, and (ii) providing an account that elucidates the intuition that *a posteriori* essential properties of an artifact are essential to the artifact *simpliciter*. In this chapter, we shall analyze what is wrong in the paradoxical inferences, and try to reveal the intuition that supports the absolute essentialist view. We start our discussion by considering Forbes' solution to Chisholm's Paradox.

4.1 Forbes' Counterpart Solution to Chisholm's Paradox

Graeme Forbes developed a "counterpart solution" to Chisholm's Paradox in which S5 is saved. In his solution, Forbes holds that (i) the essentialist principles are true[92] and (ii) the Tolerance Principle is an intuitively correct principle. We have distinguished two versions of the Tolerance Principle, the weak version **(T)** and the strong version **(ST)**. We want to see which version is the Tolerance Principle in Forbes' mind. Forbes' statement of the Tolerance Principle is the following:

[Passage (18)]

> Necessarily, any artifact could have originated from a slightly different collection of parts from any one collection from which it could have originated.[93]

This statement of the Tolerance Principle may be interpreted as the weak Tolerance Principle **(T)** stated in Section 1.2. Namely, the phrase "could have originated from a slightly different collection of parts" means "could have originated from some slightly different collection of parts," but not "could have originated from any slightly different collection of parts." We have explained that the weak Tolerance Principle **(T)** does not assert a sufficient condition for inferring any possible matter of the table. Given that table α is made from β and β' is sufficiently overlapping β, it is still not sufficient to infer, from **(T)** and the given empirical information, that it is possible for table α to be made from β'. However, in presenting Chisholm's

[92] Here we mean Forbes' essentialist principles similar to principles **(N)** and **(C)**. Forbes discusses series of these kind of principles in Forbes [1985], Chapters 5-7. The essentialist principles concerning artifacts is discussed in Chapter 7.

[93] Forbes [1984], 161.

Paradox, Forbes infers from the Tolerance Principle a sequence of possibilities for table α's original construction in exactly the same way as the sequence of possibilities inferred from **(ST)** in Salmon's presentation of Chisholm's Paradox. The weak reading of the Tolerance Principle in Passage (18) is not sufficient to legitimize such an inference. Suppose that it is given that table α has only two possible original hunks of matter, β and β′, such that β′ is slightly different from β in their collections of components. The weak reading of Forbes' Tolerance Principle about table α, "table α could have originated from some slightly different collection of parts from any collection from which it could have originated," is satisfied by this assumed situation of table α. Thus the weak version of the Tolerance Principle does not guarantee that there is such an infinite sequence of possibilities. Therefore, Forbes must rely on a stronger Tolerance Principle when he infers an infinite sequence of possibilities. Thus we have reason to think that Forbes has in mind something like the strong version **(ST)**.

With regard to the nature of Chisholm's Paradox, Forbes holds that in S5 modal logic,

$$\boldsymbol{S}(\beta',\beta)\supset\Box(\boldsymbol{M}(\alpha,\beta)\supset\Diamond\boldsymbol{M}(\alpha,\beta'))$$

is equivalent to

$$\boldsymbol{S}(\beta',\beta)\supset(\Diamond\boldsymbol{M}(\alpha,\beta)\supset\Diamond\boldsymbol{M}(\alpha,\beta')),$$

and the intuition of tolerance is more naturally expressed by the conditional $\Diamond\boldsymbol{M}(\alpha,\beta)\supset\Diamond\boldsymbol{M}(\alpha,\beta')$ in the latter formulation, rather than the necessity $\Box(\boldsymbol{M}(\alpha,\beta)\supset\Diamond\boldsymbol{M}(\alpha,\beta'))$ in the former formulation. Forbes holds that the latter formulation assimilates Chisholm's Paradox to the standard form of classical Sorites Paradox in the following way:

(0) $\Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_1)$

(1) $\Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_1) \supset \Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_2)$

......

(n-1) $\Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_{n-1}) \supset \Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_n)$

(n) $\Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_n)$

With this presentation of Chisholm's Paradox, Forbes holds that the well-known treatment of ordinary Sorites Paradoxes can be applied. This treatment modifies the two-valued semantics by introducing a range of intermediate degrees of truths between the absolute truth and the absolute falsehood. Forbes argues that "Now it is much less clear that the problem arises because of some fallacious modal inference, since there is no modal logic in a standard sorites."[94] His point is this: if Chisholm's Paradox can be solved without involving modal logic, it is unlikely that the Intransitive Accessibility Solution, which criticizes S5 modal logic, provides the correct answer to the paradox. Forbes' solution makes use of the counterpart theory originally proposed by David Lewis.[95] Forbes explains the reason for his approach in the following passage:

[Passage (19)]

> The motivation for this approach comes from taking Chisholm's Paradox to be a modal paradox of vagueness. Tolerance arises because of vagueness or fuzziness in the limits of the range of sums of wood which possibly constitute α: there is no sharp distinction between those

94 Forbes [1984], 172-173.
95 Lewis [1968]. Forbes' own elaboration of counterpart theory is in Forbes [1985], 57-64.

> sums which could, and those which could not, constitute α. Given that there is no fuzziness in the boundaries of particular sums of wood or in the constitution relation, it seems that this vagueness must arise from an underlying vagueness in the concept of possibly being identical to α; however, in standard modal semantics, such vagueness could only be represented by vagueness in α's transworld identity conditions, and solution of the paradox in which we think of identity as vague would be rather unappealing. But it does make sense to think of similarity as being vague, ... Since the counterpart relation is fixed by similarity considerations ... and similarity admits of degrees, the degree-theoretic resolution of non-modal paradoxes may be transcribable, and in fact can be transcribed, into the modal logical context.[96]

One restriction of counterpart theory is that the domains of possible worlds are disjoint. With this restriction, an individual object can exist in only one possible world. In Forbes' Counterpart Solution to Chisholm's Paradox, the real table is denoted by α, and the table made from hunk $\boldsymbol{h}_i$ in world $\boldsymbol{w}_i$ is denoted by α_i, where $\boldsymbol{h}_i$ and $\boldsymbol{w}_i$ are as described in Chisholm's Paradox in Section 1.3. The real table α exists only in the real world and for each i, α_i exists only in $\boldsymbol{w}_i$. Forbes assigns each α_i a counterparthood of a certain degree to α. Table α itself has α-counterparthood of degree 1 (the absolute similarity) and α_n, which shares no components with α, has α-counterparthood of degree 0 (the absolute dissimilarity in term of their components). For each α_i, where $1<i<n$, α_i has a slightly higher degree of α-counterparthood than the α-counterparthood α_{i+1} has.[97]

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[96] Forbes [1984], 173-174.

[97] The statement here is an over-simplification. Forbes has pointed out that degrees of similarity relation between possible tables should be determined by various aspects of the tables. But in the present problem only the constitutional components are allowed to vary and this justified the simplification. See Forbes [1985], Chapter 7.

Hence each sentence "α_i is a counterpart of α" has a truth value of 1-i/n. The conditional

$$\Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_i) \supset \Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_{i+1})$$

is translated in counterpart theory as follows:

$$\exists \boldsymbol{u}(\mathbf{C}(\alpha_i, \alpha, \boldsymbol{u}) \& \boldsymbol{M}(\alpha_i, \boldsymbol{h}_i, \boldsymbol{u})) \supset \exists \boldsymbol{v}(\mathbf{C}(\alpha_{i+1}, \alpha, \boldsymbol{v}) \& \boldsymbol{M}(\alpha_{i+1}, \boldsymbol{h}_{i+1}, \boldsymbol{v})).$$

where $\mathbf{C}(\alpha_i, \alpha, \boldsymbol{u})$ means that α_i is a counterpart of α in world $\boldsymbol{u}$. This translation says that if there is a world $\boldsymbol{u}$ in which α_i is a counterpart of α and α_i is made from $\boldsymbol{h}_i$, then there is world $\boldsymbol{v}$ in which α_{i+1} is a counterpart of α and α_{i+1} is made from $\boldsymbol{h}_{i+1}$.

The truth value of the conjunctions and conditionals are determined by the following two evaluation rules:

(Conj): $\text{val}[A \ \& \ B] = \min \{ \text{val}[A], \text{val}[B] \}$

(Cond): $$\text{val}[A \supset B] = \begin{cases} 1 - (\text{val}[A] - \text{val}[B]), & \text{if val}[A] > \text{val}[B]; \\ 1, & \text{otherwise.} \end{cases}$$

According to these evaluation rules, for each premise of the form $\Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_i) \supset \Diamond \boldsymbol{M}(\alpha, \boldsymbol{h}_{i+1})$ in the Sorites argument, the consequent is slightly less true than the antecedent. Hence, none of the premises is wholly true. Since the consequent of the preceding premise is the antecedent of the following premise, the slight decreases in the degree of truth are preserved and accumulated so that, at the end of this inference, $\boldsymbol{M}(\alpha, \boldsymbol{h}_n)$ has truth value of degree 0. Forbes says that "each application of *modus ponens* in the standard paradox commits the 'fallacy of detachment'," and hence "Chisholm's Paradox is shown to turn on the fallacy of detachment, just as the paradox of smallness does."[98]

[98] Forbes [1984], 175.

Salmon has raised some objections to Forbes' counterpart approach. He criticizes the Counterpart Solution as "just a particularly inflexible brand of essentialism."[99] Salmon argues that, by denying the existence of a possible state of affairs in which the very table α is made from ever so slightly different matter, the counterpart theorist holds that it is absolutely impossible for table α to be made from matter with even one atom different.

As I see it, Forbes' intention of introducing counterpart relation to the problem of Chisholm's Paradox is to give the truth-values to the ordinary modal claims about table α that agree with his intuition. Forbes is certainly not arguing for inflexible essentialism. In the Counterpart Solution, the counterpart relation is a semantical apparatus for the treatment of transworld identity—table α_1 in world $\boldsymbol{w}_1$ is a representative of table α in $\boldsymbol{w}_@$. It is disputable whether such a semantic device is acceptable. But even if the semantical device of counterparthood is not acceptable, it does not follow that the Counterpart Solution is an inflexible essentialist solution—it only means that this particular counterpart device does not work in solving the Paradox. According to Passage (19) quoted above, Forbes' motivation of introducing counterpart relation is to treat the vagueness in table α's possible matter. It seems to me that whether the counterpart device is acceptable depends upon whether the vagueness in table α's possible matter is, as Forbes suggested, a vagueness of the kind that admits of degrees. That is, whether the vagueness in table α's possible matter is like the vagueness in the

99 Salmon [1981], 236. Also, Salmon [1986], part VI, 95-96.

concepts of "short" or "bald" which can be said as true in a certain degree.

Salmon also raises an objection to Forbes' account on where the vagueness is located. He disagrees with the view that the transworld identity relation is vague. He argues that, though Chisholm's Paradox and other modal paradoxes can be formulated in terms of cross-world identity, they can just as easily be formulated without identity. Salmon said that "Formally, the crucially vague expression involved in [Chisholm's Paradox], according to Forbes' formal treatment, is '$\Diamond$ ... α ...,' or 'it might have been that α ...;' the crucially vague concept is the one designated by '$\lambda F \Diamond F(\alpha)$.' ... Hence, if there is any vagueness relevantly involved in the modal paradoxes, it resides in the modal operators themselves, ..." Salmon argues that standard possible-world semantics can accommodate the vagueness in modal operators precisely in the way suggested by his Intransitive Accessibility Solution: "one should treat the accessibility relation between worlds as itself vague ... When fully worked out, this involves intransitivity in the accessibility relation via a region of indeterminacy, ..." The accessibility approach "affords a solution to the modal paradoxes that accommodates vagueness precisely where it must arise, ..."[100]

Salmon's point is that the vagueness in the expression "$\lambda F \Diamond F(\alpha)$" can be viewed as vagueness in the accessibility relation, and by doing so, the vagueness in trans-world identity relation become apparent and can be eliminated. For Salmon, this view

100 All quotations in this paragraph is in Salmon [1986], 93-95.

warrants his accessibility approach to the paradoxes. But to me, this is problematic. Given Salmon's view that possible worlds are determined by possibilities and his definition of accessibility relation, if it is vague whether a world is possible relative to world w, it must be case that there are some modal propositions such that whether they belong to world w is vague in the first place. Then, we ask what is vague in these propositions. Salmon says that the vagueness resides in the modal operators themselves. What could this mean? There should be no vagueness in the meaning of "possible." A proposition is possible if it is not bound to be false. Maybe Salmon means that the mode of "$\Diamond$" and "$\Box$" is vague, that is, it is vague which modal system characterizes "$\Diamond$" and "$\Box$." If this is the case, I cannot see how such a vagueness warrants Salmon's accessibility account. We showed in last chapter that Salmon's Intransitive Accessibility Solution presupposes relative essentialist conceptual system, or say in other words, the intransitive relative possibility relation between worlds. If the mode of "$\Diamond$" and "$\Box$" (the mode of metaphysical possibility) is vague, the Intransitive Accessibility Solution just begs the question to have such a presupposition. Or perhaps he means that the truth of the possible proposition $\Diamond F\alpha$ is vague, that is, whether the modal proposition $\Diamond F\alpha$ belongs to the real world is vague, which in turn gives rise to the vagueness of the accessibility relation. If this is the case, the same thing can be restated as: whether α has the modal property $\Diamond F$ is vague. Thus we are talking about α's transworld identity—whether α could have the property F without losing its identity. Then, contrary to Salmon's claim, that the accessibility relation is vague in the first place and the vagueness in trans-world identity relation is apparent and can be eliminated, we find that the vagueness in the accessibility relation is derivatively based upon the vagueness in α's transworld identity.

My criticism of Forbes' counterpart approach to Chisholm's Paradox is different from those of Salmon's. I do not think that the view that the transworld identity relation is vague is unacceptable, given that some transworld identity relations of artifacts may be unknowable to human mind. In other words, since we are not always in the position to successfully pick out references for an identity statement, there is nothing wrong to treat theoretically those identity relations as vague and avoid giving any definite answers about the truths of them.[101]

My main objection to Forbes' Counterpart Solution is that it is not a satisfactory solution to the paradoxes because it fails to conform our intuition that in some situation we have definite yes-no answers to the questions about whether a certain hunk of matter could have constituted the given artifact, and because it fails to explain the absolute essentialist intuition that *a posteriori* necessities and possibilities of an artifact belong to the artifact *simpliciter*.

Let us consider the following question: can we ever give a definite truth value to modal assertions such as "this possible artifact constructed in world w is numerically identical to the actual artifact x in the real world $w_@$" in non-extreme cases?

[101] Robert Stalnaker has argued for the indeterminacy of identity in terms of indeterminacy of reference. See Austin [1988], 349-360.

The extreme cases concerning artifact *x*'s transworld identity are: (i) the cases in which a possible artifact is made from exactly the same matter as the actual matter of *x* and is made according to the same plan in the same historical situation as the one by which *x* is constructed in the real world, and (ii) the cases in which a possible artifact is made from a completely different matter from the one from which *x* is made in the real world. We assume that it is not controversial that in these extreme cases, the modal assertions about *x*'s transworld identity are either definitely true or definitely false. By "definitely true" and "definitely false" we mean simply "true" and "false;" the word "definitely" is used in contrast with "true-in-a-certain-degree" and "false-in-a-certain-degree." The non-extreme cases about *x*'s transworld identity are the cases in which a possible artifact is made from a hunk of matter different but not completely different from *x*'s original matter in the real world.

Suppose that Forbes' answer to the question is "No, we cannot." According to this view, there can never be definite yes-no answers to the questions about *x*'s transworld identity with respect to *x*'s original matter except in the extreme cases. Thus, one cannot really talk about the transworld identity of *x* in an absolute sense except in the extreme cases. But, saying that a possible artifact is identical to *x* to a certain degree does not make good sense because identity relation does not admits of degrees. Hence one can only say in non-extreme cases that a particular possible artifact is similar to *x* in a certain degree. If Forbes' use of counterpart relation is thus motivated, the replacement of transworld identity relation by similarity relation between counterparts is natural for his view. However, our intuition is that though there are non-extreme cases in which no definite answers can be given to the questions about whether a certain hunk of matter could have constituted ***x***, there are some non-extreme cases for which definite answers can be given to such questions. It seems to me, it is counter-intuitive to deny the existence of those non-extreme cases for which the questions about table ***x***'s possible matter are definitely answerable.

Suppose, on the other hand, that Forbes agrees with our intuition and holds that though in some non-extreme cases definite answers cannot be given to questions concerning the possible matter of the given artifact, there are non-extreme cases in which definite answers can be given to such questions. But how can the Counterpart Solution treat these non-extreme cases? To treat these cases, the counterpart relation has to be taken to represent identity relation. One option is to divide degrees of similarity into two or three regions such that each region represents identity, distinctness and indeterminate respectively. But this approach, whether it is workable or not, is really not an option for Forbes, because he holds in Passage (19) that there is no sharp cutoff threshold between the collection of components which could constitute the table and the collection of components that could not. However, one needs to determine a sharp cutoff threshold, whether the threshold is a point or an interval, in order to represent definite identity and distinctness by the counterparthood. The other option is to view identity as having degrees. This is conceptually incorrect and, as we have seen in

Passage (19), unacceptable to Forbes himself. Identity relation may be vague in some other sense, for example, identity relation may be vague in certain cases from epistemic point of view. But it is fundamentally incorrect to view the vagueness in identity relation as the same as the vagueness in the concept of being "short," which admits of degrees and is such that there is no sharp cutoff point between being short and being not short. Thus, the Counterpart Solution simply cannot treat the non-extreme cases in which the questions about possible hunks of matter of a given artifact are definitely answerable.

In Forbes' Counterpart Solution, the question about transworld identity of artifacts is avoided rather than answered. Chisholm's Paradox is dismissed—there is no paradox about table α's transworld identity—because we do not talk about table α's transworld identity any more. No explanation for the absolute essentialist intuition, that *a posteriori* essential properties of an artifact are essential to the artifact *simpliciter*, can be extracted from Forbes' account. S5 modal logic is saved by talking about similarity instead of identity. But in our view, such a solution cannot be a real solution to the paradox because the problem of transworld identity of artifacts cannot be so dismissed.

4.2 A Rejection of the Strong Tolerance Principle (ST)

We analyzed in Section 1.3 two versions of the Tolerance Principle, the weak version **(T)** and the strong version **(ST)**. The weak version **(T)** asserts only the existence of the possibility that table ***x*** be made from some hunk of matter distinct from its actual original matter (namely, table ***x*** be made from a hunk of matter which has the same quantity and quality as table ***x***'s original matter but the collection of components is slightly different from ***x***'s actual collection of components). The property of "slightly different from ***x***'s original matter" in **(T)** is not a sufficient condition for a hunk of matter to be a possible matter of table ***x***. Thus, **(T)** is not a principle for inferring further possibilities for table ***x*** from a known possibility of ***x***. On the other hand, the property of "slightly different from ***x***'s original matter" in the strong version **(ST)** is a sufficient condition for a hunk of matter to be a possible matter of table ***x***. Given a known possible matter of table ***x***, one can infer in S4 or S5 modal logic a sequence of possibilities for ***x*** from **(ST)**. The Four Worlds Paradox and Chisholm's Paradox arise from the inferences drawn from the Strong Tolerance Principle **(ST)** and principles **(N)** and **(C)** in S4 or S5 modal logic.

Salmon claims that the three necessary principles, **(N)**, **(C)** and **(ST)** are all true principles. But we can see that there is no inherent connection between **(N)** and **(ST)**, or **(C)** and **(ST)**. In principle **(N)**, the property of not being made from a hunk of matter substantially different from its actual matter is a necessary condition for table ***x*** to be itself. Principle **(N)** says nothing about what is sufficient to infer further possible hunks of matter for table ***x*** from any known possible matter of ***x***. Principle **(C)** states that no other table could be made according to the same plan from a known possible hunk of matter of table ***x***. This is to say that "made according to the same plan" and "made from a possible matter of ***x***" form an individual essence of ***x***, that is, a sufficient condition for being ***x***. The content of principle **(C)** has nothing to do with how to infer a possible hunk of matter for table ***x***'s construction. Hence, the point of view stated in **(ST)** and the points of view stated in principles **(N)** and **(C)** are mutually independent. One who believes in principles **(N)** and **(C)** need not believe in **(ST)**. In the following, we shall argue that the Strong Tolerance Principle **(ST)** is not a correct principle. Principle **(ST)** is repeated below:

> **(ST)** If a wooden table ***x*** is originally made from a hunk of matter ***y*** according to a certain plan ***P***, then ***x*** might have been made according to the same plan ***P*** from any hunk of matter ***y'*** instead of from ***y***, where ***y'*** has the same quantity and quality as ***y*** but the collection of components of ***y'*** is slightly different from that of ***y***.

We first ask where this Strong Tolerance Principle comes from and what is the rationale for those philosophers who accept this principle as a correct principle.

The considerations about measuring possible variations in the original matter of an artifact by a certain quantity of change may be thought as initially from Kripke's remarks in his famous lectures:

> [Passage (20)]
>
> If a chip, or molecule, of a given table had been replaced by another one, we would be content to say that we have the same table. But if too many chips were different, we would seem to have a different one.[102]

[102] Kripke [1980], 51, footnote 18.

One might think that Kripke's remark in Passage (20) suggests a quantitative criterion for the transworld identity of artifacts. However, if we relate this passage to its original context, we shall see that there is no indication that Kripke has the intention to suggest a sufficient condition for inferring possibilities or impossibilities for the artifact. The following passage is stated before Passage (20) in the same context:

> [Passage (21)]
>
> ... in concrete cases we may be able to answer whether a certain bunch of molecules would still constitute *T* [a table], though in some cases the answer may be indeterminate.[103]

A passage closely following Passage (20) is the following one:

> [Passage (22)]
>
> ... it is not assumed that necessary and sufficient conditions for what kinds of collections of molecules make up this table are possible; this fact I just mentioned.[104]

The words, "this fact I just mentioned," refer back to passage (21). It seems to me that in the context (20), (21) and (22), it is more appropriate to understand the passage (20) as describing two situations in which we are likely to have definite answers to questions about whether the given table is possible to be constructed from a certain hunk of matter. Given passage (22), it is inappropriate to understand passage (20) as suggesting a sufficient condition for inferring possibilities for the table.

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[103] Kripke [1980], 51.
[104] Kripke [1980], 51-52.

A clear statement of **(ST)** is found in Salmon [1986]. In his earlier works, he actually uses **(ST)** in his discussions of the Four Worlds Paradox, though **(ST)** is not explicitly stated. Salmon simply takes principle **(ST)** as an intuitively correct principle without any examination to the intuition. But, we do not think that the intuition stated in **(ST)** is so strong as self-evident so that no rationale is needed to demonstrate its correctness. Indeed, we do have doubts about this intuition. Let us borrow an example from W. Carter to convey our objection to **(ST)**.

Here is Carter's story:

[Passage (23)]

> Wilma is a retired clock-maker with time on her hands. Realizing that she has more detached clock parts than are required for the job, Wilma decides to pass the time by constructing a clock. The clock that Wilma proceeds to build—let us call it "Ben"—has never before been constructed. Ben is constructed out of clock parts c_1, c_2, ..., c_{100}. ... At the completion of the construction process, Wilma is left with three unused parts, c_{101}, c_{102} and c_{103}. As it happens, c_{101} is functionally equivalent to c_1, c_{102} is functionally equivalent to c_2, and c_{103} is functionally equivalent to c_3.[105]

We assume that the plan according to which Ben is constructed includes the design, the available material, the maker and the specific project of assembling a clock from the available material according to the design. We also assume that Wilma's choice of using one of c_1 and c_{101}, one of c_2 and c_{102}, and one of c_3 and c_{103} in the construction of the clock is accommodated in the plan. That is, given the plan thus assumed, using either c_1 or c_{101}, either c_2 or c_{102}, and

[105] Carter [1983], 225-226.

either c_3 or c_{103} to construct a clock are all considered as constructing a clock according to the same plan. The counterfactual worlds can be stipulated exactly the same as the real world except of realizing some other chances allowed in the plan. Namely, instead of having $\{c_1, c_2, \ldots c_{100}\}$ as its collection of components, the clock made in a counterfactual world has another subset of the set $\{c_1, c_2, \ldots c_{100}, c_{101}, c_{102}, c_{103}\}$ (the set of total 103 clock-parts) to be its collection of components, which is quantitatively and qualitatively identical to $\{c_1, c_2, \ldots c_{100}\}$. In the situation described in Carter's story, if the clock in a counterfactual world is made according the same plan as the one by which Ben is made, the maximum change in the original matter is a difference of three components. A difference of three components can be counted as a fairly small amount of change in the given situation. Thus, nothing intuitively essential to Ben's identity is violated in those counterfactual worlds. We seem to have a case in which we can definitely answer the question whether a certain collection of components still constitutes Ben—the collections which are subsets of $\{c_1, \ldots, c_{103}\}$ and qualitatively identical to $\{c_1, \ldots, c_{100}\}$ are possible collections of components for Ben.

We have shown that Ben could be made according to the same plan from some other collection of components which is slightly different from but qualitatively and quantitatively the same as its actual collection of components. This is the intuition expressed in principle **(T)**. Now, we consider the following question: given the plan of making Ben as assumed above, is it the case that for any collection of components, if it is slightly different from but qualitatively and quantitatively equivalent to Ben's actual matter, then this collection of components could have been made into a table according to the plan of making Ben?

Let us imagine two other counterfactual worlds. In the first counterfactual world, the clock made by Wilma is constructed from the parts c_{201}, c_{202}, c_{203}, c_4, ..., c_{100} where c_{201}, c_{202}, c_{203} are qualitatively and functionally equivalent to c_{101}, c_{102} and c_{103}. However, they are parts that Wilma obtained through a mail-order. In the second counterfactual world, the clock made by Wilma is constructed from the parts c_{301}, c_{302}, c_{303}, c_4, ... c_{100} where c_{301}, c_{302}, c_{303} are qualitatively and functionally equivalent to c_{101}, c_{102} and c_{103}. But the three parts c_{301}, c_{302}, c_{303} are preciously obtained from another planet, say from the moon, through an extraordinary exploration on the moon accomplished by Wilma. We want to ask whether the clocks in the two counterfactual worlds are made according to the same plan as the plan from which Ben is made. Given the assumption of the plan of making Ben in the real world, in both counterfactual worlds the clocks are not made according to the very same plan. We have assumed that the plan according to which Ben is made includes the design, the available material, the maker and the specific project of how to assemble a clock from the available material according to the design. In the two counterfactual worlds, the clock is not made from the available material $\{c_1, ..., c_{103}\}$ and not according to the specific project. By substituting Ben for the table x in principle **(ST)**, we produce a counterexample to **(ST)**. We showed that in both counterfactual worlds, a clock is made from a collection of components which is qualitatively and quantitatively the same as Ben's actual matter and differs from Ben's actual matter in no more than three components. The two collections of components, $\{c_{201}, c_{202}, c_{203}, c_4, \ldots, c_{100}\}$ and $\{c_{301}, c_{302}, c_{303}, c_4, \ldots, c_{100}\}$, can be considered both as slightly different from the collection of components that actually constitutes Ben. But no clock can be made from them according to the plan by which Ben is constructed. Hence, there are some collections of components, which are slightly different from but qualitatively and quantitatively the same as Ben's actual collection of components, such that Ben could not be made from these collections of components according to the same plan. This is a denial of principle **(ST)**. In other words, let $\{c_1, \ldots, c_{100}\}$ be β and $\{c_{301}, c_{302}, c_{303}, c_4, \ldots, c_{100}\}$ be δ. The collection δ is sufficiently similar to β ($\mathbf{S}(\delta,\beta)$) and Ben is made from β according to plan $\boldsymbol{P}$ ($\boldsymbol{M}(\mathrm{Ben},\beta)\&\boldsymbol{P}_{\mathrm{Ben}}$) in the real world. But it is impossible that Ben is made from δ according to plan $\boldsymbol{P}$ ($\neg\Diamond(\boldsymbol{M}(\mathrm{Ben},\delta)\&\boldsymbol{P}_{\mathrm{Ben}})$) relative to the real world. This is a denial of the following instance of **(ST)**: $\mathbf{S}(\delta,\beta)\supset\Box((\boldsymbol{M}(\mathrm{Ben},\beta)\&\boldsymbol{P}_{\mathrm{Ben}})\supset\Diamond(\boldsymbol{M}(\mathrm{Ben},\delta)\&\boldsymbol{P}_{\mathrm{Ben}})$.

This counterexample to principle **(ST)** is given based on our assumption about the plan of constructing Ben. One may hesitate to agree with this counterexample and question what we mean by a "plan" and how plausible it is to fix the plan of making Ben as the way we assumed. Roughly speaking, the word "plan" in principle **(C)** and principle **(ST)** is used to include (at least) all the essential aspects of the table except the original matter. Salmon's explanation of his usage of the word "plan" in his principle (V′) (which corresponds to our principle (**C**)) is this: the phrase "according to the same plan" means that the tables in question are constructed in precisely the same way so that the same molecule goes to the corresponding spot in the involved tables.[106] He also thinks that one may fix "factors as the artisan who constructs [the table], the artisan's reason for constructing [the table], the time and place of the construction, and so on."[107] There is no disagreement in principle between his usage of the word "plan" in his principles (V′), (I) and (II) and our usage of the word "plan" in principles (**C**), (**T**) and (**ST**).[108] Given Salmon's explanation of his usage of the "plan," the plan in his principle (V′) and principles (I) and (II) is assumed even closer to the actual situation.

The question whether the plan of constructing Ben should be the way as we assumed, or it should allow Wilma's mail-order or even Wilma's exploration on the moon, is a question about possible tolerances in other essential properties of Ben. There might be tolerances in other essential aspects of Ben. But the consideration about tolerable variations in the plan of making Ben does not in principle change our counterexample. As long as it is not the case that all changes, no matter how dramatic they are, are always tolerable, our rejection to principle (**ST**) will remain the same. Presumably we can always find counterexamples to (**ST**) when the tolerable variations in the plan are specified. For example, if the mail-

[106] Salmon [1981], 210-211.
[107] Salmon [1981], 211.
[108] Principle (V′) is given in Salmon [1981], 211. Principles (I) and (II) are given in Salmon [1986], 75 and 77.

order is a tolerable variation of the plan but the moon-exploration is not, then the collection $\{\boldsymbol{c}_{301}, \boldsymbol{c}_{302}, \boldsymbol{c}_{303}, \boldsymbol{c}_4, \ldots, \boldsymbol{c}_{100}\}$ will not be a possible collection for Ben. Thus, Ben could be made according to the same plan from some collections of components which are qualitatively and quantitatively the same as but differ in less than or equal to three components from Ben's actual matter, but it is not the case that Ben could be made according to the same plan from any collection of components which are qualitatively and quantitatively the same as but differs in less than or equal to three components from Ben's actual matter. This is to say, principle **(T)** is true of the case of Ben but principle **(ST)** is false. Do we have a reason to believe that not all changes in the plan of Ben are tolerable? The answer is yes. The intuition here is simple: if all the changes in the plan are allowable, or if the plan of Ben might vary dramatically, how can it be at the same time essential to Ben?

Given that the two paradoxes, Chisholm's Paradox and the Four Worlds Paradox, are drawn from the three principles **(N)**, **(C)** and **(ST)**, our rejection of **(ST)** results in an immediate resolution to these two paradoxes. We hold that the arguments in the paradoxes are unsound because the premise **(ST)** is an incorrect principle. It is simply not the case that if the given artifact is possible to be made according to plan ***P*** from collection **C**, then it is possible for the artifact to be made according to the same plan ***P*** from any collection of components which is qualitatively and quantitatively the same as, but having one (or two, or three, or ...) component different from, the given possible collection **C**.

4.3 Considerations that Support the Rejection of (ST)

In this section, we shall consider some viewpoints that we hold to be correct. Principle **(ST)** is inconsistent with these views. If the views expounded below are indeed correct, the inconsistence between principle **(ST)** and these views will support our rejection of **(ST)**.

(1) The physical compatibilities in the possible constructions of an artifact.

Kripke has commented on how the questions about the essences (or essential properties) of individual objects should be asked:

> [Passage (24)]
>
> Ordinarily, when we ask intuitively whether something might have happened to a given object, we ask whether the universe could have gone on as it actually did up to a certain time, but diverge in its history from that point forward so that the vicissitudes of that object would have been different from that time forth. *Perhaps* this feature should be erected into a general principle about essence. Note that the time in which the divergence from actual history occurs may be sometime before the object itself is actually created. For example, I might have been deformed if the fertilized egg from which I originated had been damaged in certain ways, even though I presumably did not yet exist at that time.[109]

Kripke's suggestion in passage (24) is that in talking about the essences or the essential properties of an individual object, we must first be able to pick out the object, that is, to be clear about which one this object is. We then consider counterfactual situations that

[109] Kripke [1980], 115, footnote 57.

branch out from the actual world at a certain time closely relevant to the existence of this actual object and ask how the divergence in the history of the actual world affects the existence of this actual object.

I think that this view is correct. A question about a particular object's individual essence can be asked meaningfully when the object can be successfully picked out and when the divergence in a counterfactual situation is comprehensible. Let us say that the "historical condition" with respect to a certain phase of the existence of an actual object is the state of history of the real world at the time when the object is in that phase (it may be a period before but closely relevant to the existence of the object). If we limit the meaningful questions about individual essences to those counterfactual situations as Kripke suggested, we actually assume a fixed historical condition for what could happen to the individual object in question. Namely, we assume that the history of a counterfactual world is the same as the history of the real world up to a point closely related to the origination of the object. The divergence in a counterfactual world must be physically compatible with the given historical condition; otherwise, the world could not be realized as a whole. In other words, if we use "possible world" in Salmon's notion (a possible world is a maximal abstract entity that could have been realized), a metaphysically possible counterfactual world must be such that the corresponding *P*-world (the material entity which would exist if the counterfactual world, the maximal abstract entity, had been realized) is a physically compatible one.[110]

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110 "*P*-world" is introduced in Section 3.4 above.

Since there is a unique *P*-world corresponding to each counterfactual world (the actual physical world is the *P*-world corresponding to world $\boldsymbol{w}_{@}$, the maximal description of the actual world), it should be clear when we say that the counterfactual worlds must be physically compatible.

In the discussion of Chisholm's Paradox and the Four Worlds Paradox, the essential properties of the given actual table are presumably viewed in those counterfactual worlds in which the history of the worlds is the same up to a time closely relevant to the construction of the table. Thus, the divergence in these counterfactual worlds must be physically compatible with the historical condition at the time the divergence occurs. Furthermore, the discussion assumes that the tables in those counterfactual worlds are constructed according to the plan by which the given actual table is constructed. Then, in addition to the historical condition, the discussion actually assumes that the aspects of the counterfactual worlds with respect to the plan are the same as those of the real world. Thus, the plan imposes an additional restriction on what can in fact be metaphysically possible—the divergence must be physically compatible not only with the historical condition but also with the aspects of the worlds related to the plan. If **(ST)** is a correct principle, then the possibilities inferred from this principle obey these restrictions.

Salmon has suggested that the counterfactual worlds can be stipulated to be exactly the same as the real world except that in each counterfactual world the table is made from a different hunk of matter. Given that the counterfactual situations are thus stipulated, the historical condition and the aspects of the counterfactual worlds related to the plan are the same as in the real world. Then Principle **(ST)** is false, for it is not the case that for any hunk of matter qualitatively and quantitatively the same as but differing slightly from the table's actual matter in their collections of components, there is always a possible counterfactual world in which it is physically compatible that the given table (or some other table) is made from this hunk of matter according to the plan based on the historical condition. For example, when the counterfactual worlds are stipulated maximally resembling the real world, it is not physically compatible that Ben be made from the collection $\{\mathbf{c}_{301}, \mathbf{c}_{302}, \mathbf{c}_{303}, \mathbf{c}_{4}, \ldots, \mathbf{c}_{100}\}$ according to the same plan.

(2) The physical necessities that are also metaphysically necessary.

One might suspect that in the above discussion we are saying that if it is physically impossible for the table in question to be made from a certain hunk of matter according to the given plan based on the historical condition, then it is metaphysically impossible for the table to be so made. Indeed, we do hold that some physical necessities of an artifact, including the physical necessities (impossibilities) in the original construction of the artifact, are also metaphysically necessary. Kripke has expressed the following view:

[Passage (25)]

> ... characteristic theoretical identifications like "Heat is the motion of molecules," are not contingent truths but necessary truths, and here of course I don't mean just

> physically necessary, but necessary in the highest degree—whatever that means. (Physical necessity, *might* turn out to be necessity in the highest degree. ... At least for this sort of example, it might be that when something is physically necessary, it always is necessary *tout court*.)[111]

Kripke also mentioned "Water is H_2O," "Gold has atomic number 79" and "Light is a stream of photons" as examples of physical necessities that are also necessary in the highest degree. Kripke's explanation is that these scientific discoveries reveal the nature of the substances and the phenomena in question.[112] These substances and phenomena are theoretically identified with their physical characteristics; they could not have been the very substance or the very phenomenon without having their physical characteristics. A necessity of the highest degree must also be metaphysically necessary. Given Forbes' view that metaphysical necessities are fundamentally necessities of conceptual truths, we may restate Kripke's view in terms of conceptual truths as follows: the scientific discoveries mentioned by Kripke are metaphysical necessities because the discovered physical characteristics form the content of our concepts of these substances and natural phenomena.

Our claim, that some physical necessities of a given artifact are also metaphysically necessary, is made for a similar reason. If it is physically impossible for the given artifact to be made from a certain collection of components according to its plan under the historical condition, it is metaphysically impossible for the given artifact to be so made, because physical compatibility is part of the content of our

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[111] Kripke [1980], 99.
[112] Kripke [1980], 125.

concept of an artifact's being made according to the given plan under the given historical condition.

(3) Necessities and possibilities that are discovered empirically.

Kripke has said: "One might very well discover essence empirically."[113] Kripke's examples are those scientific discoveries mentioned above: "Heat is the motion of molecules," "Water is H_2O," "Gold has atomic number 79," "Light is a stream of photons" and so on. What is heat or what is water is a matter of scientific discovery—it can only be discovered through empirical scientific investigation. According to Kripke, we can imagine discovering heat and water differently, but once we discovered that heat is the motion of molecules and water is H_2O, it is essential for heat to be the motion of molecules and water to be H_2O. Kripke expresses a similar view about individual artifacts: from what substance the given table is made is a matter of empirical investigation. We can imagine discovering the substance of the given table differently, but if we in fact discover that the table is originally made of wood, it is essential for the table to be originally made of wood.[114] Given that these physical necessities of the table can only be known empirically, it follows that some metaphysical necessities must be empirically discovered.

In our opinion, the metaphysical necessities and possibilities about the original matter of a certain artifact belong to the kind of necessities and possibilities that must be discovered empirically. In Salmon's and Forbes' discussion of the two paradoxes, it is suggested

[113] Kripke [1980], 110.
[114] Kripke [1980], 113-115.

that the reason for the necessities and possibilities about the original matter of a given table to be *a posteriori* is that an empirical knowledge about the artifact's actual matter is involved in the inferences of the necessities and possibilities drawn from the essentialist principles. But, in saying that the necessities and possibilities about the original matter of a given artifact are discovered empirically, we mean more than the involvement of the knowledge of the actual matter in the modal inferences. Given that physical compatibility is a necessary condition in claiming possible matters for a given artifact, one needs empirical knowledge on how the historical condition and the plan are determined in order to know what the physical compatibilities are in this particular situation. The possible collections of components for the original construction of the artifact may be determined after acquiring this knowledge. If our view is correct, the Strong Tolerance Principle (**ST**), which claims that further possible matters of a table can be inferred purely from a known possible matter of the table and a standard for quantitative change in the components, must be rejected, because this principle is incorrect on how the possibilities are determined.

4.4 About the Vagueness of the Threshold

In Passage (19) quoted in Section 4.1, Forbes expresses his view about the vagueness of the threshold between tolerable variations and intolerable variations in the original matter of artifacts: "Tolerance arises because of vagueness or fuzziness in the limits of the range of sums of wood which possibly constitute α: there is no sharp distinction between those sums which could, and those which could not, constitute α." According to this passage, the boundary between the tolerable variations and intolerable variations in the original matter of α is itself vague or fuzzy. Moreover, let us combine Passage (19) with the following passage:

[Passage (26)]

> To specify the essences of such entities, we need to find some way of representing the thought that if an entity of this sort is made up (without leftovers) of parts from a given set, then as we consider sets of parts which have less and less in common with the given set, it becomes less and less possible for the entity to ave been constructed from the set under consideration.[115]

We see that, according to Forbes, the hunks of matter sharing more parts with the original matter of the given table is more possible for the original construction of the table, whereas the hunks of matter sharing less parts with the original matter of the given table is less possible for the original construction of the table. There is no sharp boundary separating possible matters and impossible matters for the given table.

Salmon's intuition of vagueness is different from that of Forbes, as we can see from the following passage:

[Passage (27)]

> The first two principles [equivalent to our principle (**T**) and (**M**1)] ... imply that a certain amount of variation is possible in the original constitution of a table, whereas principle (0) [a principle weaker than our principle (**N**) by letting **z** be a hunk of matter which shares no

[115] Ibid.

component with table ***x***'s original matter ***y***] implies that the amount of allowable variation is something short of total. ... It follows that there is some threshold ... such that one more change in original constitution must by necessity result in a numerically distinct table. ... even if the threshold is some exact and very precise amount of overlap, from an epistemic point of view we can never be in a position to specify with adequate justification just what the threshold is—except by means of some vague locution like 'sufficiently substantial overlap'."[116]

In Salmon's view, the threshold itself is not vague. Salmon does say that "it seems more realistic to suppose that the threshold consists in some interval, perhaps some range of numbers of shared molecules."[117] But, whether the threshold is a point or an interval, there is a definite threshold such that:

[Passage (28)]

For any hunk of matter ***y'*** that shares a greater number of molecules with the actual matter ***y*** of the table ***x*** than any number in this range[the threshold] ... is determinately true of ***x*** that it might have originated from ***y'*** instead of from ***y***. For any hunk of matter **z** sharing fewer molecules with ***y*** than any number in the range, it is determinately true of ***x*** that it could not have originated from **z**. For any hunk of matter ***y''*** whose number of shared molecules with ***y*** lies within the range, it is indeterminate ... whether ***x*** could have originated from ***y''*** instead of from ***y***.[118]

We can see from Passage (28) above that in viewing the threshold as an interval, Salmon has in mind a threshold with two sharp cutoff points that separate all hunks of matter into three regions—the hunks from which table ***x*** could have originated, the hunks from

116 Salmon [1986], 76-77.
117 Salmon [1986], 76.
118 Salmon [1986], 76-77.

which whether *x* could have originated is indeterminate and the hunks from which *x* could not have originated. Salmon uses the word "vague" to refer to the epistemological indeterminacy of the truth value of the proposition that "*x* could have originated from *y*"." But this is a different issue from the question of whether the threshold itself is vague. The cognitive indeterminacy of the truth value of those propositions does not mean that the threshold is itself not clear. Salmon actually argues that there must be a cutoff threshold between possible hunks of matter and impossible hunks of matter of an given artifact.[119] For his, what is vague about the threshold is only the human cognition of the threshold—we can never be in a position to specify with adequate justification what the threshold is.

Despite the difference between the views of Salmon's and Forbes', these two views have something in common:

(i) They both hold that we can never find out where the threshold lies. For Forbes, it is because there is no such a thing in reality. For Salmon, it is because human cognition is limited.

(ii) They both hold that the possibilities regarding the original matter of a given table can be determined solely by a quantitative measure of the variations in the original matter of the table. For Salmon, any collection with a number of changed components smaller than the quantitative threshold is a possible hunk of matter for the given table, and any collection with a number of changed components greater than the threshold is an impossible hunk of matter for the table. For Forbes, a hunk with less variations from the

[119] Salmon [1986], APPENDIX, 110.

actual matter is more possible and a hunk with more variations is less possible for the origination of the given table.

We have already rejected principle **(ST)** in Section 4.2. The example of Ben shows that it is not the case that Ben could be made according to the same plan from any collection of components that is slightly different from but qualitatively and quantitatively the same as Ben's actual collection of components. There are some such collections from which Ben could not be made according to the same plan. It seems to me that it is the intuitions stated in (ii) in the previous paragraph that lead Forbes and Salmon to their belief in principle **(ST)**. The example of Ben shows that Forbes and Salmon's intuitions stated in (ii) are incorrect. The clock Ben, which is actually constructed from collection $\{c_1, c_2, \ldots c_{100}\}$, is possible to be constructed according to the same plan from collection $\{c_{101}, c_{102}, c_{103}, c_4 \ldots c_{100}\}$, but not possible to be constructed according to the same plan from collection $\{c_{301}, c_{302}, c_3, \ldots c_{100}\}$. Hence, though it is true that any variation with more changed components than the threshold is an impossible hunk of matter for Ben, it is not the case that any collection with fewer changed components than the threshold is a possible hunk of matter for Ben, and neither it is the case that a hunk with less variations from the actual matter is more possible for the origination of the given table but a hunk with more variations is less possible. The threshold of tolerable variations in Ben's original matter, a maximum difference of 3 components, is a necessary condition for determining possible collections, or say, is sufficient for determining impossible collections of components, but it is not by itself sufficient for determining possible collections for Ben. Any possible collection of components of Ben must satisfy this quantitative measure, but not all collections of components that satisfy this quantitative measure are possible collections for Ben.

We now consider the views of Forbes and Salmon stated in (i) above that the threshold is unknowable. It seems to me that there is a reasonable consideration from which we may agree that the threshold is unknowable: since there are tolerable variations in various related essential aspects of artifacts and we are likely not to be able to achieve a complete certainty about what these variations are, we may never know for sure the threshold of tolerable variations in the original matter of a given artifact—at least it may be too complicated for human mind to figure it out. But notice that this consideration is not the reason by which Forbes and Salmon think that the threshold is unknowable. In the discussion of two modal paradoxes, the plan, namely the set of essential properties of the given artifact other than the original matter, is given and assumed to be correct. Therefore, more precisely, Salmon and Forbes hold that even if the plan of making the artifact is correctly given or is known, the threshold for the tolerable variations in the original matter of the artifact is still unknowable. Specifically, we may understand Forbes as saying that since in reality there does not exist clear threshold between tolerable variations and intolerable variations, we can never find it out; and we may understand Salmon as saying that we can never have sufficient reason to identify the threshold even if the plan is explicitly given. We shall see that both of these views are incorrect in the case of Ben.

Recall our discussion of Carter's story. We concluded that the case of making Ben is one in which we can definitely answer the question whether a certain collection of components could have constituted Ben. The clock Ben, made actually according the plan from the collection $\{c_1, c_2, \ldots c_{100}\}$, could have been made according to the same plan from other collections of components which are subsets of $\{c_1, c_2, \ldots c_{100}, c_{101}, c_{102}, c_{103}\}$ and qualitatively and quantitatively the same as $\{c_1, c_2, \ldots c_{100}\}$. Given our assumptions about the plan and a fixed historical condition, an empirical investigation will discover that any collection containing a component not in the set $\{c_1, c_2, \ldots c_{100}, c_{101}, c_{102}, c_{103}\}$ is a collection of components from which Ben could not have been made according to the same plan under the fixed historical condition. In this case, any hunk of matter differing from the original matter of Ben in more than three components is an impossible matter of Ben. Thus, disagreeing with Forbes, we conclude that there is a sharp cutoff threshold, namely a maximum difference of 3 components, so that any possible variation must be within this threshold and any variation surpassing the threshold is an impossible variation. In other words, given the plan and the historical condition, the threshold for the possible variations in the original matter of Ben, that is, a maximum difference of 3 components, is itself clear, not vague. Furthermore, the threshold is knowable from philosophical analyses and empirical investigations. That the threshold is knowable in the case of Ben implies that there is a sufficient reason in the case of Ben for identifying the threshold. In our view, the sufficient reason for identifying the threshold for the tolerable variations in Ben's original matter is the defendableness of the threshold. Given the plan and the historical condition, we can show why it is the maximum difference of 3 components, but not 2 components or 4 components, that is the threshold.

We believe that there are concrete cases like the case of Ben, and we think that these cases are the only ones in which we can talk about transworld identity of artifacts. But we do not claim that every concrete case is like Ben's case—there might be cases such that even if the historical condition and the plan of making the artifact in question are given, it is still vague what are the possible variations in the original matter of the given artifact. This may be shown by the following revised story of Carter's: The retired clock-maker Wilma decides to pass her time by constructing a clock from the detached clock parts available in her hands. As a matter of fact, the clock, call it Ben*, is constructed out of clock parts $\boldsymbol{c}_1$, $\boldsymbol{c}_2$, ..., $\boldsymbol{c}_{100}$. After the completion of the work, however, she is left with twenty, instead of three, unused parts, $\boldsymbol{c}_{101}$, ..., $\boldsymbol{c}_{120}$, and each $\boldsymbol{c}_{100+i}$ is qualitatively and functionally equivalent to $\boldsymbol{c}_i$.

As we did in the case of Ben, we assume that the following is the plan of making Ben*: the plan according to which Ben* is constructed includes the design, the available material, the maker and the specific project of how to assemble a clock from the available material according to the design. We also assume, similar to the assumption given in the case of Ben, that Wilma's choice of using one of $\mathbf{c}_1$ and $\mathbf{c}_{101}$, one of $\mathbf{c}_2$ and $\mathbf{c}_{102}$, ... and one of $\mathbf{c}_{20}$ and $\mathbf{c}_{120}$ in the construction of her clock is accommodated in the plan. This is to say that given the plan thus assumed, using either $\mathbf{c}_1$ or $\mathbf{c}_{101}$, either $\mathbf{c}_2$ or $\mathbf{c}_{102}$, ... either $\mathbf{c}_{20}$ or $\mathbf{c}_{120}$ in the construction of clock are all considered as constructing a clock according to the same plan.

In this revised case, given the assumption of the plan of making Ben*, Wilma is possible to assemble a clock according to the plan from any other collection which is a subset of $\{\mathbf{c}_1, ..., \mathbf{c}_{120}\}$ and qualitatively and quantitatively the same as Ben*'s actual matter. Thus, there is a counterfactual world in which Wilma assembles a clock according to the plan from the collection of components, $\{\mathbf{c}_{101}, ..., \mathbf{c}_{120}, \mathbf{c}_{21}, ..., \mathbf{c}_{100}\}$. The question is whether this clock is numerically identical to Ben*. A twenty-component difference in a collection of one hundred components is not likely to be a slight difference. Given Principle **(N)**, one might think that this clock is not Ben*, or at least, one may suspect that this clock is not Ben*. Thus we encounter a situation in which even if the plan is unambiguously given, it is still vague what counts as possible variations in the original matter of the artifact in question.

Perhaps the plan is fixed too loosely.[120] That is, perhaps not all the choices of using $\mathbf{c}_i$ or $\mathbf{c}_{100+i}$ in the construction of Ben* should be accommodated into the plan. It is conceivable that if the plan is fixed close enough to the actual situation, the physically compatible

[120] In assuming the plan of making Ben*, we did exactly the same as we did in assuming the plan of making Ben. But because of the particularity of each individual object, there is no reason to suppose that the essential properties in the plan of Ben* must correspond to the essential properties in the plan of Ben.

variations will be limited to a fairly small amount. However, whether or not the plan of making Ben* should be specified closer to the actual situation, it seems that we have no convincing reason to suppose that whenever we encounter a case like the case of Ben*, there is always a justified reason to tighten the plan.

We have shown that the situation of Ben is a situation in which the threshold of tolerable variations in the original matter of the artifact is itself clear and knowable, whereas the situation of Ben* is a situation in which the threshold of tolerable variations in the original matter of the artifact is vague—either the threshold is itself vague or we lack the cognitive ability to recognize it.

It may be true that when the original matter of a given artifact is considered alone, we can only have a rough idea about what might count as slight change in the original matter of the artifact and what might be the threshold. But this vague conception can be sharpened in some concrete situation when various essential aspects of the artifact are given and considered in connection. A situation in which our vague conception about the threshold of tolerable variations in the original matter of a given artifact can be sharpened is a situation in which our two conceptions concerning the identity of the artifact, the conception about how the plan should be determined and the conception about what may count as slight change in the original matter of the artifact, conform with each other. In the example of Ben, given the intuition of how the plan should be determined, the physically compatible variations in the original matter of Ben are fairly small. Our initial idea about the plan of making Ben and the allowable variations in the original matter of Ben conform with each other. The characteristic of Ben's kind of cases is that a sufficient defense of the threshold can be given. Namely, given the historical condition and the plan as thus fixed, we can explain convincingly why any possible variation in the original matter of the given artifact within this threshold is, and why any variation surpassing this threshold is not a possible variation for the artifact.

On the other hand, the situation of Ben* is a situation in which our vague conception about the threshold of tolerable variations in the original matter of the artifact cannot be sharpened, or cannot be sharpened to the extent such that the threshold can be identified. In Ben*'s kind of situation, our conception about how the plan should be determined and our conception about what may count as a slight change in the original matter of the artifact cannot conform with each other. In the situation of Ben*, given the plan as we assumed, the physically compatible variations can still be considerably significant. Thus our conception about the plan and our conception about the tolerable variations do not conform with each other. Whichever number i, where $1 \leq i \leq 20$, that one may think is the threshold of the tolerable variations for the original matter of Ben*, it seems that there is no convincing argument to show why the threshold is i, not i-1 or i+1. We remain unclear about which collection is possible for Ben* and which is not, even if the plan is clearly determined.

One might object to our view on the distinction between Ben's case and Ben*'s case. One may say that if the plan of making Ben is loosened in some way, then the situation of Ben may become similar to the situation of Ben*. Thus, we may not really have the distinction, and in general we may not really have the kind of cases in which our vague conception about the threshold of tolerable variations in the original matter of the given artifact can be sharpened. It is true that by loosening the plan, some of Ben's kind of situations will become a Ben*'s kind of situation. Our reply to this objection is this: The presupposition of the discussion about possible variations in artifacts' original matter is that the plan is given according to our best understanding of what is essential to the artifact in consideration. We claim that once the plan is determined as such, there can be two kinds of situations, Ben's kind and Ben*'s kind. Just as it is not reasonable to suppose that it is always justified to tighten the plan closer to the actual situation, it is equally not reasonable to suppose that it is always justified to loosen the plan further from the actual situation. Especially, it is not plausible to suppose that it is always justified to loosen the plan to the extent of making a case of Ben's kind as one of Ben*'s kind.

The cases of Ben's kind are the cases in which we are able to determine (to know) which collection of components is possible for the given artifact, whereas the cases of Ben*'s kind are those in which we are not able to determine (to know) which collection is possible for the given artifact. In Ben's kind of cases, we are also able to determine impossible matters for the artifact, whereas in Ben*'s case, we are able to determine impossible matters for some hunks of matter but not for all. Our view is that if we are able to determine both possibilities and impossibilities about the original matter of the given artifact, our conception of the threshold is clear, not vague—at least, our conception is clear to the extent that the threshold can be defended. On the other hand, if our conception of the threshold is vague, we are simply not in the position to determine possibilities and impossibilities for the original matter of the artifact.

4.5 The Absolute Essentialist Intuition

By distinguishing the two kinds of cases, Ben's kind and the Ben*'s kind, the absolute essentialist intuition that the *a posteriori* essential properties of an artifact are essential to the object *simpliciter* can be defended. In order to defend the absolute essentialist point of view, one must show that the possibilities and impossibilities for the original matter of the given artifact will be determined the same in every counterfactual world regardless what is the forming matter of the artifact in each world in which the artifact exists. This is exactly what we can see in the Ben's kind of cases. In the Ben's kind of cases, our vague conception about the tolerable variations in Ben's original matter is sharpened by investigating the historical condition and considering the plan and the physical compatibility in the situation. A set of collections of components, which satisfy our intuition about the individual essence of Ben, is identified by these philosophical analyses and empirical investigations. The actual matter of the given artifact is just one collection in this set that happens to be realized. The realization of the actual matter cannot in any way affect the discovery of the set of possible matters for the given artifact. The same holds for the other collections in the set in the corresponding counterfactual worlds. The historical condition, the plan and the physical compatibility are assumed to be the same in the counterfactual worlds. Thus, had the artifact been made from some other possible collection, the set of possible collections for the artifact would still be the same. In the example of Ben, given the historical condition fixed up to the time when Wilma uses the available clock-parts to assemble a clock and the plan of making Ben as we assumed earlier, the possible collections of components of Ben determined in the real world $\boldsymbol{w}_@$ are those subsets of the total 103 clock-parts that are qualitatively and quantitatively equivalent to Ben's actual collection $\{\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_{100}\}$. Consider a counterfactual world $\boldsymbol{w}_i$, possible relative to the real world $\boldsymbol{w}_@$, in which Ben is made from a collection of components $\{\boldsymbol{c}_{101}, \boldsymbol{c}_2, ..., \boldsymbol{c}_{100}\}$ which is different from $\{\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_{100}\}$ in one component. In $\boldsymbol{w}_i$, the historical condition, the plan and the relevant physical (or natural) laws are the same as they are in $\boldsymbol{w}_@$. The only difference is that the $\boldsymbol{w}_@$-possibility of making Ben from $\{\boldsymbol{c}_{101}, \boldsymbol{c}_2, ..., \boldsymbol{c}_{100}\}$ is a truth of $\boldsymbol{w}_i$. Imagine conducting a similar philosophical analysis and empirical investigation on the historical condition, the plan and physical compatibility for $\boldsymbol{w}_i$ in order to determine the set of possible collections of components for Ben's original matter in $\boldsymbol{w}_i$. We can see that the set of possible collections of components for Ben found in $\boldsymbol{w}_i$ will be the same as the set found in $\boldsymbol{w}_@$, because nothing involved in the determination of the set in $\boldsymbol{w}_@$ is changed in $\boldsymbol{w}_i$. The same holds for counterfactual worlds other than $\boldsymbol{w}_i$, in which Ben is made from some other collection in the set. Thus, if it is possible for Ben to be made from collection such-and-such, it is possible for Ben *simpliciter*. On the other hand, if it is impossible for Ben to be made from collection such-and-such, it is impossible for Ben *simpliciter*. In the case of Ben, given that our conception about the threshold is clear, we may substitute the word "slightly different" in the instance of principle **(N)** concerning Ben by the identified threshold—in this case the threshold is 3 (also make corresponding substitutions in the principle so that the principle becomes one about clocks). Given the above discussion, any *a posteriori* necessities derivable from the instance of **(N)** concerning Ben is necessary to Ben *simpliciter*. A similar consideration can be given to the *a posteriori* necessities derivable from principle **(C)** as well.

What about the case of Ben*? The case of Ben* is one in which our vague conception about the threshold of tolerable variations in the original matter of the artifact cannot be sharpened. We have stated the view that if our conception about the threshold cannot be sharpened, we are simply not in the position to determine the transworld identity of the given artifact in term of its matter. However, Salmon seems to hold that even if our conception about the threshold is vague, we can still determine transworld identity for the given artifact by inferences drawn from the Strong Tolerance Principle **(ST)**. In the following we shall explain our reason for disagreeing with his view.

It is easy to see that the Strong Tolerance Principle **(ST)** is incorrect even in the cases like the the case of Ben*. Given what the plan is, the collection $\{c_{301}, c_2, ..., c_{100}\}$, where c_{301} is a component from the moon, is not a possible hunk of matter for Ben* even though it has only one component different from Ben*'s actual matter. But suppose that "a wooden table" is replaced by "a clock" and the quantifier "any" in principle **(ST)** is restricted as ranging over only all the collections that are subsets of $\{c_1, ..., c_{120}\}$ and are qualitatively and quantitatively equivalent to the collection $\{c_1, ..., c_{100}\}$. That is, the restricted principle **(ST)** about clocks says: If a clock x is originally made according to plan P from a collection y, then x might have been made according to the same plan P from any collection y' such that y' is a subset of the set $\{c_1, ..., c_{120}\}$ and has the same quality and quantity as y, but the collection of components of y' is slightly different from that of y. Is this restricted **(ST)** a correct principle for the case of Ben*?

Paradoxical arguments similar to Chisholm's Paradox and the Four Worlds Paradox can be produced in this specific situation by the restricted **(ST)** in S4 or S5 modal logic. According to the restricted **(ST)**, if the clock were made from the collection $\{c_{101}, c_2, ..., c_{100}\}$, it would be Ben* because one component difference from Ben*'s original matter is a slight difference and the collection $\{c_{101}, c_2, ..., c_{100}\}$ is a subset of the set $\{c_1, c_2, ..., c_{120}\}$ and is qualitatively and quantitatively equivalent to Ben*'s original matter. But if so, by the same consideration, the clock made from the collection $\{c_{101}, c_{102}, c_3, ..., c_{100}\}$ would also be Ben* since it is identical to the clock made from the collection $\{c_{101}, c_2, ..., c_{100}\}$, and that clock is Ben*. Repeating this argument 19 times, one will conclude that the clock made from the collection, $\{c_{101}, \ldots, c_{120}, c_{21} \ldots, c_{100}\}$. is Ben*. But at the same time, one may suspect that the clock made from $\{c_{101}, \ldots, c_{120}, c_{21} \ldots, c_{100}\}$ is not Ben* because a difference of 20 components is not likely to be a slight difference.

But we must consider carefully that given that the threshold for the tolerable variations in the original matter of Ben* is vague, whether it can be the case that the possibilities inferred from this restricted **(ST)** for Ben* are all true possibilities of Ben*. Here we need not consider whether the possibilities inferred by the transitivity and the symmetry of S5 modal logic are the true possibilities of Ben*; we may just consider that whether the possibilities inferred directly from the restricted **(ST)** for Ben* are all true possibilities of Ben*. In other words, we may think that the logic of the inferences from **(ST)** is system T and consider whether the possibilities inferred from the restricted **(ST)** for Ben* in T are all true possibilities of Ben*.

As we explained earlier, Salmon holds that there is in reality a definite threshold separating the possible collections of components from the impossible ones, but epistemically we cannot know where the threshold lies. This view implies that there is a missing link in our cognition of reality. Whether the restricted **(ST)** is correct for the case of Ben* depends on whether it is the case that the threshold separates the possible collections of components from the impossible ones in such a way that all the collections with a smaller amount of change from Ben*s actual matter than the amount set by the threshold are possible collections for Ben*, and all the collections with a greater amount of changes impossible ones. Our concern about inferring from the restricted **(ST)** in the case of Ben* is that the threshold for the tolerable variations in Ben*'s matter may not separate the collections in the way described. In the case of Ben, we see that the threshold eliminates all collections with variations surpassing the threshold as impossible matters for Ben, but not every collection which differs from Ben's actual matter in a smaller amount is a possible matter of Ben. There may be a similar situation concerning Ben*. It may be the case that the threshold of Ben* eliminates all collections which differ from Ben*'s actual matter in a greater amount than the threshold as impossible matter for Ben*, but not every collection which is a subset of the set $\{\mathbf{c}_1, \ldots \mathbf{c}_{120}\}$ and with a smaller amount of variations is a possible matter of Ben*. If this is in fact the case, the inferences from the restricted **(ST)** in modal logic system T may derive false possibility for Ben*. According to Salmon's view, we lack the ability to know what is the threshold. If so, we seem to lack the ability to know how the threshold separates the possible collections from the impossible ones as well. Thus, the correctness of the restricted **(ST)** is at best unknowable for the case of Ben*, if what is the threshold for the tolerable variations in Ben*'s original matter is unknowable in Salmon's sense. The same holds for other cases like the case of Ben*. We conclude that there is no justified reason for using the restricted **(ST)** to infer possible matters for Ben*. The case of Ben* is one in which the question which collections of components could have constituted the artifact has no definite answer, at least from epistemic point of view. Given the above discussion, we think that cases of Ben*'s kind raise no real challenge to S5 modal logic.

Finally we have some comment on Forbes' Counterpart Solution with respect to the restricted **(ST)**. We explained earlier that Forbes holds that there is in reality no sharp cutoff threshold between the collections of components that could have constituted the given artifact and the collections that could not. The more components a collection has in common with the actual matter of the given artifact, the more possible for the given artifact to be constructed from the collection; the less components a collection has in common with the actual matter of the given artifact, the less possible for the given artifact to be constructed from the collection. Given this view, no definite possible matter of Ben* other than the actual matter can be inferred from the Tolerance Principle. In fact, in Forbes' Counterpart Solution to Chisholm's Paradox, the possibilities inferred from the Tolerance Principle are viewed as true in a percentage less than wholly true. With respect to this aspect, it does not really make a difference whether to use **(ST)** or the restricted **(ST)** in the inference. Of course, the Counterpart Solution with **(ST)** will assign a truth value of true in 79 percent to the proposition "Ben* is possible to be constructed from a collection which is qualitative and quantitatively the same as but differs in 21 components from Ben*'s original matter," whereas with restricted **(ST)** the Solution will assign truth value 0 to the same proposition. We have rejected Forbes' view that there is in reality no definite threshold between possible matters and impossible matters by the case of Ben. But we want to consider whether this Forbes' view is correct with respect to the particular kind of cases like the case of Ben*. We hold that the case of Ben* is one for which the threshold for possible matters of Ben* is unidentifiable, but that is not sufficient to show that the threshold does not exist. To show that the threshold does not exist, one needs to show that if there is a threshold, there will be a contradiction. But we fail to see what this contradiction can be, so we are at least not convinced that Forbes' view is correct in Ben*'s case.

On the other hand, even if it is correct that there is no threshold in Ben*'s case, we do not think that it makes a good sense to view necessity and possibility as true in a percentage less than 1. We have argued that it is counter-intuitive to view identity as true in a certain percentage. Similarly we think that possibilities and necessities are either true or false in a certain world, but cannot be true in a certain percentage and at the same time false in a certain percentage in the same world.

If the degree-treatment of possibility has any plausibility, it has to be understood other than attributing real modal properties to artifacts. In our view, the degree-treatment of possibility may mean the following. The intuition that the possible collections of components for Ben* should differ from Ben*'s actual matter in a small amount may support a kind of probabilistic thinking that the more components a collection has in common with the actual matter of Ben*, the more probable for the collection to be a possible matter of Ben*, whereas the less components a collection has in common with the actual matter of Ben*, the less probable for the collection to be a possible matter of Ben*. There is no certainty about which collection is or is not a possible matter of Ben*. If one wishes, one may use a degree-based similarity relation to depict this probabilistic feeling about the collections. But this similarity relation is not supposed to be a technical device for representing identity relation—it is similarity literally. Also, the degree of similarity is not a scientific probability; it can only be taken as an illustration of the intuition that the possible collections for Ben* should differ from Ben*'s actual matter in a small amount.

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