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Measuring the Acceleration of Falling Objects

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Introduction

Earth's gravity pulls all objects toward its center, and near the Earth's surface. Objects in free fall accelerate at 9.8 m/s^2 vertically downward, provided air resistance is negligible. This value of acceleration is often referred to as "*g*". There are many ways to measure this rate of acceleration, and most require a timing device. Typically, stopwatches are the least expensive technology, so these are commonly available in high school science classrooms. With the recent addition of movie cameras on cell phones and digital cameras, another timing tool is available at moderate cost (free if the school policy allows students to use their personal cell phones during class time). The following set of experiments provide four ways to calculate *g* using (*1*) a 10-meter (25-yard) tape measure, (*2*) a brightly colored ball, (*3*) a stop watch, (*4*) a digital camera with movie mode, (*5*) a tripod, and (*6*) free software called *MovieTracker* (http://www.lawrencehallofscience.org/gss/rev/ip/index.html).

The Math of Falling Objects

The following are the kinematic equations of objects falling vertically downward when air resistance is negligible:

$$y_f = y_0 + v_0 * t_f + 0.5 * g * t_f^2 \qquad (1)$$

$$v_f = v_0 + g * t_f \qquad (2)$$

$$v_f^2 = v_0^2 + 2 * g * (y_f - y_0) \qquad (3)$$

where y_0 = starting y position (or height) where y increases downward toward the Earth's surface
y_f = final y position (or height)
v_0 = initial vertical velocity (assumed to be 0 m/s in these experiments
v_f = final vertical velocity
t_f = time the object took to fall from y_0 to y_f.

Question 1: Which of the three equations allows direct calculation of *g* provided the heights and time to fall will be measured?

Activity 1: Rewrite this equation so *g* becomes the dependent variable of the equation. Remember, v_0 = 0 m/s.

Question 2: You may use motion sensors to measure velocities at discrete times to calculate *g* directly. If you have access to one of these sensors, which of the three kinematic equations would you use to calculate *g*?

Collecting the Data

All four experiments may be carried out at the same time - so organizing the experiment ahead of time is essential. Key is to understand how the data will be used from each experiment.

Experiment 1: Using as many stop watches as possible, measure how long it takes for the ball to fall a measured distance.

Experiment 2: Using the digital camera in movie mode, where frames are 1/30 second apart, measure how long it takes for the ball to fall the measured distance.

Experiment 3: Using the digital camera in movie mode and the MovieTracker software, calculate the height of the ball every 1/30 second during its descent. Use the following equation, calculate the average velocity of the falling ball.

$$v_{ave} = (y_i - y_{i-1}) / (t_i - t_{i-1}) \qquad (4)$$

Plot the average velocity versus the time $(t_i + t_{i-1})/2$ and calculate the slope of the best fit line of regression.

Experiment 4: Using the data from Experiment 3 and the following equation, calculate the average g for each set of velocity data.

$$g_{ave} = (v_i - v_{i-1}) / (t_i - t_{i-1}) \qquad (5)$$

So, find a place where a large ball can be dropped from at least 4 meters where a digital camera may film and team of people with stop watches can easily observe when the ball is dropped and when it hits the ground. Place the camera on a tripod so it is steady when filming. Consider a practice drop or two to help those using stop watches to be consistent when to start and stop their timers.

Question 3: How many ball drops would be best to calculate

Analyzing the Data

Consider the Uncertainty of the Data

Very few data in this world are perfectly accurate (an example of perfectly accurate data would be the number of people in your classroom at a given time), especially those based on tools that have a finite precision. In these experiments in calculating the magnitude of g, a number of the tools had limited precision. Several good articles about uncertainty and propagating uncertainty exist on the web. Suggested articles are:

Measurement Good Practice Guide
(http://www.wmo.int/pages/prog/gcos/documents/gruanmanuals/UK_NPL/mgpg11.pdf)

Uncertainties and Error Propagation - includes questions with answers section
(http://www.rit.edu/cos/uphysics/uncertainties/Uncertaintiespart1.html)

Examples of Uncertainty Calculations
(http://spiff.rit.edu/classes/phys273/uncert/uncert.html)

Notes on Data Analysis and Experimental Uncertainty
(http://courses.washington.edu/phys431/uncertainty_notes.pdf)

Activity #: List the variables measured during the experiment, then estimate the precision of the tool used to measure the variable. Also estimate the uncertainty introduced by the limitations of the operator.

Variable Tool Precision Uncertainty due to Operator

Four sets of results of the value of g have been calculated, but how "good" is each technique - which technique produced the most accurate and precise measurements?

Rules for the Propagation of Uncertainty

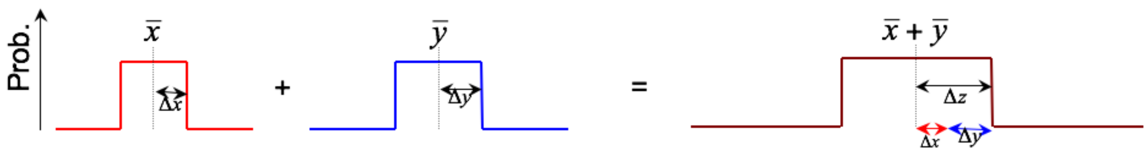
Function	**Rectangular Distribution**	**Normal (Gaussian) Distribution**
$z = x + a$ *where* a = constant	$\Delta z = \Delta x$	$\Delta z = \Delta x$
$z = b*x$ *where* b = constant	$\Delta z = \|b\| * \Delta x$	$\Delta z = \|b\| * \Delta x$
$z = b*x^2$ *where* b = constant	$\Delta z / z = 2 * \Delta x / x$	$\Delta z / z = 2 * \Delta x / x$
$z = b*x^n$ *where* b = constant	$\Delta z / z = \|n\| * \Delta x / x$	$\Delta z / z = \|n\| * \Delta x / x$
$z = x + y$	$\Delta z = \Delta x + \Delta y$	$\Delta z = \sqrt{}(\Delta x^2 + \Delta y/^2)$

z = x - y	$\Delta z = \Delta x + \Delta y$	$\Delta z = \sqrt{(\Delta x^2 + \Delta y/^2)}$
z = x * y	$\Delta z/z = \Delta x/x + \Delta y/y$	$\Delta z/z = \sqrt{((\Delta x/x)^2 + (\Delta y/y)^2)}$
z = x / y	$\Delta z/z = \Delta x/x + \Delta y/y$	$\Delta z/z = \sqrt{((\Delta x/x)^2 + (\Delta y/y)^2)}$

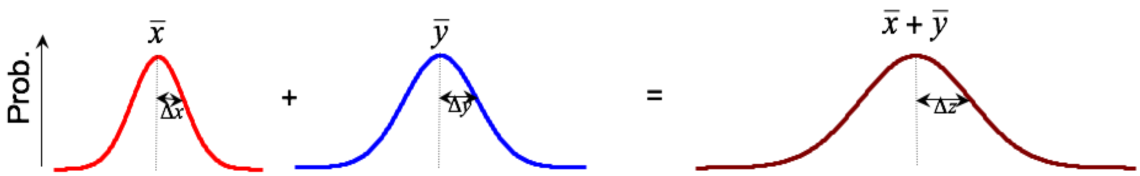
Where

Δ = uncertainty

Rectangular Distribution



Gaussian Distribution



Calculating the *Total* Uncertainty using LINEST Data and Error Bars

Uncertainty values (*U*) of *slope* (m) and *y-intercept* (b) using LINEST in Excel (see http://www.colby.edu/chemistry/PChem/notes/linest.pdf for a thorough review of LINEST) are the uncertainties based on the scatter of the data points about the regressed line (graph below):

Um_{LINEST} = 0.306 and Ub_{LINEST} = 0.177 where the LINEST output was

LINEST Data for v vs t

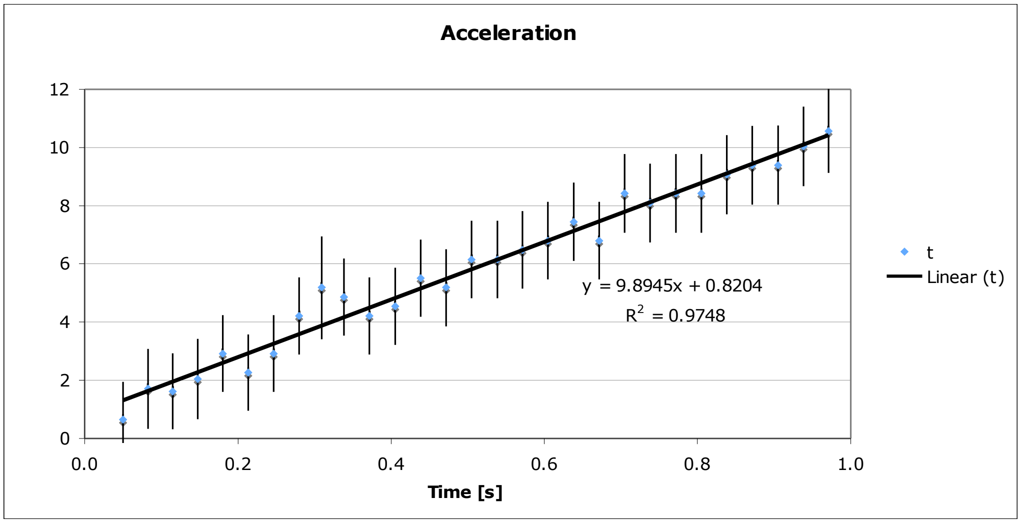
9.895	0.820
0.306	0.177
0.975	0.454
1043	27
215	5.574

Use the velocity versus time graph with the error bars *that are based on the uncertainties in your original data that you have propagated through your calculations* (sample below) to calculate and draw the maximum and minimum sloped lines that fall within the majority of data point's error bars.

a) Plot two points (select 2 representative points and add/subtract the associated error) to find the maximum and minimum slopes,

b) Plot a trend lines through those two sets of points, and

c) Add two additional data series to the graph in Excel. The difference of the lines' slopes and y-intercepts represent the *systematic* uncertainty of your experiment.



$Um_{systematic} = (m_{max} - m_{min})/2$ and $Ub_{systematic} = (b_{max} - b_{min})/2$

3) Calculate the total uncertainty in slope and y-intercept:

$Um_{total} = \sqrt{(Um_{LINEST}{}^2 + Um_{systematic}{}^2)}$

$Ub_{total} = \sqrt{(Ub_{LINEST}{}^2 + Ub_{systematic}{}^2)}$

4) Check to see if expected value was within the range of values based on the total uncertainties:

$m_{calculated} - Um_{total} < m_{expected} < m_{calculated} + Um_{total}$

$b_{calculated} - Ub_{total} < b_{expected} < b_{calculated} + Ub_{total}$