UNIVERSITY OF LEEDS

This is a repository copy of *Decentralized Resource Allocation for Heterogeneous Cellular Networks*.

White Rose Research Online URL for this paper:
http://eprints.whiterose.ac.uk/118548/

Version: Accepted Version

Proceedings Paper:

Oguejiofor, O orcid.org/0000-0002-4989-8734, Zhang, L and Alhabo, M (2017) Decentralized Resource Allocation for Heterogeneous Cellular Networks. In: 2017 International Symposium on Wireless Communication Systems (ISWCS). 2017 International Symposium on Wireless Communication Systems (ISWCS), 28-31 Aug 2017, Bologna, Italy. IEEE , pp. 158-163. ISBN 978-1-5386-2913-0

https://doi.org/10.1109/ISWCS.2017.8108102

(c) 2017 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

Reuse
Unless indicated otherwise, fulltext items are protected by copyright with all rights reserved. The copyright exception in section 29 of the Copyright, Designs and Patents Act 1988 allows the making of a single copy solely for the purpose of non-commercial research or private study within the limits of fair dealing. The publisher or other rights-holder may allow further reproduction and re-use of this version - refer to the White Rose Research Online record for this item. Where records identify the publisher as the copyright holder, users can verify any specific terms of use on the publisher's website.

Takedown
If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.

White Rose
university consortium
Universities of Leeds, Sheffield & York

eprints@whiterose.ac.uk
https://eprints.whiterose.ac.uk/

Decentralized Resource Allocation for Heterogeneous Cellular Networks

Obinna Oguejiofor, Li Zhang and Mohanad Alhabo
School of Electronic and Electrical Engineering
University of Leeds, Leeds. LS2 9JT United Kingdom.
Email: {eloso, l.x.zhang, elmdj}@leeds.ac.uk

***Abstract*—Heterogeneous cellular Network (HetNet) is a promising technology for 5th generation mobile networks (5G) that can potentially improve spatial resource reuse and extend coverage, therefore allowing it to achieve significant higher data rates than single tier networks. However, the performance of HetNet is limited by co-channel (inter-UE, inter-cell) interference. Hence, resource allocation is carefully done in this paper to ensure that the likely loss in achievable data rate due to interference doesn't diminish the gain in the achievable data rate due to higher spatial reuse. The resources which we consider in this paper are the spatial resource (unit-beamformer) and the power resource. We formulate our distributed spatial resource allocation problem as a quadratic optimization problem with non-convex quadratic constraints and solved it by exploiting stationarity karush-Kuhn-Tucker (KKT) conditions. While our proposed power resource allocation scheme is formulated as a convex optimization problem and is solved by exploiting karush-Kuhn-Tucker (KKT) conditions. Simulation results of our proposed method when compared with other existing methods show significant improvement.**

I. INTRODUCTION

Resource allocation (RA) is all about how the best radio resources such as frequency, time, transmit powers, spatial directions (unit norm beamformers), e.t.c., can be properly allocated to user equipments (UEs) in a system in order to maximize the system spectral efficiency (SE). RA is especially important in system such as heterogeneous network (HetNet) [1] which is limited by co-channel interference rather than noise [2]. The basic problem facing RA is the issue of coupling among UEs. UEs are coupled due to interference (inter-UE, inter-cell) and power constraints. This paper is focused on the optimal distributed resource allocation procedure in two-tier HetNet such that UEs in the cell range expansion (CRE) [3], [32] area of the pico cells will not experience a loss in throughput due to the higher level of interference received from the macro base station (MBS). By decentralized RA, we mean that the RA algorithm is computed at each BS without exchanging or sharing control variables or channel state information, unlike in a centralized system. In homogeneous cellular network, UE is usually served and connected to the strongest base station in downlink hence interference from other signals are received with a lower power than the desired signal. In contrast and in order to enable cell splitting gain, some UEs in HetNet may be served and connected to the strongest base station (BS) in uplink (i.e low powered BS) even though the received power from an MBS could be higher [4]. This method of cell selection in HetNet always cause high level of interference from the MBS to such UEs which are usually located at the CRE of the low powered BS. Apart from this, they also suffer from enormous signal attenuation from their home (serving) BS. These problems therefore cause them to exhibit poorer performance than the interior UEs thereby degrading the system aggregate sum-rate.

One of our objectives in this paper is to find ways to manage interference experience among UEs in HetNet effectively. Many Inter-cell Interference Coordination (ICIC) [5]–[7] schemes in long term evolution (LTE) have been proposed. In LTE releases 8 and 9, fractional frequency reuse [8] is proposed to deal with interference affecting cell-edge UEs. In LTE-Advanced Releases 10 and 11, multiple carrier components (CCs) [9], [10] were introduced and the proposed techniques were categorized into time, frequency and power domain. Traditionally, interference is mitigated by assigning all links orthogonal resources in frequency, time or code. This methods decouples all interferences from the links. However, this comes at the expense of the achievable SE of the system. Because of the high demand for data rate and scarcity of spectrum, universal frequency reuse [11] have been an attractive strategy considered for future generation mobile networks. In this perspective, we introduce an alternative solution to the problem of interference in HetNet based on the use of multi-antenna technology to suppress the leakage powers to other UEs in the system. The inter-UE and inter-cell interferences are reduced using the directivity of the antenna and this approach increases the SE of the network if the spatial dimension is utilized to serve UEs in parallel. Note, that increased radio network capacity can be achieved by improving the SE of the network. Coordinated multi-point (CoMP) [12]–[15] is a multi-antenna inter-cell cooperation technology that mitigates inter-cell interference and increases the rates of UEs at the cell edge by allowing both the UE's serving cell and other cooperating cells to communicate with these UEs simultaneously. We differ a little from this approach which usually required data to be shared and synchronized among cells in HetNet. In our case all cells in the HetNet will not transmit data to this UE simultaneously. Rather each BS will transmit data to its served UEs but might as well cause interference to other UEs in the network. Also the physical (PHY) layer will be based on space division multiple access (SDMA) which enables spatial separation of co-channel UE waveforms.

A. *Prior Works and Contributions*

Some notable works have considered interference leakage suppression in single cell [16]–[18], or single-tier multiple cells [19]–[22]. In these aforementioned works, the spatial resource allocation problem solved by them and some literatures cited therein are different from the one we are solving in this paper in terms of the objective behind the resource allocation. Furthermore, their unit-norm beamformers are obtained by maximizing the signal-to-leakage-and-noise ratio which is usually formulated as a generalized Rayleigh quotient. Consequently the eigenvector corresponding to the largest eigenvalues gives the optimum solution of the optimization problem. We differ from this method by formulating our spatial RA optimization problem as quadratic optimization problem with non-convex quadratic constraints. Which aim to minimize the total leakage caused to other UEs in the system while satisfying a fixed received power for the desired UE when transmitting to it. Also our methods are tailored to underlay HetNets [23] which have more

dominant interference scenarios than single-tier networks which are considered by other works. Also, HetNet has different propagation characteristics, deployments and cell selection procedures compared with single-tier cellular networks which will necessitate additional simulation parameters during simulation. We also differ from these authors and others cited in current literatures on how we formulate and obtain the powers that will be allocated to UEs in the system.
HetNet favours coordinated processing but done in a distributed fashion unlike CoMP transmission [14]. Each BS will make RA decisions and be sure that no uncoordinated interference exist from the cell. Our distributed spatial resource allocation problem which is informally formulated as selecting the unit-norm beamformers that will cause the least total leakage power from each transmitter subject to a receive signal power threshold at each UE, can be implemented without the requirement of any exchange among the cells in HetNet provided that time division duplex (TDD) based local channel state information (CSI) is available at each BS. Our proposed power resource allocation scheme is formulated as convex optimization problem and solved by exploiting karush-Kuhn-Tucker (KKT) conditions. The resources which we consider as the optimization variables in this paper are the powers and spatial (beamforming) directions. These are selected and assigned/allocated by each BS to UEs in its coverage in order to satisfy UEs at CRE with the minimum quality of experience (QoE)[1] [24] and improve the overall SE of the system.

B. Paper organization

The rest of this paper is organized as follows. In section II we present the system model of the considered HetNet. Section III presents the optimization problem formulation for the spatial resource and power resource allocations and how they are solved. Simulation results and discussions are provided in section IV, and conclusions are given in the last section.
Notations: $(\cdot)^H$ is the transpose-conjugate operation, $(\cdot)^T$ is the transpose operation, $||\cdot||_2$ denotes the Euclidean norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{E}\{\cdot\}$ is the statistical expectation over a random variable, Tr($\mathbf{X}$) denotes the the trace of a square matrix $\mathbf{X}$ and **card**($\mathcal{D}$) denotes the cardinality of set $\mathcal{D}$. We use upper-case boldface letters for matrices and lower-case boldface for (column) vectors and either upper-case or lower-case letters without boldface for scalars.

II. SYSTEM MODEL

We consider the downlink of a two-tier HetNet with P pico cells underlaid in a single macro-cellular coverage, making it a total of K_t cells in the system. All cells in the HetNet uses the same carrier frequency as the macro-cell. The jth BS is denoted BS$_j$ which can be any of the BSs (PBS or MBS) and is assumed to have N antennas with which it serves U UEs with single receive antenna[2] each. The set of UEs served by BS$_j$ is denoted by $\mathcal{G}_j \subseteq \{1,\ldots,U\}$ while the set of UEs that BS$_j$ causes interference to in the network is denoted $\mathcal{C}_j \subseteq \{1,\ldots,\bar{U}\}$. We assume that BS$_j$ knows the CSI of all UEs in $\mathcal{C}_j$ while the CSI of any UE $i \notin \mathcal{C}_j$ and interfered by BS$_j$ is assumed to be negligible and need not to be known, rather is treated as Gaussian noise. The complex-baseband received data signal at UE u is $y_u \in \mathbb{C}$ and given by

$$y_u = \sum_{j=1}^{K_t} \sqrt{g_{j,u}} (\mathbf{h}^s_{j,u})^H \mathbf{x}_j + n_u, \tag{1}$$

where $\sqrt{g_{j,u}}$ is the large-scale path-loss from BS$_j$ to UE u. Also $\mathbf{h}^s_{j,u} \in \mathbb{C}^{N\times 1}$ is the small scale (fading) channel vector from BS$_j$ to UE u. The downlink channel matrix from BS$_j$ to all its served UEs in the same cell is given by

$$\mathbf{H}_j = \begin{bmatrix} \mathbf{h}^H_{j,1} \\ \vdots \\ \mathbf{h}^H_{j,U} \end{bmatrix} \in \mathbb{C}^{U\times N} \tag{2}$$

where $\mathbf{h}^H_{j,u} \triangleq \sqrt{g_{j,u}}\mathbf{h}^s_{j,u}$ represent the rows of $\mathbf{H}_j$. Similarly, $\bar{\mathbf{H}}_j \in \mathbb{C}^{\bar{U}\times N}$ represent channel matrix towards UEs $\{k : k \in \mathcal{C}_j\}$ which BS$_j$ interferes. Also, we define this channel matrix $\bar{\mathbf{H}}_{j,u} = [\mathbf{h}_{j,1},\ldots,\mathbf{h}_{j,u-1},\mathbf{h}_{j,u+1},\ldots,\mathbf{h}_{j,U},\bar{\mathbf{H}}^T_j]^T \in \mathbb{C}^{(U-1+\bar{U})\times N}$ as the channel from BS$_j$ to its U-1 served UEs other than UE u as well as the $\bar{U}$ UEs $\in \mathcal{C}_j$. While $n_u \in \mathbb{C}$ is the additive noise from the surrounding and is modelled as circularly symmetric complex Gaussian, distributed as $n_u \sim \mathcal{CN}\ (0, \sigma^2)$, where σ^2 is the variance of the noise. $\mathbf{x}_j \in \mathbb{C}^{N\times 1}$ is the transmit signal vector from BS$_j$ in each cell with average power constraint $q_j = \mathbb{E}[\mathrm{Tr}(\mathbf{x}_j\ \mathbf{x}^H_j)]$. To enable spatial separation of data symbols s_u from BS$_j$ to UEs $u \in \mathcal{G}_j$, the transmitted signal vector is represented as a linear function of the symbols or linear combination of the beamforming vectors in the form

$$\mathbf{x}_j = \sum_{u\in\mathcal{G}_j} \mathbf{w}_u s_u, \tag{3}$$

where $\mathbf{w}_u \in \mathbb{C}^{N\times 1}$ corresponds to the transmit beamformers for each symbol meant for the UE u.

III. RESOURCE ALLOCATION

Resource allocation (RA) involves strategies and algorithm for controlling and sharing radio resource parameters such as frequency, time, transmit powers and spatial directions among UEs in the HetNet to maximize the system SE. The critical problem in RA facing HetNet is the issue of interference (inter-cell interference and inter-UE interference). This paper aims at allocating powers and spatial (beamforming) directions optimally to UEs such that UE in CRE will not experience loss in throughput due to the higher level of interference received from the MBS. Traditionally BS$_j$ unilaterally makes resource allocation decisions by allocating spatial directions and powers to its served UEs. Any resource allocation made without due consideration to UEs $\in \mathcal{C}_j$ will certainly diminish the SE gains in the network. We solve our RA optimization problem for spatial directions and powers for UEs in different steps not jointly.

A. Problem formulation

This section aims at designing fixed distributed beamforming directions that will spatially seperate the data symbols sent to UEs in each cell. This will spatially control the inter-UE interference caused in each cell and the interference caused to UEs $\in \mathcal{C}_j$, hence, implicitly solving the problem of inter-cell interference caused in the HetNet.

BS$_j$ serves UEs in $\mathcal{G}_j$, while coordinating interference towards UEs in $\mathcal{C}_j$. By coordinating interference, we mean that the propagation channels from BS$_j$ towards these set of UEs are also considered as input to the beamformer design algorithm in BS$_j$ during the design of its beamformers. We formulate our spatial RA problem

---

[1] QoE is a subjective measure of the quality of service (QoS) provided by the network operator and perceived by end-users. It is related to QoS but differs in the sense that, in QoS, the measure of the service provided for the end-users is solely determined by the network operator or service provider for the overall value of the service provided.

[2] We limit each UE to have a single antenna for practical reasons, such as, reducing the UE hardware complexity, it requires less CSI knowledge at the transmitter and also preserving of battery life.

informally as selecting the optimal beamformers that will cause the least interference to UEs in the same cell and UEs $\in \mathcal{C}_j$ while fulfilling the desired received power constraint (threshold). This threshold is not constant for every UE but will depend on the propagation characteristics of the cell. Assuming BS$_l$ is the serving BS of UE u, the desired signal received at UE u is

$$y_u^{des} = \mathbf{h}_{l,u}^H \mathbf{w}_u s_u, \tag{4}$$

while the leakage signal $\mathbf{y}_u^{leak} \in \mathbb{C}^{(U-1+\bar{U})}$ directed away from this UE is given by

$$\mathbf{y}_u^{leak} = \bar{\mathbf{H}}_{l,u} \mathbf{w}_u s_u. \tag{5}$$

Mathematically, the optimization problem can be stated as

$$\mathbf{w}_u^{opt} = \underset{\{\mathbf{w}_u\} \forall u \in \mathcal{G}_l}{\text{argmin}} \sum_{u \in \mathcal{G}_l} ||\mathbf{y}_u^{leak}||^2, \tag{6}$$

subject to

$$|y_u^{des}|^2 = \tau_u \quad \forall u \in \mathcal{G}_l. \tag{7}$$

To elucidate the optimization problem in beamforming terms, (6) and (7) will be stated as

$$\mathbf{w}_u^{opt} = \underset{\{\mathbf{w}_u\}}{\text{argmin}} \sum_{u \in \mathcal{G}_l} \mathbf{w}_u^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_u, \tag{8}$$

subject to

$$\mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u = \tau_u \quad \forall u \in \mathcal{G}_l. \tag{9}$$

The constraint for the received signal power for each UE in each cell can be defined as $\tau_u \triangleq \frac{q_l}{U}\, \text{Tr}(\mathbf{R}_{l,u})$. Where $\frac{q_l}{U}$ represent fixed uniform power allocation to all UEs in each cell and $\text{Tr}(\mathbf{R}_{l,u})$ gives the sum of the diagonal of the array covariance matrix of UE u. We assume that the different data symbols are uncorrelated and have normalized power $\mathbb{E}[|s_u|^2] = 1$, also $\mathbf{R}_{l,u} = \mathbf{h}_{l,u}\mathbf{h}_{l,u}^H$ is the rank one array covariance matrix for the desired UE. While $\bar{\mathbf{R}}_{l,u} = \bar{\mathbf{H}}_{l,u}^H \bar{\mathbf{H}}_{l,u}$ is the rank one array covariance matrix for UEs affected by the leakage power. Both $\mathbf{R}_{l,u}$ and $\bar{\mathbf{R}}_{l,u}$ are positive definite (PD) matrices which means that $\mathbf{w}_{l,u}^H \mathbf{R}_{l,u} \mathbf{w}_{l,u} > 0$ and $\mathbf{w}_{l,u}^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_{l,u} > 0$. In what follows, we show detailed analysis on how the optimal beamformers can be obtained. The optimal beamformer solutions can be computed by solving the following non-convex problem

$$\begin{aligned} \underset{\{\mathbf{w}_u\}}{\text{minimize}} \quad & \sum_{u \in \mathcal{G}_l} \mathbf{w}_u^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_u, \\ \text{subject to} \quad & \mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u = \tau_u \ \forall u \in \mathcal{G}_l. \end{aligned} \tag{10}$$

It is non-convex because only affine functions[3] are allowed to have equality constraints. But $\mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u$ is a quadratic function with a PD matrix $\mathbf{R}_{l,u}$ which makes it a convex function. Therefore, the equality constraint in (10) makes the optimization problem non-convex [25].

We obtain the Lagrangian function of (10) as

$$\mathcal{L}(\mathbf{w}_u, \beta_u) = \sum_{u \in \mathcal{G}_l} \mathbf{w}_u^H \bar{\mathbf{R}}_{l,u} \mathbf{w}_u - \sum_{u \in \mathcal{G}_l} \beta_u (\mathbf{w}_u^H \mathbf{R}_{l,u} \mathbf{w}_u - \tau_u), \tag{11}$$

where $\beta_u \geq 0$ is the Lagrange multiplier associated with τ_u. To solve (11), we exploit the stationarity karush-kuhn-tucker (KKT) conditions [26] which say that $\partial \mathcal{L}/\partial \mathbf{w}_u = \mathbf{0}$, at the optimal solution. The outcome of this derivative yields the following relationship

$$(\bar{\mathbf{R}}_{l,u} - \mathbf{R}_{l,u}\beta_u)\mathbf{w}_u = \mathbf{0} \ \forall u, \tag{12}$$

Note, that if $(\bar{\mathbf{R}}_{l,u} - \mathbf{R}_{l,u}\beta_u)$ in (12) is not a PD matrix, then it is possible to get a set of $\{\mathbf{w}_u\}$ that will give unbounded direction, which could cause the dual function not to have a finite value but tend towards -∞. By adding $\mathbf{R}_{l,u}\mathbf{w}_u$ to both sides of (12) and simplifying further gives us the following relationship

$$\bar{\mathbf{R}}_{l,u}\mathbf{w}_u + \mathbf{R}_{l,u}\mathbf{w}_u = \mathbf{R}_{l,u}\beta_u\mathbf{w}_u + \mathbf{R}_{l,u}\mathbf{w}_u. \tag{13}$$

Further regrouping of terms in (13) yields

$$(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})\mathbf{w}_u = \beta_u \left(1 + \frac{1}{\beta_u}\right) \mathbf{R}_{l,u}\mathbf{w}_u. \tag{14}$$

We decompose parameter "$\mathbf{R}_{l,u}$" in the right hand side (RHS) of (14) to get this relationship

$$(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})\mathbf{w}_u = \mathbf{h}_{l,u}\beta_u \left(1 + \frac{1}{\beta_u}\right) \mathbf{h}_{l,u}^H\mathbf{w}_u. \tag{15}$$

Finally our beamforming vector can be expressed as

$$\mathbf{w}_u = (\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u} \underbrace{\beta_u \left(1 + \frac{1}{\beta_u}\right) \mathbf{h}_{l,u}^H\mathbf{w}_u}_{\text{scalar term}}. \tag{16}$$

In (16) the scalar term is a single value and can be ignored because it can only contribute to the magnitude but doesn't affect the direction of the beamformer. Therefore, the unit norm beamforming vectors $\{\tilde{\mathbf{w}}_1 \cdots \tilde{\mathbf{w}}_U\}$ are

$$\tilde{\mathbf{w}}_u = \frac{(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u}}{||(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u}||} \quad \forall u \in \mathcal{G}_l. \tag{17}$$

B. Power Allocation

Since the major interference problem has been tackled in the previous section by designing unit-norm beamformers $\{\tilde{\mathbf{w}}_u\} \forall u \in \mathcal{G}_l$ that will spatially separate data symbols when transmitting to UEs. Any negligible interference in the system will be modelled as part of the background noise. What is left to be done is to select the power allocation coefficient $\{p_u\} \forall u \in \mathcal{G}_l$ which will act as optimum scale factors to each spatial directions $\{\tilde{\mathbf{w}}_u\} \forall u \in \mathcal{G}_l$ in order to maximize the SE of the system as well as satisfying each UE with a minimum QoE. We propose a power allocation scheme, which will maximize the sum-rate of the system.

Note, the relationship between the power allocation coefficients and the beamforming directions is given as

$$\mathbf{w}_u = \sqrt{p_u}\tilde{\mathbf{w}}_u \quad \forall u \in \mathcal{G}_l, \tag{18}$$

we proceed by formulating the first power RA problem that will maximize the sum-rate of each cell as

$$\begin{aligned} \underset{\{p_u\} \forall u \in \mathcal{G}_l}{\text{minimize}} \quad & -\sum_{u \in \mathcal{G}_l} log_2 \left(1 + p_u \frac{|\mathbf{h}_{l,u}^H \tilde{\mathbf{w}}_u|^2}{\sigma_u^2}\right) \\ \text{subject to} \quad & \sum_{u \in \mathcal{G}_l} p_u = q_l, \\ & p_u \geq 0 \ \ \forall u \in \mathcal{G}_l. \end{aligned} \tag{19}$$

Where the utility function represents the sum-rate achievable by UEs in each cell, q_j is the power limit at BS$_j$. The power RA problem is convex [26], therefore can be solved efficiently. We obtain the Lagrangian function of (19) as

$$\begin{aligned} \mathcal{L}(p_u, \lambda_u, \nu_j) = & -\sum_{u=1}^{U} log_2 (1 + p_u\rho_u) \\ & + \nu_j \left(\sum_{u \in \mathcal{G}_j} p_u - q_j\right) - \sum_{u=1}^{U} \lambda_u p_u. \end{aligned} \tag{20}$$

---

[3] A function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be affine if its domain is an affine set, and if, for all x, y $\in \mathbb{R}^n$ and $\theta \in \mathbb{R}$, $f(\theta x + (1-\theta)y) = \theta f(x) + (1-\theta)f(y)$.

where $\rho_u = \frac{|\mathbf{h}_{l,u}^H \tilde{\mathbf{w}}_u|^2}{\sigma_u^2}$ represents the signal to noise ratio (SNR) and $\lambda_u \geq 0$ is the Lagrange multiplier associated with UE u power limit (lower bound), while ν_j is the Lagrange multiplier associated with BS$_j$ power limit. To solve (19), we exploited the KKT optimality conditions which are

$$\sum_{u\in\mathcal{G}_j} p_u = q_j, \tag{21a}$$

$$p_u \geq 0 \quad \forall u, \tag{21b}$$

$$\lambda_u \geq 0 \quad \forall u, \tag{21c}$$

$$\lambda_u p_u = 0 \ \ \forall u, \tag{21d}$$

$$-\frac{\rho_u}{(1+p_u\rho_u)\ln 2} + \nu_j - \lambda_u = 0, \ \forall u. \tag{21e}$$

Where (21a) to (21e) represent primal feasibility, primal feasibility, dual feasibility, complementary slackness, and stationarity conditions respectively. We can easily prove that strong duality holds for this problem because the objective and constraint functions are convex and differentiable, also slater's constraint qualification [27] is satisfied. Therefore, KKT conditions are both necessary and sufficient for the optimal solution of this power RA problem. We proceed further by rearranging terms in (21e) and noting that λ_u performs as a slack variable which can easily be eliminated. Consequently, we form an equivalent representation of (21d) and (21e) as

$$p_u\left(\nu_j - \frac{\rho_u}{(1+p_u\rho_u)\ln 2}\right) = 0 \ \forall u \in \mathcal{G}_l \tag{22a}$$

$$\nu_j \geq \frac{\rho_u}{(1+p_u\rho_u)\ln 2} \ \forall u \in \mathcal{G}_l. \tag{22b}$$

The inequality in (22b) should also hold with equality in order not to violate the complementary slack condition. As a consequence, we establish the following relationships

$$p_u = \frac{1}{\grave{\nu}_j} - \frac{1}{\rho_u} \ \forall u, \tag{23}$$

where $\grave{\nu}_j = \nu_j \ln 2$. From (23) one can observe that the optimal power coefficients $\{p_u\}\ \forall u$ is dependent on the SNR $\{\rho_u\}\ \forall u$ of individual UE channels. If $\grave{\nu}_j < \rho_u\ \forall u \in \mathcal{G}_l$, positive values of p_u will be allocated to UEs whose channel SNRs are $\rho_u\ \forall u$ else non-positive values of p_u will be allocated which is not proper. Therefore the power RA problem is solved by

$$p_u = \begin{cases} \frac{1}{\grave{\nu}_j} - \frac{1}{\rho_u}, & \grave{\nu}_j < \rho_u, \\ 0, & \grave{\nu}_j \geq \rho_u. \end{cases} \tag{24}$$

We can also find the Lagrange multipliers $\grave{\nu}_j$ by rearranging some terms in (23) which will give us this relationship

$$\grave{\nu}_j = \left(\frac{q_j + \sum_{u\in\mathcal{G}_j}^{U} \frac{1}{\rho_u}}{U}\right)^{-1}. \tag{25}$$

This power RA will allocate powers to UEs based on individual channel gain. At high SNR, the values of $\frac{1}{\rho_u}$ are far less compared to $\grave{\nu}_j$, thus uniform power is allocated to each UE, while at low SNR, the values of $\frac{1}{\rho_u}$ are far more compared to $\grave{\nu}_j$, hence full power is allocated to the UE with the best channel.

C. Achievable rates for UEs in HetNet

We want to calculate the achievable data rate for each UE after allocating the spatial directions and powers accordingly. The data signal received at UE u is given by

$$y_u = y_u^{des} + y_u^{int} + n_u, \tag{26}$$

Algorithm 1 Distibuted Allocation of spatial directions and powers for each UE in HetNet

```
Input and variables
$\mathcal{G}_j$ : set of UEs served by BS$_j$;
$\bar{\mathbf{R}}_{l,u}$ : array covariance matrix for UEs affected by leakage;
$\bar{\mathbf{R}}_{l,u}$ : covariance matrix for the desire UE served by BS$_l$ ;
$U$ : total number of UEs in each cell;
$\rho_u$ : SNR of UE $u$ ;
$\nu_u$ : Lagrange multiplier associated with BS$_l$ power limit;
procedure
for UEs $\in \mathcal{G}_j$ i.e. u = 1 to $U$ do
    compute $\mathbf{w}_u$ from $(\bar{\mathbf{R}}_{l,u} + \mathbf{R}_{l,u})^{-1}\mathbf{h}_{l,u}$ using (16);
    obtain the unit-norm beamformers $\tilde{\mathbf{w}}_u$ using (17);
    compute $p_u$ from $\frac{1}{\grave{\nu}_j} - \frac{1}{\rho_u}$ using (23) and;
end for
BS$_j$ transmits $\mathbf{x}_j = \sum_{u\in\mathcal{G}_j}^{U} \sqrt{p_u}\tilde{\mathbf{w}}_u s_u$
```

where y_u^{des}, y_u^{int} and n_u represent the desired signal which is obtained by combining (4) and (18), interference signal and noise respectively. The received interference is given by

$$\begin{aligned} y_u^{int} =& \sum_{k\in\mathcal{G}_l, k\neq u}^{U} \mathbf{h}_{l,u}^H \sqrt{p_k}\tilde{\mathbf{w}}_k s_k \\ &+ \sum_{j\neq l}^{K_t} \sum_{k\in\mathcal{G}_j}^{U} \mathbf{h}_{j,u}^H \sqrt{p_k}\tilde{\mathbf{w}}_k s_k, \end{aligned} \tag{27}$$

these are signals that are destined for other UEs apart from the desired UE in HetNet. The first term in (27) is the inter-UE interference while the second term is the inter-cell interference. The achievable data rate for UE u is given by

$$r_u = log2\left(\frac{G_{y_u^{des}}}{G_{y_u^{int}} + G_{n_u}}\right) \ \forall u, \tag{28}$$

where G denotes the power aspect of the signal received.

D. Simulation setting

We consider a simple simulation setting with randomly distributed PBSs deployed at hotspot locations in the coverage area of MBS as illustrated in Fig 1. The minimum distance among pico sites is set to 40m, and we assume that all PBSs are not geometrically separated, hence interference among PBS is possible and therefore considered. The minimum distance from the macro site to the pico sites is 75m. We assume that the UEs in the HetNet are randomly distributed and are located at the CRE such that each UE will receive significant intercell interference (ICI). The UEs served by PBS are uniformly distributed between 35m and 55m from the PBS. Similarly, the UEs served by MBS are uniformly distributed between 220m and 260m from the MBS, also, the distance between the macrocell UEs and the PBS is roughly between 40m and 45m, while the distance between the picocell UEs and the MBS is between 230m and 270m. Other system parameters are also based on the 3GPP simulation baseline parameters and can be found in [29]. The total BS transmit powers for MBS and PBS are 46dBm and 30dBm respectively, while the receiver noise power is -75dBm, assuming a 10MHz bandwidth. The channel vector between BS$_j$ and UE u is generated by this formulation $\mathbf{h}_{j,u}^H \triangleq \sqrt{g_{j,u}}\mathbf{h}_{j,u}^s$, where $\sqrt{g_{j,u}}$ is the large-scale pathloss from BS$_j$ to UE u, also $\mathbf{h}_{j,u}^s \in \mathbb{C}^N$ is the small scale (fading) channel vector from BS$_j$ to UE u and is zero-mean complex gaussian distributed with

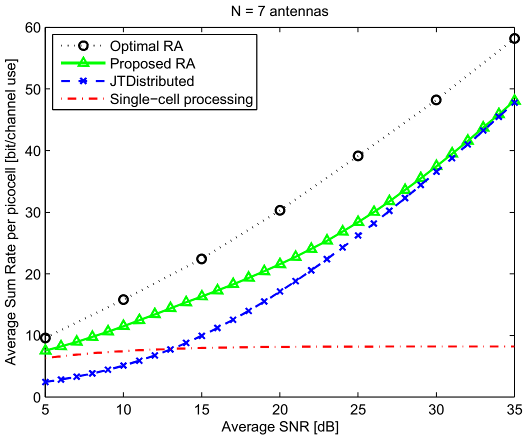

Fig. 1. Average sum-rate as a function of SNR for different RA strategies, when $N = 7$, $U = 4$ and $\mathbf{card}(\mathcal{C}_j) = 3$ (i.e., 2 macro-cell UEs and one adjacent pico-cell UE).

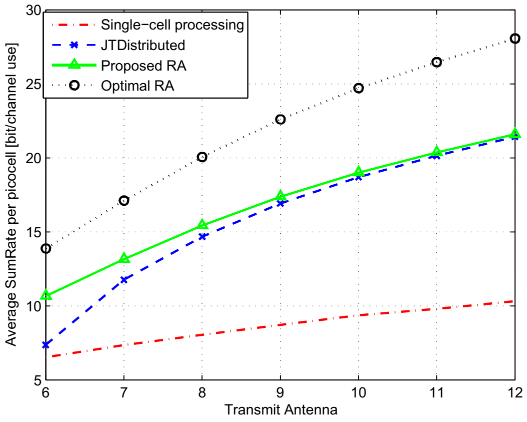

Fig. 2. Average sum-rate as a function of transmit antenna for different RA strategies, when $SNR = 10dB$, $U = 6$ and $\mathbf{card}(\mathcal{C}_j) = 3$ (i.e., 2 macro-cell UEs and one adjacent pico-cell UE).

covariance $\mathbf{R}$, or $\mathbf{h}^s_{j,u} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, and the large scale pathloss in linear scale is expressed as

$$\sqrt{g_{j,u}} = \frac{\psi}{d^n_{j,u}}, \tag{29}$$

where ψ is a constant which accounts for system losses, n is the path-loss exponent, typically $n > 3$, while $d_{j,u}$ is the distance between BS$_j$ and UE k. The large-scale path loss model in dB for the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6log(\frac{d_{j,u}}{10^3})$ and $PL(dB) = 140.7 + 36.7log(\frac{d_{j,u}}{10^3})$. This simulation settings will be used except otherwise indicated.

E. centralized vs decentralized

HetNet favours coordinated processing, but should be done in a distributed fashion to enable practicability and also to avoid computational complexity. Our proposed RA method is computed in a distributed fashion by BS$_j$ using only local CSI whereas the optimal RA depicted in Fig. 1 utilizes the B&B method [30] which favours coordinated processing but is implemented in a centralized fashion at a super BS that has the aggregate CSI of all BS in HetNet. B&B method is practically infeasible for large scale networks because of high computational complexity. Considering the trade off between performance and computational complexity and also, possible hardware failures which might lead to coordination failure for a centralized scheme, our distributed RA is hereby recommended. We also compare our proposed RA strategy with the Joint transmission (JT) distributed RA proposed in [31] and we found out that our proposed strategy gives better performance and this is because JT can only maximize its potential if there are exchange of control signaling among BSs. The least performed RA strategy in Fig. 1 is the single-cell processing, this is because it only consider its served UEs while designing the beamformers without coordinating interference to other UEs in the system thereby treating the out-of-cell interference as noise. This improper treatment of interference lead to severe performance loss when compared to other strategies.

F. Multiple antenna: Key component for the design of 5G

Multiple antenna at BS can help meet high-capacity demands in downlink, also it can help provide fast and reliable transmission without bandwidth expansion or increase in transmit power. Under ideal circumstances, data rate should increase linearly with the number of transmit antenna, i.e., if the spatial dimension is utilized to serve UEs in parallel. Increase in the number of transmit antenna also helps in improving beamforming resolution. Fig. 2 shows the average sum-rate as a function of the transmit antennas, from this result we observe that the optimal RA strategy has the best performance because it is centralized but is practically infeasible for large scale. We also note that for the distributed strategies, our proposed RA gives the best performance followed by the JTdistributed and then single-cell processing.

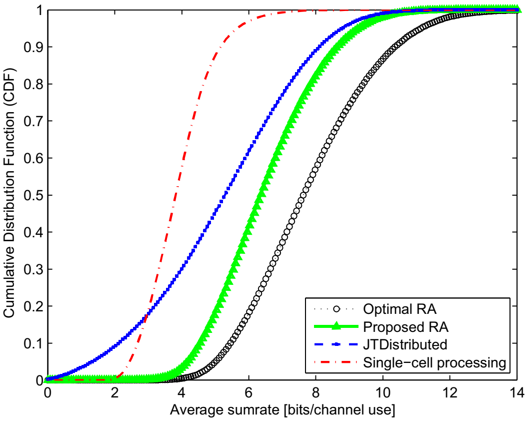

Fig. 3. Cumulative distribution function (CDF) of average sum-rate for different RA strategies, when $N = 8$, $U = 3$ and $\mathbf{card}(\mathcal{C}_j) = 3$ (i.e., 2 macrocell UEs and one adjacent picocell UE).

The CDFs of the average sum-rate are shown in Fig. 3. The optimal RA gives the best performance because of its centralized nature. Among the distributed strategies compared, our proposed RA outperforms both JTdistributed and single-cell processing strategies.

IV. CONCLUSION

In this paper, we have developed a decentralized RA strategy for UEs in HetNet such that UEs in the CRE will not experience huge loss in rate due to higher interference received from the MBS. The resources allocated to UEs are the spatial resource (unit beamformer) and the power resource. We formulate the spatial RA optimization problem as selecting the optimal beamformers that will cause the least interference to UEs in the same cell and UEs $\in \mathcal{C}_j$. While the power RA is formulated as selecting the optimal powers that when allocated to UEs will maximize the sum-rate of each cell subject to a total power constraint for each cell and individual power constraint. Both were solved by exploiting the KKT optimality conditions. Results obtained show that our decentralized RA strategy outperforms other decentralized RA strategies such as JTdistribued in [31] and the single-cell processing strategy. Our strategy is the closest in performance to the optimal RA strategy which is centralized.

REFERENCES

[1] A. Damnjanovic et al., "A survey on 3GPP heterogeneous networks," *IEEE Wireless Commun.,* vol. 18, no. 3, pp. 10-21, 2011.

[2] W. Nam et al., "Advanced interference management for 5G cellular networks," *IEEE Commun. mag.,* vol. 52, no. 5, pp. 52-60, 2014.

[3] O. Kenta et al., "Pico cell range expansion with interference mitigation toward LTE-Advanced heterogeneous networks," in *proc. IEEE Int. Conf. Commun., (ICC)*, Tokyo, 2011.

[4] D. López-Pérez et al., "Enhanced intercell interference coordination challenges in heterogeneous networks," *IEEE Wireless Commun.,* vol. 18, no. 3, pp. 22-30, 2011.

[5] G. Boudreau et al., "Interference coordination and cancellation for 4G networks," *IEEE Commun. Mag.* vol. 47, no. 4, pp. 74-81, 2009.

[6] I. Guvenc., "Dynamic Interference Management," in *Heterogeneous Cellular Networks: Theory, Simulation and Deployment.* New York, USA: Cambridge University Pressch. 7, pp. 179-216, 2013.

[7] *Summary of the Description of Candidate eICIC Solutions.* 3GPP Standard Contribution (R1-104968) (Aug. 2010).

[8] M. Assaad, "Optimal fractional frequency reuse (FFR) in multicellular OFDMA system," in *proc. IEEE VTC*, pp. 1-5, 2008.

[9] Z. Shen et al., "Overview of 3GPP LTE Advanced carrier aggregation for 4G wireless communications," *IEEE Commun. Mag.*, vol. 50, no. 2, pp. 122-130, 2012.

[10] M. Simsek et al., "Dynamic inter-cell interference coordination in HetNets: A reinforcement learning approach," in *proc. IEEE Global commun. Conf. (GLOBECOM)*, pp. 5446–5450, 2012.

[11] K. kim et al., "A universal frequency reuse system in a mobile cellular environment," in *proc. IEEE Veh. Tech. Conf. (VTC)*, pp. 2855–2859, 2007.

[12] R. irmer et al., "Coordinated multipoint: Concepts, performance, and field trial results," *IEEE Commun. Mag.*, vol. 49, no. 2, pp. 102-111, Feb. 2011.

[13] M. Sawahashi et al., "Coordinated multipoint transmission/reception techniques for LTE-advanced [Coordinated and Distributed MIMO]," *IEEE Wireless Commun.*, vol. 17, no. 3, pp. 26-34, 2010.

[14] D. lee et al., "Coordinated multipoint transmission and reception in LTE-advanced: deployment scenarios and operational challenges," *IEEE Commun. Mag.*, vol. 50, no. 2, pp. 148-155, 2012.

[15] D. Gesbert et al., "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1380-1408, 2010.

[16] A. Tarighat et al., "A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-to-leakage ratios," in *proc. IEEE Int. Conf. Commun., (ICASSP)*, Vol. 3, pp. 1129-1132, mar. 2005.

[17] M. Sadek et al., "A leakage-based precoding scheme for downlink multiuser mimo channels," *IEEE Trans. wireless Commun.*, vol. 6, no. 5, pp. 1711-1721, May 2007.

[18] P. cheng et al., "A new SLNR-based linear precoding for downlink multiuser multi-stream MIMO systems," *IEEE Commun. Lett.*, vol. 14, no. 11, pp. 1008-1010, 2010.

[19] W. Winston et al., "Decentralize Precoding for multicell mimo downlink," *IEEE Trans. Wireless Commun.*, vol. 10, no. 6, pp. 1798-1809, June 2011.

[20] O. Oguejiofor and L. Zhang, "Heuristic Coordinated Beamforming for Heterogeneous Cellular Network," in *proc. IEEE 83rd Veh. Technol. Conf., (VTC Spring)*. Nanjing, China, pp. 1-5, May 2016.

[21] B. Lee et al., "Interference-aware decentralized precoding for multicell MIMO TDD systems," in *proc. IEEE Global Commun. Conf.*, New Orleans, LA, 2008.

[22] Y. Huang et al., "Distributed multicell beamforming with limited intercell coordination,"*IEEE Trans. Signal Process.*, vol. 59, no. 2, pp.728-738, 2011.

[23] M. Peng et al., "Recent advances in underlay heterogeneous networks: Interference control, resource allocation, and self-organization," *IEEE Communications Surveys & Tutorials*, vol. 17, no. 2, pp. 700-729, 2015.

[24] K. laghari and N. crespi, "Toward total quality of experience: A QoE model in a communication ecosystem," *IEEE Commun. Mag.*, vol. 50, no. 4, pp. 58-65, April. 2012.

[25] A.M. Rubinov et al., "Lagrange-type functions in constrained nonconvex optimization," Springer Science, 2013.

[26] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[27] V. Jeyakumar et al., "A new closed cone constraint qualification for convex optimization," *Applied Mathematics Report AMR*, vol. 4, no. 8, 2004.

[28] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming." Available: http://cvxr.com/cvx, April 2011.

[29] 3rd Generation partnership project, " Technical specification group radio access network; evolved universal terrestrial radio access (E-UTRA); further advancements for E-UTRA physical layer aspects," 3GPP TR 36.814, 2010.

[30] O. Oguejiofor, Li Zhang, "Global optimization of weighted sum-rate for downlink heterogeneous cellular networks," in *proc. IEEE 23rd Int. Conf. Telecommun., (ICT)*. Thessaloniki, Greece, pp. 1-6, may 2016.

[31] E. Bjornson et al., "Cooperative multicell precoding: Rate region characterization and distributed strategies with instantaneous and statistical CSI," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4298-4310, 2010.

[32] Qualcomm, *Importance of serving cell selection in Heterogeneous Networks (R1-100701),* (Valencia, 2010).