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Recursive State-Space Identification of Non-Uniformly Sampled-Data Systems Using QR Decomposition*

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Abstract - A recursive least-squares (LS) state-space identification method based on the QR decomposition is proposed for non-uniformly sampled-data systems. Both cases of measuring all states and only the output(s) are considered for model identification. For the case of state measurement, a QR decomposition-based recursive LS (QRD-RLS) identification algorithm is given to estimate the state matrices. For the case of only output measurement, another identification algorithm is developed by combining the QRD-RLS approach with a hierarchical identification strategy. Both algorithms can guarantee fast convergence rate with low computation complexity. An illustrative example is shown to demonstrate the effectiveness of the proposed methods.

Index Terms - non-uniform sampling; state-space model identification; QR decomposition; recursive least-squares.

I. INTRODUCTION

A multirate system is labeled by the existence of multiple non-uniform sampling rates in terms of an overall period denoted by T for system operation, namely, a non-uniformly sampled-data system (NUS), which has been widely practiced in many industrial and chemical applications [2-6]. In the past two decades, multirate systems have been increasingly studied for model identification and control design [7-11].

In fact, a number of different model identification methods have been developed for various NUSs. A few identification methods [12-15] have been devoted to obtaining an autoregressive (AR) or autoregressive exogeneous (ARX) models. Owing to the convenience of describing multivariable systems, state-space model identification methods have attracted a lot of attentions in the recent years. Ding et al. [16] proposed a state-space identification algorithm for dual-rate systems by using a hierarchical identification strategy, and subsequently, derived a recursive state-space identification algorithm based on the recursive least-squares (RLS) approach for multirate sampling systems [17]. Despite the superiority of convergence and accuracy, the RLS algorithm [17] involved with relatively high computation complexity of $O(n^3)$ (where n is the number of sampled data), in comparison with the QR decomposition-based RLS (QRD-RLS) identification methods [18-20].

In this paper, two recursive identification algorithms based on the QRD-RLS and a hierarchical identification strategy are proposed for the identification of NUSs, as inspired by a recent study on using the singular value decomposition approach for recursive state-space identification [21]. When all the states of an NUS can be measured, an identification method based on QRD-RLS is given to estimate the model parameter matrices. When only the system output(s) can be measured, another identification algorithm using the hierarchical identification strategy [16] is provided which contains two steps, the first step using the approximation Kalman filter algorithm to estimate the state vectors and second step using QRD-RLS to estimate the state matrices. The rest of the paper is organized as follows. In Section 2, the state-space model description of NUSs is briefly introduced. Section 3, the proposed state-space identification algorithms are presented. A numerical example is given to illustrate the proposed algorithms in Section 4. Conclusions are drawn in Section 5.

II. MODEL DERIVATIONS

Consider a non-uniformly sampled system depicted in Fig. 1, of which the input updating and output sampling pattern are shown in Fig. 2,

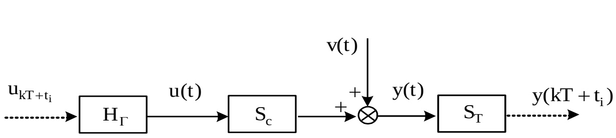

Fig. 1 The non-uniformly sampling system

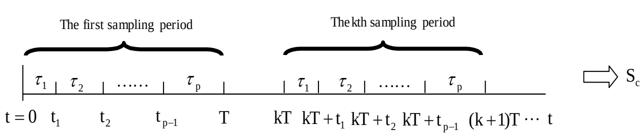

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Fig. 2 Block diagram of a non-uniformly sampling system

where S_c is a continuous-time process, H_Γ and S_T denote the zero-order-holder (ZOH) and sampler, respectively, corresponding to the following input and output format,

$$u(t)=\begin{cases} u(kT), & kT \le t < kT+t_1 \\ u(kT+t_1), & kT+t_1 \le t < kT+t_2 \\ & \vdots \\ u(kT+t_{p-1}), & kT+t_{p-1} \le t < (k+1)T \end{cases}$$

$$y(kT+t_i)=y(t)\big|_{t=kT+t_i},\ i=1,2,...,p$$

where $k=0,1,2,...,$ $t=kT+t_i$ are the sampling moments, $T=\tau_1+\tau_2+\dots+\tau_p=t_p$ is the overall period for system operation, $\{\tau_1,\tau_2,\dots,\tau_p\}$ are the sampling intervals, $\tau_0=0$, $t_i=t_{i-1}+\tau_i=\tau_1+\tau_2+\dots+\tau_i$.

A non-uniformly sampled continuous-time system with state-space representation is studied herein,

$$S_c:\begin{cases}\dot{x}(t)=\mathbf{A}x(t)+\mathbf{B}u(t)\\ y(t)=\mathbf{C}x(t)+\mathbf{D}u(t)+v(t)\end{cases} \tag{1}$$

where $x(t)\in\mathbf{R}^{nx}$, $u(t)\in\mathbf{R}^{nu}$, and $y(t)\in\mathbf{R}^{ny}$ are the state, input and output vectors, respectively. The state matrices defined by $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$. The input and output data are collected in the form of $\{u(kT+t_i),y(kT+t_i)\}$, $i=0,1,2,...,p$, $k=0,1....$

With the operational period T, we have

$$\begin{aligned} x(kT+T)&=e^{\mathbf{A}T}x(kT)+\int_{kT}^{kT+T}e^{\mathbf{A}(kT+T-\tau)}\mathbf{B}u(\tau)d\tau \\ &=e^{\mathbf{A}T}x(kT)+\sum_{i=1}^{p}e^{\mathbf{A}(T-t_i)}\int_0^{\tau_i}e^{\mathbf{A}t}d\tau\mathbf{B}u(kT+t_{i-1}) \\ &=\mathbf{A}_T x(kT)+\sum_{i=1}^{p}\mathbf{B}_i u(kT+t_{i-1}) \qquad (2)\\ &=\mathbf{A}_T x(kT)+\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \cdots & \mathbf{B}_p\end{bmatrix}\begin{bmatrix}u(kT)\\ u(kT+t_1)\\ \vdots \\ u(kT+t_{p-1})\end{bmatrix}\end{aligned}$$

where $\mathbf{A}_T=e^{\mathbf{A}_cT}\in\mathbf{R}^{nx\times nx}$, $\mathbf{B}_i=e^{\mathbf{A}_c(T-t_i)}\mathbf{B}_{\tau_i}$.

From eq. (1), $x(kT+t_i)$ can be derived as

$$\begin{aligned} x(kT+t_i)&=e^{\mathbf{A}t_i}x(kT)+\int_{kT}^{kT+t_i}e^{\mathbf{A}(kT+t_i-\tau)}\mathbf{B}u(\tau)d\tau \\ &=\mathbf{A}_i x(kT)+\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \cdots & \mathbf{B}_i\end{bmatrix}\begin{bmatrix}u(kT)\\ u(kT+t_1)\\ \vdots \\ u(kT+t_{i-1})\end{bmatrix} \end{aligned} \tag{3}$$

Correspondingly, the output can be rewritten as

$$\begin{aligned} y(kT+t_i)&=\mathbf{C}x(kT+t_i)+\mathbf{D}u(kT+t_i)+v(kT+t_i) \\ &=\mathbf{C}\mathbf{A}_i x(kT)+\begin{bmatrix}\mathbf{C}\mathbf{B}_1 & \mathbf{C}\mathbf{B}_2 & \cdots & \mathbf{C}\mathbf{B}_i\end{bmatrix}\begin{bmatrix}u(kT)\\ u(kT+t_1)\\ \vdots \\ u(kT+t_{i-1})\end{bmatrix} \\ &\quad+\mathbf{D}u(kT+t_i)+v(kT+t_i) \\ &=\mathbf{C}_i x(kT)+\begin{bmatrix}\mathbf{D}_1 & \mathbf{D}_2 & \cdots & \mathbf{D}_i & \mathbf{D}\end{bmatrix}\begin{bmatrix}u(kT)\\ u(kT+t_1)\\ \vdots \\ u(kT+t_i)\end{bmatrix}+v(kT+t_i)\end{aligned}$$

where $\mathbf{C}_i=\mathbf{C}\mathbf{A}_i$, $\mathbf{D}_i=\mathbf{C}\mathbf{B}_i$, $\mathbf{A}_i=e^{\mathbf{A}_ct_i}$.

Hence, a lifted form of the NUS can be written by

$$\begin{bmatrix}x(kT+T)\\ \tilde{y}(kT)\end{bmatrix}=\begin{bmatrix}\mathbf{A}_T & \mathbf{B}_T\\ \mathbf{C}_T & \mathbf{D}_T\end{bmatrix}\begin{bmatrix}x(kT)\\ \tilde{u}(kT)\end{bmatrix}+\begin{bmatrix}0\\ \tilde{v}(kT)\end{bmatrix} \tag{4}$$

where $\mathbf{A}_T=e^{\mathbf{A}_cT}\in\mathbf{R}^{nx\times nx}$, $\mathbf{B}_T=[\mathbf{B}_1,\mathbf{B}_2,...,\mathbf{B}_p]\in\mathbf{R}^{nx\times pnu}$, $\mathbf{C}_T=[\mathbf{C}_1,\mathbf{C}_2,...,\mathbf{C}_p]^T\in\mathbf{R}^{pny\times nx}$, $\mathbf{D}_T\in\mathbf{R}^{pny\times pnu}$.

$$\mathbf{D}_T:=\begin{bmatrix}\mathbf{D} & 0 & 0 & \cdots & 0\\ \mathbf{D}_1 & \mathbf{D} & 0 & \cdots & 0\\ \mathbf{D}_1 & \mathbf{D}_2 & \mathbf{D} & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ \mathbf{D}_1 & \mathbf{D}_2 & \cdots & \mathbf{D}_{p-1} & \mathbf{D}\end{bmatrix};\quad \tilde{y}(k)=\begin{bmatrix}y(kT)\\ y(kT-t_1)\\ \vdots\\ y(kT-t_{p-1})\end{bmatrix};$$

$$\tilde{u}(kT)=\begin{bmatrix}u(kT)\\ u(kT-t_1)\\ \vdots\\ u(kT-t_{p-1})\end{bmatrix};\quad \tilde{v}(kT)=\begin{bmatrix}v(kT)\\ v(kT-t_1)\\ \vdots\\ v(kT-t_{p-1})\end{bmatrix}.$$

where $x(t)\in\mathbf{R}^{nx}$, $\tilde{u}(kT)\in\mathbf{R}^{pnu}$, and $\tilde{y}(t)\in\mathbf{R}^{pny}$.

In the presence of measurement noise, the model (4) can be written by the following form including the white noise terms $\{\tilde{w}(kT),\tilde{v}(kT)\}$,

$$\begin{cases}x(kT+T)=\mathbf{A}_T x(kT)+\mathbf{B}_T\tilde{u}(kT)+\tilde{w}(kT)\\ \tilde{y}(kT)=\mathbf{C}_T x(kT)+\mathbf{D}_T\tilde{u}(kT)+\tilde{v}(kT)\end{cases} \tag{5}$$

III STATE ESTIMATION AND PARAMETER IDENTIFICATION

With the above formulation, we express the non-uniform periodic sampling system by

$$\begin{bmatrix}x(kT+T)\\ \hline \tilde{y}(kT)\end{bmatrix}=\begin{bmatrix}\mathbf{A}_T & \mathbf{B}_T\\ \mathbf{C}_T & \mathbf{D}_T\end{bmatrix}\begin{bmatrix}x(kT)\\ \hline \tilde{u}(kT)\end{bmatrix}+\begin{bmatrix}\tilde{w}(kT)\\ \tilde{v}(kT)\end{bmatrix} \tag{6}$$

For the convenience of computation, we define the parameter matrix $\boldsymbol{\theta}$, the information vector $\phi_0(kT)$, the augmented output vector $Z_0(kT)$ and the noise vector $e_0(kT)$, respectively, as

$$\boldsymbol{\theta}^T=\begin{bmatrix}\mathbf{A}_T & \mathbf{B}_T\\ \mathbf{C}_T & \mathbf{D}_T\end{bmatrix};\qquad e_0(kT)=\begin{bmatrix}\tilde{w}(kT)\\ \tilde{v}(kT)\end{bmatrix} \tag{7}$$

$$Z_0(kT)=\begin{bmatrix}x(kT+T)\\ \tilde{y}(kT)\end{bmatrix};\quad \phi_0(kT)=\begin{bmatrix}x(kT)\\ \tilde{u}(kT)\end{bmatrix} \tag{8}$$

The system (6) can thus be transformed into a linear regression,

$$Z_0(kT)=\boldsymbol{\theta}^T\phi_0(kT)+e_0(kT) \tag{9}$$

where $\boldsymbol{\theta}^T\in\mathbf{R}^{(nx+pny)\times(nx+pnu)}$, $\phi_0(kT)\in\mathbf{R}^{nx+pnu}$, $Z_0(kT)\in\mathbf{R}^{nx+pny}$, $e_0(kT)\in\mathbf{R}^{nx+pny}$.

A. The case of state measurement

Given a persistent excitation to the process input, if the system states $x(kT)$ are measured (i.e. $Z_0(kT)$ and $\phi_0(kT)$ are known), then we can estimate the parameter matrix $\boldsymbol{\theta}$ by QRD-RLS as below.

Define the sampled data sequence,

$$\phi(NT)=\{\phi_0(T),\phi_0(2T),\cdots,\phi_0(NT)\}\in\mathbf{R}^{(nx+pnu)\times N}$$

and so is for the output sequence $\mathbf{Z}(NT)\in\mathbf{R}^{(nx+pny)\times N}$ and noise sequence $\mathbf{e}(NT)\in\mathbf{R}^{(nx+pny)\times N}$.

The following objective function for model fitting is considered here,

$$\varepsilon(NT)=\sum_{i=0}^{N}(Z_0(iT)-\boldsymbol{\theta}^T(iT)\phi_0(iT))^2 \tag{10}$$

The information matrix $\phi(NT)$ can be transformed into a lower triangular matrix through QR decomposition,

$$\phi(NT)\mathbf{Q}(NT)=[\mathbf{L}(NT)\quad \mathbf{0}_{(nx+pnu)\times(N-nx+pnu)}] \tag{11}$$

where $\mathbf{L}(NT)\in\mathbf{R}^{(nx+pnu)\times(nx+pnu)}$, $\mathbf{Q}(NT)\in\mathbf{R}^{N\times N}$ is a unitary matrix that is obtained in terms of the Givens rotations (GR), $\mathbf{Q}(NT)=\mathbf{g}_1\mathbf{g}_2\cdots\mathbf{g}_r$ ($r = N$) [22, 23], where $\mathbf{g}_r$ is given by

$$\mathbf{g}_r(h,k,\theta)=\begin{array}{c}1\\ \\ h\\ \\ k\\ \\ N\end{array}\begin{pmatrix}1&\cdots&0&\cdots&0&\cdots&0\\ \vdots&\ddots&\vdots&&\vdots&&\vdots\\ 0&\cdots&\cos(\theta)&\cdots&\sin(\theta)&\cdots&0\\ \vdots&&\vdots&&\vdots&&\vdots\\ 0&\cdots&-\sin(\theta)&\cdots&\cos(\theta)&\cdots&0\\ \vdots&&\vdots&&\vdots&&\vdots\\ 0&\cdots&0&\cdots&0&\cdots&1\end{pmatrix} \tag{12}$$

where $h=1:nx+pnu$, $k=1:N$, $\mathbf{g}_i\in\mathbf{R}^{N\times N}$.

Post-multiplying $\mathbf{Z}_{1Q}(NT)$, $\phi(NT)$ by $\mathbf{Q}(NT)$ yields

$$\begin{aligned}e_Q(NT)&=\mathbf{Z}(NT)\mathbf{Q}(NT)-\boldsymbol{\theta}^T\phi(NT)\mathbf{Q}(NT)\\ &=\left[\mathbf{Z}_{1Q}(NT)\quad \mathbf{Z}_{2Q}(NT)\right]-\boldsymbol{\theta}^T(NT)[\mathbf{L}(NT)\quad \mathbf{0}]\end{aligned} \tag{13}$$

It is obvious that minimizing $\varepsilon(NT)$ can be realized by minimizing $\|e_Q(NT)\|^2$. In fact, $\|e_Q(NT)\|^2$ can be minimized by choosing $\boldsymbol{\theta}^T(NT)$ such that $\mathbf{Z}_{1Q}(NT)-\boldsymbol{\theta}^T(NT)\mathbf{L}(NT)$ equals zero. Therefore, we obtain the parameter estimation,

$$\boldsymbol{\theta}^T(NT)=\mathbf{Z}_{1Q}(NT)\mathbf{L}^{-1}(NT) \tag{14}$$

For the update, $\boldsymbol{\theta}\left((N+1)T\right)$ can be estimated through

$$e_Q((N+1)T)=\left[\begin{bmatrix}\mathbf{Z}_{1Q}(N)\\ \mathbf{Z}_{2Q}(N)\\ Z_0(N+1)\end{bmatrix}^T-\boldsymbol{\theta}^T(N+1)\begin{bmatrix}\mathbf{L}(N)\\ \mathbf{0}_{(nx+pnu)\times(N-nx+pnu)}\\ \phi_0(N+1)\end{bmatrix}^T\right]\mathbf{g}_{N+1}$$

$$=\left[\begin{bmatrix}\mathbf{Z}_{1Q}((N+1)T)\\ \mathbf{Z}_{2Q}((N+1)T)\end{bmatrix}^T-\boldsymbol{\theta}^T((N+1)T)\begin{bmatrix}\mathbf{L}((N+1)T)\\ \mathbf{0}_{(nx+pnu)\times(N+1-nx+pnu)}\end{bmatrix}^T\right]$$

Hence, $\boldsymbol{\theta}\left((N+1)T\right)$ can be expressed as:

$$\boldsymbol{\theta}^T((N+1)T)=\mathbf{Z}_{1Q}((N+1)T)\mathbf{L}^{-1}((N+1)T) \tag{15}$$

For convenience, $\mathbf{Z}_{1Q}((N+1)T)$ and $\mathbf{L}((N+1)T)$ can be updated only using the new data $\phi_0\left((N+1)T\right)$, $Z_0((N+1)T)$ and the one-step ahead decomposition, $\mathbf{Z}_{1Q}(NT)$ and $\mathbf{L}(NT)$ [18], i.e.

$$\begin{bmatrix}\mathbf{L}((N+1)T)\\ 0\end{bmatrix}^T=\begin{bmatrix}\mathbf{L}(NT)\\ \phi_0((N+1)T)\end{bmatrix}^T\mathbf{Q}((N+1)T) \tag{16}$$

$$\begin{bmatrix}\mathbf{Z}_{1Q}((N+1)T)\\ \mathbf{Z}_{2Q}((N+1)T)\end{bmatrix}^T=\begin{bmatrix}\mathbf{Z}_{1Q}(NT)\\ \mathbf{Z}_0((N+1)T)\end{bmatrix}^T\mathbf{Q}((N+1)T) \tag{17}$$

where $\mathbf{Q}((N+1)T)=\mathbf{g}_{nx+pny+1}\in\mathbf{R}^{(nx+pny+1)\times(nx+pny+1)}$.

Hence, the proposed QRD-RLS algorithm is summarized as follows:

1). Collect the identification data $\{\phi_0(t),Z_0(t)\}$, and take the data length N;

2). Initialize $t=1$, $\mathbf{L}(0)=0$, and $\mathbf{Z}_{1Q}(0)=0$;

3). Calculate $\mathbf{Q}(t)$ through GR as defined in (12), and compute $\mathbf{L}(t)$ and $\mathbf{Z}_{1Q}(t)$ using (16) and (17);

4). Estimate $\boldsymbol{\theta}(t)$ from (15). If $t<N+1$, then increase t by one and go to step 3.

To demonstrate the merit of the proposed method, the computation load for each step is listed in Table 1 in comparison with that of ref. [17], where the computation efforts of addition, multiplication and division are counted for each step, respectively, and n is the row number of $\phi_0(kT)$. It is seen that the proposed algorithm uses obviously less computational effort. Moreover, the initial values of the proposed QRD-RLS may be set as zero, in contrast to the initial values of the RLS algorithm in ref. [17] which need to be chosen as large numbers.

Table 1. Comparison of the computation load

Methods	Addition	Multiplication	Division
Proposed	$4n^2+n$	$8n^2+n$	n^2
Ref.[17]	$3n^2+3n+1$	n^3+4n^2+n	n

B The case of only output measurement

If only the output(s) can be measured, it can be seen that (9) contains both the unknown state vector $x(kT)$ included in $\phi_0(kT)$ and the unknown parameter matrix $\boldsymbol{\theta}$. Thus, the above QRD-RLS algorithm cannot be directly used to identify the model parameters. To circumvent the problem, we propose a modified state and parameter estimation algorithm using the hierarchical identification principle [16]. The key idea is that when estimating $\boldsymbol{\theta}$, $x(kT)$ in $\phi_0(kT)$ is replaced by its estimate $\hat{x}(kT)$, and when estimating $x(kT+T)$, the unknown $\boldsymbol{\theta}$ is replaced by its estimate $\hat{\boldsymbol{\theta}}(kT)$. Hence, the following QRD-RLS algorithm is proposed,

$$\hat{\boldsymbol{\theta}}^T(kT)=\begin{bmatrix}\hat{\mathbf{A}}_T(kT) & \hat{\mathbf{B}}_T(kT)\\ \hat{\mathbf{C}}_T(kT) & \hat{\mathbf{D}}_T(kT)\end{bmatrix} \tag{18}$$

$$\begin{aligned}\hat{x}(kT+T)=&\hat{\mathbf{A}}_T(kT)\hat{x}(kT)+\hat{\mathbf{B}}_T(kT)\tilde{u}(kT)+\\ &\rho(kT)\hat{\mathbf{C}}_T^T(kT)\left[y(kT)-\hat{\mathbf{C}}_T(kT)\hat{x}(kT)-\hat{\mathbf{D}}_T(kT)\tilde{u}(kT)\right]\end{aligned} \tag{19}$$

$$\begin{bmatrix}\mathbf{L}((N+1)T)\\ 0\end{bmatrix}^T=\begin{bmatrix}\mathbf{L}(NT)\\ \phi_0((N+1)T)\end{bmatrix}^T\mathbf{Q}((N+1)T) \tag{20}$$

$$\begin{bmatrix}\mathbf{Z}_{1Q}((N+1)T)\\ \mathbf{Z}_{2Q}((N+1)T)\end{bmatrix}^T=\begin{bmatrix}\mathbf{Z}_{1Q}(NT)\\ \mathbf{Z}_0((N+1)T)\end{bmatrix}^T\mathbf{Q}((N+1)T) \tag{21}$$

$$\boldsymbol{\theta}^T((k+1)T)=\mathbf{Z}_{1Q}((k+1)T)\mathbf{L}^{-1}((k+1)T) \tag{22}$$

$$\begin{bmatrix}\hat{\mathbf{A}}_T(kT) & \hat{\mathbf{B}}_T(kT)\\ \hat{\mathbf{C}}_T(kT) & \hat{\mathbf{D}}_T(kT)\end{bmatrix}=\hat{\boldsymbol{\theta}}^T((k+1)T) \tag{23}$$

where $\rho(kT)$ denotes the convergence rate, $\hat{x}(kT)$ is an estimate of $x(kT)$. $\hat{\mathbf{A}}_T(kT)$, $\hat{\mathbf{B}}_T(kT)$, $\hat{\mathbf{C}}_T(kT)$ and $\hat{\mathbf{D}}_T(kT)$ are the estimates of $\mathbf{A}_T$, $\mathbf{B}_T$, $\mathbf{C}_T$ and $\mathbf{D}_T$, respectively.

Note that the modified estimation algorithm shown by (18)-(23) is implemented through hierarchical computation since $\hat{x}(kT+T)$ depends on $\hat{x}(kT)$ and $\hat{\boldsymbol{\theta}}(kT)$ while $\hat{\boldsymbol{\theta}}(kT+T)$ depends on $\hat{\boldsymbol{\theta}}(kT)$ and $\hat{x}(kT)$.

IV. ILLUSTRATION

Consider a non-uniformly sampled system studied in Ref. [17],

$$S_c=\frac{s+0.8}{s^2+0.8s+0.8}$$

which corresponds to the following state space realization,

$$\begin{cases}\dot{x}(t)=\begin{bmatrix}-0.8 & -0.8\\ 1 & 0\end{bmatrix}x(t)+\begin{bmatrix}1\\ 0\end{bmatrix}u(t)\\ y(t)=\begin{bmatrix}1 & 0.8\end{bmatrix}x(t)\end{cases}$$

Let $T=1s$, $p=2$, $\tau_1=\sqrt{2}-1s$, $\tau_2=2-\sqrt{2}s$ ($t_1=\tau_1=\sqrt{2}-1s$, $t_2=1s$), as assumed in ref. [17]. The resulting discrete state-space model of the system is obtained as

$$\begin{aligned}x(kT+T)=&\begin{bmatrix}0.22659 & -0.48086\\ 0.60107 & 0.70745\end{bmatrix}x(kT)+\\ &\begin{bmatrix}0.15403 & 0.44665\\ 0.22129 & 0.1444\end{bmatrix}\begin{bmatrix}u(kT)\\ u(kT+t_1)\end{bmatrix}+w(kT)\end{aligned}$$

$$\begin{aligned}\begin{bmatrix}y(kT)\\ y(kT+t_1)\end{bmatrix}=&\begin{bmatrix}1 & 0.8\\ 0.93905 & 0.47557\end{bmatrix}x(kT)+\\ &\begin{bmatrix}0 & 0\\ 0.40553 & 0\end{bmatrix}\begin{bmatrix}u(kT)\\ u(kT+t_1)\end{bmatrix}+v(kT)\end{aligned}$$

The input $u(kT)$ and $u(kT+t_1)$ are taken as random sequences with zero mean and unit variance, $w(kT)$ and $v(kT)$ as white noise sequences with zero mean and variance δ.

Here define the parameter estimation error (PEE) by

$$\sigma:=\left\|\hat{\boldsymbol{\theta}}(kT)-\boldsymbol{\theta}\right\|/\left\|\boldsymbol{\theta}\right\|$$

where $\boldsymbol{\theta}$ is the true parameter matrix, and $\hat{\boldsymbol{\theta}}(kT)$ is an estimation of $\boldsymbol{\theta}$.

Apply the proposed QRD-RLS algorithm to estimate the state matrices under different noise levels, $\delta=0$, $\delta=0.01$, $\delta=0.04$ and $\delta=0.09$, respectively. The corresponding PEE results are shown in Fig. 3. The estimated system state-space matrices are listed in Table 2.

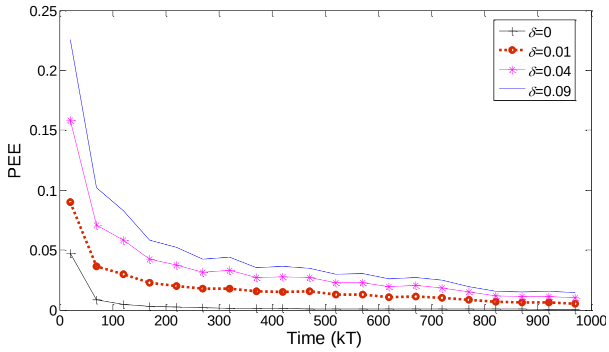

Fig.3. Plots of PEE when all the states are measured

The estimated results using the proposed QRD-RLS are similar to those of using the RLS algorithm in ref.[17], while the initial values of the QRD-RLS are taken as $\mathbf{L}(0)=0$ and $\mathbf{Z}_{1Q}(0)=0$, compared with the initial values of the RLS taken as $\mathbf{P}(0)=p_0\mathbf{I}$ (where $\mathbf{P}$ is the covariance matrix and p_0 is a large positive number, e.g., $p_0=10^6$). Fig. 3 shows that all the PEE results under different noise levels converge to smaller values when increasing t. Table 2 shows that all the state matrices under different noise levels converges to their true values. These results indicate that the proposed QRD-RLS algorithm can give good identification results with fast convergence rate and low computation complexity.

When only the output(s) can be measured, apply the proposed QRD-RLS algorithm using the hierarchical identification strategy, obtaining the parameter estimation results shown in Fig. 4, which again demonstrates that using the proposed algorithm can give good performance.

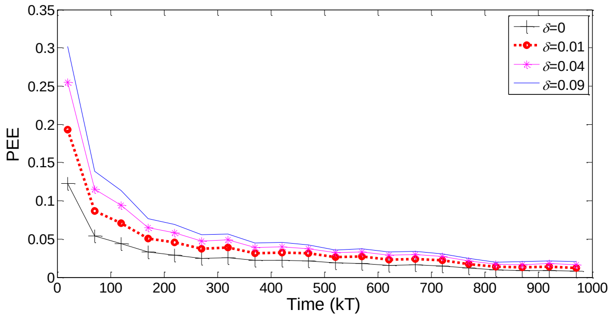

Fig.4. Plots of PEE when only the output is measured

V CONCLUSION

Based on the QR decomposition, two recursive state-space identification algorithms have been proposed for NUSs, one is for the case of state measurement and the other for the case of only output(s) measurement. Both algorithms can give good identification accuracy and convergence rate with less computation effort, compared with recently developed RLS algorithms (e.g. ref. [17]). Moreover, the proposed QRD-RLS algorithm can improve numerical behavior for ill-conditioned NUSs. Simulation results have demonstrated the effectiveness of the proposed methods.

TABLE 2. Estimation of the measured states state matrices under different noise levels

True values	$\mathbf{A}_T = \begin{bmatrix} 0.22659 & -0.48086 \\ 0.60107 & 0.70745 \end{bmatrix}$	$\mathbf{B}_T = \begin{bmatrix} 0.15403 & 0.44665 \\ 0.22129 & 0.1444 \end{bmatrix}$	$\mathbf{C}_T = \begin{bmatrix} 1 & 0.8 \\ 0.93905 & 0.47557 \end{bmatrix}$	$\mathbf{D}_T = \begin{bmatrix} 0 & 0 \\ 0.40553 & 0 \end{bmatrix}$
$\delta = 0$	$\hat{\mathbf{A}}_T = \begin{bmatrix} 0.22661 & -0.48085 \\ 0.60108 & 0.70745 \end{bmatrix}$	$\hat{\mathbf{B}}_T = \begin{bmatrix} 0.15393 & 0.44606 \\ 0.221 & 0.14405 \end{bmatrix}$	$\hat{\mathbf{C}}_T = \begin{bmatrix} 1. & 0.8 \\ 0.93906 & 0.47558 \end{bmatrix}$	$\hat{\mathbf{D}}_T = \begin{bmatrix} 0 & 0 \\ 0.40522 & 0.0003 \end{bmatrix}$
$\delta = 0.01$	$\hat{\mathbf{A}}_T = \begin{bmatrix} 0.2264 & -0.4808 \\ 0.6013 & 0.7066 \end{bmatrix}$	$\hat{\mathbf{B}}_T = \begin{bmatrix} 0.1539 & 0.4462 \\ 0.2209 & 0.1440 \end{bmatrix}$	$\hat{\mathbf{C}}_T = \begin{bmatrix} 0.9993 & 0.7999 \\ 0.9389 & 0.4765 \end{bmatrix}$	$\hat{\mathbf{D}}_T = \begin{bmatrix} 0.0002 & 0.0001 \\ 0.4054 & 0.0004 \end{bmatrix}$
$\delta = 0.04$	$\hat{\mathbf{A}}_T = \begin{bmatrix} 0.2258 & -0.4904 \\ 0.6021 & 0.7038 \end{bmatrix}$	$\hat{\mathbf{B}}_T = \begin{bmatrix} 0.1538 & 0.4467 \\ 0.2207 & 0.1438 \end{bmatrix}$	$\hat{\mathbf{C}}_T = \begin{bmatrix} 0.9975 & 0.7997 \\ 0.9385 & 0.4790 \end{bmatrix}$	$\hat{\mathbf{D}}_T = \begin{bmatrix} 0.0007 & 0.0004 \\ 0.4060 & 0.0006 \end{bmatrix}$
$\delta = 0.09$	$\hat{\mathbf{A}}_T = \begin{bmatrix} 0.2244 & -0.4788 \\ 0.6041 & 0.6991 \end{bmatrix}$	$\hat{\mathbf{B}}_T = \begin{bmatrix} 0.1538 & 0.4477 \\ 0.2203 & 0.1434 \end{bmatrix}$	$\hat{\mathbf{C}}_T = \begin{bmatrix} 0.9949 & 0.8002 \\ 0.9379 & 0.4821 \end{bmatrix}$	$\hat{\mathbf{D}}_T = \begin{bmatrix} 0.0017 & 0.0009 \\ 0.4071 & 0.0010 \end{bmatrix}$

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