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**Improved Acquisition for System Sustainment: Multiobjective
Tradeoff Analysis for Condition-Based Decision-Making**

21 October 2013

Dr. Kash Barker, Assistant Professor

University of Oklahoma

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Prepared for the Naval Postgraduate School, Monterey, CA 93943.



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Abstract

The aim of this research work is to develop a framework for making coordinated acquisitions decisions, integrating: (i) schedules of maintenance based on the condition of system components, and (ii) trigger acquisition of multiple vendor maintenance, repair, and overhaul (MRO) supplies and services. To do so, we develop a multi-objective optimization maintenance/acquisition scheduling algorithm where tradeoffs are made between two competing objectives: maximizing condition-based component reliability while maximizing the sensitivity of condition-based reliability to changes in the component's condition (e.g., degradation). This scheduling algorithm integrates with an acquisition algorithm to addresses vendor lead-time. Case studies broadly inspired by Tinker Air Force Base, the largest Air Force MRO hub in the US, in Oklahoma City, OK, illustrate the framework.

Keywords: maintenance, repair, and overhaul; proportional hazards; scheduling; greedy heuristic



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About the Author

Dr. Kash Barker—Kash Barker joined the School of Industrial and Systems Engineering in the fall of 2008 as a lecturer, with an assistant professor position commencing in the fall of 2011. He completed his PhD in the Department of Systems and Information Engineering at the University of Virginia, where he worked with Dr. Yacov Haimes and the Center for Risk Management of Engineering Systems. This followed BS and MS degrees in industrial engineering from the University of Oklahoma (OU). His research interests have dealt primarily with (i) analyzing risk in interdependent industry and infrastructure sectors, and (ii) enhancing data-driven decision making for large-scale system sustainment. Research projects have been funded by the National Science Foundation, Federal Highway Administration, Army Research Office, and Naval Postgraduate School. He is an Associate Editor for *IIE Transactions* (Government, Policy, and Society department) and in on the Editorial Board of *Risk Analysis*.

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21 October 2013

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Disclaimer: The views represented in this report are those of the author and do not reflect the official policy position of the Navy, the Department of Defense, or the federal government.



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Table of Contents

Research Summary	1
Research Output.....	2
Task 1. Formulate the Multiobjective Condition-Based Decision-Making Approach for Triggering MRO Operations	3
Introduction and Motivation	3
Methodological Background	4
Proportional Hazards Models.....	4
Degradation Models.....	6
Uncertainty Sensitivity Index Method	7
Methodological Development	9
Modeling Reliability, Maintenance, and Degradation	9
Formulating the Multiobjective Framework	12
Identifying Pareto Optimal Frontier Boundaries	14
The Pareto Frontier and Reiteration of the Decision Framework.....	15
Criteria for Determining the Value of the Decision Variable (<i>Tmj</i>).....	16
Illustrative Example	17
Specification of Input Parameters for Modeling Reliability and Degradation.....	17
Implementation of the Multiobjective Decision Framework	19
Assessment of Performance.....	22
Concluding Remarks	24
Task 2. Expand Task 1 to Multi-Component Systems	25
Introduction	25
Methodological Development	27
Total Penalty Minimization Problem.....	30
Greedy Heuristic With Local Search Algorithm	32
Phase 1: Initial Partial Solution Construction	35
Phase 2: Greedy Heuristic-Based Improvement.....	37
Phase 3: Local Search-Based Improvement	39
GHLSA Experimental Results	40



Problem Specification	41
Baseline Case.....	41
Sensitivity of γ and δ Parameters	42
Sensitivity of Penalty Terms.....	43
Concluding Remarks	46
Task 3. Develop an Algorithm to Coordinate Multiple Vendor Acquisition of Component Parts	47
Introduction	48
Methodological Background	49
Reliability: Conditional and System.....	49
Proportional Hazard Model	50
Weibull Distribution	51
Methodological Development	52
Illustrative Example	53
Performance Metrics.....	56
Conclusions.....	56
References	59



List of Figures

Figure 1.	Monotonic Degradation Examples, Increasing and Decreasing, for $x_i(t)$	7
Figure 2.	A General Representation of the Pareto-Optimal Frontier for the Multiple Objectives of Equation 6	9
Figure 3.	The Change in $R_0(t)$ From (a) Varying λ While Keeping k Constant and (b) Vice Versa	10
Figure 4.	The Trajectory of $R(t, x(t))$ for Increasing β	11
Figure 5.	The Value of $R(t, x(t))$ When $xTmj = \mathbf{0}$ Is Restored to $R_0(t)$	11
Figure 6.	The Degradation Path Following Maintenance Operations at Tmj	12
Figure 7.	Portrayal of the Bounds of t	14
Figure 8.	Solving for t_{max} With $LM = 0$	15
Figure 9.	The Pareto-Optimal Frontier Generated for Equation 16	16
Figure 10.	Depiction of the Iterative Maintenance Decision Framework	16
Figure 11.	The Loss in Objective Values With Respect to t on the Pareto Frontier	17
Figure 12.	The Trajectories of Degradation for Each State Variable	18
Figure 13.	The Functions (a) $R(t, x(t))$, (b) $\psi_{ct, xt2}$, and (c) $\partial\psi_{ct, xt2}/\partial t$ Simulated for $j = \mathbf{0}$	20
Figure 14.	The (b) Pareto-Optimal Frontier Defined With $tmax1$ to Remove the Inferior Solutions From (a)	21
Figure 15.	The Progression of the Pareto Optimal Frontier Over j	22
Figure 16.	Example Recommended Maintenance Times for Components A, B, and C	28
Figure 17.	Example Earliness and Tardiness System Maintenance Times for Components A, B, and C	28
Figure 18.	Pseudo-Code Overview of the Proposed Greedy Heuristic With Local Search Algorithm	33
Figure 19.	Pseudo-Code for GHLSA Phase I, the Partial Solution Construction Phase	36
Figure 20.	Pseudo-Code for Improving Combination Stage	38



Figure 21.	Pseudo-Code for GHLSA Phase II, the Greedy Heuristic-Based Improvement Phase	39
Figure 22.	Pseudo-Code for the GHLSA Phase III, the Local Search Phase ..	40
Figure 23.	Graphical Depiction of the Recommended Repair Times for the 10 Components (MTBF), and the System Maintenance Schedules Found From the Baseline Case and the GHLSA, Over 100 Time Periods	42
Figure 24.	Change in Percent Deviation Value With δ	43
Figure 25.	Percent Deviation Values for Different Setup Penalties Across Values of γ	44
Figure 26.	Percent Deviation Values for Different Downtime Penalties Across Values of γ	44
Figure 27.	Percent Deviation Values for Different Deviation Penalties Across Values of γ	45
Figure 28.	Depiction on When to Order Without Consideration Variability or Conditional Reliability With a Failure Defined as $R = 0.3$	49
Figure 29.	Depiction of Reliability and Conditional Reliability Over Time	50
Figure 30.	Depiction of a Situation Where Degradation Is Faster Than the Baseline	51
Figure 31.	Failure Time Probability Distribution Function	52
Figure 32.	(a) Depiction of the Effect of the Covariate Estimation and Its Interval on the Estimated Reliability; (b) The Covariate Estimate Was Increased by 0.5 to 1.0.....	53
Figure 33.	Component Baseline Reliabilities	54
Figure 34.	Case Study Output.....	56



List of Tables

Table 1.	Functional Mean Trajectory of State Variable Degradation With Standard Deviations	18
Table 2.	CPHM Coefficient Estimates	19
Table 3.	Summary of Results for Each Iteration.....	21
Table 4.	Performance Comparisons at a Baseline of <i>Tm6</i>	23
Table 5.	Parameters of the Illustrative Example.....	41
Table 6.	Optimal δ Values for Different Setup Penalty Instances Across γ .	46
Table 7.	Optimal δ Values for Different Deviation Penalty Cost Instances Across γ	46
Table 8.	Supplier Characteristics	54
Table 9.	Regression Coefficients	54
Table 10.	Acquisition Times Without Covariate Input.....	55
Table 11.	Acquisition Time Incorporating Covariate Information and Conditional Reliability	55
Table 12.	Corresponding Points of Single Supplier, Single Acquisitions Time	56
Table 13.	Subset of Pareto Points	56



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Improved Acquisition for System Sustainment: Multiobjective Tradeoff Analysis for Condition-Based Decision-Making

Research Summary

Large-scale systems across the global landscape are aging, in many cases well past their intended useful life. This is a significant concern for the Department of Defense (DoD), particularly the Air Force, as budgets tighten and the replacement of weapons systems becomes infeasible. Acquisition for system sustainment is a chief budgetary and performance concern. This research addressed the issue of maintenance, repair, and overhaul (MRO) acquisition with three tasks, summarized as follows:

Task 1. Formulate the multiobjective condition-based decision-making approach for triggering MRO operations. MRO activities are very important for the sustainment of aging systems, the failure of which can have widespread, adverse consequences. This paper presents an approach for maintenance scheduling that models condition data and system degradation with a proportional hazards model, which is commonly used for describing reliability from time-based and condition-based perspectives but has little use in maintenance decision-making. We introduce a sensitivity function for describing the extent to which reliability is impacted by degrading state variables, thus providing a temporal multiobjective maintenance optimization framework, trading off reliability and sensitivity. We also relax the common assumption of a finite condition state space governed by a stochastic process with the use of fitted models of state variable degradation.

Task 2. Expand Task 1 to multi-component systems. As many large-scale systems age, and due to budgetary and performance efficiency concerns, there is a need to improve the decision-making process for system sustainment, including MRO operations and the acquisition of MRO parts. To help address the link between sustainment policies and acquisition, this work develops a greedy heuristic-based local search algorithm to provide a system maintenance schedule for multi-component systems, coordinating recommended component maintenance times to reduce system downtime costs, thereby enabling effective acquisition.

Task 3. Develop an algorithm to coordinate multiple vendor acquisition of component parts. As large-scale systems such as aviation fleets continue operation well beyond planned use, the costs associated with maintainability greatly



depend on the quality of allocating resources. This work uses a reliability model of system and environmental covariates incorporating information at the component-through the system-fleet levels to provide a decision-making approach for MRO and inventory operations to collaboratively address multiple components. We explore a conditional reliability calculation, driven by the Cox proportional hazards model, to determine lead-time requirements and suggest a dynamic maintenance trigger. Such information will inform MRO and inventory decisions.

Task 4. Illustrate the above developments with a small case study addressing U.S. Air Force system sustainment. Illustrative examples for Tasks 2 and 3 have been inspired by conversations with employees at Tinker Air Force Base, though due to the sensitive nature of such systems, no real data could be used.

Research Output

The remainder of this report provides the primary methodological developments and research findings of the funded work, provided in the form of scholarly journal manuscripts. Complete references across all tasks are provided in the References at the end.

In total, the following papers and presentations were submitted or are still in progress (with support acknowledged):

1. Chapel, T., Barker, K., & Ramirez-Marquez, J. E. (in press). *Conditional reliability for acquisition decision making*. Manuscript submitted for publication.
2. Chapel, T., & Barker, K. (2013, May). *Maintenance, repair, and overhaul inventory decision making for multi-component systems*. Paper presented at the Industrial and Systems Engineering Research Conference, San Juan, PR.
3. Kalam, S., Barker, K., & Ramirez-Marquez, J. E. (2013b). *Scheduling multi-component maintenance with a greedy heuristic local search algorithm*. Manuscript submitted for publication.
4. Kalam, S., Barker, K., & Ramirez-Marquez, J. E. (2013a). Improving multi-component maintenance acquisition with a greedy heuristic local search algorithm. In *Proceedings of the Ninth Annual Acquisition Research Symposium*. Monterey, CA: Naval Postgraduate School.
5. Wilson, K. J., & Barker, K. (2012). *Multiobjective sensitivity analysis for condition-based sustainment decision making*. Manuscript submitted for publication.



Also, the following two theses were completed as a result of this work:

1. Chapel, T. (2013, Spring). *Maintenance, repair, and overhaul inventory decision making for multi-component systems* (Master's thesis). Norman, OK: University of Oklahoma.

Tiara Chapel is currently employed by MITRE Corporation.

2. Kalam, S. (2013, Spring). *Improving multi-component maintenance with a greedy heuristic local search algorithm* (Master's thesis). Norman, OK: University of Oklahoma.

Sifat Kalam is currently employed by Boeing.

Task 1. Formulate the Multiobjective Condition-Based Decision-Making Approach for Triggering MRO Operations

This section is based on the following:

Wilson, K. J., & Barker, K. (2012). *Multiobjective sensitivity analysis for condition-based sustainment decision-making*. Manuscript submitted for publication.

Introduction and Motivation

Maintenance, repair, and overhaul (MRO) activities are very important for the Department of Defense (DoD), the Federal Aviation Administration, and private industry organizations that deal with sustainment of aging systems (Charles, Raman, & Starly, 2011). The failure of these systems has widespread adverse consequences, from the safety of individual aircraft, to issues of homeland security, to the success of defense missions. For example, most major commercial airlines and U.S. Air Force (USAF) have extended the life of their air fleet with few replacements planned in the near future due to decreasing budgets, including a USAF KC-135 fleet that is reaching 50 years in age and must be viable until 2040. And while the previous discussion concerns aerospace systems (e.g., aircraft, radar systems), the problem of sustaining aging systems is a widespread concern across many infrastructure systems.

Performing MRO operations typically comes in the form of repairing existing components or replacing worn or obsolete components. Proactive MRO operations often result from preventive maintenance or scheduled downtime, which occurs periodically (e.g., number of cycles, amount of service time). In terms of replacement, this could occur in the form of "block upgrades," when substitutions are delayed until a specified number of components become obsolete (Concho, Ramirez-Marquez, Herald, & Sauser, 2011). Predictive maintenance, or condition-



based maintenance, occurs when the system failure signatures are assessed before failure actually occurs. Understanding the impact that MRO operations have on the reliability of systems and their components is important for decision-making (Martorell, Sanchez, & Serradell, 1999). Similar to Yacout, Ghasemi, and Ouali's (2007) work, our work suggests that MRO decision-making should incorporate condition data for scheduling maintenance well in advance.

Much of the emphasis of existing condition-based maintenance optimization is limited to single-objective analysis, neglecting consideration of competing objectives (e.g., MRO costs versus availability). Constraints are often imposed within single-objective optimization to account for multiple criteria, which presents a disadvantage because of an inability to assess tradeoffs between objectives in order to provide a solution that best represents the preference of the decision-maker (Tian, Lin, & Wu, 2012). In maintenance optimization, common optimality criterion include reliability, availability, and maintenance cost (Sharma & Yadava, 2011). This paper provides a multiobjective maintenance optimization approach where tradeoffs are made between two competing objectives: (i) maximizing the condition-based reliability function, and (ii) maximizing the sensitivity of the condition-based reliability function to changes in the state-of-the-system. A proportional hazards model, which is very useful in the study of repairable systems (Bendell, Wightman, & Walker, 1991), describes reliability here. The multiobjective framework in this paper is multistage, as maintenance is performed more than once, and provides an in-advance maintenance scheduling framework.

Methodological Background

This section describes some of the modeling ideas that comprise our methodological approach, including the condition-based proportional hazards model, degradation models, sensitivity metrics, and multiobjective optimization.

Proportional Hazards Models

Models describing failure probability often only incorporate time at failure, thereby modeling reliability, $R(t)$, only as a function of time. Such suggests that the primary driver of failure is duration of use (e.g., time, repetitions, cycles). The Cox proportional hazards model (CPHM; Cox, 1972) provides a means to model reliability (or survival) with the incorporation of covariate effects, or the effects that the state of the system, not only time, have on hazard function and thus reliability. Provided in Equation 1, the hazard function describing the rate at which failures occur is a function of a time-driven baseline hazard function, $h_0(t)$, and the state of the system, vector $\mathbf{x}(t)$. $\boldsymbol{\beta}$ is a vector of regression coefficients reflecting the effect of the state-of-the-system on the hazard function. The reliability function can then be found as $R(t) = \exp\left(-\int_0^t h(x)dx\right)$.



$$h(t; \mathbf{x}(t)) = h_0(t)\exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \quad (1)$$

The use of CPHM is widespread in medical research (Crowley & Hu, 1977; Prentice, Williams, & Peterson, 1981; Lee & Wang, 2003) and has had several applications in reliability engineering (Dale, 1985; Kumar & Klefsjo, 1993; Ansell & Phillips, 1997; Leemis, 2009). A primary reason for its popularity is that it allows the ability to assess reliability with a nonparametric approach, not needing to specify a baseline hazard, $h_0(t)$. As such, the typical use of the CPHM has been descriptive in nature (i.e., identifying the factors that significantly impact reliability), not prescriptive (i.e., actually estimating reliability given a set of covariates), which requires a specific $h_0(t)$ (e.g., Krivtsov, Tananko, & Davis, 2009).

However, as a Weibull hazard function is often used to describe the failure of engineered components, such a function is a nature choice for $h_0(t)$ when such an assumption can be made. A baseline Weibull hazard distribution is provided in Equation 2, where k and λ are the shape and scale parameters, respectively, of the Weibull distribution. Other explorations into baseline hazard functions include log-logistic (Hutton & Solomon, 1997), Gompertz (Bender, Augustin, & Blettner, 2005), and constant (Kalbfleisch & Prentice, 1973), among others. Note that we continue to use the term *CPHM* for this model, as is often the case in the reliability literature, though (i) we are specifying a baseline hazard function and not using the traditional Cox nonparametric approach, and (ii) as Allison (1995) pointed out, the analysis of time-varying covariates is technically a non-proportional hazards model.

$$h_0(t) = \frac{k}{\lambda} \left(\frac{t}{\lambda}\right)^{k-1} \quad (2)$$

The baseline Weibull reliability function $R_0(t)$, which models reliability only as a function of time and without effects of degrading state variables, is shown in Equation 3. Reliability as a function of both t and $\mathbf{x}(t)$, the result of the CPHM, is shown in Equation 4. With $k = 1$, the failure rate is constant, and increasing with time when $k > 1$.

$$R_0(t) = \exp\left(-\left(\frac{t}{\lambda}\right)^k\right) \quad (3)$$

$$R(t, \mathbf{x}(t)) = \exp\left(-\left(\frac{t}{\lambda}\right)^k \exp(\boldsymbol{\beta}^T \mathbf{x}(t))\right) \quad (4)$$

For CPHM to be useful in condition-based maintenance applications, a baseline hazard must indeed be chosen. Several works have explored a Weibull baseline hazard function in CPHM for maintenance decision making strategies, whether they be replacement decisions (Jardine, Banjevic, Makis, & Ennis, 2001); Wu & Ryan, 2011), inspection interval decisions (Chen, Chen, Li, Zhou, & Sievenpiper, 2011), procurement decisions (Louit, Pascual, Banjevic, & Jardine,



2011), and maintainability decisions (Barabadi, Barabady, & Markeset, 2011). Yacout et al. (2007) suggested that further research should be done for preventative maintenance that benefits from condition data with the power to schedule in advance.

Degradation Models

The classification of covariates with the stochastic processes described previously makes the assumption of a finite state space, potentially leading to subjectivity in how states and their transition probabilities are defined. Alternative approaches to modeling the degradation of the state-of-the-system include Brownian motion and other stochastic processes (Doksum & Hoyland, 1992; Liao, Elsayed, & Chan, 2006) and Bayesian methods for remaining life distributions (Gebraeel, Lawley, Li, & Ryan, 2005), among others (Shiau & Lin, 1999; Yuan & Pandey, 2009). The approach used here describes the state of the system, $\mathbf{x}(t)$, with a continuous fitted degradation model. System condition is not updated at discrete inspection intervals as with a finite state space stochastic process but degrades continuously. Such is an assumption that has more flexibility in describing how characteristics degrade over time but has no flexibility for modeling very sudden, pronounced degradation.

Degradation is assumed to be a monotonic function, either increasing or decreasing, as shown in Figure 1. For example, if $x_i(t)$ describes the diameter of an inspected part, one would expect to see a decreasing function for $x_i(t)$, whereas an increasing function $x_j(t)$ may describe the amount of metal filings in motor oil over time. It is assumed that the models in Figure 1 are fitted from observation data, though it could represent physical models if such are known (Rathod, Yadav, Rathore, & Jain, in press).



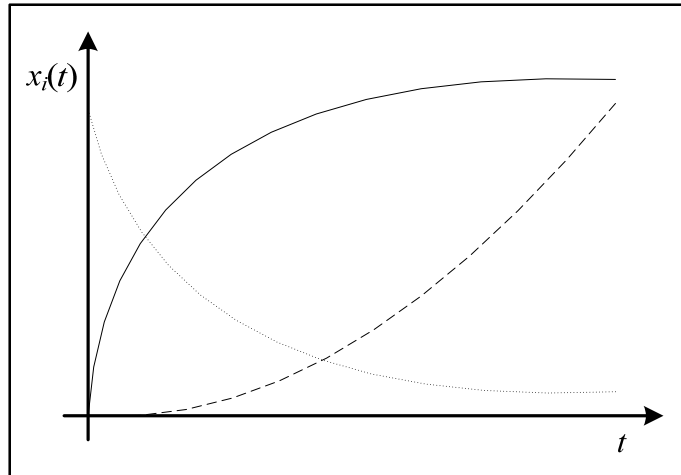


Figure 1. Monotonic Degradation Examples, Increasing and Decreasing, for $x_i(t)$

In this work, we assume that $\mu_i(t)$, the mean of $x_i(t)$, follows the trajectory by a monotonic function, like in Figure 1, though with some randomness introduced. Similar to Rathod et al. (in press), a probabilistic approach is taken to assume the actual value of $x_i(t)$.

Uncertainty Sensitivity Index Method

The Uncertainty Sensitivity Index Method (USIM; Haines & Hall, 1977; Li & Haines, 1988) addresses the sensitivity of optimal response of a model to potential variation in the parameters of the model, allowing a decision-maker to compare options that optimize an objective function while minimizing sensitivity to uncertainty with a multiobjective formulation.

We assume $f(\mathbf{u}, \boldsymbol{\alpha})$ is an objective function to be minimized subject to a set of constraints, $\mathbf{g}(\mathbf{u}, \boldsymbol{\alpha})$, where \mathbf{u} represents a vector of decision variables and $\boldsymbol{\alpha}$ a vector of parameter in the model. Sensitivity analysis of such an optimization problem would typically involve the study of changes to the optimal solution of $f(\mathbf{u}, \boldsymbol{\alpha})$ with respect to changes in the bounds of $\mathbf{g}(\mathbf{u}, \boldsymbol{\alpha})$. However, with the USIM, sensitivity of $f(\mathbf{u}, \boldsymbol{\alpha})$ is measured with respect to changes in model parameters, $\boldsymbol{\alpha}$, with sensitivity function, $\psi(\mathbf{u}, \boldsymbol{\alpha})$.

For a single parameter α_i , the choice of $\psi(\mathbf{u}, \boldsymbol{\alpha})$ from Haines and Hall (1977) squares the partial derivative of the objective function with respect to α_i , or $\psi(\mathbf{u}; \alpha_i) = [\partial f(\mathbf{x}; \alpha_i) / \partial \alpha_i]^2$. Li and Haines (1988) show that a change in an objective function of multiple uncertain parameters, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)$, is minimized when the sensitivity index in Equation 5 is minimized.

$$\psi(\mathbf{u}, \boldsymbol{\alpha}) = \sum_{i=1}^n \left[\frac{\partial f}{\partial \alpha_i}(\mathbf{u}, \boldsymbol{\alpha}) \right]^2 \quad (1)$$

Adding sensitivity to the single objective $f(\mathbf{u}, \boldsymbol{\alpha})$ results in the multiobjective problem in Equation 6. The objective function and sensitivity index are both minimized with respect to decision \mathbf{u} and are evaluated at a nominal value $\hat{\boldsymbol{\alpha}}$ (e.g., the original estimate of $\boldsymbol{\alpha}$). It is assumed that $\boldsymbol{\alpha}$, though its true value may be uncertain, varies in the neighborhood of $\hat{\boldsymbol{\alpha}}$.

$$\begin{aligned} & \min_{\mathbf{u}} f(\mathbf{u}, \hat{\boldsymbol{\alpha}}) \\ & \min_{\mathbf{u}} \psi(\mathbf{u}, \hat{\boldsymbol{\alpha}}) \\ & \text{s. t. } \mathbf{g}(\mathbf{u}, \hat{\boldsymbol{\alpha}}) \leq \mathbf{0} \end{aligned} \quad (2)$$

Multiobjective optimization problems can be challenging to solve because the objectives are often competing where any improvement in one objective comes only at the expense of another objective. Common approaches to solving multiobjective optimization problems include (i) combining the multiple objectives into a single objective with a lone optimal solution, and (ii) generating a set of non-inferior, or Pareto-optimal, solutions. Often, objectives are not only competing but also non-commensurate, and as such, devising a single objective may not be appropriate (e.g., through a weighted-sum method, goal programming, utility theory). The advantages of a Pareto-optimal frontier, generated through the ϵ -constraint method or multiple realizations of the weighted-sum method (Chankong & Haimes, 2008), allows the decision-maker to directly view tradeoffs between objectives for actual feasible solutions to those objectives. Once the Pareto-optimal frontier is generated, a decision-maker can choose a non-inferior solution that most appropriately balances his/her preferred tradeoffs for the two objectives. Figure 2, for example, depicts the Pareto-optimal frontier for the multiobjective optimization problem in Equation 6, with the instantaneous tradeoff among $f(\cdot)$ and $\psi(\cdot)$ illustrated for a particular decision-maker-defined optimal decision \mathbf{u}^* . Examples of reliability-based multiobjective decision-making from a Pareto-optimal frontier include those by Huang, Tian, and Zuo (2005); Certa, Galante, Lupo, and Passananti (2011); and Taboada, Baheranwala, Coit, and Wattanapongsakorn (2007).



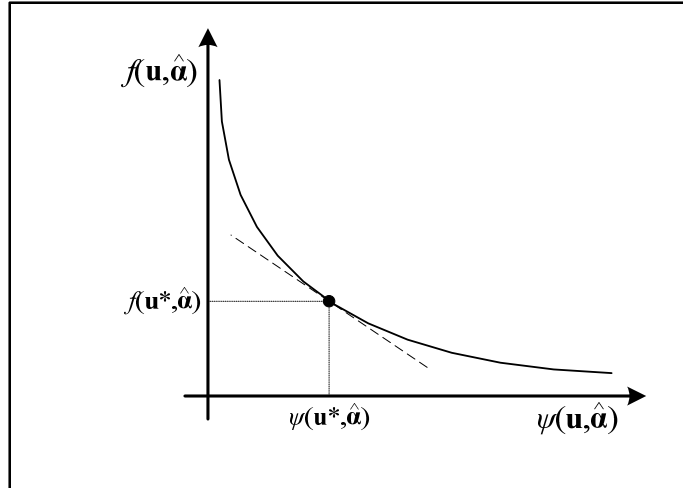


Figure 2. A General Representation of the Pareto-Optimal Frontier for the Multiple Objectives of Equation 6

Methodological Development

In this paper, we adapt sensitivity analysis to understand the contribution of states-of-the-system that lead to fluctuations in reliability. These fluctuations can be recognized in three forms: (i) the states-of-the system evolve in the form of degradation; (ii) the states-of-the system are changed with maintenance; and (iii) the estimated mathematical expression for predicting degrading states-of-the-system is not exact, and, as a result, uncertainty is introduced. An ideal maintenance trigger is one that is most sensitive to changes in states-of-the system, thus preventing significant drops in reliability due to degradation, increasing the influence of maintenance, and preventing large error in predicted reliability versus the actual reliability that may exist from uncertainty in the degradation models.

We assumed that maintenance is performed to improve the condition of $x_i(t)$ to a nominal observable state, its “preferred condition.” For example, the solid-line trajectory in Figure 1, which takes the form of t^{d_i} for $0 < d_i < 1$, suggests degradation occurs as $x_i(t)$ increases; therefore, the preferred condition would occur when $x_i(t) = 0$. The methodology is founded on ability to explicitly model maintenance impact in this way without the assumption of renewal upon repair. The maintenance decision-making framework is dynamic in nature; thus, as maintenance occurs at a point in time, it is denoted by T_m^j , the decision variable for the j th maintenance iteration.

Modeling Reliability, Maintenance, and Degradation

The CPHM is used here for modeling reliability for two reasons: (i) the model allows for consideration of time-dependent covariates (e.g., $x_i(t)$) and (ii) the use of

the model prevents the need of a “renewal upon repair” assumption, illustrated with the baseline reliability function in Equation 3.

A Weibull baseline hazard function is adopted here, and Figure 3 illustrates how the baseline reliability function varies in k and λ . Recall, $k > 1$ results in an increasing failure rate, and $\lambda > 1$ from parameter restriction for a Weibull distribution. The shape of the baseline reliability, defined with the specification of k , dictates many of the properties within the methodology and will provide better understanding for methods later mentioned.

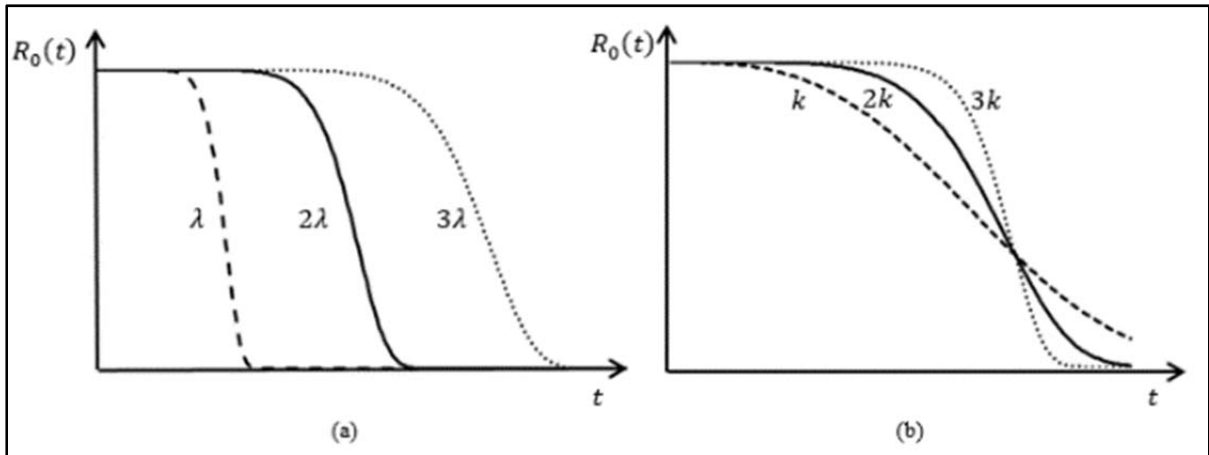


Figure 3. The Change in $R_0(t)$ From (a) Varying λ While Keeping k Constant and (b) Vice Versa

The general form of the CPHM-driven reliability function is found in Equation 7. Assuming degradation of the form t^{d_i} for $0 < d_i < 1$, the $\lim_{x(t) \rightarrow 0} \exp(\beta^T \mathbf{x}(t)) = 1$. This suggests that when the state of the system is at its preferred value, $x_i(t) = 0$, then $R(t, \mathbf{x}(t)) = R_0(t)$. Baseline reliability $R_0(t)$ can then be considered an upper bound for reliability at any point in time, and repair can only achieve at best $R_0(t)$; improving this value would require replacement.

$$R(t, \mathbf{x}(t)) = R_0(t) \exp(\beta^T \mathbf{x}(t)) \quad (7)$$

The behavior of $R(t, \mathbf{x}(t))$ with a decreasing $\beta^T \mathbf{x}(t)$ term converges to $R_0(t)$. Figure 4 represents how $R(t, \mathbf{x}(t))$ behaves with an increasing $\beta^T \mathbf{x}(t)$ term. Three functions with the same value for $\mathbf{x}(t)$ and different β vectors will result in a differing rate of decline in reliability. The influence of $x_i(t)$ is defined with β_i . In order for degradation to increase hazard (thereby decreasing reliability), $\beta_i > 0$. Therefore, $R(t, \mathbf{x}(t)) = R_0(t)$ only when $\mathbf{x}(t) = 0$. This demonstrates the usefulness of using the CPHM for modeling maintenance paired with the assumption that a nominal condition can be defined at 0.

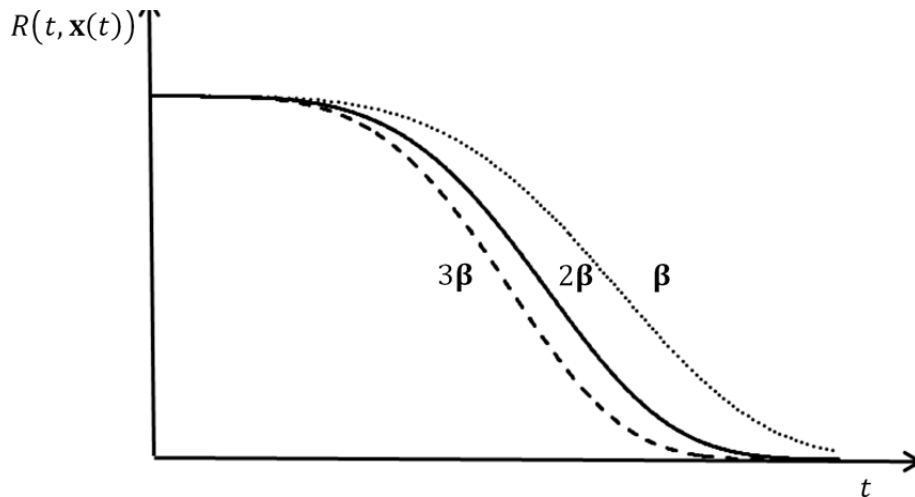


Figure 4. The Trajectory of $R(t, \mathbf{x}(t))$ for Increasing β

The time at which the j th iteration of maintenance is performed is denoted by T_m^j . Maintenance returns the state variables to their preferred condition, or $\mathbf{x}(T_m^j) = \mathbf{0}$. As such, component reliability is increased to its baseline reliability, or reliability calculated only from age, with no observable degradation but is not renewed, as illustrated in Figure 5 for two maintenance iterations. It is assumed here that the entire state vector is restored to zero, $\mathbf{x}(T_m^j) = \mathbf{0}$, but maintenance could easily target specific state variables, $x_i(T_m^j) = 0$. If specific state variables are selected at T_m^j rather than the entire vector, reliability will approach, but not equal, $R_0(t)$.

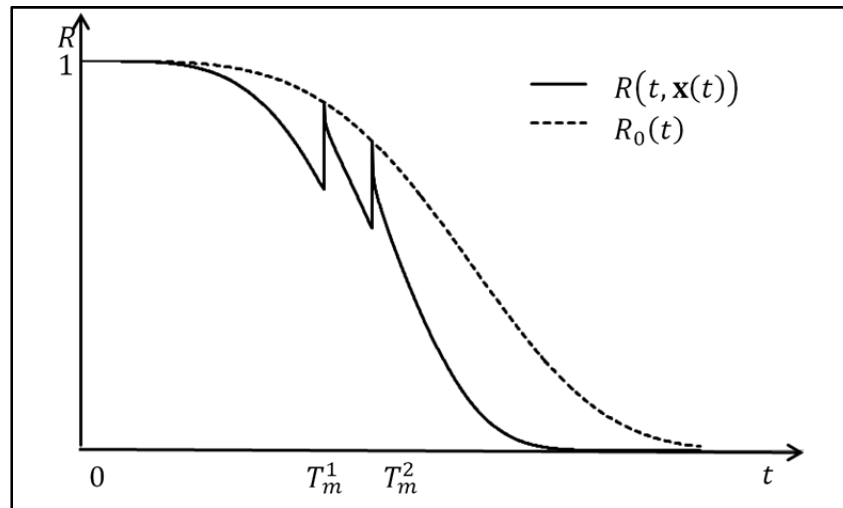


Figure 5. The Value of $R(t, \mathbf{x}(t))$ When $\mathbf{x}(T_m^j) = \mathbf{0}$ Is Restored to $R_0(t)$

Stated previously, when a maintenance operation takes place at time T_m^j , $x_i(t)$ returns to the preferred condition of $x_i(t) = 0$, but the original rate of degradation

should persist. If degradation is assumed to be modeled with $x_i(t) = t^{d_i}$ for $0 < d_i < 1$, then a different representation of degradation would have to account for maintenance at T_m^j . This is found in Equation 8, effectively allowing the “time” immediately after maintenance to taken on the value zero. Iterations of maintenance following Equation 8 are illustrated in Figure 6.

$$x_i(t) = x_{i0}(t - T_m^j), \text{ for } T_m^j < t < T_m^{j+1} \quad (8)$$

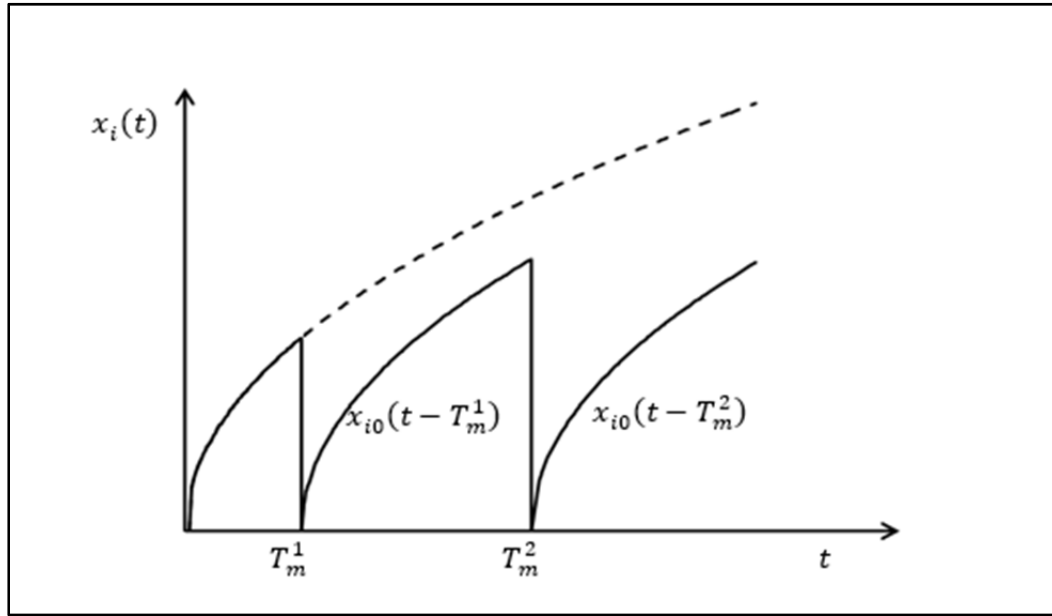


Figure 6. The Degradation Path Following Maintenance Operations at T_m^j

Formulating the Multiobjective Framework

The contribution of this methodology lies with the use of a sensitivity metric for concentrating maintenance efforts to determine T_m^j . This is done by calculating the sensitivity of reliability to changes in $x_i(t)$, thus providing (i) the states of the system that have the largest influence on the degradation of reliability, and (ii) the time at which these states have this influence. The USIM approach is used to incorporate sensitivity; thus, the partial derivative of the reliability calculation with respect to $x_i(t)$ must be found, as in Equation 9.

$$\frac{\partial R(t, \mathbf{x}(t))}{\partial x_i} = -\beta_i \left(\frac{t}{\lambda}\right)^k \exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \exp\left(-\left(\frac{t}{\lambda}\right)^k\right) \exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \quad (9)$$

Equation 10 provides the sensitivity metric $\psi_i^R(t, \mathbf{x}(t))$, describing the sensitivity of component reliability to changes in particular state variable $x_i(t)$. One would anticipate a sensitivity metric much less than one, as reliability is on $[0, 1]$. A sensitivity function for all state variables, modeled from Equation 5 for p state variables, is defined in Equation 11.

$$\psi_i^R(t, \mathbf{x}(t)) = \left[-\beta_i \left(\frac{t}{\lambda}\right)^k \exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \exp\left(-\left(\frac{t}{\lambda}\right)^k \exp(\boldsymbol{\beta}^T \mathbf{x}(t))\right) \right]^2 \quad (10)$$

$$\psi^R(t, \mathbf{x}(t)) = \sum_{i=1}^p \psi_i^R(t, \mathbf{x}(t)) \quad (11)$$

The multiobjective formulation is defined in Equation 12, maximizing both the component reliability and maximizing sensitivity (or finding the time at which reliability is most sensitivity to changes in $\mathbf{x}(t)$). Only non-negativity constraints are considered in Equation 12, though other possibilities include cost and/or reliability constraints (Tian et al., 2012). The states-of-the system are ultimately a function of t ; therefore, the decision variable is the time at which maintenance should be performed. Recall, as maintenance scheduling is multistage (e.g., maintenance is a repeated event), the time at which maintenance is scheduled for iteration j is T_m^j .

$$\begin{aligned} & \max_t R(t, \mathbf{x}(t)) \\ & \max_t \psi^R(t, \mathbf{x}(t)) \\ & \text{s. t. } t > 0 \end{aligned} \quad (12)$$

The sensitivity metric for the i th state variable, $\psi_i^R(t, \mathbf{x}(t))$, has a common component regardless of $x_i(t)$. This term is defined as $\psi_c(t, \mathbf{x}(t))$, found in Equation 13. ψ_i^R is then only a function of β_i and ψ_c , shown in Equation 14. Similarly, the sensitivity metric for all state variables is provided in Equation 15.

$$\psi_c(t, \mathbf{x}(t)) = \left(\frac{t}{\lambda}\right)^k \exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \exp\left(-\left(\frac{t}{\lambda}\right)^k \exp(\boldsymbol{\beta}^T \mathbf{x}(t))\right) \quad (13)$$

$$\psi_i^R(t, \mathbf{x}(t)) = [-\beta_i \psi_c(t, \mathbf{x}(t))]^2 \quad (14)$$

$$\psi^R(t, \mathbf{x}(t)) = [\psi_c(t, \mathbf{x}(t))]^2 \sum_{i=1}^p \beta_i^2 \quad (15)$$

As β_i is a constant with respect to t , $\psi^R(t, \mathbf{x}(t))$ is maximized with maximum $\psi_c(t, \mathbf{x}(t))^2$. As such, the multiobjective formulation is updated in Equation 16.



$$\begin{aligned}
& \max_t R(t, \mathbf{x}(t)) \\
& \max_t \psi_c(t, \mathbf{x}(t))^2 \\
& \text{s. t. } t > 0
\end{aligned} \tag{16}$$

Identifying Pareto Optimal Frontier Boundaries

Understanding the form of $R(t, \mathbf{x}(t))$ and $\psi_c(t, \mathbf{x}(t))^2$ will assist in determining the boundaries for the generation of the Pareto-optimal frontier balancing R and ψ_c^2 . Figure 3 presents the shape of $R(t, \mathbf{x}(t))$ dictated by the bounds of the parameter k , and a similar statement is made about the shape of $\psi_c(t, \mathbf{x}(t))^2$. Figure 7 illustrates the shape of both functions on shared axes. The value on the horizontal axis will vary between the current t , which is 0 initially, and t_{\max} . Beyond $\psi_c(t_{\max}, \mathbf{x}(t_{\max}))^2$, both functions decrease in t , therefore, the two objectives no longer compete (thus not generating a Pareto-optimal frontier).

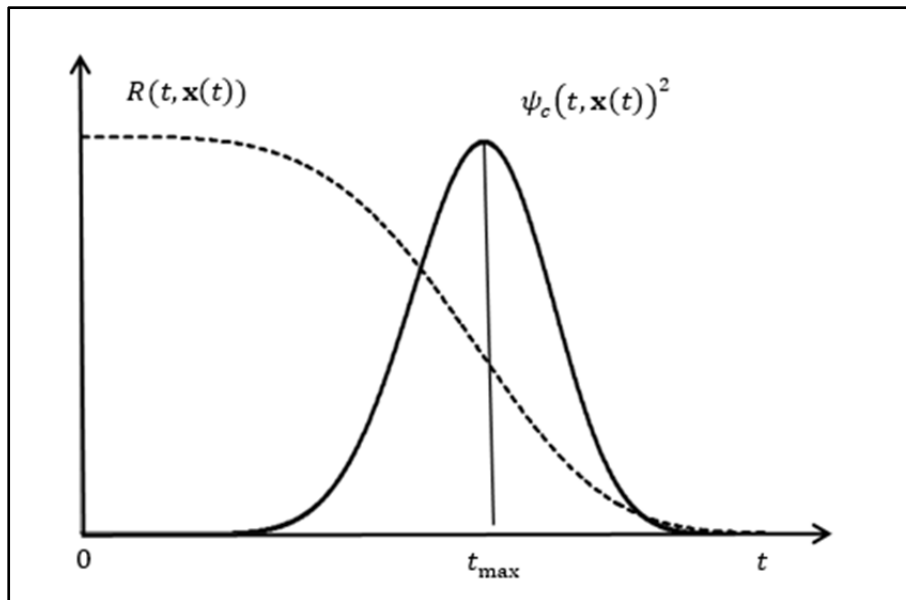


Figure 7. Portrayal of the Bounds of t

The upper bound of t , t_{\max} , is found by maximizing $\psi_c(t, \mathbf{x}(t))^2$, or setting its partial derivative with respect to t to zero and solving for t_{\max} . This derivative yields a complex equation decomposed into components L and M , or $\partial \psi_c(t, \mathbf{x}(t))^2 / \partial t = LM$, where LM is graphed in Figure 8 with shape defined from the bounds of k , as in Figure 3. Components L and M are found in Equations 17 and 18, where $\mathbf{d} = (d_1, \dots, d_i, \dots, d_p)$, the degradation coefficients for all state variables. An optimization algorithm (e.g., Newton's method) can be used to solve for t_{\max} . Although time is a

continuous variable, the value of t_{\max} is more realistically rounded to an integer value.

$$L = \frac{2R_0(t)^2 \exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \exp(2\boldsymbol{\beta}^T \mathbf{x}(t)) \left(\frac{t}{\lambda}\right)^{2k}}{t} \quad (17)$$

$$M = k + [\text{diag}(\mathbf{d})\boldsymbol{\beta}]^T \mathbf{x}(t) - k \exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \left(\frac{t}{\lambda}\right)^k + [\text{diag}(\mathbf{d})\boldsymbol{\beta}]^T \exp(\boldsymbol{\beta}^T \mathbf{x}(t)) \ln(R_0(t)) \quad (18)$$

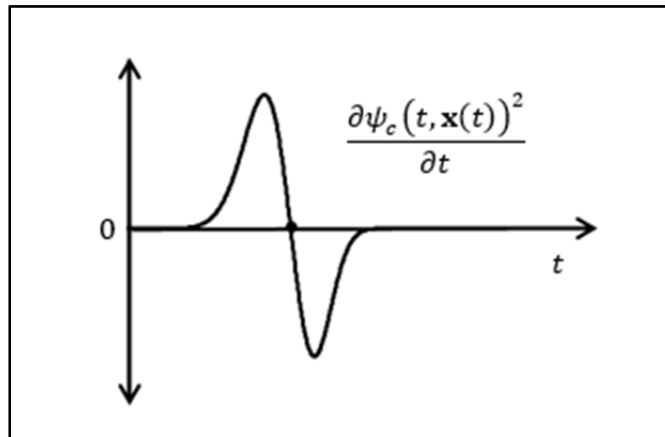


Figure 8. Solving for t_{\max} With $LM = 0$

The Pareto Frontier and Reiteration of the Decision Framework

A Pareto-optimal frontier is defined within the boundary of 0 and t_{\max} . Figure 9 illustrates the Pareto-optimal frontier from which T_m^1 will be selected and represents the competition among the objectives where $R(t, \mathbf{x}(t))$ decreases in t while $\psi_c(t, \mathbf{x}(t))^2$ increases in t . We know that the maintenance trigger will lie on $[0, t_{\max}]$.

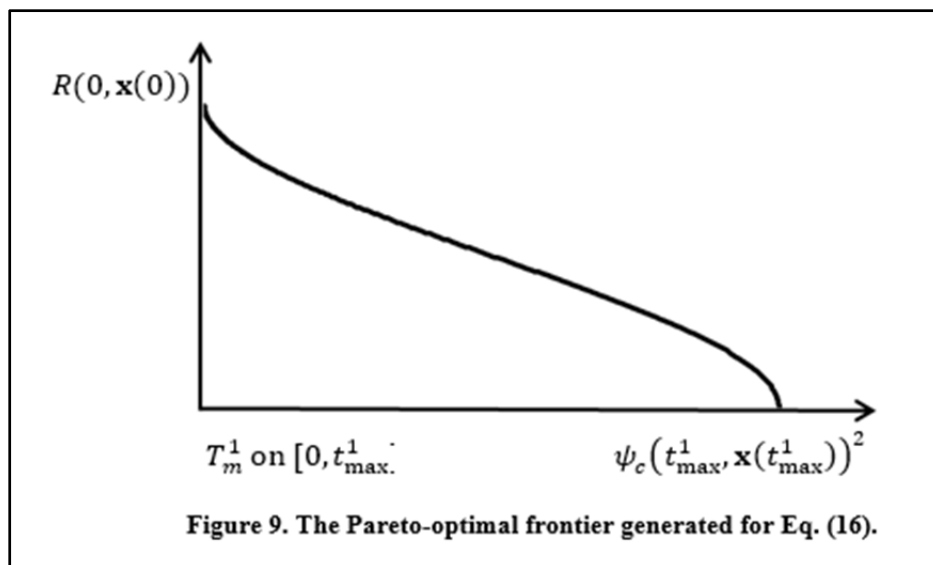


Figure 9. The Pareto-optimal frontier generated for Eq. (16).

Figure 9. The Pareto-Optimal Frontier Generated for Equation 16

Figure 10 provides the depiction of the changing Pareto optimal frontier with each iteration. The reliability decreases with each iteration. Replacement of the component may be preferable at T_m^j if $R(T_m^j, \mathbf{x}(T_m^j) = 0)$ is too low based on decision-maker preference.

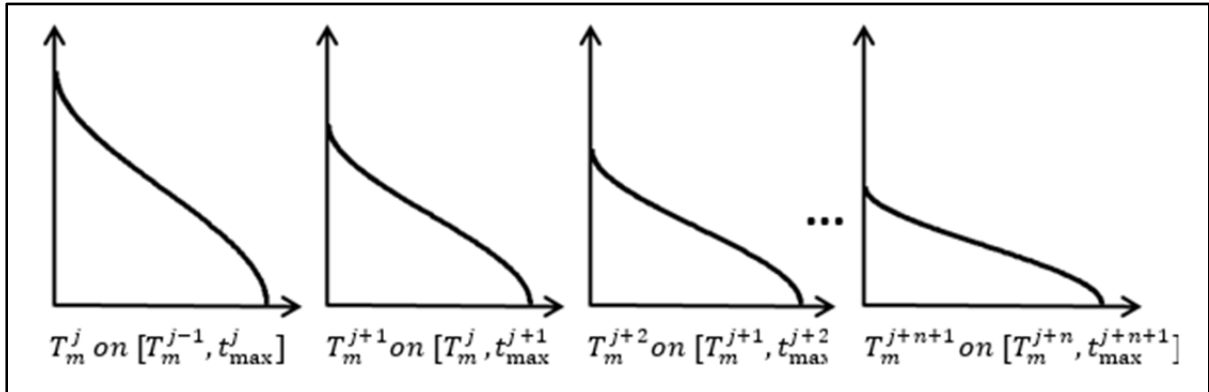


Figure 10. Depiction of the Iterative Maintenance Decision Framework

Criteria for Determining the Value of the Decision Variable (T_m^j)

Using Figure 11, the maximum value of $\psi_c(t, \mathbf{x}(t))^2$ occurs at t_{\max}^j while the maximum value of $R(t, \mathbf{x}(t))$ occurs at T_m^j on the Pareto-optimal frontier. Knowing $T_m^j < t < t_{\max}^{j+1}$, we illustrate the loss in each objective function with respect to t in Figure 11. Although both functions still depend on $\mathbf{x}(t)$, the equations used in Figure 11 are only for demonstration of loss in each objective with respect to t . A value for t such that a compromise among the loss in each objective is achieved will label T_m^{j+1} .

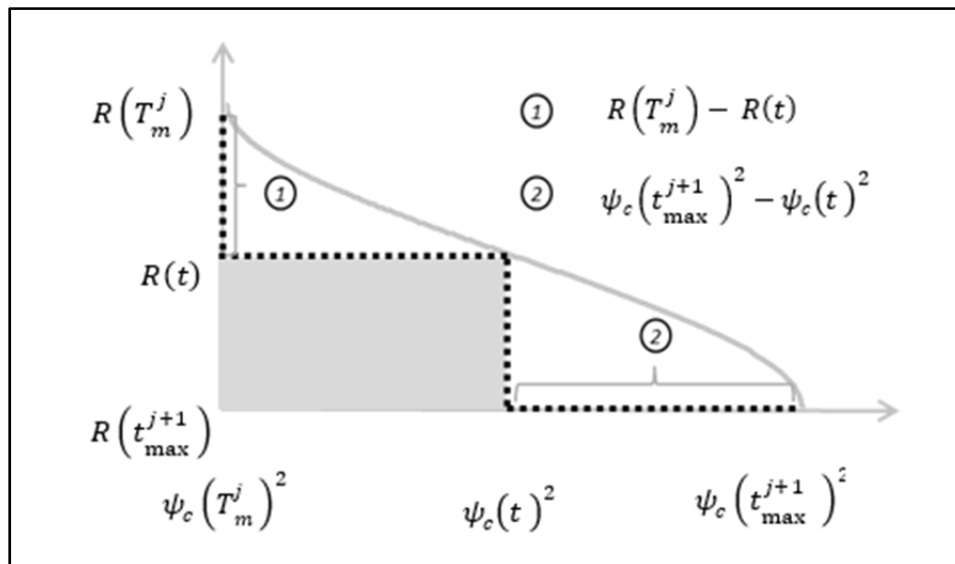


Figure 11. The Loss in Objective Values With Respect to t on the Pareto Frontier

Let $\lambda_\psi(t)$ be a unitless ratio of the deviation from the maximum functional value of $\psi_c(t, \mathbf{x}(t))^2$ over t divided by the total possible loss in $\psi_c(t, \mathbf{x}(t))^2$. And similarly, let $\lambda_R(t)$ be a unitless ratio of the deviation from the maximum functional value of $R(t, \mathbf{x}(t))$ over t divided by the total possible loss in $R(t, \mathbf{x}(t))$. We represent $\lambda_\psi(t)/\lambda_R(t)$ with $\lambda_{\psi R}(t)$ as in Equation 19, which provides a means to quantify the tradeoff between the two non-commensurable objectives at time t . The value of this ratio is subject to the decision-maker: some decision-makers may give more focus to changes in reliability over time, while others may emphasize the sensitivity of reliability to degradation in $\mathbf{x}(t)$. This point of compromise gives value to T_m^{j+1} , the time at which the $j+1$ maintenance iteration is performed. For example, a risk neutral decision-maker may assume $\lambda_{\psi R}(t) = 1$, an equal balance between the two objectives: this work does not elicit decision-maker preference and therefore assumes risk neutrality for determining each maintenance trigger.

$$\lambda_{\psi R}(t) = \frac{\left(\frac{\psi_c(t_{\max}^{j+1}, \mathbf{x}(t_{\max}^{j+1}))^2 - \psi_c(t, \mathbf{x}(t))^2}{\psi_c(t_{\max}^{j+1}, \mathbf{x}(t_{\max}^{j+1}))^2 - \psi_c(T_m^j, \mathbf{x}(T_m^j))^2} \right)}{\left(\frac{R(T_m^j, \mathbf{x}(T_m^j)) - R(t, \mathbf{x}(t))}{R(t_{\max}^{j+1}, \mathbf{x}(t_{\max}^{j+1})) - R(T_m^j, \mathbf{x}(T_m^j))} \right)} \quad \text{for } T_m^j < t < t_{\max}^{j+1} \quad (18)$$

Illustrative Example

This section provides an illustrative example of implementation of the methodological components for scheduling maintenance on an aircraft engine.

Specification of Input Parameters for Modeling Reliability and Degradation

Due to the proprietary nature of the problem at hand, we provide simulated data for five general state variables, $\mathbf{x}(t) = (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t))$, where the unit of time is measured in hours flown. The degradation paths of these state variables are assumed to follow the monotonically increasing functions in Table 1, and the trajectories are depicted graphically in Figure 12. In order to simulate states of the system, it is assumed that $x_i(t)$ is normally distributed with mean $\mu_i(t)$ and constant standard deviation σ_i . Likely, σ_i would be small at early time periods and increase over time.



Table 1. Functional Mean Trajectory of State Variable Degradation With Standard Deviations

State Variable	$\mu_i(t)$	σ_i
$x_1(t)$	$t^{1/3}$	3
$x_2(t)$	$t^{1/4}$	3
$x_3(t)$	$t^{1/5}$	2
$x_4(t)$	$t^{1/7}$	2
$x_5(t)$	$t^{1/10}$	2

From historic data, a Weibull baseline hazard function was fit with parameters $k = 4$ and $\lambda = 816.75$. Failure data were then simulated from this Weibull baseline: actual observations were not used. The time scale is limited to 1200 flight hours, which is the point where failure is assumed to occur with a likelihood of 1.

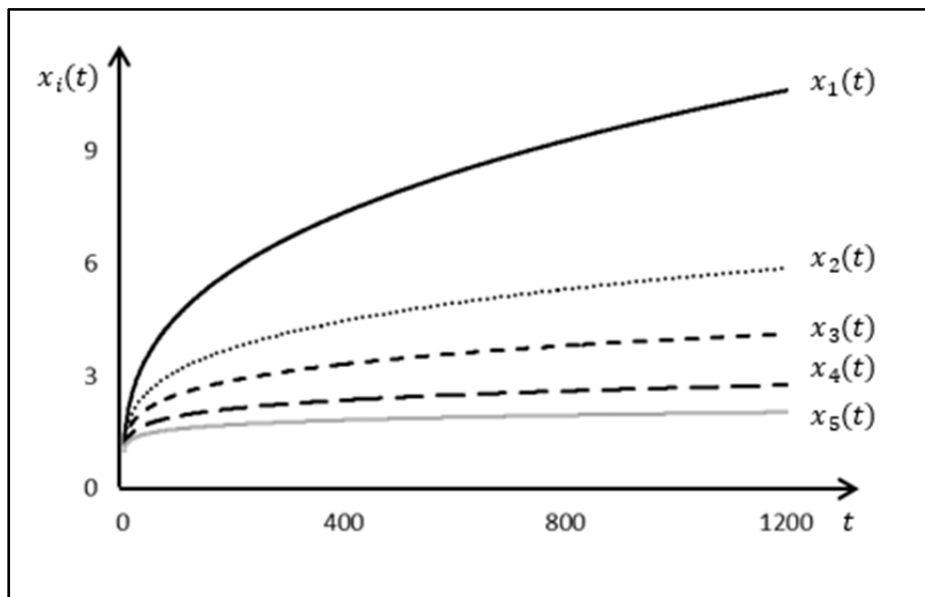


Figure 12. The Trajectories of Degradation for Each State Variable

A CPHM was fit to the data simulated and the maximum likelihood estimates of the coefficients are provided in Table 2. All state variables were found to be significant, and each coefficient estimate is positive indicating decreasing reliability with degradation as expected.

Table 2. CPHM Coefficient Estimates

State Variable	β_i	Standard Error	p -value
$x_1(t)$.0400	.0200	.0459
$x_2(t)$.0471	.0239	.0484
$x_3(t)$.0667	.0333	.0457
$x_4(t)$.1006	.0464	.0301
$x_5(t)$.0878	.0434	.0431

Implementation of the Multiobjective Decision Framework

The reliability function, $R(t, \mathbf{x}(t))$ given the set of input parameters, is simulated over 0 to 1200 flight hours, resulting in Figure 13. The sensitivity function, $\psi_c(t, \mathbf{x}(t))^2$, as well as its derivative with respect to t , is simulated in Figure 13 to aid in determining t_{\max}^1 . The value for t such that $\psi_c(t, \mathbf{x}(t))^2$ is maximized, where a maximum is implied by $\partial\psi_c(t, \mathbf{x}(t))^2/\partial t = 0$, is 584 flight hours. Therefore, $t_{\max}^1 = 584$ and is used for forming the Pareto optimal frontier for iteration $j = 0$.



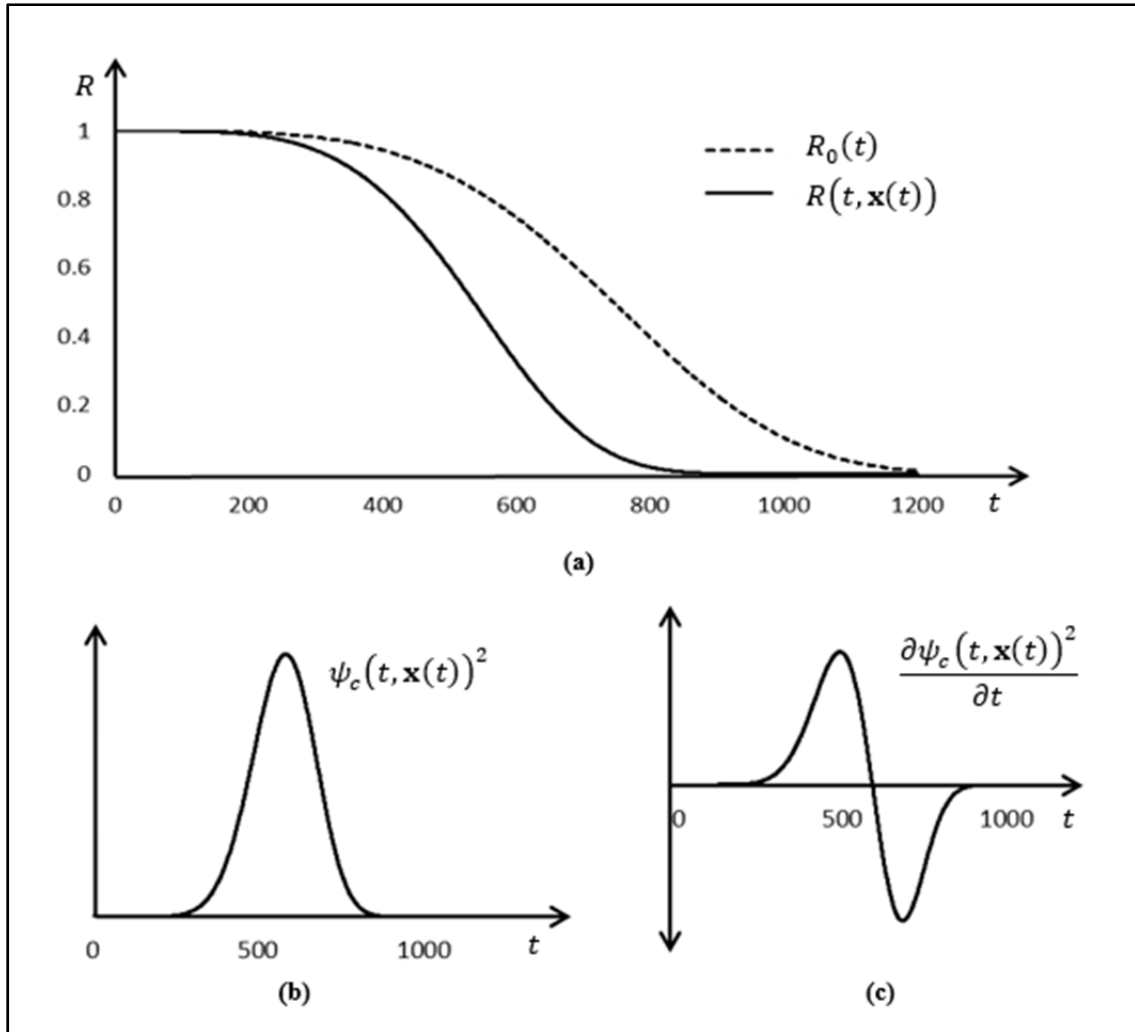


Figure 13. The Functions (a) $R(t, \mathbf{x}(t))$, (b) $\psi_c(t, \mathbf{x}(t))^2$, and (c) $\frac{\partial \psi_c(t, \mathbf{x}(t))^2}{\partial t}$ Simulated for $j = 0$

The plot of $R(t, \mathbf{x}(t))$ versus $\psi_c(t, \mathbf{x}(t))^2$ is displayed in Figure 14. In addition, Figure 14 provides the Pareto optimal frontier that is defined with $t_{\max}^1 = 584$. We can now express the range of values of the first maintenance trigger, T_m^1 , with the interval $[T_m^0 = 0, t_{\max}^1 = 584]$.

In order to determine T_m^1 , $\lambda_{\psi_R}(t)$ is used for uncovering t such that a compromise among the loss in each objective is attained. The ideal maintenance trigger is mathematically defined as the value of t such that $\lambda_{\psi_R}(t) = 1$. This maintenance trigger represents the point in time that the ratio of the loss in both objectives is equivalent. Using $\lambda_{\psi_R}(t) = 1$, T_m^1 is rounded to 466. Maintenance is performed by restoring degradation levels to nominal values and is modeled mathematically using $\mathbf{x}(T_m^1 = 466) \approx 0$.

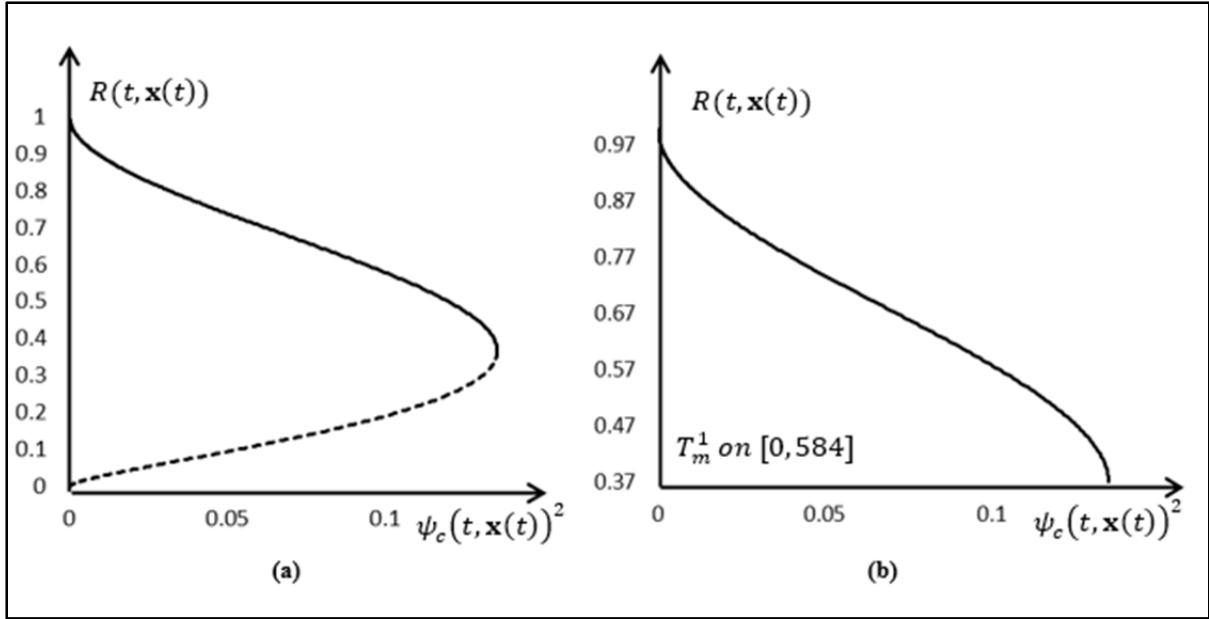


Figure 14. The (b) Pareto-Optimal Frontier Defined With t_{max}^1 to Remove the Inferior Solutions From (a)

Table 3 presents a summary of the results of the decision-making framework using the steps illustrated for $j = 0$. As previously mentioned, $T_m^1 = 466$ and the reliability at T_m^1 before maintenance is $R(466, \mathbf{x}(466)) = 0.68$. After performing maintenance, $R(466, \mathbf{x}(466)) = 0.89$ and is equal to the baseline reliability, $R_0(466)$. For the next iteration $j = 1$, $R(t, \mathbf{x}(t))$ is simulated over the interval $T_m^1 = [466, 1200]$. The point of maximum sensitivity, $\psi_c(t, \mathbf{x}(t))^2$, is found to be $t_{max}^2 = 633$; therefore, T_m^2 is on the interval $[T_m^1 = 466, t_{max}^2 = 633]$. The second maintenance trigger defined using $\lambda_{\psi_R}(t) = 1$, is $T_m^2 = 533$ as provided in Table 3. The same steps are implemented in subsequent iterations for determining each maintenance trigger in order to create a full maintenance schedule.

Table 3. Summary of Results for Each Iteration

j	t_{max}^{j+1}	T_m^{j+1}	$R(T_m^{j+1}, \mathbf{x}(T_m^{j+1}))$	$R_0(T_m^{j+1})$
0	584	466	0.68	0.89
1	633	553	0.60	0.81
2	651	576	0.62	0.78
3	657	591	0.60	0.76
4	662	602	0.59	0.74
5	665	611	0.58	0.73

The evolution of the Pareto-optimal frontier is presented in Figure 15. The time between each maintenance trigger decreases with each iteration, and this trend is mathematically realized using the $\lim_{j \rightarrow \infty} (T_m^{j+1} - T_m^j) = 0$. Additionally, the $\lim_{j \rightarrow \infty} (T_m^j - t_{\max}^{j+1}) = 0$; therefore, the final maintenance trigger T_m^{j+n+1} is on the interval $[T_m^{j+n}, t_{\max}^{j+n+1} = T_m^{j+n}]$, and the action at T_m^{j+n+1} is restricted to replacement rather than maintenance. Replacement is signified mathematically by setting $t = 0$. The point where this occurs is at 718 flight hours, but we also know that intervals between maintenance triggers are decreasing. Realistically, maintenance intervals can be considered infeasible. A simple example of infeasibility due to an interval that is too small is when the maintenance interval is less than the minimum required hours of flight. Reliability in this illustrative example is heavily influenced by degradation causing maintenance intervals to lessen rapidly. The maintenance interval between $j = 4$ and $j = 5$ is only nine flight hours. The decision to replace at an earlier time than the forced replacement due to the natural progression of the decision framework is up to the decision-maker for ensuring feasibility of maintenance intervals. The final maintenance trigger selected for this example is T_m^6 as the maintenance intervals lessen.

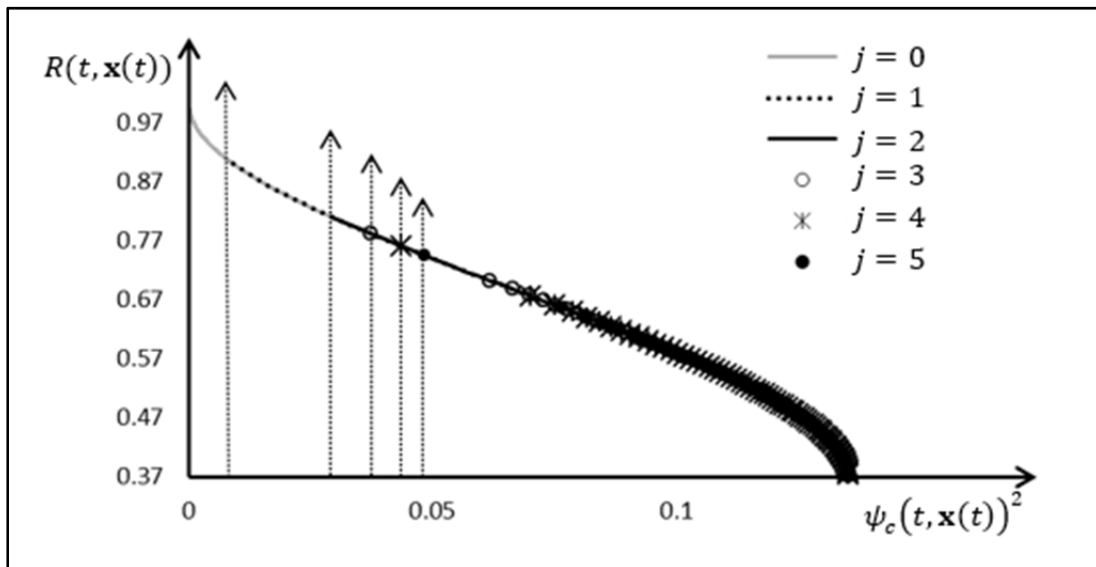


Figure 15. The Progression of the Pareto Optimal Frontier Over j

Assessment of Performance

An ideal maintenance schedule will increase an engine's life cycle, decrease the amount of maintenance required, and decrease the risk of failure. It is common practice for maintenance optimization models to impose reliability constraints, imposing that maintenance operations be performed prior to some minimum reliability level. As such, we compare the method presented in this research with

maintenance scheduling solely based on a predetermined reliability requirement. Given the required level of reliability, maintenance is scheduled as soon as the requirement is violated.

A simplified version of a cost function for a maintenance schedule can be thought of as the sum of the cost of maintenance, and the expected cost of failure to the schedule is terminated at $j = 5$ because of the decreasing time and reliability increments between maintenance triggers. As such, we compare the different scheduling approaches at a reference point of six maintenance tasks, which allows us to assume that the cost of maintenance on the engine is the same for each scheduling procedure. Although the cost of maintenance between the different methods is equivalent due to a baseline of six maintenance tasks, the expected failure cost is different between the approaches due to the likelihoods of failure differing between the approaches. As such, the differing failure likelihoods provide insight for distinguishing between the costs of each method in a relative manner without need to enumerate explicit monetary values. $\bar{R}(T_m^j)$ is derived by averaging the reliability at each discrete maintenance trigger, thus providing insight on the differing failure costs between the approaches using $1 - \bar{R}(T_m^j)$. As the actual cost of failure is constant between the methods, the expected failure cost (e.g., $(1 - \bar{R}(T_m^j))C_{\text{Failure}}$) differs by the value of $1 - \bar{R}(T_m^j)$. Other performance measures include the age of the engine at T_m^6 represented by t , the life cycle of the engine represented with $\max t$, the average maintenance interval $\overline{\Delta T_m^j}$, and lastly the average maintenance interval after the first maintenance task.

Table 4. Performance Comparisons at a Baseline of T_m^6

Goal	Performance	$R = 0.5$	$R = 0.6$	$R = 0.7$	$R = 0.8$	ψR
Min	$1 - \bar{R}(T_m^j)$	0.5	0.4	0.3	0.2	0.3883
Max	t	656	612	558	498	611
Max	$\max t$	745	690	631	561	718
Max	$\overline{\Delta T_m^j}$	109	102	93	83	102
Max	$\overline{\Delta T_m^j}$ from $T_m^1 : T_m^6$	24	22	20	17	23

The number of maintenance tasks between the methods is the same due to the baseline of T_m^6 ; therefore, it may appear that one should choose a reliability constraint of $R = 0.8$ to decrease the expected cost of failure. Although we used this baseline for a means of comparison between the approaches, it is incorrect to assume that each approach requires the same extent of maintenance. The value of t provides the engine age at T_m^6 ; thus, t provides a means for reasonably comparing



the amount of required maintenance between the approaches. We wish to maximize each of the performance measures, with the exception of failure likelihood, in order to decrease both maintenance and failure cost.

A comparison of values of t at T_m^6 shows that a low threshold of reliability (e.g., $R = 0.5$), not surprisingly the risk of failure is increased while the maintenance required is decreased. By imposing more stringent reliability constraints (e.g., $R = 0.8$), the risk of failure is decreased while the maintenance requirements are increased. In addition, the $R = 0.8$ constraint converges to an infeasible maintenance interval faster than the $R = 0.5$ constraint. By applying a reliability constraint, one naturally imposes higher amounts of required maintenance or higher risk of failure necessitating one to have a preset idea of failure cost versus maintenance cost in order to reach a desirable tradeoff. Further, when implementing reliability constraints, one must also have an idea of when replacing is more cost effective than additional maintenance actions to increase engine life cycle.

It is not sufficient to impose reliability constraints without fully understanding the ramifications from doing so. The method presented in this paper, denoted by ψR in Table 4, does not require that a fixed level of reliability be preserved. This method performance closely matches a middle ground reliability constraint as with the 0.6 constraint, but does not require replacement to occur as quickly. Using this method allows one to model reliability as an objective rather than constraint to avoid some of the impacts of incorporating a rigid reliability constraint, the impacts of which include (i) inherent inflation in maintenance cost or (ii) inherent rise in failure cost, and (iii) a constraint driven replacement schedule. Future work lies in using this new method paired with cost constraints for determining an economical maintenance schedule.

Concluding Remarks

This paper contributes a novel methodology that integrates (i) models of reliability that incorporate time-varying covariates with (ii) models of degrading state variables that (iii) lead to maintenance scheduling designed around reliability and sensitivity. We adopt degradation models for describing state variables in the proportional hazards model, where many previous uses of the CPHM rely on a stochastic process approach with a finite state space.

Further, the use of condition variables allows for the direct modeling of maintenance impact with the assumption that a nominal value, perfect diagnostic condition, exists. With this information, maintenance can be scheduled as well as planned for what degree of maintenance is required. There is no “renewal upon repair” assumption within this work.

Sensitivity analysis is commonly used for understanding uncertainty impact, but in this paper it is uniquely paired with reliability for a more mathematical



(multiobjective) treatment of prompting the time at which maintenance occurs. A Pareto-optimal frontier is found during each iteration of the multistage problem where a tradeoff function is used to define the decision variable representative of a best compromise solution. Future research will be done to assess the usefulness of the method with elicited preference from a decision-maker.

Moreover, in many optimization problems, reliability is often implemented as a constraint. By doing so, one must understand the relationship between maintenance cost and failure cost beforehand, which can be very difficult to do as maintenance cost depends on the amount of maintenance required. This paper provides the foundation for implementing reliability as an objective with future work centered on incorporating a cost constraint to create an economical maintenance schedule in order to avoid inherent influences on performance from imposing an inflexible reliability constraint.

Task 2. Expand Task 1 to Multi-Component Systems

This section is based on the following:

Kalam, S., Barker, K, & Ramirez-Marquez, J. E. (in press). *Scheduling multi-component maintenance with a greedy heuristic local search algorithm*. Manuscript submitted for publication.

Introduction

Large organizations such as the Department of Defense (DoD) have to devote a significant amount of their budgets to system maintenance. According to a 2007 Government Accountability Office (GAO) report, the DoD spends approximately 40% of its budget on operations and management (O&M) activities to ensure system readiness (\$209.5 billion in 2005). The GAO reported that since fiscal year 2001, the DoD's O&M costs are increasing, and the Air Force, in particular, had to increase its O&M cost by 29%. As many large-scale DoD systems age, and due to budgetary and performance efficiency concerns, there is a need to improve the decision-making process for system sustainment, including maintenance, repair, and overhaul (MRO) operations and the acquisition of MRO parts.

The DoD's acquisition costs have seen growth in recent years (GAO, 2013). The GAO (2013) recommended that the DoD improve its strategic management plan to make maintenance supply chain operations more cost effective. Further, the DoD was advised to "link acquisition and sustainment policies" for depot maintenance improvement and ultimate cost efficiency (GAO, 2011). To help address the link between sustainment policies and acquisition, this work develops a framework to provide a system maintenance schedule for multi-component systems. As the



multiple components of a system have their own life cycles, an efficient means to schedule overall system maintenance should consider these individual components to maximize long-term availability of the system. This framework coordinates recommended maintenance times, such as those found as a result of reliability centered maintenance (RCM) or from original equipment manufacturer (OEM) suggestions, to formulate a system-level maintenance schedule for a finite time horizon. Such a framework will increase the acquisition efficiency of components with a more effective system level maintenance schedule.

With recent computational advances, several preventive maintenance models have been proposed for complex multi-component systems considering component interactions. In the preventive maintenance scheduling problem (PMSP), different kinds of component interactions are taken into account (Dekker, Wildeman, & van der Duyn Schouten, 1997).

Several model formulations, as well as solution techniques, have been proposed to address preventive maintenance decision-making. Stinson and Basheer (1987) formulated a heuristics-based mixed integer linear program (MILP) model for a finite horizon preventive maintenance problem for maintaining machines in series. Budai, Huisman, and Dekker (2005) proposed a heuristics-based MILP solution for scheduling railroad network maintenance. Recently, Roux, Duvivier, Quesnel, and Ramat (2010) developed a hybrid model by integrating optimization and simulation to minimize unavailability of the system due to preventive block maintenance policy. Other few noteworthy approaches are the Bayesian network model (Celeux, Corset, Lannoy, & Ricard, 2006), goal programming for a multiobjective problem (Bertolini & Bevilacqua, 2006), and dynamic programming (Dekker, Wildeman, & Van Egmond, 1996).

In terms of algorithm development, Dekker, Smit, and Losekoot (1991) presented an optimal maintenance model using set-partitioning algorithm for multiple maintenance activities. One downside of their model was that they considered each activity time to be negligible relative to the total planning horizon. Later, Dekker et al. (1996) solved the above-mentioned problem with a dynamic programming formulation, concluding that the dynamic algorithm is a good heuristic for rolling horizon based problems which can incorporate short-term system information for decision support. Dekker et al. (1997) provided a review of maintenance models for multi-component systems which covered economically dependent systems. The Markov decision chain-based approach was also studied by Dekker et al. (1996) for the multi-activities maintenance problem, which was applicable to system consisting of many components. Previous Markov chain-based models were limited to few components. An opportunistic maintenance policy was modeled by Gürlér and Kaya (2002) and van der Duyn Schouten and Vanneste (1993) for identical multi-



component system. Sheu and Jhang (1996) modeled a similar kind of problem with a two-stage opportunistic policy. In case of non-identical components maintenance, the tradeoff of repair cost of one component versus another should be considered, including the resulting increase in the complexity of the model.

PMSP remains a very active area of research. Little work in this field has used heuristics and meta-heuristics–based methodologies to model preventive maintenance framework (Nicolai & Dekker, 2008). A new meta-heuristic based on a genetic algorithm was applied in train maintenance scheduling problems by Sriskandarajah, Jardine, and Chan (1998), primarily optimizing cost. Nicolai and Dekker (2008) presented a review of preventive maintenance and recommended that more researches need to be done in this area developing more heuristic and meta-heuristic approaches like simulated annealing and local search. Meta-heuristic–based algorithms have proven very successful for flowshop scheduling problems (Pan & Ruiz, 2012), which has similar characteristics to preventive maintenance scheduling.

This work presents a greedy heuristic-based local search algorithm for preventive maintenance of multiple components, which would be a new contribution in this field of research. This work develops a local search-based algorithm with the objective in mind to minimize the total maintenance cost over a finite planning horizon.

Methodological Development

Here we develop a new formulation and solution algorithm to address preventive maintenance scheduling for a multi-component system. It is assumed that maintenance results in a “good-as-new” condition. Baseline individual component maintenance times for planning horizon T (i.e., system-in-use time) are known and recommended based on a mean time between failure (MTBF) calculation (e.g., by RCM calculations or OEM recommendations). We assume these recommended component maintenance times are given in their in-use-time or up-time. The k th recommended maintenance time for component l , or T_l^k , is illustrated in Figure 16 for components A, B, and C.



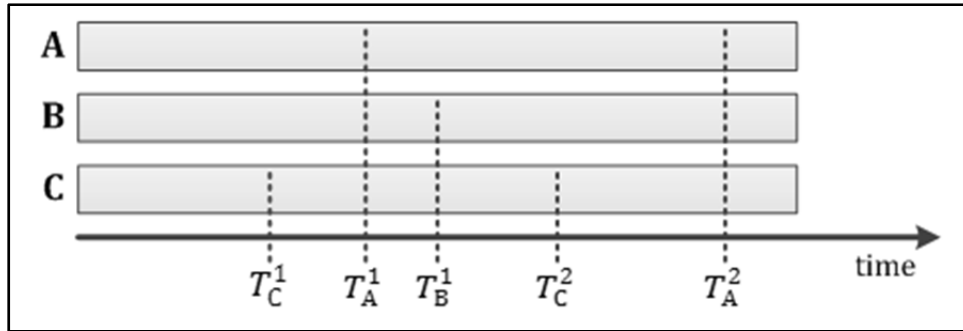


Figure 16. Example Recommended Maintenance Times for Components A, B, and C

Our goal is to recommend altering the recommended maintenance schedule for a multi-component system in a joint manner for as many components as possible. Performing many individual component maintenance events at once can potentially lead to cost savings due to reduced setup costs and reduced downtime. However, varying too far from recommended MTBF guidance can lead to unnecessary maintenance (in the earliness situation) and risk of failure (in the tardiness situation). *Earliness* refers to the performance of maintenance earlier than recommended, with *tardiness* representing the performance of maintenance at a time later than recommended. As such, there are penalties associated with both earliness and lateness, as well as a penalty for system downtime while maintenance is being performed. If the j th system maintenance is performed at time t^j , Figure 17 provides an example of earliness and tardiness for three components relative to their recommended individual component maintenance times, T_l^k .

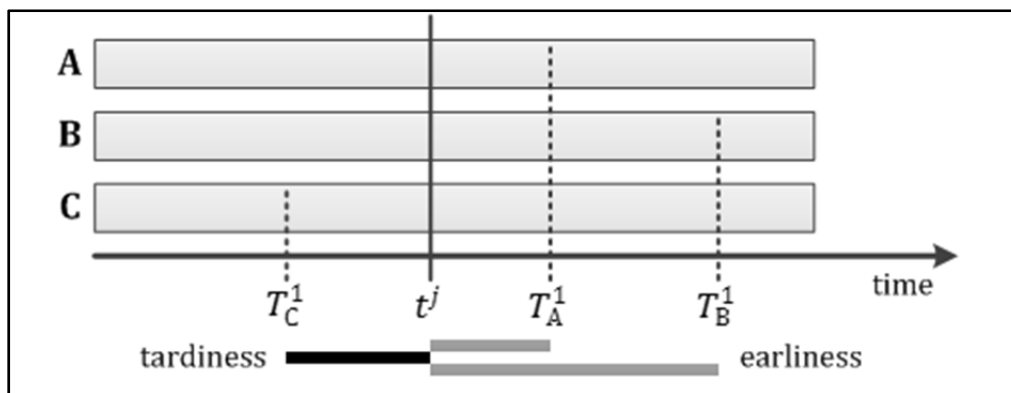


Figure 17. Example Earliness and Tardiness System Maintenance Times for Components A, B, and C

Different potential maintenance schedules can be compared and evaluated using a penalty function approach (Yousefi & Yusuff, 2013). In this approach, without being explicit on the dollar values associated with “costs,” a penalty function quantifies (i) setup related costs, (ii) downtime costs, and (iii) the expenses of

earliness (i.e., costs of unnecessary maintenance) and costs potential failure due to tardiness. By implementing this approach, a maintenance schedule can be found that will minimize these penalties. These penalties, as well as other notation, are defined as follows:

T	planning horizon
n	number of components in the system
T_l^k	k th recommended maintenance time for component l
C_S	system setup penalty per maintenance
C_D	system downtime penalty per unit time
$C_{E,l}$	earliness penalty for component l , per unit time
$C_{L,l}$	tardiness penalty for component l , per unit time
D_l	component maintenance duration for component l
Δ^j	deviation of individual component recommended maintenance times from j th system maintenance time
T_c	construction phase time-span
T_c^{\max}	maximum possible construction phase time-span
δ	construction phase time-span parameter where $\delta \in (0, 1]$
γ	joint maintenance time parameter $\gamma \in (0, 1]$
\mathcal{T}_1	set of first component recommended maintenance times
\mathcal{T}_2	set of second component recommended maintenance times
π^c	candidate solution
π^d	discard solution
S^c	candidate combination set
S^d	candidate discard set
S	algorithm solution vector

Decision variables for the scheduling formulation include the following:

t^j	j th system maintenance time
M	total number of system maintenance events scheduled in planning horizon T
x_l^j	if feature earliness is present in component l for maintenance j ($x_l^j = 1$) or not ($x_l^j = 0$)
y_l^j	if feature tardiness is present in component l for maintenance j ($y_l^j = 1$) or not ($y_l^j = 0$)
z_l^j	if component l should be repaired at time t^j ($z_l^j = 1$) or not ($z_l^j = 0$)

Performing joint repair has the potential to save maintenance cost, because for many multi-component systems it is possible to perform component maintenance



simultaneously. Thus, total repair time for joint maintenance depends on individual instance and can be predicted from previous system maintenance data.

Total Penalty Minimization Problem

One objective is to minimize system downtime due to maintenance over a finite time horizon T . As a proxy for downtime, several penalty functions comprise the objective function, ultimately presented in Equation 24. This total penalty function consists of system set-up penalty, system downtime time penalty, and penalty for any deviation of individual component maintenance times from system maintenance time. Note that the cost of performing actual repair, including the cost of acquisition, and the cost of labor, among others, is not included under the assumption that this cost would be the same whether individual component repair or joint repair is performed.

The setup penalty in Equation 20 accounts for the time to arrange system for maintenance. System setup penalty penalizes all associated costs for maintenance setup, charged only once regardless of the number of multiple components involved in a maintenance work, thus incentivizing the repair of multiple components at one system maintenance operation. Not included is component set-up time, as that is not expected to be a factor in determining individual or joint maintenance: any maintenance performed on a component would require component setup time. There is fixed system penalty per maintenance work C_S for each of the total number of system maintenance operations M .

$$\text{System setup penalty} = C_S M \quad (20)$$

There is a penalty for executing the component maintenance at a time other than the recommended schedule (e.g., from OEM suggestions, RCM calculations). If system maintenance is scheduled earlier than the recommended time, then there is a penalty for early maintenance work for that component. This penalty attempts to penalize the performance of maintenance unnecessarily too far in advance of the recommendation, and it is a function of (i) the total amount of earliness determined by $|T_l^k - t^j|$, (ii) the earliness penalty $C_{E,l}$, and (iii) whether or not component l maintenance is performed early at the j th system maintenance iteration, determined by x_l^j .

$$\text{Earliness penalty} = \sum_{l=1}^n \sum_{j=1}^M |T_l^k - t^j| C_{E,l} x_l^j \quad (21)$$

If system maintenance is scheduled later than individual component maintenance, then there is a penalty for late maintenance work for that component.



This penalty is a function of deviation of recommended individual component maintenance times from actual system maintenance time. The penalty is higher for tardiness than earliness here due to aversion to performing maintenance later than recommended, which could lead to a higher potential for component failure. This aversion is represented, in part, by the square on the amount of tardiness time, $(T_i^k - t^j)^2$, though a stronger penalty (larger exponent) could be imposed. Other elements include tardiness penalty $C_{L,l}$ and whether component l maintenance is performed after the recommended maintenance time during the j th system maintenance iteration, determined by y_l^j .

$$\text{Tardiness penalty} = \sum_{l=1}^n \sum_{j=1}^M (T_i^k - t^j)^2 C_{L,l} y_l^j \quad (22)$$

There is a cost associated with system downtime due to an unproductive or idle system. The system downtime penalty per unit time C_D is known. Expected component maintenance duration for component l is parameterized as D_l . Parameter γ represents the percentage of total expected component maintenance duration (i.e., $\sum D_l$ for all l that are present in j th system maintenance) that would be the expected joint maintenance duration for j th system repair. We assume this γ value to be constant for all iterations. Value of joint maintenance time parameter γ can be chosen from historical data of related system such that $\gamma \in (0, 1]$. The larger the value of γ , the higher the downtime maintenance cost would be. Higher γ values suggest less time savings in joint repair relative to separate, individual maintenance. Expected downtimes are same for joint repair and separate repair when $\gamma = 1$. This value defines joint maintenance times for a multi-component system. The term z_l^j determines whether the j th maintenance operation for component l is performed.

$$\text{System downtime cost} = \sum_{l=1}^n \sum_{j=1}^M \gamma C_D D_l z_l^j \quad (23)$$

Equation 24 presents the objective function and constraints incorporating the previous penalty functions.



$$\begin{aligned} \min \quad & C_S M + \sum_{l=1}^n \sum_j^M |T_l^k - t^j| C_{E,l} x_l^j + \sum_{l=1}^n \sum_j^M (T_l^k - t^j)^2 C_{L,l} y_l^j \\ & + \sum_l^n \sum_j^M \gamma C_D D_l z_l^j \end{aligned} \quad (24)$$

$$\begin{aligned} \text{s. t.} \quad & M, t^j > 0 \\ & x_l^j, y_l^j, z_l^j \in \{0,1\} \\ & \gamma \in (0,1] \end{aligned}$$

As maintenance scheduling is multistage (i.e., maintenance is a repeated event), one of the decision variables is the system-in-use time, t^j , at which the j th system maintenance (or iteration j) should be performed. Equation 24 is solved over a finite time horizon for several MRO iterations, finding a series of t^j values at which maintenance should occur. Individual component maintenance occurrence time recommendations are denoted by T_l^k for component l . Here, t^j values attempt to coordinate the downtime of several components to maximize long-term availability of the system (or equivalently, minimize downtime). Equation 24 also attempts to improve upon T_l^k to minimize the deviation of individual component maintenance times from system maintenance time, found in the neighborhood of T_l^k . As such, maintenance schedule for the system is provided, determining whether the j th maintenance operation will repair an optimal subset of the n components in the system.

Greedy Heuristic With Local Search Algorithm

The maintenance optimization model described above is solved with a proposed iterative greedy heuristic with local search algorithm (GHLA). The proposed algorithm is similar to the general structure of the greedy randomized adaptive search procedure (GRASP; Feo & Resende, 1995). In contrast to the two phases of GRASP, our proposed algorithm has three phases: (i) a construction phase, (ii) an improvement phase, and (iii) a local search phase. In the GRASP algorithm, the initial solution is constructed using a randomized sampling technique, whereas our algorithm uses a greedy heuristic to construct initial partial solution. We also use an additional improvement phase, where the greedy heuristic-based improvement ends. An overview of proposed algorithm is presented in Figure 18.




```

procedure GHLSA ()
  begin
    I ← InputInstance { };
    for GHLSA stopping criterion not satisfied →
       $\pi_j^0 \leftarrow \text{InitialPartialSolution} (I, \delta);$ 
       $\pi_j' \leftarrow \text{GHBI} (\pi_j^0);$ 
       $\pi_j'' \leftarrow \text{LocalSearch} (\pi_j');$ 
      UpdateSolution ( $\pi_j''$ );
    endfor
    return OptimalSolutionFound;
  end GHLSA;

```

Figure 18. Pseudo-Code Overview of the Proposed Greedy Heuristic With Local Search Algorithm

In brief, the three phases of the algorithm achieve the following:

1. The *construction phase* determines how many components in the system should be initially examined to include in system maintenance of multiple components and an initial estimate for the time at which that multi-component maintenance operation should occur. The window of time considered during the construction phase is a proportion δ of T_c^{\max} , and shown later, algorithm results are sensitive to δ .
2. The *improvement phase* improves the construction phase result by dividing the set of multiple components into two sets (a *candidate set* and a *discard set*) to determine whether dividing the maintenance operation will produce lower penalties than the construction phase. This phase iterates by removing a component out of the candidate set one at a time and placing it in the discard set and calculating penalty improvement.
3. The *local search phase* focuses on the resulting candidate set from the improvement phase and iterates across the different times associated with recommended component maintenance to balance the penalties of earliness and tardiness of individual components.

These three phases are performed at each system maintenance iteration j , thereby resulting in (i) the set of components involved in the j th system maintenance operation and (ii) the time at which the j th system maintenance operation should be performed. The algorithm stopping criterion is the pre-determined planning horizon

7. Let I be the set of discrete time periods where each element represents recommended (e.g., from RCM or OEM suggestions) repair times of a component during planning horizon T .

The final solution of this algorithm is essentially an $M \times 1$ vector for all system maintenance operations, where each element of the vector represents the recommended j th system maintenance. The result of each iteration j is referred to as the j th partial solution of the over final solution. Each element of the algorithm solution is comprised of two parts: $\pi_j[0]$ refers to the set of repair times $\{T_{A_1}^{a_1}, \dots, T_{A_n}^{a_n}\}$ of components to be performed jointly at the j th system maintenance operation (where $T_{A_n}^{a_n}$ is the a_n th maintenance operation for component A_n), and $\pi_j[1]$ refers to the recommended time t^j at which the j th system maintenance is to be performed. For example, $\pi_j = [\pi_j[0], \pi_j[1]] = [\{T_A^a, T_B^b, T_C^c\}, t^j]$ suggests that the a th maintenance operation of component A, the b th maintenance of component B, and the c th maintenance of component C will all be performed jointly at time t^j , the time chosen for the j th system maintenance operation to occur. Thus, during each iteration of this algorithm, an element that we refer to as a partial solution for algorithm solution set is found. At each iteration j , the three phases of the algorithm are performed, each of which is subsequently explained in detail. Through these three phases of construction and improvement, a partial solution is found, and this partial solution is then added to the solution set to update the algorithm solution for the scheduling maintenance problem. This iterative process is completed when the solution is found for the given planning horizon.

Using input instance I and chosen value δ , an initial partial solution π_j^0 is created in the construction phase. During the improvement phase, this initial partial solution π_j^0 is improved using greedy heuristic based procedure GHBI. This improved partial solution is represented by π_j' . During local search phase of the j th iteration, partial solution π_j' is further improved using the LocalSearch procedure, and the third phase returns the final partial solution π_j'' . After finding the best partial solution π_j'' in the third phase, we need to update the existing algorithm solution S and input set I . This partial solution π_j'' is then added as the j th element to solution vector S , to update the algorithm solution All scheduled component maintenance times T_i^k , at iteration j are removed from set I for next $(j + 1)$ st iteration and rest of the unscheduled component repair times of set I is updated according to their earliness or tardiness deviation for j th system maintenance.



Phase 1: Initial Partial Solution Construction

At each iteration j , the first phase is a construction phase where the initial partial solution is generated. General pseudo-code for this partial solution construction phase is presented in Figure 19. T_c^{\max} is the time duration that expresses the maximum time-span that includes all the component repair times to be initially considered for repair during j th system maintenance. The construction phase time-span is selected according to the δ value, which reduces the length of time originally considered by proportion δ . All component repair times T_i^k during time-span T_c are included in the joint repair component set for the initial partial solution π_j^0 for iteration j . This constructs the first part of the initial partial solution, $\pi_j^0[0]$.

Step 1.1. Calculate T_c^{\max} . The maximum time-span of construction phase T_c^{\max} represents the time duration between the recommended time for the earliest first repair of all components and the recommended time for the earliest second repair. Let sets of first and second repair times of each component out of all unscheduled maintenance times be \mathcal{T}_1 and \mathcal{T}_2 , respectively. The minimum value of set \mathcal{T}_1 is denoted by EarliestFirstRepairTime and the minimum value of set \mathcal{T}_2 is expressed by EarliestSecondRepairTime in the pseudo-code in Figure 19. The absolute value of their difference is the value of time-span T_c^{\max} .



```

procedure InitialPartialSolution (I,δ)
begin
     $\pi_j^0 \leftarrow [ ]$ ;
     $\pi_j^0[0] \leftarrow \{ \}$ ;
     $T_c^{\max} \leftarrow |\text{EarliestFirstRepairTime} - \text{EarliestSecondRepairTime}|$ ;
     $n_1 \leftarrow |\mathcal{J}_1|$ 
     $T_c \leftarrow \delta T_c^{\max}$ 
    for  $i \leftarrow 1$  to  $n_1$  do
        if  $\mathcal{J}_1[i] < T_c$  then
             $\pi_j^0[0] \leftarrow \pi_j^0 \cup \mathcal{J}_1[i]$  ;
        endif
    Endfor
     $\pi_j^0 \leftarrow [ \pi_j^0[0], \min(\pi_j^0[0]) ]$ 
    return  $\pi_j^0$ ;
end InitialPartialSolution;

```

Figure 19. Pseudo-Code for GHLSA Phase I, the Partial Solution Construction Phase

Step 1.2. Calculate T_c . Construction phase time-span, T_c can be calculated by multiplying the value of the maximum time-span of construction phase T_c^{\max} by δ . In a sense, δ is the scope of granularity. A small value of δ suggests a tight granularity of maintenance option set, meaning that a shorter time frame will be considered for T_c with which to consider multiple component maintenance options. For a larger value of δ , T_c approaches T_c^{\max} value. And T_c is equal to T_c^{\max} when $\delta = 1$.

Step 1.3. Partial Solution Component Set. Insert all recommended component maintenance times T_i^k that are originally scheduled during construction phase time-span T_c , into joint repair component set $\pi_j^0[0]$ of initial partial solution π_j^0 . If there are n_1 elements in set \mathcal{J}_1 , then it would take n_1 iterations to construct the initial partial solution component set.

The time at which system maintenance is performed on the components in $\pi_j^0[0]$ constitutes the second part of the initial partial solution, $\pi_j^0[1]$, which can be chosen according to several heuristics including

- the mid-point of time-span T_c ,
- a component repair time of component set $\pi_j^0[0]$ where the deviation Δ_j is minimized, or
- the earliest component repair time (i.e., the minimum value of component set $\pi_j^0[0]$).

In our implementation, the third heuristic is used to construct the later part of initial partial solution. That is, the second part of the initial partial solution, $\pi_j^0[1]$, is chosen according to the heuristic convention of scheduling system repair at earliest component repair time. Thus, this phase schedules all possible component maintenance during time-span T_c at the earliest possible time to produce an initial partial solution.

Phase 2: Greedy Heuristic-Based Improvement

During the second phase of iteration j , the algorithm improves the initial partial solution π_j^0 constructed in Phase 1, focusing primarily on the components in $\pi_j^0[0]$ to be repaired jointly (e.g., $\{T_A^a, T_B^b, T_C^c\}$). A search is performed in the neighborhood of π_j^0 to find a better partial solution. This combination of component repair times is improved according to a greedy heuristic of removing the last-one-out (i.e., latest component repair time) from existing combinations.

Let initial partial solution π_j^0 be the existing partial solution π_j' (i.e., j th solution element). If there are n_p elements in joint repair component set ($\pi_j^0[0]$) of existing partial solution, then there would be n_p possible combinations of component sets that can be created according to the last-one-out greedy heuristic. The best combination set among n_p possible combinations is selected in $(n_p - 1)$ iterations. At each iteration of the $(n_p - 1)$, two temporary partial solution elements called *candidate solution* π^c and *discard solution* π^d (i.e., temporary j th and $(j + 1)$ st) are generated from existing partial solution π_j' . The best candidate solution is selected as new existing partial solution π_j' according to an acceptance criterion. Each iteration of this greedy heuristic based improvement method, which is the ImproveCombination procedure in Figure 20.

Step 2.1. Determining $\pi_j^0[0]$. The first part of a solution element presents the component repair times to be repaired jointly. An improved combination of this joint repair component set is searched using the last-one-out heuristic. To generate an improved combination of the j th solution element, two sets (i.e., candidate combination set S_c and discard set S_d) are created from the existing joint repair component set. The candidate set will eventually be repaired during the j th iteration, and the discard set will be saved for the $(j + 1)$ st iteration or beyond. Let the existing joint repair component set be the initial value of candidate combination set S_c . By applying the last-one-out greedy heuristic (i.e., latest component repair time), a new discard set S_d is created. To generate the discard set S_d , the latest component repair time (i.e., $\max S_c$) is removed from candidate solution set S_c and inserted into discard set S_d . Candidate set S_c and discard set S_d construct the first part of the candidate solution π^c and discard solution π^d , respectively (i.e., $\pi^c[0]$ and $\pi^d[0]$).



Step 2.2. Determining $\pi_j'[1]$. The time at which the elements of the candidate solution $\pi^c[0]$ are repaired is found from the earliest component repair time heuristic for the set (i.e., $\min S_c$). This time of repair is $\pi^c[1]$. Similarly, the components in discard solution $\pi^d[0]$ are repaired at $\pi^d[1]$, or $\min S_d$. Other heuristics that could be used in this step were presented in Step 1.3 of the previous phase.

Step 2.3. Acceptance Criterion. The candidate solution is selected as the existing partial solution π_j' , according to the acceptance criterion of the minimum penalty function. The existing candidate solution is chosen as the partial solution π_j' if the combined penalty function value of candidate and discard solutions is less than the penalty function value of the existing partial solution π_j' . Figure 20 presents the procedure of developing new combination set according to the greedy heuristic.

```

procedure ImproveCombination (  $\pi_j^0$  )
begin
    CurrentPenalty  $\leftarrow$  PenaltyFunction (  $\pi_j^0$  );
     $\pi_j' \leftarrow \pi_j^0$  ;
     $\pi_c \leftarrow [ ]$  ;
     $\pi_d \leftarrow [ ]$  ;
     $S_c \leftarrow \pi_j^0[0]$  ;
     $S_d \leftarrow \{ \}$  ;
     $n_p \leftarrow |\pi_j^0[0]|$  ;
    for i  $\leftarrow$  1 to (  $n_p - 1$  ) do
         $S_c \leftarrow$  remove latest component repair time and insert it in  $S_d$  ;
         $\pi_c \leftarrow [ \{ S_c \}, \min ( S_c ) ]$  ;
         $\pi_d \leftarrow [ \{ S_d \}, \min ( S_d ) ]$  ;
        NewPenalty  $\leftarrow$  PenaltyFunction (  $\pi_c$  ) + PenaltyFunction (  $\pi_d$  );
        if NewPenalty < CurrentPenalty then    % Acceptance criterion
             $\pi_j' \leftarrow \pi_c$  ;
            CurrentPenalty  $\leftarrow$  PenaltyFunction (  $\pi_c$  ) ;
        endif
    endfor
    return  $\pi_j'$  ;
end ImproveCombination ;

```

Figure 20. Pseudo-Code for Improving Combination Stage

As long as the number of elements n_p of existing partial solution π_j' is greater than 1 and minimizes the penalty function value, π_j' is divided into two new parts: candidate solution π^c and discard solution π^d . This iterative improvement is done

while in the loop presented in procedure GHBI. Figure 21 describes the procedure GHBI using pseudo-code.

```

procedure GHBI ( $\pi_j^0$ )
begin
 $\pi_j' \leftarrow \pi_j^0$ ;
CurrentPenalty  $\leftarrow$  PenaltyFunction ( $\pi_j^0$ );
 $n_p \leftarrow |\pi_j^0[0]|$ ;
NewPenalty  $\leftarrow$  0;
while (NewPenalty < CurrentPenalty and  $n_p > 1$ ) do
    CurrentPenalty  $\leftarrow$  PenaltyFunction ( $\pi_j'$ );
 $\pi_j' \leftarrow$  ImproveCombination ( $\pi_j'$ );    % Using heuristic last-one-out
    NewPenalty  $\leftarrow$  PenaltyFunction ( $\pi_j'$ );
 $n_p \leftarrow |\pi_j'[0]|$ ;
endwhile
return  $\pi_j'$ ;
end GHBI;

```

Figure 21. Pseudo-Code for GHLSA Phase II, the Greedy Heuristic-Based Improvement Phase

Phase 3: Local Search-Based Improvement

In the last phase of system maintenance iteration j , an improved partial solution is selected by searching the neighborhood of current partial solution π_j' , building the best candidate set of components repair at the j th iteration. Let this improved partial solution be π_j'' and its initial value be π_j' . Emphasis in this third phase is placed primarily on searching different values of t^j in the neighborhood of $\pi_j'[1]$ to determine when the j th maintenance operation should occur. Pseudo-code for this local search phase is shown in Figure 22. During this improvement phase, t^j iteratively takes the values of component maintenance time generated from the final combination set $\pi_j'[0]$ during previous phase and create a temporary partial solution. During this iterative process, the partial solution is updated according to the penalty. According to our selected method, it takes n_p iterations to search the neighborhood of $\pi_j'[1]$, if number of elements in combination set $\pi_j'[0]$ is n_p . At each iteration of n_p , a new temporary partial solution called *temp* is generated. Steps of each iteration are as follows:



Step 3.1. Determining $\pi_j''[0]$. The joint repair component set comprising $\pi_j''[0]$ takes the value of the final combination set (i.e., $\pi_j'[0]$) found in the second phase.

Step 3.2. Determining $\pi_j''[1]$. During this improvement phase, $temp[1]$ (i.e., t^j) iteratively takes the values of component maintenance time generated from the final combination set $\pi_j'[0]$ during the previous phase. At iteration n_p , t^j would take the value of n_p th element of combination set $\pi_j'[0]$.

Step 3.3. Acceptance Criterion. The acceptance criterion is the value of penalty function presented. At each iteration of n_p , the temporary partial solution $temp$ is selected as new existing partial solution only if the new temporary partial solution minimizes the penalty function value.

At the end of n_p iterations, the LocalSearch procedure returns the best value found in the search. The return value, π_j'' , of this local search-based improvement is the partial solution representing the j th element of the final solution vector.

```

procedure LocalSearch ( $\pi_j'$ )
begin
     $\pi_j'' \leftarrow \pi_j'$  ;
    CurrentPenalty  $\leftarrow$  PenaltyFunction ( $\pi_j'$ );
    NoOfElement  $\leftarrow$   $|\pi_j'[0]|$ ;
    if NoOfElement > 1 then
        for  $i \leftarrow 1$  to NoOfElement do
             $temp \leftarrow \pi_j''$  ;
             $temp[1] \leftarrow \pi_j'[0][i]$ ;
            NewPenalty  $\leftarrow$  PenaltyFunction ( $temp$ );
            if NewPenalty < CurrentPenalty then
                 $\pi_j''[1] \leftarrow \pi_j'[0][i]$ ;
                CurrentPenalty = PenaltyFunction ( $\pi_j''$ );
            endif;
        endfor;
    endif;
    return  $\pi_j''$  ;
end LocalSearch;

```

Figure 22. Pseudo-Code for the GHLSA Phase III, the Local Search Phase
GHLSA Experimental Results

An example problem briefly illustrates the algorithm and some insights.



Problem Specification

Our example problem examines a series system composed of ten components. We assume the initial start time TNOW is zero. We assumed earliness penalty and tardiness penalty value to equal and same for all components (i.e., deviation penalty C_p). Maintenance duration D_l is assumed to be five time units for all components. The recommended individual maintenance times of these components are assumed here to be the MTBF from a two-parameter Weibull distribution with individual shape and scale parameters, β and η , respectively. The assumed values of the planning horizon, setup cost, downtime cost per unit time, earliness penalty and tardiness penalty values are presented in Table 5.

Table 5. Parameters of the Illustrative Example

Component	η	β	Other values
A	15	2	TNOW =0
B	20	3	$T = 200$ time unit
C	15	3	$C_S = 30,000$
D	17	4	$C_D = 5,000$
E	23	5	$C_p = C_{E,l} = C_{L,l} = 500$, for all l
F	37	4	$D_l = 5$ time unit, for all l
G	30	7	
H	22	3	
I	19	2	
J	26	4	

Baseline Case

The baseline case follows a simple procedure for maintenance. Each system maintenance operation is performed at the earliest component repair time (i.e., $\min T_l^k$), out of unscheduled component maintenance times. It is assumed that all repair times are in the system repair window (i.e., $\min [T_l^k + D_l]$) will be scheduled to be repaired at the same time. We used the same penalty function to calculate the objective function value for the baseline case. Note that the tardiness penalty will always be zero in baseline case instance, as system maintenance is done at earliest component repair time and there is no push back of component maintenance.

Figure 23 graphically depicts the recommended individual maintenance schedules for the 10 components, plus the baseline system maintenance schedule and GHLSA system maintenance schedule.



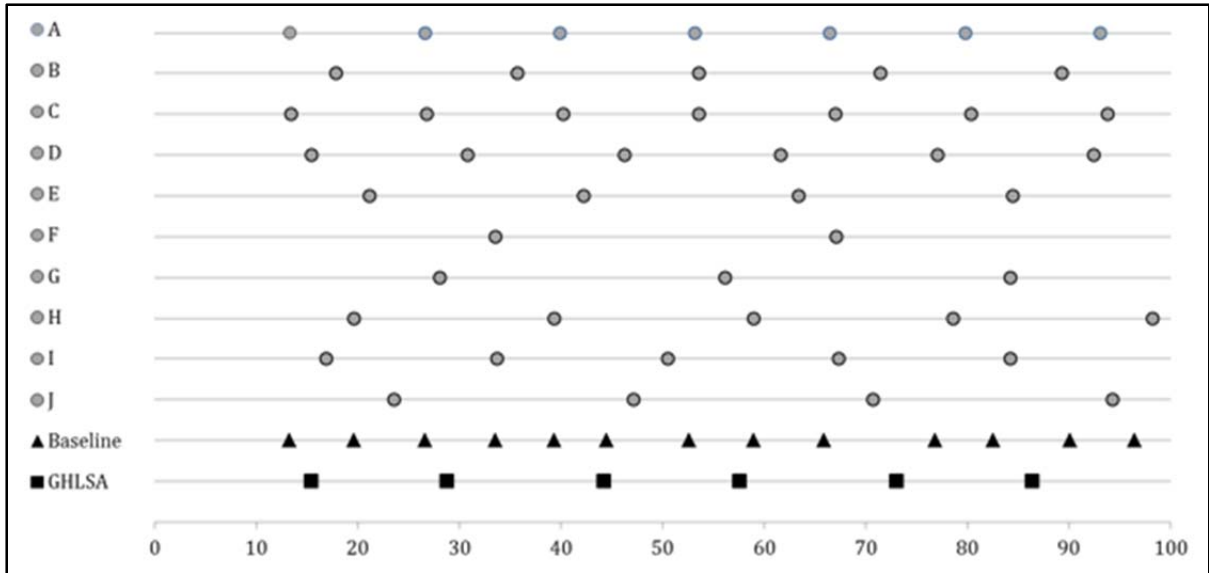


Figure 23. Graphical Depiction of the Recommended Repair Times for the 10 Components (MTBF), and the System Maintenance Schedules Found From the Baseline Case and the GHLSA, Over 100 Time Periods

Sensitivity of γ and δ Parameters

We solved the problem described above with the baseline case and the GHLSA. Experimental penalty function data were transformed into percent deviation value (PD). We calculated the PD with Equation 25, where Obj_{base} represents penalty function value for the baseline case and Obj_{GHLSA} represents the penalty function value produced using GHLSA procedure. A positive PD suggests that the objective function value has improved (reduced) using the proposed algorithm and vice versa. All calculated results for different δ values are depicted graphically in Figure 24.

$$\text{Percentage Deviation (PD)} = \frac{Obj_{base} - Obj_{GHLSA}}{Obj_{base}} \times 100 \quad (25)$$

Sensitivity Analysis on γ

Figure 24 shows that for a given instance, the proposed algorithm produced very high objective function values that resulted in negative PD value for lower γ values (i.e., $\gamma = 0.1$ to $\gamma = 0.3$). For γ greater than 0.3, calculated PD resulted in positive values. As such, for larger γ values (i.e., to $\gamma > 0.3$), the best solutions found using the GHLSA improved the objective function value of baseline case. As γ increases, the PD value decreases for both positive and negative deviation trends. Results were not very sensitive to γ value. The trend of PD remained the same, and the objective function value changed very little with a change in γ .

4.3.2. Sensitivity Analysis on δ

For all γ values, objective function percent deviation change was logarithmic with δ , shown in Figure 24. For smaller values of δ , GHLSA produced some negative deviation. As δ increased, it generated a positive deviation, as the objective function value decreased with higher δ value.

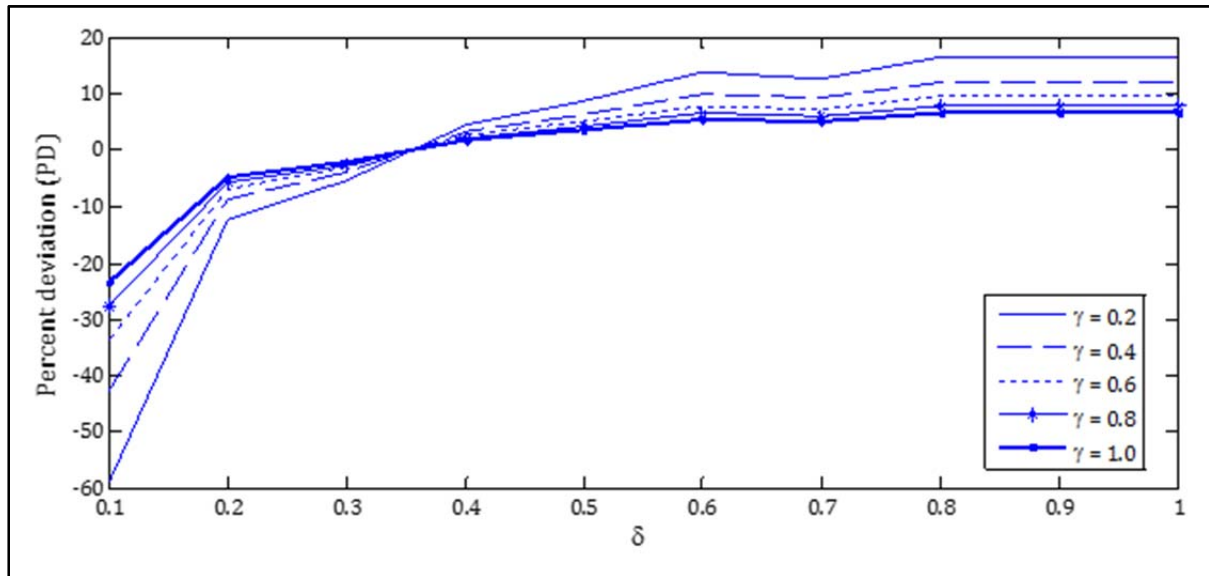


Figure 24. Change in Percent Deviation Value With δ

Sensitivity of Penalty Terms

We performed comparative study of baseline case and GHLSA-based results by generating different instances by changing given value of C_S , C_D , and C_P .

Sensitivity Analysis on Setup Penalty C_S

The GHLSA proved to be more effective (i.e., better improvement of objective function value) in the cases where joint maintenance of components has higher potential of minimizing system downtime and where system maintenance is more likely to incur higher setup cost. Results for the baseline and GHLSA cases for different instances for six different setup costs are presented in Figure 25. For all values C_S , GHLSA was able to improve (i.e., positive PD values) the penalty function value relative to the baseline case for the same γ . For a given C_S , penalty function value increased as γ decreased. Here, γ is the fraction of sum of all recommended component maintenance times representing the total joint repair time for j th system maintenance. Results signify the effectiveness of the proposed meta-heuristic-based algorithm where joint maintenance time is more likely to reduce the system downtime. It showed an increasing trend in PD value with increasing C_S , for any given γ , demonstrating the potential of this GHLSA for multi-component system maintenance where setup cost is comparatively high.

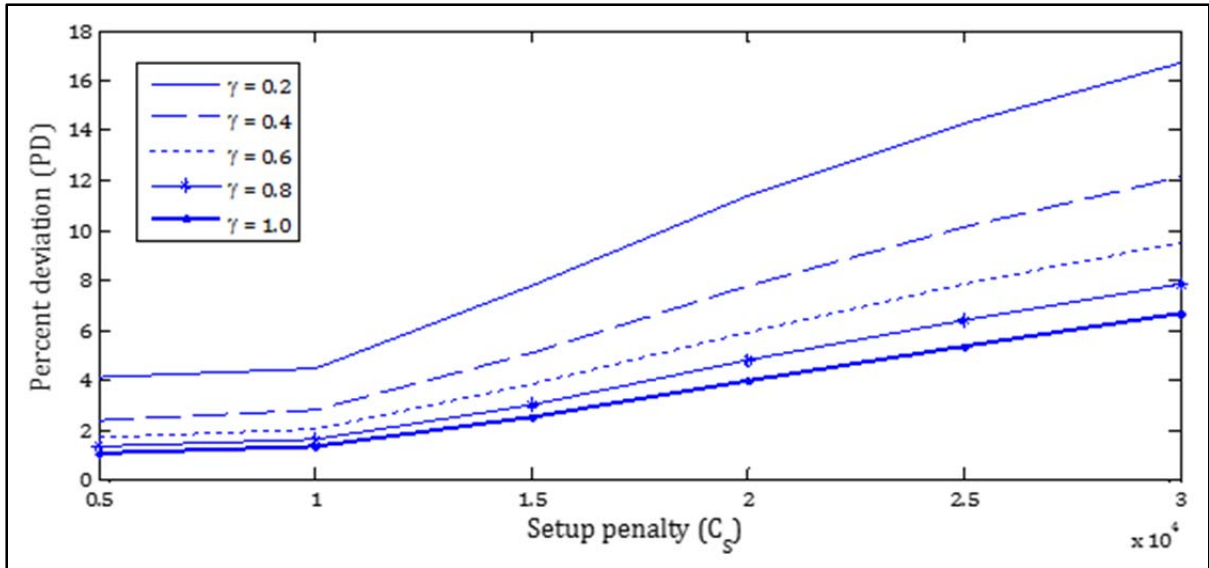


Figure 25. Percent Deviation Values for Different Setup Penalties Across Values of γ

Sensitivity on Downtime Penalty C_D

Results were generated for 10 different C_D values ranging from 1,000 to 10,000. Percentage deviation values are representative of the best solution found using proposed GHLSA at granularity level $\delta = 0.2$, shown in Figure 26. The decreasing nature of PD with increasing C_D suggests that the benefit of performing GHLSA decreases as the downtime is penalized more. The PD value ranged from 3.80% to 23.88%, suggesting that when $\gamma = 0.2$, the improvement relative to the baseline case is at its largest.

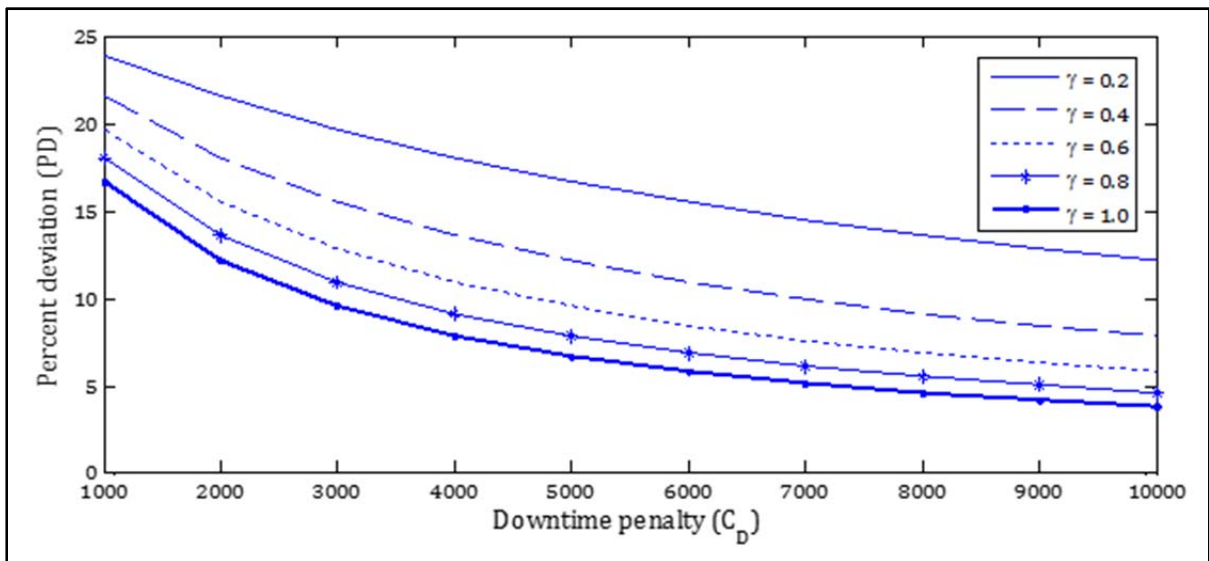


Figure 26. Percent Deviation Values for Different Downtime Penalties Across Values of γ



Sensitivity on Deviation Penalty C_p

Different C_p values ranging from 100 to 1000 were used to generate experimental instances, depicted in Figure 27. As the penalty increased, the PD of the performance of the GHLSA relative to the baseline decreased. Such a result would be expected, as the more the penalty placed on when system maintenance iteration deviates from the individual component maintenance recommendation, the more likely one is to stay away from earliness or tardiness when performing maintenance.

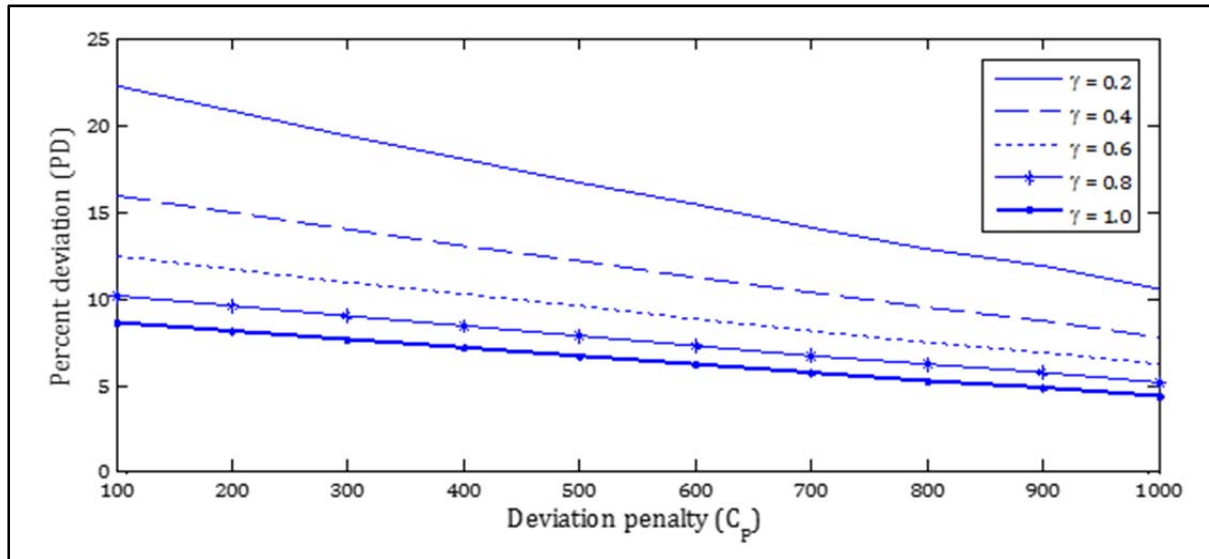


Figure 27. Percent Deviation Values for Different Deviation Penalties Across Values of γ

Optimal δ Value

Table 6 and Table 7 present the optimal δ values at granularity level 0.1 for generated instances for C_S and C_P . Note that optimal δ values for downtime instances were not reported in tables. For all 100 instances for C_D , generated optimal delta value was 0.8-1.0 at granularity level 0.1. For setup cost, all instances for cost ranging from 15,000 to 30,000, optimal δ was 0.8-1.0. For 10,000 setup penalty instances, optimal delta value was 1.0, and it decreased to 0.5 for lowest setup penalty 5,000.



Table 6. Optimal δ Values for Different Setup Penalty Instances Across γ

Table 6. Optimal δ values for different setup penalty instances, across γ .

C_s	γ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
5,000	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
10,000	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
15,000-30,000	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0

Table 7. Optimal δ Values for Different Deviation Penalty Cost Instances Across γ

C_p	γ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
100-800	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0
900	0.7-1.0	0.7-1.0	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
1000	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0	0.8-1.0

For deviation penalty instances, the optimal δ value was constant for all values of C_p , except 900. For $C_p = 900$ and for lower γ values (i.e. $\gamma = 0.1-0.2$), δ value for best found results was 0.7-1.0; and for remaining γ values, it was 0.7. According to the PD analysis, δ is a significant factor in finding a good solution when implementing this greedy heuristic based methodology. Experimental results suggest that higher values of δ , at granularity level 0.1 is a safer choice, when penalty function for all the δ values cannot be evaluated. In those cases, our recommended δ value would be 0.5-1.0, at granularity level 0.1.

Results were very sensitive to δ . The tuning of this construction phase time-span parameter depends on the number of components in the system and available computation power. If possible, initial tuning can be done at granularity level 0.1. Granularity level 0.1 suggests changing the scope of granularity δ value by 0.1. If T_i^k values result in a very small T_c^{\max} , then higher granularity level may not produce an improved result, as the number of components repair times may remain same for resulted construction phase time-span T_c .

Concluding Remarks

The problem addressed here is one where many (or all) of the multiple components of a system each have different recommended maintenance times. Such recommended maintenance times could be established from OEM suggestions or through reliability-centered maintenance (RCM) program, or likely some



combination thereof. As the expense of performing maintenance at each of those recommended schedules may be too great, in terms of budget or downtime, a means to schedule system downtime for maintenance is needed. Such a problem exists regularly in DoD weapons systems.

We address this problem with the development of a greedy heuristic local search algorithm for multi-component preventive maintenance scheduling problems. The proposed algorithm provides a coordinated system maintenance schedule for multi-component system according to minimizing maintenance penalties, including penalty functions for earliness, tardiness, system setup, and downtime. This work contributes a new heuristic and meta-heuristics based algorithm in the field of preventive maintenance scheduling problems.

We used a sensitivity analysis to understand the impact of joint repair time parameter γ , construction phase time-span parameter δ , setup penalty C_S , downtime penalty C_D , and deviation penalty C_P on the objective function value. These analyses suggest that the optimal value of construction phase time-span parameter δ depends on the granularity level and provided the importance of tuning parameter δ .

The proposed novel algorithm has been shown to make significant improvement of the objective function criterion of penalty value, compared to a baseline case where maintenance is performed at earliest recommended maintenance times. We have implemented the presented GHLSA for both objective function criteria, for 260 generated instances and found remarkable results. Deviation analysis showed significant improvement of objective function value for all 260 problem instances for penalty criterion. The greedy heuristic based algorithm appears to be promising in solving preventive maintenance scheduling problems. Experimental results have demonstrated the prospect of the GHLSA for multi-component system maintenance where setup cost is comparatively high and where joint component maintenance has the potential to reduce system downtime due to repair activities.

Depending on the interest of the maintenance decision-maker, an alternative objective to minimizing downtime could be maximizing system reliability. Future work will explore this second objective in a multiobjective framework. It is hypothesized that a system reliability objective may change the maintenance schedule, particularly when the system schedule suggests that some components be maintained after their recommended maintenance times (tardiness), potentially resulting in an undesired system reliability.

Task 3. Develop an Algorithm to Coordinate Multiple Vendor Acquisition of Component Parts

This section is based on the following:



Chapel, T., Barker, K, & Ramirez-Marquez, J. E. (2013). *Conditional reliability for acquisition decision making*. Unpublished manuscript.

Introduction

As aging aircraft fleets continue in operation well beyond their planned use, operational costs are primarily composed of the support operations, such as maintenance, repair, and overhaul in order to maintain the fleet. The United States Air Force (USAF) has cited that its O&S (operations and sustainment) costs are increasing twice that of the rate of inflation. According to the GAO, many aspects of the DoD acquisitions process are currently on the “High Risk List” and have been there since 1995, including DoD’s Business Systems Modernization, Supply Chain Management, and Weapon Systems Acquisitions. The GAO has recommended on several occasions that military departments and the Defense Logistics Agency “improve demand forecasting” and move toward “automated methods,” and DoD responded by setting goals to reduce excess inventory categorized as on-order and on-hand. Aside from the GAO, much of the documented milestones and future plans of the DoD are provided by the Defense Acquisitions University (DAU). According to the DAU, the current maintenance policy of the DoD is condition-based maintenance–Plus (CBM+), though several other maintenance policies and many maintenance metrics are have been implemented with various degrees of success.

Common in literature when analyzing a CBM or reliability-centered maintenance (RCM)–driven maintenance framework, system analyses and optimization primarily consider only a single objective. The primary hurdle seems to be with noncommensurate and competing objectives; because “optimal” solutions do not exist with multiple objectives, decision-maker preferences are elicited to appropriately balance such competing objectives. When operating a government-owned fleet, there are many performance metrics that must be incorporated because the DoD does not, and should not, determine a policy’s success or failure on a single metric. If a multiple objective analysis is developed, it is for a single spare part of a fleet; and so it seems that an analysis must choose to optimize for either multiple parts by a single objective or a single objective allowing for multiple parts. Recently, there has been some interest in merging the two topics of spare part allocation, reliability allocation for research due to their interactions within an operational system. Similar approaches have been discussed in the context of nuclear power plants, reservoirs, and various industrial processes.

This work addresses the gap between (i) CBM/RCM modeling approaches and (ii) acquisition of MRO components and parts by introducing an acquisitions trigger that integrates conditional reliability and the acquisitions lead time for a given component and supplier. We model conditional reliability with the well-documented Proportional Hazards Model (PHM) (Cox 1972), treating suppliers as covariate



effects. In addition, this research also encompasses the possibility of multiple suppliers because it is not often that the DoD is solely dependent on a single supplier due to the nature of performance-based contracts.

The two following sections lay the background and development of the described approach. This is followed by reviewing several examples from literature that highlight various steps in the approach followed by an illustrative example that shows the potential of the ideas outlined. Finally, we give several concluding remarks that will help with future research.

Methodological Background

The following sections describe several aspects that together will aid in the development of the proposed methodology.

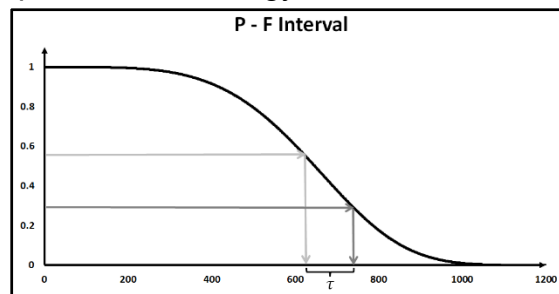


Figure 28. Depiction on When to Order Without Consideration Variability or Conditional Reliability With a Failure Defined as $R = 0.3$

Reliability: Conditional and System

Due to the emphasis on the order of events a conditional reliability calculation can be found and can be projected onto a planning horizon to allow for temporal decisions to be made. From Bayes' theorem, Equation 26 can be defined for the conditional reliability of τ time beyond the current time. Starting with Bayes' theorem of $P(A|B) = P(B|A) * P(A)/P(B)$ and assigning event A as surviving the time interval of $\tau + t$ and event B being survival to time t .

$$R(t + \tau | t, \mathbf{x}) = \frac{R(\tau + t, \mathbf{x})}{R(t, \mathbf{x})} \quad (26)$$

Equation 26 shows the conditional reliability or the unreliability of the component surviving the time next time interval of τ . Figure 29 is an example of a comparison between the baseline reliability, reliability given covariate information, and finally reliability based on a conditional reliability.

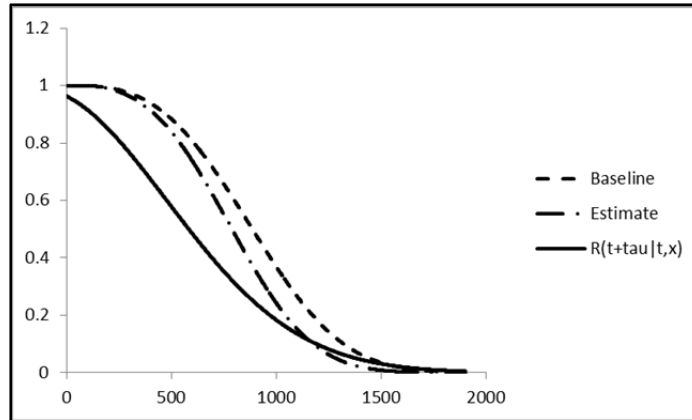


Figure 29. Depiction of Reliability and Conditional Reliability Over Time

To translate the multiple component reliabilities into the systems reliability several assumptions must be made. Using a reliability block diagram, we can assume the critical components both fail individually and the system can be described as a system in series the following equation can be used to calculate the probability that the system will survive the next time interval of τ .

$$R_s(t + \tau|t, \mathbf{x}) = \prod_{n=1}^N R_n(t + \tau|t, \mathbf{x}) \quad (27)$$

Proportional Hazard Model

The PHM provides a means to incorporate covariates into the reliability (survivability) of a given interest (e.g., component, aircraft, and fleet). Reliability is most often provided as a function of time that is less accurate than a model utilizing covariate information. The CPHM primarily depicted through the hazard function that is the instantaneous conditional probability in a small time period is shown in Equation 28.

$$h(t, \mathbf{x}) = h_0(t)\exp(\boldsymbol{\beta}^T \mathbf{x}) \quad (28)$$

Where baseline hazard function is $h_0(t)$ and $\boldsymbol{\beta}^T$ and \mathbf{x} are the transpose of the regression coefficients and covariate vector. The robustness of the model has allowed its wide use in medical research. Though the CPHM is primarily used as a descriptive model, this research hopes to show that through the model's assumptions (primarily the hazard ratio being a constant value), the model in the application of aviation maintenance can provide a useful estimation of reliability at multiple levels. Adjusted survival curves or reliability curves are provided as step functions like those commonly viewed in the Kaplan–Meir method. Reliability, which is primarily calculated with sole dependence on time, it can be redefined from

$R(t) = \exp\left(-\int_0^t h(x)dx\right)$ to Equation 2 so as to utilize the additional information. When reliability is of an engineered component, the baseline reliability is initially assumed to be $R_0(t) = \exp\left(-\left(\frac{t}{\lambda}\right)^k\right)$. Through an investigation of how the CPHM updates the originally assumed baseline (still assuming the choice of the parametric distribution is correct). Equation 29 provides the use of this in the use of the reliability calculation.

$$R(t, \mathbf{x}) = R_0(t) \exp(\beta^T \mathbf{x}) \quad (29)$$

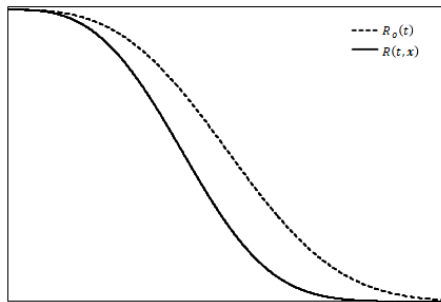


Figure 30. Depiction of a Situation Where Degradation Is Faster Than the Baseline

Weibull Distribution

Though there are many possible distributions in which to describe the behavior of components, the Weibull distribution has been shown as the best starting point from which to describe the degradation and failure of aviation components. Though much of the instrumentation in use may use software, which is commonly described by the exponential function, that is not the focus of this research because the nature of the exponential function and components with similar failure characteristics do not allow for the opportunity to make inventory decisions due to the durations necessary for the acquisitions processes.

There are two key parameters that drive the characteristics of the Weibull distribution: k , the shape parameter, and λ , the scale parameter.

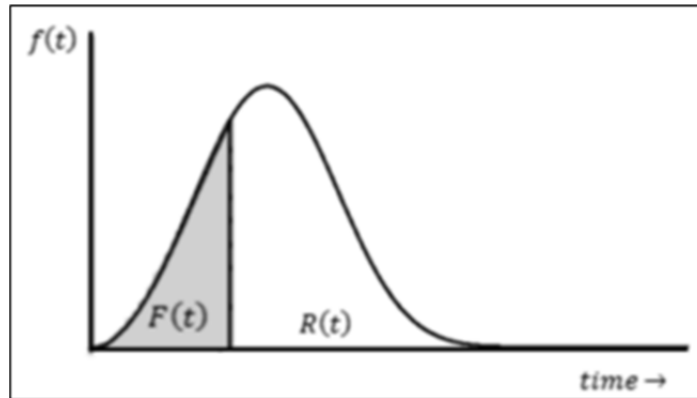


Figure 31. Failure Time Probability Distribution Function

Figure 31 provides an illustration of the PDF over time and expresses that the overall behavior of the distribution is more sensitive to changes in the shape parameter compared to the scale parameter. When increasing the shape parameter, the distribution is more sensitive to changes in parameters and the rate of change experienced is much higher than with similar changes in the scale parameter (when holding the other parameter constant). When changing the scale parameter makes the overall curve less sensitive to change in and increased to an extremely large number, the slope of such a function would have an asymptote of zero. In comparing the two parameters, the scale parameter is more accepting of large amounts of variability, and so it is chosen later in the illustrative example to allow for a larger difference in the scale parameter among the multiple components.

Methodological Development

The following development is a product of the building blocks set and is outlined in the previous section. It is also how this research proposes integration for such an application to allow for a decision-making aid for the supporting operations of MRO and spare parts acquisitions. To narrow the presented ideas into a specific application, we make several additional assumptions to proceed with the methodology.

Assumptions:

1. $Lead\ time_{ij} \leq \tau_{ij}$. If not, then reducing spare parts cannot be a recommendation because the system would be in a constant state of unnecessary risk.
2. Suppliers are willing to provide necessary information. Such an assumption can be made because the process of manufacturing and delivery of spare parts is a type of service; thus, the dynamics of the provider user take place. The dynamics of the service industry are grounded on a competitive nature to be the provider deemed by the

user as the best, fastest, cheapest, highest quality, most flexible, and others.

3. A model can be abstracted from the system, and such an abstraction does not introduce enough variation as to invalidate an analysis of the system through the model.

Because there is inherent error in point estimates, we propose to use an interval of the covariate found through the regression model. Figure 32 portrays the range of the estimated reliability changes over time.

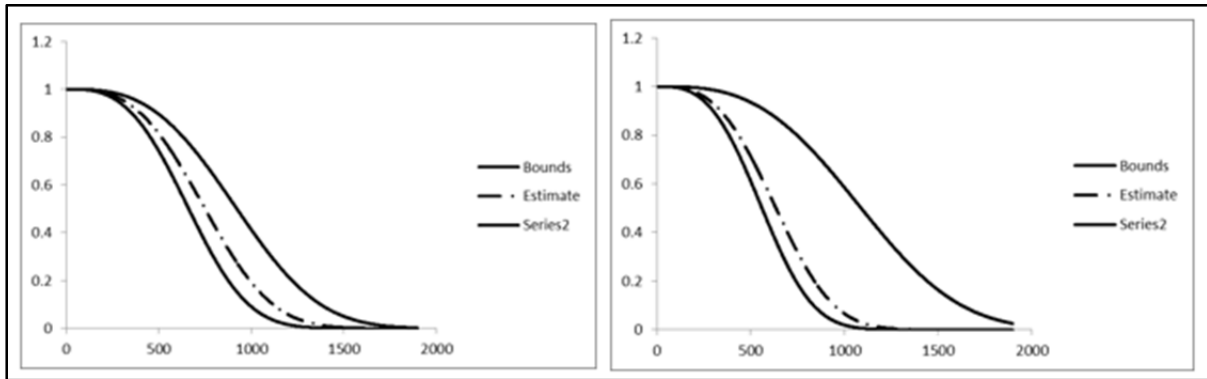


Figure 32. (a) Depiction of the Effect of the Covariate Estimation and Its Interval on the Estimated Reliability; (b) The Covariate Estimate Was Increased by 0.5 to 1.0

As the covariate estimate increases, the upper and lower values of estimated reliability widen, which shows that the system must generate covariate information low enough for acceptable ranges on reliability calculations. For the illustration of the methodology it is assumed that

$$[\hat{\beta}_1, \hat{\beta}_2] = \pm \hat{\beta} + Z_{\alpha/2} \sigma \quad (30)$$

Illustrative Example

The following illustrative example is of a very simple and theoretical system and is provided purely as an exercise of the methodology developed in the preceding sections. Though assumptions in the previous sections have been discussed to allow for applications in aviation, MRO operations, and support operations, additional assumptions are necessary as input for the methodology. The abstraction of the system is described as a two component in a series subsystem with three similar subsystems described as the systems in parallel. Additionally, there are three suppliers with the ability to supply both components in use by the subsystems, and there are no restrictions on how many parts are to be supplied by which supplier or whether the parts of various suppliers create a conflict in the same

system. Though the components have been tested and can be defined as both reliable and maintainable, the conflict that arises is that MRO acquisitions costs are too high and that a need for more optimal decisions are necessary given the constraints of a budget. This work is interested in using provided information describing the various suppliers that provide a more detailed picture than just the price of a component, such as the details described in Table 8. Each supplier has shown its components meet certain requirements of reliability (i.e., their shape and scale parameters to which the parts have been built are the same) but there seems to be a difference in their operational performance. The scale parameters are shown as 1000, 1800; and the shape parameters are 3 and 6.

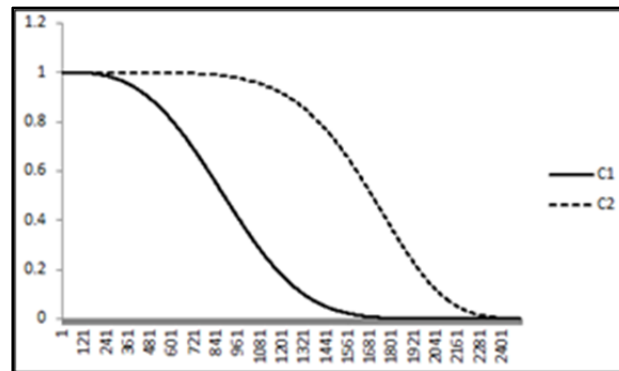


Figure 33. Component Baseline Reliabilities

Each supplier has a different price and lead time.

Table 8. Supplier Characteristics

	Supplier 1		Supplier 2		Supplier 3	
Lead Time, Variability	350	90	200	85	520	80
Cost (x2000)	780	820	660	770	520	840

The following regression coefficients were found by simulating data based on the scale and shape parameters 100 times for each supplier. Though the parameters of the Weibull distribution were the same, there were differences in their performance, which is visible in comparing with the regression values.

Table 9. Regression Coefficients

	Supplier 1	Supplier 2	Supplier 3
Component 1	0	0.0360	0.0386
Component 2	0	0.0587	-0.0652



Given that the conditional reliability maintenance trigger has happened, it is now necessary to determine the optimal maintenance decisions. At the decision point, several items must be determined: which operations should take place; and if overhaul is an operation, how many and which components will be overhauled, and from which supplier will they be replaced?

Table 10. Acquisition Times Without Covariate Input

	Supplier 1	Supplier 2	Supplier 3
Component 1	882	1276	603
Component 2	1326	1962	1388

Table 11. Acquisition Time Incorporating Covariate Information and Conditional Reliability

	Supplier 1		Supplier 2		Supplier 3	
Component 1	870	909	1288	1339	589	621
Component 2	1582	1624	1874	1921	1346	1386

It is clear there are differences in the actual times in which to start the acquisitions process depending on the current supplier of the part in use and on the possible supplier options when a component is to be overhauled. This is clearer in calculating the costs associated with and without the incorporation of covariate information.

The objectives considered at the time of the decision are reliability, maintainability, and cost. Although in many DoD documents, availability is an added objective, it is not seen as necessary to optimize because it is dependent on reliability and maintainability values. Operations, maintenance, repair, and overhaul can be described in their ratios of time necessary, costs associated, and effect on reliability.



Performance Metrics

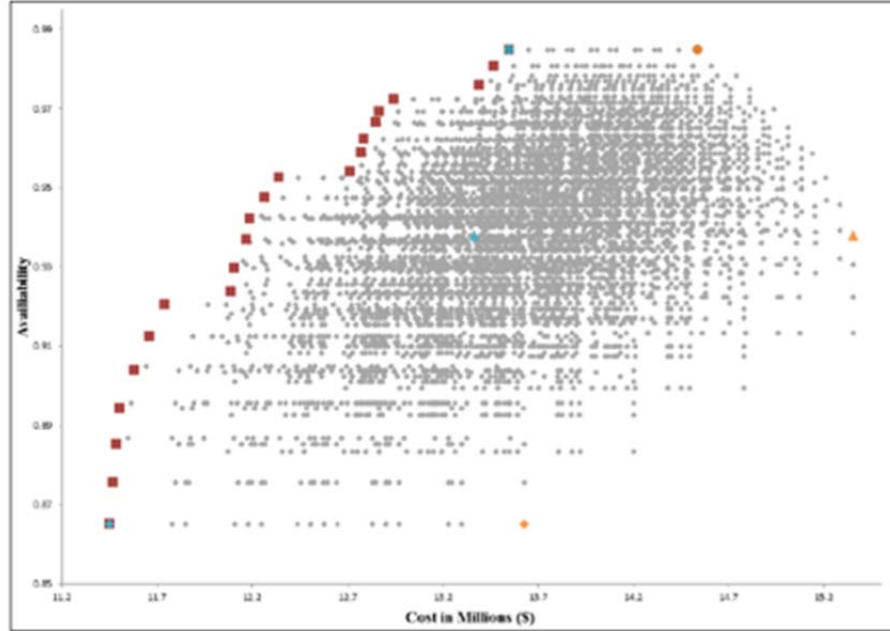


Figure 34. Case Study Output

Table 12. Corresponding Points of Single Supplier, Single Acquisitions Time

Database Number	Cost	Ao	Action
1	13,361,504	0.93304	Supplier 1 – Early
9332	15,351,984	0.93304	Supplier 1 – Late
18663	13,546,161	0.97971	Supplier 2 – Early
27994	14,537,604	0.97971	Supplier 2 – Late
37325	11,450,866	0.86009	Supplier 3 - Early
46656	13,624,902	0.86009	Supplier 3 – Late

Table 13. Subset of Pareto Points

Database Point	Cost	Availability	C11	C12	C13	C21	C22	C23
18,579	13,389,793	0.97101	2, Early	2, Early	2, Early	1, Early	1, Early	2, Early
19,009	12,708,485	0.94897	2, Early	2, Early	3, Early	1, Early	1, Early	1, Early
19,025	12,769,545	0.95384	2, Early	2, Early	3, Early	1, Early	2, Early	3, Early
21,673	12,183,543	0.93716	2, Early	3, Early	3, Early	2, Early	1, Early	1, Early
37,239	11,736,786	0.91546	3, Early	3, Early	3, Early	2, Early	2, Early	2, Early

At this point, the analysis would be given to the decision-maker or group for consideration, if the analysis were to continue using a utility where the decision-maker is interested in maximizing the gain of availability compared to increased cost.

Conclusions

Instead of designing out maintenance, because the benefits cannot outweigh the costs, design instead in maintenance where the maintenance schedules of critical components align. This alignment can be reached either through design or by



varying the suppliers. For example, for a two-component system, supplier 1 can provide both with renewal times of 4 and 7; and supplier 2 provides times of 2 and 3. Respectively, the suppliers have a least-common factor of the renewal process of 28 and 6. 28 may be too many cycles to go with an unbalanced schedule with supplier 1, but supplier 2 may be too often. In mixing the acquisitions process and ordering component 1 from supplier 1 and component 2 from supplier 2, the schedules line up every 12 cycles. Of course, this is a point estimate, but this idea may be of use to fleet managers. Additionally, being able to coordinate the components maintenance jointly will improve the achieved availability of the system. Furthermore, in looking at the subset where all components are supplied by a single supplier at a single time, the early triggering times always outperformed the latter trigger times. This may be due to the costs associated with failure and holding; but based on the actual system, such a conclusion has the possibility of changing. As changing the metrics to represent systems may lead to alternate collusions, so too could the introduction of additional metrics, especially those concerned with the experienced service of the supplier.

The other more dominant insights of this approach come from its development. The goal was to incorporate the acquisitions process and dynamic maintenance schedule (based on RCM) into a cohesive model. With an integrated model, it will be easier for the managers of the acquisitions department to communicate with those in charge of maintenance procedures. Through this model, the analysis develops and incorporates the needs of both departments.



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