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# Acquisition Management for System of Systems: Requirement Evolution and Acquisition Strategy Planning

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**PROJECT FINAL REPORT**

**“Acquisition Management for System of Systems: Requirement Evolution and  
Acquisition Strategy Planning”**

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## **Synopsis**

Our research is focused upon new methods to support decision-making on acquisitions in a system of systems (SoS). We have improved upon prior work by introducing both an extension to the previously developed Computational Exploratory Model (CEM) and a decision-analysis framework that addresses issues of identifying optimal collections of interconnected systems. An Agent-Based Model (ABM) approach extends prior CEM capabilities by enabling the development of a simulation to model interdependencies between constituent systems in a SoS. A robust investment portfolio framework complements the approach by providing a means to objectively balance expected performance, cost, and risks in identifying optimal collections of systems that constitute a SoS.

## **Executive Summary**

The Department of Defense (DoD) has recognized the need for effective tools in managing collections of systems that, when operating in a networked manner, serve to provide an overarching set of capabilities. Acquisition management of these systems of systems (SoSs) is difficult due to the complex interdependencies that exist between constituent systems, both across the programmatic and technical phases of development. The identification of developmental risks, their consequences, and expected payoffs (potential gains in SoS-wide capabilities) are equally important in identifying an optimal SoS architecture.

Research efforts during this report period have focused on extending previous frameworks of the Computational Exploratory Model (CEM) and Markov-based analytic approach. The CEM and Markov approaches were developed as means of comparing different networks of systems and quantifying cascading modes of risks that propagate due to the interconnectivities that exist between constituent systems. The cascading risks transcend both technical and programmatic dimensions of acquisition management and often produce developmental delays. The research in this report assumes a two-pronged approach. The first introduces an Agent-Based Model (ABM) that extends the exploratory nature of the prior CEM and Markov approaches by considering the dynamics and propagating risks beyond their purely probabilistic setting. ABMs are extremely well suited to simulating complex interactions of a SoS, and can incorporate a large range of types of information that drive the interactive behavior between participating “agents.”

The second approach complements the exploratory nature of the aforementioned work with a decision-analysis framework that is centered on an investment portfolio approach. The acquisition strategy proposed in our project is based on the 16 technical management and technical system-engineering processes outlined in the *Defense Acquisition Guidebook* (DAG; U.S. DoD, 2008a), often referred to as the 5000-series guide, and the DoD *Systems Engineering Guide for Systems of Systems* (SoS-SE; U.S. DoD, 2008b) guide. Decisions on system acquisitions are accomplished by adapting algorithmic advances in robust portfolio optimization techniques from financial engineering and operations research, to deal with uncertainties that can affect optimal selection of “portfolios” of systems in a SoS. The report details an approach to SoS acquisition in the form of a robust optimization problem that balances rewards (potential gain in SoS-wide capabilities) against developmental risks and acquisition cost. The approach is illustrated by using a simple concept problem based on the Littoral Combat Ship (LCS) platform.

## **Outreach and Collaboration**

Work documented in this report has resulted in a conference publication and a presentation at the NPS Ninth Annual Acquisition Research Symposium in 2012. Resulting interactions have produced very valuable feedback on the merits of our current results and potential further development of the portfolio approach. The symposium especially allowed us to foster closer ties and exchanges with various members of the NPS community. In particular, we have been involved in collaborative exchanges with Dr. Tom Housel and Dr. Jonathan Mun, who are actively involved in the development of portfolio-based approaches for acquisition and risk management. The work in this report has also been presented at the INFORMS 2012 Conference, October 14–17, in Phoenix, AZ. The conference presentation generated useful feedback from attending practitioners in the operations research and financial engineering communities, focused on ways to improve and further develop the framework presented in this report.

## **Introduction**

A system of systems (SoS) consists of a network of operationally and managerially independent systems that work synergistically in achieving an overarching capability (Maier, 1998). This confluence of multiple entities gives rise to an emergent behavior that may not be explicitly apparent from the individual constituents. The SoS paradigm has motivated the development of new acquisition strategies and integration of individual systems to better address the issue of achieving a set of overall capabilities instead of requirement-specific metrics. Acquisition efforts to obtain SoS capabilities for projects such as the U.S. Army's Future Combat Systems (FCS; Gilmore, 2006) and the U.S. Coast Guard's Deepwater System have been met with a large degree of developmental difficulties. This includes a combination of vulnerabilities in the developmental stage and poor management oversight that often leads to costly schedule overruns and, ultimately, cancellations. These large-scale systems are often developed incrementally with system-level requirements being the immediate focus of attention. Implicit consideration is given to the overarching objectives of the intended SoS capability. The decoupled and decentralized nature of architecting SoS across multiple hierarchies of interdependencies has given rise to a range of inefficiencies and warranted the pursuit of better systems engineering practices to encompass SoS principles.

The recognition of the need for improved methods in architecting and acquiring systems that comprise a SoS has led to further research in developing frameworks that maximize SoS-wide capabilities whilst minimizing cost and mitigating risk. Our prior efforts under NPS Acquisition research program funding have introduced the concept of a Computational Exploratory Model (CEM)—a discrete event simulator for the development and acquisition process. The CEM is based on the 16 basic technical management and technical systems-engineering processes outlined in the *Defense Acquisition Guidebook* (U.S. DoD, 2008a)—referred to as the 5000-series guide. The method also considers the modified processes in accordance with the *Systems Engineering Guide for System-of-Systems* (SoS-SE; U.S. DoD, 2008b) that adapts the 5000-series guide processes to the SoS framework. Research work in the present reporting period sought to provide a complementary decision-making tool that focuses on balancing capability development against cost and interdependent risks through the use of Modern Portfolio Theory (MPT).

## **Extending CEM and Markov: Agent-Based Modeling Approach**

Research conducted in the previous reporting period extended the CEM work to include a Markov perspective (Mane, DeLaurentis, & Frazho, 2011) that utilizes network-level metrics and computes conditional probabilities that relate the transmission of cascading risks between nodes (systems) in an interconnected SoS network. The Markov extension to the CEM approach allows for the comparison of alternate SoS architectures, with the intention of mitigating cascading risks and identifying SoS architectures with desired overarching capabilities. However, both the CEM and Markov approaches require inputs on interaction and performance in the general SoS setting. Our current work extends the CEM and Markov approaches to now include an Agent-Based Model (ABM) that allows for the modeling of interacting entities as discrete “agents.” Each agent has a predefined set of behaviors and rules of interaction. The modeling and simulation of these interacting agents allows for a more generalized computational framework that is not predominantly based on fixed probabilities, but rather, on a much wider range of possible metrics for simulation. The fidelity of the ABM is tuned such that its outputs are well suited to the required inputs for our CEM, Markov-based, and robust portfolio models. We motivate this addition with the simulation of a naval warfare scenario that is based on the Littoral Combat Ship (LCS) platform.

### **Example Application: Naval Warfare SoS Agent-Based Model**

Full-scale littoral battle scenarios involve a Littoral Combat Ship (LCS) squadron with a mix of ships and accompanying helicopters, unmanned aerial and surface vehicles, and a host of antagonist units. Such scenarios exhibit all the characteristics of a SoS problem with each individual asset possessing operational and managerial independence, with spatial distribution, and evolving with time to exhibit emergent behavior as a result of system interaction. Agent-Based Modeling (ABM) is ideally suited to tackle SoS problems, and Purdue University has its very own MATLAB-based simulation framework called Discrete Agent Framework (DAF). Hence an agent-based simulation model was developed using DAF to capture the performance of the Littoral Combat Ship (LCS) in a warfare scenario involving multiple mission threats. The ABM itself was based on the models described in three masters’ theses from the U.S. Naval Postgraduate School (Abbot, 2008; Ozdemir, 2009; Jacobson, 2010). Military and industry experts have verified that the agents, that constitute the SoS, are based on realistic and well-reviewed information.

### **Enemy Forces**

In order to accurately capture how LCS will perform in a stressing operational environment, a robust scenario containing three different types of antagonist units—missile boats, submarines, and mines—was developed. Each type of enemy unit was assigned a home position at the start of the scenario. The missile boats and submarines exhibit different patrol, detection, and engagement strategies, while the mines used in this simulation simply detonate whenever an agent is within its proximity radius.

### **Friendly Forces**

The LCS was designed primarily to take advantage of the interchangeable modularized onboard packages. Each of the following packages was modeled in the simulation scenario: Surface Warfare

(SUW), Anti-Submarine Warfare (ASW), and Mine Countermeasures (MCM). Apart from the primary mission package, the protagonist forces were augmented by the presence of a host of supporting entities which included—MH-60R, Unmanned Aerial Vehicles (UAVs), Unmanned Surface Vehicles (USVs), and Remote Mine-Hunting Vehicles (RMVs). Each of the units has different capabilities and tackles different antagonist units, depending on the LCS mission package to which it is affiliated.

### Neutral Forces

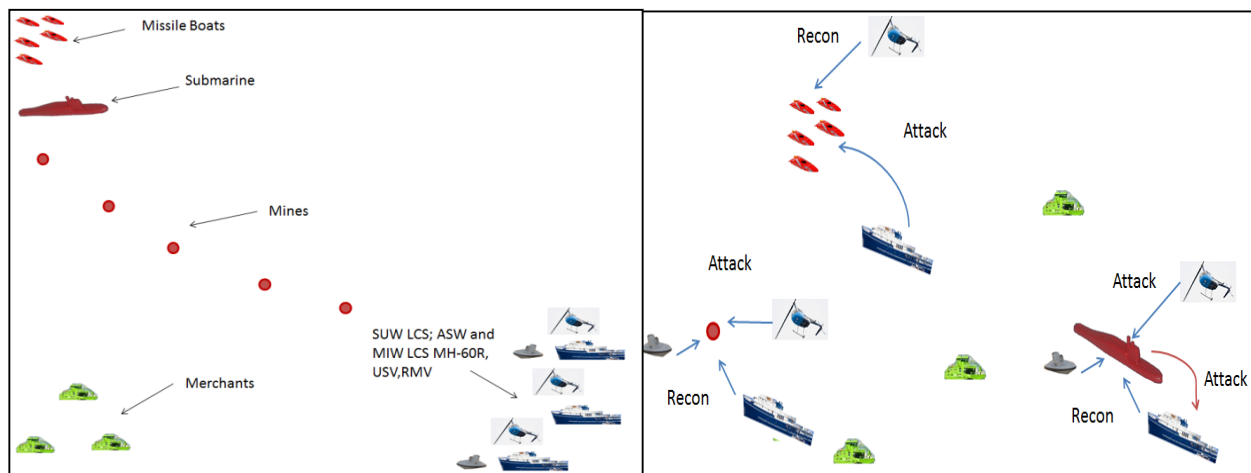
Merchant traffic was included to emulate realism in the scenarios and to add to the surface clutter, thereby making detection more difficult for both the protagonist and antagonist units.

### Goal

The mission of the LCS fleet was to clear the waters of all threats, while incurring the minimum number of friendly casualties. The factors that played an important role in this simulation are the number of enemy platforms, the number and type of LCS, the detection probability for the friendly sensors, and the kill probability of friendly weapons.

### Results

Multiple simulation runs were carried out in order to analyze the battle scenario and the agent behavior. Figure 1 displays the starting and the emergent scenario that arises for a typical run case.



**Figure 1, Naval Warfare Simulation Scenario**

The Naval Warfare Simulation, depicted in Figure 1, provides a good insight into the workings of agent-based modeling and serves as an ideal exploratory model to glean the mission performance for a naval warfare scenario. The model captures salient emergent behavior and properties that result from the collective interaction among the system components and provides inroads in capturing the performance and the shortcomings of the LCS.

Multiple simulation runs were carried out in order to analyze the battle scenario and the agent behavior. Table 1 depicts the results from a typical simulation run.

**Table 1. Results From a Typical Simulation Run**

<b>Agent Terminated</b>	<b>Agent Location</b>	<b>Time Step</b>	<b>Terminator</b>	<b>Terminator Location</b>	<b>Weapon Used</b>
Mine 5	(75,40)	38	MIW MH60	(78.85,32.18)	Clearance
Mine 4	(65,50)	200	MIW MH60	(68.7, 42.9)	Clearance
Mine 3	(55,60)	272	MIW MH60	(58.70,49.89)	Clearance
Mine 2	(35,65)	621	MIW MH60	(40.58,65.77)	Clearance
MBS 1	(17.8,111.18)	623	SUW LCS	(30.88,96.62)	NLOS
MBS 2	(17.96,110.09)	635	SUW LCS	(28.25,99.63)	NLOS
MBS 3	(18.65,112.08)	642	SUW LCS	(26.72,101.39)	NLOS
MBS 4	(19.2,112.56)	651	SUW LCS	(24.75,103.66)	NLOS
MBS 5	(18.10,108.88)	659	SUW LCS	(22.98,105.66)	NLOS
Mine 1	(15,75)	665	MIW MH60	(20.32,73.5)	Clearance
ASW LCS	(44.72,79.73)	763	Submarine	(44.69, 79.69)	Torpedo

The simulation results reveal the expected outcome for missile boats and mines based on intuition, and an unanticipated outcome for the ASW LCS due to emergent properties. The SUW MH-60R helicopter detects missile boats early in the scenario due to its high velocity and detection radius. The SUW MH-60R maintains a standoff distance of 20 nautical miles (nm) from the detected missile boats and passes on the position information to the SUW LCS. Guided by the antagonist’s position information, the SUW LCS was able to terminate each of the missile boats using its Non-Line-of-Sight Launch System from distances ranging from 6–20 nm without suffering any fire in return. The missile boats could not get close enough to the LCS to use their weapons, which had a very short range of 1 nm. The MIW LCS detects the mines and passes on the position information to the MIW MH-60 and the other LCSs, so that they can avoid colliding with the mines. The MIW LCS maintains a standoff distance of 3 nm from the detected mines, while the MIW MH-60, guided by the antagonist’s position information, terminates each of the mines using its clearance missiles.

The termination of the ASW LCS at the hands of the submarine was an emergent and non-intuitive result. The ASW LCS does not have the means to terminate submarines and is restricted to maintain a standoff distance of 10 nm from a detected submarine and to pass on the submarine’s location



to the accompanying MH-60R. Once the detected submarine is within the weapon range of the ASW MH-60R, it fires torpedo missiles on the submarine; however, because of the limited number of torpedo missiles onboard the ASW MH-60R, and the probability of termination associated with the torpedo missiles, the ASW MH-60R either missed its target or could only injure the submarine, before it ran out of ammunition. The submarine was then able to terminate the ASW LCS before the ASW MH-60R could reload its torpedo missiles. Once the ASW LCS was terminated, the ASW MH-60R could no longer reload and the submarine was able to escape.

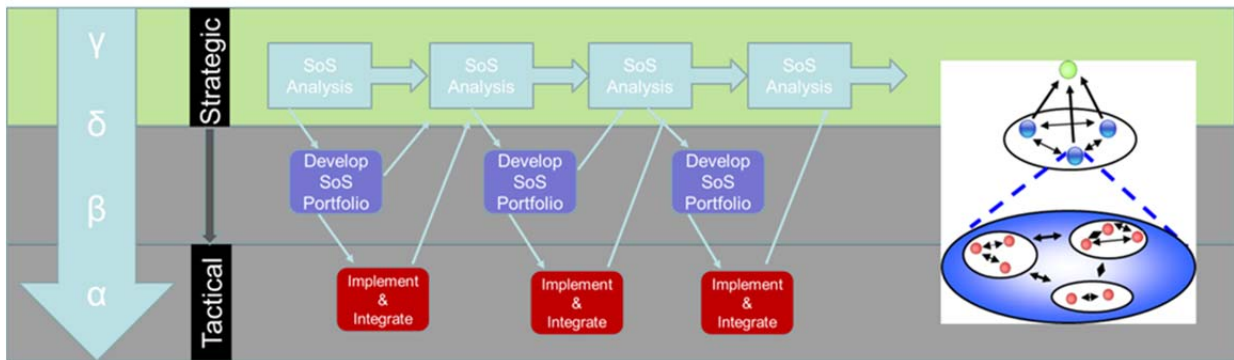
The Naval Warfare LCS example illustrates the salient features and versatility of modeling complex SoS architectures using an agent-based approach. The ABM approach facilitates analysis of complex dynamics of interacting agents while providing the overarching capabilities that emerge from the interaction of complex agents. The DAF implementation used in this research allows for easy instantiation of scenarios, be they operational (such as in this LCS example), or even developmental. The simulation of developmental scenarios would be representative of the CEM and Markov frameworks that have focused on programmatic disruptions. The agent model simulated in our LCS scenario, while operational in nature, can easily be extended to include such simulation as well.

### **SoS Acquisitions: Robust Investment Portfolio Approach**

Portfolio management techniques have been successfully used to address strategic-level asset acquisition and are extendable to include multi-period considerations. *Real options analysis*, for example, has shown effectiveness across various industries to evaluate discrete, long-term investment strategies. The work by Komoroski, Housel, Hom & Mun (2006) has developed a methodology that addresses strategic financial decisions through an eight-phase process using a toolbox of financial techniques—including portfolio optimization techniques. Such frameworks are geared towards financial uncertainty considerations of strategic projects and do not explicitly address technical architecture and/or evolving SoS-wide capabilities.

Figure 2 is a simplified adaptation of the “Wave model” structure that has been advocated as a conceptual model for guiding SoSE in the literature (Dahmann, Rebovich, Lowry, Lane, & Baldwin, 2011). The purpose of the Wave model is to identify SoS artifacts and their employment in the engineering of SoS architectures (and, ultimately, in acquisition of component systems). Our research employs a lexicon, denoted by Greek letters, that indicates hierarchical levels in a system architecture, with the alpha ( $\alpha$ -level) being the lowest level of operation, and moving up towards higher levels of the hierarchy from there—the lexicon is adapted from literature on a SoS taxonomy guide (DeLaurentis, Crossley, & Mane, 2011).

The Wave model is adapted here to include hierarchy and time scale that ranges from the broad, overarching objectives that are strategic in nature ( $\gamma$ -level) to the tactical aspects of individual system acquisition ( $\alpha$ -level). Research in this report addresses the  $\beta$ -level SoS portfolio development stage that evaluates candidate systems and, consequently, selects a portfolio of interdependent systems to fulfill overarching SoS capability objectives. The idea is to still maintain compliance with the “top-down integration, bottoms-up implementation” paradigm that is part of the Wave model implementation. Finding optimal portfolios assists in SoS architecting (e.g., ensuring capabilities are met), while also improving acquisition decisions for key component systems.



**Figure 2. The Context of the “Wave Model” (Dahmann, 2011) in Acquisition Decision-Making**

The framework proposed in this report does not attempt to replace, but rather to complement, existing methodologies by more directly addressing issues of integration and acquisition from a *robust portfolio theory* standpoint. Robust portfolio management methodologies have been widely used by financial engineering practitioners to manage portfolios in the face of market volatility and uncertainties. In the present context, such quantitative guidance is important for providing acquisition groups with the means of performing acquisition, integration, and development decision-making in the mist of evolving capability requirements.

Our approach seeks to leverage tools from robust portfolio methods to improve acquisition, integration, and development decisions while retaining advantages in balancing systems acquisition against evolving capability requirements. The research complements the notion of *open architectures* and *modularity* by providing a means of managing the possible combinations of connectible assets, and indirectly promoting competitive development of these interchangeable assets.

#### *Open Architectures, Competition, and Modularity*

Open architecture (OA) involves the design and implementation of systems that conform to a common and unified set of technical interfaces and business standards. This form of architecture results in the development of modular systems and increases opportunities for innovation and rapid development of new technologies that can be readily integrated/swapped into current architectures. The Littoral Combat Ship (LCS) program, for example, has recognized the need for multi-vendor acquisitions and OA implementations to ensure greater technological adaptability. The LCS program exploits the benefits of dual award contracting under fixed-price initiatives (FPI), along with rapid technology insertion processes and open architectures, to fulfil evolving technological and mission requirements of littoral warfare. The combination of dual contracting and system modularity helps achieve the necessary cost reductions while maintaining a greater degree of adaptability towards changing mission requirements (“LCS,” 2011; U.S. GAO, 2007b; Gansler & Lucyshyn, 2008). Although the platform is not, strictly speaking, a SoS, it nevertheless is a representative microcosm of what constitutes a SoS. The LCS platform carries many comparable salient features such as the confluence of multiple (sub) systems that work cohesively to achieve required capabilities.



**Figure 3. Littoral Combat Ship Layout**  
 (“LCS,” 2011)

The benefits of open architectures and competitive contracting are intuitively clear and have been shown to generate notable cost savings as exhibited in previous development projects such as Joint Direct Attack Munitions (JDAM; U.S. GAO, 2007b). However, system integrators and program managers are often faced with the challenge of leveraging the potential benefits of introducing new and improved systems against potential risks associated with developmental disruptions and cost considerations. Although the LCS program had significant success through the dual contracting scheme, it still experienced cost overruns due to a variety of problems. The problems included risks from a simultaneous design and build strategy due to schedule constraints, unrealistic budget expectations, and market risk from the greatly increased price of steel during the development period (O’Rourke, 2011). There have also been revisions in the requirements of fleet capabilities and refocusing of intended capabilities (O’Rourke, 2011). The robust investment portfolio approach, we believe, as formulated in this research work, provides a beneficial framework that takes full advantage of the “open architecture” approach to system and capability development. It offers the combinatorial nature of “mixing and matching” feasible collections of systems while balancing risk (developmental), rewards (potential capabilities), and cost of the overall SoS.

*Concept Acquisition Portfolio: Littoral Combat Ship Example*

Littoral Combat Ships are designed and developed by two primary contractors—General Dynamics and Lockheed Martin—as a result of the Navy’s dual contract award strategy that seeks to minimize costs through competitive contracting. The ships are designed to serve as primary units in close coastal littoral warfare and take advantage of modularized onboard packages (systems) that are interchangeable for different operational requirements. These packages include the Anti-Submarine Warfare (ASW), Mine Countermeasure (MCM) and Surface Warfare (SUW) packages. More recent developments have seen the introduction of an irregular warfare package for assistance and general support missions. We use this LCS example to demonstrate our work since it is a case where the objective is to achieve desired combat effectiveness and operational capabilities while minimizing cost and development risk. The simple model inputs and characteristics are described in Table 1.

**Table 2. Individual System Information for LCS Example**

		System Capabilities				System Req.			Dev. Time	Acq. Cost	
		Weapon Strike Range	Threat Detection Range	Anti Mine Detection Speed	Comm. Capacity	Air/Sea State Capacity	Air/Sea State	Comm.	(Years)	(\$)	
Package											
ASW	Variable Depth	0	50	0	0	0	0	250	125	3	3000000
	Multi Fcn Tow	0	40	0	0	0	0	150	75	2	2000000
	Lightweight tow	0	30	0	0	0	0	100	50	4	4000000
MCN	RAMCS II	0	0	40	0	0	3	200	100	1	1000000
	ALMDS (MH-60)	0	0	30	0	0	4	100	50	2	2000000
SUW	N-LOS Missiles	25	0	0	0	0	0	200	100	3	3000000
	Griffin Missiles	3	0	0	0	0	0	100	50	4	4000000
Seaframe	Package System 1	0	0	0	400	4	0	0	0	3	3000000
& Combat	Package System 2	0	0	0	300	4	0	0	0	4	4000000
Manage,	Package System 3	0	0	0	250	3	0	0	0	5	5000000

Table 2 is a hypothetical and simplified catalogue of individual systems available to the Navy in its pursuit of achieving desired capabilities. Although the numbers are hypothetical and do not explicitly illustrate real data, the salient features of considering capabilities, requirements, and risk in acquisition problems are still preserved. Table 2 lists systems that are available for each of the three mission packages—ASW, MCM, SUW—along with an individual rating of system capabilities and requirements for the systems to operate. Additionally, Table 2 provides the system development time and associated acquisition costs. Systems that are unable to provide a particular capability (or do not have a particular requirement) have a zero entry. Although the sea frame is typically a single system, the current sample problem couples the sea frame with battle management software as a base system that provides intra system capabilities. The development of these systems is based on a projected time schedule that is inherently subject to overruns and risk. This element is captured in the covariance matrix shown in Table 3.

**Table 3. System Interdependency and Development Risk (Covariance)**

	Variable Depth	Multi Fcn Tow	Lightweight tow	RAMCS II	ALMDS (MH-60)	N-LOS Missiles	Griffin Missiles	Package System 1	Package System 2	Package System 3
Variable Depth	0.1	0	0	0	0	0	0	0	0	0
Multi Fcn Tow	0	0.6	0	0	0	0	0	0.1	0.1	0
Lightweight tow	0	0	0.2	0	0	0	0	0.3	0	0.2
RAMCS II	0	0	0	0.3	0.1	0	0	0	0.2	0
ALMDS (MH-60)	0	0	0	0.1	0.1	0	0	0	0	0.3
N-LOS Missiles	0	0	0	0	0	0.5	0.2	0	0.1	0
Griffin Missiles	0	0	0	0	0	0.2	0.3	0	0	0
Package System 1	0	0.1	0.3	0	0	0	0	0.5	0	0
Package System 2	0	0.1	0	0.2	0	0.1	0	0	0.3	0
Package System 3	0	0	0.2	0	0.3	0	0	0	0	0.2

Table 3 shows the risk and interdependency aspects of the decision process. The diagonal terms represent the variance (degree of deviation from expected time) in development time. The off-diagonal terms are the variances due to interdependencies between individual systems that have commonly developed subsystems. For example, since the N-LOS and Griffin missile systems are both developed by Northrop Grumman, it is conceivable that they have common parts or undergo similar processes in development and manufacturing. The covariance value of 0.2 in the non-diagonal, therefore, represents joint development risk in terms of development time variance (e.g., 0.2 years), due to interdependencies between the two systems.

Estimation of the covariance matrix entries can come directly from manufacturing and development data. In the case of new systems, the quantities can be estimated heuristically using basic rules similar to those used in project management techniques such as PERT and other CPM methods (Blanchard & Fabrycky, 2005). The CEM and Markov analytical frameworks from prior research efforts present a means of generating surrogates of these covariance matrices, through identification of interdependencies (covariance) between systems.

The entries of the matrix in Table 2 are typically inferred from data; in this case, the values are hypothetically developed for the concept example problem. Most of the individual systems do not bear many interdependencies, with the exception of the sea frame and combat management support systems that are interlinked more explicitly to other listed systems in Table 2.

### Investment Model Formulation and Solution

The problem statement for the given acquisition problem is formulated as a mathematical optimization problem, which requires the definition of the objective function and constraints. The objective function is the equation that describes the primary metric to be optimized. This typically translates to, for example, the maximization of profits or minimization of costs/risk in the commercial sense. The second important aspect deals with the formulation of constraints, which are equations that typically describe resource (e.g., time, cost) constraints on the system and can be manipulated to reflect the salient conditions of the problem to be solved. The investment portfolio problem presented in the formulation shown in Equations 1–9 (also known as the Markowitz formulation) seeks to maximize the aggregate capabilities of a SoS architecture while minimizing the cumulative effect of cost, developmental time, and integration risks. The mathematical model for the concept problem can be written as the following:

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left( X_q^F \right)^T \Sigma_{ij} X_q^F - \sum_q \left( C_q X_q^B \right) \right) \quad (1)$$

$$X_q^F = \frac{X_q^B C_q}{\text{Budget}} \text{ (Portfolio Fractions)} \quad (2)$$

$$\sum_q C_q X_q^B + \varepsilon = \text{Budget} \text{ (Budget Constraint)} \quad (3)$$

$$\sum_q S_{qc} X_q^B \geq \sum_q S_{qr} X_q^B \text{ (Satisfy All System Requirements)} \quad (4)$$

$$X_1^B + X_2^B + X_3^B = 1 \text{ (ASW System Compatibility)} \quad (5)$$

$$X_4^B + X_5^B = 1 \text{ (MCM System Compatibility)} \quad (6)$$

$$X_6^B + X_7^B = 1 \text{ (SUW System Compatibility)} \quad (7)$$

$$X_8^B + X_9^B + X_{10}^B = 1 \text{ (Package System Compatibility)} \quad (8)$$

$$X_q^B \in \{0,1\} \text{ (binary)} \quad (9)$$

- where:  $S_{qc}$  : system ( $q$ )'s value for capability ( $c$ )  
 $S_{qr}$  : system ( $q$ )'s value for requirement ( $r$ )  
 $R_c$  : reference value for baseline desired SoS capability ( $c$ )  
 $w$  : weighting factor vector that weights importance across desired SoS capabilities  
 $X_q^B$  : decision variable of selecting system ( $q$ )  
 $X_q^F$  : decision variable of budget fraction that selecting system ( $q$ ) represents  
 $C_q$  : monetary cost of acquiring system ( $q$ )

- $\Sigma_{ij}$  : covariance matrix of risk (variances of development and integration time for systems)  
 $\lambda$  : risk aversion factor  
 $\varepsilon$  : budget remainder  
 Budget : total acquisition budget

The mathematical model shown by Equations 1–9 represents the formulation of a traditional single-stage optimization problem that is typical of operations research and financial engineering circles. The current form for the portfolio model at hand is known as a quadratic integer program (QIP) and is based on the Markowitz formulation that seeks to generate optimal portfolios that balance potential expected rewards against risk. Equation 1 is the objective function. The objective is to maximize overall capability while minimizing cost and development risk. Equation 2 is the fraction of the budget invested in individual systems. Equation 3 is the budgeting constraints, where the sum of all investments in individual systems (and savings) must be equal to the total budget allotted. Equation 4 ensures that all requirements of individual systems must be met. Equations 5–7 are the individual system compatibilities. In Equations 5–7, this translates to the selection of one system from each mission package (ASW, MCM, SUW) and a sea frame and combat management package that services the mission modules. These packages are mutually exclusive and, therefore, warrant a total selection of summation equal to 1, which ensures that no two packages per category are selected to satisfy the respective requirements. The covariance matrix, as denoted by  $\Sigma_{ij}$ , represents variations in development time due to system interdependencies.

The mathematical programming formulation of Equations 1–9 requires the covariance matrix to be positive definite (positive eigenvalues); in the context of financial applications, this means that there should not be a combination of assets that result in a zero net risk condition. Given that the entries of the covariance matrix are estimated individually, there is a possibility that the resulting matrix can violate the positive definite constraint; however, this can be corrected using a procedure that attempts to “find” the closest positive definite covariance matrix that can represent the existing covariance matrix. The procedure requires solution of the following optimization problem:

$$\min d_F(\Sigma, \hat{\Sigma}) \quad (10)$$

$$l_{ij} \leq \Sigma_{ij} \leq u_{ij} \quad (11)$$

$$\Sigma \in C_s^n \quad (12)$$

Equation 10 is the objective function that seeks to minimize the difference between the provided covariance matrix,  $\hat{\Sigma}$ , and the closest positive definite matrix,  $\Sigma$  (decision variable). Equation 11 enforces the condition that the entries of the matrix are bounded between some pre-determined upper and lower bounds. Equation 12 ensures that the covariance matrix exists in the cone of positive semi-definite matrices,  $C_s^n$ . The formulation presented in Equations 10–12 can be further altered through introduction of relevant constraints, to further tailor the characteristics of the covariance matrix. For example, constraints that enforce non-connections between systems that are known to not have any direct

connectivity can be enforced to ensure that the closest positive definite covariance matrix also adheres to such a condition.

### *Investment Portfolio Robustification*

It is well known in financial engineering circles that the Markowitz formulation, as used in the simplified LCS scenario, is sensitive to changes in estimated quantities of the covariance matrix (system interdependencies) and expected return (system performance). The sensitivity due to poor covariance estimations can result in highly inefficient portfolios due to errors in estimation or market shifts. Such sensitivity issues have prompted the development of a variety of robust methods in portfolio analysis to ensure that the chosen portfolio of assets is stable against potential changes in market conditions/expected volatility.

The current portfolio formulation in Equations 1–9 can be reformulated using robust optimization techniques; this includes Semi-Definite Programming (SDP) approaches (Fabozzi, Kolm, Pachamanova, & Focardi, 2007; Tutuncu & Cornuejols, 2007) that are extensions of modern portfolio and control theory. The reformulation allows for possible changes in estimated quantities (e.g., due to market shifts in pricing, volatility, system interdependencies) to be accounted more explicitly as uncertainty sets. The resulting portfolio allocation will not change appreciably even if salient estimated quantities or benefits change (within prescribed limits). In the context of an acquisition problem, the use of a robust formulation translates to reduced costs associated with capability estimation errors, development time volatility, and changing requirement conditions.

The general form of the portfolio problem in this research can be reposed as a robust optimization problem, given by the following form (Fabozzi et al., 2007; Tutuncu & Cornuejols, 2007):

$$\max_x \left\{ \min_{\mu \in U_\mu} \{ \mu^T x \} - \lambda \max_{\Sigma \in U_\Sigma} \{ x^T \Sigma x \} \right\} \quad (13)$$

$$AX \geq B \quad (14)$$

$$CX = D \quad (15)$$

$$\bar{\Lambda} \geq 0, \underline{\Lambda} \geq 0 \quad (16)$$

$$U_\delta(\hat{\mu}) = \{ \mu \mid \mu_i - \hat{\mu}_i \leq \delta_i, i = 1, \dots, N \} \quad (17)$$

$$\Sigma_{ij}^L \leq \Sigma \leq \Sigma_{ij}^U \quad (18)$$

Although the complexity of the optimization problem increases, it is nevertheless very amenable to a collection of numerical methods that provide good computational performance for realistic portfolio problems, especially portfolios with high volatility (Fabozzi et al., 2007). Equation 13 denotes the robust form of the objective function in (Equation 1). Equations 14 and 15 are the generalized linear form that represents the linear relationships in Equations 6–11. Equations 17 and 18 are the uncertainty bounds of the performance (capability) and operational risk due to interdependencies in each system. The portfolio formulation of the LCS sample problem as shown in Equations 1–9 is rewritten to now incorporate the uncertainties that are associated with estimation of the covariance matrix and the effective capabilities of individual systems. The demonstration LCS problem assumes an uncertain covariance matrix and utilizes the conversion methodology as detailed in literature (Fabozzi et al., 2007) to convert the problem into an



SDP. Equations 1–9 are adapted into the robust framework in Equations 13–18 to yield the following SDP:

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left\{ \langle \bar{\Lambda} \bar{\Sigma} \rangle - \langle \underline{\Lambda} \underline{\Sigma} \rangle \right\} - \sum_q (C_q X_q^B) \right) \quad (19)$$

$$X_q^F = \frac{X_q^B C_q}{\text{Budget}} \text{ (Portfolio Fractions)} \quad (20)$$

$$\sum_q C_q X_q^B + \varepsilon = \text{Budget} \text{ (Budget Constraint)} \quad (21)$$

$$\sum_q S_{qC} X_q^B \geq \sum_q S_{qR} X_q^B \text{ (Satisfy All System Requirements)} \quad (22)$$

$$X_1^B + X_1^B + X_1^B = 1 \text{ (ASW System Compatibility)} \quad (23)$$

$$X_4^B + X_5^B = 1 \text{ (MCM System Compatibility)} \quad (24)$$

$$X_6^B + X_7^B = 1 \text{ (SUW System Compatibility)} \quad (25)$$

$$X_8^B + X_9^B + X_{10}^B = 1 \text{ (Package System Compatibility)} \quad (26)$$

$$\begin{bmatrix} \bar{\Lambda} - \underline{\Lambda} & X_q^F \\ X_q^F & 1 \end{bmatrix} \pm 0 \text{ (Linear Matrix Inequality)} \quad (27)$$

$$X_q^B \in \{0,1\} \text{ (binary)} \quad (28)$$

$$X_q^F \in \square^n \quad (29)$$

where

$\bar{\Lambda}, \underline{\Lambda}, \bar{\Sigma}, \underline{\Sigma}$  : dual variables associated with covariance term

$\langle \bar{\Lambda} \bar{\Sigma} \rangle$  : denotes trace of the product of  $\bar{\Lambda}$  and  $\bar{\Sigma}$

Equations 19–28 retain most off the original form from Equations 1–9. The exceptions are that the uncertainty in the covariance matrix and absolute value bounds on the capability weighting vector,  $w$ , are reintroduced via exploitation of the dual form of the problem, now an SDP. The reparametization of the problem as an SDP manifests as additional terms in the objective function and constraints, namely, Equations 27 and 28. Equation 27 is a linear matrix inequality and enforces the condition of positive definiteness due to the symmetry of the matrix and positive values of all variables in the problem.

The uncertainty set of the covariance, as defined in Equation 15, is assumed to be +/- 10% of each respective entry in the matrix. This is arbitrarily chosen for this LCS demonstration problem; however, real-world problems will require the estimation of these bounds from statistical measures such as through the use of, say, confidence intervals. Additional measures such as *factor models* can be used

to estimate the values of the covariance with respect to relevant drivers; for a SoS problem, metrics such as the TRL and SRL, which directly relate to project risk, may be used.

*Littoral Combat Portfolio: Results*

The primary results are shown by varying the risk aversion parameter,  $\lambda$ , each time to generate the robust performance efficiency frontier. Increasing this parameter increases the portfolio's aversion to risk. The increase in  $\lambda$  forces the portfolio to select systems that have lower risk and, consequently, results in a lower performance of the SoS (Figure 4).

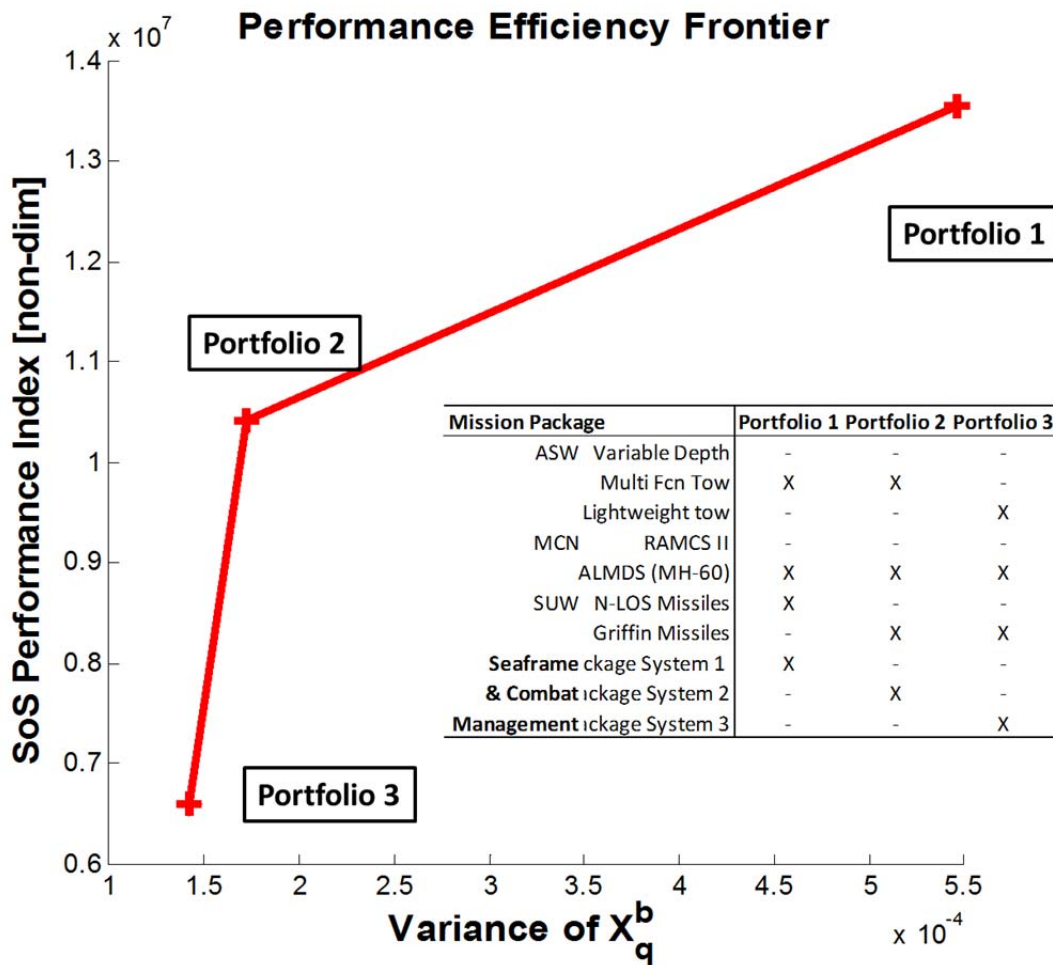
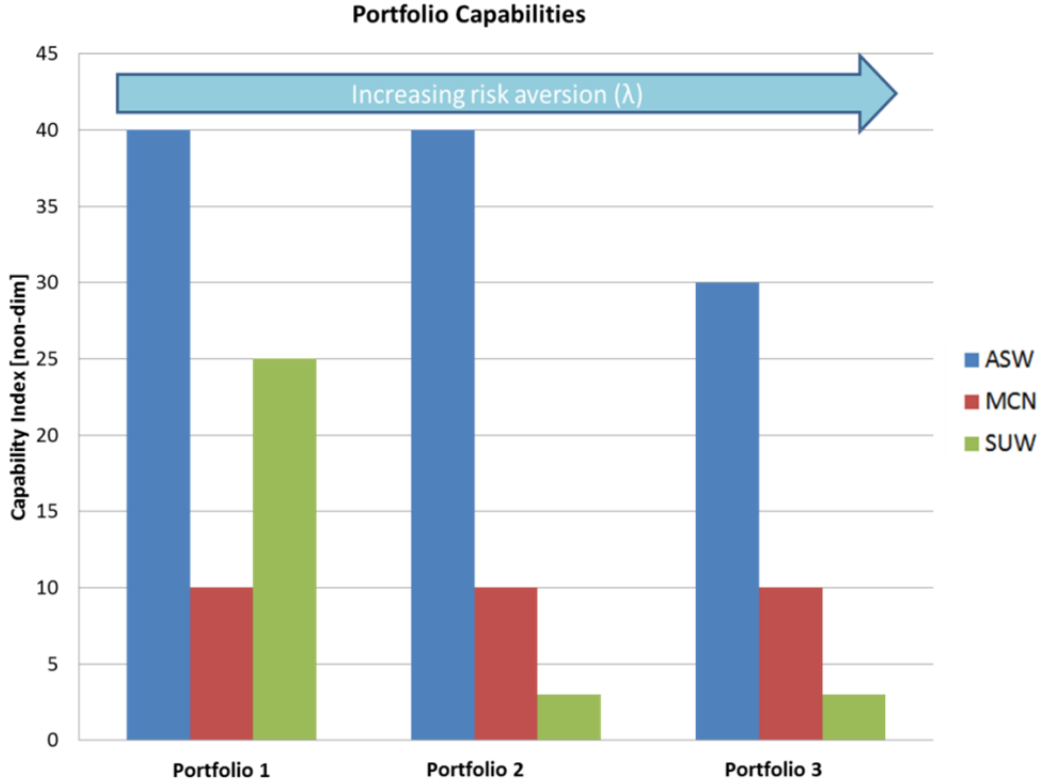


Figure 4. Robust Portfolio Efficiency Frontier



**Figure 5. System Capabilities**

In Figure 4, each point on the frontier is a portfolio that corresponds to a chosen level of risk aversion and shows the amount of variance associated with it. The higher the risk aversion, the lower the expected SoS performance due to the trade-off in choosing say, older, more reliable technology over newer technologies with lower TRL values. The table within Figure 4 shows the portfolio allocation for each of the three critical points on the frontier, with the corresponding breakdown in capabilities for each portfolio of systems shown in Figure 5. Typical efficiency frontiers will have more points due to the combinatorial possibilities of systems available to the portfolio selection process. One system is common across all three portfolios, which indicates that this system has high performance and relatively low risk. Some systems, however, exhibit increased SoS-wide performance but with added intersystem risk. The trade-off between individual SoS capabilities (ASW, MCM, SUW) and risk is shown in Figure 4. The analysis as shown in Figures 4 and 5 is useful for acquisition practitioners to determine the appropriate balance of SoS-wide performance against developmental risk.

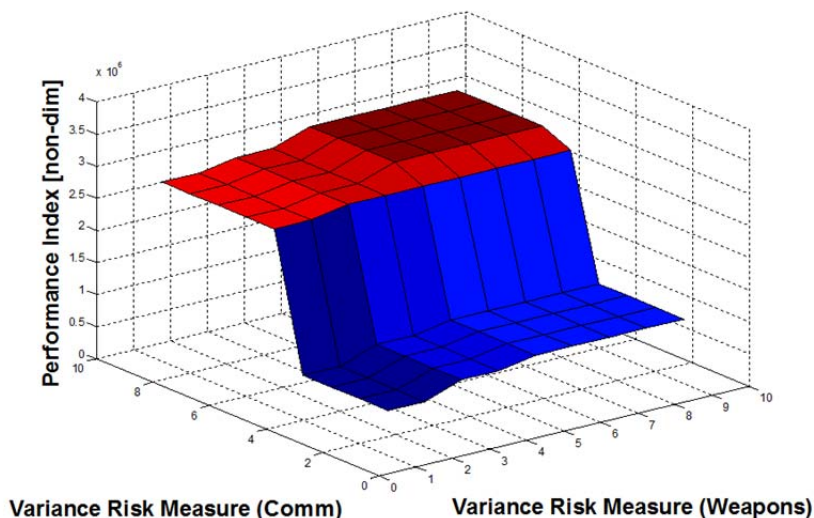
#### *Multiple Measures of Risk*

The portfolio analysis method in this work can be extended to include multiple measures of risk, where multiple covariance matrices are used to represent the connectivity between systems, based on different layers of risk in developing capabilities. For example, in the case of considering risks in the *weapons* and *communications* layers separately, the formulation can be adapted to include the following constraints:

$$\sqrt{\mathbf{X}_i^F \sum_{\text{weapons}} \mathbf{X}_i^F} \leq \sigma_{\text{weapons}} \quad (30)$$

$$\sqrt{X_i^F \sum_{communications} X_i^F} \leq \sigma_{communications} \quad (31)$$

Equations 30 and 31 represent the risk as denoted in Equation 1, but now for two separate forms of risk. Each equation is subject to a limit of variance,  $\sigma_{weapons}/\sigma_{communications}$  that is determined by the acquisition practitioner. Figure 6 shows (using arbitrary covariance matrices to represent risks for each layer) the topology of performance against each measure of risk (weapons/communications) for the LCS case. An increase in risk for each dimension of capability presents a potential increase in the SoS-wide capability. The discrete jumps in performance are reflective of the discrete nature of including individual systems in the portfolio of systems that constitutes the SoS architecture.

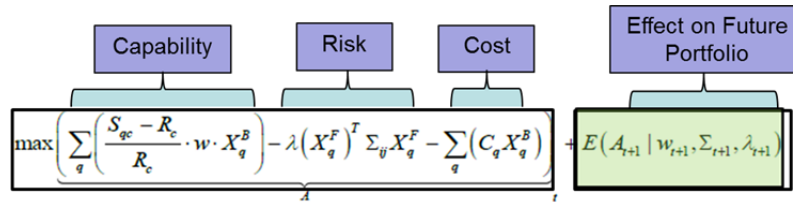


**Figure 6. Example Portfolio Assessment With Two Risks**

## Future Work

### *Formulating a Dynamic Investment Policy*

The multi-period portfolio problem draws upon a rich history of algorithmic development, as noted in literature both in the operations research and financial engineering circles. Sequential decision-making is known more broadly as *dynamic programming*, and adapts control theory methodologies to dynamic management of resources in the interest of maximizing (or minimizing) some given metric. The construction of an appropriate dynamic policy, in the context of an acquisition management problem, translates to balancing decisions on acquisitions and their implications of risk and potential capability over a specified time horizon.



**Figure 7. Dynamic Portfolio Extension**

Figure 7 illustrates the generalized extension of the objective function in the robust portfolio formulation shown in Equation 19 to include the potential effect of current decisions on expectations of future portfolio performance. Decisions can include the myriad contracting and pricing initiatives used to control costs, improve warfighter portfolio performance, and mitigate unnecessary risk throughout the acquisition management phase. A good policy will also exploit potential adaptive traits in collections of military assets where the associated costs of evolving requirements are minimized due to the adaptability of the chosen set of assets. The complexity arises in the interlinked nature of sequential decisions where the consequences of earlier decisions can affect the efficiency, cost effectiveness, and ability of military systems to adapt to new requirements.

## Biographies of Investigators



Daniel DeLaurentis is an associate professor in the School of Aeronautics and Astronautics Engineering, Purdue University. He received his PhD from Georgia Institute of Technology in aerospace engineering in 1998. His current research interests are in mathematical modeling and object-oriented frameworks for the design of system of systems, especially those for which air vehicles are a main element; approaches for robust design, including robust control analogies and uncertainty modeling/management in multidisciplinary design.



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## References

- Abbot, B. P. (2008). *Littoral Combat Ship (LCS) mission packages: Determining the best mix* (Unpublished master's thesis). Naval Postgraduate School, Monterey, CA.
- Blanchard, B., & Fabrycky, W. (2005). *Systems engineering and analysis* (4th ed.). Upper Saddle River, NJ; Prentice Hall.
- Dahmann, J., Rebovich, G., Lowry, R., Lane, J., & Baldwin, K. (2011). An implementer's view of systems engineering for system of systems. Paper presented at the IEEE Systems Conference, Montreal, Quebec.
- DeLaurentis, D., Crossley, W., & Mane, M. (2011). Taxonomy to guide system-of-systems decision-making in air transportation problems. *AIAA Journal of Aircraft*, 48(3), 760–770.
- Department of Defense (DoD). (2005). *Operation of the defense acquisition system* (DoDI 5000.2). Washington, DC: Author.
- Fabozzi, F., Kolm, P. N., Pachamanova, D., & Focardi, S. M. (2007). *Robust portfolio optimization and management* (1<sup>st</sup> ed.). Hoboken, NJ: John Wiley & Sons.
- Gansler, J., & Lucyshyn, W. (2008, May). Competition in defense acquisitions. In *Proceedings of the Fifth Annual Acquisition Research Symposium*. Retrieved from <http://www.acquisitionresearch.net>
- Gilmore, J. M. (2006, April). *The Army's future combat system program: CBO testimony: Statement of J. Michael Gilmore, assistant director, before the Subcommittee on Tactical Air and Land Forces, Committee on Armed Services, U.S. House of Representatives*. Retrieved from <http://www.cbo.gov/ftpdocs/71xx/doc7122/04-04-FutureCombatSystems.pdf>
- Jacobson, K. R. (2010). *The Littoral Combat Ship (LCS) surface warfare (SUW) module: Determining the best mix of surface-to-surface and air-to-surface missiles* (Unpublished master's thesis). Naval Postgraduate School, Monterey, CA.
- Komoroski, C. Housel, T., Hom, S., Mun, J., (2006). A methodology for improving the shipyard planning process: Using KVA analysis, risk simulation and strategic real options. In *Proceedings of the Third Annual Acquisition Research Symposium*. Monterey, CA: Naval Postgraduate School.
- LCS: The USA's new Littoral Combat Ships. (2011, December 19). *Defense Industry Daily*. Retrieved from <http://www.defenseindustrydaily.com/the-usas-new-littoral-combat-ships-updated-01343/>
- Maier, M. (1998). Architecting principles for system-of-systems. *Systems Engineering*, 1(4), 267–284.
- Mane, M., & DeLaurentis, D. (2009, May). Acquisition management for systems of systems: Exploratory model development and experimentation. In *Proceedings of the Sixth Annual Acquisition Research Symposium*. Retrieved from <http://www.acquisitionresearch.net>
- Mane, M., DeLaurentis, D., & Frazho, A. (2011). A Markov perspective on development interdependencies in networks of systems. *Journal of Mechanical Design*, 133(10).



- Odzemir, O. (2009). *Evaluation and comparison of freedom class LCS and other frigates/corvettes against small boat, FPB and submarine threats in confined waters* (Unpublished master's thesis). Naval Postgraduate School, Monterey, CA.
- O'Rourke, R. (2012, August 10). *Navy Littoral Combat Ship (LCS) program: Background and issues for Congress*. Retrieved from <http://www.fas.org/sgp/crs/weapons/RL33741.pdf>
- Tutuncu, R., & Cornuejols, G. (2007). *Optimization methods in finance* (1st ed.). New York, NY: Cambridge University Press.
- U.S. Army. (2008). Future combat systems. Retrieved from <https://www.fcs.army.mil/>
- U.S. Department of Defense. (2008a). *Defense acquisition guidebook*. Retrieved from <https://dag.dau.mil>
- U.S. Department of Defense. (2008b). *Systems engineering guide for system-of-systems*. Retrieved from <http://www.acq.osd.mil/se/docs/SE-Guide-for-SoS.pdf>
- U.S. General Accounting Office. (2003, August 13). *Issues facing the Army's future combat systems*. Retrieved from <http://www.gao.gov/new.items/d031010r.pdf>
- U.S. Government Accountability Office. (2007a, March 27). *Future combat system risks underscore the importance of oversight*. Retrieved from <http://www.gao.gov/new.items/d07672t.pdf>
- U.S. Government Accountability Office. (2007b, June). *Role of lead systems integrator on future combat systems program poses oversight challenge*. Retrieved from <http://www.gao.gov/new.items/d07380.pdf>
- U.S. Government Accountability Office. (2012, March). *Defense acquisitions: Assessments of selected weapon programs*. Retrieved from <http://www.gao.gov/assets/590/589695.pdf>



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