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Single-case Probabilities and the Case of Monty Hall: Levy´s View

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Abstract: In Baumann (2005) I argued that reflections on a variation of the Monty Hall problem throws a very general sceptical light on the idea of single-case probabilities. Levy (2007) puts forward some interesting objections which I answer here.

The notion of probability is so important that we cannot do without it, neither in science nor in everyday life nor in philosophy. At the same time there is more than one interpretation of this notion (cf., e.g., Gillies 2000) and definitely many controversies about basic issues. One of them has to do with the question whether there is such a thing as single-case probabilities. In Baumann (2005) I argued that considerations concerning a problem that has been discussed widely and not just within philosophy, namely the Monty Hall Problem (cf., e.g., vos Savant 1992, 199-209), suggest that one should be skeptical with respect to the idea that there are single-case probabilities at all. After an exposition of the argument (I) I will discuss some interesting objections (II-IV) against my argument which have recently been made by Ken Levy in Levy (2007). My conclusion (V) will be that Levy´s objections can be answered. The argument against single-case probabilities remains intact. But first to the exposition of my argument.

I.

What is the Monty Hall Problem? It is based on a tv game show ("Let´s Make a Deal!") which was popular in the US some decades ago. Here is the basic outline. The player is confronted with three closed doors. Behind one door is a prize he wants but there is nothing he would want behind the other two doors. The player can pick one door and keep what is behind it. Unfortunately, the player does not know which door is the "winning door". After the player has picked one door, the host, Monty Hall (MH), opens another door with nothing of interest behind it. The player then has the choice between making his initial choice his final one and getting what is behind the chosen door plus $100 ("sticking") or "switching" to the other, remaining door, getting what is behind it but no additional $100. All this is common knowledge between the player and the host. What would a rational player who is interested in the best possible outcome for himself do – stick or switch?

The intuitive answer for most people is "Stick". It is based on the idea that the probability that the prize is behind the originally chosen door equals the probability that it is behind the remaining door (1/2 in both cases). However, one can show relatively easily that this intuition is mistaken. The player can originally choose between three doors and the prize can be behind any of these three doors. This gives us nine equally probable scenarios which are both mutually exclusive and jointly exhaustive. Let numbers stand for the different doors and "W" and "L" for "winning" and "losing" respectively:

Door chosen	Prize behind	MH opens	switch	stick
1	1	2 or 3	L	W
1	2	3	W	L
1	3	2	W	L
2	1	3	W	L
2	2	1 or 3	L	W
2	3	1	W	L
3	1	2	W	L
3	2	1	W	L
3	3	1 or 2	L	W
			6/9 WINS	3/9 WINS

Sticking only gives one a 1/3 chance of winning whereas switching gives one a 2/3 chance of winning. Hence, the rational player will switch. Application of Bayes´ Principle leads to the same result but is less intuitive.[i]

There is no doubt that with respect to a large enough sample of Monty Hall games the player should switch. But what if we look at a single game (cf. Moser and Mulder 1994 and Horgan 1995)? I want to argue that we run into serious problems if we apply probabilistic notions and arguments like the one above to a single Monty Hall game. The application of such notions and arguments to a single case (a single game) does not make sense; hence, there is no answer to the question what the rational player should do in an isolated case, at least no probabilistic answer. My argument involves a variation of the original scenario for two players.

Suppose there are two players. No player knows what the other player is choosing. Nothing changes with respect to the original Monty Hall scenario if both happen to choose the same door first. If they choose different doors, then Monty Hall will open the third door, no matter whether there is a prize behind it. If Monty Hall opens the door with a prize behind it, then both players can go for the full prize. What I am going to call "The modified sticking strategy" tells the player to stick with the original choice but when Monty Hall opens another door with the prize behind it, then the strategy recommends to go for the winning door. What I am going to call "The modified switching strategy" tells the player to switch from the original choice, but when Monty Hall opens the door with the prize behind it, then the strategy recommends to go for the winning door. These explanations of "modified switching" and "modified sticking" are meant as definitions of those terms when used for modified Monty Hall scenarios.[ii] It is important not to confuse this with the use of those terms in the context of the original Monty Hall game. Again, the assumption is that all players are rational and self-interested and that all this is common knowledge between players and host. What should players do?

Let us assume door 1 wins (obviously, we can neglect the other two possibilities without affecting the overall probabilities). Again, we get nine possibilities (equally probable, mutually exclusive and jointly exhaustive), here represented with outcomes for one of the players (“ego” rather than “alter”):

Ego chooses	Alter chooses	MH opens door	switch	stick
1	1	2 or 3	L	W
1	2	3	L	W
1	3	2	L	W
2	1	3	W	L
2	2	3	W	L
2	3	1	W	W
3	1	2	W	L
3	2	1	W	W
3	3	2	W	L
			6/9 WINS	5/9 WINS

The situation is symmetrical between ego and alter. Given the definitions of "switching" and "sticking" for our modified Monty Hall game (see above), it should be clear that ego's final choice of door 1 in the 6th and the 8th case in the chart above is recommended by both the modified sticking and the modified switching strategy.[iii] But now we have a serious problem: What if they have chosen different doors? Well, they don´t know this so their (epistemic) probabilities will remain unaffected: both 2/3 for switching and 5/9 for sticking. If ego has initially chosen door 1 and alter door 2, then we will get two incompatible probabilities for door 2 being the winning one: 2/3 (ego) and 5/9 (alter) (similar problems arise, of course, for door 1). If both switch or both stick, then the probabilities that one of the players will win do not add up to 1: 4/3 in the first case and 10/9 in the second case. All this seems clearly absurd.

One obvious response is that we are dealing with epistemic probabilities here, not with objective probabilities. If two persons in the same situation have different information, then their subjective or epistemic probabilities may differ (cf. Freund 1965 and Shafer 1985). And indeed: Both players know something the other does not know, namely which door they have initially chosen.

The problem with this reply lies in the question whether that information is relevant here. This is an important point and everything hinges on it here. If the informational differences between both players are not relevant, then both players should assign the same probabilities to the same choices. Otherwise they would violate an intuitive condition of non-arbitrariness: If two persons have non-arbitrary (reasonable) probability assignments concerning a set of possible outcomes and if they share the same relevant information, then they should assign the same probabilities to the same outcomes (cf. for more detail Baumann 2005, 73). There are, I think, good reasons to believe that both players have the same relevant information. Assume both players immediately forget which door they have originally chosen. It is obvious that this does not affect the nature of the choice situation: They could just tell Monty Hall that they want to switch (or stick). Hence the information with respect to which they differ ("I have chosen x") is not relevant (for more detail on this cf. Baumann 2005, 73-74). But then we run into absurdity by using notions of single-case probability in our modified Monty Hall Problem. It does not even make sense to ask whether the rational player in an isolated single case would switch or stick.[iv]

I have explained the problem with respect to a variation of the Monty Hall Problem for two players. The same things hold, *mutatis mutandis*, for a variation which involves one actual player and his counterfactual "counterpart" (cf. Baumann 2005, 76-77). The basic problem is the same in both variations. Since both players are rational they can reflect on all this. If they apply probabilities to a single case, and if they also reflect on how things look from the perspective of the other player, then they must come to the conclusion that their own assignment of probabilities is arbitrary; this leads, as I have argued and will not repeat here, straight into Moore-Paradoxality (cf. Baumann 2005, 77). The latter problem is just another aspect of the same basic problem. The overall conclusion is quite general: The use of single-case probabilities leads to absurdity at least in some cases. This shows that there is something very basic wrong with the idea of a probability in a single case.

II.

Ken Levy (2007) makes a couple of critical remarks about my argument. First a subtlety. He says: "Baumann ... defends our initial intuition that there is no advantage to switching over sticking in any single game" (3-4). Now, I certainly do not defend the intuition of stickers, neither for the original Monty Hall scenario nor for my modified Monty Hall scenario. Do I think that there is "no advantage of switching over sticking in any single game"? I do not hold that switching "is not necessarily the more rational strategy in the Monty Hall situation where I am playing only one game" (4). To hold that view would be to presuppose what I argue against: that probabilities can be meaningfully applied to single cases. Only on that basis could one make judgments about the relative merits of the two strategies in single cases. In other words, I am not saying that one of the strategies is better than or as good as the other. Levy, however, says at the beginning of his paper that according to me "it is not necessarily rational to switch doors in a single game" (2; cf. 7, 12). If "not rational" means "irrational", then I certainly disagree. I would not even qualify a decision in a single case as "rational" or "not rational" if it is based on probabilistic reasons like the ones we are discussing here.

Levy holds that my main argument "splits into two arguments" (4). Let us see whether that is correct. According to Levy my "first argument" concludes that "probabilities cannot be meaningfully applied to a single game" (5) - the reason being simply that the probabilities would not add up in the right way. However, as my original paper (as well as section I above) makes clear, this is not the whole argument: Of crucial importance is the additional point that the two players do not have relevantly different information. So, what Levy calls "Baumann's First Argument" was never offered by me as a complete argument.

Levy goes on to make two objections against this "argument". Interestingly, he does not raise the problem I myself raise (both in my original paper and in section I above): that epistemic probabilities of different people do not necessarily have to add up to 1; different people can assign different probabilities to the same propositions if they have different relevant background information. Levy rather objects that "if probabilities do not 'add up' in a single game, then they should not any more 'add up' in an extended series. It is difficult to see - and Baumann fails to make it clear - how the problematic probabilities in a single game would be 'ironed out' merely by the repetition of games."(5). Well, even though this is rather a side issue here I will sketch an explanation why single cases differ from a series of cases in that respect.

It is a basic assumption of probability theory that the probabilities of several propositions only add up to 1 if those propositions are mutually exclusive and jointly exhaustive. In a single case of the kind of modified Monty Hall game I am discussing here (or in Baumann 2005, 72), the players have exactly two options: switching or sticking.[v] Take a case where both players pick the same door initially; then it is obvious that the two options they have are in this single case mutually exclusive and jointly exhaustive. Hence, the two probabilities of winning would have to add up to 1 (but they do not, as it turned out). However, the case of an extended series of games is different because the condition of mutual exclusivity is not met. “Exclusivity” means different things when applied to single cases or to a series of cases: In series of cases the possibility of cases where both strategies recommend the same action suffices for lack of exclusivity whereas in a single case it does not. Take a sufficiently long series of the modified Monty Hall game presented here. As one can see easily from the table above, there are 2 out of 9 cases in which the two strategies give the same advice. Hence, the exclusivity condition is not met for the series of cases and the simple addition theorem of probability theory does not apply. This explains why we get absurd results for single games but not for series of games. Note: If one leaves the two "overlapping" cases just mentioned aside and just considers the remaining 7 cases, then exclusivity holds and the probabilities add up again in the right way (4/7 + 3/7 =1).

Levy's second objection to what he calls "Baumann's First Argument" is this: "Second, Baumann's chart is simply mistaken. If the prize is behind door 1, then A does *not* win by sticking with door 2 in Situation 6 or by sticking with door 3 in Situation 8.

This is plain error. Instead, A should *switch* to door 1 in both cases. Once we correct these errors in the chart, we find that the probabilities are not 6/9 and 5/9 but rather 6/9 and 3/9. So (2) no longer follows, and Baumann's First Argument collapses." (5). What Levy means by "Situation 6" and "Situation 8" is simply the sixth and eighth row of the second table above (cf. my 2005, 72).

I think Levy is confused here about words. Whoever follows the modified switching strategy in a particular situation in the modified Monty Hall game counts as switching, according to my definition. And whoever follows the modified sticking strategy in a particular situation in the modified Monty Hall game counts as sticking, according to my definition. Levy, however, is using the terms "switching" and "sticking" in a different sense, namely in the sense that is appropriate only to the original Monty Hall game. Later Levy says that when the winning door 1 is opened the "game is over". Well, not according to my definition of the modified Monty Hall game (see above).

Levy moves on to what he calls "Baumann's Second Argument". His exposition of the argument (cf. 5-6) captures much better the essence of my real argument. Interestingly, Levy now mentions - but only very shortly - the crucial point about relevant information (cf. his premis (9)).

His critique of this argument, more or less the argument I am really making, is not easy to identify. He says:

> "Contrary to Baumann, the addition of a second player to the game leads the probability for both closed doors to rise to 1/2. ... Consider again the one-player scenario. When A chooses door 1 and Monty Hall then opens door 3,

> we assume that door 3 was opened because it is part of the pair of doors that is opposed to the door that A initially chose. And because there was a 2/3 probability that the car was behind one of the two doors in this pair, either door 2 or door 3, the probabilities rise to 2/3 for the only door remaining in the pair, door 2. ... But when A competes against B, we may no longer assume that door 3 was opened because it is part of the pair of doors that is opposed to the door that A initially chose - i.e., doors 2 and 3. For it might just as easily have been opened because it is part of the pair of doors that is opposed to the door that B initially chose - i.e., doors 1 and 3. Since we have no reason to place door 3 in either pair, it follows that doors 1 and 2 oppose each other individually; neither opposes the other as part of a pair with door 3. Therefore the probability for either door does not rise to 2/3. Instead the probabilities for both doors rise to 1/2." (6).

I do not see how this conclusion follows. In the original Monty Hall game, door 3 was not just opened because it was part of a pair of doors A did not initially choose but also because it was empty. Furthermore, it is misleading to say that in the modified Monty Hall game door 3 was chosen as much because it is part of one pair of doors (2 and 3) as because it is part of another pair of doors (1 and 3). No, it was chosen because neither A nor B initially chose it (see my description of the modified Monty Hall game). Hence, Levy's "indifference" argument for equal probabilities of 1/2 does not even get off the ground.[vi]

III.

Levy moves on to what he calls "Baumann's Rigidity Argument" (6). The first point to note here is simply that I do not think that there is an additional argument at all (this is one of the rare occasions where an author defends himself against a critic by pointing out that he has fewer arguments than the critic thinks he has!). The points I made (cf. Baumann 2005, 75-76) about rigidity were rather supposed to clarify a tricky and potentially misleading point. I will try to keep it short (since Levy is attacking an argument I am not making).

There are two ways of referring to doors here: via definite descriptions like “the other door” or “the originally chosen door” or via rigidly referring proper names like “door 1”, “door 2” or “door 3”. Suppose I am the only player and Monty has just opened door 3 after I picked door 1. In this case, the expressions “the other door” and “door 2” are co-referring but certainly don´t have the same meaning. This has important implications. The expressions “the probability that the other door will win” and “the probability that door 2 will win” are not synonymous. They are not even co-referential: the first one has an interpretation with respect to a larger number of games (it is, of course, 2/3). The second one does not make sense with respect to large numbers of games. Because of this and also because of the intensionality of epistemic probability (and we are dealing only with that kind of probability here), it is not legitimate for a player to infer from “the probability that the other door will win is 2/3” that “the probability that door 2 will win is 2/3”. This blocks an obvious objection in favor of single-case probabilities ("Isn't there a 2/3 probability that door 2 will win in this single case?"). It also avoids the problem of incoherent probabilities (see my main argument above). But it is not an argument leading to my main conclusion. Why not?

I argue that talk about winning by "choosing door 1" or "door 2" does not make sense. However, I am not using this assumption as a premise; on the contrary, it is the conclusion of my argument. The form of the argument is rather a reductio: If one assumes that single-case probabilities can be assigned, then one will have to accept that the single-case probabilities differ from player to player; since there is an independent argument that players share the same relevant information, this constitutes a violation of a crucial non-arbitrariness condition. Therefore, one should give up the idea of single-case probabilities. This leads back to the points mentioned in the first section above.

What does Levy object to what he calls "Baumann's Rigidity Argument"? He says: "First, I have already given compelling reasons to reject Baumann's First and Second Arguments. To the extent that the Rigidity Argument depends on these arguments, we may reject it as well." (7). Unsurprisingly, this does not impress me much, given that I do not find Levy's criticism of my argument above compelling. Here is Levy's second point: "It makes perfect sense to say that the probability for door$_{otherclosed}$ in this particular game, which will not be repeated, is 2/3. And if probability statements like this may be meaningfully applied to single games, then it is difficult to see why probability itself cannot meaningfully apply to single games." (7). Isn't Levy just asserting and assuming here what I am arguing against? I am not saying that Levy's objection does not make sense; rather, he simply assumes what I doubt.

IV.

Finally, in the context of what Levy calls “The Second Switching Argument” he argues for an alternative interpretation of probability, namely an “intrinsic interpretation” (using an analogous 3-card example with 1 king and 2 queens): “It is not that the 1/3 probability of drawing a king in a single instance derives from the statistical results over an extended series – as if these statistical results were nothing more than an accident or brute fact. Rather, the statistical results over an extended series derive from the 1/3 probability in each particular instance.” (8). Levy adds that this probability derives from the “causal structure” in the individual case: “the fact that (a) only one of the three cards is a king; (b) I have no evidence, and therefore must simply guess, which face-down card this is; and therefore (c) only one of the three possible guesses available to me can be successful.” (8). First of all, it is not quite clear what is meant by “causal structure” here; Levy does not give any further explanations. All he tells us is that the causal structure is whatever explains a given probability distribution. This is not very informative: we need to know more about this *explanans*. Levy certainly wants to avoid explanations of the kind of "Dormative powers explain sleeping behaviour". He would have to say much more before one can start to discuss his proposal. The three facts (a-c) mentioned do not help at all for an understanding of the nature of “intrinsic probability”; no adherent of a statistical or frequentist interpretation of probability would have to deny (a-c). Furthermore, one has to be careful with talk about “causal structure” because causality might imply a reference to a large number of cases (at least according to some standard theories of causality). By the way, I was not and am not arguing for a statistical or frequentist interpretation of probability; I was just arguing against any interpretation that allows for single-case probabilities. I did not start with a statistical or frequentist interpretation of probability (or any other for that matter); rather, my argument might, perhaps, lead to it.

Is Levy´s own argument, the "Second Switching Argument" (cf. 8-9) successful? Does it pose a threat against my argument? First of all, it is not clear how this argument bears on the problem of single-case probabilities. Apart from that, Levy makes an assumption that would need much more argumentative support in order to seem acceptable. He says that when I initially pick door 1 and Monty Hall opens door 3, then I do not learn anything new about door 1 which would give me a reason to update the prior probability from 1/3 to a new value. However, he thinks that I do learn something new about door 2 which forces the rational player to update the prior probability from 1/3 to 2/3. It is not so clear how exactly the need for an update follows from the kind of information the player gets about the three doors; not all information is relevant. Apart from that: new information about one of the doors is also, indirectly, new information about the other doors. If I find out that door 2 is the unique F, then I can easily infer (as a rational person) that door 1 is not the unique F. This is new information about door 1, too. Levy would have to say much more about the concept of information and on how it impacts the updating of probabilities before one can discuss the Second Switching Argument in more detail. Levy's second switching argument would deserve a whole paper;

given that he could only shortly sketch it in his paper, I should restrict myself to these brief remarks here.

V.

Does my result for the modified Monty Hall game also apply to the original Monty Hall game? Does it tell us something more general? I think it does. I have offered a counterexample against the idea that there is no problem with the idea of single case probabilities. Since there appear to be no relevant differences between the two versions of the Monty Hall game (relevant to the idea of single-case probabilities), I am justified in extending the result to the original Monty Hall game. A particular case can teach more general lessons. Nobody to my knowledge has presented an argument to the effect that there is a fundamental difference between one-player and two-player situations such that my results are restricted to two-player situations. Lacking such an argument, I take my claim to be justified in its generality.

Levy ends with a somewhat surprising turn: "But I think that the most convincing approach is merely to step back and look at the matter with a healthy dose of common sense, free from the complications of the Monty Hall problem." (12). Common sense, according to Levy, recommends switching in series of games as well as in single cases (12). He thinks that this is "perhaps the most convincing" reason to reject my view (cf. 12). I disagree, again. What people tend to call "common sense" has a mixed history and has often been shown to be wrong. For instance, those who propose to stick in the original

Monty Hall Game often claim to have common sense on their side. To defer a tricky problem to common sense rather than to argument, does not strike me as a good idea at all. However, I take it that in the end Levy and I might agree on this. Otherwise it would be hard to understand why he wrote a whole paper in response to mine.

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Notes

i Here is another way to make the point. Consider a series of 300 Monty Hall games. How often does the initial choice pick the winning door? Obviously (roughly) in 1/3 of the cases, that is in 100 games. The remaining cases are, obviously again, cases where the initial choice is not a winning one but where the one remaining door is one with the prize behind it. In other words, switching will lead on to a win in (roughly) 2/3 of the cases, that is in 200 games.

ii For the sake of simplicity of expression, I will at times use the terms "switching" and "sticking" for the modified strategies defined above. I will do that when it is clear that I am talking about the modified Monty Hall game. In those cases, what is meant by the terms "switching" and "sticking" differs from what is meant by them in the context of talk about the original Monty Hall game.

iii This does not mean that the strategies are not really distinct; two different strategies can recommend the same things in some (though not all) possible cases. Even though the principle of expected utility and the dominance principle might conflict in some cases (e.g., Newcomb situations) they recommend the same choice in many cases; this, however, does not mean that the principles are not very different.

iv What is relevant, according to me, is the information whether the other player has chosen the same or a different door (the probability of winning by sticking, given that the other player has initially chosen the same door, is 1/3 whereas it is 2/3, given that the other player has initially chosen a different door – as a look at the second table above shows). It is, however, irrelevant whether the door the other player has chosen is door 2 (or any other particular door) or not (cf. Baumann 2005, 74).

v Remember that I am using the terms "switching" and "sticking" in the sense explained for the case of the modified Monty Hall game.

vi Interestingly, Levy seems to admit being slightly uncomfortable with his objection when he says that "this argument will become clearer in Sect. 6" (6).