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MEASUREMENTS OF THE DRAG ON SPHERES FALLING THROUGH THE AIR

A problem often investigated in undergraduate labs on classical mechanics is the effect of air resistance on falling objects. One of the most interesting of these involves the drag force on spheres falling through the air. Frequently in these experiments the time of fall of the sphere is measured; this time is then used in a drag equation to calculate the distance fallen, for comparison with the measured distance. This distance measurement is a critical one and, for distances on the order of a few meters or less, must be accurate to within 0.5 mm. Measurements to this precision are difficult with the equipment and techniques available to the average undergraduate laboratory and, when several distances are involved, can present a large source of error. The experiment to be described eliminates the need for this distance measurement by using a phototransistor timing circuit to obtain the velocities at various times of flight. These measured velocities are then compared to the velocities calculated from a drag equation.

For many problems in fluid mechanics it is useful to define the dimensionless drag coefficient C_d in terms of the drag force F_d by the equation $C_d = (F_d/A)/(\frac{1}{2}\varrho V^2)$, where A is the cross sectional area of the body normal to the direction of motion, ϱ is the density of the medium, and V is the speed of the body. Thus, for a sphere falling through the air, $F_d = \frac{1}{2}C_d\pi r^2\varrho_a V^2$, where ϱ_a is the density of air and r is the radius of the sphere. Another dimensionless quantity which relates the inertial forces on a body to the viscous forces is the Reynolds number R which is defined by $R = 2rV/\nu$, where ν is the kinematic viscosity of the medium. For spheres, the relationship between C_d and R is well known from experiment; a value of $C_d \approx \frac{1}{2}$ for R between 10^3 and 10^5 is commonly accepted (H. Rouse, *Elementary Mechanics of Fluids,* Dover Publishing, 1978, p. 249). For air at room temperature, $\nu = 0.15$ cm/s², and for a sphere of radius r = 2 cm, this range corresponds to velocities between 38 cm/s and 3800 cm/s. Since all velocity measurements in this experiment are made in this range, the drag force may be approximated by:

$$F_d = \frac{1}{4}\pi r^2\varrho_a V^2 \ . \qquad [1]$$

From Newton's 2nd Law,

$$M\frac{dV}{dt} = Mg - F_d = Mg - \frac{1}{4}\pi r^2\varrho_a V^2 \ , \qquad [2]$$

and

$$\frac{dV}{dt} = g - (\frac{1}{4}\pi r^2\varrho_a V^2)/(\frac{4}{3}\pi r^3\varrho_s) \ , \qquad [3]$$

with ϱ_s = the density of the sphere, so that

$$\frac{dV}{dt} = g(1 - (3\varrho_a V^2)/(16\varrho_s gr)) = g(1 - V^2/V_0{}^2) \qquad [4]$$

where $V_o^2 = (16\varrho_s gr)/(3\varrho_a)$. From this we find

$$\frac{dV}{V_0{}^2 - V^2} = g\frac{dt}{V_0{}^2} \ . \qquad [5]$$

Integrating both sides and setting V = 0 at t = 0, we have

$$(1/V_0)\tanh^{-1}(V/V_0) = gt/V_0{}^2 \ , \qquad [6]$$

and

$$\tanh^{-1}(V/V_0) = gt/V_0 \ . \qquad [7]$$

With this approximation the velocity of a sphere falling through air is given by

$$V = V_0\{\tanh(gt/V_0)\} \ . \qquad [8]$$

The device used for dropping the spheres was constructed by modifying the apparatus used in a typical undergraduate falling body lab. An electromagnet is mounted on a stand approximately 1.7 meters tall. Positioned in front of the magnet is a bracket designed to hold the ball before it drops without influencing its motion when it is released. The ball is held against the bracket by a metal bar which is hinged to the stand under tension by a spring, and held in place by the electromagnet. Timing was done with a Merlan Micro Series Computimer #30M100 controlled by a Commodore CBM Model 8032 Computer. A double pole-single throw switch is used to control the electromagnet as well as the timer. When

Table 1. Velocities of falling spheres at various fall times.

Time of fall (msec)	Ideal Velocity (cm/s)	Calculated Velocity (cm/s)	Measured Velocity (cm/s)
146	142.97	142.89	142.95
236	231.46	231.13	230.73
278	272.21	271.67	271.98
324	317.17	316.33	316.50
370	362.41	361.15	360.80
406	398.14	396.47	396.23
455	445.58	443.24	443.58
484	473.72	470.91	470.57
515	504.80	501.40	501.06
551	539.80	535.65	535.77

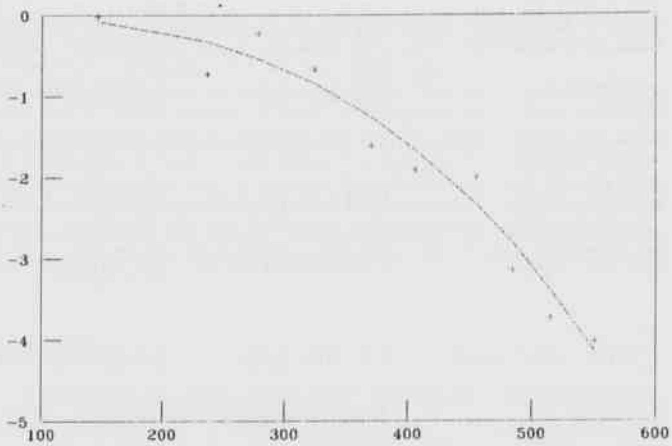

Figure 1. Differences from the ideal falling body case (dashed line - calculated values; plusses - measured values).

the switch is thrown, the power to the magnet is cut off, allowing the bar to fall and release the ball. At the same time, the timer cycle begins. Located in the path of the ball is a photogate consisting of a laser beam directed onto a phototransistor connected to the timing system. When the ball breaks the beam, the phototransistor is turned off and the timer stops. In addition, the computer can be set to measure the time the phototransistor is obstructed by the sphere. In this mode, a microsecond timer begins when the leading edge of the sphere breaks the beam and ends when the phototransistor is no longer darkened.

The experiment was conducted using a plastic sphere of diameter d = 4.379 cm and mass M = 57.80 g. Since a photogate may not be obstructed by all of an object which passes through it, a preliminary test was made and the effective diameter of the sphere was found to be 4.27 cm. The main timing procedure involved first setting the timer to the microsecond time-through-gate mode and determining the time taken for the ball to travel through the photogate. Great care was taken through numerous horizontal adjustments of the photogate system to insure that the sphere passed through the beam along its diameter, since this path will give the longest possible time measurement for the pass through the gate. When this time was achieved consistently, it was recorded as the gate time and the timer was reset to the (millisecond) time-of-flight mode. The ball was then dropped to determine the time taken to fall from rest to the photogate. This entire procedure was followed for 10 different heights at approximately 15 cm increments.

The average velocity of an object travelling a distance d in a time t can be written as $V_{av} = d/t$. If the velocity increases linearly with time, then the average velocity is equal to the velocity at t/2. For the time interval during which the sphere passes through the gate, the acceleration is very nearly constant, so these approximations are valid for calculating the velocity at a particular time. Thus, the experimental velocity, which is just the average velocity of the sphere through the photogate, may be calculated from 4.27 cm/(gate time) and corresponds to a total travel time equal to the time of flight plus one half the gate time. Using this travel time, the theoretical velocity was calculated from the drag equation (Eq. 8). A value for g for the local latitude and elevation equal to 979.7 cm/s^2 and an air density of $\varrho_a = 1.2 \times 10^{-3}$ g/cm^3 were used. A computer program was written to determine the lag time between the start of the timer and the actual dropping of the sphere to obtain the best agreement between the theoretical and calculated velocities; the lag was found to be 3 msec. The velocity if there were no air resistance is referred to as the ideal velocity and can be calculated from V = gt. Figure 1 shows the differences from the ideal case for the measured velocities and the velocities calculated from Eq. 8 for various times of flight. It is seen from the figure that the experimental velocities correspond very well with the calculated velocities.

We have shown that this timing technique can be used to provide measurements of the effects of drag forces on falling bodies to relatively high precision. This can be useful in the undergraduate laboratory to supplement the usual free-fall measurement of the gravitational acceleration and to introduce the concept of drag forces in viscous media.

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EXTRALIMITAL HUMMINGBIRDS IN ARKANSAS

The Ruby-throated Hummingbird *(Archilochus colubris)* is the only species of hummingbird known to nest in Arkansas or anywhere else in eastern North America (James and Neal, 1986; A.O.U., 1983). Until 1985 it was the only hummingbird species ever identified in Arkansas, where it is a common summer resident and migrant in all parts of the state (James and Neal, 1986).

Before 1985 there were several well documented Arkansas reports (notably at Little Rock in December of 1978 and January of 1979 and at North Little Rock in October of 1984) of unidentified hummingbirds that clearly belonged to other species (Arkansas Audubon Society files). All these were identified as members of the genus *Selasphorus*, most likely Rufous Hummingbirds *(S. rufus)*, a western species that normally winters in Mexico, though small numbers of Rufous Hummingbirds regularly overwinter along the Gulf Coast of the U.S. (A.O.U., 1983). Positive identification of the Arkansas birds could not be made on the basis of field observations or photographs. Only in the plumage of the adult male (with an all-rufous back) is *S. rufus* distinguishable in the field from the very similar Allen's Hummingbird *(S. sasin)*. In female and immature plumages,