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# Low Speed Current Bearing Anti-force Waves

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Running Title: Low Speed Current Bearing Anti-force Waves

## Abstract

For theoretical investigation of electrical breakdown of a gas, we apply a one-dimensional, steady profile, constant velocity, three-component (electrons, ions and neutral particles) fluid model. Our fluid model consists of the equations of conservation of mass, momentum and energy, coupled with the Poison's equation. The set of equations is referred to as the electron fluid dynamical equations (EFD). This investigation involves breakdown waves with a substantial current behind the wave front, and waves for which the electric field force on electrons is in the opposite direction of the wave propagation (anti-force waves – lightning return stroke). Therefore, the set of electron fluid dynamical equations need to be modified. For a low wave speed, we intend to find current values, and also the maximum current, for which solutions for our set of electron fluid dynamical equations become possible.

## Introduction and Model

For anti-force waves the electric field force on electrons is in the opposite direction of the wave propagation; however, the electron gas pressure is considered to be large enough to provide the driving force. The leading edge of the wave is treated as a shock front followed by a thin dynamical transition region, referred to as the sheath region of the wave. Following the sheath region of the wave is a relatively thicker region, in which the electron gas cools down through further ionization of the heavy particles. This region is referred to as the quasi-neutral-region of the wave. In the sheath region, the electric field starting with its maximum value at the shock front reduces to zero at the trailing edge of the wave; and the electrons, starting with an initial speed at the wave front, slow down to speeds comparable to those of ions and neutral particles.

To analyze breakdown waves, we use the set of equations which were developed by Fowler et al.

(1984). This set of equations describes pro-force waves and has proven to be successful (1984). The set of equations consists of the equations of conservation of mass, momentum, and energy plus the Poisson's equation, and they respectively are

$$\frac{d(n\upsilon)}{dx} = n\beta, \tag{1}$$

$$\frac{d}{dx}[mnv(\upsilon - V) + nkT_e] = -enE - Kmn(\upsilon - V), \tag{2}$$

$$\frac{d}{dx}[mn\upsilon(\upsilon - V)^2 + nkT_e(5\upsilon - 2V) + 2en\upsilon\Phi - \frac{5nkT_e}{mK}\frac{dT_e}{dx}]$$
$$= -3(\frac{m}{M})nkKT_e - (\frac{m}{M})Kmn(\upsilon - V)^2, \tag{3}$$

$$\frac{dE}{dx} = \frac{e}{\varepsilon_0}n(\frac{v}{V} - 1). \tag{4}$$

where $n$, $v$, $T_e$, $e$ and $m$ represent the electron number density, velocity, temperature, charge, and mass, respectively, and $M$, $E$, $E_0$, $V$, $k$, $K$, $x$, $\beta$ and $\phi$ represent the neutral particle mass, electric field within the sheath region, electric field at the wave front, wave velocity, Boltzmann's constant, elastic collision frequency, position within the sheath region, ionization frequency and ionization potential of the gas.

To reduce the set of electron fluid dynamical equations to a non-dimensional form, Fowler et al. (1984), introduced the following set of dimensionless variables:

$$\eta = \frac{E}{E_o}, \nu = (\frac{2e\phi}{\varepsilon_o E_o^2})n, \psi = \frac{v}{V}, \theta = \frac{T_e k}{2e\phi}, \xi = \frac{eE_o x}{mV^2},$$

$$\alpha = \frac{2e\phi}{mV^2}, \kappa = \frac{mV}{eE_o}K, \mu = \frac{\beta}{K}, \omega = \frac{2m}{M},$$

in which $\eta$, $\nu$, $\psi$, $\theta$, $\mu$ and $\xi$ represent the dimensionless net electric field of the applied field plus the space charge field, electron number density, electron velocity, electron gas temperature, ionization rate, and position within the sheath region, while $\alpha$ and $\kappa$

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represent wave parameters. Substituting these dimensionless variables in equations 1-4 yields

$$\frac{d(\nu\psi)}{d\xi} = \kappa\mu\nu, \qquad (5)$$

$$\frac{d}{d\xi}[\nu\psi(\psi-1)+\alpha\nu\theta] = -\nu\eta - \kappa\nu(\psi-1), \qquad (6)$$

$$\frac{d}{d\xi}[\nu\psi(\psi-1)^2 + \alpha\nu\theta(5\psi-2)+\alpha\nu\psi+\alpha\eta^2 -$$

$$\frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}] = -\omega\kappa\nu[3\alpha\theta + (\psi-1)^2], \qquad (7)$$

$$\frac{d\eta}{d\xi} = \frac{\nu}{\alpha}(\psi-1). \qquad (8)$$

To solve for anti-force problems, we will use the set of non-dimensional variables developed by Hemmati (1999), in which all quantities including κ are positive and ξ is positive backward. The set of non-dimensional variables for anti-force waves are

$$\eta = \frac{E}{E_o}, \nu = (\frac{2e\phi}{\varepsilon_o E_o^2})n, \psi = \frac{\nu}{V}, \theta = \frac{T_e k}{2e\phi}, \xi = -\frac{eE_o x}{mV^2},$$

$$\alpha = \frac{2e\phi}{mV^2}, \kappa = -\frac{mV}{eE_o}K, \mu = \frac{\beta}{K}, \omega = \frac{2m}{M}.$$

For breakdown waves with a significant current behind the shock front, in addition to the Poisson's equation and equation of conservation of energy, the boundary condition on electron temperature at the shock front needs to be modified as well. For theoretical investigation of anti-force waves with a significant current behind the shock front, we will use Hemmati et. al's (2011) modified set of electron fluid dynamical equations.

$$\frac{d}{d\xi}[\nu\psi] = \kappa\mu\nu, \qquad (9)$$

$$\frac{d}{d\xi}[\nu\psi(\psi-1)+\alpha\nu\theta] = \nu\eta - \kappa\nu(\psi-1), \qquad (10)$$

$$\frac{d}{d\xi}[\nu\psi(\psi-1)^2 + \alpha\nu\theta(5\psi-2)+\alpha\nu\psi - \frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}$$

$$+\alpha\eta^2] = 2\eta\kappa\iota\alpha - \omega\kappa\nu[3\alpha\theta + (\psi-1)^2], \qquad (11)$$

$$\frac{d\eta}{d\xi} = \kappa\iota - \frac{\nu}{\alpha}(\psi-1). \qquad (12)$$

Where, with $I_1$ representing the current behind the shock front,

$$\iota = \frac{I_1}{\varepsilon_0 K E_0}, \qquad (13)$$

is the dimensionless current behind the wave front.

**Results and Discussion**

Uman and McLain (1970) derived an expression to calculate the current for stepped leader (pro-force waves) in lightning. Their calculated values for current were in the range of 800 to 5000 amperes. With optical observations and measuring currents at the lightning channel base, Rakov et al. (1998) reported a stepped leader current value of 5 kA and return stroke (anti-force) peak current value of 10 kA. In their study of lightning attachment processes in rocket-triggered lightning strokes, for return-strokes, Wang et al. (1999) reported a current peak value of about $12 \approx 21kA$. Determining K from experimental curves (McDaniel 1964), at a temperature of $10^5$ K, K will be $2.4 \times 10^9$ for helium and $9 \times 10^9$ for nitrogen. In our formulas $E_0$, K, and β are scaled with electron pressure, P, and the applied fields are of the order of $10^5$ *V / m*. For $I_1 = 10kA$, using the values of $I_1$, $\varepsilon_0$, $E_0$, and K, one can estimate the value of the dimensionless current, $\iota$, which is on the order of one. Using helium-filled discharge tubes with different diameters, Asinovsky et al. (1994) measured breakdown wave speeds ranging from $10^7$ m/s to $6 \times 10^7$ m/s.

We use a trial and error method to integrate equations (9-12). For a given wave speed, α, at the wave front a set of values of wave constant, κ, electron velocity, $\psi_1$, and electron number density, $\nu_1$, were selected and equations (9-12) were integrated with that set. The values of κ, $\psi_1$, and $\nu_1$ were changed repeatedly in integrating equations (9-12), until the process leads to a conclusion in agreement with the expected conditions at the trailing edge of the wave.

Using Hemmati et al.'s (2011) modified electron temperature at the shock front,

$$\theta_1 = \frac{\psi_1(1-\psi_1)}{\alpha} - \frac{\kappa\iota}{\nu_1}, \qquad (14)$$

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for several current values and for a relatively low wave speed, we have been able to integrate equations (9-12) through the sheath region of the wave. Our solutions meet the expected physical conditions at the trailing edge of the wave. However, for low wave speeds, integration of the set of equations became possible for lower current values only. For wave speed value of $5.93 \times 10^6$ m/s ($\alpha = 0.25$), successful solutions required the following boundary values

$$\iota = 0.0, \kappa = 0.3883, \psi_1 = 0.96, \nu_1 = 0.985$$

$$\iota = 0.25, \kappa = 0.36115, \psi_1 = 0.9205, \nu_1 = 0.91405$$

$$\iota = 0.7, \kappa = 0.332, \psi_1 = 0.87, \nu_1 = 0.8342$$

$$\iota = 1.5, \kappa = 0.308, \psi_1 = 0.7805, \nu_1 = 0.723$$

Figure 1 is a graph of the electric field as a function of electron velocity. As the graph shows, dimensionless current value of 0.7 seems to be the maximum value for which solutions for the set of electron fluid dynamical equations come to a successful conclusion $(\eta_2 \rightarrow 0, \psi_2 \rightarrow 1)$.

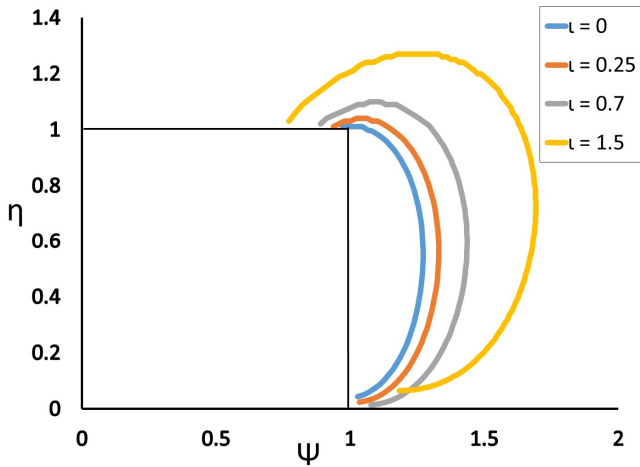

Figure 1. Electric field, η, as a function of electron velocity, Ψ, within the sheath region of current bearing anti-force waves for a wave speed value of α=0.25 and for current values 0, 0.25, 0.7 and 1.5.

Figure 2 is a graph of the electric field as a function of position within the sheath region of the wave. As the graphs show, for larger current values the sheath thickness becomes larger as well.
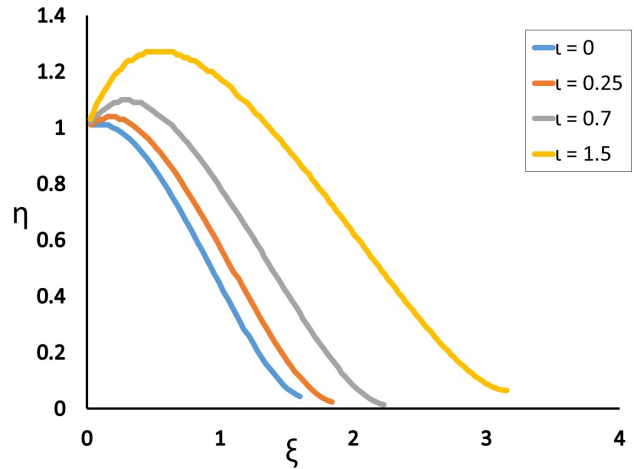


Figure 2. Electric field, η, as a function of position, ξ, within the sheath region of current bearing anti-force waves for a wave speed value of α=0.25 and for current values 0, 0.25, 0.7 and 1.5.
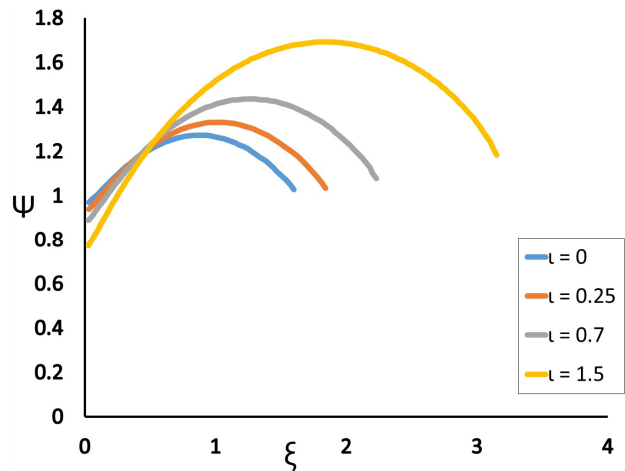


Figure 3. Electron velocity, Ψ, as a function of position, ξ, within the sheath region of current bearing anti-force waves for a wave speed value of α=0.25 and for current values 0, 0.25, 0.7 and 1.5.

Figure 3 is a graph of the dimensionless electron velocity as a function of dimensionless position within the sheath region of the wave. Figure 4 is a graph of the dimensionless electron number density as a function of dimensionless position within the sheath region of the wave. For current values for which solutions to the set of EFD equations become possible, 0.8 seems to be the average dimensionless electron number density within the sheath region of the wave. Dimensionless electron number density of 0.8 is equivalent to $8.85 \times 10^{15}$ *elc / m³*. In his fluid model simulations of a 13.56-MHz rf discharge, David Graves (1987) reports electron number density values between $5 \times 10^{15} / m^3$ and $2 \times 10^{16} / m^3$.
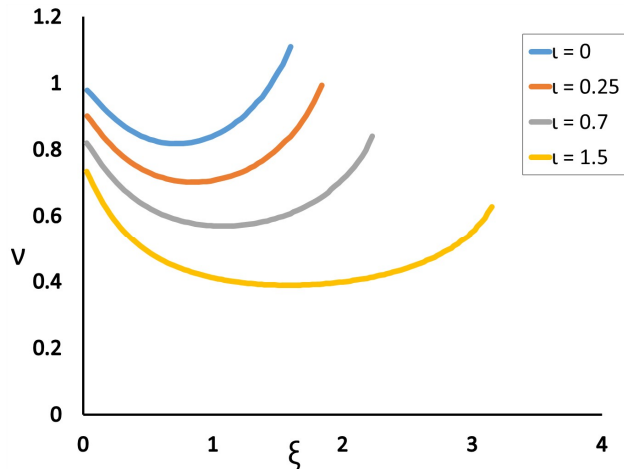
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Figure 4. Electron number density, ν, as a function of position, ξ, within the sheath region of current bearing anti-force waves for a wave speed value of α=0.25 and for current values, 0, 0.25, 0.7 and 1.5.
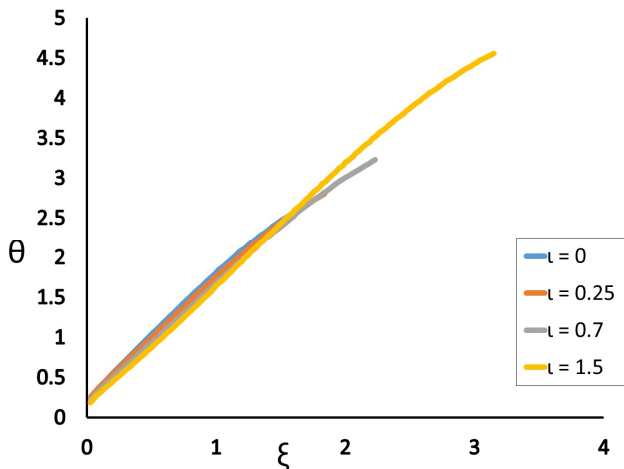


Figure 5. Electron temperature, θ, as a function of position, ξ, within the sheath region of current bearing anti-force waves for a wave speed value of α=0.25 and for current values 0, 0.25, 0.7 and 1.5.

Figure 5 shows a graph of the dimensionless electron temperature as a function of dimensionless position within the sheath region of the wave. Our average dimensionless electron temperature value of 2 is equivalent to an approximate temperature of $1.44x10^6$ K. For ionizing waves propagating counter to strong electric fields (anti-force waves), Sanmann and Fowler (1975) reported that the electron temperature increases rapidly away from the wave front until it reaches a peak value of around $3.17x10^7$ K.
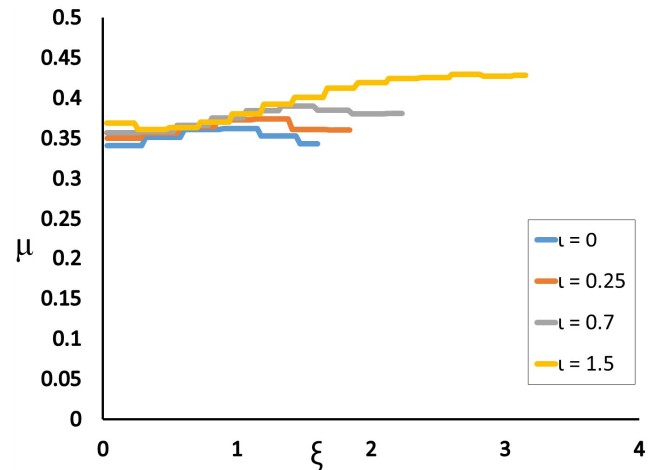


Figure 6. Ionization rate, μ, as a function of position, ξ, within the sheath region of current bearing anti-force waves for a wave speed value of α=0.25 and for current values 0, 0.25, 0.7 and 1.5.

Figure 6 is a graph of the dimensionless ionization rate as a function of dimensionless position with the sheath region of the wave. Earlier studies considered the ionization rate to be a function of temperature only; however, in our numerical integration of the set of electron fluid dynamical equations, the ionization rate was calculated considering both random and directed motion of the electrons (Fowler 1983). The graphs indicate that the ionization rate remains almost constant at the beginning of the sheath; however, it varies slightly as we traverse through the sheath.

**Conclusions**

For low wave speeds, solutions for the set of electron fluid dynamical equations became possible for smaller current values only. For low wave speeds, integration of the set of electron fluid dynamical equations becomes very time consuming and difficult. However, for larger current values, the sheath thickness becomes larger and makes the integration of the set of equations even harder. However, for wave speed value of $5.93x10^6$ m/s (α = 0.25), dimensionless current value of 0.7 ($I_1 \leq 10kA$) seems to be the cut-off point for current. For fast moving waves, $v = 10^8$ *m/s*, we have been able to find solutions for dimensionless current values as high as 10. Our results are in good agreement with the results reported by other investigators; this is another confirmation of the validity of our set of electron fluid dynamical equations and boundary conditions.

## Acknowledgement

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