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SOME CLASSES OF NONPARAMETRIC TESTS FOR SPECIAL TWO-SAMPLE LOCATION PROBLEM BASED ON SUBSAMPLE EXTREMES

Parameshwar V. Pandit[1] **and Deepa R. Acharya**[2]

[1]Department of Statistics
Bangalore University
Bengaluru 560 056, India
e-mail: panditpv@bub.ernet.in; panditpv12@gmail.com

[2]Department of Statistics
Government Science College
Bengaluru 560 001, India
e-mail: deeparacharya@yahoo.co.in

Abstract

The special two-sample location problem is an important problem which is useful in comparing the performance of two measuring instruments. The problem of comparing the performances of two packing machines in which one machine may underfill the packets and the other may overfill the packets on an average, fits into special two-sample location setup wherein one wishes to test for the point of symmetry versus an appropriate alternative. The only test available in the literature to the best of our knowledge is the class of tests due to Shetty and Umarani [13] which is based on *U*-statistics. In this paper, two classes of test statistics are proposed which are based on extremes of subsamples. The performances of the proposed classes of tests are

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evaluated in terms of Pitman asymptotic relative efficiency with respect to the test due to Shetty and Umarani [13]. It is observed that the members of proposed classes of tests perform better than the test due to Shetty and Umarani [13], for those distributions considered for evaluation.

1. Introduction

One of the important problems that have been widely studied in statistical inference is the two-sample location problem. This problem arises when one would like to know whether two samples come from the same distribution or they differ only in location, that has applications in many fields such as economics, botany, medicine, psychology, etc. However, a special type of location problem is useful in some situations which is described below, received less attention from researchers. This type of problem is quite commonly encountered while comparing the performance of two measuring devices. The problem of testing for point of symmetry against an appropriate alternative fits into a two-sample location setup, mentioned above.

One can find many nonparametric tests in the literature for the two-sample location problem. A popular nonparametric procedure for this problem is Mann-Whitney test [6]. Mood’s median (M) test [8] is effective in detecting shift in location in populations whose distributions are symmetric and heavy tailed whereas, in detecting shifts in moderately heavy tailed distributions, Gastwirth’s *H* and *L* tests [2] are effective. The normal scores (NS) test [3] is effective in detecting a shift in the normal distribution. The RS test due to Hogg et al. [4] is effective in detecting shifts in distributions that are skewed. The SG test proposed by Shetty and Govindarajulu [11] based on subsample medians takes care of two suspected outliers at the extremes of both the samples. A generalization of test due to Mathisen [7] is considered by Shetty and Bhat [12]. Their relative efficiency and suitability depend on the nature of the (unknown) underlying distribution. Ahmad [1] proposed a generalization of Mann-Whitney test for this problem based on subsample extremes. However, the special two-sample location problem received less attention from the researchers though it has potential applications. The only test available in the literature for this special type of two-sample location is due to Shetty and Umarani [13].

In this paper, we consider two classes of distribution free tests for the special type of two-sample location problem. The proposed classes of tests are given in Section 2. Section 3 deals with the distributional properties of the statistics and Section 4 gives the asymptotic relative efficiency comparisons. Remarks and conclusions are given in Section 5.

2. Proposed Classes of Tests

Let $X_1, X_2, ..., X_m$ and $Y_1, Y_2, ..., Y_n$ be independent random samples from absolutely continuous distribution functions with cdf $F(x + \theta)$ and $F(x - \theta)$, respectively. We wish to test the hypothesis $H_0 : \theta = 0$ versus the alternative $H_1 : \theta > 0$. It is assumed that $F(0) = \frac{1}{2}$. The two classes of test statistics are proposed for testing H_0 against H_1.

The proposed classes of test statistics are defined as

$$V_1(k, k) = \frac{1}{\binom{m}{k}\binom{n}{k}} \sum_A h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k),$$

$$V_2(k, k) = \frac{1}{\binom{m}{k}\binom{n}{k}} \sum_A h_2(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k),$$

where

$$h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k) = \begin{cases} 1 & \text{if } X_{(1)} \le 0 \le Y_{(k)}, \\ 0 & \text{otherwise,} \end{cases}$$

$$h_2(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k) = \begin{cases} 1 & \text{if } X_{(k)} \le 0 \le Y_{(1)}, \\ 0 & \text{otherwise,} \end{cases}$$

$$X_{(1)} = \min(X_1, X_2, \ldots, X_k),$$

$$X_{(k)} = \max(X_1, X_2, \ldots, X_k),$$

$$Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_k),$$

$$Y_{(k)} = \max(Y_1, Y_2, \ldots, Y_k),$$

$1 \le k \le \min(m, n)$ and $\sum_A$ indicates the sum over all subsamples of size k drawn without replacement from X and Y samples. Here $V_1(k, k)$ and $V_2(k, k)$ are the two, two-sample U-statistics. Large values of $V_1(k, k)$ and $V_2(k, k)$ are significant for testing H_0 against H_1.

3. Distributional Properties of the Statistics

The mean of $V_1(k, k)$ is given by

$$\begin{aligned} E(V_1(k, k)) &= E(h_1(X_1, X_2, \ldots, X_k; Y_1, Y_2, \ldots, Y_k)) \\ &= P(X_{(1)} < 0 < Y_{(k)}) \\ &= (1 - \overline{F}^k(\theta))(1 - F^k(-\theta)). \end{aligned}$$

Under H_0, $E(V_1(k, k)) = \left(1 - \left(\frac{1}{2}\right)^{2k}\right)^2.$

The mean of $V_2(k, k)$ is given by

$$\begin{aligned} E(V_2(k, k)) &= E(h_2(X_1, X_2, \ldots, X_k; Y_1, Y_2, \ldots, Y_k)) \\ &= P(X_{(k)} < 0 < Y_{(1)}) \\ &= F^k(\theta)\overline{F}^k(-\theta). \end{aligned}$$

Under H_0, $E(V_2(k, k)) = \left(\frac{1}{2}\right)^{2k}.$

The computation of exact variance of $V_1(k, k)$ and $V_2(k, k)$ is very tedious for arbitrary k. Hence we obtain the asymptotic distribution of $V_1(k, k)$, $V_2(k, k)$ using generalized U-statistics theorem due to Lehmann [5], which is given as below.

Theorem 3.1. *Let* $(X_1, X_2, ..., X_m; Y_1, Y_2, ..., Y_n)$ *denote independent random samples from populations with distribution functions* $F(x)$ *and* $G(y)$, *respectively. Let* $h(\cdot)$ *be a symmetric kernel for an estimable parameter* γ *of degree* (r, s). *If* $E[h^2(X_1, X_2, ..., X_r; Y_1, Y_2, ..., Y_s)] < \infty$, *then* $\sqrt{N}(V(X_1, X_2, ..., X_m; Y_1, Y_2, ..., Y_n) - \gamma)$ *has a limiting normal distribution with mean* 0 *and variance* $\frac{r^2\xi_{10}}{\lambda} + \frac{s^2\xi_{01}}{1-\lambda}$, *provided this variance is positive, where* $0 < \lambda = \lim_{N\to\infty} \frac{m}{n} < 1$ *and* $N = m + n$, *where*

$$\xi_{10} = Cov[h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$
$$h_1(X_1, X_{k+1}, ..., X_{2k-1}; Y_{k+1}, ..., Y_{2k})]$$

and

$$\xi_{01} = Cov[h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$
$$h_1(X_{k+1}, ..., X_{2k}; Y_1, Y_{k+1}, ..., Y_{2k-1})].$$

From Theorem 3.1, it follows that $\sqrt{N}(V_1(k, k) - E(V_1(k, k)))$ has asymptotically normal distribution with mean zero and variance $\frac{k^2\xi_{10}}{\lambda} + \frac{k^2\xi_{01}}{1-\lambda}$ and $\sqrt{N}(V_2(k, k) - E(V_2(k, k)))$ has asymptotically normal distribution with mean zero and variance $\frac{k^2\xi^*_{10}}{\lambda} + \frac{k^2\xi^*_{01}}{1-\lambda}$, where

$$\xi_{10} = Cov[h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$
$$h_1(X_1, X_{k+1}, ..., X_{2k-1}; Y_{k+1}, ..., Y_{2k})]$$

$$\xi_{01} = Cov[h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$
$$h_1(X_{k+1}, ..., X_{2k}; Y_1, Y_{k+1}, ..., Y_{2k-1})]$$

$$\xi_{10}^{*} = Cov[h_2(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$
$$h_2(X_1, X_{k+1}, ..., X_{2k-1}; Y_{k+1}, ..., Y_{2k})]$$

and

$$\xi_{01}^{*} = Cov[h_2(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$
$$h_2(X_{k+1}, ..., X_{2k}; Y_1, Y_{k+1}, ..., Y_{2k-1})].$$

$N = m + n$ and $0 < \lambda = \lim_{N\to\infty} \frac{m}{n} < 1.$

Under H_0,

$$\xi_{10} = Cov_{H_0}[h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$
$$h_1(X_1, X_{k+1}, ..., X_{2k-1}; Y_{k+1}, ..., Y_{2k})]$$
$$= \int_{-\infty}^{\infty} [P(\min(x, X_2, ..., X_k) < 0 < \max(Y_1, Y_2, ..., Y_k))]^2 dF(x)$$
$$- [E_{H_0}(V_1(k, k))]^2. \tag{3.1.1}$$

Now

$$P(\min(x, X_2, ..., X_k) < 0 < \max(Y_1, Y_2, ..., Y_k))$$
$$= a_1 + a_2 + a_3 + a_4 + a_5 + a_6 \tag{3.1.2}$$

with

$$a_1 = P[x < \min(X_2, ..., X_k) < 0 < \max(Y_1, Y_2, ..., Y_k)]$$
$$= \left[\bar{F}^{k-1}(x) - \left(\frac{1}{2}\right)^{k-1}\right]\left[1 - \left(\frac{1}{2}\right)^{k}\right] I_{(-\infty, 0)}(x),$$

$$a_2 = P[x < 0 < \min(X_2, ..., X_k) < \max(Y_1, Y_2, ..., Y_k)]$$

$$= k\left\{\left(\frac{1}{2}\right)^{k-1}\int_{0.5}^{1} x^{k-1}dx - \int_{0.5}^{1} x^{k-1}(1-x)^{k-1}dx\right\} I_{(-\infty, 0)}(x),$$

$$a_3 = P[x < 0 < \max(Y_1, Y_2, ..., Y_k) < \min(X_2, ..., X_k)]$$

$$= k\left\{\int_{0.5}^{1} x^{k-1}(1-x)^{k-1}dx\right\} I_{(-\infty, 0)}(x),$$

$$a_4 = P[\min(X_2, ..., X_k) < x < 0 < \max(Y_1, Y_2, ..., Y_k)]$$

$$= \left[1 - \bar{F}^{k-1}(x)\left(1 - \left(\frac{1}{2}\right)^{k}\right)\right] I_{(-\infty, 0)}(x),$$

$$a_5 = P[\min(X_2, ..., X_k) < 0 < x < \max(Y_1, Y_2, ..., Y_k)]$$

$$= \left[(1 - F^{k}(x))\left(1 - \left(\frac{1}{2}\right)^{k-1}\right)\right] I_{(0, \infty)}(x),$$

$$a_6 = P[\min(X_2, ..., X_k) < 0 < \max(Y_1, Y_2, ..., Y_k) < x]$$

$$= \left\{\left(1 - \left(\frac{1}{2}\right)^{k-1}\right)\left(F^{k}(x) - \left(\frac{1}{2}\right)^{k}\right)\right\} I_{(0, \infty)}(x),$$

where

$$I_C(x) = \begin{cases} 1 & \text{if } x \in C, \\ 0 & \text{otherwise.} \end{cases}$$

From (3.1.2), we have

$$P(\min(x, X_2, ..., X_k) < 0 < \max(Y_1, Y_2, ..., Y_k)) = A + B,$$

where

$$A = \left[1 - \left(\frac{1}{2}\right)^{k}\right] I_{(-\infty, 0)}(x)$$

and

$$B = \left[\left(1 - \left(\frac{1}{2}\right)^k\right)\left(1 - \left(\frac{1}{2}\right)^{k-1}\right)\right] I_{(0,\infty)}(x).$$

Therefore, from (3.1.1), we get

$$\xi_{10} = \int_{-\infty}^{\infty} [P(\min(x, X_2, \ldots, X_k) < 0 < \max(Y_1, Y_2, \ldots, Y_k))]^2 dF(x)$$

$$- [E_{H_0}(V_1(k, k))]^2$$

$$= \frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^k\right]^2 \left[1 + \left(1 - \left(\frac{1}{2}\right)^{k-1}\right)^2 - 2\left(1 - \left(\frac{1}{2}\right)^k\right)^2\right]. \qquad (3.1.3)$$

Next,

$$\xi_{01} = Cov_{H_0}[h_1(X_1, X_2, \ldots, X_k; Y_1, Y_2, \ldots, Y_k)$$

$$h_1(X_{k+1}, \ldots, X_{2k}; Y_1, Y_{k+1}, \ldots, Y_{2k-1})]$$

$$= \int_{-\infty}^{\infty} [P(\min(X_1, X_2, \ldots, X_k) < 0 < \max(y, Y_2, \ldots, Y_k))]^2 dF(x)$$

$$- [E_{H_0}(V_1(k, k))]^2. \qquad (3.1.4)$$

Here,

$$P(\min(X_1, X_2, \ldots, X_k) < 0 < \max(y, Y_2, \ldots, Y_k))$$

$$= b_1 + b_2 + b_3 + b_4 + b_5 + b_6,$$

where

$$b_1 = P[\min(X_1, \ldots, X_k) < 0 < y < \max(Y_2, \ldots, Y_k)]$$

$$= \left[1 - \left(\frac{1}{2}\right)^k\right][1 - F^{k-1}(y)] I_{(0,\infty)}(y),$$

$$b_2 = P[\min(X_1, ..., X_k) < 0 < \max(Y_2, ..., Y_k) < y]$$

$$= \left[1 - \left(\frac{1}{2}\right)^k\right]\left[F^{k-1}(y) - \left(\frac{1}{2}\right)^{k-1}\right] I_{(0, \infty)}(y),$$

$$b_3 = P[\min(X_1, ..., X_k) < \max(Y_2, ..., Y_k) < 0 < y]$$

$$= (k-1)\left\{\int_{-\infty}^{0} F^{k-2}(t)[1 - \bar{F}^k(t)]dF(t)\right\} I_{(0, \infty)}(y),$$

$$b_4 = P[\max(Y_2, ..., Y_k) < \min(X_1, ..., X_k) < 0 < y]$$

$$= k\left\{\int_{-\infty}^{0} F^{k-1}(t)[\bar{F}^{k-1}(t)]dF(t)\right\} I_{(0, \infty)}(y),$$

$$b_5 = P[\min(X_1, ..., X_k) < y < 0 < \max(Y_2, ..., Y_k)]$$

$$= \left[1 - \left(\frac{1}{2}\right)^{k-1}\right][1 - \bar{F}^k(y)] I_{(-\infty, 0)}(y),$$

$$b_6 = P[y < \min(X_1, ..., X_k) < 0 < \max(Y_2, ..., Y_k)]$$

$$= \left[1 - \left(\frac{1}{2}\right)^{k-1}\right]\left[\bar{F}^k(y) - \left(\frac{1}{2}\right)^k\right] I_{(-\infty, 0)}(y).$$

From equation (3.1.4), ξ_{01} is obtained as

$$\xi_{01} = Cov_{H_0}[h_1(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$

$$h_1(X_{k+1}, ..., X_{2k}; Y_1, Y_{k+1}, ..., Y_{2k-1})]$$

$$= \int_{-\infty}^{\infty} [P(\min(X_1, X_2, ..., X_k) < 0 < \max(y, Y_2, ..., Y_k))]^2 dF(x)$$

$$- [E_{H_0}(V_1(k, k))]^2$$

$$= \int_{-\infty}^{0} (A)^2 dF(y) + \int_{0}^{\infty} (B)^2 dF(y) - \left[\left(1 - \left(\frac{1}{2}\right)^k\right)^2\right]^2,$$

where

$$A = \left[1 - \left(\frac{1}{2}\right)^{k-1}\right]\left[1 - \overline{F}^{k}(y) + \overline{F}^{k-1}(y) + \left(\frac{1}{2}\right)^{k}\right],$$

$$B = \left[1 - \left(\frac{1}{2}\right)^{k}\right]\left[1 - \left(\frac{1}{2}\right)^{k-1}\right] + (k-1)F^{k-2}(y)(1 - \overline{F}^{k}(y))$$

$$+ k\overline{F}^{k-1}(y)F^{k-1}(y).$$

Thus, $\sqrt{N}(V_1(k, k) - E(V_1(k, k)))$ has asymptotically normal distribution with mean zero and variance $\sigma_1^2 = \frac{k^2\xi_{10}}{\lambda} + \frac{k^2\xi_{01}}{1-\lambda} = \frac{k^2\xi}{\lambda(1-\lambda)}$, when H_0 is true.

On similar lines, ξ_{10}^* is evaluated as

$$\xi_{10}^* = Cov_{H_0}[h_2(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$

$$h_2(X_1, X_{k+1}, ..., X_{2k-1}; Y_{k+1}, Y_2, ..., Y_{2k})]$$

$$= \int_{-\infty}^{\infty} [P(\max(x, X_2, ..., X_k) < 0 < \min(Y_1, Y_2, ..., Y_k))]^2 dF(x)$$

$$- [E_{H_0}(V_2(k, k))]^2$$

$$= \int_{-\infty}^{0} \left[\left(\frac{1}{2}\right)^{2k-1}\right]^2 dF(x) - \left[\left(\frac{1}{2}\right)^{2k}\right]^2 = \left(\frac{1}{2}\right)^{4k} \quad (3.1.5)$$

and

$$\xi_{01}^* = Cov_{H_0}[h_2(X_1, X_2, ..., X_k; Y_1, Y_2, ..., Y_k)$$

$$h_2(X_{k+1}, ..., X_{2k}; Y_1, Y_{k+1}, Y_2, ..., Y_{2k-1})]$$

$$= \int_{-\infty}^{\infty} [P(\max(X_1, X_2, ..., X_k) < 0 < \min(y, Y_2, ..., Y_k))]^2 dF(y)$$

$$- [E_{H_0}(V_2(k, k))]^2$$

$$= \int_{-\infty}^{0} \left[1 - \left(\frac{1}{2}\right)^{k}\right]^{2} dF(x) - \left[\left(\frac{1}{2}\right)^{2k}\right]^{2} = \left(\frac{1}{2}\right)^{4k}. \tag{3.1.6}$$

Thus, $\sqrt{N}(V_2(k, k) - E(V_2(k, k)))$ has asymptotically normal distribution with mean zero and variance $\sigma_2^2 = \frac{k^2\xi_{10}^*}{\lambda} + \frac{k^2\xi_{01}^*}{1-\lambda} = \frac{k^2\xi^*}{\lambda(1-\lambda)}$ when H_0 is true.

4. Asymptotic Relative Efficiency

Pitman [9] defined the asymptotic relative efficiency of one test P relative to another test Q as the limiting ratio of sample sizes required to obtain the same limiting power for a sequence of alternatives converging to null hypothesis. By Noether's theorem, it follows that

$$ARE(P, Q) = \left[\frac{eff(P)}{eff(Q)}\right]^2,$$

where

$$eff(P) = \frac{\frac{dE(P)}{d\theta}\Big|_{\theta=\theta_0}}{\sqrt{NVar_{H_0}(P)}}.$$

Under the assumption that $F(x)$ is differentiable at 0 with pdf $f(0) \neq 0$, we have

$$\frac{dE(V_1(k, k))}{d\theta}\Big|_{\theta=\theta_0} = 2kf(0)\left[1 - \left(\frac{1}{2}\right)^{k}\right]\left(\frac{1}{2}\right)^{k-1},$$

$$\frac{dE(V_2(k, k))}{d\theta}\Big|_{\theta=\theta_0} = 2kf(0)\left(\frac{1}{2}\right)^{k-1}.$$

Hence,

$$eff(V_1(k, k)) = \frac{2kf(0)\left[1 - \left(\frac{1}{2}\right)^{k}\right]\left(\frac{1}{2}\right)^{k-1}}{\sqrt{\frac{k^2\xi}{\lambda(1-\lambda)}}}$$

and

$$eff(V_2(k, k)) = \frac{2kf(0)\left(\frac{1}{2}\right)^{k-1}}{\sqrt{\frac{k^2\xi^*}{\lambda(1-\lambda)}}}.$$

The asymptotic relative efficiencies (AREs) of $(V_1(k, k))$ relative to $(V_2(k, k))$ for different values of k are presented in Table 1. It is clear from Table 1 that $(V_1(k, k))$ and $(V_2(k, k))$ are equivalent in Pitman asymptotic relative efficiency sense.

Table 1. Asymptotic relative efficiency of $(V_1(k, k))$ relative to $(V_2(k, k))$

k	ARE of $(V_1(k, k))$ relative to $(V_2(k, k))$
1	1
2	1.0000014
3	0.9999974
4	0.9994261
5	0.9992940
6	0.9999256
7	1.0013510
8	0.9888483
9	0.9895973
10	1.0116094

The asymptotic relative efficiency (ARE) of $(V_1(k, k))$ and $(V_2(k, k))$ relative to t-test is evaluated for different values of k and presented in Table 2.

Table 2. Asymptotic relative efficiency of $(V_1(k, k))$ and $(V_2(k, k))$ relative to t-test

Distribution	ARE of $(V_1(k, k))$ and $(V_2(k, k))$ relative to t-test
Uniform (–1, 1)	1.3333
Triangular	5.3333
Normal	2.5465
Logistic	5.4413
Laplace	8.0001
Cauchy	1.6211
Parabolic	6.7082

Here, we compare the efficiencies of members of our class of tests $(V_1(k, k))$ with the members of classes of tests due to Shetty and Umarani [13] $(U_1(k)$ and $U_2(k))$. The various values of AREs of $(V_1(k, k))$ relative to $U_1(k)$ and $U_2(k)$ are given in Table 3 and Table 4, respectively.

Table 3. Asymptotic relative efficiency of $V_1(k, k)$ and $V_2(k, k)$ relative to $U_1(k)$

$\lambda \to k \downarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2	0.4500	0.6500	0.8500	1.0500	1.2500	1.4500	1.6500	1.8500	2.0500	2.2500
3	0.3625	0.6625	0.9625	1.2625	1.5625	1.8625	2.1625	2.4625	2.7625	3.0625
4	0.3656	0.7156	1.0656	1.4156	1.7656	2.1156	2.4656	2.8156	3.1656	3.5156
5	0.3789	0.7539	1.1289	1.5039	1.8789	2.2539	2.6289	3.0039	3.3789	3.7539
6	0.3886	0.7760	1.1635	1.5510	1.9385	2.3259	2.7135	3.1009	3.4885	3.8760
7	0.3940	0.7877	1.1815	1.5752	1.9691	2.3627	2.7565	3.1504	3.5440	3.9378
8	0.3971	0.7941	1.1911	1.5881	1.9851	2.3821	2.7791	3.1761	3.5731	3.9701
9	0.3985	0.7969	1.1953	1.5938	1.9922	2.3906	2.7891	3.1875	3.5860	3.9844
10	0.3992	0.7985	1.1977	1.5969	1.9961	2.3953	2.7946	3.1938	3.5930	3.9922

Table 4. Asymptotic relative efficiency of $V_1(k, k)$ and $V_2(k, k)$ relative to $U_2(k)$

$\lambda \to k \downarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2	2.0500	1.8500	1.6500	1.4500	1.2500	1.0500	0.8500	0.6500	0.4500	0.2500
3	2.7625	2.4625	2.1625	1.8625	1.5625	1.2625	0.9625	0.6625	0.3625	0.0625
4	3.1656	2.8156	2.4656	2.1156	1.7656	1.4156	1.0656	0.7156	0.3656	0.0156
5	3.3789	3.0039	2.6289	2.2539	1.8789	1.5039	1.1289	0.7539	0.3789	0.0039
6	3.4885	3.1009	2.7135	2.3259	1.9385	1.5509	1.1635	0.7759	0.3885	0.0009
7	3.5440	3.1503	2.7565	2.3628	1.9690	1.5753	1.1815	0.7877	0.3939	0.0002
8	3.5731	3.1761	2.7791	2.3821	1.9850	1.5888	1.1911	0.7905	0.3971	0.00006
9	3.5859	3.1875	2.7898	2.3906	1.9922	1.5938	1.1953	0.7968	0.3984	0.000015
10	3.5931	3.1938	2.7946	2.3953	1.9961	1.5969	1.1977	0.7984	0.3992	0.000003

5. Remarks and Conclusions

1. Two classes of test statistics for special two-sample location problem are proposed assuming the underlying distribution of the sample drawn to be symmetric.

2. The performances of few members of the proposed class are evaluated in terms of asymptotic relative efficiencies (AREs).

3. It has been observed that $V_1(k, k)$ and $V_2(k, k)$ are equally efficient for all the values of *k*. Here, it is to be noted that for $k = 1$, $V_1(1, 1)$ is asymptotically equivalent to $U_1(1)$ and $V_2(1, 1)$ is asymptotically equivalent to $U_2(1)$.

4. It has been shown that the performance of the proposed tests is better than the only test existing in the literature due to Shetty and Umarani [13] for the distributions uniform, triangular, normal, logistic, Laplace, Cauchy, parabolic. Both the proposed tests perform better than *t*-test for all the distributions.

5. Further, it is observed that ARE of $V_1(k, k)$ and $V_2(k, k)$ relative to $U_1(k)$ increases as *k* increases for $\lambda \geq 0.3$ and ARE of $V_1(k, k)$ and $V_2(k, k)$ relative to ARE of $U_2(k)$ decreases as *k* increases for $0 \leq \lambda \leq 0.7$.

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