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RELIABILITY ESTIMATION IN MULTI-COMPONENT PARETO STRESS-STRENGTH MODELS

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Abstract

A Stress-Strength model is formulated for a multi-component system consisting of k identical components. The k components of the system with random strengths $(X_1, X_2, \ldots, X_k)$ are subjected to one of the r random stresses (X_{k+1}, X_{k+2}). The estimation of system reliability based on maximum likelihood estimates (MLEs) in k components series system is considered with the assumption that strengths and stresses follow Pareto distribution.

1. Introduction

The problem of estimating reliability $P[X > Y]$ in a stress-strength model has been discussed in the literature extensively when X and Y have some specified distribution. Enis and Geiser [2], Tong [9], Kelley et al. [7] have considered this problem when X and Y follow independent exponential distributions. Beg and Singh

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[1] considered this problem when X and Y follow two parameter exponential distribution. Further, the problem of estimation of $P[X_3 > Max(X_1, X_2)]$ has been considered by Hanagal [3] when (X_1, X_2) follow bivariate exponential models and X_3 follow independent exponential distribution. Hanagal [4] considered the estimation of $P[X_k < \text{Min}(X_1, X_2, ..., X_{k-1})]$ when $(X_1, X_2, ..., X_{k-1})$ follow multivariate Pareto distribution. Hanagal [6] considered the problem of estimating $P[X_{k+1} < \text{Min}(X_1, X_2, ..., X_k)]$ and $P[X_{k+1} < \text{Max}(X_1, X_2, ..., X_k)]$ when $X_1, X_2, ..., X_k$ are strengths subjected to a common stress X_{k+1}, assuming that $X_1, X_2, ..., X_{k+1}$ follow independent two parameter exponential distributions. Hanagal [5] obtained maximum likelihood estimators for

$$P[X_{k+1} < \text{Min}(X_1, X_2, ..., X_k)] \text{ and } P[X_{k+1} < \text{Max}(X_1, X_2, ..., X_k)]$$

when $X_1, X_2, ..., X_k$ are strengths subjected to a common stress X_{k+1}, assuming that $X_1, X_2, ..., X_{k+1}$ follow independent Pareto or Weibull or Gamma distribution.

In this paper, we consider the estimation of

$$R_s = P[\text{Max}(X_{k+1}, X_{k+2}) < \text{Min}(X_1, X_2, ..., X_k)]$$

when $X_1, X_2, ..., X_k$ are strengths subjected to one of the stresses X_{k+1}, X_{k+2} assuming that $X_1, X_2, ..., X_{k+2}$ follow independent Pareto distributions.

In Section 2, we derive the expression for system reliability of series system for Pareto stress-strength model. The MLEs for the parameters and reliability functions with their asymptotic distributions are derived in Section 3. Some remarks and conclusions are given in Section 4.

2. System Reliability

Consider a multi-component system with k identical components. Here, we assume that strengths of k components are subjected to one of the two stresses. Let $X_1, X_2, ..., X_k$ be strengths, subjected to one of the stresses X_{k+1}, X_{k+2}. We assume that X_i, $i = 1, 2, ..., k+2$ follow Pareto distribution.

The p.d.f. of X_i is given by

$$f(x_i) = \frac{a_i \mu^{a_i}}{x_i^{a_i+1}} \quad i = 1, 2, \ldots, k+2, \; x_i > \mu, \; a_i, \; \mu > 0.$$

The distribution function of $V = \text{Max}(X_{k+1}, X_{k+2})$ is given by

$$H(v) = P[V < v]$$

$$= \prod_{i=k+1}^{k+2} \left[1 - \left(\frac{\mu}{v}\right)^{a_i} \right]$$

$$= 1 - \left(\frac{\mu}{v}\right)^{a_{k+1}} - \left(\frac{\mu}{v}\right)^{a_{k+2}} + \left(\frac{\mu}{v}\right)^{a_{k+1}+a_{k+2}}$$

Now in series system, the system reliability is

$$R_s = P[V < \text{Min}(X_1, X_2, \ldots, X_k)]$$

$$= \int_{\mu}^{\infty} \prod_{i=1}^{k} \left(\frac{\mu}{v}\right)^{a_i} \left[\left(\frac{\mu^{a_{k+1}} a_{k+1}}{v^{a_{k+1}+1}} + \frac{\mu^{a_{k+2}} a_{k+2}}{v^{a_{k+2}+1}} - \frac{\mu^{a_{k+1}+a_{k+2}} (a_{k+1} + a_{k+2})}{v^{a_{k+1}+a_{k+2}+1}} \right) \right] dv$$

$$= \frac{a_{k+1}}{\sum_{i=1}^{k+1} a_i} + \frac{a_{k+2}}{\sum_{i=1}^{k} a_i + a_{k+2}} - \frac{(a_{k+1} + a_{k+2})}{\sum_{i=1}^{k+2} a_i}$$

which is independent μ.

3. Maximum Likelihood Estimators for Parameters and Reliability

Consider a k-component system, in which components are subjected to two stresses. Let $X_{i,1}, X_{i,2}, \ldots, X_{i,k} (i = 1, 2, \ldots, n)$ be a random sample of strengths of n systems, that are following Pareto distribution with parameters a_i $(i = 1, 2, \ldots, k)$, μ and $X_{i,k+1}$, $X_{i,k+2}$ $(i = 1, 2, \ldots, n)$ be a random sample of stresses corresponding to n systems, that are following Pareto distribution with parameters a_i $(i = k+1, k+2)$ and μ.

The MLE of R_s based on $\underline{a} = (a_1, a_2, \ldots, a_{k+2})$ and μ are given by

$$\hat{R}_s = R_s(\underline{\hat{a}}), \text{ where } \underline{\hat{a}} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{k+2}).$$

The MLEs of $\hat{a}_i$ and $\hat{\mu}$ are obtained as

$$\hat{a}_i = \left[\ln \left(\frac{\left(\prod_{j=1}^{n} x_{ij} \right)^{1/n}}{\hat{\mu}} \right) \right]^{-1} \quad \text{and} \quad \hat{\mu} = \min_{i,j} (x_{ij}).$$

The asymptotic variances of the MLEs of $\hat{a}_i$ are given by

$$V(\hat{a}_i) = \frac{a_i^2}{n} \quad i = 1, 2, \ldots, k+2 \text{ and } Cov(\hat{a}_i, \hat{a}_j) = 0 \text{ for } i \neq j = 1, 2, \ldots, k+2.$$

Therefore, the estimate of R_s based on MLEs of a_i $(i = 1, 2, \ldots, k+2)$ is

$$\hat{R}_s = \frac{\hat{a}_{k+1}}{\sum_{i=1}^{k+1} \hat{a}_i} + \frac{\hat{a}_{k+2}}{\sum_{i=1}^{k} \hat{a}_i + \hat{a}_{k+2}} - \frac{(\hat{a}_{k+1} + \hat{a}_{k+2})}{\sum_{i=1}^{k+2} \hat{a}_i}.$$

As $\hat{\mu} \xrightarrow{P} \mu$, we get $\sqrt{n}(\hat{\mu} - \mu) \xrightarrow{P} 0$. Also using the result that $\sqrt{n}(\underline{\hat{a}} - \underline{a}) \xrightarrow{L} AMVN(0, \Sigma)$, where $\Sigma = diag(a_1^2, a_2^2, \ldots, a_{k+2}^2)$.

Thus

$$\sqrt{n}(\hat{\mu} - \mu) + \sqrt{n}(\hat{\mu} - \mu) \xrightarrow{L} AMVN(0, \Sigma).$$

Hence the distribution of $\hat{R}_s$ is $AN(R_s, B'\Lambda B/n)$, where

$$B' = \left(\frac{\partial R_s}{\partial a_1}, \frac{\partial R_s}{\partial a_2}, \ldots, \frac{\partial R_s}{\partial a_{k+2}} \right) \text{ and } \Lambda = \frac{1}{n} diag(a_1^2, a_2^2, \ldots, a_{k+2}^2).$$

Here,

$$\frac{\partial R_s}{\partial a_i} = -\frac{a_{k+1}}{\left(\sum_{i=1}^{k+1} a_i \right)^2} - \frac{a_{k+2}}{\left(\sum_{i=1}^{k} a_i + a_{k+2} \right)^2} + \frac{a_{k+1} + a_{k+2}}{\left(\sum_{i=1}^{k+2} a_i \right)^2} \text{ for } i = 1, 2, \ldots, k,$$

$$\frac{\partial R_s}{\partial a_{k+1}} = \frac{a_{k+1}}{\left(\sum_{i=1}^{k+1} a_i\right)^2} + \frac{1}{\sum_{i=1}^{k+1} a_i} - \frac{1}{\sum_{i=1}^{k+2} a_i} + \frac{a_{k+1} + a_{k+2}}{\left(\sum_{i=1}^{k+2} a_i\right)^2}$$

and

$$\frac{\partial R_s}{\partial a_{k+2}} = \frac{a_{k+2}}{\left(\sum_{i=1}^{k} a_i + a_{k+2}\right)^2} + \frac{1}{\sum_{i=1}^{k} a_i + a_{k+2}} - \frac{1}{\sum_{i=1}^{k+2} a_i} + \frac{a_{k+1} + a_{k+2}}{\left(\sum_{i=1}^{k+2} a_i\right)^2}.$$

4. Some Remarks and Conclusions

(i) In this paper, we have considered Pareto stress-strength models for estimating the parameters and reliability functions.

(ii) In case of Pareto stress-strength models, the system considered here has k strengths and two stresses. The ML estimators for the parameters are derived and the estimator for reliability function in case of series system is obtained. The asymptotic distributions of the estimators are obtained.

(iii) The uniformly minimum variance unbiased estimators for reliability of the series system is under progress.

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