

## Perchance to Dream: Art, Mathematics, and Shakespeare

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## Perchance to Dream: Art, Mathematics, and Shakespeare

### Cover Page Footnote

This article is dedicated to the Salisbury University Mathematics and Computer Science Department, for inspiring generations of learners and thinkers.

# Perchance to Dream: Art, Mathematics, and Shakespeare

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## Abstract

Visual representation of textual works has often aided in the understanding of sophisticated concepts [3]. In the Digital Age this is particularly true, given the advent of natural language processing, the ubiquity of general programming languages, and the maturation of digital visualization. In this article, we eschew the traditional disciplinary boundaries to view and analyze Shakespeare's works in various ways. Our point of departure is *Hamlet*, where we first examine the play as a unity — both graphically and analytically. We then focus on Act III, Scene 1, where we analyze one of the most famous passages in English Literature: Hamlet's famous "To be or not to be" soliloquy. We study this passage with a sequence of progressively sophisticated content analysis software packages, each of which renders a useful artistic visual representation of the text. In these studies, we have gathered preliminary sets of context-free content data which allow us to give illustrations of a wide range of analytic tools. Throughout our explorations, we use new modes of linguistic exploration, synthesized from elements of philosophy, literature, mathematics, computer science, psychology, and the arts. Many of the methods, which help to form these new modes of exploration, may appear individually as Cartesian in nature, yet when collectively synthesized into a holism, form a fundamentally phenomenological study of human creativity.

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## 1. Introduction

As they are arranged, the words that comprise Shakespeare's *Hamlet* are rich, profound, and inexhaustibly interesting. We well know that these words and their arrangements go beyond a simple literary work: they have become part

of our culture [4] and a part of human consciousness [5]. This single play has inspired numerous theatrical productions, literary reinterpretations, movies, operas, music, and other works of art. Just as importantly, this single play has challenged the intellects of scholars, students, thinkers, and critics for several hundred years. In a fundamental way, *Hamlet* has revealed itself as a muse for a limitless series of artistic, psychological, and philosophical human phenomena.

Our present intent is to continue the exploration of the object ‘Hamlet’, and of Shakespeare’s works in general, doing so in new and novel ways. We wish to dis-close new avenues of discovery to people who may have, at some earlier stage in life, found that Shakespeare’s literature seemed ‘closed-off’ or inaccessible. That is to say, our purpose is to un-conceal<sup>1</sup> aspects of the text that seemed distant, opaque, or reserved for the expert. In this disclosure and un-concealing, we also wish to arouse interest in the scientific, humanistic, and educational fields to re-examine their possible roles in the future understanding, exploration, and reinterpretation of great works in literature.

*“Well, you know or don’t you kennet or haven’t I told you  
every telling has a taling and that’s the he and the she of it . . .”*

–James Joyce, *Finnegans Wake*

When telling or reading a tale, such activities involve the development and utilization of cognitive fluencies at many different levels [12, 19]. These levels are manifold and various, many of which may be accessed through the use of supporting visual and auditory information. Some literary works are truly difficult to apprehend due to their inherent complexity, even upon multiple readings. Yet, such works may have other access points, or formats, which help in their apprehension. For example, when Joyce reads aloud the passage “Anna Livia Plurabelle” from the complex *Finnegans Wake* [8], there are

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<sup>1</sup>Martin Heidegger, *Parmenides* [6]. The use of un-conceal in our present studies is strongly in the sense of Heidegger, discovered in his quest to understand “truth” as the Greeks understood it in the days of the Pre-Socratic philosophers. Throughout the present investigations, the “truth” we seek consists in such un-concealment, and not in the sense of correctness or rectitude. We similarly use the term “dis-close”, which too provides an appropriate emphasis on access.

clear, artistic patterns in his verbal re-telling; patterns which are harder to detect when the story is available only in written form. Shakespeare's works are likewise beautifully complex, and it is our purpose herein to describe new access points for their exploration. Hence, as a propaedeutic to a more detailed discussion, we first present a visualization of a global perspective of *Hamlet*, using structural divisions as indicated by a five-act play:

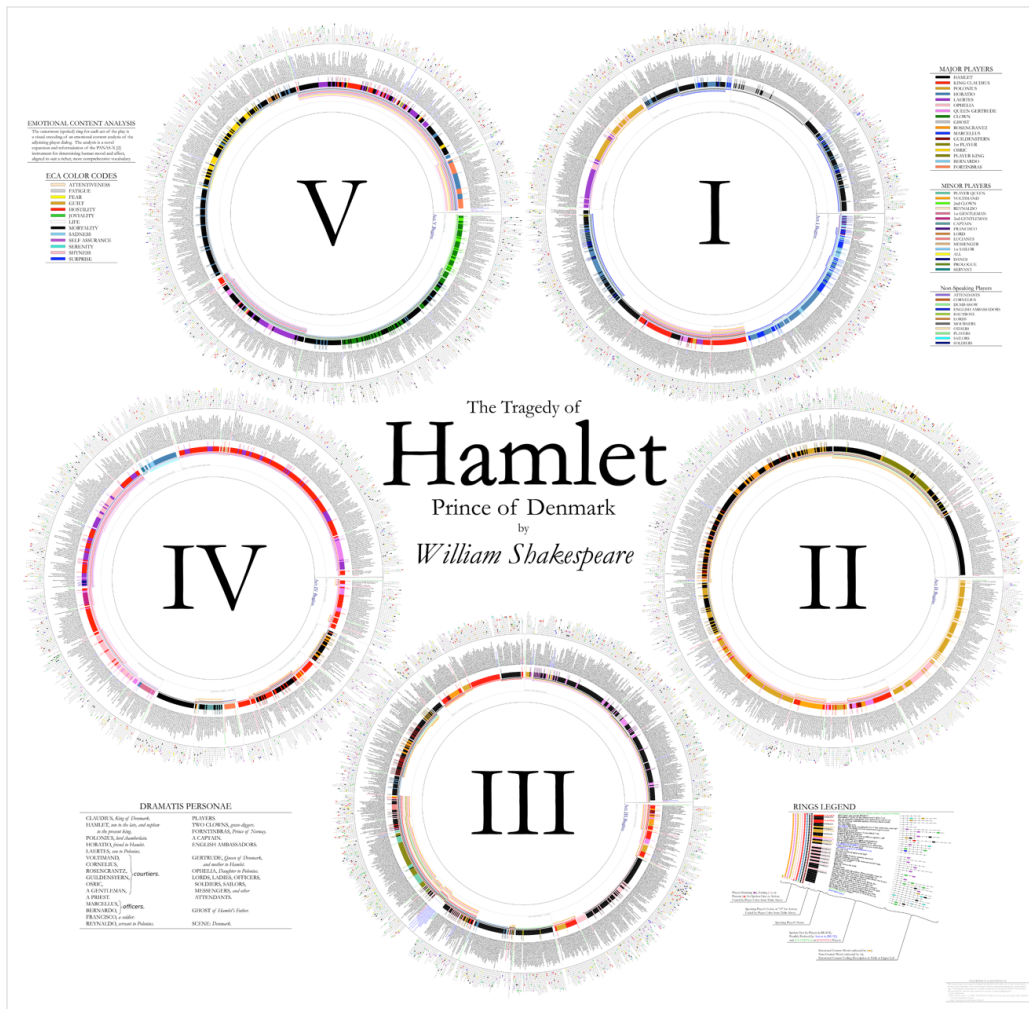


Figure 1: *Hamlet*, by William Shakespeare.

In Figure 1, *Hamlet* appears in its entirety.<sup>2</sup> The complete literary text is present, with attending visualizations of several analyses. The ‘rings’ structure is an intentional choice, designed to emphasize unity and cohesion of the act structure, as well as to facilitate inter-act (and inter-play) comparison. Once the barest of descriptions are given of the meaning of colors in the rings, many interesting aspects of the entire play may be seen at a glance. Furthermore, once the content and structure of *Hamlet* are understood in this format, other plays by Shakespeare may be brought into direct, visual comparison with *Hamlet*, and therefore also to each other.

To create the image in Figure 1, a public domain copy of *Hamlet* was retrieved from the MIT Shakespeare website [16], and custom software was written in the Python programming language to analyze and visualize the play. Each of the explorations in the present article uses this version of *Hamlet*, which is a modified Second Quarto text. Physically, the image in Figure 1 is quite large (8 ft. by 8 ft. at 96 ppi), which, when displayed full-size at full resolution, lends itself to direct exploration by interested viewers. In the smaller format necessary in this article, we explore smaller sections of the image in Figure 1, examining interesting features at the scene and passage levels. Throughout these successive examinations of smaller sections of this primary image, we refer back to it as a reminder of those examinations’ place within the unified whole.

Because we examine the passage in such detail, and in numerous ways, the full text of Hamlet’s soliloquy in Act III, Scene 1 is provided below for ready reference and comparison:

*“To be, or not to be: that is the question:  
Whether ‘tis nobler in the mind to suffer  
The slings and arrows of outrageous fortune,  
Or to take arms against a sea of troubles,  
And by opposing end them? To die: to sleep;  
No more; and by a sleep to say we end  
The heartache and the thousand natural shocks  
That flesh is heir to, ‘tis a consummation  
Devoutly to be wish’d. To die, to sleep;  
To sleep: perchance to dream: ay, there’s the rub;*

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<sup>2</sup>All visualizations in this article are hyperlinked to higher-resolution versions, available to the reader when connected to the internet.

*For in that sleep of death what dreams may come  
 When we have shuffled off this mortal coil,  
 Must give us pause: there's the respect  
 That makes calamity of so long life;  
 For who would bear the whips and scorns of time,  
 The oppressor's wrong, the proud man's contumely,  
 The pangs of despis'd love, the law's delay,  
 The insolence of office and the spurns  
 That patient merit of the unworthy takes,  
 When he himself might his quietus make  
 With a bare bodkin? who would fardels bear,  
 To grunt and sweat under a weary life,  
 But that the dread of something after death,  
 The undiscover'd country from whose bourn  
 No traveller returns, puzzles the will  
 And makes us rather bear those ills we have  
 Than fly to others that we know not of?  
 Thus conscience does make cowards of us all;  
 And thus the native hue of resolution  
 Is sicklied o'er with the pale cast of thought,  
 And enterprises of great pith and moment  
 With this regard their currents turn awry,  
 And lose the name of action.—Soft you now!  
 The fair Ophelia! Nymph, in thy orisons  
 Be all my sins remember'd.”*

NOTE: Much of the work herein is rooted in the relatively new field of *Digital Humanities*. From the use of digital forensics to discover that Christopher Marlowe co-authored several of Shakespeare's plays [18], to the use of cultural analytics to study historical changes in literature and art [11], the exciting and trans-disciplinary nature of Digital Humanities offers new realms for exploration across all academic domains.

## 2. Setting the Stage

As a point of departure for a more detailed study, we limit our initial focus to the preliminary visualization and analysis of the emotional content of the start of Act III, Scene 1 of *Hamlet*. Later, we fully describe this preliminary analysis, and then move structurally outward (and inward) for a more comprehensive discussion of the methods at hand.



Figure 2: Act III, Scene 1 of *Hamlet*.

Some remarks are necessary regarding the contents of Figure 2. The large colored rectangles to the left of the main text indicate which character is speaking. We will refer to these rectangles as Speaker Indicator Blocks (SIB). For example, King Claudius’ SIB are indicated in red, Ophelia’s are pink, and Hamlet’s appear as black. The dotted rays projecting to the right of the text indicate the number of words in the associated line of text, as well as a word-by-word Emotional Content Analysis (ECA).

We also note that the data gathered as an emotional ‘dictionary’ for all studies herein are preliminary, generated by a survey of colleagues. Nevertheless, the emotions assigned to words were done so completely independent of context. In particular, the data were gathered as a single-blind survey, so that participants had no knowledge of the source of the words (i.e., *Hamlet*) or how the assignments were to be used.

The underlying principles for the emotional content analysis were inspired by work in psychological understanding and detection of emotion, mood, and affect, including: State-Trait Emotion Measure (STEM) [9], Positive and Negative Affect Schedule (PANAS-X) [20], and Plutchik’s Model [13] for emotional classification. It should be noted that even the best of such



psychological studies and instrumentation are limited in scope, utility, and granularity, with respect to the emotional analysis of literature. For example, the PANAS-X instrument is limited to a vocabulary of 60 moderately-simple words, originally intended for human subjects to self-assess their moods and affects. In order to apply similar detection to literature, it was necessary to extend such instrumentation well beyond their present, intended uses, particularly with respect to the detection vocabulary.

### *2.1. Initial Discoveries*

Limiting our initial studies to the structure of the ECA information for Hamlet's soliloquy in Act III, Scene 1, we make several discoveries in Figure 2. We notice a pervasive theme of mortality, especially in the first half of the soliloquy. There is an interlude of hostility mid-way through the passage, which resolves into a more attentive, guilty, and fearful mood. A fascinating revelation is the lack of emotional content detected in what is the most famous of lines in all of English literature: "To be or not to be; that is the question". Yet, this lack of emotional content indicates the need for other Content Analysis Engines: engines that can detect potential existential and philosophical elements in text. We describe such a Philosophical Content Analysis (PCA) engine later in this article, and some early results when applied to this same soliloquy.

Although this initial ECA approach is word-based, its perspective is not atomistic. That is, within the ECA software, a single emotion is assigned to a given word, and done so outside of any explicit context. Yet, when these same words are then visualized within the context of a literary work, we take notice of emotional patterns and structures that occur therein. Just as no single word in any language is wholly removed from other words and concepts, the atomistic assignment of single emotions to individual words dissipates in these visualizations, resolving into the context of larger structures and themes. Nevertheless, useful extensions and explorations of these preliminary analyses can be made, beyond the single-word-single-emotion assignment; they are discussed in Section 3 below.

Moving outward slightly from the soliloquy, we notice additional emotional structure occurring around Hamlet's Act III, Scene 1 soliloquy, as visualized in Figure 3.



Two discoveries are immediately apparent in Figure 3: the nature of the emotional content verbalized by the characters in their discussion prior to the soliloquy, and the change in emotional structure in Hamlet’s dialogue immediately following the soliloquy.

In the passage preceding the soliloquy, the emotional content is somewhat sparse, is wholly without structure, and does not have elements of hostility or mortality. Theatrically, this may serve to set up an emotional contrast to Hamlet’s soliloquy, in order to further emphasize the dramatic emotional structure therein. The most substantive of the two discoveries is the emotional structure of the dialogue that immediately follows Hamlet’s soliloquy. Hamlet’s brooding and mortally hostile mood within the soliloquy gives way to jovial self-assurance and surprise, once he realizes he is not alone on stage. Even if Hamlet’s emotional transition is disingenuous, it is detected and visualized by the ECA software, with readily apparent structure and clear differentiations.

There is further visual information in Figure 3, which plays an important role in understanding the organization of the play and performing an effective character analysis. There are small circles and triangles, directly to the left of the Speaker Indicator Blocks, which indicate what characters are entering, exiting, or presently on stage. This information is useful in understanding which of the characters are contributing to, or in receipt of, the emotional content of the dialogue.

At this point in our discussion, it is useful to direct our view to the ‘Rings Legend’ in Figure 4 on the next page, which provides full instruction on how to interpret the visual and analytic information present in the full image of Figure 1.

## *2.2. A Step Back*

We move outward again, to the act level. From this perspective, we are best served through additional knowledge about the layout of the Act Ring, and about the Speaker Indicator Blocks (SIB) — particularly, that colors indicate which of the characters is speaking, and when. The major characters’, or players’, SIB color codes are given in the table on the following page (see Figure 5).



Graphically, the play’s dialogue for an act begins on the right-hand side of its respective Act Ring, or three o’clock on a clock face, and proceeds clockwise around the Act Ring — as seen in Figure 6 below. Within the figure, there are Act and Scene start lines, which occur as those particular elements of the play start, moving again clockwise around the Act Ring.

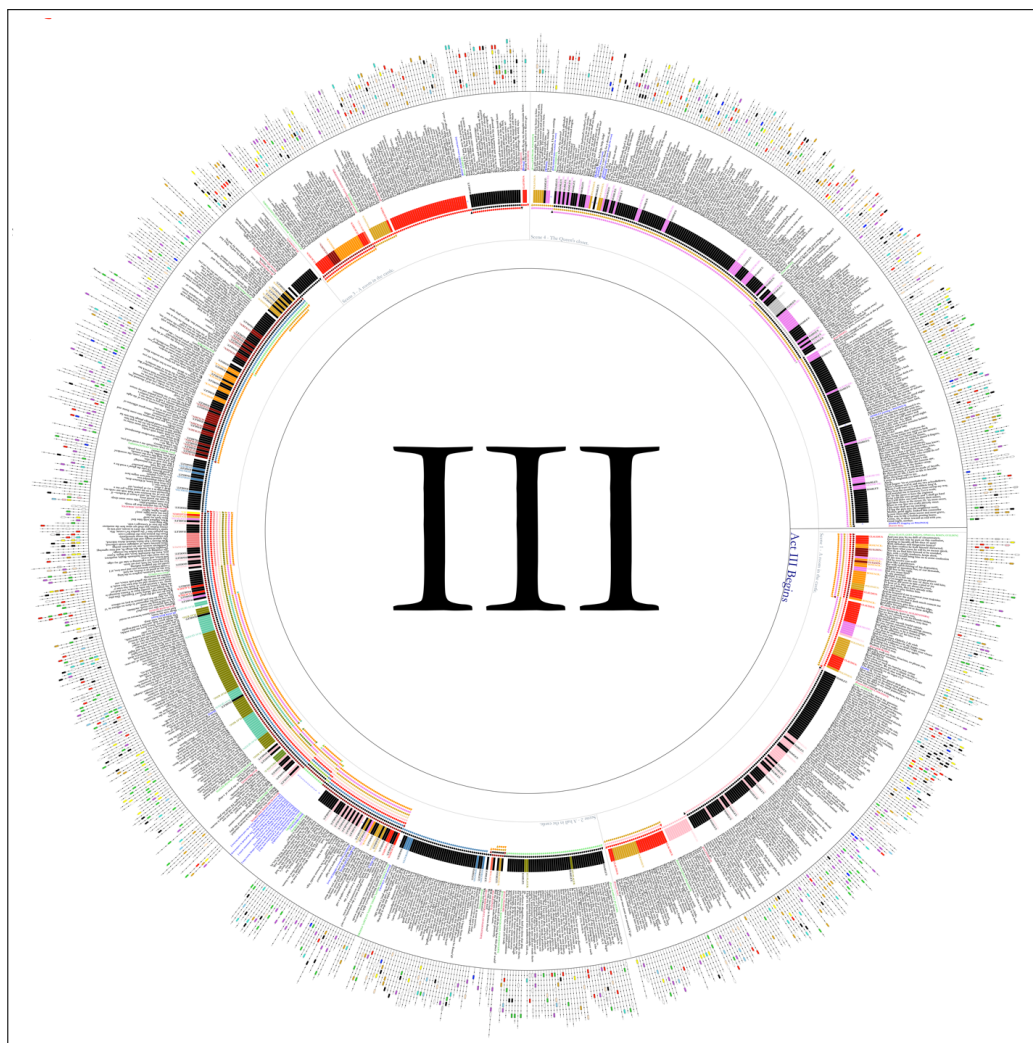


Figure 6: *Hamlet*, Act III.

With this information, we may now take a discerning view of all of Act III of *Hamlet*, and notice several basic features of the play's dialogue. From this act-level perspective, we see a prevalence of black-colored Speaker Indicator Blocks (SIB), which indicate that Act III is largely determined by the dialogue of Hamlet. Furthermore, because many of the black SIB occur in unbroken sequences, it is clear Hamlet has several monologues within Act III, most of which occur in the presence of other characters. We notice that there are many alternating sequences of Hamlet's SIB with another character's SIB, each of which is indicating that Hamlet is exchanging dialogue with another character. For example, we see such interchanges mid-way through Scene 1 with Ophelia and Hamlet (pink and black), and throughout the last scene, between Queen Gertrude and Hamlet (violet and black). We notice also that King Claudius (red SIB) plays an active role in the dialogue of Act III, given his discussions with Gertrude and Polonius in Scene 1, and in his famous monologue during Scene 4.

### 2.3. *Another Introduction*

Maintaining our act-level perspective, but now better attuned to the visualizations under study, we examine Act I of *Hamlet*, found in Figure 7 on the following page.

It is interesting to notice that, when armed with simple color information, we may make immediate discoveries about the fundamental nature of Act I. First, we see large contiguous blocks of color, which reveal that many of the major characters in the play have an opportunity to speak at length, usually in the presence of other characters. Yet, in this act, it is only Hamlet who presents dialogue while alone on stage — a discovery visually accessible by exploring Figure 7. Furthermore, because each of the major characters is given such long, unbroken opportunities for dialogue, we see that this first act is truly an introductory exposition, wherein the major characters are given the early opportunity to express their motivations and roles.

Moving outward once more, we look again at Figure 1, and consider the five acts of *Hamlet* simultaneously. Even in the reduced size and format required in this article's format, we may make several general observations about the play's overall composition. For example, none of the five acts seem to have the same dialogue structure, as indicated by their respective Speaker Indicator Block (SIB) Rings. The character Hamlet appears only in

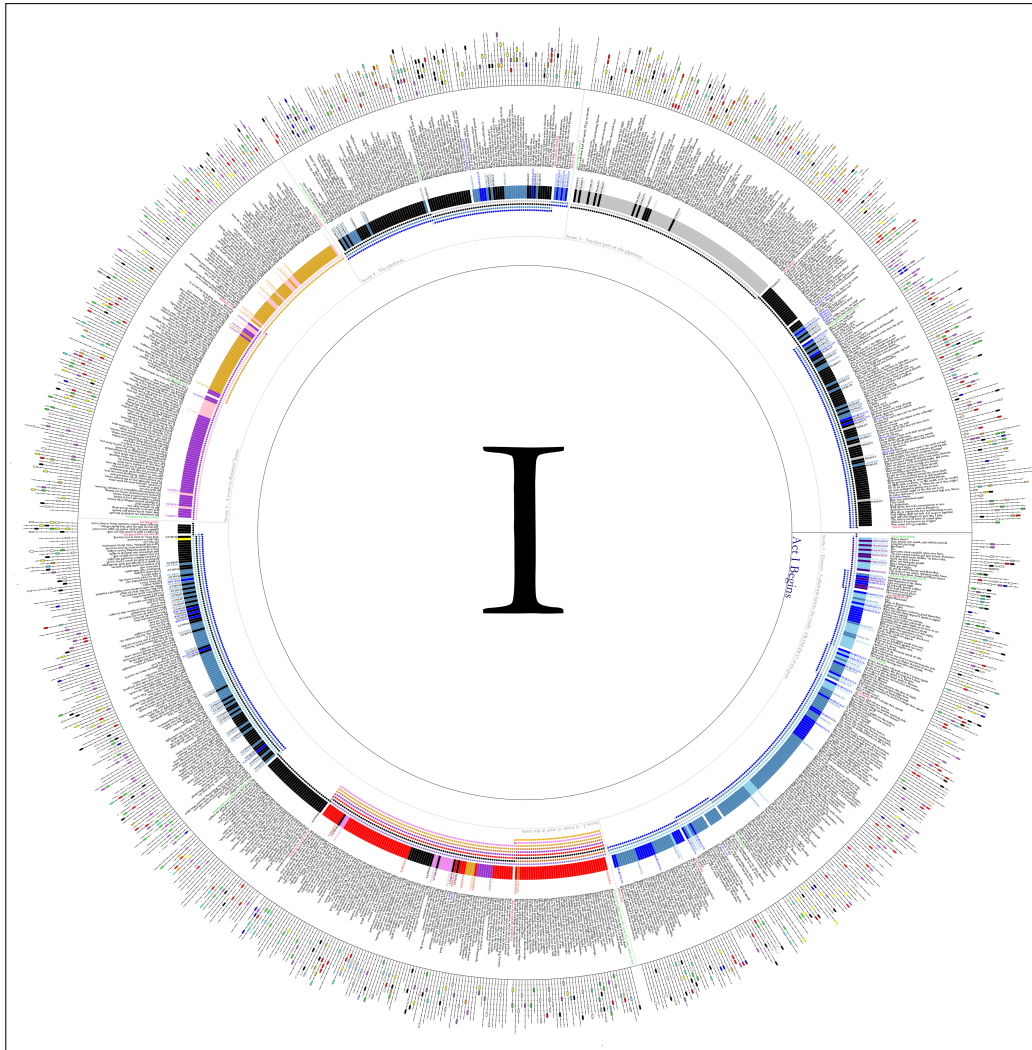


Figure 7: *Hamlet*, Act I.

the second half of Act II, and is mostly absent in Act IV. The SIB Ring in Act V reveals a predominance of inter-character dialogue, as seen in the rapid alternation of SIB colors between two or more characters. Just as fascinating are the appearances of the gravediggers in Act V, Scene 1, therein indicated by shades of green — colors that make no appearance anywhere else in the play, just as the gravediggers themselves.

There are many further discoveries made through the exploration of the preceding visualizations. We encourage readers to make some of their own. In lieu of describing further some of these discoveries, we move our discussion to alternative explorations, visualizations, and analyses.

### 3. Explorations

In coding the words for the Emotional Content Analysis (ECA) software engine, it is necessary to make certain choices about what emotions are to be associated with which words. To create the visualizations described in the preceding discussion, an unusual mathematical function is used.

#### 3.1. An Emotional Content Function

We call this mathematical function  $E$ , which connotes the Emotion  $E(w)$  of word  $w$ . This function has as its domain the set of all words  $w$  in Hamlet, and is denoted by:

$$HW = \{to, be, not, beware, question, suffer, carefully, murder, mock, \dots\}.$$

$E$  then maps the words  $w$  from  $HW$  into the much smaller range set of ECA Indicator Words ( $EIW$ ); the set given by:

$$EIW = \{sadness, fear, mortality, hostility, \dots\}.$$

Mathematically we would write this as:  $E : HW \rightarrow EIW$ .

Since  $E$  is a function, it maps a given word from  $HW$  to a single emotion. For example,  $E$  maps *beware* to *fear*, which may be alternatively denoted by:

$$E(\textit{beware}) = \textit{fear}.$$

Two further examples of this function in action are given by:

$$\begin{aligned} E(\textit{murder}) &= \textit{hostility}; \\ E(\textit{mock}) &= \textit{hostility}. \end{aligned}$$

The latter two assignments immediately bring to the foreground the first of several important and natural questions: should not some words in  $HW$  be assigned a greater weight, with respect to their mapped emotions? Should



not *murder* be designated as indicating a greater intensity of hostility, and therefore a higher level (or weight) of hostility, relative to *mock*? It seems reasonable and natural to do so, and we may certainly explore these ideas as opportunities to deepen our study of *Hamlet*. As motivation toward further study of such ideas, we now present results of the direct application of a new weighted function  $E_2$  to the soliloquy of Act III, Scene 1. We will rigorously define this new function,  $E_2$ , on the next page.

In advance of a description of the technical aspects of Figure 8 and its associated mathematical function, we take note of the emotional structure indicated therein. The hostility and mortality structures strongly resemble those found previously, in Figures 2 and 3.

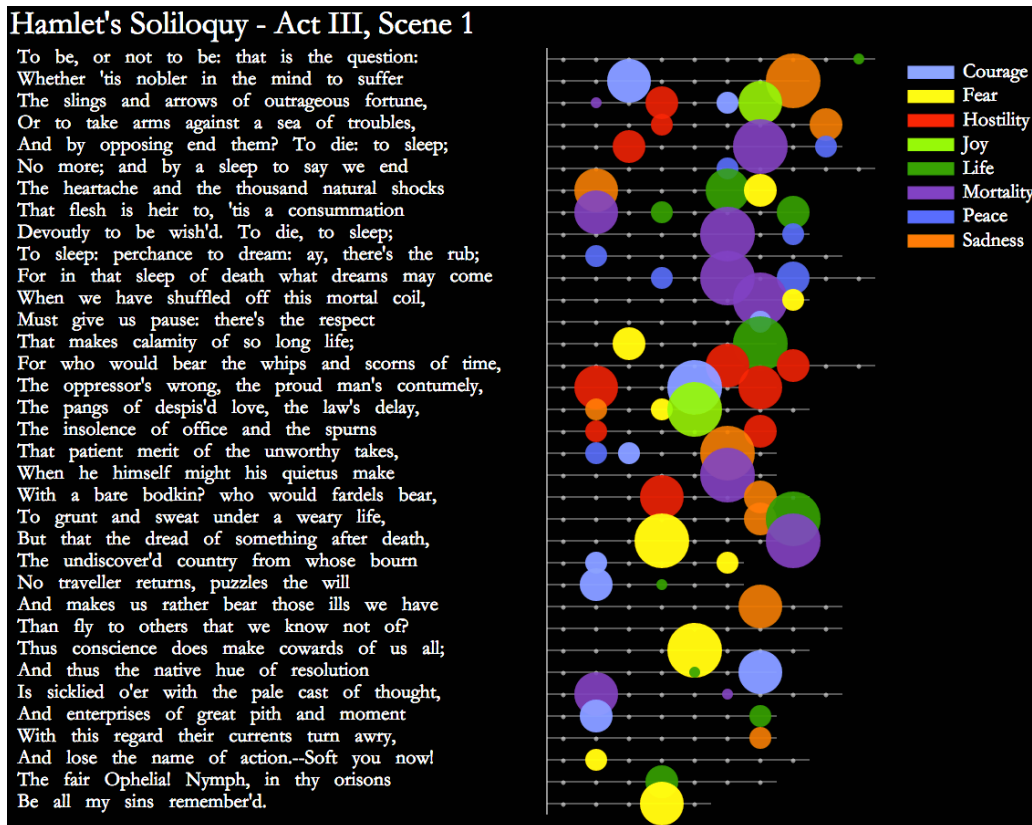


Figure 8: *Hamlet*, Act III, Scene 1, Weighted Emotion Analysis — Audience 1.

Again the emotions in the soliloquy seem to resolve into a state of fear and sadness. Hence, even though we have moved to a different visual representation of the emotional content, many of the same structures appear to be, at some level, invariant with respect to a particular mathematical treatment. Stylistically, the new visualization in Figure 8 maintains a correspondence between the words in the dialogue via the dotted ray configurations, which indicate relative word position and emotional content.

In the mathematical analysis that is coupled with this new visualization, we reduce the number of ECA Indicator Words to eight, so that our new word set for these indicators is

$$EIW_2 = \{courage, fear, hostility, joy, life^3, mortality, peace, sadness\}.$$

With this new word set, we establish a new function  $E_2$  which maps a word  $w$  from  $HW$  to  $E_2(w)$  which lives in a new mathematical space of weighted emotions. Emotional weights are assigned on a (linear) whole-number scale, from one to five, where a value of one indicates the weakest intensity of an emotion and a five indicates the strongest. In mathematical terms, we denote this function by:

$$E_2 : HW \rightarrow EIW_2 \times \{1, 2, 3, 4, 5\}.$$

For example, in examining the original data used to create Figure 8, we have the following emotion/weight pair assignments:

$$E_2(coward) = (fear, 5);$$

$$E_2(bodkin) = (hostility, 4);$$

$$E_2(consummation) = (life, 3).$$

With such weighted assignments of emotions to words, we gain additional sensitivity to emotional content detection – at least in the sense of the detection of intensity levels with which some words connote emotion.

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<sup>3</sup>Note: the ECA Indicator word *life* is shorthand for ‘life-affirming’ and ‘life-reinforcing’ sentiments and emotions.

### 3.2. *Social Revelation*

It is important to note that the emotional content data and weight assignments used to create the visualization in Figure 8 were not created by the present author, but were instead gathered anonymously at a talk given by the author in a general university setting. The audience at this talk consisted mostly of university faculty and students from a range of disciplinary backgrounds. The words from Hamlet's soliloquy were revealed, one at a time, to the audience as a whole, then assigned emotion/weight pairs, completely without context. In fact, these assignments were made without foreknowledge of how or in what context such assignments would be applied. The full experiment runs as follows:

- (i) Reveal a random word  $w$  (from the soliloquy) to the audience;
- (ii) Show the audience the Emotion Indicator Words ( $EIW_2$ ) and possible weights;
- (iii) Wait for the audience to arrive at a consensus for an emotion and weight assignment for  $w$ ;
- (iv) Record the audience's consensus on emotion/weight assignment;
- (v) Repeat Steps 1-4 for all words in the soliloquy;
- (vi) Create a weighted-emotion visualization from the data gathered from Steps 1-5.

Custom software was written in Python to facilitate this experiment, from word display, to emotion/weight choice display, to data storage, to the  $E_2(w)$  function application, and finally to the visualization of the results.

This experiment was repeated at a second university, during a colloquium presentation, again in front of an audience from a wide range of disciplines, using precisely the same methods and software. The results of this second experiment appear in Figure 9 on the following page. Note that the dotted rays are not shown in Figure 9, which lends a better aesthetic to the visualization, and a more unified, uncluttered emotional representation.

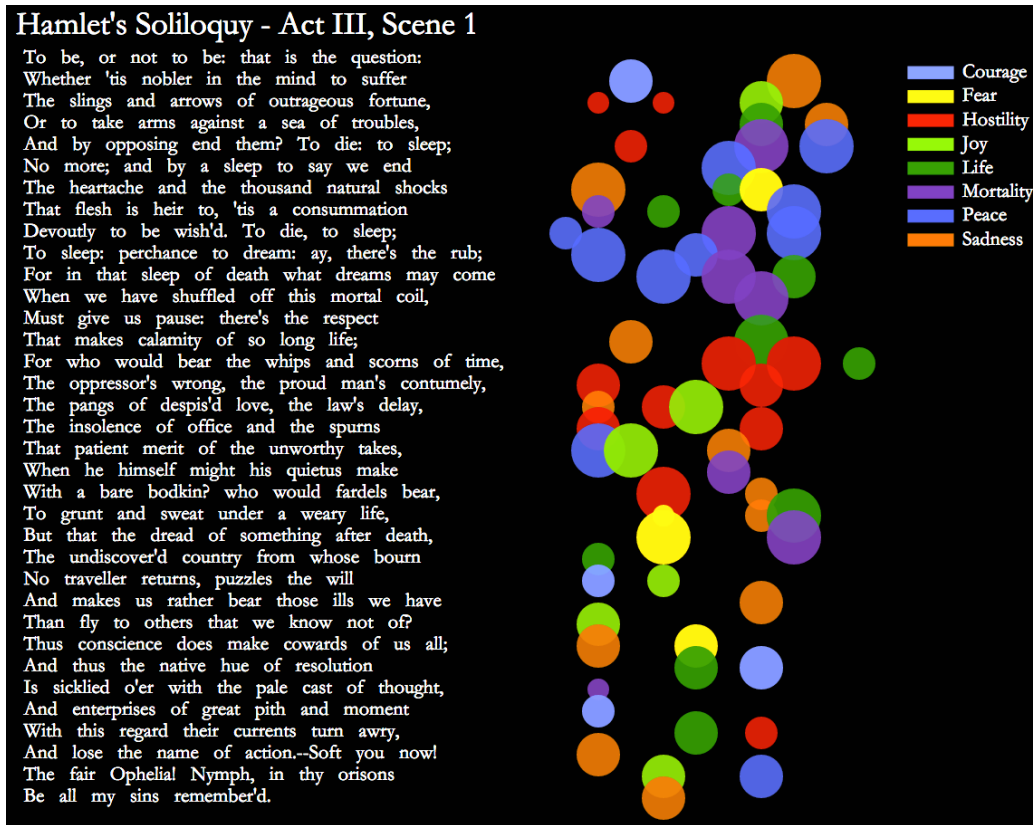


Figure 9: *Hamlet*, Act III, Scene 1, Weighted Emotion Analysis — Audience 2.

When comparing the results of the two different emotion/weight assignments determined by the two university audiences, we notice that some emotional structures still seem to remain invariant, such as the centralized hostility feature and the early mortality structure. There are also differences between the two graphics, such as the sparseness of content detection in Figure 8 relative to Figure 9, and the generally higher emotional weights assigned to all emotions in the second of the two figures. Further similarities and differences may be discerned between the two graphics, which indicate that further study is necessary in order to better understand how certain populations interpret Shakespeare and assign emotions to content.

In the vernacular of mathematics, the data from each university audience define their own, unique  $E_2$  function, through the assignment of particular emotion/weight combinations to the words in the soliloquy. When such emotion/weight choices are made, be they word-level or otherwise, they indicate a particular perspective or interpretation of the literary work at hand. This is similar to particular choices made by a director or actor in a theatrical or film production of a play.

### 3.3. *Emotional Content Functions – Revisited*

In the preceding discussion, we demonstrated one method by which we could extend the Emotional Content Analysis function  $E$ . In that case, the extension was to allow for the inclusion of weights, giving a weighted-emotion function  $E_2$ . This allowed us to indicate more sensitively what emotions are detected and to what intensity. Specifically, this extension was made by the assignment of weights of single emotions to words from a literary passage. We next indicate a further refinement to this function, which will allow multiple emotions and weights to be assigned to individual words.

Some words in literature are intended by the author to convey multiple emotions simultaneously. Additionally, a reader may interpret some words as having multiple moods, as indicated by their own experience, world-view, or education. For example, the assignment  $E_2(\textit{murder}) = (\textit{hostility}, 5)$  would wholly suffice and seem a reasonable assignment, yet so would  $E_2(\textit{murder}) = (\textit{mortality}, 5)$ . Perhaps a better approach in emotional content detection would allow the flexibility of assigning a series of weights and emotions to single words. Hence, it seems necessary to create an instrument that is more sensitive to the detection of multi-emotion, word-based content detection. Since some words may not contain certain emotions, at any weight, we choose a new set of emotional weights:

$$\Gamma = \{0, 1, 2, 3, 4, 5\}.$$

We then create a new function  $E_3$  which maps words  $w$  in  $HW$  to weighted emotion vectors; this can mathematically be represented as follows:

$$E_3 : HW \rightarrow \underbrace{\Gamma \oplus \Gamma \oplus \Gamma \oplus \Gamma \oplus \Gamma \oplus \Gamma \oplus \Gamma \oplus \Gamma}_{\text{eight copies – one for each emotion in } EIW_2} = \bigoplus_{i \in EIW_2} \Gamma.$$

In other words, the vector assigned to  $w$  is an ordered 8-tuple, denoted by  $E_3(w)$ , with each position in the tuple indicating a respective weight for the eight emotions. That is, the ordered list of the elements of

$$EIW_2 = \{courage, fear, hostility, joy, life, mortality, peace, sadness\}.$$

For example, we see that the following assignment for murder indicates both hostility and fear at weight 5:

$$E_3(murder) = (0, 0, 5, 0, 0, 5, 0, 0).$$

Another example is in the assignment:

$$E_3(sicklied) = (0, 3, 0, 0, 0, 2, 0, 4),$$

which assigns the non-zero weights 3, 2, and 4 to *sicklied* for the respective emotions of fear, mortality, and sadness. We refer to the function  $E_3$  as the linguistic spectrogram function for emotion detection on single words. From this, we may make deeper, more exacting, and more technical analyses of literary texts. In Figure 10 below, we re-examine the Act III, Scene 1 soliloquy from *Hamlet*:

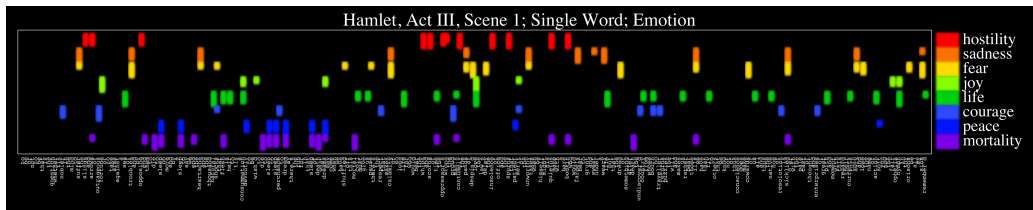


Figure 10: *Hamlet*, Act III, Scene 1: Linguistic Spectrogram, Emotions, Single Word.

We refer to the visualization created by the application of function  $E_3$  to a literary passage as a *linguistic spectrogram*. More precisely, Figure 10 is a linguistic spectrogram for emotional content detection on single words in Hamlet's soliloquy from Act III, Scene 1. The words in the soliloquy are listed along the bottom of the visualization, from left to right, in the order in which they occur in the passage. Each word is assigned a weighted emotion vector, which is then visualized directly above the word. The similarities are clear between the seemingly invariant hostility and mortality emotion structures in this analysis and the previous analyses.

With this spectrographic method, we now have access to the possibility of discovering new, richer structures, otherwise undetected and unseen in the earlier analyses and visualizations. For example, we notice a semi-periodic note of fearful sadness occurring throughout the soliloquy, a periodicity which becomes quite regular during the second half of the passage. There also seems to be the foil of an active thread of animated life – the hints of which we see throughout the soliloquy. Regarding the latter, our findings align with the critical interpretations of Asimov, wherein he judges the soliloquy as one about active choices [1], rather than about suicide. To provide confirmation of this assertion, certainly more research and study would be needed; some related discussion appears below.

### 3.4. *Further Context*

As readers, we often determine the meanings of words by their context. This is sometimes referred to as *context-sensitivity* [2]. With this in mind, we extend the analysis above into the domain of context-sensitivity, with respect to the emotional content of words. We do not assemble here the mathematical machinery and functions necessary to make this extension, since to do so is a straightforward adjustment of the work above. Suffice it to declare that the required functions will map *sequences* of words, from a literary text, to weighted emotion vectors.

For example, the first such sequence-extended function maps bi-grams (two successive words in text) to weighted emotion vectors, the next function maps tri-grams (three successive words in text) to weighted emotion vectors, and so on, so that a generalized function maps  $n$ -grams ( $n$  successive words in text) to weighted emotion vectors. With the construction of such functions, we are able to retrieve some emotional context from the larger grammatical structures in literature. Figure 11 below illustrates such analyses, and sets single-word, bi-gram, and tri-gram analyses together for comparison.<sup>4</sup> One consequence of the comparison of the single-word, bi-gram, and tri-grams analyses of Hamlet's soliloquy is the detection of much of the emotional content structure within the single-word analysis. That is to say, we retrieve much of the emotional content structure in the single-word analysis, which

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<sup>4</sup>Note that only  $n$ -gram sequences of successive words unbroken by punctuation are used in these analyses.

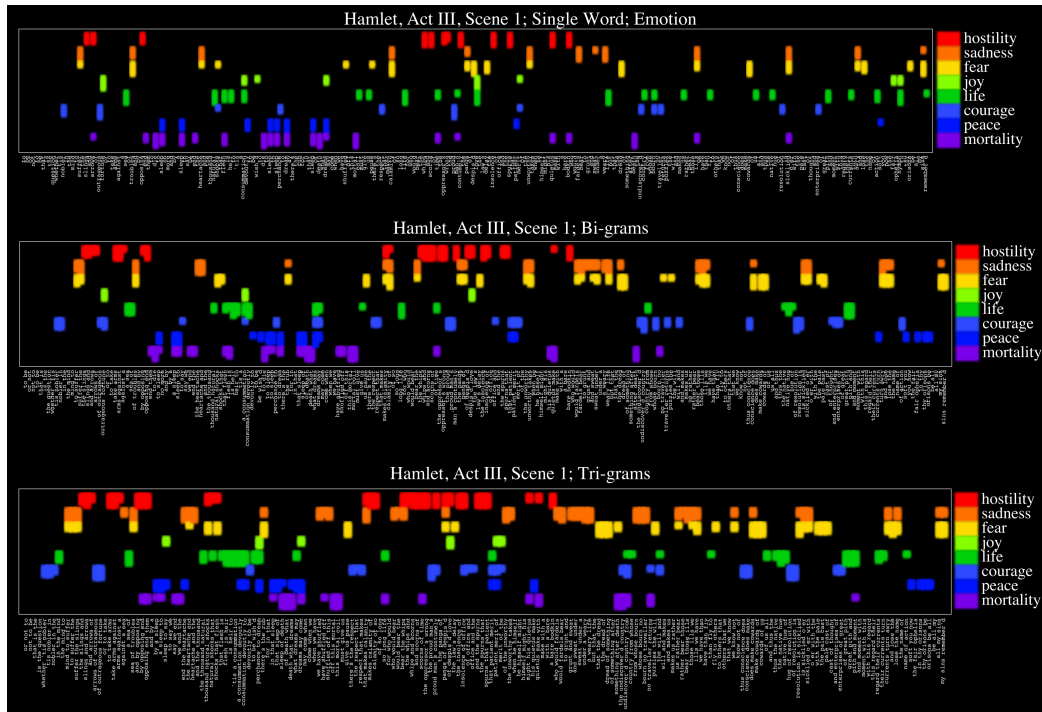


Figure 11: Spectral Analysis of Emotion in *Hamlet*, Act III, Scene 1.

is independent of any word context. Nevertheless, as we move contextually outward to bi-grams and tri-grams, we see the deepening of some of the single-word analysis emotional features, and the refinement of others. An interesting question for future exploration is: for what literary works (or other material) does this comparative analysis yield context-sensitive emotional information only in tri-gram analysis, or higher  $n$ -gram analysis, but not in single-word analysis?

### 3.5. Other Domains of Inquiry

In pursuit of the study of an earlier idea, that of content detection of philosophical elements in text, we apply the above-mentioned methods into domains outside of psychology. We begin this process by searching for an entry-point into partitioning philosophy into a finite set of individual subdomains. Guided by general historical studies of western philosophy [14], we



use the following set of Philosophic Indicator Words<sup>5</sup> (*PIW*) to characterize the fundamental areas of inquiry for philosophy:

$$PIW = \{Logic, Epistemology, Metaphysics, Ethics, Aesthetics, Religion\}.$$

There are other natural divisions and subdivisions of philosophy, but we use this set for our initial studies. Instead of mapping a word  $w$  (or sequence of words) from a literary passage to a vector of emotions and their respective weights, we now create a function  $P(w)$  which maps a word  $w$  to a vector of philosophical categories, with respective weights. We analyze Hamlet's soliloquy with this new function on single words, which yields the image in Figure 12. In this figure, the Emotional Content Analysis of the same passage is shown again with the Philosophical Content Analysis (PCA), for reference.

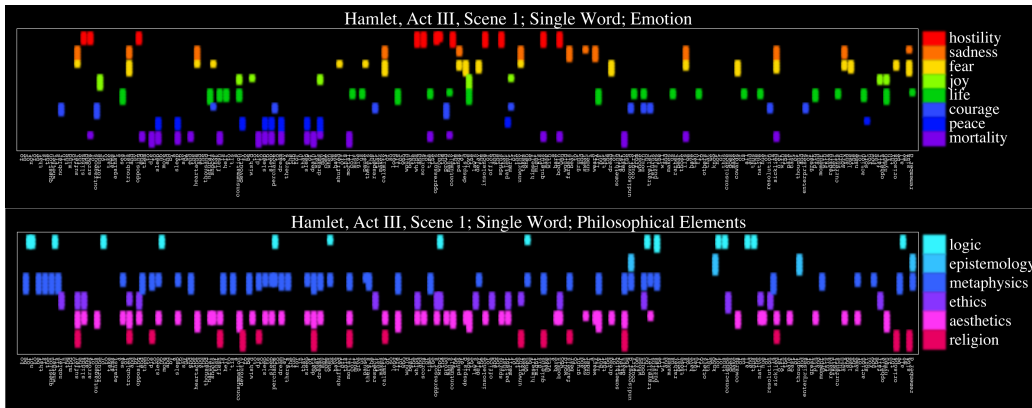


Figure 12: *Hamlet*, Act III, Scene 1 Soliloquy – Philosophical Content Detection.

As might be expected, we see consistent metaphysical structures throughout the soliloquy, including a leading metaphysical associated with the initial “To be or not to be”. Also unsurprisingly, a strong sense of aesthetics is detected as running throughout the passage, largely due to the consistent occurrence of emotion-oriented words. Other interesting features are manifest in the PCA spectrogram, such as the detection of ethical/moral content, which occur twice in conjunction with the ECA hostility structures.

<sup>5</sup>The categorization of *Ethics* herein includes morality and political philosophy.

### 3.6. *Macbeth*

We may study any passage of text using the instrumentation described above. As an example of such study, the single-word ECA spectrogram of Macbeth's early soliloquy in *Macbeth*, Act II, Scene 1 is given in Figure 13 below.

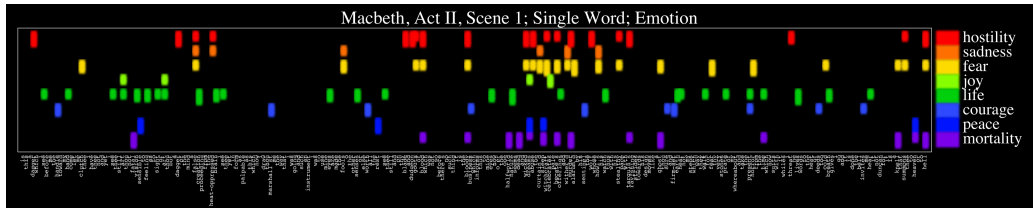


Figure 13: *Macbeth*, Act II, Scene 1 Soliloquy: Single-word ECA Spectrogram.

There are clear differences between the ECA spectrogram for Macbeth's soliloquy and the single-word ECA spectrogram for Hamlet's soliloquy, which is expected. Perhaps of primary importance is that the ECA spectrographic information from each soliloquy, as visualized, demonstrates just how different the emotional structures are in each of the two passages. We provide below the full text of Macbeth's Act II, Scene 1 soliloquy [17] for reference:

*“Is this a dagger which I see before me,  
The handle toward my hand? Come, let me clutch thee.  
I have thee not, and yet I see thee still.  
Art thou not, fatal vision, sensible  
To feeling as to sight? or art thou but  
A dagger of the mind, a false creation,  
Proceeding from the heat-oppressed brain?  
I see thee yet, in form as palpable  
As this which now I draw.  
Thou marshall'st me the way that I was going;  
And such an instrument I was to use.  
Mine eyes are made the fools o' the other senses,  
Or else worth all the rest; I see thee still,  
And on thy blade and dudgeon gouts of blood,  
Which was not so before. There's no such thing:  
It is the bloody business which informs  
Thus to mine eyes. Now o'er the one halfworld  
Nature seems dead, and wicked dreams abuse*

*The curtain'd sleep; witchcraft celebrates  
 Pale Hecate's offerings, and wither'd murder,  
 Alarum'd by his sentinel, the wolf,  
 Whose howl's his watch, thus with his stealthy pace.  
 With Tarquin's ravishing strides, towards his design  
 Moves like a ghost. Thou sure and firm-set earth,  
 Hear not my steps, which way they walk, for fear  
 Thy very stones prate of my whereabouts,  
 And take the present horror from the time,  
 Which now suits with it. Whiles I threat, he lives:  
 Words to the heat of deeds too cold breath gives.  
 I go, and it is done; the bell invites me.  
 Hear it not, Duncan; for it is a knell  
 That summons thee to heaven or to hell."*

## 4. General Considerations

### 4.1. Philosophical Considerations

In each visual study above, we create a representation of an impression of a literary work, or some portion of a literary work. These representations are not intended to be definitive, but intended instead to give us new information about, and perspectives on, literary passages as determined by certain individuals, populations, or audiences. The variety and form of such impressions is inexhaustible, largely due to the inexhaustibility of the number of interpretations, and therefore the number of representations, of literary passages. This is a crucial notion, which indicates the fundamental richness of literature and human creativity.

As indicated by Sartre, Husserl, and other 20<sup>th</sup> century continental philosophers, when studying a particular object we should not assume some underlying secret essence that determines that object. Husserl's perspective [7], as summarized by Sartre [15], is that an object is the unity of the infinite number of impressions, perspectives, and possible appearances of that object — the totality of which determines the nature of that object. That is to say, when studying an object, we have an infinite series of perspectives associated with the object, and it is this series which we refer to collectively as the object itself. An analogy to this kind of thinking may be summoned from the world of mathematics.

For many people, the number one, or “1”, indicates the first symbol which we use to count collections of objects. For example, many books begin their main content with a page numbered “1”, then move through a familiar sequence of counting numbers until all successive pages of the main content are each enumerated. In primary school, we learn additional facts about the number one, such as: any number multiplied by one retains its original value, that  $\frac{1}{2} + \frac{1}{2} = 1$ , and  $\frac{1}{6} + \frac{1}{3} + \frac{1}{2} = 1$ , and so on. However, some of us receive additional mathematical training, in college or otherwise, which permits access to a host of new characterizations of the number “1”; many of which are incredibly rich and complex. An example relevant to our discussion at-hand is that of an infinite series, where the word ‘series’ is quite similar to that as used by Sartre. For mathematics, an infinite series may be considered initially as a type of infinite sum.<sup>6</sup> Informally, we may write a new, infinite representation of the number one, through the use of the following sequence of familiar symbols:

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \quad (1)$$

**Equation (1).** Informal Representation of Unity.

The ellipses at the end of Equation (1) indicate a continuation of the pattern established in the terms of the sum, and imply that the addition operation occurs between an infinite number of terms. This is not entirely satisfactory, so we use other notation to express this idea more precisely:

$$1 = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{2^i}. \quad (2)$$

**Equation (2).** An Infinite Series Representation of Unity.

As some readers may know, the large sigma ( $\sum$ ) in this representation indicates a sum, and the symbols “i=1” and “n” indicate that we are adding a total of  $n$  terms, where  $n$  is a whole number and  $i$  is the indexing variable. So, if  $n = 5$ , we add the first five terms of Equation (2), which as a sum are close to equaling unity:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{31}{32}.$$

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<sup>6</sup>More precisely, an infinite series is the limit of an infinite sequence of partial sums, where the terms for a partial sum are taken from a given preliminary infinite sequence.

If we instead take  $n = 10$  and add the first ten terms of the series, our sum would equal  $\frac{1023}{1024}$ , which is a number that more closely approximates unity. Lastly, the symbols “ $\lim_{n \rightarrow \infty}$ ” indicate the infinite nature of the infinite series representation, pushing  $n$  to infinity, and therefore the sum, to unity.

Connecting our thoughts: without any one of the single terms in the infinite sum on the right-hand side of Equation 2, our sum would no longer equal 1. It would no longer be unity. When considering similar infinite processes for the number of possible perspectives (appearances) of a given object, Sartre takes Husserl’s line of thinking further [15], arguing that the nature of that object transcends even this type of infinite process. In agreement with this, we may find the supreme mathematical metaphor to such thinking in an infinite series representation of the transcendental number  $\pi$ ; a number which is algebraically transcendent [10] and for which an infinite representation is fundamental to its nature. To get a good sense of the fundamental nature of  $\pi$ , we may look at the finite sums of terms of  $\pi$ , just as we did above for the number 1. The more terms we have of this series for  $\pi$ , the better our understanding of, and therefore a better approximation of, we have of the number  $\pi$ . An example of an infinite series representation of  $\pi$  appears below in Equation (3).

$$\pi = \lim_{n \rightarrow \infty} 4 \sum_{i=1}^n \frac{(-1)^{i+1}}{2i-1}. \quad (3)$$

**Equation (3).** An Infinite Series Representation of  $\pi$ .

In an analogous way, we cannot truly understand the essence of a literary object without passing to a transcendent unity of the infinite series of appearances of that object. We suggest that in our present studies, we aim to achieve similar goals with respect to the understanding of Shakespeare’s works. That is to say: through new digital representations and analyses, such as those described in the first three sections of this article, we come to reveal more fully the transcendent infinite series of remarkable terms which form the inexhaustible and magnificent unity found in the works of Shakespeare. Our goal herein, therefore, is connected essentially to such a phenomenological perspective: we wish to reveal new terms in the series of interpretations and impressions of Shakespeare’s works, through digital revelation and representation.

#### 4.2. *Scientific Considerations*

The mathematical and computational techniques described above are general enough, and flexible enough, to apply to a large number of explorations, well outside the realm of Shakespearean studies. Indeed, the techniques are flexible enough for the study of any textual material, especially when coupled with well-programmed experimentation and well-chosen elementation of domains. For example, in the preceding we first partitioned the emotions into elements to form a finite set of emotional content indicators, and then proceeded to do the same with areas of inquiry for philosophy. With some effort, we could do the same for socio-political studies, for studies in bias and prejudice, and for studies of the basic domains of science. In addition, we are free to choose a sub-domain of any of those listed above, and from it, partition that sub-domain to create an indicator set of its most salient characteristics.

In scientific studies, we often look to establish general laws to describe the widest number of cases. This course of action is open to the studies above. For example, in Figures 8 and 9 above, we gathered data from two distinct audiences, and thereafter described some seemingly invariant features found as common to both analyses. We may continue this process and look for emotional structures which appear to be invariant across several experiments, populations, or literary passages. That is, we may look to statistical averages and establish emotional structures which are common to any interpretation of a literary passage. That is, we may normalize the results.

Instead, and perhaps more interestingly, we may choose to study the richness and variances found in the differences in how populations read the same passages. That is, we learn to better understand how different populations react emotionally, philosophically, and socio-politically to the same words. From such studies, we may find therein an opportunity to learn new ways to communicate with each other that are better informed and more sensitive.

#### 4.3. *Technological Considerations*

All software mentioned above was written from scratch by the author, in pure Python<sup>7</sup> with no external library, package, or toolkit dependencies.

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<sup>7</sup>Python is freely available at: <http://www.python.org/>.

This includes software for natural language processing, content analyses, and the various visualizations. The flexibility, power, and ease-of-use of Python cannot be overstated. An illustrative diagram, which accurately represents the workflow of the various software systems necessary for this work, appears below:

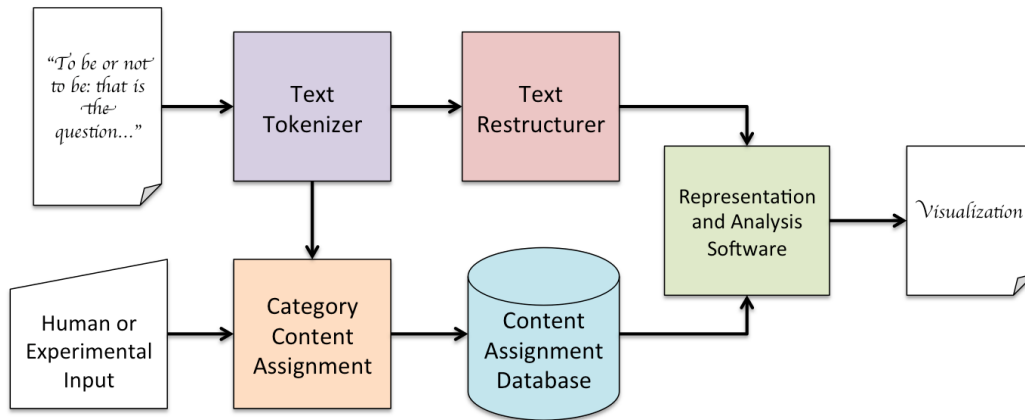


Figure 14: Content Analysis and Visualization Software Workflow.

The final product in the chain of software components (indicated by colored blocks in Figure 14) is a Scalable Vector Graphics<sup>8</sup> (SVG) formatted document, or \*.svg visualization file. SVG documents are extraordinarily flexible in their ability to scale naturally, independent of image size, and support for rendering them efficiently is found in most modern Internet browsers. Additionally, SVG files can be reformatted as PDF files, via the Open-Source software package Inkscape,<sup>9</sup> or further post-processed digitally with the Open-Source package GIMP<sup>10</sup>. The ready (and cost-free) availability of high-quality programming languages and image manipulation programs make studies, such as those found herein, possible.

<sup>8</sup>The Scalable Vector Graphics specification may be found at: <http://www.w3.org/Graphics/SVG/>, accessed last on June 11, 2017.

<sup>9</sup>Inkscape's main website is: <http://inkscape.org/>, accessed last on June 11, 2017.

<sup>10</sup>The GIMP (GNU Image Manipulation Program) may be found at: <http://www.gimp.org/>, accessed last on June 11, 2017.

## 5. New Perspectives

The works of Shakespeare are vibrant phenomena that transcend particular words their arrangements. Throughout the centuries, the innumerable interpretations of these works have indicated that they each possess a certain kind of transcendent immortality. Indeed, the very innumerability of the interpretations of *Hamlet* exist as terms of an infinite series, the whole of which indicates the authentic phenomenon ‘Hamlet’. Our present studies are simply more terms in this series, additional interpretations of the transcendental object ‘Hamlet’, as manifested in new modes of visualization, communication, and analyses.

The research described throughout this article is predominantly phenomenological in approach, yet it relies heavily upon the mathematical and computational sciences. This is true not only in the development of a series of mathematical (and respectively, computational) content analysis functions, but also in the choices made in the geometry for each image’s layout and graphic design. Combining the (mathematical and the computational) sciences and the humanities in this way has the potential to open new doors and foster new discoveries about literature, art, and music. Indeed, when mathematics is coupled with the humanities, it may be successfully employed as the foundation upon which to visualize and explore new territory across all human endeavors. To this end, we appeal to academic communities for a more holistic approach to all of learning and research, that we shake off the shackles of artificial disciplinary boundaries presently and persistently maintained in academia, yet erected so long ago.

The custom software written for the explorations described above is designed to permit digital interaction between the visual analyses and interested students or researchers. Our next step in these studies is to perfect the software, in order to permit a manifold of tactile, visual, and auditory interactions between researchers and literary works. We may research the way different actors interpret the emotional content of certain words, and we may do this with directors, or any combination of theatrical company members. This may reveal creative differences or similarities, each of which adds richness to the study and interpretation of Shakespeare’s works. We also hope to extend our knowledge of how various populations interpret Shakespeare’s works, emotionally, philosophically, and socio-politically. In this, we have the facility to create databases of how populations relate to one another in



such readings, how they differ, and how they change over time. Such work is easily extended outside the realm of Shakespearian studies, and outside of literary analysis altogether.

As technological advances in computational sciences and mathematics continue, the ease by which we are able to manipulate and visualize large-scale data sets has increased dramatically. Our preliminary work has leveraged these advances, allowing us to develop new modes of linguistic exploration, synthesized from elements of philosophy, literature, mathematics, computer science, psychology, and the arts. Already, these new modes of exploration and analysis permit us to begin to answer such questions as: How do different populations groups interpret the same passage from a literary work? In what ways are their reactions and interpretations different, psychologically, philosophically, and socially? In what ways does the recognition and understanding of such differences affect these populations' abilities to effectively communicate and hold meaningful discourse about general human concerns and the common good?

Throughout all of this, retaining a mindfulness of the benefits of new analytic tools is useful. This includes maintaining an active awareness of the ready availability of easy-to-use programming languages, the consistency and utility of modern mathematics, and the ongoing refinement of visualization technologies. Good graphic design choices not only allow researchers to explore literary works in intuitive ways, they facilitate the natural transmission of otherwise arcane mathematical analyses. All of these things being true, we may perchance to dream further about the mathematics of Shakespeare.

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